Collusion under asymmetric information and institutional incompleteness*

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Abstract

We study collusion between a regulated firm and an unregulated competitor selling differentiated goods in a common market. There exists an institutional incompleteness that prevents the regulator from contracting with the competitor. Due to this, a simple form of collusion may appear in equilibrium, intended at concealing information from the regulator. With a more sophisticated collusive agreement between the firms, we show that the unique regulator is not able to benefit from asymmetries of information within the coalition if she offers contracts that induce truthful revelation of her firm's cost. Collusion entails large departures from the full information allocation (bunching may appear) casting some doubt on the optimality of collusion-proof mechanisms: The institutional incompleteness may make it better for the regulator to allow collusion in equilibrium.

When firms are regulated each by a different body, the lack of coordination between regulators to fight collusion still entails a loss.

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1 Introduction and motivations

Opening regulated markets to competition has been a major policy advice of international organizations such as the World Bank or the IMF in the last decade[1]. Liberalization of entry has occurred in many sectors. Network industries, such as telecommunications, electricity, water delivery... are typical examples. But a striking fact is that regulatory structures have not been perfectly adapted to the new competitive conditions. Incumbent firms now compete with entrants that are much less regulated, or with foreign firms that are subject to different national regulatory constraints. The incompleteness in regulation that stems from the existing institutions can give rise to some concerns. The purpose of this paper is to assess its consequences on the efficiency of regulation, in particular when competing firms can enter into collusive agreements.

Opening markets to competition entails both costs and benefits. As for costs, authorizing entry in formerly protected monopolies may discourage expenditures in non contractible specific investments by the incumbent and, under some conditions, in innovation. It may also prevent cross-subsidization of small users[2] since competitors would concentrate on profitable niches of the market. As for benefits, competition may allow (developing) countries to benefit from transfers of technologies from foreign firms. It also increases product variety. Moreover it fosters cost efficiency improvements. According to Stiglitz [1996], "one way to provide more effective incentives [...] is to extend the scope for competition". Last, a benefit of competition related to this incentive issue is that it alleviates informational problems. The extraction of informational rents in particular is strongly affected by the presence of unregulated competitors. Observing the behavior of firms with correlated types may indeed enable the regulator to extract informational rents with yardstick mechanisms, as has been shown by Shleifer [1985], Crémer and McLean [1988] or Auriol and Laffont [1995] provided that the regulator has a full set of instruments at her disposal.

Nevertheless the inadequacy of institutions may prevent the regulator from fully reaping this informational benefit As stressed by Laffont [1998], the sequencing of reforms is very important: "Efforts to impose these reforms [privatization and liberalization] before a credible set of institutions - regulation, competition policy, financial regulation - has been designed will yield disappointing results". If institutions are not modified so as to accompagny reforms, regulators may be restricted in the set of instruments they can use, for instance because they are not allowed to regulate new firms. This institutional incompleteness can be particularly costly for regulators in the fight against collusion between competing firms. Collusion is indeed a concern, especially in industries, such as network industies, in which production is concentrated in a few firms. Moreover firms will have particularly strong incentives to coordinate their behavior if the regulator uses their outputs to extract their profits.

The issue arises for a whole range of industries. Nationally regulated entreprises may compete with firms that are under the control of foreign regulatory agencies, as for telephony or electricity. In the same way, differentiated goods competing on the same market may be produced in

[1]See the World Bank annual reports for 1996, 1997 and 1998.

[2]Gasmi, Laffont and Sharkey [1999] characterize conditions under which urban-to-rural cross-subsidies are useful tools to finance universal service obligations, in which case competition should be limited by the creation of exclusive territories. They show that these conditions are more likely to be met in developing countries.

several sectors, that are subject to a different regulation. This is the case in the transportation industry for instance, where some lack of coordination exists both between sectors and between countries: Although a Ministry of Transportation usually organizes some coordination of the regulatory decisions concerning wares transportation (freight railroads, trucking, and so on), that coordination cannot be considered perfect. At the European level, the creation of freight corridors between countries has been decided but national regulations remain different. Moreover universal service obligations may justify that a firm, or a specific activity within the firm, remains regulated while competitors, that are not subject to the USO, are unregulated. The postal service, in particular parcel delivery, is a sector in which the natural monopoly argument is no longer applied to prevent entry from unregulated firms, yet the incumbent is still submitted to USO, and therefore regulated. These markets being highly concentrated, collusion is an issue.

Notice that cooperation between firms may be a concern for regulators even if it is not illegal. It may be legal when it consists in organising coordination between different divisions within the same firm. Indeed a unique firm may have several relatively independent subsidiaries, one of which is regulated, due to some public service characteristic, and not the others. This is the case in the telecommunications industry for instance: Fixed telephony is under tight control in the incumbent firm, contrary to mobile telephony. Fixed and mobile telephony are managed by independent divisions. The head council of the incumbent operates some coordination of global policies but it has to take into account informational asymmetries within the organization. The same sort of issues can arise in a multinational firm whose activities take place in different countries and are gathered in independent divisions. Divisions are then submitted to different local regulations. The head manager has to coordinate decisions. When doing so, he may attempt to extract rents from the regulatory contracts offered to each division. Coordination between producing units that are subjected to different regulatory structures seems therefore a widespread phenomenon.

We assume that firms produce substitute goods and have correlated marginal costs. This assumption seems reasonnable when goods are relatively similar. We focus on the cases in which the competitor of the regulated firm is unregulated ('unilateral regulation') and in which the competitor is regulated by another agency ('bilateral regulation'). Our main results are given below.

Unilateral incomplete regulation This case rather corresponds to markets being opened to entry so that the regulated firm is an incumbent. We therefore assume that the regulator is a Stackelberg leader of the Nash game with the competing firm.

Observing the behavior of a competitor, without directly intervening in its output choices, is enough to extract the informational rents of the regulated firm provided that the regulator be able to condition the incentive contract she offers on the quantity put on the market by the competitor. Yet, the incompleteness in the scope of regulation can give rise to large distortions when collusion between the regulated and the unregulated firms has to be prevented. For instance, both firms may enter into an agreement stipulating that the quantity produced by the unregulated firm be uninformative: By committing to an ex post inefficient behavior, the unregulated firm destroys the ability of the regulator to use yardstick mechanisms. The regulator then has to leave a rent to the firm when it is efficient and distorts downward the quantity produced by the inefficient regulated firm. The regulated firm always benefits from such a collusive agreement, and the unregulated firm is willing to participate for some parameter values, when the cost of not adjusting its output to its type is small relative to the benefit of inducing a lower production of the regulated firm (goods being substitutes).

Another way to model collusion is to use the methodology developed by Laffont and Martimort [1997, 1998]. With such a modelling, we show that the cost associated to a collusion-proof mecanism, that induces truthful revelation by the regulated firm, can be very large. It may force the regulator to implement pooling quantities. Obtaining the relevant information by the regulated firm might then not be an optimal policy: Letting collusion happen might allow more differentiation in output levels. We show moreover that, because the regulator does not have a full set of instruments at her disposal, she cannot benefit from asymmetries of information at the level of the coalition.

Bilateral regulation In a setting of competing hierarchies, yardstick mechanisms can be used if the contracts prevailing in the other hierarchy are observable. Since regulations and decrees affect the profitability and competitiveness of firms on the common market, regulation becomes strategic. The externalities induced by regulatory contracts affect the fight against collusion and the lack of coordination entails additional costs for the regulators compared with a centralized situation.

Sections 2 and 3 respectively set up the model and characterizes the equilibrium without collusion. Section 4 describes the effects of a commitment ability allowing to conceal information on the type of the unregulated firm. A collusive behavior of firms under asymmetric information is described in section 5. We derive in section 6 a "collusion-proof" equilibrium in which inducing truthful revelation of the type of the regulated firm is enough to prevent distortions in the output level chosen for the unregulated firm. In section 7, we consider the issues associated with competing hierarchies and observable contracts. The outcome under bilateral regulation is compared with the one under centralization. Section 8 discusses informally the validity of the Collusion-Proofness Principle and section 9 concludes.

2 The model: Notations and definitions

We consider a market in which consumers can buy two differentiated goods, a and b, produced respectively by firm F^a and firm F^b. Gross consumers surplus when a quantity q^a of good a and a quantity q^b of good b are sold on the market is given by

$$d_a q^a + d_b q^b - \frac{1}{2}(q^a)^2 - \frac{1}{2}(q^b)^2 + c q^a q^b.$$

Parameter $c \in (-1, 0)$ measures the degree of substitutability between the two goods. The demand functions for goods a and b are therefore given respectively by

$$\begin{cases} p^a(q^a, q^b) = d_a - q^a + c q^b \\ p^b(q^a, q^b) = d_b - q^b + c q^a. \end{cases}$$

Each firm $i = a, b$ has a constant marginal cost θ^i that is private information. It is common knowledge that (θ^a, θ^b) takes its value in $\Theta^2 = \{\underline{\theta}, \overline{\theta}\}^2$ according to the joint probability distribution $p(.)$. Throughout the paper, superscripts (letters) will be used to denote the goods and firms whereas subscripts (numbers) denote the state of nature realized. We use the following notations: $p_{11} = p(\theta^a = \underline{\theta}, \theta^b = \underline{\theta})$, $p_{12} = p(\theta^a = \underline{\theta}, \theta^b = \overline{\theta}) = p(\theta^a = \overline{\theta}, \theta^b = \underline{\theta})$ and $p_{22} = p(\theta^a = \overline{\theta}, \theta^b = \overline{\theta})$. The same subscripts will be used to distinguish quantities and transfers according to the state of nature. The degree of correlation is then defined by $\rho = p_{11}p_{22} - (p_{12})^2$. We assume $\rho > 0$ which is the relevant case for the industries we are interested in.

Firm F^a is regulated by a (national) regulator R^a, while firm F^b, on the reverse, is not regulated. We adopt the accounting convention that the regulated firm F^a receives all the revenues derived from the sale of the good it produces but pays a transfer t^a to its regulator. Total ex post profits for F^a and F^b respectively are therefore equal to

$$\begin{cases} \pi^a & = [p^a(q^a, q^b) - \theta^a]q^a - t^a \\ \pi^b & = [p^b(q^a, q^b) - \theta^b]q^b. \end{cases}$$

Regulator R^a maximizes the social welfare of her country. For notational simplicity we assume that the profits of the firms do not enter the social welfare function of the regulator, so that this objective is reduced to net consumers surplus[3] plus the transfer paid by the regulated firm. Net consumers surplus is equal to gross consumers surplus minus expenditures, that is

$$\frac{1}{2}(q^a)^2 + \frac{1}{2}(q^b)^2 - cq^aq^b.$$

Social welfare can therefore be written

$$\begin{aligned} SW^a &= \frac{1}{2}(q^a)^2 + \frac{1}{2}(q^b)^2 - cq^aq^b + t^a \\ &= -\frac{1}{2}(q^a)^2 + \frac{1}{2}(q^b)^2 + (d_a - \theta^a)q^a - \pi^a. \end{aligned}$$

3 Asymmetric information without collusion

3.1 The timing

Regulator R^a is uninformed on the marginal cost of the firm she regulates as on that of the unregulated competitor. The timing we adopt is the following.

1. Nature draws the private information $\theta^i \in \Theta$, $i = a, b$.
2. Each firm privately learns its cost.
3. The regulator R^a proposes a contract (q^a, t^a) to its firm F^a.
4. The regulated firm decides to accept or reject this contract. In case of refusal, it gets a reservation gain, exogenously normalized to 0. If it accepts, then it produces a quantity q^a and receives a transfer t^a.

[3]The results can be easily extended to positive weights of profits, $\alpha_i \in [0, 1)$, $i = a, b$.

5. Firm F^b chooses q^b.

It should be stressed that this timing places explicitly the regulator R^a in a Stackelberg leader position, while the unregulated firm is the Stackelberg follower of our game. Consequently, because marginal costs are constant and drawn from the same support, the unregulated firm could be forced to exit the market (for example if goods are perfect substitutes) in some states of nature. For expositional purposes, we will only consider situations in which F^b produces a strictly positive quantity in each state of nature.

From the regulator's point of view, we can apply the Revelation Principle[4] to characterize the set of contracts she can use: There is no loss of generality in restricting the regulator to use direct mechanisms which ensure truthful revelation of the regulated firm's private information.

3.2 The role of observability

In our model, regulator R^a is able to observe ex post the quentity q^b put on the market by the unregulated firm. Moreover she is also able to commit to contracts contingent on the information.

The next point concerns the inferences that can be made by the regulator: Is she able to infer from q^b the true type θ^b of the unregulated firm? In our setting, assume that firm F^a is, say, efficient. It is straightforward that when the quantity produced by F^b varies with its type, that is when $q^b_{11} \neq q^b_{12}$, then R^a can perfectly infer θ^b from q^b. Accordingly, she can distinguish two states of nature for a given type θ^a of F^a.

This is clearly no longer possible when $q^b_{11} = q^b_{12}$ because she will then be forced to 'pool' the quantities proposed to her regulated firm (when F^a is efficient). Unless otherwise specified, we only consider cases where, ex post, the quantities of the unregulated firm enable the regulator to perfectly infer θ^b ($q^b_{11} \neq q^b_{12}$ and $q^b_{21} \neq q^b_{22}$).

This informational linkage has important consequences for rent extraction. Indeed we know from the work of Crémer and McLean (1988) that in a correlated environment the complete information allocation might be optimally implementable despite the informational asymmetries. In our model, if R^a can obtain ex post a verifiable signal correlated with the private information of her firm, then she can use it ex ante (i.e. at the time of writing the contract) to condition the couple (q^a, t^a) offered to F^a on this information. Constructing such lotteries and exploiting the risk neutrality of the regulated firm enable the regulator to punish or to reward the firm in order to obtain truthful revelation at no expected cost. As incentive problems disappear, the full information allocation can be implemented.

Indeed, in order to obtain truthful revelation of θ^a, the Bayesian incentive compatibility constraints have to be met[5], that is

$$\begin{array}{ll} BIC^a(\underline{\theta}) & p_{11}\pi^a_{11} + p_{12}\pi^a_{12} \geq p_{11}\pi^a_{21} + p_{12}\pi^a_{22} + \Delta\theta(p_{11}q^a_{21} + p_{12}q^a_{22}) \\ BIC^a(\overline{\theta}) & p_{12}\pi^a_{21} + p_{22}\pi^a_{22} \geq p_{12}\pi^a_{11} + p_{22}\pi^a_{12} - \Delta\theta(p_{12}q^a_{11} + p_{22}q^a_{12}). \end{array}$$

Finally, R^a must ascertain that the regulated firm is willing to participate to the contract. This

[4]See Green and Laffont (1977) or Myerson (1981) among others.

gives two Bayesian individual rationality constraints, that is

$$\begin{array}{ll} BIR^a(\underline{\theta}) & p_{11}\pi^a_{11} + p_{12}\pi^a_{12} \geq 0 \\ BIR^a(\overline{\theta}) & p_{12}\pi^a_{21} + p_{22}\pi^a_{22} \geq 0. \end{array}$$

In the appendices, we show that R^a can satisfy the four constraints as equalities whenever the degree of correlation is non null.

