Labor Earnings Inequality and Learning about Individual Ability: Theory and Evidence from Japan and the United States[1]

Futoshi Yamauchi K[2]
Institute of Economic Research,
Kyoto University

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[2]Email: yama_kawa@kier.kyoto-u.ac.jp, Phone:+81-75-753-7137, Fax:+81-75-753-7198, Address: Institute of Economic Research, Kyoto University, Sakyo-ku, Kyoto 606-8501, Japan

Abstract

This paper examines the evolution of labor earnings inequality in an environment where individuals learn about their own ability (productivity) from wage realizations and decide their effort levels. It is shown that innate ability heterogeneity and idiosyncratic income shock variance have distinct effects on emergence patterns of earnings inequality. Structural parameters are estimated using data from Japan and the United States. It is found that wage is more directly linked with individual ability in the United States than Japan. The weak linkage of wage to individual ability in Japan slows down the speed by which agents learn about ability, and makes the evolution of both cross-agent effort and earnings variability later in lifetime in the country.

JEL Classification Codes: J2, J3, D3, D8

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1 Introduction

The evolution of earnings inequality has long been in the center of research agenda for economics profession as well as policy makers.[1] Labor earnings, of the largest share among different categories of income sources, show various patterns of inequality emergence across countries (e.g. Deaton and Paxson, 1994; Ohtake and Saito, 1998; etc.). However, the way to interpret the observed inequality of labor earnings, from which to design an justifiable redistribution scheme, depends on what proportion of the variations is attributed to innate ability heterogeneity and to stochastic nature of life, i.e. luck. If most of earnings inequality is attributed to ability heterogeneity, it can be understood that the inequality is cross-agent productivity differentials revealed in labor markets, but pre-determined prior to the entry to labor force. On the other hand, if luck plays a major role in the determination of inequality, it is ex post consequences of stochastic income process. The aim of this paper is to empirically identify, using cross-agent earnings variability by various ages computed from Japanese and U.S. data, the effects of ability heterogeneity and of income risks on the pattern in which labor earnings inequality emerges as people age.

Ability heterogeneity determines the time-invariant variations of earnings, while income shocks determine the time-varying variations. In a perfect-information stationary world, therefore, their proportion should not affect on average the earnings inequality. However, once ability is ex ante unknown and agents optimize their efforts sequentially, this proposition does not hold. In a theoretical framework, I set up a model in which individual workers dynamically learn about their abilities (productivity) on their job and decide their effort levels sequentially. Through the learning behavior and sequential effort decisions, it is found that ability heterogeneity and income-shock variance have distinct and identifiable effects on the evolutionary pattern of within-cohort labor earnings inequality; ability heterogeneity makes the inequality emergence earlier in career while income

[1] Levy and Murane (1992) surveys issues of earnings inequality in the U.S., and Gottschalk and Smeeding (1997) provide an extended survey on cross-country comparisons of earnings and income inequality. Gottschalk and Joyce (1998) provide a comprehensive analysis of rises in earnings inquality in OECD countries and explain the changes from market and institutional factors. Murphy and Welch (1992), Katz and Murphy (1992), and Juhn, Murphy and Piece (1993) examine the sources for the widely documented increase in wage inequality in the 1980s U.S. Most of the empirical studies on earnings (income) distribution focus on the causes for rises in the inequality observed in U.S., U.K., and other developed countries.

shocks make the emergence later in career. Using this asymmetry in their effects, it is possible to explain different patterns of inequality emergence actually observed in different societies and cohorts.

Although employer's learning about workers' abilities and their wage-setting behavior have been examined in the literature (Farber and Gibbons, 1996; Gibbons and Murphy, 1992; etc.), the role of individual ability-learning and sequential effort decisions in determining earnings inequality has not been addressed and tested. Whether employers or workers learn about workers' abilities, however, one common implication would be that the correlation between ability and earnings is increasing as people age. Farber and Gibbons (1996) find that time-invariant variables, correlated with unobserved ability, become more strongly correlated to wages as workers experience. In their arguments, it is because employers learn about employees' abilities over time and adjust wage rates. From a different perspective, Behrman, Hrubec, Taubmen and Wales (1980) using a sample of twin-individuals find that the earnings correlation between twins is about 0.56 for white male veterans of about age 50. Since the estimated correlation has not been adjusted for differences in environments with which the twin individuals are provided, it should be regarded as the lower bound of the contribution of ability to earnings variance. The contribution of ability to earnings increases as people age.

Another finding in the literature is that different economies have different proportions of permanent and transitory components in labor earnings. As for the attempt to decompose permanent and transitory components of earnings, Blundell and Preston (1998) examine the composition of permanent and transitory components in household income shocks in the U.K., and conclude that an increase in transitory income shock variance contributed to a rise in the consumption inequality in the 1980s.[2] For Germany and the U.S., Burkhauser, Holtz-Eakin and Rhody (1997) show that individual-specific fixed components mainly contribute to the inequality of labor earnings in the U.S., while persistence of income shocks contributes to labor earnings inequality in Germany. For the U.S., Geweke and Keane (1997) show that about 60 to 70 percent of the variations of the log of earnings is accounted for by transitory income shocks and that about 60 percent of the variation of lifetime earnings is attributed to unobserved permanent individual characteristics uncorrelated with race, age and education.[3]

The implications of this paper are consistent with the above two facts. The behavior of

[2] They use consumption as a measure of inequality as it measures permanent component of income more precisely than income does. Note also that in their benchmark framework, all the shocks (permanent and transitory) are idiosyncratic. They show that the introduction of correlated shocks to households within a cohort does not change the identification problem of permanent and transitory components.

[3] In a related paper using the same data set, Gottschalk and Moffit (1993) found that, within age-education groups, earnings variations due to differences in permanent component are much larger than that attributed to transitory shock component.

labor earnings inequality emergence depends on the extent to which wage signals contain noise (transitory wage shocks). In an environment where wage signals contain relatively large amount of noise, agents could at best learn their ability slowly. Thus the cross-agent effort (and earnings) variations emerge in late career. If ability heterogeneity is relatively large, the sensitivity of effort decision to wage signal realizations rises because workers learn their ability from current wage realizations and are willing to adjust their effort levels based on their updated perceptions. This results in an early emergence of earnings inequality.[4] Therefore, different proportion of permanent and transitory components of earnings now determines evolutionary patterns of earnings inequality. In addition to this outcome, since workers learn their ability over time, earnings becomes correlated to ability more strongly as workers age.

But, can we observe different patterns of earnings inequality emergence? For example, can we observe different timing of within-cohort inequality emergence in Japan and the U.S.? Although it is perceived that the inequality emerges in relatively early career in the U.S. and it emerges in later career in Japan, empirical examinations have been provided only recently. In a seminal paper by Deaton and Paxson (1994), within-cohort earnings and consumption inequality increase with age in Taiwan, U.S., and U.K., but patterns of earnings (not necessarily consumption) inequality emergence are different across the three countries. For example, earnings inequality emerges intensively around age of 50 in Taiwan, but it emerges earlier in the U.S. The pattern for Japan looks more like Taiwan or U.K. cases[5] (Iwamoto, 1999; Ohtake and Saito, 1998). To motivate us on this issue, Section 2 illustrates different emerging patterns of within-cohort earnings inequality observed in the U.S. and Japan.

Section 3 sets up a model, and characterizes intertemporal changes in effort and labor-earnings inequality. Section 4 provides some estimation results from Japan and the U.S. and shows a contrasting nature of the two economies. Concluding remarks are following in the final section.

[4] In similar spirit, Prendargast and Stole (1996) examine the role of individual-specific noise variance (defined as ability in their paper, also private information) in investment behavior. In their context, therefore, the sensitivity of investment decisions to signals reveals the individual-specific noise variance, which contributes to the formation of market-wide reputation agents concern. Similarly in this paper, the time-varying responsiveness of individual effort decisions to wage realizations in micro levels plays an important role in determining the timing and size of the emergence of earnings inequality in aggregate level.

[5] Cohort-specific varinaces of household income in U.K. are computed by Blundell and Preston (1998, table 1) using the *Family Expenditure Survey* (FES) 1968-1992. Ten-year bands for age of birth of household head are used for defining cohorts. Income inequality for the cohorts in the1920s, 1930s, 1940s, and 1950s.take similar convex shapes. Particularly, income variances of the 1930s and 1940s cohorts rise in late career. Deaton and Paxson (1994) use the same data and derive similar curvatures of age-earnings inequality relationship for the country.

2 Some Evidence from Japan and the US

This section shows observed evolutionary patterns of earnings or income inequality in Japan and the U.S. Different economies share a common phenomenon that earnings inequality increases as people age, but convexity (or concavity) of the shapes is different across countries.

