Altruism with Endogenous Labor Supply[*]

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Abstract

This paper proposes a model of altruism with endogenous labor supply. A full characterization of the family's choices of consumption and leisure is provided. Initially, work effort is assumed to be publicly observed; this assumption is later relaxed, allowing for privately observed actions. It is shown that the "redistributive neutrality" property commonly associated with altruism holds only with respect to non-labor income sources. Failing to control for labor income amounts to an inadequate specification of empirical tests of the neutrality hypothesis. Further, when effort is privately observed, the need to convey incentives causes neutrality to break down entirely.

Keywords: Altruism, endogenous labor supply, private information, redistributive neutrality.

JEL Classification: D13, D82, J22.

1. Introduction

In the altruism literature, it has been frequently postulated that the distribution of resources within the family should not affect the allocation of consumption across family members (see, for example, Altonji, Hayashi and Kotlikoff [1]). This conclusion, the very essence of Ricardian Equivalence (Barro [5] and Becker [6]), stems

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[*] Section 5 borrows considerably from my University of Chicago dissertation.

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from the fact that, in altruistic families, resources are pooled and family members face a common budget constraint. Consequently, as long as total resources are constant, the family may attain its preferred consumption allocation and the way in which the total is decomposed into parcels, representing the contribution of individual family members, should not matter. The invariance of familial resource allocation to the distribution of resources within the family has been empirically tested in a number of ways. One implication of the theory, for families engaging in financial transfers, is that taking one dollar from the transfer recipient's income and adding it to the donor's income should be compensated by an increment of exactly one dollar in the initial transfer. Therefore, the initial consumption allocation would not be displaced. This implication will be referred throughout as "redistributive neutrality." Contrary to the predictions of the theory, empirical evidence has unanimously rejected such a fact: while transfers display the pattern predicted by altruism, flowing from richer family members to poorer ones, they do not neutralize income redistribution (for the most complete treatment on this issue, see Altonji et al. [3]).

This paper proposes a model of altruism with endogenous labor supply. First, I revise the standard labor-leisure choice of an individual consumer. This consumer later impersonates the "child" member of the altruistic family, once joined together with the parental figure and decision maker of the Barro-Becker literature. Following the Barro-Becker approach, I initially confine all decision making ability to the parent: he may choose consumption and the time to spent at work of all family members. The introduction of endogenous labor supply brings an important qualification to the implications of income redistribution within the family. In fact, the neutrality of income redistribution applies only with respect to non-labor income sources: exogenous income sources are perfect substitutes in providing utils; changes in wage rates, on the other hand, induce readjustments in labor force participation. To the extent that family members earn different wages and spend different amounts of time in the labor market, a one dollar change in their labor income induces different adjustments in terms of labor force participation. This fact, alone, would suffice to break "redistributive neutrality." It is worth noting that this holds regardless of whether or not utility is separable in consumption and leisure, as changes in wages still induce adjustments in labor choices in the separability case.

I proceed by relaxing the centralized decision making ability, allowing the child to chose her preferred effort level and simultaneously preventing the parent from imposing negative transfers on the child. I analyze the strategic interaction between parent and child and characterize the unique Nash-equilibrium of this static game. The outcome of the game with two decision makers replicates the Barro-Becker choices when transfers are strictly positive. When transfers are zero, the child chooses effort just like the typical consumer deciding between labor and leisure, and the parent simply consumes his endowment. When positive transfers take place, the model maintains the neutrality result concerning redistribution of non-labor income sources, provided transfers are still positive after redistribution takes place. As before, income redistribution with respect to wages is an ill-defined experiment. When transfers are zero, income redistribution is effective: taking one dollar from one family member and giving it to another, when transfers remain at zero, implies that this additional dollar will be entirely spent in higher consumption and less working time by the recipient. Other implications of this model include differences in the wage and income elasticities of consumption and labor supply, across transfer regimes. For example, the model predicts a less elastic response of labor supply to changes in the child's income when transfers are positive.

Finally, I consider the possibility that the child's effort is privately observed by her alone. As is standard in agency theory, I model the child's income as being drawn from a probability distribution conditional on the effort level she exerted. It is shown that income redistribution is not neutral, in this case. In fact, different realizations of the child's income convey different inference concerning the likelihood that the child put in the effort level which the principal (the parent) wishes to implement. This different inference is the source of neutrality breakdown: in the redistribution experiment of comparing the initial consumption allocation to one where the child's income is subtracted of one dollar and the parent's income is incremented by the same amount, the differences in the child's income additionally convey differences in information. In this sense, they are not neutral.

The last section of the paper is devoted to confronting results in the empirical literature with the predictions of the model. I make two different sets of remarks. Under the assumption that families have — in the real world — the ability to adjust labor supply in response to income and wage changes, testing redistributive neutrality by the vague experiment of comparing familial resource allocation before and after income redistribution is an inadequate procedure. As stated above, endogenous labor supply implies that this experiment is only a proper test of neutrality when applied to non-labor income sources. Consequently, unless the income source is correctly specified and wages are held fixed in the comparison, one should not expect neutrality to hold. The models proposed in this paper provide an expression relating the actual empirical estimates and the parameter of interest, from the point of view of redistributive neutrality. These two numbers differ only when labor supply responds to changes of wages and income. As shown below, it is possible that the empirical rejection of redistributive neutrality stems from a sufficiently large labor supply elasticity with respect to the wage rate. The other set of comments concerns additional reasons for properly defined tests of neutrality to reject the null. In the setup of the agency model of section 5, changes in the child's total income can be interpreted as changes in her wage rate. Even in this case, when changes in "wages" are not associated with adjustments in the labor supply, it is shown that the typical trade-off between insurance and incentives causes neutrality to break down. Under mild assumptions, the theoretical prediction for transfer responses after income redistribution (transfers are expected to compensate only partially the one dollar redistribution experiment) corresponds, at least qualitatively, to the empirical findings.

2. The Child's Problem

In this section, I present the labor-leisure choice faced by a consumer who will later impersonate the "child" member of the altruistic family. Independent consideration of the child's problem makes clear what assumptions on the child's utility function are needed to require normality of consumption and leisure, say, and what role these assumptions will play later, when the family is enlarged to consider the parent.

2.1. The Model

All variables in this section have subscript c, indicating they refer to the "child." Although such a distinction is unnecessary here, this is intended to make notation consistent across sections. The child has one unit of time she may devote to work or leisure. Time spent at work is denoted e_c whereas the child's consumption is written c_c, $c_c \in R_+$. Preferences are given by U_c:

$$U_c = u(c_c; 1 - e_c);$$

where $u(\cdot)$ is C^2, strictly increasing and strictly concave with respect to both arguments. The direct utility function $u(\cdot)$ is also assumed to verify, for $e \in [0; 1]$, $u_1(0; 1 - e) = \infty$, $\lim_{c \to \infty} u_1(c; 1 - e) = 0$, and for $c \in R_+$, $u_2(c; 0) = \infty$.

When going into the labor market, the child receives wage w_c. She also has exogenous income I_c. Her problem is then to

$$\max_{e_c \in [0,1]} u\left(I_c + w_c e_c, 1 - e_c\right).$$

The first-order condition is:

$$u_1\left(I_c + w_c e_c, 1 - e_c\right) w_c - u_2\left(I_c + w_c e_c, 1 - e_c\right) \leq 0. \tag{2.1}$$

Although there is nothing to preclude the optimal choice of labor time to be not to work at all ($e_c = 0$), I will assume throughout that this corner solution is not reached so that (2.1) always holds at equality. Formally ignoring the corner solution makes the formulation of the familial decision problem, in the sections to follow, much less cumbersome (as well as the proofs concerning the properties of optimal transfers, consumption and effort). For the purpose of analyzing the response of parental transfers to changes in income or wages, taken up below, this carries no loss in generality.

Concerning the child's utility function, I make the following assumptions:

Assumption 1 For all $(I_p, w_c) \in \mathbb{R}^2_{++}$ and all effort levels $e_c \in [0, 1)$, i) $u_{11} w_c^2 - 2u_{12} w_c + u_{22} < 0$, ii) $u_{21} - u_{11} w_c > 0$, iii) $u_{12} w_c - u_{22} > 0$.

Part i) of assumption 1 ensures that the child's second-order conditions are satisfied (condition i) is sufficient for a maximum). Assumption 1, parts i) and ii) ensure that leisure is a normal good, while i) and iii) deliver the normality of consumption.

Assumption 2 For all $(I_p, w_c) \in \mathbb{R}^2_{++}$ and all effort levels $e_c \in [0, 1)$, $(u_{21} - u_{11} w_c) e_c - u_1 \leq 0$.

Assumption 2 ensures that work effort varies positively with the wage. In turn, this guarantees that consumption is also higher when the wage is higher.

The previous assumptions are, in a sense, too restrictive, since they have been imposed as global conditions. In fact, assuming that leisure and consumption are normal goods, for example, should be a local condition, holding "close" to optimal choices of effort. In the statement of assumptions 1 and 2, however, it has been assumed that the required local conditions also hold for any possibly non-optimal choice of effort. I have chosen this restrictive conditions for simplicity. The global form of assumptions 1 and 2 will only be used to ensure that the solutions to the optimization problems in sections 2 and 3 are unique, as well as in the proof of lemma 4.3 (to deliver uniqueness and stability of the Nash-equilibrium of the game played between parent and child). For all other results, “local” statements of these assumptions suffice.

The following lemmas are straightforward implications of the properties of the utility function $u(\cdot)$ and the previous assumptions.