Proposition 1 *Under asymmetric information, the regulator optimally implement the full information allocation despite her informational disadvantage. The quantities at the Nash equilibrium of the game between the regulator and her firm on one hand, and the unregulated firm on the other hand, are*

$$\begin{cases} q^{a*}(\theta^a, \theta^b) & = \frac{1}{4-c^2}[4(d_a - \theta^a) + c(d_b - \theta^b)] \\ q^{b*}(\theta^a, \theta^b) & = \frac{2}{4-c^2}[(d_b - \theta^b) + c(d_a - \theta^a)]. \end{cases}$$

The regulated firm F^a has no rent in equilibrium, whereas the unregulated firm makes positive profits.

The quantities of F^b obtained are indeed differentiated according to θ_b's type[6]. The ex post signal, which reveals perfectly the type of the unregulated firm, enables the regulator, through the correlation, to extract completely the rent of her firm.

4 Committing to inefficient behaviour to conceal information

We add to the initial timing a new stage taking place before the report of F^a to its regulator. In that stage, firms have the possibility to delegate the choice of q^b to an agent that is uninformed on firms' types. If they agree, in the final stage, after q^a has been produced, q^b is chosen by this third party.

By committing to delegate the choice of q^b to an uninformed agent, firm F^b ensures that q^b will not reveal any useful information to regulator R^a.

4.1 The collusive choice of q^b

Let us solve the problem of the uninformed agent that has to choose q^b after having observed the contract[7] selected by F^a. The program of the uninformed agent is

$$\max_q \mathbf{E}_{(\theta^b/\theta^a)}\{\pi^b\},$$

[5] We multiplied all the constraints referring to a θ-type firm by $p(\theta)$.

[6] Our results concerning the cost of collusion under institutional incompleteness could be easily extended to situations in which firm F^b's quantities are not differentiated. The main difference is that the outcome under asymmetric information and no collusion would already show distortions.

[7] We assume that this contract is separating and reveals θ^a and we check this assumption ex post.

which obviously gives

$$q^b(q^a) = \frac{1}{2}[d_b - \mathbf{E}\{\theta^b/\theta^a\} + cq^a].$$

Everything happens as if F^b had a 'common knowledge' cost $\mathbf{E}\{\theta^b/\theta^a\}$. The problem of the regulator can now be solved anticipating this best-response function.

4.2 The optimal regulatory contract

From the Revelation Principle we know that R^a can restrict attention to direct and truthful contracts. We denote by

$$\pi^a(\theta^a, \tilde{\theta}^a) = [d_a - q^a(\tilde{\theta}^a) + cq^b(\tilde{\theta}^a) - \theta^a]q^a(\tilde{\theta}^a) - t^a(\tilde{\theta}^a)$$

the rent of a firm F^a that is of type θ^a when it announces $\tilde{\theta}^a$ to R^a. Besides $\underline{\pi}^a$ and $\overline{\pi}^a$ denote the rents of an efficient and of an inefficient firm respectively at a truthful equilibrium. Incentive compatibility constraints can be written as follows

$$\begin{array}{ll} BIC^a(\underline{\theta}) & \underline{\pi}^a \geq \overline{\pi}^a + \Delta\theta\overline{q}^a \\ BIC^a(\overline{\theta}) & \overline{\pi}^a \geq \underline{\pi}^a - \Delta\theta\underline{q}^a. \end{array}$$

Individual rationality constraints are

$$\begin{array}{ll} BIR^a(\underline{\theta}) & \underline{\pi}^a \geq 0 \\ BIR^a(\overline{\theta}) & \overline{\pi}^a \geq 0. \end{array}$$

R^a's objective function is now defined by

$$\mathbf{E}_{\theta^a}\{SW^a\} = p(\underline{\theta})[-\frac{1}{2}(\underline{q}^a)^2 + \frac{1}{2}(q^b(\underline{q}^a))^2 + (d_a - \underline{\theta})\underline{q}^a - \underline{\pi}^a]$$
$$+ p(\overline{\theta})[-\frac{1}{2}(\overline{q}^a)^2 + \frac{1}{2}(q^b(\overline{q}^a))^2 + (d_a - \overline{\theta})\overline{q}^a - \overline{\pi}^a].$$

With this strategic delegation, R^a cannot obtain a correlated signal. Consequently, she can only distinguish between two states of nature depending on θ^a, and we are back to the standard analysis of a Principal-Agent situation under adverse selection.

As usual, the binding constraints are the incentive compatibility constraint of an efficient firm, $BIC^a(\underline{\theta})$, and the participation constraint of an inefficient one, $BIR^a(\overline{\theta})$. The remaining constraints are checked ex post. Solving the program of the regulator yields the following proposition.

Proposition 2 *When firm F^b chooses to have q^b uninformative on its type, the optimal quantities are given by*

$$\begin{cases} \underline{q}^a & = \frac{1}{4-c^2}[4(d_a - \underline{\theta}) + c(d_b - \mathbf{E}\{\theta^b/\underline{\theta}\})] \\ \overline{q}^a & = \frac{1}{4-c^2}[4(d_a - \overline{\theta}) + c(d_b - \mathbf{E}\{\theta^b/\overline{\theta}\}) - 4\frac{p(\underline{\theta})}{p(\overline{\theta})}\Delta\theta] \\ q^b(\underline{q}^a) & = \frac{2}{4-c^2}[d_b - \mathbf{E}\{\theta^b/\underline{\theta}\} + c(d_a - \underline{\theta})] \\ q^b(\overline{q}^a) & = \frac{2}{4-c^2}[d_b - \mathbf{E}\{\theta^b/\overline{\theta}\} + c(d_a - \overline{\theta}) - c\frac{p(\underline{\theta})}{p(\overline{\theta})}\Delta\theta]. \end{cases}$$

Moreover the regulated firm obtains a strictly positive rent when efficient.

Revelation by the efficient firm F^a can now be obtained only by leaving a costly informational rent to that firm. Since that rent increases with the quantity produced by the inefficient firm F^a, that latter quantity is distorted downward (with respect to the situation in which R^a would be informed on θ^a but not on θ^b).

The regulated firm is unambiguously better off with such a behaviour of the unregulated firm since it obtains a positive rent. Whether or not firm F^b will be willing to strategically delegate its choice of quantity depends on two effects. Given the timing, it will decide to delegate if its interim gain is larger than when q^b is informative. The cost of the srtategic manipulation stems from the fact that q^b is not adjusted according to its information (θ^b). This loss is smaller the larger the correlation between the cost parameters of the firms. However, a benefit arises since the regulator chooses underproduction (to trade off rent extraction and efficiency). This gain is larger the higher the degree of substitutability between the goods produced by the firms.

In appendices, in a simple example, we show that when goods are strongly substitutes and when the correlation is sufficiently large, then both types of unregulated firm are willing to commit to delegation[8].

This section highlights the fact that contracts that use information provided by the actions of competitors (such as yardstick mechanisms) may not be immune to simple collusive agreements. Indeed, those competitors might use devices that enable them to commit to an ex post inefficient behaviour in order to blur the information transmitted.

Of course, if firms could collude and use bribes then the likelihood of such strategic delegation would increase as the regulated firm could share its informational rent to compensate the unregulated firm for its inefficient behaviour. One issue that arises then is how to model collusion between firms that are uninformed on each other's type. The purpose of the next section is to analyze collusion under asymmetric information and institutional incompleteness.

5 Coalition formation

We will now depart from our crude modelling of collusion and use the methodology proposed in Laffont and Martimort (1997, 1998). We want to evaluate the impact of the institutional incompleteness on the possibilities of collusion under asymmetric information and on the optimal regulatory response arising from the threat of collusion.

5.1 The timing of the game and collusion modelling

The timing is the following.

1. Nature draws the type θ^i of firm F^i, $i = a, b$. Each firm only learns its own type.

2. Regulator R^a proposes a contract to her firm. This contract is composed of a quantity q^a to be produced and a transfer t^a to be paid by F^a.

[8] This may be compared to results of Fershtman, Kalai and Judd (1991). They show that firms may have incentives to hire agents and to delegate to them production decisions. It is a commitment device used to sustain a collusive behavior. In our setting, the unregulated firm may hire an uninformed agent in order to commit to conceal information.

3. Firm F^a decides to accept or to reject this contract. If it refuses, it then receives its reservation gain normalized to 0, and the next steps do not occur.

4. A third party T proposes a side contract SC to *both* firms. $SC = \{\phi(.), q^b(.), \{y^i(.)\}_{i=a,b}\}$ where $\phi(.)$ is the manipulation function of the announcement of the regulated firm, $q^b(.)$ is the quantity produced by the unregulated firm and $\{y^i(.)\}_{i=a,b}$ is the vector of side transfers paid by each firm to T. The side transfers are subject to be budget balanced $(\sum_{i=a,b} y^i(.) = 0)$ in each state of nature. If one firm at least refuses SC then F^a chooses non cooperatively in the contract offered by R^a while F^b non cooperatively sets its quantity in a following step[9].

5. Report into R^a's contract is sent by F^a (according to the manipulation function if the side contract has been accepted).

6. Production of q^b is done according to the side contract. The quantity and transfer proposed by the regulator take place, as well as the side transfers promised by T.

The third party is uninformed on the firms' types. Its objective is to maximize the sum of the expected rents of the firm, $\mathbf{E}_{(\theta^a,\theta^b)} \sum_{i=a,b} \pi^i(\theta)$. These gains are obtained by playing the composition of the contract of R^a with the side contract proposed by T.

Notice that the Revelation Principle, which applies at the level of coalition formation, tells us that there is no loss of generality in restricting SC to be a direct and truthful side contract[10].

5.2 The program of the third party

Before considering the resolution of the program of the third party, we introduce the following definition.

Definition 1 *The null side contract SC_0 is the side contract such that there are no collective manipulations of the announcement of the regulated firm and of the quantity produced by the unregulated firm, and there is no side transfer:* $SC_0 = \{\phi^* = Id_\Theta,\ q^b \in \arg\max_q \pi^b,\ y^i = 0,\ i = a, b\}$

Notice that whatever the contract offered by R^a to F^a, the third party can always implement the null side contract: In equilibrium there will always be formation of the coalition. The problem is to determine whether that coalition will be active (i.e. choose $\phi^* \neq Id$ and/or $q^b \notin \arg\max_q \pi^b$ for instance) or not in equilibrium.

Let us assume that R^a wants to ensure truthful revelation of the information held by F^a (we will dsicuss in a later section the relevance of this assumption). To characterize the additional

[9] In order to make the analysis of the coalition formation game tractable, we assume that beliefs are passive: In the case of refusal of the side contract by firm i the beliefs of firm j on the type of firm i are unchanged and equal to the common knowledge probability distribution. We refer the reader to Laffont and Martimort (1998) for the analysis of collusion when this assumption is relaxed.

[10] Notice that even if firm F^a knows if it colludes or not, any attempt by the principal to elicit this information can be countered by T who can punish a firm that deviates from the side contract as much as what the principal promised to this firm for the revelation of this information. We break this indifference in favor of the third party.

constraints to be satisfied by R^a, we must solve the program $(\mathcal{P}_T)$ of the third party. Denoting by $\Pi^a(\theta^a,\tilde{\theta}^a) = \mathbf{E}_{(\theta^b/\theta^a)}\{[d_a - q^a(\phi(\tilde{\theta}^a,\theta^b)) + cq^b(\tilde{\theta}^a,\theta^b) - \theta^a]q^a(\phi(\tilde{\theta}^a,\theta^b)) - t^a(\phi(\tilde{\theta}^a,\theta^b)) - y^a(\tilde{\theta}^a,\theta^b)\}$ and $\Pi^b(\theta^b,\tilde{\theta}^b) = \mathbf{E}_{(\theta^a/\theta^b)}\{[d_b - q^b(\tilde{\theta}^b,\theta^a) + cq^a(\phi(\theta^a,\tilde{\theta}^b)) - \theta^b]q^b(\theta^a,\tilde{\theta}^b) - y^b(\theta^a,\tilde{\theta}^b)\}$ the expected gain of both firms with type θ^i announcing $\tilde{\theta^i}$ to the third party, the program of T can be formulated as follows,

$$(\mathcal{P}_T)\begin{cases} \max\limits_{\{\phi,q^b,y^a,y^b\}} \mathbf{E}_{(\theta^a,\theta^b)}\{\sum_{i=a,b}\Pi^i(\theta^i,\theta^i)\} \\ \text{subject to} \\ BIR^i(\theta^i) \quad \Pi(\theta^i,\theta^i) \geq \hat{\Pi}(\theta^i) \quad \forall\theta^i \in \Theta, i=a,b \\ BIC^i(\theta^i) \quad \Pi(\theta^i,\theta^i) \geq \Pi(\theta^i,\tilde{\theta}^i) \quad \forall(\theta^i,\hat{\theta}^i) \in \Theta^2, i=a,b \\ BB(\theta^a,\theta^b) \quad \sum_{i=a,b} y^i(\theta^a,\theta^b) = 0 \quad \forall(\theta^a,\theta^b) \in \Theta^2 \end{cases}$$

where $\hat{\Pi}^i(\theta^i)$ is the profit obtained by firm F^i when it refuses the collusive agreement and chooses its quantity or report non cooperatively, given the contract proposed by regulator R^a.

The resolution of this program is provided in appendices.

6 Equilibrium allocations with collusion

Writing the conditions such that it is optimal for the third party not to manipulate the report of the regulated firm enables us to obtain the 'collusion-proofness' constraints. We choose to use the term collusion-proofness although in the present context the regulator cannot influence directly the choice of the unregulated quantity and consequently cannot ensure that the firms will not be colluding by manipulating the quantity of the unregulated firm. Nonetheless, if R^a ensures the revelation of θ^a then we will show that this strongly constrains the collusive choice of the unregulated quantity.

We denote by δ_i^T and $\underline{\nu}_i^T$ the Lagrange multipliers associated respectively to the Bayesian incentive compatibility constraint of an efficient firm i and to the Bayesian individual rationality constraint of an inefficient firm i in the program of the third party [11].

6.1 Conditions for truthful revelation by the regulated firm

Under complete information at the level of the coalition, the regulated firm F^a will be willing to reveal truthfully its private information if the gains earned by the coalition are greater when it tells the truth than when it lies. In appendices, we show that under incomplete information between the colluding members the constraints for truthful revelation by the regulated firm are as follow:

[11] Notice that we only consider the incentive constraints of the *efficient* firms F^a and F^b in the program of the third party. This may appear restrictive in our setting but we will show later that this simplification has nevertheless no impact on the results.

