The Drawbacks of Electoral Competition

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Abstract

We examine the effect of the number of candidates and the impact of ideology on the efficiency of the electoral process. We show that the tendency to focus on policies that provide particularistic benefits increases with the number of candidates to the expense of policies that benefit the population at large. Thus, the efficiency of policies provided in an electoral equilibrium worsens when the number of candidates increases.

We next show that partisan voters are disadvantaged in the process of redistributive politics, and that the larger the fraction of voters who vote ideologically, the less efficient the political process. This is because electoral competition focuses on swing voters, increasing the values of policies with targetable benefits.

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1 Introduction

This paper has two goals. First, we examine the effect of the number of candidates on the efficiency of the electoral process. Second, we investigate the effect of ideology on efficiency and on redistributive politics in a model with more than two candidates.

In most democratic political systems, electoral competition takes place between more than two parties. Whether a large number of parties is desirable is subject to debate (see, for instance, Lijphart (1984)). One view is that, in societies that are divided along many ethnic, religious, and ideological lines, multi-party systems are beneficial because they allow a more diverse representation. A different view is that multi-party systems suffer from "indecisiveness", i.e., an inability to reach important decisions that comes from the post-electoral bargaining among coalitions. Moreover, multi-party systems are viewed as suffering from cabinet instability.

We provide a complementary perspective, that abstracts from issues of ideological divisions and post-electoral politics, to focus on the following simple idea. When there are many parties, each party tries to please a narrow constituency, ignoring the majority of the electorate. Thus, the larger the number of parties, the greater the appeal of particularistic (or pork-barrel) projects that can easily be targeted to subsets of the electorate. Instead, policies that benefit the population at large may be provided less effectively than when there are few parties. Thus, this tradeoff between targetability and efficiency worsens with the number of parties.

This viewpoint affords a natural perspective on the efficiency of electoral competition that is absent in the views described above. We analyze a model of redistributive politics with N candidates, and study how the equilibrium policy outcomes depend on the number of candidates. This gives insights into the effects of the number of parties on the efficiency of electoral competition, in the spirit of the results on the effect of the number of firms in economic markets.

We build on Lizzeri-Persico (1998), where there are only two candidates who compete for office by making a (binding) electoral promise to each voter. Candidates only care about the outcome of the election. Voters are homogeneous: each voter will vote for the candidate promising him the most utility. Candidates have a budget of one dollar per voter and are faced with only two possible choices: investing all the money in the public good, or redistributing the money. In the first case all voters get utility G (in money units). If $G > 1$ efficiency requires that the public good be provided. Money (local public goods, pork-barrel projects), however, can be targeted to subsets of voters. Even when efficiency requires provision of the public good, it may not be an equilibrium for candidates to do so; when $G < 2$, offering transfers of more than G to 51 percent of the voters is feasible, and is a superior strategy if the opponent promises the public good. Thus, the incentives towards tactical redistribution lead to under-provision of the public good; the equilibrium is inefficient whenever $1 < G < 2$.

Figure 1:

Efficiency in this model is given by the probability that the public good is provided in equilibrium. With two candidates in a proportional system, this is the solid line in Figure **??** (see Lizzeri and Persico (1998)).

In this model, the public good is underprovided. This is in contrast with the classic analysis of the inefficiency of provision of public goods in democracies (see for instance Stiglitz (1988)) which relies on the contrast between the policy preferred by the median voter and the Samuelsonian optimum. The incentives toward tactical redistribution which underlies some of the results in the present paper, cannot be adequately discussed by using median voter style models that rely on (exogenous) restrictions on the dimension of the policy space, such as linear taxes.

The present paper analyzes the case where there are N candidates. We show that increasing the number of candidates leads to worse outcomes: the provision of public goods is less efficient when the number of candidates is increased. The intuition is the following. Suppose that all candidates offer the public good. Then, they receive an equal share of $1/N$ of the vote. Now, if $G < N$, offering transfers of more than G to more than $1/N$ of the voters is feasible, and is a superior strategy. Thus, equilibrium is inefficient for $1 < G < N$. The value of the targetability of transfers becomes more important as the number of candidates increases. Consider for example the case of three candidates. In Section 4 we show that, at the equilibrium with equal treatment, the probability that the public good is provided is given by the dashed line in Figure **??**.

Next, we introduce an ideological dimension in voters' preferences: we allow for the possibility that some voters are partisans in the sense that they tend to favor one candidate. We show that partisan voters tend to be disadvantaged in the process of redistributive politics. This is because partisan voters are less responsive to transfers, and so competition for those voters is less intense. This feature of our model parallels a finding by Lindbeck and Weibull (1987), who obtain a similar result in a model with two candidates. One advantage of our model is that it remains tractable when we consider more than two candidates.

Ideological biases are a new source of inefficiency in our model. We show that even if these ideological preferences have no economic dimension, there is an impact on the efficiency of policy making that results from electoral competition. This is due to the fact that electoral competition focuses on swing voters, creating an even stronger incentive for candidates to provide pliable policies targeted to these voters, instead of a public good whose services benefit swing and partisan voters equally. This leads to lower provision of the public good, relative to the case where voters have no ideological bias.

The main body of our analysis concentrates on the (unique) equilibrium with equal ex-ante treatment of voters. When there are more than two candidates, a new phenomenon emerges: equilibrium coordination by candidates on particular sets of voters. Each candidate chooses to focus its attention on a particular set of voters and ignore other voters. This is despite the fact that voters are ex-ante identical from the perspective of each candidate. Thus, there are equilibria where candidates cater to different groups of voters. We call this phenomenon "fragmented competition."