For U.S., Farber and Gibbons (1996, their table 1) computed standard deviations of wages for each experience group for relatively young workers, using the *National Longitudinal Survey of Youth* (NLSY)1979-1988. Those who were the ages of 14 to 21 on January 1, 1979 are in the sample. Figure 1 shows the experience path of wage variance. It is found that the wage variance rises as years of experience increase, but the rate of increase is the highest in the onset of their career and decreases as workers experience. Contrary to the findings for the U.K., the path of wage inequality exhibits a concave shape in the U.S. Deaton and Paxson (1994, figure 6) using the *Consumer Expenditure Survey* 1980-1990 show that age effect on the variance of log earnings exhibits a *concave* shape particularly in the ages of 20-50, consistent with Figure 1 (in early career up to at most 11 years of experience).[6]

I estimated the variances of log transformed labor earnings[7] using the *Panel Study of Income Dynamics, 1990-1997,* and estimated the age effects following Deaton and Paxson (1994) method. Figure 2 shows the estimated age effects for age 25-55. The earnings

[6] Geweke and Keane (1997) in their study using PSID provide some interpretable evidence on age-varying magnitudes of income risks, and of education effect. First, disturbance variance in income determination is large when young and it is decreasing as people age. In other words, stochastic mobility is large when young, not when old. Disturbance variances for young men in one version of model are 0.614 (age 25), 0.455 (age 30), 0.442 (age 45), and 0.442 (age 60), and those in another version are 0.599 (age 25), 0.473 (age 30), 0.445 (age 45), and 0.445 (age 60). Second, education effects also vary by ages. The marginal effects on earnings of 16 year education relative to 12 year education for young men in the first version are 0.195 (age 25), 0.341 (age 35), 0.374 (age 45), and 0.284 (age 55), and those in the second are 0.173 (age 25), 0.450 (age 35), 0.389 (age 45), and 0.400 (age 55). The first effect works for widening earnings inequality in relatively early career, but the second effect contributes to widening the inequality in late career.

[7] For the self-employed, labor earnings include income from their assets. Therefore, it is possible to take a negative value. The observations of negative value are dropped from the sample. The age groups of age less than 24 and more than 56 were not used in the estimation because the sample siges are too small and possibly cause biases in the variance estimates. Compared with Deaton and Paxson (1994) estimates for the U.S., my variance estimates for the ages more than 56 certainly show excessively large numbers.

inequality rises in the 20-30s and it increases linearly thereafter, which is consistent with the findings in Farber and Gibbons (1996).

For Japan, though data source is limited in the country, *Wage Structure Survey* 1961 and 1976 can be used for a comparison of within-cohort inequality of earnings between the two years. Atoda and Tachibanaki (1991), using this data, compute variances of log earnings in different birth cohorts sorted by educational attainment. Strikingly, the inequality had decreased as workers age in that period for all the cohorts they investigate.[8] However, more recently, Ohtake and Saito (1998, figures 3-2 and 4-1) use the *National Survey on Family Income and Expenditure* 1979, 1984, and 1989 and show a more comprehensive picture of within-cohort log-income variance dynamics, in which age effect on income variance is found to be positive and convex. Iwamoto (1999), on the other hand, also decomposed the variance of log income into age and cohort effects, using merged large-sample cross-sectional household data from 1989 to 1995 (*Comprehensive Survey of Living Condition of the People on Health and Wealth*). Figure 3 shows the estimated age effects from the Iwamoto estimates of log income variances (Iwamoto, 1999). An increasing and convex age-curve is depicted for the age 25-55. From the last two studies, the income (earnings) inequality of Japanese households is smaller than those for the U.S. and U.K., and it emerges slower[9]. For the case of Taiwan, Deaton and Paxson (1993, figure 6) use the *Personal Income Distribution Surveys* 1976-1990 and find that earnings variance is convex in age. The pattern is similar to the case of Japan, but the inequality emerges more intensively around the age of 50.

The observations from these countries motivate us to formulate a basic framework for understanding the mechanism for generating different patterns of earnings inequality emergence. The model in the next section provides some interpretations for a variety of patterns in which earnings inequality emerges as workers age. In the core of our motivations is to answer why different societies and different cohorts exhibit different patterns of earnings-inequality emergence.

[8] This observation is the only one which shows a negative age effect on earnings inequality. Compared to other studies about Japan and other OECD countries, I conclude that the generality of this finding is questionable.

[9] For Germany, I estimated the variance of log earnings from the *German Socio-Economic Panel* (GSOEP)1984-1989 (before the German unification). The cohort effects of 10 year band are controlled for. The interesting finding for Germany is that earnings inequality rises intensively in the age 35-45. It increases in a convex way before the age of 40 and in a concave way after the age. In this sense, it is a hybrid type of Japan and the U.S. Different from the countries previously surveyed, the case of Germany alarms that, to begin with our investigation, it is important to recognize heterogeneities across countries in labor-market institutions which generate earnings inequality.

3 Framework

3.1 Set up

Individual i in a cohort (or simply agent i), uniformly distributed over $[0,1]$, decides his/her effort level e_t^i before observing wage rate w_t^i in each time. Production shocks affect the marginal productivity of labor. Wage rate w_t^i is a sum of individual ability (endowment) and a stochastic shock[10][11]:

$$w_t^i = \theta_i + \varepsilon_t^i$$

where θ_i is individual ability[12], and ε_t^i is idiosyncratic shock. True productivity θ_i is not known to both employers and individual workers. ε_t^i is a real productivity shock to worker's output. For this benchmark case, assume that $\varepsilon_t^i \sim^{iid} N\left(0, \sigma_\varepsilon^2\right)$ where $\sigma_\varepsilon^2 > 0$. The extent to which ability can be inferred from wage observations (negatively related to σ_ε^2) differs apparently across labor market institutions. Utility function is assumed to be separable over time and additive for consumption and leisure.[13]

$$\begin{aligned} U_t^i &= u\left(c_t^i\right) - v\left(e_t^i\right) \\ &= w_t^i e_t^i - \frac{1}{2}\left(e_t^i\right)^2, \end{aligned}$$

[10] It is assumed here that wage rate is a linear function of individual ability, not of teams. A rather simple production technology such as this is assumed so that the other workers' abilities do not affect his/her marginal productivity.

[11] For example, if wage is log normally distributed, i.e. $\ln w_t^i = \theta_i + \varepsilon_t^i$, and utility function is $w_t^i \left(e_t^i\right)^\lambda - \left(e_t^i\right)^\eta$, then we can set up a framework in which agents learn about θ_i from signal $\ln w_t^i$. Now, agents set their efforts in a way that $\frac{\eta}{\lambda}\left(e_t^i\right)^{(\eta-\lambda)} = \exp\left(E_t\left[\theta_i\right] + \frac{1}{2}\sigma_\varepsilon^2\right)$ and adjust $\ln e_t^i$ sequentially. By Bayesian, the variance of log earnings is given as $Var\left(\ln y_t\right) = \sigma_\varepsilon^2 + \frac{\lambda^2}{(\lambda-\eta)^2}\left(\left(\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} + 1\right)^{-1} + 1\right)\sigma_\theta^2$. However, the curvature of this function turns out to be always concave. The fit to the estimated age-effects is not generally good.

[12] Constancy of θ_i implies that the mean of wage does not change over time. I abstract human-capital accumulation from this model, simply because the focus of this paper is placed on the evolution of earnings inequality, not on age-profile of mean wage or earnings. However, by assuming arbitrary low value of initial ability estimate (therefore, of initial effort level), it is possible to incorporate an increasing age-profile of mean earnings, but not mean wage, in this model

[13] As long as the cost function is increasing and convex, the qualitative results coming below hold. $E\left(\theta|\Omega_t^i\right) = c'\left(e_t^{i*}\right) \equiv g\left(e_t^{i*}\right)$. Then, $e_t^{i*} = g^{-1}\left(E\left(\theta|\Omega_t^i\right)\right)$.

Some reservations on the form of our wage equation follow. First, for simplicity, years of schooling, experience, on-the-job training, and other determinants of individual productivity are normalized to be zero.[14] Second, returns on ability is normalized to be one (constant); it is assumed that $q = 1$ in $w_t^i = q\theta_i + \varepsilon_t^i$. In general, the value of q (market price of ability or skill) depends on demands for abilities and varies over time. The production in this section has a simplified structure in that individual marginal productivity depends only on his or her own ability plus an idiosyncratic shock. It is also important to recognize that different labor-market institutions have different functioning for determining q. The framework also does not exclude a possibility that q differs across countries, i.e. magnitudes of earnings inequality attributed to ability heterogeneity are different. If we attempt to decompose the earnings inequality into $q\theta$ and ε variations, it is important to identify the metric of ability in wage terms (the value of q). However, since the aim of this paper is to disentangle the patterns, not the magnitude, of cohort-specific inequality evolution, it is thought to be a minor issue.[15]

Assume that there is no publicly observable correlates of individual ability by which employers (or market) can infer individual ability, but individual output is measurable at each time with inclusion of transitory shocks. Since relevant information for ability learning is the history of individual wage realizations exogenously given in market, the evolution of Ω_t^i does not depend on effort decisions. Thus at each time, each worker optimizes his/her effort level so as to maximize contemporaneous utility subject to the information set.