Lemma 2.1. The optimal effort choice $e_c = e_c(I_c; w_c)$ is a continuously differentiable function in all arguments[1]. Moreover, $e_{c;1} < 0$ and $e_{c;2} > 0$.

Lemma 2.2. The optimal consumption choice $c_c(I_c; w_c)$ is a continuously differentiable function in all arguments. Moreover, $c_{c;1} > 0$ and $c_{c;2} > 0$.

Expressions for the derivatives of effort and consumption with respect to income and wages are given in the appendix.

3. The Barro-Becker Model with Effort

This section looks back at the model of the family introduced by Barro [5] and Becker [6], extensively used in the altruism literature[2]. In a static environment, I enlarge the model to include non-separable effort choices. The model is used to analyze how the single-decision maker — the parental figure of this altruism literature — divides total income among family members, and the properties of the inherent transfers which put the consumption allocation in place. The novelty, in this section, is the ability conveyed to the same decision maker of selecting the effort intensity of family members, and the effect of work intensity choices on the optimal transfer, consumption and effort allocations.

The results of this section will be used as a benchmark with which to compare the properties of familial equilibrium allocations with those emerging from more elaborate models of familial interaction, discussed in sections 4 and 5.

[1] If effort had been allowed not to be strictly positive, then for some income values and wage rates, the function describing the optimal choice of effort would have a kink. Consequently, it would only be differentiable away from those income and wage pairs. The derivatives presented here can be interpreted as the derivatives of the more general effort function for income and wage rates such that the optimal choice of hours is strictly positive. Since optimal consumption choices inherit the properties of effort, the same remark applies to the statement of the next lemma. This is also true of the results presented in lemmas 3.1 through 3.4, and 4.1.

[2] For an excelent survey of the altruism literature, see Laitner [15].

3.1. The Model

Consider a family formed of parent and child, the child being the individual described in section 2. For simplicity, it is assumed that only the child works (consequently, only the child suffers disutility from work). The subscript p indicates parental variables. Let the constant λ take values in $[0.5, 1]$. Given a familial consumption pair (c_p, c_c), and the child's effort e_c, the parent's total utility U_p is:

$$U_p = \lambda U(c_p) + (1 - \lambda) u(c_c, 1 - e_c),$$

where the direct utility function $U(\cdot)$ is C^2, strictly increasing and strictly concave. Further, it is assumed that $U'(0) = \infty$ and $\lim_{c \to \infty} U'(c) = 0$. The properties of the child's direct utility function $u(\cdot)$ have been stated above.

The parent receives exogenous income I_p whereas the child's total endowment is the sum of the exogenous component I_c and the labor payments $w_c e_c$. Given I_p, I_c and the market wage w_c, the parent chooses the (possibly negative) amount of resources he may transfer to the child, denoted T_p, as well as the child's working hours, e_c. The child's consumption is then:

$$c_c = I_c + w_c e_c + T_p, \tag{3.1}$$

while the parent consumes

$$c_p = I_p - T_p. \tag{3.2}$$

The transfer and working hours solve

$$\max_{T_p, e_c} \lambda U(I_p - T_p) + (1 - \lambda) u(I_c + w_c e_c + T_p, 1 - e_c).$$

First-order conditions are:

$$\lambda U'(I_p - T_p) = (1 - \lambda) u_1(I_c + w_c e_c + T_p, 1 - e_c) \tag{3.3}$$

$$u_1(I_c + w_c e_c + T_p, 1 - e_c) w_c = u_2(I_c + w_c e_c + T_p, 1 - e_c). \tag{3.4}$$

Let $T_p(I_p, I_c, w_c)$ and $e_c(I_p, I_c, w_c)$ denote solutions to (3.3) and (3.4). From the properties of $U(\cdot)$ and $u(\cdot)$, the optimal choices of transfers and effort are continuously differentiable functions of the arguments (I_p, I_c, w_c). Substituting the optimal choices in (3.1) and (3.2), we get the corresponding familial consumption choices, $c_c(I_p, I_c, w_c)$ and $c_p(I_p, I_c, w_c)$.

Let A denote the Hessian matrix associated with the parental problem. This matrix and its determinant are derived in the appendix. I make the following assumptions:

Assumption 3 For all $(I_p; I_c; w_c) \in R^3_{++}$, all transfers $T_p \in R$, $T_p < I_p$, and all effort levels $e_c \in [0; 1)$, $u_{11}u_{22} - u_{12}^2 > 0$.

Assumption 3 serves two purposes. Together with assumption 1, part i), it ensures that $|A| > 0$. From the properties of $u(\cdot)$ and $U(\cdot)$, we know that the system of first-order conditions (3.3) and (3.4) has a solution. $|A|$ being positive implies that this solution is a maximizer. From the fact that these assumptions hold globally (for all endowment and wage rate values as well as effort choices), it follows that the solution is also unique. Assumption 3 additionally ensures that parental consumption is a normal good.

Assumption 4 For all $(I_p; I_c; w_c) \in R^3_{++}$, all transfers $T_p \in R$, $T_p < I_p$, and all effort levels $e_c \in [0; 1)$, $-u_1((1-\lambda)u_{11} + \lambda U'') - \lambda U'' e_c (u_{11}w_c - u_{21}) > 0$.

Assumption 4 ensures that the parent will choose longer working hours for the child when her wage goes up.

Assumption 5 For all $(I_p; I_c; w_c) \in R^3_{++}$, all transfers $T_p \in R$, $T_p < I_p$, and all effort levels $e_c \in [0; 1)$, $\lambda U''[(u_{22} - u_{12}w_c) e_c - u_1 w_c] - u_1 u_{12}(1-\lambda) > 0$.

Assumption 5 ensures that the child's consumption goes up with her wage.

The comment made in section 2 concerning the global and, therefore, restrictive nature of the assumptions stated there applies to assumptions 3 through 5, as well.

3.2. Results

The following properties of the family's optimal choices are formally shown in the appendix.

Lemma 3.1. The optimal parental transfer $T_p(I_p; I_c; w_c)$ is a continuously differentiable function in all arguments, with $T_{p;1} > 0$, $T_{p;2} < 0$ and $T_{p;3} < 0$. Moreover, $T_{p;1} - T_{p;2} = 1$.

Lemma 3.2. The child's optimal effort $e_c(I_p; I_c; w_c)$ is a continuously differentiable function in all arguments, with $e_{c;1} < 0$, $e_{c;2} < 0$ and $e_{c;3} > 0$. Moreover, $e_{c;1} = e_{c;2}$.

Lemma 3.3. Optimal parental consumption $c_p = c_p(I_p; I_c; w_c)$ is a continuously differentiable function in all arguments, with $c_{p;1} > 0$, $c_{p;2} > 0$ and $c_{p;3} > 0$. Moreover, $c_{p;1} = c_{p;2}$.

Lemma 3.4. The child's optimal consumption $c_c = c_c(I_p; I_c; w_c)$ is a continuously differentiable function in all arguments, with $c_{c;1} > 0$, $c_{c;2} > 0$ and $c_{c;3} > 0$. Moreover, $c_{c;1} = c_{c;2}$.

The fact that $T_{p;1} - T_{p;2} = 1$ holds implies that, for a given wage w_c, only the total $(I_p + I_c)$ matters for familial resource allocation. For a constant w_c, the effect of subtracting one dollar from the child's exogenous income I_c and adding it to the parent's I_p will be a transfer increment of exactly the same amount, leaving familial consumption and effort choices unchanged. Consequently, redistribution of exogenous income across family members does not affect the optimal choices of consumption and effort. In other words, I_p and I_c are perfect substitutes in providing utils. This is confirmed by the fact that the derivatives of consumption and effort are identical with respect to either source of exogenous income (e.g. $c_{c;1} = c_{c;2}$).

The property $T_{p;1} - T_{p;2} = 1$ is the redistributive neutrality result. Since wages typically induce adjustments in labor force participation, there is no reason to expect consumption and effort choices to remain constant after wage variation. A detailed discussion of this issue is provided in section 6.2.

It is important to point out that the result $T_{p;1} - T_{p;2} = 1$ is obtained simply by differentiating the system of first-order conditions (3.3) and (3.4). Consequently, it only relies on the optimality of that solution, delivered by assumption 1, part i), and assumption 3. The same applies to the results $e_{c;1} = e_{c;2}$, $c_{p;1} = c_{p;2}$ and $c_{c;1} = c_{c;2}$.

4. A Family with Two Decision Makers

The Barro-Becker model had all the decision making ability concentrated on the parent. The purpose of this section is to modify this feature of the choice process. Here, I model the interaction between parent and child in a strategic form, as a static game[3]. The child decides on the amount of hours to work and the parent chooses transfers. I maintain the utility functions just as before, so that it is also

[3] Lindbeck and Weibull [16] first modelled the interaction between two altruistic agents as a game.

natural now to prevent the parent from imposing negative transfer upon a selfish child.

It is shown that this static game has a unique Nash-equilibrium. Moreover, as a function of the exogenous income pairs and wage rate $(I_p; I_c; w_c)$, the outcome of this game between parent and child replicates exactly the choices of the Barro-Becker framework, when transfers are strictly positive. When transfers are zero, the child continues to act like the decision maker of section 2 and the parent simply consumes the totality of his endowment.