- For an efficient regulated firm F^a

$$\begin{array}{ll} CPC_1 & \pi^a_{11} + \pi^b_{11} \geq \pi^a_{21} + \pi^b_{21} + \Delta\theta q^a_{21} \\ CPC_2 & \pi^a_{12} + \pi^b_{12} \geq \pi^a_{22} + \pi^b_{22} + \Delta\theta q^a_{22} - \dfrac{p_{11}}{p_{12}}\epsilon^b \Delta\theta (q^b_{22} - q^b_{12}). \end{array}$$

- For an inefficient regulated firm F^a

$$\begin{array}{ll} CPC_3 & \pi^a_{21} + \pi^b_{21} \geq \pi^a_{11} - \Delta\theta q^a_{11} + \pi^b_{11} - \dfrac{p_{11}}{p_{12}}\epsilon^a \Delta\theta (q^a_{11} - q^a_{21}) \\ CPC_4 & \pi^a_{22} + \pi^b_{22} \geq \pi^a_{12} - \Delta\theta q^a_{12} + \pi^b_{12} - f(\epsilon^a)\Delta\theta (q^a_{12} - q^a_{22}) - f(\epsilon^b)\Delta\theta (q^b_{12} - q^b_{22}). \end{array}$$

where $\epsilon^i = \frac{\delta^T_i}{1+\underline{\nu}^T_i + \delta^T_i}$ and $f(\epsilon^i) = \frac{p^2_{12}\epsilon^i}{p_{12}p_{22}+\epsilon^i \rho}$ for $i = a, b$. It will later turn out to be important that $\epsilon^i = 0$ if and only if $\delta^T_i = 0$.

The ϵs reflect the asymmetry of information at the firms' level. In order to obtain truthful revelation by the firms, the third party has to make distorsions with respect to the situation where collusion could take place under complete information. These informational problems at the level of the third party can be the source of inefficiency from the viewpoint of the coalition[12].

In this model, because the two firms behave in a different way they are a priori treated differently by the third party. This explains the asymmetry in the ϵs which did not appear in the symmetric models of Laffont and Martimort (1997, 1998). Notice that ϵ^a (resp. ϵ^b) refers to multipliers associated to the constraints of firm F^a (F^b) in the program of the third party.

6.2 The modified reaction function of the unregulated firm

Collusion will modify the reaction function of the unregulated competitor. As for the regulated firm, the new reaction function must be distorted by the third party because (i) it is now aimed at maximizing the sum of the gains of the colluding members and (ii) there are informational problems from the perspective of the third party. In appendices, we show that the best-response function of F^b becomes 'more monopolistic'.

However to be willing to delegate the choice of its quantity, the unregulated firm must be compensated with some monetary side transfer paid by F^a to the third party. Indeed, let us assume that regulator R^a proposes a contract to her firm, and let us consider the behavior of F^a as fixed. If F^b accepts the agreement, q^b will be chosen so as to maximize the *sum* of the gains of the coalition. On the other hand, if it refuses the side contract, it earns the gains obtained by playing non cooperatively and setting q^b so as to maximize its *own* profit.

If it is not compensated with positive side transfers, the unregulated firm will therefore always prefer to behave non cooperatively (for a fixed regulated quantity q^a). This simple observation is important for the following. Indeed let us assume that the regulator proposes a contract such that the optimal manipulation function is the identity: The regulated firm reveals

[12] One can see immediately that for a given contract offered by regulator R^a, if $\epsilon^a = 0$ then everything happens as if the third party were under complete information vis- vis both firms. However this is not equivalent to a situation in which the third party knows the private information of the firms. In that latter case, $\epsilon^a = 0$ for *any* contract proposed by the regulator. But the regulator could then try to take advantage of this.

truthfully. Now consider the decision of the regulated firm F^a to accept or not the side contract. If it accepts, then it reveals truthfully its information, earns the corresponding rent from the regulator and pays some monetary side transfers to the third party; now if it refuses, then it earns the same gain (as the third party recommends to reveal truthfully) but does not have to pay side transfers!

Therefore if the regulator proposes a contract that induces truthful revelation by the regulated firm, then the third party cannot make this firm pay side transfers. But this in turn implies that the unregulated firm cannot be compensated for the change in its reaction function. This yields the following lemma.

Lemma 1 *Assume that regulator R^a proposes to the regulated firm a contract such that the third party finds it optimal not to manipulate the announcement of F^a. Then the only side contract implementable by the third party is the null side contract.*

The important point here is that, by destroying the incentives of the third party to manipulate the report on the type of F^a, the regulator also destroys the possibility for the third party to modify the quantity produced by the unregulated firm. If, in a two-member coalition, one member is not willing to pay the third party, then the other member will not be willing to distort its action in a way that makes it more favorable for the coalition but less for him.

It should be noticed that in the setting of Laffont and Martimort (1997, 1998), contracts that induce truthful revelation are equivalent by definition to collusion-proof contracts. Here, collusion-proofness should not be a priori restricted to truthful revelation of the type of the regulated firm F^a. Yet if a contract induces truthful revelation of θ^a, it necessarily implies that it is collusion-proof.

Notice that things may be quite different if the coalition were formed with at least two unregulated firms. Indeed in this case, even if the regulated firm is not willing to participate unless it has no side transfers to pay, there is a gain for the unregulated firms to let the third party choose their production levels and redistribute the gains among them. In equilibrium then, there could be collusion among the unregulated members, justifying for instance an antitrust intervention. We leave this issue for future research.

Finally, this proposition implies that along the equilibrium path in which the regulator proposes a contract such that the optimal manipulation function is the identity, the participation constraints of both firms will be binding in the program of the third party. As a result the complementarity slackness conditions do not bring more information on the multipliers associated to *these* constraints in $(\mathcal{P}_T)$. In Laffont and Martimort (1998) the same reasoning holds for the incentive compatibility constraints in the program of the third party implying that ϵ^a and ϵ^b are *a priori* choice variables for the regulator. We show nevertheless in the following that in our setting, and contrary to the results obtained by Laffont and Martimort (1997, 1998), those variables are constrained in a very strong way.

6.3 Can the regulator benefit from the asymmetric information inside the coalition?

When regulator R^a offers a contract such that the third party finds it optimal not to manipulate the report of F^a then the side transfers become incompatible with participation constraints in the program of the third party, and the null side contract is proposed. But for this contract, since the quantity produced by the unregulated firm is such that it maximizes its profits, firm F^b has no incentive to misreport on its cost! Therefore the incentive constraint of the efficient firm F^b is not binding in the program of the third party and the Lagrange multiplier attached to it are zero, i.e. $\delta^b_{TP} = 0$. This implies the following lemma.

Lemma 2 *Assume that the third party offers the null side contract SC_0 to the firms. Then the Bayesian incentive compatibility constraints of the unregulated firm F^b are strictly satisfied in the program of the third party. This implies that $\epsilon^b = 0$.*

Let us now turn to the value of ϵ^a. We will show that for a collusion-proof contract proposed by the regulator, the Bayesian incentive compatibility constraint of an efficient firm is not binding in the program of the regulator. Moreover, when the third party chooses to implement the null side contract, the Bayesian incentive compatibility constraints are exactly the same as the ones in the program of the third party. Therefore we necessarily have $\delta^a_{TP} = 0$[13]. This implies the following lemma.

Lemma 3 *Assume that regulator R^a proposes to the regulated firm a contract such that the third party finds it optimal not to manipulate the announcement of F^a. If the incentive constraints of the regulated firm F^a are strictly satisfied in the program of the regulator then they are also strictly satisfied in the program of the third party. In this case $\epsilon^a = 0$.*

In the next subsection, we will show that the Bayesian incentive compatibility constraint of an efficient firm and of an inefficient firm will be strictly satisfied in equilibrium. From the previous lemma, this implies that ϵ^a is zero.

The lemmas show that the regulator will not be able to play on the asymmetries of information at the level of the coalition to decrease the cost of the relevant collusion-proofness constraints if she wants to eradicate collusion.

Proposition 3 *When one firm is left unregulated, if regulator R^a proposes to the regulated firm a contract such that the third party finds it optimal not to manipulate the announcement of F^a, then the third party chooses to implement the null side contract, but the regulator cannot take advantage of the potential inefficiencies in collusion generated by asymmetric information between firms.*

Observe that in Laffont and Martimort (1997, 1998) the regulator can always choose the values of the ϵs, and sometimes finds it optimal to set them equal to zero. Our institutional incompleteness destroys this possibility.

The gain of the regulator when proposing a collusion-proof contract is to force the third party to implement the quantity q^b that would have been chosen by F^b if it were behaving non cooperatively.

We will see in a separate section that this result does not extend to the case of bilateral regulation, where firm F^b is regulated by an independant regulator R^b. The asymmetry linked to a model of incomplete unilateral regulation has therefore strong consequences on the possibility of efficient regulation.

[13] The same logic applies had we also considered in the program of the third party the Bayesian incentive compatibility constraints of inefficient firms F^a and F^b.

6.4 The allocation with collusion-proof regulation

As is usual in adverse selection models, for efficiency reasons there will be at least one participation constraint binding. Indeed, the interim, and even ex post, individual rationality constraint(s) of an inefficient firm F^a will be binding at the optimum.

The two collusion-proofness constraints CPC_1 and CPC_2 will be binding at the optimum. They prevent an efficient firm F^a from lying and announcing to the regulator that it is inefficient, and we show in the appendices that the Bayesian incentive compatibility constraints are strictly satisfied in equilibrium. From the previous lemmatas, this implies that ϵ^a and ϵ^b are equal to zero.

Proposition 4 *Assume that the regulator R^a proposes to the regulated firm a contract such that the third party finds it optimal not to manipulate the announcement of F^a. The optimal quantities for the regulated firm are*

$$\begin{cases} q_{11}^a &= \frac{1}{4-3c^2}[4(d_a-\underline{\theta})+3c(d_b-\underline{\theta})] \\ q_{12}^a &= \frac{1}{4-3c^2}[4(d_a-\underline{\theta})+3c(d_b-\overline{\theta})] \\ q_{21}^a &= \frac{1}{1-\frac{c^2}{2}(\frac{1}{2}-\frac{p_{11}}{p_{12}})}[d_a-\overline{\theta}+\frac{c}{2}(\frac{1}{2}-\frac{p_{11}}{p_{12}})(d_b-\underline{\theta})-\frac{p_{11}}{p_{12}}\Delta\theta] \\ q_{22}^a &= \frac{1}{1-\frac{c^2}{2}(\frac{1}{2}-\frac{p_{12}}{p_{22}})}[d_a-\overline{\theta}+\frac{c}{2}(1-\frac{p_{12}}{p_{22}})(d_b-\overline{\theta})-\frac{p_{12}}{p_{22}}\Delta\theta]. \end{cases}$$

All the remaining constraints will be satisfied if[14]

$$\mathcal{C}_1 \quad c^2(d_a-\underline{\theta})+c(d_b-\underline{\theta}) \geq -(4-3c^2)\frac{\Delta\theta}{2}.$$

The quantity q^b is set non cooperatively by the unregulated firm so as to maximize its profit.

The standard no distorsion at the top result does not hold in our context: The cost of collusion-proofness for the regulator depends on the quantites produced by the unregulated firm, and therefore indirectly on the quantities produced by the regulated one. The regulator therefore distorts quantities, even if the regulated firm is efficient, in order to affect the choice of q^b. Moreover because the regulator R^a has to ensure the truthful revelation of an efficient firm F^a, the quantities produced by an inefficient regulated firm have to be downward distorted. This is the usual trade-off between efficiency and rent extraction.

Moreover, because the binding collusion-proofness constraints are CPC_1 and CPC_2, decreasing q_{21}^a and q_{22}^a, that is the quantities produced by an inefficient regulated firm, enables the regulator to decrease the cost associated to these binding collusion-proofness constraints.

In the centralized models of regulation of Laffont and Martimort (1997, 1998), the binding constraints turn out to be the two collusion-proofness constraints corresponding to a coalition composed of two efficient firms and a coalition composed of one efficient firm and one inefficient firm that announce to be less efficient, the Bayesian participation constraint of an inefficient firm and the Bayesian incentive compatibility constraint of an inefficient firm.

[14]Positivity of the quantities of the regulated firm and the unregulated firm is ensured if d_a is sufficiently large and c is sufficiently small.

In our model, as the regulator only controls one firm it becomes very demanding to satisfy the collusion-proofness constraints. They are even more costly than dominant strategy incentive constraints, let alone Bayesian incentive constraints. This contrasts strongly with the Laffont and Martimort (1997, 1998) case. Finally, the two ex post rents of an inefficient firm are used to ensure the participation of an inefficient firm.

The remaining collusion-proofness constraints CPC_3 and CPC_4 will be satisfied at the optimum if $q^a_{11} \geq q^a_{21}$ and $q^a_{12} \geq q^a_{22}$. In the appendices, we show that the first condition, which can be rewritten as $\mathcal{C}_1$ is more demanding than the second one. If $d_b - \underline{\theta}$ is large then the externality created by an efficient regulated firm when it lies to the regulator on the unregulated firm is large: The stake of collusion is then large and the regulator is forced to offer its firm a pooling contract in order to destroy the stakes of collusion. When the cost differential is large however, the condition $\mathcal{C}_1$ is likely to be satisfied. In this case, an efficient regulated firm will be less willing to lie because in this case it has to produce a much smaller quantity, earning consequently a much smaller rent.

Condition $\mathcal{C}_1$ is clearly a restrictive condition and it is legitimate to study the threat of collusion when this condition does not hold. This is the purpose of the next proposition.

Proposition 5 *Assume that the regulator R^a proposes to the regulated firm a contract such that the third party finds it optimal not to manipulate the announcement of F^a. Assume that condition $\mathcal{C}_1$ does not hold.*

- *The optimal quantity for the regulated firm when its competitor is efficient is*

$$q^a(\theta^a, \theta^b = \underline{\theta}) = \frac{1}{4 - c^2}[4(d_a - \overline{\theta}) + \frac{c}{2}(d_b - \underline{\theta})] \quad \forall \theta^a \in \Theta.$$

- *If the following condition holds*

$$\mathcal{C}_2 \quad c^2(d_a - \underline{\theta}) + c(d_b - \overline{\theta}) \geq -(4 - 3c^2)\frac{\Delta\theta}{2}$$

  then the optimal quantities when the unregulated firm is inefficient are

$$\begin{cases} q^a_{12} &= \frac{1}{4-3c^2}[4(d_a - \underline{\theta}) + 3c(d_b - \overline{\theta})] \\ q^a_{22} &= \frac{1}{1-\frac{c^2}{2}(\frac{1}{2}-\frac{p_{12}}{p_{22}})}[d_a - \overline{\theta} + \frac{c}{2}(1 - \frac{p_{12}}{p_{22}})(d_b - \overline{\theta}) - \frac{p_{12}}{p_{22}}\Delta\theta]. \end{cases}$$

- *If that condition does not hold, the optimal quantity for the regulated firm when the unregulated firm is inefficient is*

$$q^a(\theta^a, \theta^b = \overline{\theta}) = \frac{1}{4 - c^2}[4(d_a - \overline{\theta}) + \frac{c}{2}(d_b - \overline{\theta})] \quad \forall \theta^a \in \Theta.$$

All the remaining constraints are satisfied at the optimum. The quantity q^b of the unregulated firm is set non cooperatively so as to maximize its profit.