In the beginning, agents only know the population distribution of $\theta_i \sim N\left(\mu_\theta, \sigma_\theta^2\right)$. Let $N\left(\mu_\theta, \sigma_\theta^2\right)$ also denote the initial prior for all agents. We assume that noise variance in wages is large relative to the prior variance. Specifically, assume $2 < \frac{\sigma_\varepsilon^2}{\sigma_\theta^2} < +\infty$.

3.2 Learning and Effort Variability

As agents do not know their abilities ex ante, they necessarily need to to learn it.[16]

[14] In other words, ability (productivity) is assumed to be constant. This assumption is necessary for exclusively focusing on ability learning and resulting effort decisions. However, when we assess earnings data empirically, it is necessary to incorporate some frameworks for distinguishing ability learning and productivity increase due to human capital investments inside and outside firms.

[15] Of course, if the variances of ability are compared across societies, we need to take into accout the contribution of α variations.

[16] If agents have full information on θ_i, it is optimal to set the effort level equal to ability: $e_t^{i*} = \theta_i$. Thus, θ_i is regarded as a target value for effort decision for i. An alternative interpretation of θ_i and e_t^i is that θ_i measures types of occupation in R and individuals look for a perfect match of her occupational choice e_t^i and her most suitable occupation θ_i. In each period, individual receives some signal about her

Bayesian updating provides the law of motion for the subjective mean $\mu^i_{\theta|t} \equiv E\left(\theta_i|\Omega^i_t\right)$:

$$\mu^i_{\theta|t} = \mu^i_{\theta|t-1} + \omega\left(\sigma^2_\theta, \sigma^2_\varepsilon, t\right)\left[w^i_{t-1} - \mu^i_{\theta|t-1}\right].$$

where $\omega\left(\sigma^2_\theta, \sigma^2_\varepsilon, t\right) = \frac{\sigma^2_{\theta|t}}{\sigma^2_\varepsilon + \sigma^2_{\theta|t}}$, identical for all individuals. We know at this stage that $e^{i*}_t = \mu^i_{\theta|t-1}$. Given the updating of variance prior: $\sigma^2_{\theta|t} = \frac{\sigma^2_\varepsilon \sigma^2_{\theta|t-1}}{\sigma^2_\varepsilon + \sigma^2_{\theta|t-1}}$, $\omega\left(\sigma^2_\theta, \sigma^2_\varepsilon, t\right) = \left(\frac{\sigma^2_\varepsilon}{\sigma^2_\theta} + t\right)^{-1}$. The second term is an adjustment of worker i's perception on θ, which is the deviation of wage from his/her previous perception, multiplied by learning weight $\omega\left(\sigma^2_\theta, \sigma^2_\varepsilon, t\right)$. That's, as the learning weight increases, the updation of perception (i.e. $\mu^i_{\theta|t} - \mu^i_{\theta|t-1}$) becomes more responsive to wage surprise ($w^i_{t-1} - \mu^i_{\theta|t-1}$) and learning thus gets faster. Effort decision follows $\mu^i_{\theta|t}$ dynamics:

$$e^{i*}_t = e^{i*}_{t-1} + \left(\frac{\sigma^2_\varepsilon}{\sigma^2_\theta} + t\right)^{-1}\left[\theta_i + \varepsilon^i_{t-1} - e^{i*}_{t-1}\right]. \qquad (1)$$

Learning weight, $\omega\left(\sigma^2_\theta, \sigma^2_\varepsilon, t\right)$, measures the speed of adjustment in sequantial effort decision, in the context of (1). The conditional variance of e^{i*}_t given e^{i*}_{t-1} is $Var\left(e^{i*}_t|e^{i*}_{t-1}, \theta_i\right) = \left[\mathbf{t} + \left(\frac{\sigma^2_\varepsilon}{\sigma^2_\theta}\right)\right]^{-2} \sigma^2_\varepsilon$, which is decreasing in t given σ^2_θ and σ^2_ε. Note that the variance above is *objective* in the sense that the variance is conditional on θ_i (constant), i.e. deterministic although agents do not know. If their guess was actually correct but ambiguous (i.e. with subjective uncertainty); $e^{i*}_{t-1} = \theta_i$, then $Var\left(e^{i*}_t|e^{i*}_{t-1}, \theta_i\right) > 0$ since $0 < \frac{\sigma^2_\varepsilon}{\sigma^2_\theta} < +\infty$.

We first examine the effects of income shock volatility on $Var\left(e^{i*}_t|e^{i*}_{t-1}, \theta_i\right)$, and then proceed to characterizing $Var\left(e^{i*}_t\right)$. The derivative of the conditional variance of e^{i*}_t given e^{i*}_{t-1} and θ_i with respect to σ^2_ε is $\frac{\partial Var\left(e^{i*}_t|\, e^{i*}_{t-1}, \theta_i\right)}{\partial \sigma^2_\varepsilon} = (\omega_t)^4\left[t^2 - \left(\frac{\sigma^2_\varepsilon}{\sigma^2_\theta}\right)^2\right]$. Therefore, an increase in σ^2_ε decreases $Var\left(e^{i*}_t|e^{i*}_{t-1}, \theta_i\right)$ in early periods, but increases $Var\left(e^{i*}_t|e^{i*}_{t-1}, \theta_i\right)$ in later periods. This is different from a monotonically increasing relationship, stated in most of literature. A rise in wage uncertainty may lessen the conditional fluctuations of effort. Formally, the condition is:

$$\frac{\partial Var\left(e^{i*}_t|e^{i*}_{t-1}, \theta_i\right)}{\partial \sigma^2_\varepsilon} \gtreqless 0 \;\Leftrightarrow\; t \gtreqless \frac{\sigma^2_\varepsilon}{\sigma^2_\theta}.$$

best occupation, $\theta_i + \varepsilon^i_t$.

There is a non-monotonic relationship between income shock variance and effort variability.

We are interested in the lifetime path of effort variations and the dynamics of its cross-individual variations, which is the time-varying unconditional variance of effort. Since the population is uniformly distributed over $[0,1]$, the distinction of sample and population does not matter. Given the above preliminary results, we characterize the unconditional variance of e_t^{i*}.

Theorem 1

$$Var\left(e_t^*\right) = t\omega\left(\sigma_\theta^2, \sigma_\varepsilon^2, t\right)^2 Var\left(\varepsilon\right) + \left[t\omega\left(\sigma_\theta^2, \sigma_\varepsilon^2, t\right)\right]^2 Var\left(\theta\right). \tag{2}$$

Proof.

$$\begin{aligned} Var\left(e_t^{i*}|\theta_i\right) &= E\left(Var\left(e_t^{i*}|e_{t-1}^{i*}, \theta_i\right)|\theta_i\right) + Var\left(E\left(e_t^{i*}|e_{t-1}^{i*}, \theta_i\right)|\theta_i\right) \\ &= Var\left(e_t^{i*}|e_{t-1}^{i*}, \theta_i\right) \; + \left(1-\omega_t\right)^2 Var\left(e_{t-1}^{i*}|\theta_i\right). \end{aligned}$$

By recursively substituting,

$$Var\left(e_t^{i*}|\theta_i\right) = \sum_{s=1}^{t} \phi_s Var\left(e_s^{i*}|e_{s-1}^{i*}, \theta_i\right)$$

where $\phi_t = 1$ and $\phi_s = \prod_{q=s}^{t-1}\left(1-\omega_{q+1}^i\right)^2$ if $s < t$. By the definition of w_s^i, it is easy to show

$$\begin{aligned} \phi_s &= \prod_{q=s}^{t-1}\left(1-\omega_{q+1}^i\right)^2 \\ &= \left\{\frac{\sigma_\varepsilon^2 + s\sigma_\theta^2}{\sigma_\varepsilon^2 + t\sigma_\theta^2}\right\}^2. \end{aligned}$$

Then,

$$
\begin{aligned}
Var\left(e_t^{i*}|\theta_i\right) &= \omega_t^2 Var\left(\varepsilon_{t-1}^i\right) \\
&\quad + \sum_{s=0}^{t-2}\left\{\frac{\sigma_\varepsilon^2 + s\sigma_\theta^2}{\sigma_\varepsilon^2 + t\sigma_\theta^2}\right\}^2 \left\{\frac{\sigma_\theta^2}{\sigma_\varepsilon^2 + s\sigma_\theta^2}\right\}^2 Var\left(\varepsilon_s^i\right) \\
&= \omega_t^2 Var\left(\varepsilon_{t-1}^i\right) + \sum_{s=0}^{t-2}\left\{\frac{\sigma_\theta^2}{\sigma_\varepsilon^2 + t\sigma_\theta^2}\right\}^2 Var\left(\ \varepsilon_s^i\right) \\
&= \omega_t^2 Var\left(\varepsilon_{t-1}^i\right) + \sum_{s=0}^{t-2}\left(\omega_t^i\right)\ ^2 Var\left(\varepsilon_s^i\right) \\
&= \omega_t^2 \sum_{s=0}^{t-1} Var\left(\varepsilon_s^i\right) \\
&= t\omega_t^2 Var\left(\varepsilon_t^i\right).
\end{aligned}
$$

$$
\begin{aligned}
Var\left(e_t^*\right) &= E_\theta\left(Var\left(e_t^{i*}|\theta_i\right)\right) + Var_\theta\left(E\left(e_t^{i*}|\theta_i\right)\right) \\
&= t\omega_t^2 Var\left(\varepsilon\right) + Var_\theta\left(\theta_i \sum_{s=1}^{t}\omega_s \prod_{j=s+1}^{t}\left(1-\omega_j\right) + \prod_{s=1}^{t}\left(1-\omega_s\right) E\left(e_0^{i*}|\theta_i\right)\right) \\
&= t\omega_t^2 Var\left(\varepsilon\right) + \left(t\omega_t\right)^2 Var\left(\theta\right),
\end{aligned}
$$

where $e_0^{i*} = \mu_\theta$ for all i by assumption.