4.1. The Model

The setup of the model is the same of section 3, and all the variables defined similarly. For convenience, the expressions for the parent and the child's total utility are reproduced below:

$$U_p = \lambda U(c_p) + (1 - \lambda)\, u(c_c; 1 - e_c),$$

$$U_c = u(c_c; 1 - e_c).$$

Parent and child interaction is now modelled as a non-cooperative, static game. The parent chooses a non-negative transfer to the child, T_p, also constrained not to exceed parental resources: $T_p \in [0; I_p]$. The child's action is the amount of time to spend at work, e_c, constrained to lie in the unit interval: $e_c \in [0; 1]$. Given income values $(I_p; I_c)$, wage rate w_c and actions $(T_p; e_c)$, consumption values are determined as before. The game $\Gamma(I_p; I_c; w_c) = \{\{p; c\}; \{T_p; e_c\}\, \{U_p; U_c\}\}$ is now fully specified.

Nash-equilibria of this game are given by $(T_p; e_c)$ pairs such that:

$$T_p \in \arg\max_{T_p} \left\{ \lambda U\left(I_p - T_p\right) + (1 - \lambda)\, u\left(I_c + w_c e_c + T_p; 1 - e_c\right) \right\}$$

$$e_c \in \arg\max_{e_c} \{u(I_c + w e_c + T_p; 1 - e_c)\}:$$

The optimal choices of parent and child, conditioning on the other player's action, solve the following first-order conditions:

$$-\lambda U'(I_p - T_p) + (1 - \lambda)\, u_1(I_c + w_c e_c + T_p; 1 - e_c) \le 0; \tag{4.1}$$

where (4.1) holds at equality when $T_p > 0$, and

$$u_1(I_c + w_c e_c + T_p; 1 - e_c)\, w_c = u_2(I_c + w_c e_c + T_p; 1 - e_c): \tag{4.2}$$

Comparison of (4.1) and (4.2) with (3.3) and (3.4) reveals that the choices of effort and transfers, when the latter are positive, are exactly the same as those in the centralized decision maker model of section 3. Consequently, as long as transfers are strictly positive, the outcome of this game (the Nash-equilibrium effort and transfer choices) displays the same "local" properties as functions of familial income and wage rate as the optimal choices emerging in the Barro-Becker model. This is to say that, for small changes in (I_p, I_c, w_c), where "small" indicates that transfers remain positive after the changes takes place, equilibrium transfers, consumption and effort respond to these changes just like the optimal choices in the Barro-Becker model did.

When transfers are zero, the child is solving the problem described in section 2. In fact, the child's best-response function, characterized by (4.2), coincides with the child's effort choice in the model of section 2, given in (2.1), when transfers are set to zero. The following results follow directly from (4.1) and (4.2).

Lemma 4.1. The child's best response function, $e_c(I_p, I_c, w_c, T_p) = e_c(I_c + T_p, w_c)$, is continuously differentiable in all arguments. Moreover, $e_{c,1} < 0$, $e_{c,2} > 0$.

Lemma 4.2. The parent's best response function $T_p(I_p, I_c, w_c, e_c)$ is continuous on all its arguments and $T_{p,1} \geq 0$, $T_{p,2} \leq 0$, $T_{p,3} \leq 0$ and $T_{p,4} \leq 0$. When $T_p > 0$, the previous inequalities hold strictly and $T_{p,1} - T_{p,2} = 1$. Moreover, there is $\bar{I}_p$ such that $T_p(I_p, I_c, w_c, e_c) = 0$, for $I_p \leq \bar{I}_p$, and $T_p(I_p, I_c, w_c, e_c) > 0$, for $I_p > \bar{I}_p$.

Proof. Set $\bar{I}_p$ as follows:

$$\lambda U'\left(\bar{I}_p\right) = (1 - \lambda)\, u_1\left(I_c + w_c e_c, 1 - e_c\right).$$

From the properties of $U(\cdot)$ and $u(\cdot)$, $\bar{I}_p$ exists and is unique. The rest of the proof follows from differentiating (4.1), when $T_p > 0$, and from comparative statics, otherwise. ■

Lemma 4.2 states that positive transfers preserve the redistributive neutrality property form the Barro-Becker model. Since parental transfers compensate for the redistribution of exogenous sources of income which maintain $(I_p + I_c)$ constant, the child's best response is also not to modify the effort choice prevailing before redistribution took place. Consequently, when transfers are positive, only the total $(I_p + I_c)$ affects consumption and effort outcomes, for a given wage rate w_c.

The following result is shown in the appendix.

Lemma 4.3. The game $\Gamma(I_p; I_c; w_c)$ has a unique, stable Nash-equilibrium $(T_p; e_c)$ in pure strategies. The Nash-equilibrium transfer and e¤ort outcomes coincide with those of the Barro-Becker model, when $T_p > 0$.

5. Private Information

In the previous sections, I have analyzed the properties of familial resource allocation assuming that time spent at work is observed by parent and child. In this section, I drop the assumption that the time the child spends working, e_c, is public: in the current setting, work e¤ort is assumed to be privately observed by the child.

As it is standard in agency theory, the child's e¤ort determines the distribution of her income, higher e¤ort shifting the probability distribution towards higher income values. Putting in more e¤ort bene…ts the child in that her income will be higher, in expectation. It also bene…ts the altruistic parent, who transfers resources to the child as a function of income di¤erentials: the child's higher expected income will require lower transfers from the parent, in expectation. In this sense, the child's e¤ort can be partly interpreted as a "transfer" to the parent. Since e¤ort is costly and the child is sel…sh, she has no interest in bene…tting the parent in any way. The assumptions below will make it such that the own bene…ts from higher expected income will not su¢ ce to induce the child to put in the high e¤ort level, when she is receiving a transfer schedule similar to that of sections 3 and 4. As a consequence, the parent will have to modify this transfer schedule in order to induce higher e¤ort from the child. Because the model deals with a static setup, this deviation from the transfer schedule of previous sections is not credible: the parent always prefers to resort to the transfer arrangement of the Barro-Becker model, after output is realized. In fact, being able to deviate from it involves commitment ability, on the part of the parent.

Chami [11] and Gatti [13] have also considered private information in the context of the family. Chami compares the bene…ts for the parent of committing to a …xed payment to the child — upon observing the outcome of her e¤ort — with the transfer menu preferred by the parent without commitment. Gatti analyzes several di¤erent transfer arrangements and their implications for the child's choice of e¤ort. The current section focuses on the properties of the transfer schedule under parental commitment. Speci…cally, I will show that when e¤ort has to be induced, the familiar insurance/incentives trade-o¤ causes the "redistributive neutrality" of transfers to break-down.

5.1. The model

The child receives income stream $I_c \in R_+$. Her income is drawn from probability distribution $f(I_c; e_c)$, defined over $B(R_+)$, where e_c denotes the child's work time or effort level. For simplicity, it is assumed that effort can take values in $E = \{e_H; e_L\}$, $e_H > e_L$. The probability distribution $f(\cdot; e_H)$ first-order stochastically dominates $f(\cdot; e_L)$. Parental income $I_p \in R_+$ is now also assumed to be random and distributed according to probability density function $\mu(\cdot)$, defined over $B(R_+)$. I_p is assumed to be statistically independent from I_c.

The timing is as follows[4]. The child puts in effort level $e_c \in E$. Endowments $(I_p; I_c)$ are drawn from $\mu(\cdot) f(\cdot; e)$. After observing the endowment realizations, the parent provides a non-negative transfer $T_p(I_c; I_p)$ to the child. Given endowments and transfer decision, familial consumption is given by:

$$c_p = I_p - T_p(I_p; I_c)$$

and

$$c_c = I_c + T_p(I_p; I_c).$$

The preferences of family members are the same as before and reproduced below for convenience:

$$U_p = \lambda U(c_p) + (1 - \lambda) u(c_c; 1 - e_c);$$

$$U_c = u(c_c; 1 - e_c).$$

In this section, the constant λ is constrained to the interval $(0.5; 1]$ (see subsection 5.4).

Since transfers are given after the child's income is realized, the optimal transfer choice is given by:

$$-\lambda U'(I_p - T_p(I_p; I_c)) + (1 - \lambda) u_1(I_c + T_p(I_p; I_c); 1 - e_c) \leq 0, \tag{5.1}$$

for all $(I_p; I_c)$ pairs, with strict equality when $T_p(I_p; I_c) > 0$. In a static context, therefore, the transfer schedule given in (5.1) is the only credible transfer

[4] Although, strictly speaking, there a timing issue associated with the setup of moral hazard models, with the agent moving first and income being draw afterwards, this carries no implications for the analysis of the current section. For simplicity alone, I will maintain the simpler concept and notation of a Nash-equilibrium throughout, as opposed to that of subgame perfection.

arrangement the parent could announce. In order to give private information an interesting role, I will assume that the parent would like to commit to a different transfer payment, one that maximizes his expected utility. This if formally stated in assumptions 6 and 7, below.