The intuition is the following. Suppose that there are three candidates. Each set of voters is courted by a pair of candidates and ignored by other candidates. The reason it pays for candidate 1 to ignore voters who are courted by candidates 2 and 3 is that attracting these voters is too costly; candidate 1 would have to outbid both other candidates. In contrast, the voters he focuses on are courted by only one other candidate, and hence are attracted more cheaply. In Section 6 we show that, at the equilibrium with fragmented competition, the probability that the public good is provided is given by the dotted line in Figure **??**. The inefficiency of the electoral process becomes more pronounced relative to the case of equal treatment. The reason is that now the value of targeting is even greater, since candidates want to be able to focus their promises on a pre-specified subsection of the population.

1.1 Related Literature

The idea of the tradeoff between policies with diffuse benefits and pork-barrel projects has long been present in the mainstream political science literature; a comprehensive survey of the literature is in Persson and Tabellini (1998). Most of the literature on the inefficiency of democracy focuses on decision making in legislatures.[1] For instance, Baron (1991) models the legislative process via a sequential bargaining model. These models analyze specific, very structured legislative processes; they do not address candidates competing in large elections. In Chari *et al.* (1997), policy is determined through bargaining between a president and local representatives. This gives rise to a common pool problem whereby an excessive number of local public projects are financed from general taxation (see also Weingast *et al.*(1981)). Similarly, Persson *et al.* (1997) compare congressional and parliamentary systems. In their model, the politician choosing the level of public good provision has the option of foregoing the public good and appropriating the money for his own district; again, a common pool problem arises. Our setup differs from these models since we focus on national candidates who do not represent any specific district: when proposing to increase transfers to a district, our candidates take into account the loss of another district. Thus, nationwide candidates mitigate the common pool problem by internalizing, in part, the costs of providing pork-barrel projects.

The effect of the number of candidates on political competition has been extensively explored in the setting of spatial competition (see, for instance, Palfrey (1984), Austen-Smith and Banks (1988)). The spatial model is not naturally suited to discuss efficiency (no two points in the policy space are Pareto-comparable). For an excellent review of this strand of the literature, see Shepsle (1991). Besley and Coate (1997) and Osborne and Slivinski (1996) provide a different model of multi-candidate elections with endogenous entry by citizen-candidates who have policy preferences and do not commit to electoral platforms.

Myerson (1993) deals with redistributive issues, and compares electoral systems in terms of the inequality of redistribution. Our paper borrows the model of redistribution from Myerson, and adds a public good. This introduces an efficiency dimension, which is absent in Myerson's paper.

Besley and Coate (1998) discuss the efficiency of representative democracy where candidates are citizens who, if elected, implement their favorite policies. Theirs is a model of repeated elections, and the inefficiency depends on the dynamic nature of the model.

Persson and Tabellini (1999) discuss a two-candidates model related to Lizzeri and Persico (1998) and also provide some empirical evidence on the provision of public goods in different political systems. As they point out, there was no model addressing the provision of public goods under multi-candidate competition. Our paper provides such a model.

Alesina, Baqir, and Easterly (1999) discuss the provision of public goods in relation to ethnic divisions. Their analysis of U.S. local public goods shows that the shares of spending on public goods is inversely related to ethnic fragmentation. To explain this phenomenon, they provide a model of two-stage budgeting procedure, where first voting takes place on the

[1] An exception is Coate and Morris (1995), who focus on the fact that politicians have private information on the effects of government policy.

size of the budget, and in a second stage on its composition. Our model of Section 5 provides an alternative explanation for this phenomenon.

2 Model

2.1 Economy and Agents

There are N candidates. There is a continuum of voters with measure one. The set of voters is denoted by V.[2] There are two goods, money and a public good. The public good can only be produced by using all the money in the economy.

Each voter has an endowment of one unit of money. The public good yields a utility of G to each voter. Voters have no a priori preference for either candidate, and have linear utility over goods.

Candidates make binding promises to each voter. A candidate can offer to provide the public good (to all voters); alternatively, he can offer different taxes and transfers to different voters. Each voter votes for the candidate who promises her the greatest utility. Candidates maximize their expected vote share. This assumption on candidates' objective can be interpreted as describing a *proportional system* where the spoils of office (seats in an assembly) are divided proportionally to the share of the vote.

2.2 Game

A pure strategy for a candidate specifies whether he chooses to offer the public good or transfers (he cannot offer both). In the event he chooses transfers, a pure strategy specifies a promise of a transfer to each voter. Formally, a pure strategy is a function $\Phi : V \to [0, +\infty)$, where $\Phi(v)$ represents the consumption promised to voter v. The function Φ satisfies one of two conditions: either $\Phi(v) = G$ for all v's, signifying that the candidate offers the public good; or, $\int_V \Phi(v)dv = 1$, which is the balanced budget condition when a candidate offers transfers. In the latter case, $\Phi(v) - 1$ represents the transfer (or tax, if negative) promised to voter v.

There are two stages of the game:

Stage 1 Candidates choose offers to voters simultaneously and independently.

Stage 2 Each voter v gets offers $(\Phi_1(v), \ldots, \Phi_N(v))$ from candidates. After observing the offers, voter v votes for candidate i if, for all $j \neq i$, $\Phi_i(v) > \Phi_j(v)$. Ties are resolved by randomizing with equal probability.

[2]The assumption of infinitely many voters is useful because it allows us to invoke the law of large numbers on a number of occasions. This is meant to be an approximation for a game with a large (but finite) number of voters.