Q.E.D. ■

By Theorem 1 and the assumption that prior variance is identical to population variance, the expression for effort variance is further simplified to

$$
Var\left(e_t^*\right) = t\omega\left(\sigma_\theta^2, \sigma_\varepsilon^2, t\right) Var\left(\theta\right). \tag{3}
$$

Since $t\omega\left(\sigma_\theta^2, \sigma_\varepsilon^2, t\right) \to 1$ monotonically, $Var\left(e_t^*\right) \to \sigma_\theta^2$ monotonically. The effects of income-shock variance and ability variance on effort variations are characterized as follows.

Proposition 1 *Comparative Statics of Effort Inequality*

(i) A rise in idiosyncratic income shock variance decreases effort inequality.

(ii) Large ability variance raises effort inequality.

(iii) Effort inequality is increasing over time.

Proof. (i)

$$\frac{\partial Var\left(e_t^*\right)}{\partial \sigma_\varepsilon^2} = \frac{-t}{\left[\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} + t\right]^2} < 0.$$

(ii)

$$\frac{\partial Var\left(e_t^*\right)}{\partial \sigma_\theta^2} = \frac{t\left[2\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} + t\right]}{\left[\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} + t\right]^2} > 0.$$

(iii)

$$\frac{\partial Var\left(e_t^*\right)}{\partial t} = \frac{\sigma_\varepsilon^2}{\left[\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} + t\right]^2} > 0.$$

Q.E.D. ■

The roles of income shock variance and ability variance are distinct: while income shock variance decreases effort variations, ability variance increases its variations. First, there are two ways in which an increase in ability variance influences effort variations: i) an increase in asymptotic effort variance, and ii) an increase in learning speed (sensitivity to wage observations). The first point is a natural consequence of heterogeneities in ability and wage. The second point results in early emergence of cross-agent effort variations. Therefore, both contribute to raising effort inequality. Appendix 1 proves that ability heterogeneity is not necessary for effort variations, in a special case that agents are identical with ability, but it is unknown (rational expectations do not hold).

However, a seemingly counter-intuitive point is (i). Since wage uncertainty increases in income shock variance and effort decisions are responsive to wage realizations, it seems that an increase in income shock variance raises effort inequality. However, this reasoning does not incorporate the role of ability learning. An increase in income shock variance slows down the learning speed, and makes effort decisions less responsive to wage realizations. This results in smaller cross-agent variations of effort levels.

3.3 An Extension: Autocorrelated Case

In this section, the basic framework is extended to a case in which noise in wage is autocorrelated. I assume that agents know autocorrelation parameter for simplicity. Consider AR(1) wage process as follows.

$$\begin{aligned} w_t^i &= \theta_i + \varepsilon_t^i \\ \varepsilon_t^i &= \rho_i \varepsilon_{t-1}^i + v_t^i, \end{aligned}$$

where $\varepsilon_{i,t}$ has a population variance, $\frac{1}{1-\rho_i^2}\sigma_v^2$ (if $\rho_i = 0$, then $\sigma_\varepsilon^2 = \sigma_v^2$). In this case, signal is a weighted average of $(w_{t-1}^i,\, w_{t-2}^i)$.[17]

$$\begin{aligned} s(w_{t-1}^i, w_{t-2}^i) &= \alpha_i w_{t-1}^i + (1-\alpha_i)\, w_{t-2}^i \\ &= \theta_i + \frac{v_{t-1}^i}{1-\rho_i} \\ \alpha_i &= \frac{1}{1-\rho_i} \end{aligned}$$

The weight α_i is increasing in ρ_i (more weight on new signal), and the variance of noise, $\sigma_v^2/\,(1-\rho_i)^2$, is also increasing in ρ_i.[18] Signal consists of two-period wages, but it is unbiased, i.e. $E\left[s(w_{t-1}^i, w_{t-2}^i)\right] = \theta_i$.

In the autocorrelation case, it is optimal for workers to set $e_t^{i*} = E\left[w_t^i \middle|\, \Omega_t^i\right]$, from which

$$e_t^{i*} = (1-\rho_i)\left[e_{t-1}^{i*} + \omega(\sigma_\theta^2, \sigma_v^2, \rho_i, t)\left\{s(w_{t-1}^i, w_{t-2}^i) - e_{t-1}^{i*}\right\}\right] + \rho_i w_{t-1}^i$$

If $\rho_i = 0$, $e_t^{i*} = e_{t-1}^{i*} + \omega(\sigma_\theta^2, \sigma_v^2, \rho_i, t)[w_{t-1}^i - e_{t-1}^{i*}]$ as in the i.i.d. case. The response of effort to a new wage observation is given as

[17] The weight can be negative for $w_{i,t-2}$ if agents weight heavily on $w_{i,t-1}$.

[18] If agents are learning ρ_i as well, the weighting scheme α_i is time-varying and subjectively perceived noise variance $\sigma_v^2/\,(1-\rho_i)^2$ is also time-varying although it is difficult to compute population variance of effort and therefore of earnings due to deviations of optimal $s(w_{t-1}^i, w_{t-2}^i)$ and actual signal using an estimate of ρ_i in each period. In this case, signal is not unbiased and affected by lagged i.i.d.shocks.

$$\begin{aligned}\frac{\partial e_t^{i*}}{\partial w_{t-1}^i}|_{e_{t-1}^{i*},w_{t-2}^i} &= (1-\rho_i)\,\frac{\partial s}{\partial w_{i,t-1}}\omega(\sigma_\theta^2,\sigma_v^2,\rho_i,t)+\rho_i \\ &= \left[\frac{\sigma_v^2}{(1-\rho_i)^2\,\sigma_\theta^2}+t\right]^{-1}+\rho_i \qquad (4)\end{aligned}$$

First, learning weight $\omega_{i,t}$ is decreasing in ρ_i. The persistency of shocks makes noise variance relatively large, thus signal less informative about θ_i (noise variance $\sigma_v^2/\left(1-\rho_i\right)^2$ is increasing in ρ_i). Second, as $w_{t-1}^i$is used for predicting w_t^i, it affects the current effort decision via. autocorrelation parameter ρ_i. Next, we see that effort response to wage realization is time-varying. Differentiating (4) with respect to ρ_i,

$$\frac{\partial e_t^{i*}/\partial w_{t-1}^i}{\partial \rho_i}|_{e_{t-1}^{i*},w_{t-2}^i}=\frac{-2\phi}{[\phi+\ t]^2}+1 \qquad (5)$$

where $\phi=\frac{\sigma_v^2}{(1-\rho_i)^2\sigma_\theta^2}$. Therefore, $\frac{\partial e_t^{i*}/\partial w_{t-1}^i}{\partial \rho_i}|_{e_{t-1}^{i*},w_{t-2}^i}>0$ for all t. This result, however, does not necessarily mean that an increase in autocorrelation raises the variance of effort, since the conditional variance of w_{t-1}^i given w_{t-2}^i becomes smaller as autocorrelation increases.

An interesting implication in the above case is that effort change $\Delta e_{(t,t-1)}^{i*}$ depends on both w_{t-1}^i and w_{t-2}^i. (In the i.i.d. case, it depends on w_{t-1}^i only). Moreover, the effect of two-period lagged wage depends on the sign of ρ_i; w_{t-2}^i negatively affects $\Delta e_{(t,t-1)}^{i*}$ if $\rho_i>0$ and positively affects $\Delta e_{(t,t-1)}^{i*}$ if $\rho_i<0$ (no effect if $\rho_i=0$). If shocks are positively (negatively) correlated, a differencing (averaging) of subsequent wage observations provides information on θ_i. This condition helps test for the consistency of learning behavior and wage process.

3.4 Evolution of Labor Earnings Inequality

We go back to the i.i.d. case in this section. An advantage of the i.i.d. case is that it is possible to derive a closed-form formula of labor earnings variance. Recall that income is generated as the product of wage and effort, i.e. $y_t^i = w_t^i e_t^{i*} = \left(\theta_i + \varepsilon_t^i\right) e_t^{i*}$. The next result shows the dynamics of labor earnings variance.