Let $E_{I_p,I_c|e}$ denote the expectations operator induced by $\mu(\cdot)\, f(\cdot, e)$. The transfer schedule $T_p(I_p, I_c)$ satisfies (5.1), when $e_c = e_L$. Then we have:

Assumption 6 The direct utility functions $U(\cdot)$ and $u(\cdot)$, the constant λ, the probability distributions $f(\cdot, e_H)$, $f(\cdot, e_L)$ and $\mu(\cdot)$ are such that

$$E_{I_p,I_c|e_L} u(I_c + T_p(I_p, I_c), 1 - e_L) > E_{I_p,I_c|e_H} u(I_c + T_p(I_p, I_c), 1 - e_H). \quad (5.2)$$

Assumption 7 The direct utility functions $U(\cdot)$ and $u(\cdot)$, the constant λ, the probability distributions $f(\cdot, e_H)$, $f(\cdot, e_L)$ and $\mu(\cdot)$ are such that there exists a transfer schedule $\hat{T}(I_p, I_c)$ satisfying the following two conditions:

$$E_{I_p,I_c|e_H} u\left(I_c + \hat{T}_p(I_p, I_c), 1 - e_H\right) \geq E_{I_p,I_c|e_L} u\left(I_c + \hat{T}_p(I_p, I_c), 1 - e_L\right) \quad (5.3)$$

and

$$E_{I_p,I_c|e_H} \left\{\lambda U\left(I_p - \hat{T}_p(I_p, I_c)\right) + (1-\lambda) u\left(I_c + \hat{T}_p(I_p, I_c), 1 - e_H\right)\right\} > \quad (5.4)$$
$$E_{I_p,I_c|e_L} \{\lambda U(I_p - T_p(I_p, I_c)) + (1-\lambda) u(I_c + T_p(I_p, I_c), 1 - e_L)\}.$$

Equation (5.2) states that the child prefers the low effort level when the parent implements the transfer mechanism given in (5.1)[5]. In turn, (5.4) says that, under $\hat{T}_p(\cdot)$, the child weakly prefers to work hard. From (5.4), we know that the parent prefers transfer mechanism $\hat{T}_p(I_p, I_c)$ with hard work to $T(I_p, I_c)$, with low effort. In the remainder of this section, I will use $T^*_p(I_p, I_c)$ to denote the best compensation scheme, from the parent's perspective, satisfying the conditions in assumption 7.

[5] Assumption 6 is not entirely general in the following sense. The condition expressed in (5.1) depends on the effort level the child exerts. Consequently, transfers should depend on e_c: $T_p(I_p, I_c, e_c)$. In a more general way, assumption 6 should ensure that the child's expected utility under low effort and transfer payments $T_p(I_p, I_c, e_L)$ exceeds her expected utility under high effort and $T_p(I_p, I_c, e_H)$. This could be incorporated in assumption 6 all though I have not done so for notational simplicity.

In what follows, I will confine attention to pure effort strategies. Since (5.4) holds strictly, from the continuity of $U(\cdot)$ and $u(\cdot)$ it can be shown that there exists a compensation schedule $\hat{T}_p^{\varepsilon}(I_p, I_c)$ such that the parent still prefers $\hat{T}_p^{\varepsilon}(I_p, I_c)$ to $T(I_p, I_c)$ and the child is strictly better off selecting e_H.

The transfer $T_p(I_p, I_c)$, given in (5.1), is a dominant strategy for the parent, among all possible compensation arrangements. From (5.2), the child will select low effort once faced with this payment arrangement. The following result follows immediately.

Lemma 5.1. The static game with private information has a unique Nash-equilibrium in pure strategies, characterized by $(e_L, T_p(I_p, I_c))$, where $T_p(I_p, I_c)$ solves (5.1).

From (5.4), the parent would like to be able to commit to the transfer mechanism $\hat{T}_p(I_p, I_c)$, in order to induce e_H. In the choice of $\hat{T}_p(I_p, I_c)$, however, the incentive compatibility constraint (5.3) will bind (if private information is to have a role in this problem). In turn, since (5.3) — which conveys the incentives to high effort — was not taken into account when deriving $T_p(I_p, I_c)$, we expect $\hat{T}_p(I_p, I_c) \neq T_p(I_p, I_c)$, for some (I_p, I_c).

The transfer schedule $T_p(I_p, I_c)$ displays the same redistributive neutrality property stated in lemmas 3.1 and 4.2. When transfers are strictly positive and income redistribution between parent and child leaves the family's total income unchanged, optimal transfer will offset redistribution, leaving familial consumption intact. The transfer schedule $\hat{T}_p(I_p, I_c)$, however, is constrained to satisfy the incentive compatibility constraint, equation (5.3), in order to induce high effort. When this constraint binds, it no longer follows that income redistribution is neutral to the optimal allocation of resources, for $\hat{T}_p(I_p, I_c)$ carries the well-know trade-off between insurance and incentives. This will be formally shown in the next section.

5.2. Some Properties of Transfer Schedule $\hat{T}_p(I_p, I_c)$

Here it is shown how the parental transfer $\hat{T}_p(I_p, I_c)$ is affected by income redistribution which leaves total familial resources unchanged.

As stated above, $\hat{T}_p(I_p, I_c)$ is the transfer arrangement which maximizes the parent's expected utility, subject to the child (weakly) preferring to work hard. Formally, $\hat{T}_p(I_p, I_c)$ solves:

$$\max_{\hat{T}_p(I_p, I_c) \geq 0} E_{I_p I_c | e_H} \left\{ \lambda U\left(I_p - \hat{T}_p(I_p, I_c)\right) + (1-\lambda) u\left(I_c + \hat{T}_p(I_p, I_c), 1 - e_H\right) \right\}$$

subject to the child's incentive compatibility constraint, equation (5.3).

Pointwise maximization with respect to $T_p(\cdot)$ yields:

$$-\lambda U'(c_p) + u_1(c_c, 1-e_H)\left[(1-\lambda) + \mu\left(1 - \frac{u_1(c_c, 1-e_L)}{u_1(c_c, 1-e_H)}\frac{f(I_c, e_L)}{f(I_c, e_H)}\right)\right] \leq 0. \tag{5.5}$$

where the above expression holds at equality for $\tilde{T}_p(I_p, I_c) > 0$. The non-negative number μ is the Lagrange multiplier associated with (5.3). Denote by $F(I_c)$ the ratio of probabilities $f(I_c, e_L)/f(I_c, e_H)$, know as the likelihood ratio. Similarly, let $U_1(c_c)$ denote the ratio of marginal utilities $u_1(c_c, 1-e_L)/u_1(c_c, 1-e_H)$. For (I_c, I_p) pairs such that transfers are strictly positive, we may rewrite (5.5) as:

$$\lambda U'(c_p) = u_1(c_c, 1-e_H)\left[(1-\lambda) + \mu(1 - U_1(c_c)F(I_c))\right]. \tag{5.6}$$

When $\mu = 0$ (the incentive compatibility constraint does not bind), we naturally recover the transfer rule given in (5.1). For $\mu > 0$, departures from the preferred parental transfer arrangement occur when the product $U_1(c_c)F(I_c)$ is different than 1. The likelihood ratio $F(\cdot)$ has a familiar interpretation. The term $U_1(\cdot)$ corrects the statistical inference associated with $F(\cdot)$ by transforming it into marginal utility units. When $u_{12} > 0$, $U_1(c_c) > 1$, implying that the consequences of shirking (taking effort e_L rather than e_H) are penalized more acutely. This is so since taking the low effort level implies that the child's marginal utility is higher than when she works hard. Conversely, when $u_{12} < 0$, evidence of shirking is not punished so severely since it reduces the marginal utility from consumption. When the child's utility function is separable in consumption and leisure, $U_1(\cdot) = 1$, and this factor vanishes.

In order to see how the optimal transfer $\tilde{T}_p(I_p, I_c)$ is affected by income redistribution, I will perform the following experiment. Take an income pair (I_p, I_c) for which transfers are strictly positive and consider subtracting $\delta > 0$ from the child's income, and giving this amount to the parent. Assume that $\tilde{T}_p(I_p + \delta, I_c - \delta) > 0$. Fully differentiating (5.6) and rearranging, we get:

$$d\tilde{T}_p(I_p, I_c) = \left(1 - \frac{u_1(c_c, 1-e_H)\,\mu U_1(c_c)F'(I_c)}{D}\right)\delta, \tag{5.7}$$

where D, the denominator in the previous expression, is given by:

$$D = \lambda U''(c_p) + u_{11}(c_c, 1-e_H)\left[(1-\lambda) + \mu(1 - U_1(c_c))F(I_c)\right]$$
$$- u_1(c_c, 1-e_H)\,\mu U_1'(c_c)F(I_c).$$

In this case, therefore, neutrality does not hold: $dT_p(I_p; I_c) \neq 1$. The sign of the ratio in (5.7) hinges on the signs of $F'(\cdot)$ and $U_1'(\cdot)$. $F'(\cdot) < 0$ is a sufficient condition for $f(\cdot; e_H)$ to first-order stochastically dominate $f(\cdot; e_L)$. The derivative of the ratio $U_1(\cdot)$ does not seem to have a particularly meaningful interpretation. Having $F'(\cdot) < 0$ and $U_1'(\cdot) > 0$ is sufficient for $dT_p(I_p; I_c) < 1$. This implies that transfers less than fully compensate for income redistribution, a result that has been consistently found in the empirical literature, as discussed in section 6.