In our model, bunching seems to be more the rule than the exception and comes from the cost of collusion-proofness only. This casts some doubts on the optimality of those collusion-proof mechanisms and our analysis clearly shows that it should be possible that collusion appears in equilibrium as an optimal regulatory response[15].

Moreover if bunching is very likely to occur when markets are opened to unregulated competitors, the decision to open markets should be made only if the welfare of the country, given the cost associated to the formation of a coalition is greater than the welfare under institutional entry barriers, where the regulator cannot benefit from a correlated signal on the type of the regulated firm and cannot consequently use yardstick-like mechanism. The regulator has to trade off the amount of information brought by an increase in competition with the threat of collusion.

7 Bilateral regulation with public contracts

So far, we have only considered the case of a unilateral regulation. However, as mentioned in the introduction, many interesting real-world situations involve at least two regulated firms that are controlled by different bodies.

Competing hierarchies under adverse selection have been studied in the context of bilateral trade by Brainard and Martimort (1996), Combes, Caillaud and Jullien (1997) or Maggi (1999) for instance. These papers deal with the effect of trade on the strategic manipulation of national regulatory policies. However, none of them allows for the possibility of collusion between the two firms. Notice also that a common feature of those papers is that they exclude, by assumption, the possibility of Crémer and McLean mechanisms, even though the private informations are assumed to be correlated.

On the contrary, and in the same vein as the previous sections, we allow each regulator to commit to contract contingent on the quantity produced by the competing firm.

We slightly modify our initial setting and consider the following situations. Two countries a and b with identical demand for an homogenous good ($q^a = d - p$, $q^b = d - p$) and identical market size decide to form a common market. The resulting price is $p(q^a + q^b) = d - \frac{q^a+q^b}{2}$. Each regulator takes into account in her objective function the welfare of her consumers only and gives no weight to the national firm's profit. The objective function of regulator R^i, $i = a, b$, is then given by

$$\begin{aligned}SW^i &= [\frac{1}{2}d(q^a+q^b) - \frac{1}{4}(q^a+q^b)^2] - (d - \frac{1}{2}(q^a+q^b))\frac{1}{2}(q^a+q^b) + t^i \\ &= \frac{1}{2}(\frac{q^a+q^b}{2})^2 + (d - \frac{q^a+q^b}{2} - \theta^i)q^i - \pi^i.\end{aligned}$$

where $\pi^i = [p^i(q^a+q^b) - \theta^i]q^i - t^i$ is the ex post rent obtained by firm F^i, $i = a, b$.

7.1 The timing of the game and collusion modeling

The timing is the following.

[15] To the best of our knolewdge, there exists no formal characterization of the set of non collusion-proof mechanisms. We let this for for furhter research.

1. Nature draws the type θ^i of firm F^i, $i = a, b$. Each firm only learns its own type.

2. Each regulator R^i proposes a contract composed of a quantity q^i to be produced and a transfer t^i to be paid by firm F^i.

3. Each firm F^i decides independently to accept or to reject this contract. If it refuses, then firm i receives its reservation gain normalized to 0, and the next steps do not occur.

4. A third party T proposes a side contract SC to *both* firms. $SC = \{\phi(.), \{y^i(.)\}_{i=a,b}$ where $\phi(.)$ is a manipulation function and $\{y^i(.)\}_{i=a,b}$ is a vector of budget balanced side transfers paid by each firm to T. If one firm at least refuses[16] SC then each firm F^i chooses non cooperatively in the contract proposed by R^i.

5. A report is sent by firm i, $i = a, b$ (according to the manipulation function if the side contract has been accepted). The quantities and transfers proposed by the two regulators take place, as well as the side transfers promised by T.

As previously the third party maximizes the sum of the expected gains of both firms under incentive, participation and ex post budget balance constraints. This program is detailed in the appendices and is similar to the situation of unilateral regulation. Its resolution yields the optimal manipulation of reports for the coalition. Collusion- proofness is now defined as follows.

Definition 2 *A contract C^i proposed by R^i is collusion-proof if the side contract proposed to the colluding firms by the third party is the null side contract, i.e. the side contract such that the manipulation function is equal to the identity function, $\phi = Id_{\Theta^2}$, and no side transfers occur at the equilibrium, $y^a(.) = y^b(.) = 0$.*

7.2 Asymmetric information without collusion

As a benchmark, we assume that firms do not collude. Bayesian incentive compatibility and Bayesian individual rationality constraints for F^a are given by

$$
\begin{array}{ll}
BIC^a(\underline{\theta}) & \underline{\pi}^a \geq \overline{\pi}^a + \Delta\theta\overline{q}^a \\
BIC^a(\overline{\theta}) & \overline{\pi}^a \geq \underline{\pi}^a - \Delta\theta\underline{q}^a
\end{array}
$$

and

$$
\begin{array}{ll}
BIR^a(\underline{\theta}) & \underline{\pi}^a \geq 0 \\
BIR^a(\overline{\theta}) & \overline{\pi}^a \geq 0.
\end{array}
$$

Given the symmetry of the model, the constraints faced by regulator R^b can be immediately obtained. Indeed, it is immediate to check that if the firm i's rent is defined as

$$Vpi^i = (p(q^a + q^b) - \theta^i)q^i - t^i$$

then all the individual constraints can be rewritten as in section 3.2.

[16] As before, we assume that beliefs are passive.

As with unilateral regulation, yardstick mechanisms can be used: Indeed if the regulator in one hierarchy chooses truthful separating contracts, then the other regulator can discriminate between four states of nature and the result of Crémer and McLean applies.

Proposition 6 *Under bilateral regulation, without collusion and under asymmetric information, the quantities at the Nash equilibrium are given by*

$$\begin{cases} q_{11}^{a*} & = q_{11}^{b*} = d - \underline{\theta} \\ q_{12}^{a*} & = q_{21}^{b*} = d - \underline{\theta} + \frac{1}{2}\Delta\theta \\ q_{21}^{a*} & = q_{12}^{b*} = d - \overline{\theta} - \frac{1}{2}\Delta\theta \\ q_{22}^{a*} & = q_{22}^{b*} = d - \overline{\theta}. \end{cases}$$

Quantities are equal to the full information ones and firms get no rent.

7.3 The allocation with collusion-proof regulation

Writing the conditions such that the optimal manipulation function is the identity yields the collusion-proofness constraints.

he coalitions that matter are those that are composed of at least one efficient firm and that pretend to be wholly inefficient[17]. Consider for instance regulator R^a. If R^a wants to ensure truthful revelation of the type of her regulated firm F^a then the following constraints must be satisfied:

$$\begin{array}{rl} CPC(2,0) & \pi_{11}^a + \pi_{11}^b \geq \pi_{22}^a + \pi_{22}^b + \Delta\theta q_{22}^a + \Delta\theta q_{22}^b \\ CPC(1,0,a) & \pi_{12}^a + \pi_{12}^b \geq \pi_{22}^a + \pi_{22}^b + \Delta\theta q_{22}^a - \dfrac{p_{11}}{p_{12}}\epsilon^b\Delta\theta(q_{22}^b - q_{12}^b). \end{array}$$

Notice that those constraints depend on the (observable) contract proposed by regulator R^b to firm F^b.

Hence by symmetry, the contract offered by R^a to firm F^a affects the incentives of the coalition to manipulate the report of F^b to R^b. If R^a wants to ensure collusion-proofness then she also has to ascertain that the report of firm F^b is not manipulated by the coaltition, or that constraint $CPC(1,0,b)$ is satisfied:

$$CPC(1,0,b) \quad \pi_{21}^a + \pi_{21}^b \geq \pi_{22}^a + \pi_{22}^b + \Delta\theta q_{22}^b - \frac{p_{11}}{p_{12}}\epsilon^a\Delta\theta(q_{22}^a - q_{21}^a).$$

The incompleteness of regulation appears in the right-hand side of the constraints, where some terms (specifically, the quantities contracted by the other regulator R^j, and the value of ϵ^j) are not controlled by regulator R^i, $i = a, b$. Indeed, the program of each regulator can be solved by having five constraints binding: The Bayesian incentive compatibility constraint of the efficient type, the Bayesian individual rationality constraint of the inefficient type, and the three collusion-proofness constraints above.

As opposed to the case of unilateral regulation, each regulator can now play on the cost of collusion under asymmetric information in order to make collusion less efficient.

[17]See Pouyet (1998) for an exposition of the role of correlation and strategic interaction in the determination of the relevant collusion-proofness constraints in equilibrium in a centralized organisation.

Lemma 4 *Assume that each regulator offers a contract such that the third party finds it optimal not to manipulate the announcements of the firms. Assume that the Bayesian incentive compatibility and Bayesian individual rationality constraints are binding in the program of each regulator. Then ϵ^i can be chosen in $[0,1]$ by regulator R^i, $i = a, b$.*

Indeed, the complementarity slackness conditions derived from the program of the third party do not bring additional information on the values of ϵ^a and ϵ^b. This contrasts strongly with the case of unilateral incomplete regulation, where they were necessarily equal to zero in equilibrium, implying that the single regulator could not benefit from the asymmetries of information within the coalition. Although there may appear a coordination problem in the choice of the ϵs, incomplete information at the level of the third party has an effect on the outcome of the game.

In the program of each regulator R^i, $i = a, b$, minimization of rents allows to obtain the shadow costs of the binding constraints. But these costs, except for the Bayesian participation constraint of an inefficient firm, are defined up to a free parameter μ^i, $i = a, b$. Yet, for symmetric equilibria, the collusion-proofness constraint preventing a $\underline{\theta}$-type firm F^a from pretending being inefficient when F^b is inefficient is exactly equal to the one preventing a $\underline{\theta}$-type firm F^b to pretend being inefficient when F^a is inefficient. The Lagrange multipliers of these constraints must therefore be equal. This adds a condition on the multipliers in the program of each regulator that is such that μ^i is uniquely defined[18]. The symmetric equilibirum of the Nash game with collusion-proof contracts is then unique[19].

Proposition 7 *Under bilateral regulation, with collusion, assume that both regulators offer collusion-proof contracts. The quantities at the Nash equilibrium are given by*

$$\begin{cases} q^a_{11} & = q^b_{11} = d - \underline{\theta} \\ q^a_{12} & = q^b_{21} = d - \underline{\theta} + \frac{\Delta\theta}{2}[1 - \frac{p_{11}}{p_{12}}] \\ q^a_{21} & = q^b_{12} = d - \overline{\theta} - \frac{\Delta\theta}{2}[1 - \frac{3}{2}\frac{p_{11}}{p_{12}}] \\ q^a_{22} & = q^b_{22} = d - \overline{\theta} - \frac{\Delta\theta}{p_{22}p(\overline{\theta})}[(1 - p_{12})p(\underline{\theta}) - \frac{\rho p_{11}}{\rho + p_{12}}]. \end{cases}$$

The binding constraints are $BIC(\underline{\theta})$, $BIR(\overline{\theta})$, $CPC(2,0)$, $CPC(1,0,a)$ and $CPC(1,0,b)$. All the remaining constraints are satisfied.

7.4 Centralization vs decentralization and the cost of collusion-proofness

In this section, we compare the situation of bilateral regulation with the centralization case in which both firms are under the juridiction of an unique regulator, called R^c. This is the situation studied in Laffont-Martimort (1997,1998), in which it can be shown that there is no loss of generality in restricting the regulator to use collusion-proof contracts.

The objective of R^c is to maximize the sum of the welfare of both countries. The individual constraints are identical to the ones in the case of bilateral regulation.

Without collusion the centralized regulator uses yardstick mechanisms and implements the following output schedule.

[18] The Lagrange multiplier of the incentive compatibility constraint of an efficient firm is then equal to $\frac{p_{12}}{\rho + p_{12}}$.

[19] Notice that an asymmetric equilibrium, outputs are defined up to a free parameter μ^i that can vary in the domain for which all multipliers remain positive.

Proposition 8 *Under centralized regulation, without collusion and under asymmetric information, the optimal quantities are given by*

$$\begin{cases} q_{11}^{a*} & = q_{11}^{b*} = d - \underline{\theta} \\ q_{12}^{a*} & = q_{21}^{b*} = 2(d - \underline{\theta}) \\ q_{21}^{a*} & = q_{12}^{b*} = 0 \\ q_{22}^{a*} & = q_{22}^{b*} = d - \overline{\theta}. \end{cases}$$

Quantities are equal to the full information ones and firms get no rent.

Under centralization, the regulator chooses to shift the whole production onto the most efficient firm as goods are perfectly substitutable. Quantities therefore differ from the ones in the bilateral regulation case, in which each regulator tries to favor production by her own firm.

Let us now consider the possibility of collusion between firms. For a given contract $(q^i, t^i)_{\{i=a,b\}}$ proposed to the firms by the centralized regulator, the program of the third party is unchanged with respect to the case of bilateral regulation. The collusion-proofnes constraints write exactly as before.

Contrary to the bilateral regulation case, the centralized regulator coordinates the choice of quantities produced by the firms in order to meet the collusion-proofness constraints in the least distortive way. Adapting the methodology used before we obtain the following proposition[20].

Proposition 9 *Under centralized regulation, with collusion, the optimal quantities are given by*

$$\begin{cases} q_{11}^{a*} & = q_{11}^{b*} = d - \underline{\theta} \\ q_{12}^{a*} & = q_{21}^{b*} = 2(d - \underline{\theta}) \\ q_{21}^{a*} & = q_{12}^{b*} = 0 \\ q_{22}^{a*} & = q_{22}^{b*} = d - \overline{\theta} - \frac{p_{12}}{p_{22}}\Delta\theta. \end{cases}$$

Beyond the sole difference in objectives, the bilateral regulation case differs from the centralized one in that a given regulator can affect the behaviour of the coalition only through her choice of quantity. The multipliers associated with the binding collusion-proofness constraints differ from their values under separation of regulators.

8 The role of non collusion-proof contracts: A heuristic discussion

As shown throughout the paper, the requirements of collusion-proofness are quite strong especially when institutional incompletenesses are taken into account and letting firms actively collude might be a preferred policy. As a preliminary step toward the characterization of the set of contracts when collusion is possible, let us discuss the potential gains and losses associated to equilibrium collusion.

[20] Although objectives and full information quantitites differ from the bilateral case the stakes of collusion go in the same direction and the binding collusion-proofness constraints turn out to be the same.