Theorem 2 *Labor Earnings Variance*

$$Var\left(y_t\right) = \omega(\sigma_\theta^2, \sigma_\varepsilon^2, t)^2 \left\{t^2 \alpha\left(m\right) + \beta\left(m\right) + t\gamma\left(m\right)\right\} \qquad (7)$$

where

$$\begin{aligned}
\alpha\left(m\right) &= \lim_{t\to+\infty} Var(y_t) = Var\left(\theta^2\right) + \sigma_\varepsilon^2 E\left(\theta^2\right), \\
\beta\left(m\right) &= \lim_{t\to 1} Var(y_t) = \mu_\theta^2 \left[Var\left(\theta\right) + \sigma_\varepsilon^2\right], \\
\gamma\left(m\right) &= \sigma_\varepsilon^2 \left[E\left(\theta^2\right) + 2\mu_\theta^2 + \sigma_\varepsilon^2\right],
\end{aligned}$$

and m denotes a set of moments of θ and ε.

Proof.

$$\begin{aligned}
Var\left(y_t\right) &= Var_\theta\left[E\left(y_t|\theta_i\right)\right] + E_\theta\left[Var\left(y_t|\theta_i\right)\right] \\
&= Var_\theta\left[\theta_i E\left(e_t^{i*}|\theta_i\right) + E\left(\varepsilon_t^i e_t^{i*}|\theta_i\right)\right] \\
&\qquad + E_\theta\left[\theta_i^2 Var\left(e_t^{i*}|\theta_i\right) + Var\left(\varepsilon_t^i|\theta_i\right)\left[E\left(e_t^{i*}|\theta_i\right)^2 + Var\left(e_t^{i*}|\theta_i\right)\right]\right]
\end{aligned}$$

where $Var\left(y_t|\theta_i\right)$ was further conditioned on ε_t^i and the last term was derived. By the results of Theorem 1, it is equivalent to

$$\begin{aligned}
& Var_\theta\left[\theta_i\left(t\omega_t\theta_i + \omega_t\mu_\theta\right) + 0\right] \\
&\qquad + E_\theta\left[\theta_i^2 t\omega_t^2\sigma_\varepsilon^2 + \sigma_\varepsilon^2\left\{\left(t\omega_t\theta_i + \omega_t\mu_\theta\right)^2 + t\omega_t^2\sigma_\varepsilon^2\right\}\right] \\
= {} & \left(t\omega_t\right)^2 Var_\theta\left(\theta_i^2\right) + \left(\omega_t\mu_\theta\right)^2 Var_\theta\left(\theta_i\right) \\
&\qquad + t\omega_t^2\sigma_\varepsilon^2 E_\theta\left(\theta_i^2\right) + \sigma_\varepsilon^2\left(t\omega_t\right)^2 E_\theta\left(\theta_i^2\right) + 2t\omega_t^2\mu_\theta^2\sigma_\varepsilon^2 + \omega_t^2\mu_\theta^2\sigma_\varepsilon^2 + t\omega_t^2\sigma_\varepsilon^4 \\
= {} & \left(t\omega_t\right)^2\left[Var\left(\theta^2\right) + \sigma_\varepsilon^2 E\left(\theta^2\right)\right] \\
&\qquad + \omega_t^2\left[\mu_\theta^2\left(Var\left(\theta\right) + \sigma_\varepsilon^2\right) + t\sigma_\varepsilon^2\left[E\left(\theta^2\right) + 2\mu_\theta^2 + \sigma_\varepsilon^2\right]\right] \\
= {} & \left(t\omega_t\right)^2\alpha(m) + \omega_t^2\left[\beta(m) + t\gamma(m)\right].
\end{aligned}$$

Q.E.D. ■

Proposition 2 *(i)* $Var(y_t) \to \alpha(m)$ *as* $t \to +\infty$. *(ii)* $Var(y_1) = \beta(m)$.

Proof. (i) The result follows from that $t^2\omega\left(\sigma_\theta^2, \sigma_\varepsilon^2, t\right)^2 \to 1$, $\omega\left(\sigma_\theta^2, \sigma_\varepsilon^2, t\right)^2 \to 0$, and $t\omega\left(\sigma_\theta^2, \sigma_\varepsilon^2, t\right)^2 \to 0$ as $t \to +\infty$.

(ii) Since $e_1^{i*} = \mu_\theta$ for all i, thus $y_1^i = (\theta_i + \varepsilon_t^i)\mu_\theta$. Therefore, $Var(y_1) = \mu_\theta^2(\sigma_\theta^2 + \sigma_\varepsilon^2)$.

Q.E.D. ■

There are three components in labor earnings variance. First, the variance of earnings converges to $\alpha(m) = Var\left(\theta^2\right) + \sigma_\varepsilon^2 E\left(\theta^2\right)$. Note that θ^2 follows a chi-squared distribution. Second, $\beta(m) = \mu_\theta^2\left[Var(\theta) + \sigma_\varepsilon^2\right]$ is the earnings variance at the initial period, the effect of which decreases over time as it is associated with $\omega(\sigma_\theta^2, \sigma_\varepsilon^2, t)^2$. Third, $\gamma(m) = \sigma_\varepsilon^2\left[E\left(\theta^2\right) + 2\mu_\theta^2 + \sigma_\varepsilon^2\right]$ determines a temporally increasing and decreasing portion since $t\omega(\sigma_\theta^2, \sigma_\varepsilon^2, t)^2$ is increasing initially but converging to zero asymptotically. The three components jointly determine the dynamics of labor earnings inequality.

The next result shows its comparative statics and provides empirical predictions on the evolutionary patterns of labor earnings inequality (see also simulation results in Yamauchi, 1998).

Empirical Predictions:

(i) An increase in ability variance raises labor earnings variance.

(ii) An increase in idiosyncratic income shock variance raises labor earnings variance for large t (late career), and decreases the inequality for small t (early career), given that $\frac{\sigma_\varepsilon^2}{\sigma_{\theta^2}}$ *is large and* $\frac{\sigma_\theta^2}{\mu_\theta^2}$ *is small (noise variance is relatively large and ability variance is relatively small).*

Proof. (i) Since ω_t^2, $\alpha(m)$, $\beta(m)$, $\gamma(m)$ are strictly increasing in σ_θ^2, we have $\frac{\partial Var(y_t)}{\partial \sigma_\theta^2} > 0$.

(ii) On σ_ε^2 effect, since

$$\frac{\partial Var\left(y_t\right)}{\partial \sigma_\varepsilon^2} = \omega_t^2\left[-2\frac{\omega_t}{\sigma_\theta^2}\left\{t^2\alpha+\beta+t\gamma\right\}+\left(t^2+t\right)E\left(\theta^2\right)+\left(2+t\right)\mu_\theta^2+2t\sigma_\varepsilon^2\right],$$

$$\begin{aligned} \frac{\partial Var\left(y_t\right)}{\partial \sigma_\varepsilon^2} &\gtreqless 0 \\ &\Leftrightarrow \\ \left[\sigma_\varepsilon^2+t\sigma_\theta^2\right]\left[\left(t^2+t\right)E\left(\theta^2\right)+\left(2+t\right)\mu_\theta^2+2t\sigma_\varepsilon^2\right] &\gtreqless 2\left[t^2\alpha+\beta+t\gamma\right] \end{aligned}$$

Dividing both sides by t^3, it is equivalent to

$$\left[\frac{\sigma_\varepsilon^2}{t}+\sigma_\theta^2\right]\left[\left(1+\frac{1}{t}\right)E\left(\theta^2\right)+\left(\frac{2}{t^2}+\frac{1}{t}\right)\mu_\theta^2+2\frac{1}{t}\sigma_\varepsilon^2\right]\gtreqless 2\left[\frac{1}{t}\alpha+\frac{1}{t^3}\beta+\frac{1}{t^2}\gamma\right].$$

Note thta as $t\to+\infty$, the l.h.s$\to\sigma_\theta^2 E\left(\theta^2\right)>0$ and the r.h.s$\to 0$. Hence, $\frac{\partial Var(y_t)}{\partial \sigma_\varepsilon^2}>0$ for sufficiently large t.