5.3. Interpretation

It was shown that the transfer schedule under commitment does not have the redistributive neutrality property. The driving forces of this result are the fact that the child's income is endogenous — in the sense that the distribution of income depends on her choice of action, the non-obervability of effort — so that parental transfers do not have effort as an argument, and the parent' ability to commit. In previous sections, although the child also had to decide how much time to spend working, wages and income were exogenously given to both family members. Because the parent cares about the child in a non-distortionary way, conditioning on a transfer amount, parent and child would always agree on the amount of time the child would spend in the labor market. Consequently, the parent would not use any commitment device, even if he had access to such a possibility: the transfer schedule he would choose under commitment would coincide with that one provided in sections 3 and 4. In the current setup, higher effort has a component of transfer from the child to the parent. The (rather standard) assumptions imposed above make it such that parent and child's preferred effort levels are different. In other words, the incentive compatibility constraint binds. Additionally, if effort was observed, the parent would design a transfer schedule including effort as an argument. Given the child's effort, the parent's preferred transfer schedule (with or without commitment ability) would verify redistributive neutrality[6]. When effort is not observed, transfers depend only on observable income. Still, without commitment, once output has been realized, the parent's preferred transfer schedule verifies redistributive neutrality. It is the parent's ability to commit that enables him to implement a transfer schedule yielding a higher

[6] This requires the additional assumption that the transfer schedule with redistributive neutrality provides the child with expected utility at least as high as that of receiving no transfers at all.

expected utility. Neutrality breaks down, in this case, due to the typical trade-o¤ between insurance and incentives from agency theory: income realizations are perceived di¤erent (therefore, non-neutral) according to the likelihood that high versus low e¤ort was associated with each individual outcome.

While the parent's ability to commit is unappealing in a static setup, it can be more easily justi…ed in view of repeated interaction between parent and child. However, a …nite horizon may lead to unraveling and the repetition of the static dominant strategies, whereas dynamic games with in…nite horizons are know to have many subgame perfect equilibria. Also, although parental commitment would not have altered the outcome of the Barro-Becker model (since the parent would want to induce the e¢cient e¤ort choice from the child), the child's e¤ort choice would be di¤erent if she had commitment ability, both in the Barro-Becker setup and the private information model. Consequently, one may question the pertinence of the speci…c transfer schedule $T^1(I_p; I_c)$ proposed above. While this choice is certainly arbitrary, the properties of $T^1(I_p; I_c)$ are consistent with a number of empirical regularities discussed below.

The endogenous consideration of labor supply — sections 3 and 4 — showed that "redistributive neutrality" is an ill-de…ned experiment with respect to changes in labor income. In fact, family members adjust their labor supply decisions once faced with changes in wages. Consequently, there is no reason to expect resource allocation to be invariant to such modi…cations. The standard private information setup involves a single e¤ort decision associated with di¤erent realizations of income. In this context, changes in the child's income are not associated with changes in "wages" and subsequent labor supply adjustment. Neutrality was shown to break down due to the di¤erent "information" associated with di¤erent income realizations.

Realistically, the distribution of most sources of income (including wage income) depends on privately observed actions. As such, the insurance/incentives trade-o¤ examined in this section is likely to characterize intra-family transfers in general, including transfers targeted towards variation in labor income. In [12], I suggested "controlling" for private information by examining whether truly exogenous events, such as accidents or one's house burning down, trigger a di¤erent transfer response compared to ordinary changes in income. This issue is taken up in section 6.2, below.

5.4. Private Information and Altruism

It is important to notice that asymmetric information has the potential to affect familial resource allocation only to the extent that family members have conflicting preferences over consumption and leisure. Consider, for example, two-sided altruism, where the parent cares about the child and the child cares about the parent. One could model the child's preferences as

$$U_c = (1 - \lambda)\, U_p + \lambda\, u\,(c_c; 1 - e_c)\,;$$

so that the child attaches weight λ to her direct utility from consumption and leisure, and the complement $1 - \lambda$ to the parent's utility from consumption. When altruism is partial (family members value their direct utility more than the utility of relatives), λ exceeds 0.5. However, strong altruism, with $\lambda = 0.5$, implies that parent and child are unanimous in their choice of consumption and leisure. Consequently, the fact that the child's effort is privately observed would have no bearing on the family's consumption and effort choices. In other words, no incentive compatibility constraint would bind if parent and child had identical preferences.

6. Statistical Model

In this section, I write down the statistical model corresponding to the Barro-Becker model, presented in section 3. The model is then confronted with actual empirical estimates from the literature.

6.1. The Model

Previous sections provided a stylized theory of familial transfers and resource allocation. The models implicitly considered a "representative" family, since no mention was made to household specific characteristics. When looking at data, it is imperative to take such diversity into account. This section describes the implications of the Barro-Becker model for transfer data from a cross-section survey panel, explicitly considering household heterogeneity as described by demographic elements. Given assumptions on how demographic variables affect household choices, the model of section 3 enables full specification of the features of transfer and consumption data which would emerge from a cross-section sample of households.

Consider a statistical population of interest, for example the households living in the US, in year t. Since we want to eventually look into familial transfers, it is convenient to specialize the population of interest by confining attention to households linked by kinship. In particular, I will simplify the analysis by assuming that the population can be divided into parent and adult-child household pairs. Each parent and child household pair forms a data point of the population of interest.

Households differ according to demographic variables such as the number of household members, the age, sex and marital status of its members. The vector of demographic characteristics for family i in the population, in year t, will be denoted $X_{it} = (X_{pi;t}; X_{ci;t})$. This vector contains the demographic characteristics of the parental household, $X_{pi;t}$, as well as the child's, $X_{ci;t}$. Wage rates $W_{it} = (w_{pi;t}; w_{ci;t})$ and exogenous familial income $I_{it} = (I_{pi;t}; I_{ci;t})$ are other sources of diversity across households. Finally, households differ with respect to unobservables, subsumed in u_{it}.

Demographic characteristics, wages, exogenous income sources and unobservables are distributed in the population of families according to probability distribution function $p(X_t; W_t; I_t; u_t)$. The data available is assumed to be a random draw from the population of interest, consisting of N parent-child observations of $(X; W; I)$ for each sample year.

Let T_p^* denote the desired transfer from parent to child, the optimal transfer choice from the Barro-Becker as described in section 3. Omitting subscripts i and t, I assume that desired transfers for families in the population, have the following representation[7]:

$$T_p^* = \alpha + \beta_1 I_p + \beta_2 I_c + \gamma_1 w_p + \gamma_2 w_c + \delta_p X_p + \delta_c X_c + u \qquad (6.1)$$

The linear representation of transfers presented in (6.1) is not a limitation since non-linear functional forms can be easily accommodated by including additional regressors in the powers of the explanatory variables.

[7] The model of section 4 did not consider the choice of parental labor supply. By including the parent's wage in the empirical equation (6.1), I am considering here the more realistic generalization of the model, with parents participating in the labor market, earning wage w_p. I have derived all the results concerning how transfers, time at work and familial consumption vary with income and wages for this model, when utility is separable in consumption and leisure and parent and child have the same momentary utility function. The results are simply generalizations, in the natural way, of those presented in lemmas 3.1 through 3.4. Interesting extensions include $\partial e_p/\partial w_c < 0$ and $\partial e_c/\partial w_p < 0$.

From the altruism model with two decision makers, desired transfers are observed when actual transfers are positive. That is, if T_p is the actual parental transfer to the child, $T_p^* = T_p$ whenever $T_p > 0$.

In order to estimate the parameters of interest, one possible approach is to make an assumption concerning the distribution of the error component u. Given the censored transfer data, a Tobit model seems especially appropriate. One could therefore assume that u is normally distributed, $u \sim N(0; \sigma^2)$, and proceed to estimate the parameters in (6.1).

So far, I have implicitly assumed that $(\beta_1; \beta_2)$ (as well as $(\gamma_1; \gamma_2)$) are identical across households. They are functions of the coefficient of partial altruism λ, among other utility parameters. However, one would expect the degree of altruism to vary across families. Also, since the data concerns different families, u is likely to be heteroskedastic. Tobit estimation procedures can be modified to accommodate both random coefficients and heteroskedasticity. A more flexible estimator, robust to the functional form relating the utility of family members to consumption and to observed and unobserved preference characteristics, has been derived in Altonji and Ichimura [4] and used in Altonji et al. [3]. I will maintain the standard Tobit model as the benchmark procedure since it has been extensively used in most of the empirical work I wish to address.

The model of section 3 of course suggests that $\beta_1 > 0$ and $\beta_2 < 0$: a wealthier parent wishes to provide a higher transfer while the desired transfer is lower towards a less needy child. In fact, β_1 corresponds to the Barro-Becker transfer derivative with respect to parental income, $\partial T_p / \partial I_p$. Similarly, β_2 is the empirical model equivalent of $\partial T_p / \partial I_c$. The redistributive neutrality of the model without asymmetric information additionally imposes $\beta_1 - \beta_2 = 1$. With respect to wages, the model predicts $\gamma_1 > 0$ and $\gamma_2 < 0$.

6.2. Results from the Empirical Literature

A substantial part of recent empirical work on altruism has devoted attention to the properties of financial and time transfers between parents and their adult children. Some examples of this literature include Altonji et al. [2], [3], McGarry and Schoeni [17], [18], Cox [8], Cox and Raines [9] and Cox and Rank [10]. Of particular interest, from the point of view of altruism, is the concern about whether or not financial transfers are increasing in the income of the donor and decreasing in the recipient's. Another empirically examined property of transfers, presumed to hold under the null hypothesis of altruism, is redistributive neutrality.

The properties of transfers have been analyzed using versions of equation (6.1) of the following form:

$$T_p^* = a + b_1 I_p^S + b_2 I_c^S + d_1 X_p + d_2 X_c + v; \qquad (6.2)$$

where the superscript S indicates total income: the sum of labor and non-labor income. The child's total income, in the notation of section 3, is then $I_c^S = I_c + w_c e_c$. Thorough empirical experimentation has estimated 6.2 using several different possibilities for the income variables, including current and permanent income. All the references cited above have found evidence that the probability of a transfer being provided depends positively and in a significant way in the donor's income, and depends significantly on the recipient's income, although with a negative coefficient. Concerning amounts given, with the exception of Cox [8], Cox and Raines [9], and Cox and Rank [10][8], actual transfers were found to display the same sign pattern as the probability that one was given.