In a centralized organization, when a unique entity regulates both firms, both the regulator and the third party have the same information and, loosely speaking, the same contracting abilities. Moreover any transfers that could be implemented by the third party could be exactly replicated by the regulator at the same cost. This is the essence of the Collusion-Proofness Principle: There is no loss of generality in restricting the regulator to offer a regulatory contract such that there is no manipulation of announcements and no side transfers by the third party in equilibrium. Notice that it does not mean that collusion cannot occur in equilibrium but rather that an equilibrium in which the firms actively collude cannot perform strictly better, from a welfare point of view, than the optimal collusion-proof contract. This Principle may no longer be valid in the two structures of institutionally incomplete regulation we have considered. Indeed the third party can coordinate the rents of *both* firms and, in this sense, has more leeway than a given regulator that contracts only with her own firm. We focus in the following on the situation where one firm is left unregulated.

First, a non collusion-proof contract is such that the regulated firm F^a does not reveal truthfully its piece of information in at least one state of nature. Indeed, the manipulation function is otherwise the identity function and we are back to our analysis of collusion-proof contracts. Hence, *one cost associated to letting collusion in equilibrium is that some pooling results in the final allocation.*

Second, with a collusion-proof contract, we showed that the unregulated firm F^b produces in equilibrium the quantity that maximizes its own profit. On the contrary, *with a non collusion-proof contract, the quantity of the unregulated firm is set so as to maximize the sum of the gains of the colluding members.* With substitutable products, this tends to decrease the quantity produced by F^b (whereas the reverse holds for complementary products). Everything else being equal, this has a direct effect on consumers' welfare.

Third, notice that the quantity of the unregulated firm q^b is ex post observable by regulator R^a. However, with a collusion-proof contract, she cannot condition the transfer given to F^a on the fact that firms are colluding or not (i.e. q^b not set non cooperatively) as F^a is indifferent to collude or not in equilibrium. Suppose now that R^a offers to F^a a contract that will for sure trigger the formation of an active coalition. There will be a manipulation function different from the identity and a quantity q^b different from the one that maximizes the sole gain of F^b. However, as q^b is observable regulator R^a can now 'augment' the contract offered to F^a as follows: R^a knows neither θ^b nor θ^a but she knows that the firms will actively collude. If q^b does not belong to a certain set that she specifies then she imposes a 'fine' on the regulated firm F^a. If the firms are strictly willing to collude, then the coalition will be forced to pay the fine (which is initially imposed on the regulated firm only) or to ensure that q^b is produced according to the augmented contract. This was not possible with a collusion-proof contract. Hence *letting collusion happen in equilibrium enables the regulator to recover a partial control over the actions taken by the unregulated firm by using the information that firms are actively colluding.*

Fourth, with a collusion-proof contract we have shown that regulator R^a was not able to benefit from the informational asymmetries at the level of the coalition. This might not be the case with a non collusion-proof contract.

This discussion has been couched in a very informal way; we left for further research a full-fleged characterization of the space of implementable contracts when collusion is possible[21].

9 Conclusion

Aside from other numerous reasons(such as transfers of technology for developing countries, additional variety, increased cost efficiency, and so on), a reason to open markets to competition is to obtain information.

In our model, authorizing private firms to compete with a public firm on a given market may bring new information correlated with the information privately held by the regulated public firm. Hence, this should help to regulate the firm in a more efficient way as this new information were not previously available. However, we argue that the use of such yardstick mechanisms can also trigger the formation of collusive coalitions in response to a regulation that extracts all the rents of the regulated firm.

We have shown that when one firm is left unregulated collusion-proofness may be very costly since it can entail pooling of quantities. In such cases letting collusion happen in some states of nature may be a way to still have a differentiation of quantities according to cost levels.

This paper highlights the role of the sequencing of events in the liberalization process. Opening markets to competition should be associated to important changes in the institutional structure of the countries, so as to lessen the costs of lack of coordination and of incomplete regulation: Regulatory structures should be adapted so as to allow the regulator to control the whole industry, before entry barriers are removed. Developing countries and emerging economies in particular should therefore improve their institutions *before* introducing more competition if they want to circumvent the problems linked to institutional incompleteness of regulation.

In the context of bilateral regulation,the contract chosen by a regulator modifies the gains associated with different classes of contracts (collusion-proof or not collusion-proof contracts) for the other regulator. If a regulator chooses to ensure collusion-proofness, it may not be optimal for the second one to also offer collusion-proof contracts. +Some 'free riding' on the sharing of the costs of collusion-proofness may yield a higher surplus. Fighting collusion can indeed be viewed as a public good. Each regulator only partly internalizes the benefits associated to collusion-proofness. Even when collusion-proofness is ensured, coordination problems remain.

Non coordination of national policies can give rise to some important inefficiencies in the fight against collusion. This advocates clearly in favor of one supra-national authority that could intervene to facilitate the fight against collusion by the national regulators. The role of antitrust authorities for instance naturally emerges in settings where collusion may prevail in equilibrium. Antitrust agencies have an overlapping mandate on different sectors. They can therefore punish collusion even if the colluding firms do not belong to the same narrowly specified sector. This may enable them to correct for the partial inefficiency of a single sectorial regulator. A global, international antitrust authority would not necessarily require harmonization of domestic regulations, but would intervene when the lack of coordination enables collusion to appear.

Several topics remain open to further research. In bilateral hierarchies, the incentives of a regulator to engage into collusion-proofness are still to be clarified.

[21] In a framework of competing agencies, see the work of Epstein and Peters (1999) on the Revelation Principle.

On the theoretical side, the characterization of non collusion-proof contracts, and in particular of the optimal such ones, is as yet to be done. It would certainly allow non negligible advances in incentive theory and in organization theory.

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11 Appendices

11.1 The complete information benchmark

In the second stage of the game F^b chooses a production level conditional on the output produced by its competitor:

$$\max_{q^b}\{(p^b(q^a,q^b)-\theta^b)q^b\}.$$

The first-order condition yields $q^b=\frac{1}{2}(d_b-\theta^b+cq^a)$. The second-order condition of the unregulated firm's maximisation problem is trivially satisfied.

In the first stage of the game, the regulator offers a contract to firm F^a anticipating the best-response of the competitor, that is

$$\max_{q^a}\{SW^a(q^a,q^b(q^a))\}.$$

Under complete information on the cost parameter of firm F^a the regulator leaves no rent to the firm ($\pi^a=0$) and specifies quantities to be produced according to $q^a=(d_a-\theta^a)+\frac{c}{2}q^b$. Replacing q^b yields the equilibrium quantities.

11.2 Implementation of the complete information allocation when collusion is not possible

For a given quantity profile, let us assume that R^a wants to satisfy the four incentive and participation constraints of her firm as equalities. This is equivalent to finding a profile of ex post rent $\pi^a=(\pi^a_{11},\pi^a_{12},\pi^a_{21},\pi^a_{22})$ such that

$$\begin{pmatrix} p_{11} & p_{12}\\ p_{12} & p_{22}\end{pmatrix}\begin{pmatrix}\pi^a_{11}\\ \pi^a_{12}\end{pmatrix}=\Delta\theta\begin{pmatrix}0\\ p_{12}q^a_{11}+p_{22}q^a_{12}\end{pmatrix}$$

and

$$\begin{pmatrix} p_{11} & p_{12}\\ p_{12} & p_{22}\end{pmatrix}\begin{pmatrix}\pi^a_{21}\\ \pi^a_{22}\end{pmatrix}=-\Delta\theta\begin{pmatrix}p_{11}q^a_{21}+p_{12}q^a_{22}\\ 0\end{pmatrix}.$$

A necessary and sufficient condition for such rents to exist is that the degree of correlation is non null, i.e. $\rho\neq 0$. When the expected rent of of both types of regulated firm are zero (that is when both Bayesian individual rationality constraints are binding) then the optimization of the expected social welfare function with respect to the quantities leads to the complete information outcome.

11.3 The equilibrium with a non informative q^b

11.3.1 The optimal mechanism

We assume that the participation constraint of an inefficient firm and the incentive constraint of an efficient firm are binding, that is $\underline{\pi}^a=\Delta\theta\overline{q}^a$ and $\overline{\pi}^a=0$. Replacing these values of the rents in the social welfare function of the regulator and optimizing with respect to $(\underline{q}^a, \overline{q}^a)$ yields the optimal quantities. The remaining constraints will be satisfied in equilibrium if $\underline{q}^a \geq \overline{q}^a$, i.e.

$$4\frac{\Delta\theta}{p(\overline{\theta})} + c[\mathbf{E}\{\theta^b/\overline{\theta}\} - \mathbf{E}\{\theta^b/\underline{\theta}\}] \geq 0.$$

Immediate computations show that $\mathbf{E}\{\theta^b/\overline{\theta}\} - \mathbf{E}\{\theta^b/\underline{\theta}\} = \Delta\theta\frac{\rho}{\rho+p_{12}}$ and the previous condition boils down to $4(p_{11}+p_{12})+c\rho \geq 0$ which always holds (because it holds for $c=-1$ and $p_{12}=0$).

11.3.2 The participation decision of F^b to the collusive agreement

Let us denote $\underline{q}^b = q^b(\underline{q}^a)$ and $\overline{q}^b = q^b(\overline{q}^a)$. Immediate computations show that

$$\begin{aligned}
& d_b - \underline{q}^b + c\underline{q}^a - \underline{\theta} \\
&= d_b - \underline{\theta} - \frac{1}{2}(d_b - \mathbf{E}\{\theta^b/\underline{\theta}\}) + \frac{c}{2}\underline{q}^a \\
&= d_b - \underline{\theta} - \frac{1}{2}(d_b - \mathbf{E}\{\theta^b/\underline{\theta}\}) + \frac{1}{2(4-c^2)}[4c(d_a - \underline{\theta}) + c^2(d_b - \mathbf{E}\{\theta^b/\underline{\theta}\})] \\
&= \mathbf{E}\{\theta^b/\underline{\theta}\} - \underline{\theta} + 2c(d_a - \underline{\theta}) + \frac{2}{4-c^2}(d_b - \mathbf{E}\{\theta^b/\underline{\theta}\}) \\
&= \mathbf{E}\{\theta^b/\underline{\theta}\} - \underline{\theta} + \underline{q}^b.
\end{aligned}$$

In the same way,

$$\begin{aligned}
& d_b - \underline{\theta} - \overline{q}^b + c\overline{q}^a \\
&= \mathbf{E}\{\theta^b/\overline{\theta}\} - \underline{\theta} + \overline{q}^b.
\end{aligned}$$

The interim gain of an efficient unregulated firm is then

$$\begin{aligned}
& p11(d_b - \underline{q}^b + c\underline{q}^a - \underline{\theta}) + p12(d_b - \overline{q}^b + c\overline{q}^a - \underline{\theta}) \\
&= p_{11}[\mathbf{E}\{\theta^b/\underline{\theta}\} - \underline{\theta} + \underline{q}^b]\underline{q}^b + p_{12}[\mathbf{E}\{\theta^b/\overline{\theta}\} - \underline{\theta} + \overline{q}^b]\overline{q}^b
\end{aligned}$$

Without collusion, the reaction function of the unregulated firm is $q^b = \frac{1}{2}(d_b - \theta^b + cq^a)$ and its profit can be rewritten as $\pi^b = (p^b(q^a, q^b) - \theta^b)q^b = (q^b)^2$. The interim gain of an efficient unregulated firm without collusion can be rewritten as

$$p_{11}(q_{11}^{b*})^2 + p_{12}(q_{12}^{b*})^2.$$

This condition can be rewritten as

$$p_{11}[\mathbf{E}\{\theta^b/\underline{\theta}\} - \underline{\theta}] + p_{12}[\mathbf{E}\{\theta^b/\overline{\theta}\} - \underline{\theta}] + p_{11}(\underline{q}^b - q_{11}^{b*})(\underline{q}^b + q_{11}^{b*}) + p_{12}(\overline{q}^b - q_{21}^{b*})(\overline{q}^b + q_{21}^{b*}) \geq 0.$$

Immediate computations show that $\mathbf{E}\{\theta^b/\underline{\theta}\} - \underline{\theta} = \frac{p_{12}}{p(\underline{\theta})}\Delta\theta$, $\mathbf{E}\{\theta^b/\overline{\theta}\} - \underline{\theta} = \frac{p_{22}}{p(\overline{\theta})}\Delta\theta$, $\underline{q}^b - q_{11}^{b*} = \frac{-2}{4-c^2}\frac{p_{12}}{p(\underline{\theta})}\Delta\theta$ and $\overline{q}^b - q_{21}^{b*} = \frac{-2}{4-c^2}[\mathbf{E}\{\theta^b/\overline{\theta}\} - \underline{\theta} + 2c\frac{p(\underline{\theta})}{p(\overline{\theta})}\Delta\theta]$. The previous inequality becomes

$$\Delta\theta p_{12}\frac{1}{p(\underline{\theta})p(\overline{\theta})}(p_{11}\overline{\theta} + p_{22}\underline{\theta}) - \frac{2}{4-c^2}\frac{p_{11}}{p(\underline{\theta})}(\underline{q}^b + q_{21}^{b*}) - \frac{2}{(4-c^2)p(\overline{\theta})}(p_{22} + 2cp(\underline{\theta}))(\overline{q}^b + q_{21}^{b*}) \geq 0.$$

We consider now an inefficient unregulated firm F^b. Similar computations show that a $\overline{\theta}$-type firm F^b will accept the collusive agreement if the following condition holds:

$$p_{12}[\mathbf{E}\{\theta^b/\underline{\theta}\} - \overline{\theta} + \underline{q}^b]\underline{q}^b + p_{22}[\mathbf{E}\{\theta^b/\overline{\theta}\} - \overline{\theta} + \overline{q}^b]\overline{q}^b \geq p_{12}(q_{12}^{b*})^2 + p_{22}(q_{22}^{b*})^2.$$

Since $\mathbf{E}\{\theta^b/\underline{\theta}\} - \overline{\theta} = -\frac{\Delta\theta p_{11}}{p(\underline{\theta})}$ and $\mathbf{E}\{\theta^b/\overline{\theta}\} - \overline{\theta} = -\frac{\Delta\theta p_{12}}{p(\overline{\theta})}$, the inequality becomes

$$-\Delta\theta p_{12}(\frac{p_{11}}{p(\underline{\theta})} + \frac{p_{22}}{p(\overline{\theta})}) + p_{12}(\underline{q}^b - q_{12}^{b*})(\underline{q}^b + q_{12}^{b*}) + p_{22}(\overline{q}^b - q_{22}^{b*})(\overline{q}^b + q_{22}^{b*}) \geq 0.$$

But $\underline{q}^b - q_{12}^{b*} = \frac{2}{4-c^2}\frac{p_{11}\Delta\theta}{p(\underline{\theta})}$ and $\overline{q}^b - q_{22}^{b*} = \frac{2}{4-c^2}\frac{\Delta\theta}{p(\overline{\theta})}(p_{12} - cp(\underline{\theta}))$ and the previous inequality becomes

$$-\Delta\theta p_{12}(\frac{p_{11}}{p(\underline{\theta})} + \frac{p_{22}}{p(\overline{\theta})}) + \frac{2}{4-c^2}\frac{p_{12}\Delta\theta}{p(\underline{\theta})}(\underline{q}^b + q_{12}^{b*}) + \frac{2}{4-c^2}\frac{p_{22}\Delta\theta}{p(\overline{\theta})}(p_{12} - cp(\underline{\theta}))(\overline{q}^b + q_{22}^{b*}) \geq 0.$$

An example where the unregulated firm accepts the collusive agreement Assume that $d_b - \overline{\theta} + c(d_a - \overline{\theta}) = 0$. This implies that $q_{22}^b = q_{12}^b = 0$. Moreover, $q_{11}^b = (1+c)\Delta\theta$ and $q_{21}^b = \Delta\theta$[22]. Assume now that $c \to -1$. This implies that $\underline{q}^b = 0$.