On the sign of $\frac{\partial Var(y_t)}{\partial \sigma_\varepsilon^2}$ for small t, suppose that $t=1$. From the above inequalities, the condition for $\frac{\partial Var(y_t)}{\partial \sigma_\varepsilon^2}|_{t=1}<0$ is:

$$\begin{aligned} \left[\sigma_\varepsilon^2+\sigma_\theta^2\right]\left[2E\left(\theta^2\right)+3\mu_\theta^2+2\sigma_\varepsilon^2\right] &< 2\left[\alpha+\beta+\gamma\right] \\ \left[\sigma_\varepsilon^2+\sigma_\theta^2\right]\left[E\left(\theta^2\right)+\frac{3}{2}\mu_\theta^2+\sigma_\varepsilon^2\right] &< Var\left(\theta^2\right)+\sigma_\varepsilon^2 E\left(\theta^2\right)+\mu_\theta^2\left[Var\left(\theta\right)+\sigma_\varepsilon^2\right] \\ &\quad +\sigma_\varepsilon^2\left[E\left(\theta^2\right)+2\mu_\theta^2+\sigma_\varepsilon^2\right] \end{aligned}$$

Rearranging,

$$\sigma_\theta^4 < Var\left(\theta^2\right)+\mu_\theta^2\sigma_\varepsilon^2+\frac{3}{2}\left(\sigma_\varepsilon^2-\sigma_\theta^2\right)\mu_\theta^2.$$

It is equivalent to

$$0<\frac{Var\left(\theta^2\right)}{\sigma_\theta^2}+\mu_\theta^2\left[\frac{5}{2}\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}+2\frac{\mu_\theta^2}{\sigma_\theta^2}-\frac{\sigma_\theta^2}{\mu_\theta^2}-\frac{3}{2}\right].$$

Since $\frac{Var\left(\theta^2\right)}{\sigma_\theta^2}>0$, the sufficiency comes from

$$0<\frac{5}{2}\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}+\frac{2\mu_\theta^2}{\sigma_\theta^2}-\frac{\sigma_\theta^2}{\mu_\theta^2}-\frac{3}{2}.$$

The statement follows from this condition.

Q.E.D. ■

4 Empirical Analysis

In this section, I structurally estimate ability and noise variances for Japan and the U.S. The data used for Japan is *Iwamoto estimates* of age-specific log income variances from the *Comprehensive Survey of Living Condition of the People on Health and Wealth*, and the data for the U.S. is the *Panel Study of Income Dynamics (PSID) 1990-1997*.

Table 1 shows descriptive statistics of individual labor earnings from PSID and Table 2 shows the numbers of observations by ages in sample.[19] In Table 2, sample sizes for teenagers and the elderly (above 55) are small. To avoid excessive sampling errors in earnings variance estimates, I use the age interval of 25-55 for the U.S. Although the age-specific sample size for Japan does not have the small-size problem, I use the same age interval for estimation of age-effects on log income variances.

To compare parameter estimates between Japan and the U.S., it is necessary to standardize variance estimates. As in Section 2, earnings (or income) are log transformed and their variances are estimated by ages. Variance of log earnings is independent of price levels and, more importantly, of exchange rate between the countries. Therefore, it is possible to compare directly age effects of log earnings variance between Japan and the U.S.

Table 3 compares the estimated age-effects of log earnings (income) variance between Japan and the U.S. The effects of age 25 are normalized to be zero in Table 3, while the minimum of the age effects are normalized to be zero in the following structural estimation. We can observe that levels of the age effects are widely diverged between the countries. To strengthen our motivation, Figure 4 makes the age effects comparable between the countries by setting, in addition to the age-25 effects, the age-55 effects as one for both countries. This normalization enables us to compare the evolutionary patterns of earnings

[19] Since, in GSOEP-equivalent version of PSID, labor earnings include the asset income for the self-employed, some take negative values. For the log transformation and diffculty in its interpretation, I dropped the observations of nagative values.

inequality between the two countries. It is found in the figure that, though the patterns in the 20s are unstable in both countries, the age effects for age 32 or above in the U.S. are likely larger than those in Japan; the concavity of inequality emergence is stronger in the U.S. than in Japan.

However, because our model produces the variance of earnings (not of log earnings), it is necessary to make a transformation from the estimated age-effects of log earnings to earnings variance from which to estimate structural parameters. It is known that the distribution of earnings is well approximated by a log normal distribution. Under log-normality assumption, it is possible to compute age-specific variances of earnings from the estimated log-earnings age effects (age-specific log-earnings variances). Variance of earnings is related to the mean and variance of log earnings, as follows:

$$Var\left(y_t\right) = \exp\left(2\mu + Var\left(\ln y_t\right)\right)\left\{\exp\left(Var\left(\ln y_t\right)\right) - 1\right\}, \qquad (8)$$

where I assume $\mu = 1$ hereinafter.

Expressing the higher moments in $\alpha(m)$, $\beta(m)$ and $\gamma(m)$ in Eq.(7) in terms of mean and variance, we can have

$$\begin{aligned}
\alpha(m) &= \mu_\theta^4 + 6\mu_\theta^2\sigma_\theta^2 + 3\sigma_\theta^2 - (\mu_\theta^2 + \sigma_\theta^2)^2 + \sigma_\varepsilon^2(\mu_\theta^2 + \sigma_\theta^2),\\
\beta(m) &= \mu_\theta^2(\sigma_\theta^2 + \sigma_\varepsilon^2),\\
\gamma(m) &= \sigma_\varepsilon^2(3\mu_\theta^2 + \sigma_\theta^2 + \sigma_\varepsilon^2).
\end{aligned}$$

Assume that $\mu_\theta = 1$ (ability mean is identical in both countries). We search for the combination of σ_θ^2 and σ_ε^2 that minimizes

$$\sum_{t=25}^{55}\left[Var\left(y_t;\sigma_\theta^2,\sigma_\varepsilon^2,t\right) - \exp\left(2 + Var\left(\ln y_t\right)\right)\left\{\exp\left(Var\left(\ln y_t\right)\right) - 1\right\}\right]^2$$

where $Var\left(\ln y_t\right)$ is estimated age-t effect, and $Var\left(y_t;\sigma_\theta^2,\sigma_\varepsilon^2,t\right)$ is theoretical earnings variance.[20]

The result is summarized in Table 4. Columns 1-3 show reduced form estimates, from which to find a contrast in the patterns of earnings differential evolution between the countries. As cohort ages, the rate of inequality emergence increases in Japan and decreases in the U.S., although significance of quadratic term for the U.S. is weak. If age 53 effect is ignored from the U.S, increasing trend and concave shape of age effects are significant.

Columns 4 and 5 show estimates of σ_θ^2 and σ_ε^2. First, the variances of both ability and noise are larger in the U.S. than those in Japan. Second, asymptotic variances of earnings are 31.50 for Japan and 96.47 for the U.S. The asymptotic earnings variance of the U.S. is about three times larger than that of Japan.

Third, the ratio of ability variance to noise variance $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$ (the key determinant of inequality emergence pattern) is, however, larger in Japan (114.70) than in the U.S.(45.50). Though both ability and noise variances in absolute terms are larger in the U.S. than in Japan, the noise-ability variance ratio for Japan is nealy 2.5 times larger than that for the U.S. From our theoretical framework, it would be concluded that the large noise-ability variance ratio of Japan contributes to the relatively late emergence of earnings inequality in career in the country (vice versa for the U.S.).

Some cautions are called for in our empirical results. First, I abstract from a possibility of autocorrelation of wage shocks in the empirical framework. If shocks are positively correlated more in Japan than the U.S, the noise variance estimate could be biased upward in the former. This factor might have contributed to a relatively late emergence of earnings in Japan.

Second, relatively large estimate of noise variance in Japan may generate from inflexible turnover behavior in the labor markets, or lack of information infrastructure for

[20] As discussed in a previous section, we face an identification problem if ρ is incorporated in the empirical framework. Due to difficulty in deriving closed form of earnings variance, it is necessarily to use simulations: given a configuration of $(\sigma_\theta^2,\sigma_\varepsilon^2,\rho,\mu_\theta)$, we randomly draw θ_i from $N(\mu_\theta,\sigma_\theta^2)$ and fix them. Next, we simulate lifetime path of w_t^i from i.i.d. draws of v_t^i from $N(0,\sigma_v^2)$ and of resulting e_t^{i*} from Bayesian learning. Variance estimates of y_t^i in simulated sample provides $Var(y_t;\sigma_\theta^2,\sigma_\varepsilon^2,\rho,\mu_\theta)$. Then, repeat i.i.d. drawing v_t^i to simulate wage process R times. Conpute $u_t(\sigma_\theta^2,\sigma_\varepsilon^2,\rho,\mu_\theta)\equiv Var\left(y_t\right)-\frac{1}{R}\sum_r Var_r\left(y_t;\sigma_\theta^2,\sigma_\varepsilon^2,\rho,\mu_\theta\right)$ where $\lim_{R\to\infty}\frac{1}{R}\sum_r Var_r(y_t;\sigma_\theta^2,\sigma_\varepsilon^2,\rho,\mu_\theta) = Var(y_t;\sigma_\theta^2,\sigma_\varepsilon^2,\rho,\mu_\theta)$. Then, compute $\sum_t(u_t)^2$, sum of squared errors for $(\sigma_\theta^2,\sigma_\varepsilon^2,\rho,\mu_\theta)$. Finally search for the parameter configuration $(\sigma_\theta^2,\sigma_\varepsilon^2,\rho,\mu_\theta)$ which minimizes the sum of squared errors. Standard deviations of the parameter estimates are derived from the assumption of Normal distribution on u_t (likelihood function).

job search in the labor markets. Not only noise in wage-ability linkage in workplace, but job-search mismatch in nation-wide labor market likely affects the noise variance estimate.