In all the work cited here, although the topic is only seriously considered in Altonji et al. [3], reference has been made to the redistributive neutrality test. Redistributive neutrality has been interpreted as the statement that the difference between the transfer derivatives with respect to parent and child's income should equal unity. Using the notation of the test equation above, this translates into $b_1 - b_2 = 1$.

In section 3, when characterizing the properties of transfers, it was stressed that income redistribution within the family was neutral with respect to resource allocation only when non-labor income redistribution was considered. Under the assumption that parent and child can adjust their working hours in response to changes in wages or exogenous income sources, redistribution of labor-income is not a well-defined experiment, in general. In fact, if parent and child face different wage rates, they will engage in different adjustments of their labor force participation when faced with changes in their labor income. This is so even when utility is separable in consumption and leisure as changes in income or wages still induce adjustments in labor force participation in that particular case[9].

[8] See Fernandes [12] on estimation procedures — generalized Tobit — and potential problems with the datasets used in [8], [9] and [10].

[9] Although there is no reason to expect neutrality with respect to labor income, when labor supply is endogenously determined, I tried to check whether $\partial T_p/\partial I_c = \partial T_p/\partial w_c$, for some functional forms. For log utility, with $U(c) = \log(c)$ and $u(c; 1-e) = \log(c) + \log(1-e)$, the previous equality does indeed hold. For more general specifications, with $U(c) = c^{\alpha}$ and $u(c; 1-e) = c^{\alpha} + (1-e)^{\beta}$, $\alpha \neq \beta$, $\partial T_p/\partial I_c \neq \partial T_p/\partial w_c$.

The Barro-Becker model (section 3), enables us to relate the parameter of interest concerning redistributive neutrality, the coefficient β_2 in equation (6.1), with the parameter actually estimated, b_2, from equation (6.2). The child's total income relates to labor income as follows:

$$I_c^S = I_c + w_c e_c.$$

Suppose that the exogenous component of I_c^S is very small, so that most of the changes in I_c^S are due to changes in wages and labor force subsequent adjustment[10]. Then, changes in I_c^S relate to changes in the wage rate w_c as follows:

$$dI_c^S = \left[1 + \frac{w_c}{e_c}\frac{\partial e_c}{\partial w_c}\right] e_c dw_c. \tag{6.3}$$

Let η stand for the elasticity of labor supply with respect to changes in wage:

$$\eta = \frac{w_c}{e_c}\frac{\partial e_c}{\partial w_c}.$$

Then, we may rewrite (6.3) as follows:

$$dw_c = \frac{dI_c^S}{(1+\eta)\, e_c}. \tag{6.4}$$

From the model of section 3, desired transfers depend on the wage rate as well as on the exogenous income components, I_p and I_c. Consider the expression for the derivative of parental transfers with respect to the child's wage, derived in equation (B.13)[11]:

$$\frac{\partial T_p}{\partial w_c} = \left[\frac{\partial T_p}{\partial I_c} - \frac{\partial e_c}{\partial I_c}\frac{u_1}{\lambda U'' e_c}\right] e_c.$$

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[10] Whether or not I_c is small does not affect the substance of the results presented here, while simplifying the exposition.

[11] The more general expression, derived for the model when the parent also participates in the labor market, is as follows. The parental utility function $U(c_p)$ is now replaced by $u(c_p) + v(1 - e_p)$, where e_p denotes time spent at work by the parent and $u(\cdot)$ and $v(\cdot)$ have the usual properties. The child's utility function $u(c_c; 1 - e_c)$ is specialized to assume separability and equals the functional form of the parent's utility function, just provided. Then:

$$\frac{\partial T_p}{\partial w_c} = \left[\frac{\partial T_p}{\partial I_c} - \frac{1-\lambda}{\lambda}\frac{u_p'' u_c'}{e_c}\frac{\partial e_c}{\partial I_c} - \frac{w_p u_c'}{v_c'' e_c}\frac{\partial e_p}{\partial I_c}\right] e_c;$$

with u_p'' denoting the second derivative of the parent's utility from consumption, and similarly for the other terms. Considering instead the simpler expression provided in (6.5) is consistent with the Barro-Becker model presented in section 3 and does not qualitatively alter the subsequent discussion.

Using (6.4), we get:

$$\frac{\partial T_p}{\partial I_c^S} = \left[\frac{\partial T_p}{\partial I_c} - \frac{\partial e_c}{\partial I_c} \frac{u_1}{\lambda U'' e_c} \right] \frac{1}{(1+\eta)}. \tag{6.5}$$

Leaving aside the implications of using the functional form in (6.2) to estimate the transfer function in (6.1), one may think of the number given in (6.5) as the expression that was actually estimated[12]. Recall that the redistributive neutrality property applies to the term $\partial T_p / \partial I_c$. In fact, the model of an altruistic parent and his child predicts $\partial T_p / \partial I_p - \partial T_p / \partial I_c = 1$. Using the notation of the test equations, we may rewrite (6.5) as:

$$\frac{\partial T_p}{\partial I_c^S} = \left[\beta_2 - \frac{\partial e_c}{\partial I_c} \frac{u_1}{\lambda U'' e_c} \right] \frac{1}{(1+\eta)} \approx b_2.$$

It is worth comparing the actual estimate b_2 with β_2. The term in brackets is more negative than $\partial T_p / \partial I_c$, from the assumption that leisure is normal. On the other hand, to the extent that the elasticity of labor supply is positive (negative), this reduces (raises) the magnitude of actual estimates. If the effect of the labor supply elasticity dominates, in the sense of outweighing the effect of the second parcel of (6.5), then b_2 will be strictly smaller than β_2, in absolute value. Since neutrality tests have been performed by computing the difference $\partial T_p / \partial I_p^S - \partial T_p / \partial I_c^S$, and the estimates of $\partial T_p / \partial I_c^S$ have been found to be negative, "compressed" estimates of $\partial T_p / \partial I_p^S$ and $\partial T_p / \partial I_c^S$ due to the dampening effect of the labor supply wage elasticity could help explain the very low value of the "test" results, which have been found to be significantly below unity. Although estimates of male labor supply wage elasticities tend to be negative (around -0.1), female labor supply elasticities are positive and more elastic (around 0.2)[13]. The number η represents the wage elasticity of the child's household. As such, when head and spouse are present, it will not correspond exactly to any of these estimates but instead it reflects their joint hours and labor market participation response to wage changes.

The introduction of private information in the model proposed in section 5 was shown to cause neutrality to break down. As argued above, the distribution of most sources of income (including wage income) is likely to depend on privately

[12] The actual estimate, without the assumption that I_c is small, would be an weighted average of the coefficient presented in (6.5) and $\partial T_p / \partial I_c$.

[13] See Borjas [7], pp. 68.

observed actions. Consequently, the insurance/incentives trade-off will also be present in the transfer response to income changes. Observing transfer responses targeted to exogenous events could provide both a test of the quantitative relevance of asymmetric information as well as a test of the Barro-Becker model of altruism.

Villanueva [19] uses the PSID to test a version of equation (6.2), augmented to consider income losses due to involuntary unemployment, as well as income losses due to disability. In a Tobit regression (see table 14 of [19]), the derivative of parental transfers associated with a one dollar reduction in the income of the child's household head is 11 cents, when a dummy is included to control for income loss due to a layoff. The coefficient associated with the dummy variable is 0.2. Consequently, income loss due to a layoff raises the total transfer from 11 to approximately 30 cents, for a one dollar reduction in the child's income. When a dummy is included for income loss due to disability, instead, the initial transfer derivative of 12 cents is raised by 18 cents, again totalling 30 cents. These numbers imply that the transfer derivative goes up by a factor of 2.5-3, once "asymmetric information" is taken into account.

Going back to the expression for $\partial T_p / \partial I_c^S$, a lower bound on η, the wage elasticity of labor supply, may obtained as follows. Assumed that the response of labor to income is negligible: $\partial e_c / \partial I_c \simeq 0$. Then:

$$b_2 = \frac{\beta_2}{1+\eta}.$$

Assume further that the wage elasticity of labor supply is identical in the households of parent and child. Under these assumptions:

$$b_1 - b_2 = \frac{\beta_1 - \beta_2}{1+\eta}.$$

Villanueva's estimates of $b_1 - b_2$, corrected for "asymmetric information," are about 0.33. A similar magnitude is obtained by multiplying estimates of $b_1 - b_2$, from Altonji et al. [3], by the private information 2.5 "correcting factor." Consequently, we have:

$$0.33 = \frac{\beta_1 - \beta_2}{1+\eta}.$$

In order to square these numbers with redistributive neutrality ($\beta_1 - \beta_2 = 1$), we need $\eta \simeq 2$.

Indirect evidence on private information is additionally provided in Jensen [14]. He analyzes the response of remittances from family members who migrated from rural communities to cities, how these remittances respond to di¤erent income sources. He …nds that the derivative of transfers with respect to income receipts from public programs exceeds, by orders of magnitude, the transfer derivative when other income sources change[14]. This corroborates the possibility that incentives are an important part of transfer response to income variation.