For an efficient unregulated firm, the gain without collusion is approximately

$$p_{12}(\Delta\theta)^2,$$

whereas the gain of collusion is approximately

$$p_{12}[p_{22} + \frac{2}{3}(p_{11} + 2p_{12})](\Delta\theta)^2\frac{p_{11} + 2p_{12}}{p(\overline{\theta})^2}.$$

Assume now that $p_{11} = p_{22} \equiv p'$. Then F^b will accept the collusive agreement if

$$3(p' + p_{12})^2 \leq 2(p' + 2p_{12})[p' + \frac{2}{3}(p' + 2p_{12})].$$

Since $p' > \frac{1}{2}$ (from the positive correlation assumption), $8(2 - p' - (p')^2) \geq 8\frac{5}{4} > 9$.

From the point of view of the inefficient regulated firm, its gain without collusion is 0. Its gain in case of collusion will be positive if

$$p'[\frac{2}{3}\frac{\Delta\theta}{p(\overline{\theta})}(p' + 2p_{12})]^2 \geq \Delta\theta\frac{2p'p_{12}}{p' + p_{12}}.$$

Sufficient conditions for this inequality to hold are that $\Delta\theta$ be large enough and that the correlation be sufficiently large (p_{12} close to 0).

[22] Crmer and McLean mechanisms can still be used in this case.

11.4 Resolution of the program of T and characterization of the collusion-proof contracts

Let us start with some notations: $\phi_{11} = \phi(\theta^a = \underline{\theta}, \theta^b = \underline{\theta})$, $\phi_{12} = \phi(\theta^a = \underline{\theta}, \theta^b = \overline{\theta})$, $\phi_{21} = \phi(\theta^a = \overline{\theta}, \theta^b = \underline{\theta})$ and $\phi_{22} = \phi(\theta^a = \overline{\theta}, \theta^b = \overline{\theta})$. Now, for expositional purposes, we also introduce the rents of the firms from the point of view of the third party (side transfers are oitted for expositional purposes).

- For the regulated firm F^a

$$\pi^a_{11}(TP) = [d_a - q^a(\phi_{11}) + cq^b_{11} - \underline{\theta}]q^a(\phi_{11}) - t^a(\phi_{11})$$
$$\pi^a_{12}(TP) = [d_a - q^a(\phi_{12}) + cq^b_{12} - \underline{\theta}]q^a(\phi_{12}) - t^a(\phi_{12})$$
$$\pi^a_{21}(TP) = [d_a - q^a(\phi_{21}) + cq^b_{21} - \overline{\theta}]q^a(\phi_{21}) - t^a(\phi_{21})$$
$$\pi^a_{22}(TP) = [d_a - q^a(\phi_{22}) + cq^b_{22} - \overline{\theta}]q^a(\phi_{22}) - t^a(\phi_{22}).$$

- For the unregulated firm F^b

$$\pi^b_{11}(TP) = [d_b - q^b_{11} + cq^a_{11}(\phi_{11}) - \underline{\theta}]q^b_{11}$$
$$\pi^b_{12}(TP) = [d_b - q^b_{12} + cq^a_{12}(\phi_{12}) - \overline{\theta}]q^b_{12}$$
$$\pi^b_{21}(TP) = [d_b - q^b_{21} + cq^a_{21}(\phi_{21}) - \underline{\theta}]q^b_{21}$$
$$\pi^b_{22}(TP) = [d_b - q^b_{22} + cq^a_{22}(\phi_{22}) - \overline{\theta}]q^b_{22}.$$

11.4.1 The program of the third party

Because the side transfers are budeget balanced in each state of nature, the program of the third party can be written as follows

$$\max_{\{\phi(.), q^b(.), y(.)\}} \sum_{i,j=1,2} p_{ij}[\pi^a_{ij}(TP) + \pi^b_{ij}(TP)]$$

subject to

$$BIC^a(\underline{\theta}) \quad p_{11}[\pi^a_{11}(TP) - y^a(\underline{\theta},\underline{\theta})] + p_{12}[\pi^a_{12}(TP) - y^a(\underline{\theta},\overline{\theta})] \geq$$
$$p_{11}[\pi^a_{21}(TP) + \Delta\theta q^a(\phi_{21}) - y^a(\overline{\theta},\underline{\theta})] + p_{12}[\pi^a_{22}(TP) + \Delta\theta q^a(\phi_{22}) - y^a(\overline{\theta},\overline{\theta})]$$

$$BIC^b(\underline{\theta}) \quad p_{11}[\pi^b_{11}(TP) - y^b(\underline{\theta},\underline{\theta})] + p_{12}[\pi^b_{21}(TP) - y^b(\overline{\theta},\underline{\theta})] \geq$$
$$p_{11}[\pi^b_{12}(TP) + \Delta\theta q^b(\phi_{12}) - y^b(\underline{\theta},\overline{\theta})] + p_{12}[\pi^b_{22}(TP) + \Delta\theta q^b(\phi_{22}) - y^a(\overline{\theta},\overline{\theta})]$$

$$BIR^a(\underline{\theta}) \quad p_{11}[\pi^a_{11}(TP) - y^a(\underline{\theta},\underline{\theta})] + p_{12}[\pi^a_{12}(TP) - y^a(\underline{\theta},\overline{\theta})] \geq \underline{\tilde{\pi}}^a$$

$$BIR^b(\underline{\theta}) \quad p_{11}[\pi^b_{11}(TP) - y^b(\underline{\theta},\underline{\theta})] + p_{12}[\pi^b_{21}(TP) - y^b(\overline{\theta},\underline{\theta})] \geq \underline{\tilde{\pi}}^b$$

$$BIR^a(\overline{\theta}) \quad p_{12}[\pi^a_{21}(TP) - y^a(\overline{\theta},\underline{\theta})] + p_{22}[\pi^a_{22}(TP) - y^a(\overline{\theta},\overline{\theta})] \geq \overline{\tilde{\pi}}^a$$

$$BIR^b(\overline{\theta}) \quad p_{12}[\pi^b_{12}(TP) - y^b(\underline{\theta},\overline{\theta})] + p_{22}[\pi^b_{22}(TP) - y^b(\overline{\theta},\overline{\theta})] \geq \overline{\tilde{\pi}}^b$$

$$BB(\theta^a,\theta^b) \quad \sum_{i=a,b} y^i(\theta^a,\theta^b) = 0 \quad \forall(\theta^a,\theta^b) \in \Theta^b.$$

We denote by δ_i^{TP}, $i = a, b$, the multiplier associated to $BIC^i(\underline{\theta})$, $\underline{\nu}_i^{TP}$ the multiplier associated to $BIR^i(\underline{\theta})$, $\overline{\nu}_i^{TP}$ the multiplier associated to $BIR^i(\overline{\theta})$ and $\rho(\theta^a,\theta^b)^{TP}$ the multiplier associated to $BB(\theta^a,\theta^b)$. Computations can be immediately adapted if the Bayesian incentive compatibility of inefficient (regulated and unregulated) firms are incorporated in the program of T.

11.4.2 Optimality conditions for the side transfers

Optimizing the Lagragean associated to the previous problem with respect to the side-transfers gives the following conditions.

- First order conditions w.r.t. $y^a(\underline{\theta},\underline{\theta})$ and $y^b(\underline{\theta},\underline{\theta})$

$$\begin{aligned}-p_{11}\delta_a^{TP} - p_{11}\underline{\nu}_a^{TP} + \rho(\underline{\theta},\underline{\theta})^{TP} &= 0\\ -p_{11}\delta_b^{TP} - p_{11}\underline{\nu}_b^{TP} + \rho(\underline{\theta},\underline{\theta})^{TP} &= 0.\end{aligned}$$

- First order conditions w.r.t. $y^a(\underline{\theta},\overline{\theta})$ and $y^b(\underline{\theta},\overline{\theta})$

$$\begin{aligned}-p_{12}\delta_a^{TP} - p_{12}\underline{\nu}_a^{TP} + \rho(\underline{\theta},\overline{\theta})^{TP} &= 0\\ p_{11}\delta_b^{TP} - p_{12}\overline{\nu}_b^{TP} + \rho(\underline{\theta},\overline{\theta})^{TP} &= 0.\end{aligned}$$

- First order conditions w.r.t. $y^a(\overline{\theta},\underline{\theta})$ and $y^b(\overline{\theta},\underline{\theta})$

$$\begin{aligned}p_{11}\delta_a^{TP} - p_{12}\overline{\nu}_a^{TP} + \rho(\overline{\theta},\underline{\theta})^{TP} &= 0\\ -p_{12}\delta b^{TP} - p_{12}\underline{\nu}_b^{TP} + \rho(\overline{\theta},\underline{\theta})^{TP} &= 0.\end{aligned}$$

- First order conditions w.r.t. $y^a(\overline{\theta},\overline{\theta})$ et $y^b(\overline{\theta},\overline{\theta})$

$$\begin{aligned}p_{12}\delta_a^{TP} - p_{22}\overline{\nu}_a^{TP} + \rho(\overline{\theta},\overline{\theta})^{TP} &= 0\\ p_{12}\delta_b^{TP} - p_{22}\overline{\nu}_b^{TP} + \rho(\overline{\theta},\overline{\theta})^{TP} &= 0.\end{aligned}$$

We can deduce immediately the following relations between the multipliers:

$$\begin{aligned}\delta_a^{TP} + \underline{\nu}_a^{TP} &= \delta_b^{TP} + \underline{\nu}_b^{TP}\\ \delta_a^{TP} + \underline{\nu}_a^{TP} &= \overline{\nu}_b^{TP} - \frac{p_{11}}{p_{12}}\delta_b^{TP}\\ \delta_b^{TP} + \underline{\nu}_b^{TP} &= \overline{\nu}_a^{TP} - \frac{p_{11}}{p_{12}}\delta_a^{TP}\\ \overline{\nu}_a^{TP} - \frac{p_{12}}{p_{22}}\delta_a^{TP} &= \overline{\nu}_b^{TP} - \frac{p_{12}}{p_{22}}\delta_b^{TP}.\end{aligned}$$

$\delta_a^{TP} + \underline{\nu}_a^{TP} = \delta_b^{TP} + \underline{\nu}_b^{TP}$, $\delta_a^{TP} + \underline{\nu}_a^{TP} = \overline{\nu}_b^{TP} - \frac{p_{11}}{p_{12}}\delta_b^{TP}$, $\delta_b^{TP} + \underline{\nu}_b^{TP} = \overline{\nu}_a^{TP} - \frac{p_{11}}{p_{12}}\delta_a^{TP}$, and $\overline{\nu}_a^{TP} - \frac{p_{12}}{p_{22}}\delta_a^{TP} = \overline{\nu}_b^{TP} - \frac{p_{12}}{p_{22}}\delta_b^{TP}$.

11.4.3 Optimality conditions for the manipulation function

Using the previous conditions on the multipliers, it is immediate to derive the optimality conditions on the manipulation function $\phi(.)$.

$$\phi_{11}^* \in \arg\max_{\phi_{11}} \pi_{11}^a(TP) + \pi_{11}^b(TP)$$

$$\phi_{12}^* \in \arg\max_{\phi_{12}} \pi_{12}^a(TP) + \pi_{12}^b(TP) - \frac{p_{11}}{p_{12}}\epsilon^b \Delta\theta q_{12}^b$$

$$\phi_{21}^* \in \arg\max_{\phi_{21}} \pi_{21}^a(TP) + \pi_{21}^b(TP) - \frac{p_{11}}{p_{12}}\epsilon^a \Delta\theta q^a(\phi_{21})$$

$$\phi_{22}^* \in \arg\max_{\phi_{22}} \pi_{22}^a(TP) + \pi_{22}^b(TP) - \frac{p_{12}^2\epsilon^a}{p_{12}p_{22} + \epsilon^a\rho}\Delta\theta q^a(\phi_{22}) - \frac{p_{12}^2\epsilon^b}{p_{12}p_{22} + \epsilon^b\rho}\Delta\theta q_{22}^b$$

where $\epsilon^a = \frac{\delta_a^{TP}}{1+\underline{\nu}_a^{TP}+\delta_a^{TP}}$ and $\epsilon^b = \frac{\delta_b^{TP}}{1+\underline{\nu}_b^{TP}+\delta_b^{TP}}$. For further references, we define $f(\epsilon) = \frac{p_{12}^2\epsilon}{p_{12}p_{22}+\epsilon\rho}$.

11.4.4 Optimality conditions for the quantity of the unregulated firm

Similar computations yield immediately the following conditions.

$$q_{11}^{b*} \in \arg\max_{q_{11}^b} cq_{11}^b q^a(\phi_{11}) + [d_b - q_{11}^b + cq^a(\phi_{11}) - \underline{\theta}]q_{11}^b$$

$$q_{12}^{b*} \in \arg\max_{q_{12}^b} cq_{12}^b q^a(\phi_{12}) + [d_b - q_{12}^b + cq^a(\phi_{12}) - \overline{\theta}]q_{12}^b - \frac{p_{11}}{p_{12}}\epsilon^b \Delta\theta q_{12}^b$$

$$q_{21}^{b*} \in \arg\max_{q_{21}^b} cq_{21}^b q^a(\phi_{21}) + [d_b - q_{21}^b + cq^a(\phi_{21}) - \underline{\theta}]q_{21}^b$$

$$q_{22}^{b*} \in \arg\max_{q_{22}^b} cq_{22}^a q^a(\phi_{22}) + (d_b - q_{22}^b + cq^a(\phi_{22}) - \overline{\theta})q_{22}^b - \frac{p_{12}}{p_{22}}f(\epsilon^b)\Delta\theta q_{22}^b.$$

11.4.5 Non pooling contract

We assume that the ex post individual rationality constraints of an inefficient firm are binding, $\pi_{21}^a = \pi_{22}^a = 0$, as well as the collusion-proofness constraints preventing an efficient regulated firm from lying to the regulator, CPC_1 and CPC_2. This gives $\pi_{11}^a = \pi_{21}^b - \pi_{11}^b + \Delta\theta q_{21}^a$, $\pi_{12}^a = \pi_{22}^b - \pi_{12}^b + \Delta\theta q_{22}^a$, and $\pi_{21} = \pi_{22} = 0$.