Third, if human capital (on-the-job training) is more firm-specific

in Japan than in the U.S., wage thus endogenously diverges from individual marginal productivity for young workers. In this case, wage does not play an active role in revealing workers' ability to themselves. In our framework, it implies that wages contain large magnitude of noise although wage is not distributed around the mean marginal productivity, but diverges systematically below the productivity (positively autocorrelated). The factor contributes to a large estimate of noise variance in Japan.[21]

5 Conclusions

The model of this paper predicts that differently endowed societies show different evolutionary patterns of labor earnings inequality over time. The inequality emerges early in lifetime in a heterogeneous society where ability variance is relatively large, and the inequality likely emerges more intensively late in lifetime in a homogeneous society where income-shock (i.e. noise) variance is relatively large.

The estimation results show, consistent with the predictions of our model, that Japan has a relatively larger noise variance (compared to ability variance). In this restricted sense, the degree in which wage is linked with individual ability (productivity) is small in Japan. This makes the emergence of earnings inequality later in lifetime in the country.

Second, both ability and noise variances are larger in the U.S. than in Japan. This results in an asymptotically large earnings variance in the U.S, three times as large as that for Japan. It is possible to conclude that cross-agent ability heterogeneity as well as risks in pay determination are larger in the U.S.

However, the analysis ignores other factors which possibly generate time-varying earnings inequality. These include changes in the returns on schooling and skills, differences in on-the-job productivity increase, differences in promotion speeds across individuals, and others. Of course, although incorporating these time-varying factors of individual productivity change enriches as well as complicates our framework, it is however beyond the scope of this paper.

[21] Human capital formation is not incorporated in the model, however. I therefore cannot identify the different roles of ability-learning effect and of human-capital accumulation effect in the current framework. But I just mention the effect of the existence of specific human capital on workers' ability-learning.

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Table 1 Descriptive Statistics (PSID)

	N(obs)	labor earnings		log labor earnings	
		mean	std	mean	std
1990	7175	22456.4	28176.38	9.416688	1.36145
1991	7094	23362.22	26547.46	9.472832	1.33226
1992	7092	24574.96	30464.91	9.514024	1.348293
1993	6748	26020.6	30077.65	9.574694	1.346954
1994	5880	26160.73	34450.22	9.600195	1.358413
1995	6489	27490.86	31875.3	9.676742	1.33088
1996	6551	28094.64	34162.36	9.676348	1.391237
1997	5108	29268.21	34579.71	9.676934	1.391375

Source: Panel Study of Income Dynamics (PSID) 1990-1997. For the self-employed, labor income includes asset income. Observations of negative labor income for the self-employed are dropped from the sample. PSID-German Socio-Economic Panel (GSOEP) equivalent files are used for the computation.

Table 2 Sample Sizes by Ages

year age	1990	1991	1992	1993	1994	1995	1996	1997
15	65	52	43	39	40	66	60	36
20	149	130	149	134	89	84	77	120
25	222	99	169	134	129	141	144	123
30	242	227	227	183	162	172	178	103
35	223	228	173	186	174	213	206	137
40	169	155	188	170	171	210	235	104
45	102	121	134	156	140	164	140	109
50	75	106	88	82	82	79	102	96
55	73	68	71	78	65	65	84	68
60	80	55	73	65	55	60	37	36
65	34	28	44	42	43	46	36	32
70	10	17	25	9	15	16	10	21

Source: Panel Study of Income Dynamics (PSID) 1990-1997. For the self-employed, labor income includes asset income. Observations of negative labor income for the self-employed are dropped from the sample. PSID-German Socio-Economic Panel (GSOEP) equivalent files are used for the computation.

Table 3 Estimated Age Effects (Japan, U.S., Germany)

age	Japan 1989-1995 CSLCPHW	United States 1990-1997 PSID	(West) Germany 1984-1989 GSOEP
25	0	0	N/A
26	0.024409	-0.13172	0
27	0.026441	0.175281	-0.01496
28	0.063539	0.019030	0.015282
29	0.020109	0.096153	-0.06414
30	0.017334	0.013287	-0.02058
31	0.044591	0.467054	-0.00163
32	0.017334	0.381620	0.073316
33	0.058942	0.298204	-0.04026
34	0.051895	0.455451	0.086568
35	0.050859	0.390418	0.023339
36	0.084368	0.399558	0.203801
37	0.053424	0.454730	0.183239
38	0.091724	0.686616	0.381054
39	0.138207	0.885032	0.332285
40	0.103301	0.550869	0.514630
41	0.134624	0.499877	0.328828
42	0.132957	0.572010	0.360681
43	0.125728	0.638247	0.390192
44	0.161527	0.644096	0.406175
45	0.172817	0.601857	0.472170
46	0.124137	0.868421	0.546288
47	0.169747	0.844874	0.421225
48	0.180981	1.024274	0.580326
49	0.171785	0.835084	0.358501
50	0.212878	0.853049	0.341198
51	0.164372	0.868168	0.440541
52	0.240670	0.986194	0.421676
53	0.240089	1.469202	0.453043
54	0.281826	0.893557	0.483976
55	0.248023	0.987311	0.533816

Source: see text for Japan and the U.S. For Germany, German Socio-Economic Panel 1984-1989.

Table 4 Estimation Results

	reduced form estimates			structural estimates	
country:	**Japan**	**United States**		**Japan**	**United States**
depenedent:	estimated age effects of log y			variance of y computed from (5)	
age:	all ages	age53 not incl.	age53 incl.	all ages	age<46
age	-0.0025028	0.0903836	0.0690824		
	[0.622]	[3.952]	[2.226]		
age squared	0.0001353	-0.0007052	-0.0004038		
	[2.646]	[2.533]	[0.995]		
noise variance				24.7154	45.32838
				[0.222]	[0.402]
ability variance				0.2154891	0.9962593
				[1.570]	[12.816]
noise-ability variance ratio:				114.6944	45.4986
asymptotic variance of earnings:				31.5033	96.4685
# obs.	31	31	30	31	21

Asymptotic t values are in parentheses.

Figure 1 The Effect of Years of Experience on Wage Variance
(Gibbons and Farber estimates)

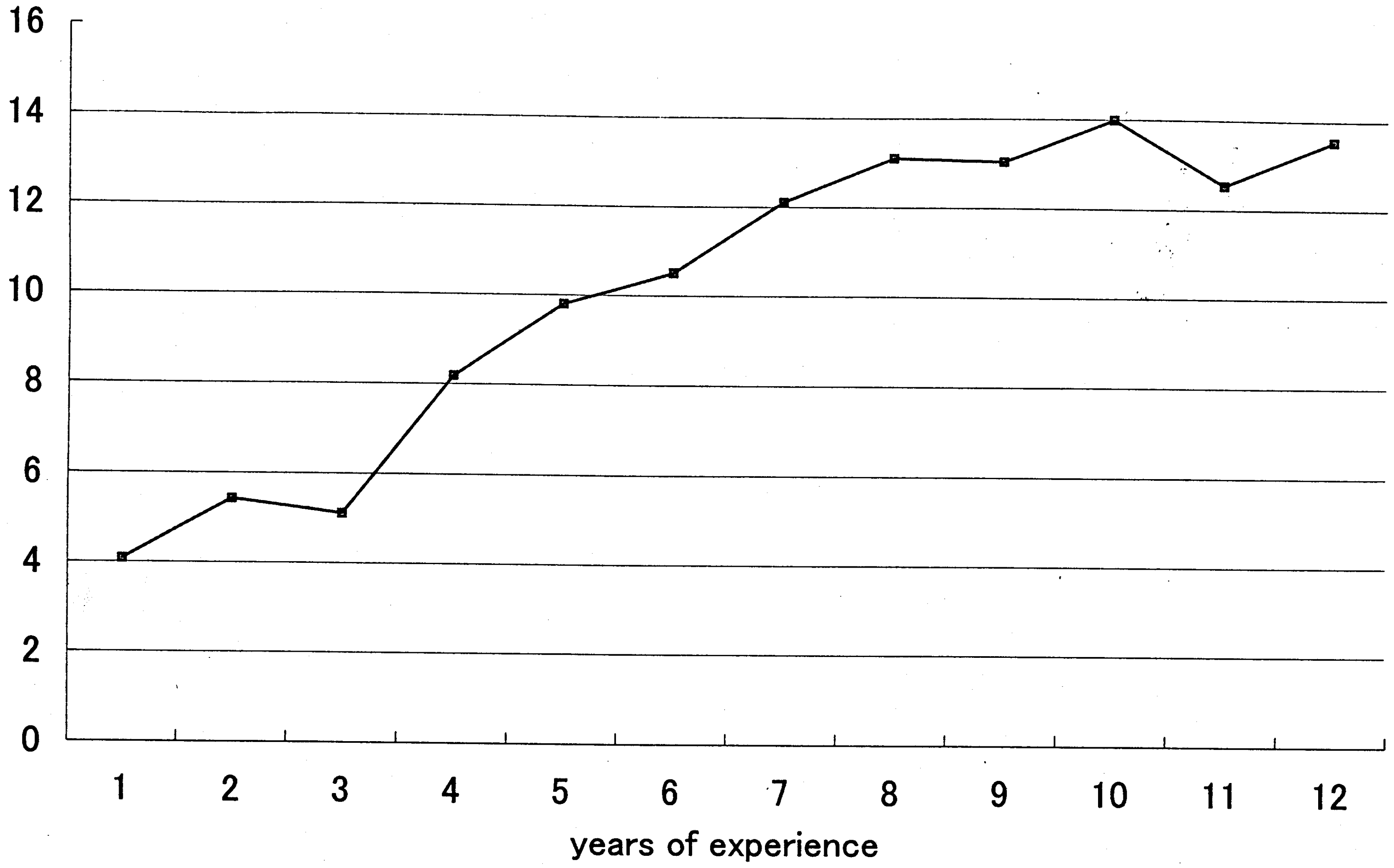

Figure 2 Age Effect on Log Earnings Variances in the United States (estimated from PSID: 1990–1997)