From the parent's best-response function in the model of two decision makers, the transfer choice which solves equation (4.1), we obtain transfers as functions of e¤ort as well as the child's total income. With e¤ort …xed, transfers display the redistributive neutrality property: the parent would compensate a one dollar reduction in the child's total income matched with a one dollar increment in his own income by raising transfers exactly one dollar. Consequently, tests of redistributive neutrality could be performed by enlarging the test equation (6.2) to include (instruments for) the time spent in the labor market by parent and child, respectively e_p and e_c. There is an attempt to control for time at work in some of the empirical work cited here. In both papers by McGarry and Schoeni, [17] and [18], the regressors include dummies for the cases in which the child is working full time or when she is not working/missing. For the parent, a dummy is included for data points where the head/spouse is not employed. The authors report that the results reject redistributive neutrality. In Altonji et al. [2], dummy variables for hours in unemployment (two categorical measures) are included for parent and child. They also report the rejection of redistributive neutrality.

In Altonji et al. [1], redistributive neutrality was tested using consumption data. Under the assumption that altruistic family members pool resources, their marginal utility from consumption would be common across family members. It could, therefore, be estimated as a …xed e¤ect. Altonji et al. regress individual consumption of family members on the family's total income, the income of the particular individual corresponding to that consumption information, as well as on a vector of demographic values. In order to take the endogenous choice of working hours into account, I would stress the results they present in Table 4, where the wage rates of husband and wife are controlled for. Still, as in all the other estimates they present, the coe¢ cient on non-labor income is positive and signi…cant. This result corroborates the empirical failure of redistributive

[14] The lowest ratio of the transfer derivative in response to a reduction in pension income (presumably publicly observed) over the transfer derivative in response to reductions in total familial income is about 2. However, he provides estimates in which this ratio is as high as 10.

neutrality.

6.3. Additional Empirical Implications

Other testable implications of the Barro-Becker model include the fact that income and wage elasticities vary with the transfer regime. Consider, for example, the response of the child's effort e_c when her exogenous income I_c changes. When transfers are zero, the child will respond to higher individual income — under a normality assumption — by reducing her labor supply and increasing consumption. When transfers are positive, the child's higher income will be "shared" by the parent in the form of a reduced transfer. In fact, assuming that parental consumption is also a normal good, the parent will reduce his initial transfer. In net terms (from the assumption that the child's consumption and leisure are normal), the child's total resources after transfers are still higher than before the change in I_c; however, they are lower than the initial increment in I_c by the amount corresponding to the increment in parental consumption. Consequently, the reduction in the child's labor supply in response to identical changes in I_c will be smaller when she receives transfers from the parent than otherwise.

This can be formally shown by comparing the derivatives $\partial e_c/\partial I_c$ when transfers are positive and when they are zero. Let $\partial e_c^0/\partial I_c$ denote the change in effort when no transfers are provided, and let $\partial e_c^+/\partial I_c$ denote the effort response when transfers are positive. Then, from (A.1), we get:

$$\frac{\partial e_c^0}{\partial I_c} = \frac{u_{21} - u_{11}w_c}{u_{11}w_c^2 - 2u_{12}w_c + u_{22}};$$

while, from (B.5) we find:

$$\frac{\partial e_c^+}{\partial I_c} = \frac{(u_{12} - u_{11}w_c)\,\lambda U''}{\lambda U''(u_{11}w_c^2 + u_{22} - 2u_{12}w_c) + (1-\lambda)(u_{11}u_{22} - u_{12}^2)}:$$

Rearranging,

$$\frac{\partial e_c^+}{\partial I_c} = \frac{(u_{12} - u_{11}w_c)}{(u_{11}w_c^2 + u_{22} - 2u_{12}w_c) + \frac{(1-\lambda)}{\lambda U''}(u_{11}u_{22} - u_{12}^2)}$$

and it follows that:

$$\left|\frac{\partial e_c^0}{\partial I_c}\right| > \left|\frac{\partial e_c^+}{\partial I_c}\right|:$$

Similarly, the child's consumption response is more elastic when she receives no parental transfers:

$$\frac{\partial c_c^0}{\partial I_c} > \frac{\partial c_c^+}{\partial I_c}.$$

The response of labor supply to wage changes is ambiguous, however:

$$\frac{\partial e_c^0}{\partial w_c} \gtreqless \frac{\partial e_c^+}{\partial w_c}.$$

Again, the difference in elasticities across transfer regimes can be empirically tested.

7. Conclusion

This paper has stressed the role of endogenous labor supply in the resource allocation of altruistic families. A full characterization of the family's choices has been provided. Initially, work effort was assumed to be public. This assumption was later relaxed to consider effort choices privately observed by individual family members.

It was shown that endogenous labor supply has important implications for the consequences of income redistribution within the family. In particular, given the allocative implications of wage rates for the choice of labor supply, income redistribution involving changes in labor income alters the family's consumption and leisure choices and, consequently, it is not neutral. Income redistribution involving exogenous income sources alone still displays "redistributive neutrality," as long as work effort is publicly observed. In fact, when effort is privately known only by the worker, redistributive neutrality breaks down entirely.

The results derived here were used to confront empirical estimates in the literature. I have claimed that, to the extent that families — in the real world — adjust labor supply to changes in income or wages, the empirical evidence does not necessarily lead to a rejection of altruism. Moreover, even when the ability to adjust labor supply is small, by causing redistributive neutrality to break-down, asymmetric information still suggests qualitative implications that correspond to the empirical findings.

Endogenous consideration of labor supply also suggests a number of testable empirical implications. In fact, the model predicts that consumption and labor supply elasticities vary with the transfer regime prevailing in the family. Stated differently, when intra-family transfers are positive, labor supply and consumption choices in one household depend on the income and wages of members of other kinship related households. One example is the less elastic response of labor supply and consumption to changes in income, when transfers are positive. Other empirical implications concern the di¤erent response of transfers with respect to hazardous events, such as personal illness or accident, versus ordinary changes in labor income.

Summing up, this paper contributes to a broader understanding of the implications of altruism for resource allocation by providing a complete description of the altruistic family's choices when labor supply is endogenously determined. It also provides a better understanding of the consequences of redistributing income across cohorts. The e¤ects of income redistribution are at the core of a number of extremely important policy issues, namely whether or not Ricardian Equivalence holds or the e¤ectiveness of welfare programs. While is it know that Ricardian Equivalence holds only when taxes and subsidies are lump-sum, by making explicit the consequences of income redistribution for labor supply, this paper helps assess the deviations from Ricardo's neutrality result for a policy involving (distortionary) personal income taxation. Similarly, the model helps us understand the consequences for labor supply and transfer decisions triggered in family members who were providing …nancial help to relatives, if their relatives become welfare recipients.

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A. The Child's Problem

I characterize here the child's choice of effort, when she acts as a single decision maker, not receiving transfers from the parent. The properties of the implied consumption values are also presented. Recall that corner solutions ($e_c = 0$) have been ruled out, for simplicity.

Fully differentiating the child's first-order condition, we get:

$$\frac{\partial e_c}{\partial I_c} = \frac{u_{21} - u_{11} w_c}{u_{11} w_c^2 - 2u_{12} w_c + u_{22}} < 0; \qquad \text{(A.1)}$$

where the inequality follows from assumption 1, parts i) and ii). As for changes in the wage rate,

$$\frac{\partial e_c}{\partial w_c} = \frac{(u_{21} - u_{11} w_c) e_c - u_1}{u_{11} w_c^2 - 2u_{12} w_c + u_{22}} > 0; \qquad \text{(A.2)}$$

the inequality following from assumption 1, part i), and assumption 2.

Given the properties of $u(\cdot)$, the optimal effort choice $e_c(I_c; w_c)$ is a continuously differentiable function on all arguments. If corner solutions had been allowed, then for income and wage rates $(I_c; w_c)$ such that the first-order condition (2.1) holds at equality with exactly zero hours at work, the optimal choice function $e_c(I_c; w_c)$ would have a kink. Consequently, it would only be differentiable away from those income and wage pairs.

The child's consumption is derived from her resource constraint:

$$c_c(I_c; w_c) = I_c + w_c e_c(I_c; w_c).$$

We have:

$$\frac{\partial c_c}{\partial I_c} = -\frac{u_{12} w_c - u_{22}}{u_{11} w_c^2 - 2u_{12} w_c + u_{22}} > 0;$$

where the inequality follows from assumption 1, parts i) and ii). Changes in the wage rate affect consumption as follows:

$$\frac{\partial c_c}{\partial w_c} = \frac{e_c\left(u_{22} - u_{12}w_c\right) - u_1 w_c}{u_{11}w_c^2 - 2u_{12}w_c + u_{22}} > 0;$$

the inequality following from assumption 1, parts i) and iii). Optimal consumption $c_c(I_c; w_c)$ is also a continuously differentiable function of all arguments. If $e_c = 0$ had been allowed, this function would have kinks for the $(I_c; w_c)$ pairs described above.

B. The Barro-Becker Model with Effort

Here, I derive the Hessian matrix of the Barro-Becker model presented in section 3.

B.1. The Hessian Matrix

Let A be the Hessian matrix associated with the Barro-Becker model, $A = \{a_{ij}\}_{i;j=1;2}$. A is as follows:

$$A = \begin{bmatrix} (1-\lambda)u_{11} + \lambda U'' & (1-\lambda)(u_{11}w_c - u_{12}) \\ u_{11}w_c - u_{12} & u_{11}w_c^2 + u_{22} - 2u_{12}w_c \end{bmatrix}:$$

From the properties of the direct utility functions $u(\cdot)$ and $U(\cdot)$, $a_{11} < 0$. After some rearranging, the determinant of A is

$$|A| = \lambda U''\left(u_{11}w_c^2 + u_{22} - 2u_{12}w_c\right) + (1-\lambda)\left(u_{11}u_{22} - u_{12}^2\right):$$

B.2. Proof of Lemmas

Proof of lemmas 3.1 through 3.4.