Voters' behavior can be made fully consistent with the usual rationality assumptions (strategic voting). Assume that the probability that the policy proposed by any given candidate is implemented is equal to this candidate's share of the vote; then, it is a dominant strategy for a voter to vote for the candidate who proposes the policy that is best for him. This interpretation of policy implementation is the one that we will adopt when computing the probability of provision of the public good in Theorems 2 and 6.

For values of G smaller than N there is no equilibrium in pure strategies. Consider for instance the case of two candidates. Suppose candidate 1's strategy was Φ_1. If $\Phi_1(v) = G$ for all v, i.e. candidate 1 promises each voter the public good, then candidate 2 can choose to promise more than G to more than 50 percent of the voters and obtain more than 50 percent of the votes. This is impossible in equilibrium. Suppose then that candidate 1 chooses to offer money. Now candidate 2 could take a set of voters V_1 with small positive measure such that $\Phi_1(v) > 0$ for $v \in V_1$, offer zero to these voters and use the saved money to finance offers of $\Phi_1(v) + \epsilon$ to all other voters. The set V_1 and the ϵ can be chosen so that candidate 2 wins with a share of the vote arbitrarily close to one hundred percent. Thus, at equilibrium both candidates will be employing mixed strategies.

A mixed strategy in this game could be a very complicated object, since the space of pure strategies is large. We discuss the case where $\Phi_i(v)$, the offer made by candidate i to voter v, is a realization from a random variable with c.d.f. $F_i^v : \Re_+ \to [0,1]$.[3] Note that, even when candidates use mixed strategies, each voter observes her realized promises before voting, not random variables.

It turns out that we only need to look at "simple" strategies of the following form: candidate i chooses to promise the public good with probability β_i and promise money with probability $1 - \beta_i$. We show that there are two types of equilibria, corresponding to the way that candidates redistribute money. One type of equilibrium displays *(ex-ante) equal treatment:* candidates draw promises to all voters from the same F_i (notice the absence of the superscript v).[4] The empirical distribution of transfers by candidate i to voters is F_i; hence, $F_i(x)$ is the fraction of supporters who receive promises below x from candidate i. By manipulating F_i, candidate i is able to target transfers to sections of the electorate. We want to stress that there is a natural interpretation for these mixed strategies: choosing F should be thought of as choosing the Lorenz curve, i.e. the empirical distribution of transfers, in the population.

In the equilibrium with fragmented competition, candidates partition the set of voters into two groups: one where voters get nothing (they are fully expropriated), the other where their offers are drawn from the same c.d.f. $F_i : \Re \to [0,1]$.

[3]We rule out correlation between offers to different voters.

[4]Of course, the fact that offers are realizations of the same random variable does not mean that each voter gets the same offer.

3 The Linearity of Returns to Transfers

In this section, we consider the case of equal (ex ante) treatment. We show that in equilibrium, candidates choose transfers from the set $[0, k] \cup (G, N]$, for some positive $k < G$. The probability of winning a voter with a promise of x on this set is shown to be x/N. This characterization is used, in the next section, to infer the distribution of transfers and the probability of provision of the public good.

Let F^* denote the equilibrium distribution of transfers. Clearly, F^* is continuous.[5] Let $H^*(x)$ denote the equilibrium probability of winning a voter with an offer of x. In equilibrium, candidates may promise the public good with positive probability, so H^* incorporates this probability; in particular, $H^*(x)$ is discontinuous at $x = G$ if the public good is offered with positive probability. For the moment, assume that the min of the support of H^* is zero, and the max is N (this will be proved below). We now show that $H^*(x) = x/N$ whenever x belongs to the support of F^* (and $x \neq G$), and $H^*(x) \leq x/N$ otherwise.

Candidate 1's problem is to choose a c.d.f F to maximize his expected vote share subject to the budget constraint. If the candidate offers x to a voter, the probability of winning his vote is $H^*(x)$; if the candidate chooses offers according to F, his expected vote share is $\int H^*(x)\, dF(x)$. So, the candidate solves

$$\max_F \int_0^N H^*(x)\, dF(x) \text{ s.t. } \int_0^N x dF(x) \leq 1.$$

The associated Lagrangean is

$$\mathcal{L} = \int_0^N \left[H^*(x) + \lambda(1 - x)\right] dF(x).$$

Let A denote the support of F^*. Since F^* maximizes the Lagrangean, it must be that $H^*(x) - \lambda^* x$ is maximal, and constant, on A. Since 0 is the inf of A, it must be that $const = H^*(0) - \lambda^* \cdot 0 = 0$. Since N is the sup of A, we have $0 = H^*(N) - \lambda^* \cdot N = 1 - N\lambda^*$. This shows that $\lambda^* = 1/N$. Thus, $H^*(x) = x/N$ for x in A. Since $H^*(x) - \lambda^* x$ is maximal, and equal to zero, on A, we have $H^*(x) - x/N \leq 0$ outside of A.

We now verify that the minimum of the support of H^* is zero, and the maximum is N. Suppose that the minimum of A is $m > 0$. Because of the budget constraint, $m < G$; therefore, because F^* is continuous, the probability of winning a vote by offering m equals zero. But then a candidate is paying m to obtain zero votes, which cannot be optimal.

Denote with M the maximum of A. At equilibrium, M has to equal N. Indeed, suppose it was smaller than N. Then, a player could deviate and give $N - \varepsilon$ to more than $1/N$ of the

[5] Suppose F is discontinuous at $\widehat{x}$. Then, a fraction $p > 0$ of voters are offered $\widehat{x}$. But this cannot be optimal. Candidate 1 gains by offering slightly more than $\widehat{x}$ to $p - \varepsilon$ of these voters, financing this deviation by offering zero to the remaining ε. This deviation profitably breaks the tie at $\widehat{x}$.