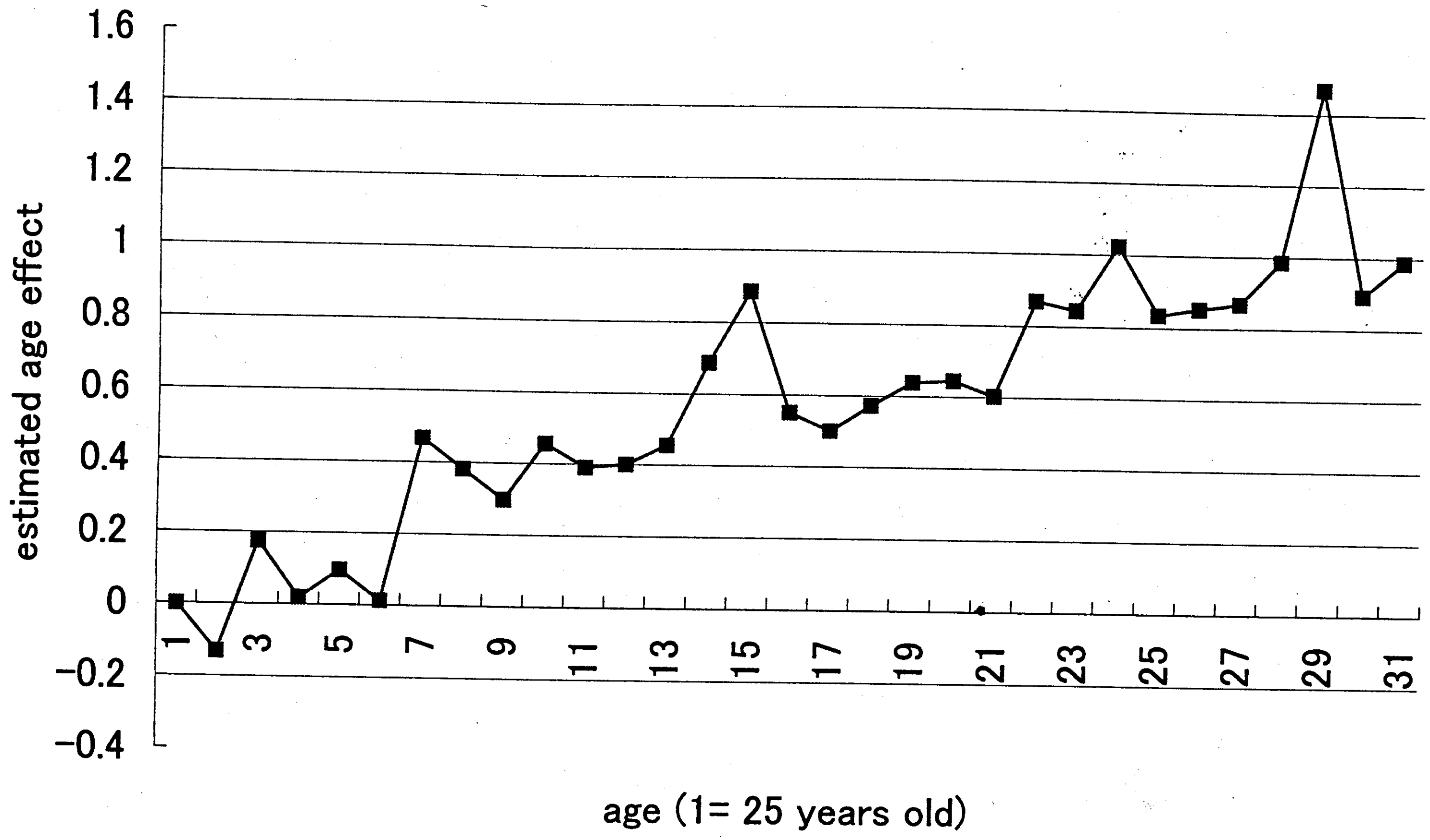

Figure 3 Age Effects on Variance of Log Income in Japan
(estimated from Iwamoto's log income variance estimates)

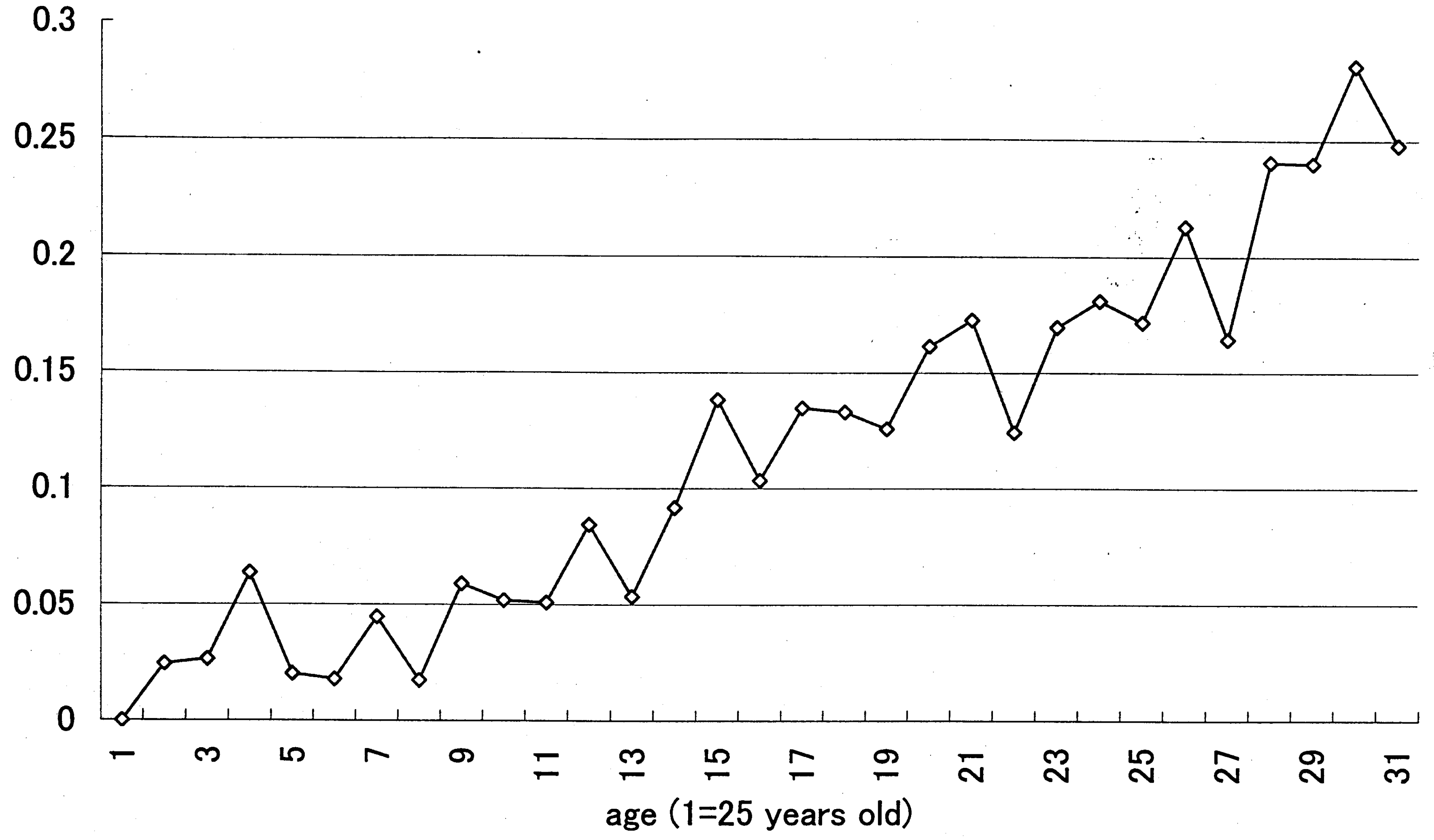

Figure 4 Normalized Age Effects in Japan and the United States