The profit of the unregulated firm F^b is $\pi^b = (d_b - q^b + cq^a - \theta^b)q^b$, and since $q^b \in \arg\max \pi^b$, we have $q^b = \frac{1}{2}(d_b - \theta^b + cq^a)$ for a given state of nature, implying $\frac{\partial \pi^b}{\partial q^a} = cq^b$ and $\frac{dq^b}{dq^a} = \frac{c}{2}$. Hence $\pi_{11}^a = \frac{d_b - \underline{\theta}}{2}c(q_{21}^a - q_{11}^a) + \Delta\theta q_{21}^a$ and $\pi_{12}^a = \frac{d_b - \overline{\theta}}{2}c(q_{22}^a - q_{12}^a) + \Delta\theta q_{22}^a$. Replacing these ex post rents in the expected social welfare function of the principal and optimizing with respect to the quantities yield the optimal quantities given in the proposition.

Replacing the values of q^a in the reaction function of q^b, we obtain the output levels produced by the unregulated firm in each state of nature given in the proposition.

We need to find the conditions under which the remaining collusion-proofness constraints are verified at the optimum. Immediate computations show that

$$\begin{aligned} CPC_3 &\Leftrightarrow q_{11}^a \geq q_{21}^a \Leftrightarrow c^2(d_a - \underline{\theta}) + c(d_b - \underline{\theta}) \geq -(4-3c^2)\frac{\Delta\theta}{2} \\ CPC_4 &\Leftrightarrow q_{12}^a \geq q_{22}^a \Leftrightarrow c^2(d_a - \underline{\theta}) + c(d_b - \overline{\theta}) \geq -(4-3c^2)\frac{\Delta\theta}{2}. \end{aligned}$$

Under the assumption that the goods are subsitutes, $c \leq 0$, the first condition implies the second one. From now on we shall note

$$\begin{aligned} \mathcal{C}_1 \quad & c^2(d_a - \underline{\theta}) + c(d_b - \underline{\theta}) \geq -(4-3c^2)\frac{\Delta\theta}{2} \\ \mathcal{C}_2 \quad & c^2(d_a - \underline{\theta}) + c(d_b - \overline{\theta}) \geq -(4-3c^2)\frac{\Delta\theta}{2}. \end{aligned}$$

We have now to check that individual constraints are satisfied in equilibrium. $BIR(\underline{\theta})$ is trivially satisfied because π_{11}^a and π_{12}^a are both positive. $BIC(\underline{\theta})$ is equivalent to $p_{11}(\pi_{11}^a - \Delta\theta q_{21}^a) + p_{12}(\pi_{12}^a - \Delta\theta q_{22}^a) \geq 0$, which is satisfied because both terms are positive. $BIC(\overline{\theta})$ is equivalent to $p_{12}(\pi_{11}^a - \Delta\theta q_{11}^a) + p_{22}(\pi_{12}^a - \Delta\theta q_{12}^a) \leq 0$. After computations we obtain that

$$\begin{aligned} &\pi_{11}^a - \Delta\theta q_{11}^a = \\ &\qquad -\frac{4(p_{11}+p_{12})(c^2(p_{11}-2p_{12})+4p_{12})}{(4-3c^2)(c^2(2p_{11}-p_{12})+4p_{12})^2}[2c(d_b-\underline{\theta}) + 2c^2(d_a-\overline{\theta}) + (4-c^2)\Delta\theta]^2 \leq 0, \end{aligned}$$

and

$$\begin{aligned} &\pi_{12}^a - \Delta\theta q_{12}^a = \\ &\qquad -\frac{4(p_{12}+p_{22})(c^2(p_{12}-2p_{22})+4p_{22})}{(4-3c^2)(c^2(2p_{12}-p_{22})+4p_{22})^2}[2c(d_b-\overline{\theta}) + 2c^2(d_a-\overline{\theta}) + (4-c^2)\Delta\theta]^2 \leq 0. \end{aligned}$$

$BIC(\overline{\theta})$ is therefore satisfied, as well as $BIC(\underline{\theta})$.

Notice finally that if d_a is sufficiently large and c is sufficiently close to zero (and negative) then the quantities produced by both firms are positive at the optimum.

11.4.6 Partially pooling contract

Let us assume now that $\mathcal{C}_1$ is not satisfied while $\mathcal{C}_2$ is satisfied. Then, the only way for the regulator to satisfy the collusion-proofness constraints CPC_1 and CPC_2 is to impose a partial pooling of the quantities. We shall note $\hat{q} = q_{11}^a = q_{21}^a$.

The binding constraints are the same. This yields the profile of ex post rents: $\pi_{22}^a = 0$, $\pi_{21}^a = 0$, $\pi_{11}^a = \Delta\theta\hat{q}$ and $\pi_{21}^a = (q_{22}^b)^2 - (q_{12}^b)^2 + \Delta\theta q_{22}^a$. Replacing these ex post rents in the expected social welfare function of the regulator and optimizing with respect to the quantities yields the quantities given in the proposition.

Immediate computations show that all the remaining collusion-proofness constraints are satisfied, as well as the individual participation and incentive compatibility constraints.

11.4.7 Completely pooling contract

Finally we assume that $\mathcal{C}_2$ is not satisfied. We shall note $\hat{q}^1 = q^a_{11} = q^a_{21}$ and $\hat{q}^2 = q^a_{12} = q^a_{22}$. The binding collusion-proofness constraints are assumed to be the same. This gives: $\pi^a_{22} = 0$, $\pi^a_{21} = 0$, $\pi^a_{11} = \Delta\theta\hat{q}^1$ and $\pi^a_{12} = \Delta\theta\hat{q}^2$. Replacing these values in the expected social welfare and optimizing with respect to the quantities yields the optimal quantities given in the proposition.

It is immediate to check that all the constraints we have neglected in a first time are satisfied at the optimum.

11.5 Bilateral intervention

11.5.1 The third party's program

Let us start with some notations: $\phi_{11} = \phi(\theta^1 = \underline{\theta}, \theta^2 = \underline{\theta})$, $\phi_{12} = \phi(\theta^a = \underline{\theta}, \theta^b = \overline{\theta})$, $\phi_{21} = \phi(\theta^a = \overline{\theta}, \theta^b = \underline{\theta})$ and $\phi_{22} = \phi(\theta^a = \overline{\theta}, \theta^b = \overline{\theta})$. Let $Q = q^a + q^b$ be the total quantity sold on the market. The program of T is

$$\begin{aligned}
\max_{\{\phi(.),y(.)\}} & p_{11}[(P(Q\circ\phi_{11}) - \underline{\theta})q^a\circ\phi_{11} - t^a\circ\phi_{11} + (P(Q\circ\phi_{11}) - \underline{\theta})q^b\circ\phi_{11} - t^b\circ\phi_{11}] \\
& + p_{12}[(P(Q\circ\phi_{12}) - \underline{\theta})q^a\circ\phi_{12} - t^a\circ\phi_{12} + (P(Q\circ\phi_{12}) - \overline{\theta})q^b\circ\phi_{12} - t^b\circ\phi_{12}] \\
& + p_{12}[(P(Q\circ\phi_{21}) - \overline{\theta})q^a\circ\phi_{21} - t^a\circ\phi_{21} + (P(Q\circ\phi_{21}) - \underline{\theta})q^b\circ\phi_{21} - t^b\circ\phi_{21}] \\
& + p_{22}[(P(Q\circ\phi_{22}) - \overline{\theta})q^a\circ\phi_{22} - t^a\circ\phi_{22} + (P(Q\circ\phi_{22}) - \overline{\theta})q^b\circ\phi_{22} - t^b\circ\phi_{22}]
\end{aligned}$$

subject to

$$\begin{aligned}
BIC^a(\underline{\theta}) \quad & p_{11}[(P(Q\circ\phi_{11}) - \underline{\theta})q^a\circ\phi_{11} - t^a\circ\phi_{11} - y^a(\underline{\theta},\underline{\theta})] \\
& \qquad + p_{12}[(P(Q\circ\phi_{12}) - \underline{\theta})q^a\circ\phi_{12} - t^a\circ\phi_{12} - y^a(\underline{\theta},\overline{\theta})] \\
& \geq p_{11}[(P(Q\circ\phi_{21}) - \underline{\theta})q^a\circ\phi_{21} - t^a\circ\phi_{21} - y^a(\overline{\theta},\underline{\theta})] \\
& \qquad + p_{12}[(P(Q\circ\phi_{22}) - \underline{\theta})q^a\circ\phi_{22} - t^a\circ\phi_{22} - y^a(\overline{\theta},\overline{\theta})]
\end{aligned}$$

$$\begin{aligned}
BIC^b(\underline{\theta}) \quad & p_{11}[(P(Q\circ\phi_{11}) - \underline{\theta})q^b\circ\phi_{11} - t^b\circ\phi_{11} - y^b(\underline{\theta},\underline{\theta})] \\
& \qquad + p_{12}[(P(Q\circ\phi_{21}) - \underline{\theta})q^b\circ\phi_{21} - t^b\circ\phi_{21} - y^b(\overline{\theta},\underline{\theta})] \\
& \geq p_{11}[(P(Q\circ\phi_{12}) - \underline{\theta})q^b\circ\phi_{12} - t^b\circ\phi_{12} - y^b(\underline{\theta},\overline{\theta})] \\
& \qquad + p_{12}[(P(Q\circ\phi_{22}) - \underline{\theta})q^b\circ\phi_{22} - t^b\circ\phi_{22} - y^b(\overline{\theta},\overline{\theta})]
\end{aligned}$$

$$\begin{aligned}
BIR^a(\underline{\theta}) \quad & p_{11}[(P(Q\circ\phi_{11}) - \underline{\theta})q^a\circ\phi_{11} - t^a\circ\phi_{11} - y^a(\underline{\theta},\underline{\theta})] \\
& \qquad + p_{12}[(P(Q\circ\phi_{12}) - \underline{\theta})q^a\circ\phi_{12} - t^a\circ\phi_{12} - y^a(\underline{\theta},\overline{\theta})] \geq \pi^a(\underline{\theta})
\end{aligned}$$

$$\begin{aligned}
BIR^b(\underline{\theta}) \quad & p_{11}[(P(Q\circ\phi_{11}) - \underline{\theta})q^b\circ\phi_{11} - t^b\circ\phi_{11} - y^b(\underline{\theta},\underline{\theta})] \\
& \qquad + p_{12}[(P(Q\circ\phi_{21}) - \underline{\theta})q^b\circ\phi_{21} - t^b\circ\phi_{21} - y^b(\overline{\theta},\underline{\theta})] \geq \pi^b(\underline{\theta})
\end{aligned}$$

$$BIR^a(\overline{\theta}) \quad p_{12}[(P(Q\circ\phi_{21})-\overline{\theta})q^a\circ\phi_{21}-t^a\circ\phi_{21}-y^a(\overline{\theta},\underline{\theta})]$$
$$+p_{22}[(P(Q\circ\phi_{22})-\overline{\theta})q^a\circ\phi_{22}-t^a\circ\phi_{22}-y^a(\overline{\theta},\overline{\theta})]\geq\pi^a(\overline{\theta})$$

$$BIR^b(\overline{\theta}) \quad p_{12}[(P(Q\circ\phi_{12})-\overline{\theta})q^b\circ\phi_{12}-t^b\circ\phi_{12}-y^b(\underline{\theta},\overline{\theta})]$$
$$+p_{22}[(P(Q\circ\phi_{22})-\overline{\theta})q^b\circ\phi_{22}-t^b\circ\phi_{22}-y^b(\overline{\theta},\overline{\theta})]\geq\pi^b(\overline{\theta})$$

$$BB(\theta^a,\theta^b) \quad \sum_{i=a,b} y^i(\theta^a,\theta^b)=0 \quad \forall(\theta^a,\theta^b)\in\Theta^b.$$

We denote by δ_i^T, $i=a,b$, the multiplier associated to $BIC^i(\underline{\theta})$, $\underline{\nu}_i^T$ the multiplier associated to $BIR^i(\underline{\theta})$, $\overline{\nu}_i^T$ the multiplier associated to $BIR^i(\overline{\theta})$ and $\rho(\theta^a,\theta^b)^T$ the multiplier associated to $BB(\theta^a,\theta^b)$. Optimizing the Lagragean associated to the previous problem with respect to the side-transfers gives the following conditions.

- First order conditions w.r.t. $y^a(\underline{\theta},\underline{\theta})$ and $y^b(\underline{\theta},\underline{\theta})$

$$-p_{11}\delta_a^T-p_{11}\underline{\nu}_a^T+\rho(\underline{\theta},\underline{\theta})^T=0$$
$$-p_{11}\delta_b^T-p_{11}\underline{\nu}_b^T+\rho(\underline{\theta},\underline{\theta})^T=0.$$

- First order conditions w.r.t. $y^a(\underline{\theta},\overline{\theta})$ and $y^b(\underline{\theta},\overline{\theta})$

$$-p_{12}\delta_a^T-p_{12}\underline{\nu}_a^T+\rho(\underline{\theta},\overline{\theta})^T=0$$
$$p_{11}\delta_b^T-p_{12}\overline{\nu}_b^T+\rho(\underline{\theta},\overline{\theta})^T=0.$$

- First order conditions w.r.t. $y^a(\overline{\theta},\underline{\theta})$ and $y^b(\overline{\theta},\underline{\theta})$

$$p_{11}\delta_a^T-p_{12}\overline{\nu}_a^T+\rho(\overline{\theta},\underline{\theta})^T=0$$
$$-p_{12}\delta b^T-p_{12}\underline{\nu}_b^T+\rho(\overline{\theta},\underline{\theta})^T=0.$$

- First order conditions w.r.t. $y^a(\overline{\theta},\overline{\theta})$ et $y^b(\overline{\theta},\overline{\theta})$

$$p_{12}\delta_a^T-p_{22}\overline{\nu}_a^T+\rho(\overline{\theta},\overline{\theta})^T=0$$
$$p_{12}\delta_b^T-p_{22}\overline{\nu}_b^T+\rho(\overline{\theta},\overline{\theta})^T=0.$$

We can deduce immediately the following relations between the multipliers: $\delta_a^T+\underline{\nu}_a^T=\delta_b^T+\underline{\nu}_b^T$, $\delta_a^T+\underline{\nu}_a^T=\overline{\nu}_b^T-\frac{p_{11}}{p_{12}}\delta_b^T$, $\delta_b^T+\underline{\nu}_b^T=\overline{\nu}_a^T-\frac{p_{11}}{p_{12}}\delta_a^T$, and $\overline{\nu}_a^T-\frac{p_{12}}{p_{22}}\delta_a^T=\overline{\nu}_b^T-\frac{p_{12}}{p_{22}}\delta_b^T$.