Figure 2:

voters. This guarantees a vote share larger than $1/N$, contradicting equilibrium. Suppose then that M was larger than N. Because zero and M are in the support of F, one optimal strategy is to offer close to M to $1/M$ of the electorate, and zero to the rest. However, this strategy yields a vote share smaller than $1/N$ if M is greater than N. Thus, $M = N$.

Finally, we show that the support of F is the union of two intervals, $[0, k]$ and $[G, N]$, for some positive $k < G$. In other words, we show that the only interval where F can be flat is $[k, G]$. Suppose that there is an interval (a, b) such that $F(b) - F(a) = 0$, $F(G) > F(b)$, and such that a and b are in the support of F. This means that the probability of obtaining a vote by offering b is the same as by offering a. However, offering b is more costly. Thus, offering b cannot be optimal. A similar logic applies to the interval $[G, N]$.

4 Equilibrium With Equal Treatment

Let us first consider the game of pure redistribution. Myerson (1993) proves the following result.[6]

Theorem 1 *(Myerson) If $G < 1$ there is a unique equilibrium with equal treatment. The public good is not provided, and candidates choose transfers according to the distribution F^**

[6]Myerson proves a more general result that treats the case of many candidates and any rank-scoring rule.

given by $F^*(x) = \left(\frac{x}{N}\right)^{\frac{1}{N-1}}$ *for* x *in* $[0, N]$.

Sketch of Proof The probability of winning a vote with an offer of x is $H^*(x) = [F^*(x)]^{N-1}$. We showed in Section 3 that $H^*(x) = x/N$.

We now consider the case where $G > 1$. In this case the public good must be provided with positive probability in equilibrium. To see this, suppose that candidates $2, ..., N$ behave according to the equilibrium described in Theorem 1. Then, if candidate 1 offers the public good, he receives a share of the vote of $G/N > 1$ for $G > 1$. Furthermore, if $G < N$ there is no equilibrium where the public good is provided with probability one; if all candidates offer the public good, offering transfers above G to more than one Nth of the voters is feasible and is a profitable deviation. Thus, in the region where $1 < G < N$, candidates randomize between promising transfers and offering the public good.

Theorem 2 *There is a unique equilibrium with equal treatment. For* $1 < G < N$, *the equilibrium is characterized by a probability* β *of providing the public good and a distribution of transfers* F^* *with support* $[0, k] \cup [G, N]$, *where* k *solves* $\left(\frac{G}{N}\right)^{\frac{1}{N-1}} \left(1 - \frac{G}{N}\right) = \left(\frac{k}{N}\right)^{\frac{1}{N-1}} \left(1 - \frac{k}{N}\right)$ *and* $\beta = \left(\frac{G}{N}\right)^{\frac{1}{N-1}} - \left(\frac{k}{N}\right)^{\frac{1}{N-1}}$. *The probability* β *is increasing in* G. *For* $G > N$ *the public good is provided with probability one.*

Proof: Equilibrium requires that if a candidate offers transfers, he cannot do better than by choosing F^*. This requires that $H^*(x) = x/N$ on $[0, k] \cup [G, N]$ (see the discussion in Section 3). When H^* has this form, the payoff of a candidate who redistributes according to F is

$$\begin{aligned} & \int_0^\infty H^*(x)\, dF(x) \\ \leq & \int_0^\infty \frac{x}{N} dF(x) = \frac{1}{N}, \end{aligned}$$

and the strict inequality holds only if F has a larger support than F^*. In particular, the payoff equals $1/N$ when $F = F^*$.

Now, let us turn our knowledge of H^* into a characterization of F^*. The probability of winning a vote with an offer of x in $[0, k]$ is

$$H^*(x) = [(1 - \beta) F^*(x)]^{N-1}.$$

Equating to x/N and solving for F^* yields

$$F^*(x) = \frac{1}{1-\beta} \left(\frac{x}{N}\right)^{\frac{1}{N-1}} \text{ for } x \in (0, k). \tag{1}$$

The probability of winning a vote with an offer of x in $(G,N]$ is

$$\begin{aligned} H^*(x) &= \sum_{j=0}^{N-1} \binom{N-1}{j} ((1-\beta) F^*(x))^j \beta^{N-1-j} \\ &= [(1-\beta) F^*(x) + \beta]^{N-1}. \end{aligned}$$

Equating to x/N and solving for F^* yields

$$F^*(x) = \frac{1}{1-\beta}\left[\left(\frac{x}{N}\right)^{\frac{1}{N-1}} - \beta\right] \text{ for } x \in (G,N). \tag{2}$$

To complete the characterization of F^* we look for conditions to pin down β and k. The first condition is given by the continuity of F^*, which requires $F^*(k) = F^*(G)$. Substituting from (1) and (2), we get

$$\beta = \left(\frac{G}{N}\right)^{\frac{1}{N-1}} - \left(\frac{k}{N}\right)^{\frac{1}{N-1}}. \tag{3}$$