In the text, it was stated that only $e_c > 0$ would be considered, for simplicity. Then, the properties of $u(\cdot)$ and $U(\cdot)$ ensure that $T_p(I_p; I_c; w_c)$ and $e_c(I_p; I_c; w_c)$ are continuously differentiable functions of all arguments. This properties carries over to c_p and c_c, given in (3.2) and (3.1).

Transfer and effort properties follow from fully differentiating the system of first-order conditions with respect to I_p; I_c and w_c. We get:

$$\frac{\partial T_p}{\partial I_p} = \frac{u_{11}w_c^2 + u_{22} - 2u_{12}w_c}{|A|}\lambda U'' > 0; \qquad \text{(B.1)}$$

where the inequality follows from assumption 1, part i), and assumption 3.

$$\frac{\partial T_p}{\partial I_c} = -\frac{(1-\lambda)}{|A|}\left(u_{22}u_{11} - u_{12}^2\right) < 0; \tag{B.2}$$

the inequality following from assumptions 3 and 5.

$$\frac{\partial T_p}{\partial w_c} = \frac{1-\lambda}{|A|}\left[-e_c\left(u_{11}u_{22} - u_{12}^2\right) + u_1\left(u_{11}w_c - u_{12}\right)\right] < 0; \tag{B.3}$$

the inequality following from assumption 1, part ii), and assumption 3.

Simple algebra shows that $T_{p;1} - T_{p;2} = 1$:

$$\frac{u_{11}w_c^2 + u_{22} - 2u_{12}w_c}{|A|}\lambda U'' + \frac{(1-\lambda)}{|A|}\left(u_{22}u_{11} - u_{12}^2\right) = \frac{|A|}{|A|} = 1.$$

Concerning the properties of the optimal effort choices, we have:

$$\frac{\partial e_c}{\partial I_p} = \frac{(u_{12} - u_{11}w_c)}{|A|}\lambda U'' < 0; \tag{B.4}$$

from assumption 1, part ii), and assumption 3.

$$\frac{\partial e_c}{\partial I_c} = \frac{(u_{12} - u_{11}w_c)}{|A|}\lambda U'' = \frac{\partial e_c}{\partial I_p}. \tag{B.5}$$

$$\frac{\partial e_c}{\partial w_c} = -\frac{1}{|A|}\left[u_1\left((1-\lambda)u_{11} + \lambda U''\right) + \lambda U'' e_c\left(u_{11}w_c - u_{21}\right)\right] > 0; \tag{B.6}$$

from assumptions 3 and 4.

Parental consumption is a normal good in terms of I_p and I_c:

$$\frac{\partial c_p}{\partial I_p} = 1 - \frac{\partial T_p}{\partial I_p} = -\frac{\partial T_p}{\partial I_c} > 0; \tag{B.7}$$

where I have used $T_{p;1} - T_{p;2} = 1$ and the result in (B.2), above.

$$\frac{\partial c_p}{\partial I_c} = -\frac{\partial T_p}{\partial I_c} > 0. \tag{B.8}$$

Concerning the effects on parental consumption of changes in the wage:

$$\frac{\partial c_p}{\partial w_c} = -\frac{\partial T_p}{\partial w_c} > 0. \tag{B.9}$$

The child's consumption is also a normal good with respect to exogenous income:

$$\frac{\partial c_c}{\partial I_p} = \frac{\partial T_p}{\partial I_p} + w_c \frac{\partial e_c}{\partial I_p} = \frac{(u_{22} - u_{12} w_c)}{|A|} \lambda U'' > 0, \tag{B.10}$$

where the inequality follows from assumption 1, part iii), and assumption 3.

$$\frac{\partial c_c}{\partial I_c} = 1 + \frac{\partial T_p}{\partial I_c} + w_c \frac{\partial e_c}{\partial I_c} = \frac{\partial T_p}{\partial I_p} + w_c \frac{\partial e_c}{\partial I_p} > 0, \tag{B.11}$$

where I have used the fact that $T_{p,1} - T_{p,2} = 1$ and $\partial e_c / \partial I_p = \partial e_c / \partial I_c$.

Finally, to see how the child's consumption responds to the wage rate:

$$\begin{aligned} \frac{\partial c_c}{\partial w_c} &= \frac{\partial T_p}{\partial w_c} + e_c + w_c \frac{\partial e_c}{\partial w_c} \\ &= \frac{\lambda U'' \left[(u_{22} - u_{12} w_c) e_c - u_1 w_c \right] - u_1 u_{12} (1 - \lambda)}{|A|} > 0, \end{aligned} \tag{B.12}$$

the inequality following from assumption 5. ■

We may use (B.2) and (B.5) to rewrite (B.3) as follows:

$$\frac{\partial T_p}{\partial w_c} = \left[\frac{\partial T_p}{\partial I_c} - \frac{\partial e_c}{\partial I_c} \frac{u_1}{\lambda U'' e_c} \right] e_c. \tag{B.13}$$

C. A Family with Two Decision Makers

Here, I characterize the properties of the best-response functions of parent and child, corresponding to the model of section 4.

C.1. Best-Response Functions

Proof of lemma 4.1. The child's best-response function is fully characterized from her first-order condition, equation (4.2). This condition is exactly the same as the child's first-order condition in the problem of section 2, equation (3.4). Consequently, the results of lemma 2.1 apply. ■

Proof of lemma 4.2. The properties $T_{p,1} \geq 0$, $T_{p,2} \leq 0$ and $T_{p,3} \leq 0$ are obvious. To show $T_{p,4} \leq 0$, consider the derivative of the child's marginal utility from consumption with respect to change in effort:

$$\frac{\partial}{\partial e_c} \left[u_1 \left(I_c + T_p + e_c w_c, 1 - e_c \right) \right] = u_{11} w_c - u_{12}.$$

From assumption 1, part ii), this derivative is negative. When $T_p = 0$, therefore, the parent's best response to increments in effort is simply to let transfers stay at zero: positive transfers would make his first-order condition even more negative. When transfers are positive, by differentiating (4.1) it can be shown that:

$$\frac{\partial T_p}{\partial e_c} = \frac{(u_{12} - u_{11}w_c)(1-\lambda)}{\lambda U'' + (1-\lambda)u_{11}} < 0;$$

where the inequality follows form assumption 1, part ii). ∎

C.2. Uniqueness and Stability of Nash-Equilibrium

Proof of lemma 4.3. Consider first the case $T_p > 0$. Differentiating (4.1), we get the parent's best response to changes in effort:

$$dT_p = \frac{(1-\lambda)(u_{12} - u_{11}w_c)}{\lambda U'' + (1-\lambda)u_{11}} de_c. \tag{C.1}$$

Similarly, differentiating (4.2), we get the child's best-response to changes in parental transfers:

$$de_c = \frac{u_{21} - u_{11}w_c}{u_{11}w_c^2 - 2u_{12}w_c + u_{22}} dT_p. \tag{C.2}$$

From assumption 1, part ii), we know that $(u_{12} - u_{11}w_c) > 0$. This implies that the slope of the parental best-response function is negative. Assumption 1, part i), ensures that the denominator of the child's best-response function is negative, which carries over to the slope de_c/dT_p, as well. Uniqueness and stability of the Nash-equilibrium requires that the modulus of the derivative dT_p/de_c, computed along the parent's best-response function (equation (C.1)), is smaller than the modulus of dT_p/de_c computed along the child's best-response function (C.2). This implies:

$$\frac{u_{11}w_c^2 - 2u_{12}w_c + u_{22}}{u_{21} - u_{11}w_c} < \frac{(1-\lambda)(u_{12} - u_{11}w_c)}{\lambda U'' + (1-\lambda)u_{11}}$$

or

$$\left(u_{11}w_c^2 - 2u_{12}w_c + u_{22}\right)(\lambda U'' + (1-\lambda)u_{11}) - (1-\lambda)(u_{21} - u_{11}w_c)^2 > 0;$$

which is exactly the condition that the determinant of the Hessian matrix in the Barro-Becker problem be positive. Assumption 1, part i), and assumption 3 deliver the previous inequality. Consider now the case when $T_p = 0$. As previously stated, the child's best-response is uniquely defined for zero transfers. For $T_p = 0$ to be optimal, it must be the case that

$$-\lambda U'(I_p) + (1-\lambda)\, u_1(I_c + w_c e_c; 1 - e_c) \leq 0.$$

Consider perturbations of the Nash-equilibrium. If e_c goes above its optimal level, then the child's marginal utility from consumption u_1 decreases. This makes the previous inequality more negative so that $T_p = 0$ remains a best-response for the parent. Transfers being zero, the child's optimal effort choice is the one prevailing before the perturbation took place. Consider now a reduction in effort, below the child's optimal choice, still with $T_p = 0$. Since lower effort raises the child's marginal utility from consumption, it may be the case that the parent wants to start providing a strictly positive transfer. This being the case, we may invoke the stability conditions verified when transfers are positive. If, however, the parent's best response to the perturbation in effort is to still set $T_p = 0$, the child's best-response is to set e_c to the effort choice which prevailed before the perturbation took place. Finally, consider a perturbation of the Nash-equilibrium by increasing transfers. The child's best response to positive transfers is to work less. The parental best response to lower effort is to reduce transfers. Therefore, the original equilibrium is restored. $\blacksquare$