Let us turn on to the optimality conditions for the manipulation function of the third party. After immediate algebra and using the conditions on the multipliers found previously, we find

$$\phi_{11}^*\in\arg\max_{\phi_{11}} \quad (P(Q\circ\phi_{11})-\underline{\theta})q^a\circ\phi_{11}-t^a\circ\phi_{11}+(P(Q\circ\phi_{11})-\underline{\theta})q^b\circ\phi_{11}-t^b\circ\phi_{11},$$

$$\phi_{12}^* \in \arg\max_{\phi_{12}} \quad (P(Q \circ \phi_{12}) - \underline{\theta})q^a \circ \phi_{12} - t^a \circ \phi_{12}$$
$$+ (P(Q(\circ\phi_{12}) - \overline{\theta})q^b \circ \phi_{12} - t^b \circ \phi_{12} - \frac{p_{11}}{p_{12}} \frac{\delta_b^T}{1 + \delta_b^T + \underline{\nu}_b^T} \Delta\theta q^b \circ \phi_{12},$$

$$\phi_{21}^* \in \arg\max_{\phi_{21}} \quad (P(Q \circ \phi_{21}) - \overline{\theta})q^a \circ \phi_{21} - t^a \circ \phi_{21}$$
$$+ (P(Q \circ \phi_{21}) - \underline{\theta})q^b \circ \phi_{21} - t^b \circ \phi_{21} - \frac{p_{11}}{p_{12}} \frac{\delta_a^T}{1 + \delta_a^T + \underline{\nu}_a^T} \Delta\theta q^a \circ \phi_{21},$$

and

$$\phi_{22}^* \in \arg\max_{\phi_{22}} \quad (P(Q \circ \phi_{22}) - \overline{\theta})q^a \circ \phi_{22} - t^a \circ \phi_{22} - \frac{p_{12}}{p_{22}} \frac{\frac{\delta_a^T}{1+\delta_a^T+\underline{\nu}_a^T}}{1 + \frac{p_{11}p_{22}-p_{12}^2}{p_{12}p_{22}} \frac{\delta_a^T}{1+\delta_a^T+\underline{\nu}_a^T}} q^a \circ \phi_{22}$$
$$+ (P(Q \circ \phi_{22}) - \overline{\theta})q^b \circ \phi_{22} - t^b \circ \phi_{22} - \frac{p_{12}}{p_{22}} \frac{\frac{\delta_b^T}{1+\delta_b^T+\underline{\nu}_b^T}}{1 + \frac{p_{11}p_{22}-p_{12}^2}{p_{12}p_{22}} \frac{\delta_b^T}{1+\delta_b^T+\underline{\nu}_b^T}} q^b \circ \phi_{22}.$$

11.5.2 The collusion-proofness constraints

Denoting $\epsilon^i = \frac{\delta_i^T}{1+\delta_i^T+\underline{\nu}_i^T}$ and writing that the identity function is the optimal manipulation function for the third party we obtain the collusion-proofness constraints: A regulatory contract will be collusion- proof if it satisfies the following set of constraints.

- For a coalition $(\theta^a = \underline{\theta}, \theta^b = \underline{\theta})$

$$(\underline{\theta}, \underline{\theta}) \to (\overline{\theta}, \underline{\theta}) \quad \pi_{11}^a + \pi_{11}^b \geq \pi_{21}^a + \pi_{21}^b + \Delta\theta q_{21}^a$$
$$(\underline{\theta}, \underline{\theta}) \to (\underline{\theta}, \overline{\theta}) \quad \pi_{11}^a + \pi_{11}^b \geq \pi_{12}^a + \pi_{12}^b + \Delta\theta q_{12}^b$$
$$(\underline{\theta}, \underline{\theta}) \to (\overline{\theta}, \overline{\theta}) \quad \pi_{11}^a + \pi_{11}^b \geq \pi_{22}^a + \pi_{22}^b + \Delta\theta q_{22}^a + \Delta\theta q_{22}^b.$$

- For a coalition $(\theta^a = \underline{\theta}, \theta^b = \overline{\theta})$

$$(\underline{\theta}, \overline{\theta}) \to (\overline{\theta}, \overline{\theta}) \quad \pi_{12}^a + \pi_{12}^b \geq \pi_{22}^a + \pi_{22}^b + \Delta\theta q_{22}^a - \frac{p_{11}}{p_{12}} \epsilon^b \Delta\theta (q_{22}^b - q_{12}^b)$$
$$(\underline{\theta}, \overline{\theta}) \to (\underline{\theta}, \underline{\theta}) \quad \pi_{12}^a + \pi_{12}^b \geq \pi_{11}^a + \pi_{11}^b - \Delta\theta q_{11}^b - \frac{p_{11}}{p_{12}} \epsilon^b \Delta\theta (q_{11}^b - q_{12}^b)$$
$$(\underline{\theta}, \overline{\theta}) \to (\overline{\theta}, \underline{\theta}) \quad \pi_{12}^a + \pi_{12}^b \geq \pi_{21}^a + \pi_{21}^b + \Delta\theta q_{21}^a - \Delta\theta q_{21}^b - \frac{p_{11}}{p_{12}} \epsilon^b \Delta\theta (q_{21}^b - q_{12}^b).$$

- For a coalition $(\theta^a = \overline{\theta}, \theta^b = \underline{\theta})$

$$(\overline{\theta}, \underline{\theta}) \to (\overline{\theta}, \overline{\theta}) \quad \pi_{21}^a + \pi_{21}^b \geq \pi_{22}^a + \pi_{22}^b + \Delta\theta q_{22}^b - \frac{p_{11}}{p_{12}} \epsilon^a \Delta\theta (q_{22}^a - q_{21}^a)$$
$$(\overline{\theta}, \underline{\theta}) \to (\underline{\theta}, \underline{\theta}) \quad \pi_{21}^a + \pi_{21}^b \geq \pi_{11}^a + \pi_{11}^b - \Delta\theta q_{11}^a - \frac{p_{11}}{p_{12}} \epsilon^a \Delta\theta (q_{11}^a - q_{21}^a)$$
$$(\overline{\theta}, \underline{\theta}) \to (\underline{\theta}, \overline{\theta}) \quad \pi_{21}^a + \pi_{21}^b \geq \pi_{12}^a + \pi_{12}^b - \Delta\theta q_{12}^a + \Delta\theta q_{12}^b - \frac{p_{11}}{p_{12}} \epsilon^a \Delta\theta (q_{12}^a - q_{21}^a).$$

- For a coalition $(\theta^a = \overline{\theta}, \theta^b = \overline{\theta})$

$$(\overline{\theta},\overline{\theta}) \to (\underline{\theta},\overline{\theta}) \quad \pi^a_{22} + \pi^b_{22} \geq \pi^a_{12} + \pi^b_{12} - \Delta\theta q^a_{12} - \frac{p_{12}}{p_{22}}\Delta\theta \sum_{i=a,b}[f(\epsilon^i)(q^i_{12} - q^i_{22})]$$
$$(\overline{\theta},\overline{\theta}) \to (\overline{\theta},\underline{\theta}) \quad \pi^a_{22} + \pi^b_{22} \geq \pi^a_{21} + \pi^b_{21} - \Delta\theta q^b_{21} - \frac{p_{12}}{p_{22}}\Delta\theta \sum_{i=a,b}[f(\epsilon^i)(q^i_{21} - q^i_{22})]$$
$$(\overline{\theta},\overline{\theta}) \to (\underline{\theta},\underline{\theta}) \quad \pi^a_{22} + \pi^b_{22} \geq \pi^a_{11} + \pi^b_{11} - \Delta\theta (q^a_{11} + q^b_{11}) - \frac{p_{12}}{p_{22}}\Delta\theta \sum_{i=a,b}[f(\epsilon^i)(q^i_{11} - q^i_{22})].$$

11.5.3 The quantities at the collusion-proof symmetric equilibria

Let us consider the problem of regulator R^a. We denote by $\underline{\delta}^a$, $\overline{\nu}^a$, $\lambda^a(2,0)$, $\lambda^a(1,0,b)$ and $\lambda^a(1,0,b)$ the Lagrange multipliers associated to the constraints $BIC^a(\underline{\theta})$, $BIR^a(\overline{\theta})$, $CPC(2,0)$, $CPC(1,0,a)$ and $CPC(1,0,b)$ respectively.

Optimizing the Lagrangean of R^a's maximisation problem with respect to the four ex post rents associated to F^a yields the following conditions,

$$\begin{aligned}
&-p_{11} + \underline{\delta}^a p_{11} + \lambda^a(2,0) = 0\\
&-p_{12} + \underline{\delta}^a p_{12} + \lambda^a(1,0,a) = 0\\
&-p_{12} - \underline{\delta}^a p_{11} + \overline{\nu}^a p_{12} + \lambda^a(1,0,b) = 0\\
&-p_{22} - \underline{\delta}^a p_{12} + \overline{\nu}^a p_{22} - \lambda^a(1,0,a) - \lambda^a(1,0,b) - \lambda^a(2,0) = 0
\end{aligned}$$

Having only four first order conditions to determine five multipliers, they are expressed as functions of $\underline{\delta}^a$. The Lagrance multipliers of the binding constraints for regulator R^a are $\overline{\nu}^a = 1 + \frac{p(\underline{\theta})}{p(\overline{\theta})}$, $\underline{\delta}^a = \underline{\delta}^a$, $\lambda^a(1,0,a) = p_{12}(1-\underline{\delta}^a)$, $\lambda^a(1,0,b) = p_{11}\underline{\delta}^a - p_{12}\frac{p(\underline{\theta})}{p(\overline{\theta})}$ and $\lambda^a(2,0) = p_{11}(1-\underline{\delta}^a)$.

To ensure the positivity of the Lagrange multipliers $\underline{\delta}$ must belong to interval $[\frac{p_{11}}{p_{12}}\frac{p(\underline{\theta})}{p(\overline{\theta})}, 1]$.

We will show below that the equilibrium quantities satisfy $q^a_{22} \geq q^a_{21}$ (and $q^b_{22} \geq q^b_{12}$) which implies that R^a (respectively R^b) finds it optimal to set ϵ^a (respectively ϵ^b) equal to 1 in order to decrease the cost associated to $CPC(1,0,b)$ (respectively $CPC(1,0,a)$). At a symmetric equilibrium, we need to have $\lambda^a(1,0,a) = \lambda^a(1,0,b)$, since the two associated constraints are identical. This implies $\underline{\delta}^a = \frac{p_{12}}{\rho + p_{12}} \in [\frac{p_{11}}{p_{12}}\frac{p(\underline{\theta})}{p(\overline{\theta})}, 1]$. The symmetric equilibrium is therefore completely defined. At a symmetric equilibrium, $\pi^a_{11} = \Delta\theta q^a_{22}$, $\pi^a_{12} = \Delta\theta q^a_{22} - \Delta\theta\frac{p_{11}}{p_{12}}(q^a_{22} - q^a_{21})$, $\pi^a_{21} = \pi^a_{22} = 0$.

We can then optimize the program of regulator R^a with respect to her quantities. We obtain the optimal quantities. Direct computations show that $q^a_{12} \geq q^a_{11} \geq q^a_{22} \geq q^a_{21}$ and $q^b_{21} \geq q^b_{11} \geq q^b_{22} \geq q^b_{12}$.

We must then check that all the constraints are satisfied at the symmetric equilibria. Individual constraints are satisfied since

$$\begin{aligned}
BIC^a(\overline{\theta}) &\Leftrightarrow 0 \geq p_{11}p_{22}(q^a_{21} - q^a_{22}) + (p_{12})^2(q^a_{22} - q^a_{11}) + p_{22}p_{12}(q^a_{22} - q^a_{12})\\
BIC^b(\overline{\theta}) &\Leftrightarrow 0 \geq p_{11}p_{22}(q^b_{12} - q^b_{22}) + (p_{12})^2(q^b_{22} - q^b_{11}) + p_{22}p_{12}(q^b_{22} - q^b_{21}),
\end{aligned}$$

which is satisfied given the ranking of the equilibrium quantities, and

$$BIR^a(\underline{\theta}) \Leftrightarrow p_{12}q^a_{22} + p_{11}q^a_{21} \geq 0$$
$$BIR^b(\underline{\theta}) \Leftrightarrow p_{12}q^b_{22} + p_{11}q^b_{12} \geq 0.$$

Moreover, in a symmetric equilibrium, the remaining collusion-proofness constraints boil down to

$$\begin{aligned}
&2\pi^a_{11} \geq \pi^a_{12} + \pi^a_{21} + \Delta\theta q^a_{21} \\
&\pi^a_{12} + \pi^a_{21} \geq 2\pi^a_{11} - \Delta\theta q^a_{11} - \frac{p_{11}}{p_{12}}\Delta\theta(q^a_{11} - q^a_{21}) \\
&2\pi^a_{22} \geq \pi^a_{12} + \pi^a_{21} - \Delta\theta q^a_{12} - \frac{p_{12}}{p_{22}}\Delta\theta f(1)(q^a_{12} + q^a_{21} - 2q^a_{22}) \\
&2\pi^a_{22} \geq 2\pi^a_{11} - 2\Delta\theta q^a_{11} - 2\frac{p_{12}}{p_{22}}\Delta\theta f(1)(q^a_{11} - 2q^a_{22}).
\end{aligned}$$

Replacing the rents by their values at the symmetric equilibrium, the previous inequalities are equivalent to

$$\begin{aligned}
&0 \geq \Delta\theta\frac{p(\underline{\theta})}{p_{12}}(q^a_{22} - q^a_{21}) \\
&0 \geq -\Delta\theta(1 + \frac{p_{11}}{p_{12}})(q^a_{11} - q^a_{22}) \\
&0 \geq \Delta\theta[-(1 + \frac{p_{12}}{p_{22}}f(1))(q^a_{12} - q^a_{22}) - (\frac{p_{11}}{p_{12}} - \frac{p_{12}}{p_{22}}f(1))(q^a_{22} - q^a_{21})] \\
&0 \geq -\Delta\theta(1 + \frac{p_{12}}{p_{22}}f(1))(q^a_{11} - q^a_{22}).
\end{aligned}$$

The first constraint is satisfied since $q^a_{22} \geq q^a_{21}$ at the equilibrium. The second and fourth constraints are also satisfied since $q^a_{11} \geq q^a_{22}$. The third constraint (4) is satisfied since $q^a_{12} \geq q^a_{22}$, $q^a_{22} \geq q^a_{21}$, and $\frac{p_{11}}{p_{12}} - \frac{p_{12}}{p_{22}}f(1) \geq 0$. Indeed $f(1) \leq 1$ so that $\frac{p_{11}}{p_{12}} - \frac{p_{12}}{p_{22}}f(1) \geq \frac{p_{11}}{p_{12}} - \frac{p_{12}}{p_{22}} = \frac{\rho}{p_{12}p_{22}}$, which is positive.