The second condition is given by the budget constraint, i.e., $\int_0^k xf(x)\,dx + \int_G^N xf(x)\,dx = 1$. To compute the budget constraint, observe that, on $[0,k] \cup [G,N]$,

$$f^*(x) = \frac{1}{1-\beta}\left(\frac{1}{N}\right)^{\frac{1}{N-1}} \frac{1}{N-1} x^{\frac{2-N}{N-1}},$$

and therefore

$$\int_a^b xf^*(x)\,dx = \frac{1}{1-\beta}\left(\frac{1}{N}\right)^{\frac{1}{N-1}} \frac{1}{N-1} \frac{x^{\frac{N}{N-1}}}{N}(N-1)\Bigg|_a^b = \frac{1}{1-\beta}\left(\frac{x}{N}\right)^{\frac{N}{N-1}}\Bigg|_a^b.$$

Using this equation, the budget constraint can be expressed as

$$\frac{1}{1-\beta}\left[\left(\frac{k}{N}\right)^{\frac{N}{N-1}} + 1 - \left(\frac{G}{N}\right)^{\frac{N}{N-1}}\right] = 1,$$

or

$$\beta = \left(\frac{G}{N}\right)^{\frac{N}{N-1}} - \left(\frac{k}{N}\right)^{\frac{N}{N-1}}. \tag{4}$$

Equations (3) and (4) form a system of two equations in the unknowns k and β.

It is possible to solve for k. Indeed, from (3) and (4), k must solve

$$\left(\frac{G}{N}\right)^{\frac{1}{N-1}} - \left(\frac{k}{N}\right)^{\frac{1}{N-1}} = \left(\frac{G}{N}\right)^{\frac{N}{N-1}} - \left(\frac{k}{N}\right)^{\frac{N}{N-1}},$$

or

$$\left(\frac{G}{N}\right)^{\frac{1}{N-1}}\left(1 - \frac{G}{N}\right) = \left(\frac{k}{N}\right)^{\frac{1}{N-1}}\left(1 - \frac{k}{N}\right). \tag{5}$$

The function $h(z) = \left(\frac{z}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{z}{N}\right)$ is single-peaked on $[0,\infty)$, has a maximum at $z=1$, and has value zero at $z=0$ and $z=N$. Because h is single peaked, equation (5) only has two solutions: one is $k=G$; the other is $k^*(G) = h^{-1}(G)$, where h^{-1} denote the inverse of h on the interval $[0,1]$. Only the second solution can be part of an equilibrium, since the first solution requires that β equals zero, and this is impossible when $1 < G < N$. Solving for β is accomplished by substituting $k^*(G)$ into equation (3), to obtain $\beta^*(G)$. Observe that $k^*(G)$ is decreasing in G since h is increasing on $(0,1)$; therefore, $\beta^*(G) = (G/N)^{\frac{1}{N-1}} - (k^*(G)/N)^{\frac{1}{N-1}}$ is increasing in G.

To verify that the pair $\beta^*(G)$ and $k^*(G)$ indeed forms an equilibrium, we need to check that the expected vote share from offering the public good, S_G, is equal to $1/N$,

$$S_G = \sum_{j=0}^{N-1} \frac{1}{j+1}\binom{N-1}{j}(1-\beta)^{N-1-j}\beta^j\left(F^*(G)\right)^{N-1-j} = \frac{1}{N}.$$

Using the fact that $\sum_{j=0}^{N-1}\frac{1}{j+1}\binom{N-1}{j}q^{N-1-j}p^j = \left[(q+p)^N - q^N\right]/pN$, we rewrite the above equation as

$$\frac{\left[(1-\beta)F^*(G)+\beta\right]^N - \left[(1-\beta)F^*(G)\right]^N}{\beta N} = \frac{1}{N}. \tag{6}$$

Since $F^*(G) = F^*(k)$ we can rewrite this as

$$\frac{\left[(1-\beta)F^*(G)+\beta\right]^N - \left[(1-\beta)F^*(k)\right]^N}{\beta N} = \frac{1}{N},$$

and after substituting from (1) and (2), we obtain

$$\frac{\left[\frac{G}{N}\right]^{\frac{N}{N-1}} - \left[\frac{k}{N}\right]^{\frac{N}{N-1}}}{\beta N} = \frac{1}{N}.$$

This equation is the same as equation (4). Thus, when $\beta = \beta^*(G), k = k^*(G)$, it is indeed the case that $S_G = 1/N$. This shows that the pair $\beta^*(G), k^*(G)$ forms an equilibrium.

Uniqueness follows from the discussion in Section 3 together with the uniqueness of the solutions of β and k. ∎

The following table presents the equilibrium values of β for different pairs G, N. For given value of G, β declines with N.

N / G	2	3	4	5	6
1.5	.5	.27	.19	.15	.12
2	1	.52	.36	.28	.23
2.5	1	.76	.52	.40	.33
3	1	1	.68	.52	.43
3.5	1	1	.84	.64	.52
4	1	1	1	.76	.61
4.5	1	1	1	.88	.71
5	1	1	1	1	.80
5.5	1	1	1	1	.90

The following figure presents the density $f^*(x)$ for the cases of $G = 1.5$ and $N = 3$ (the thin line) and $N = 4$ (the boldface line). Promises between $x = 7.9382 \times 10^{-2}$ and 3 are less likely with $N = 4$ than with $N = 3$. Thus, redistributional platforms become more unequal as N goes from 3 to 4.

5 The Costs of Ideology

We now introduce the possibility that some voters have ideological biases in favor of one of the candidates. First, we address redistributional issues: we show that partisan voters tend to be disadvantaged in the process of redistributive politics (Theorem 3). This is because partisan voters are less responsive to transfers, and so competition for those voters is less intense. We present the results for the model with three candidates, to show that the logic of Lindbeck and Weibull (1987) also applies to a model with more than two candidates. A similar result can be derived in the case with two candidates.

Next, we show that ideological biases are a source of inefficiency in our model (Theorem 4). The fact that electoral competition focuses on swing voters, creates an even stronger incentive for candidates to offer redistributive platforms targeted to these voters, instead of a public good whose services benefit swing voters and partisan voters equally. This leads to lower provision of the public good, relative to the case where voters have no ideological bias.

There are $N+1$ groups of voters: $V_S, V_1, ..., V_N$, where V_S is of size $1-\varphi$, and each V_i is of size φ/N for $i = 1, ..., N$. Voters in V_i are *partisan* for candidate i: with probability p they *vote ideologically*, i.e., they ignore electoral promises and vote for candidate i; with probability $1-p$ they vote for the candidate who makes the higher promise. Voters in V_S are *swing voters*: they have no ideological preference, they always vote for the candidate making the higher promise.

This way of modeling ideology allows for the possibility that there is another dimension of the policy space (such as war and peace, abortion rights ...), the various candidates have distinct positions on this policy issue, and that some voters have stronger preferences than others on this issue. The model can be readily extended to treat several degrees of partisanship by allowing for groups with different probabilities p_i of voting ideologically.

Theorem 3 *Assume that, for each of the three candidates, there is a fraction φ/N of partisan voters who with probability p vote ideologically; the remaining $(1-\varphi)$ are swing voters. Assume further that $G(1-\varphi) < 1$. Then, the following is an equilibrium. Each candidate chooses transfers according to a pair of distributions: offers to partisan voters are drawn from the distribution $F_P^*(x) =$ on $\left[0, \frac{N(1-p)}{1-\varphi p}\right]$; offers to swing voters are drawn from the distribution $F_S^*(x) = \left(x\frac{1-\varphi p}{N}\right)^{\frac{1}{N-1}}$ on $\left[0, \frac{N}{1-\varphi p}\right]$.*

Proof: In equilibrium, $[F_S^*(x)]^{N-1}$ and $[F_P^*(x)]^{N-1}$ must be uniform on $[0, K_S]$ and $[0, K_P]$, respectively (see the discussion in Section 3). We use the equation $[F_S^*(x)]^{N-1} = x/K_S$ to find $F_S^*(x)$,

$$F_S^*(x) = \left(\frac{x}{K_S}\right)^{\frac{1}{N-1}} \text{ on } [0, K_S]. \tag{7}$$

Analogously,

$$F_P^*(x) = \left(\frac{x}{K_P}\right)^{\frac{1}{N-1}} \text{ on } [0, K_P]. \tag{8}$$

It remains to solve for K_S and K_P. To this end, we first note that candidates must receive the same expected return from offering x to swing and partisan voters, hence

$$[F_S^*(x)]^{N-1} = (1-p)[F_P^*(x)]^{N-1}.$$

Substituting from equations (7) and (8) we rewrite the above equation as

$$\frac{K_P}{K_S} = (1-p). \tag{9}$$

Next, we note that the budget constraint requires that

$$(1-\varphi)\int_0^{K_S} xdF_S^*(x)+\varphi\int_0^{K_P} xdF_P^*(x)=1,$$

so substituting from (7) and (8) we have

$$(1-\varphi)\int_0^{K_S}\frac{1}{(N-1)(K_S)^{\frac{1}{N-1}}}x^{\frac{2-N}{N-1}}xdx+\varphi\int_0^{K_P}\frac{1}{(N-1)(K_P)^{\frac{1}{N-1}}}x^{\frac{2-N}{N-1}}xdx=1$$

Solving the integrals and simplifying we get

$$\frac{1}{N}(1-\varphi)K_S+\frac{1}{N}\varphi K_P=1. \tag{10}$$

Solving equations (9) and (10) for K_S and K_P yields $K_S=\frac{N}{1-\varphi p}, K_P=N\frac{1-p}{1-\varphi p}$. ∎

In the next theorem we show that the inefficiency in our model is proportional to the fraction of partisan voters in the electorate. To this end, we make the simplifying assumption that partisan voters are completely unresponsive to electoral promises, i.e., $p=1$. We are then able to show that, when a fraction φ of the electorate is partisan, the probability of public good provision corresponds to the probability in a model where no voter is partisan, and the public good has value $G(1-\varphi)$. Thus, the probability that the public good is provided is decreasing in the degree of polarization of the electorate.

The intuition is the following: when there are φ partisan voters and $p=1$, at equilibrium partisans receive zero transfers, thus freeing up resources for candidates to compete for swing voters. Then, candidates can allocate transfers of $1/(1-\varphi)$ per swing voter. This is more money per capita than was available in the case of no partisans. Normalizing the per capita money to 1 requires normalizing the value of the public good to $G(1-\varphi)$. After this normalization, the equilibrium can be derived from Theorem 2.

Theorem 4 *Assume that, for each of the three candidates, there is a fraction φ/N of partisan voters who with probability $p=1$ vote ideologically; the remaining $(1-\varphi)$ are swing voters. Then, the following is an equilibrium. Partisan voters never receive any trasnfers. For $1<G(1-\varphi)<N$, the equilibrium is characterized by a probability β of providing the public good and a distribution of transfers F_S^* with support $[0,w]\cup[G,N/(1-\varphi)]$, where w solves $\left(\frac{G(1-\varphi)}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{G(1-\varphi)}{N}\right)=\left(\frac{w(1-\varphi)}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{w(1-\varphi)}{N}\right)$ and $\beta=\left(\frac{G(1-\varphi)}{N}\right)^{\frac{1}{N-1}}-\left(\frac{w(1-\varphi)}{N}\right)^{\frac{1}{N-1}}$. For $G(1-\varphi)>N$ the public good is provided with probability one.*

Proof: See Appendix. ∎

Figure 3:

6 The Inefficiency of Fragmented Competition

This section deals with equilibria with fragmented competition. We show that there are equilibria where a candidate treats different voters differently, by concentrating the benefits of redistribution on a subset of the electorate which we call his *support base*. The inefficiency of the electoral process becomes more pronounced relative to the case of equal treatment covered in Section 4. The reason is that now the value of targeting is even greater, since candidates want to be able to focus their promises on a pre-specified subsection of the population.

For expositional purposes, it is useful to think of voters being distributed uniformly on a circle with perimeter equal to one. As before, candidate i chooses to promise the public good with probability β_i and promise money with probability $1-\beta_i$. Now, in the event of promising money, the candidate partitions the set of voters into two groups: one where voters get nothing (they are fully expropriated), the other where their offers are drawn from the same c.d.f. $F_i : \Re \to [0,1]$. We call the second group of voters the *support base* of candidate i.

As in Section 4, we start with a discussion of the case where the public good is provided with probability zero. In contrast with the previous analysis, the values of G for which this happens is now larger.

Theorem 5 *Assume $G < 3\left(\sqrt{2}-1\right)$. Then there is an equilibrium where candidate 1 has support base from 5 to 1 o'clock (clockwise). Candidate 2 has support base from 9 to 5 o'clock. Candidate 3 has support base from 1 to 9 o'clock. All candidates draw offers to voters in their support base from a uniform distribution on $[0,3]$.*

Proof: Denote with $F^*(x)$ the c.d.f. of a random variable distributed as a Uniform on $[0,3]$. At the proposed strategy combination, each candidate's share of the vote equals $1/3$. Suppose candidate 1 deviates and offers the public good.

For each voter in his support base (from 5 to 1), candidate 1 is only competing with one other candidate who offers transfers with c.d.f. $F^*(x) = x/3$. Therefore, the probability of winning a voter in the support base is $G/3$. For a voter outside the support base of candidate 1, the candidate competes with both opponents, leading to a lower probability $(G/3)^2$ of winning a voter. Summing up, candidate 1's total share of the vote is

$$S(F^*, F^*; G) = \frac{2}{3}\frac{G}{3} + \frac{1}{3}\frac{G^2}{9}.$$

Setting $S(F^*, F^*; G) = \frac{1}{3}$ yields $G = 3\left(\sqrt{2} - 1\right)$. Since $S(F^*, F^*; G)$ is increasing in G, we have $S(G) \leq \frac{1}{3}$ for $G \leq 3\left(\sqrt{2} - 1\right)$. In this parameter range, deviating to offering the public good does not pay.

Suppose that candidate 1 deviates and chooses an alternative distribution of transfers. First, suppose that candidate 1 considers changing his support base, i.e., he offers positive amounts to voters between 1 and 5 o'clock. Take any particular voter in this set and suppose he offers the voter x. Then the probability of getting his vote is $(F^*(x))^2$, since the offer has to be higher than the equilibrium offer by both other candidates. On the other hand, the probability of getting the vote of a voter in the support base is $F^*(x) \geq (F^*(x))^2$. Thus, for candidate 1, offering transfer outside his support base is dominated by offering transfers to voters inside his support base.

Now, suppose candidate 1 sticks to offering transfers only to his support base, but uses an alternative distribution F_1. Then, his share of the vote is

$$S(F^*, F^*; F_1) = \frac{2}{3}\int_0^\infty F^*(x)dF_1(x) \leq \frac{2}{3}\int_0^\infty \frac{x}{3}dF_1(x) = \frac{1}{3},$$

where the inequality comes from the fact that $F^*(x) = \frac{x}{3}$ on $[0,3]$ and $F^*(x) = 1 < \frac{x}{3}$ for $x > 3$; the last equality comes from the budget constraint. This shows that there is no benefit to deviating to a different distribution of transfers. ∎

Remark: There is an interesting contrast between Theorem 5 and the equilibrium described by Myerson (1993) (Theorem 1 above). In that equilbrium, in contrast with the equilibrium described in Theorem 5, all voters get the same amount from all candidates in expectation.

We now discuss the case where the public good is provided with positive probability. As in Section 4, when $G < 3$ there is no equilibrium where the public good is provided with probability one.

Theorem 6 *Assume* $3\left(\sqrt{2}-1\right) < G < 3$. *Then there is an equilibrium where candidate 1 has support base from 5 to 1 o'clock (clockwise). Candidate 2 has support base from 9 to 5 o'clock. Candidate 3 has support base from 1 to 9 o'clock. The probability* β *that the public good is provided is monotonically increasing in* G *and is characterized by the system (11).*

When $G \geq 3$ *the public good is provided with probability one.*

Proof: We guess that the equilibrium distribution of transfers F^* has a particular form: F^* increases linearly with slope γ_1 between 0 and k, with slope γ_2 between G and K, and is constant elsewhere. Thus, $K > G$ denotes the maximum transfer offered by a candidate, and $k < G$ denotes the maximum transfer below G.

We can eliminate k and K. Indeed, denote $q = 1 - F^*(G)$; using the definition of F^*, it must be $k = \frac{1-q}{\gamma_1}$ and $K = \frac{q}{\gamma_2} + G$. A further restriction on the parameters comes from the budget constraint $\int_0^\infty x dF^*(x) = 3/2$. Since $\int_0^\infty x dF^*(x) = \int_0^\infty [1 - F^*(x)] dx$, the budget constraint reads $qG + q\frac{K-G}{2} + k\frac{1-q}{2} = \frac{3}{2}$.

We want to show that equilibrium is characterized by the following system:

$$\begin{cases} \beta^2\left(\frac{2}{3}\right) + 2\beta(1-\beta)\left(\frac{1}{3} + \frac{1}{3}(1-q-\gamma_2 G)\right) + (1-\beta)^2\frac{2}{3}(1-q-\gamma_2 G) = 0 \\ (1-\beta)^2\frac{2}{3}\gamma_2 + 2\beta(1-\beta)\frac{1}{3}\gamma_2 = (1-\beta)^2\frac{2}{3}\gamma_1 \\ k = \frac{1-q}{\gamma_1} \\ K = \frac{q}{\gamma_2} + G \\ qG + q\frac{K-G}{2} + k\frac{1-q}{2} = \frac{3}{2} \\ \beta = \frac{2-4q+q^2}{(q-1)^2} \end{cases} \tag{11}$$

The third, fourth, and fifth equations in the system come from the discussion above. We now derive the remaining equations.

At equilibrium, the expected share of the vote from promising money and the public good must be equal, so that candidates are willing to offer both. To compute the expected share of the vote, start by considering the event $(G, F^*; F^*)$ where candidate 1 and candidate 3 promise to redistribute according to F^*, and candidate 2 promises the public good. On half of his support base, candidate 1 competes with candidate 2 alone, and the probability of winning a vote is q. On the other half of his support base, candidate 1 competes with both opponents. In that case, to win a vote, candidate 1's offer must (a) exceed G, which happens with probability q; AND (b) conditional on being higher than G, candidate 1's offer must exceed candidate 2's offer, which happens whenever 2's offer is smaller than G (prob. $1-q$), and with probability 1/2 when 2's offer is larger than G (prob. q), leading to a total probability of $1-q+(q/2)$. On the second half of the support base, the probability of winning a vote is therefore $q(1-q+(q/2))$. Summing up the payoff from both halves of the support base, we get

$$S(G, F^*; F^*) = \frac{1}{3}q + \frac{1}{3}q(1-q+(q/2)) = \frac{1}{3}q\left(2 - \frac{q}{2}\right).$$

Proceeding in a similar fashion, we can fill in all the entries in the following table. The table presents the share of the vote of the player playing the row action when his opponents' actions are those written in the columns.

	G,G	G,F^*	F^*,G	F^*,F^*
G	$1/3$	$\frac{2}{6}(1-q)+\frac{1}{6}$	$\frac{2}{6}(1-q)+\frac{1}{6}$	$\frac{1}{3}(1-q)^2+\frac{2}{3}(1-q)$
F^*	$\frac{2}{3}q$	$\frac{1}{3}q\left(2-\frac{q}{2}\right)$	$\frac{1}{3}q\left(2-\frac{q}{2}\right)$	$1/3$

(12)

From this table we can recover the expected share of the vote when offering the public good,

$$\begin{aligned} ES_G &= \beta^2 S(G,G;G) + 2\beta(1-\beta) S(G,F^*;G) + (1-\beta)^2 S(F^*,F^*;G) \\ &= \beta^2\left(\frac{1}{3}\right) + \beta(1-\beta)\left(\frac{2}{3}(1-q)+\frac{1}{3}\right) + (1-\beta)^2\left(\frac{1}{3}(1-q)^2+\frac{2}{3}(1-q)\right). \end{aligned}$$

Similarly, the expected share of the vote when offering transfers according to F^* is

$$\begin{aligned} ES_{F^*} &= \beta^2 S(G,G;F^*) + 2\beta(1-\beta) S(G,F^*;F^*) + (1-\beta)^2 S(F^*,F^*;F^*) \\ &= \beta^2\left(\frac{2}{3}\right)q + 2\beta(1-\beta)\left(\frac{1}{3}q\left(2-\frac{q}{2}\right)\right) + (1-\beta)^2\left(\frac{1}{3}\right). \end{aligned}$$

In equilibrium, candidates randomize between offering the public good and transfers. Thus, the expected share of the vote of a candidate who offers the public good and money must be equal,

$$ES_G = ES_{F^*}. \tag{13}$$

Using the fact that $S(G,G;F^*) = 1-2S(G,F^*;G)$ and $S(F^*,F^*;G) = 1-2S(G,F^*;F^*)$, after some algebra one obtains

$$ES_G = ES_{F^*} + 2\beta\left[S(G,F^*;G) + S(G,F^*;F^*) - \frac{2}{3}\right] + 2\left(\frac{1}{3} - S(G,F^*;F^*)\right).$$

Thus, equality of expected shares (equation (13)) is equivalent to

$$2\beta\left[S(G,F^*;G) + S(G,F^*;F^*) - \frac{2}{3}\right] + 2\left(\frac{1}{3} - S(G,F^*;F^*)\right) = 0.$$

Substituting for $S(G,F^*;G)$ and $S(G,F^*;F^*)$, and solving for β, yields

$$\beta = \frac{2-4q+q^2}{(q-1)^2}. \tag{14}$$

This is the last equation of system (11). When this equation is verified, candidates are indifferent between promising the public good and the equilibrium distribution of transfers. Let us now consider deviations to different distributions of transfers.

The share of the vote of a candidate who redistributes according to F, when the other two candidates redistribute according to the equilibrium, is

$$\begin{aligned} S &= \frac{2}{3}\left[\int_0^k \gamma_1 x dF(x) + \int_G^K (1-q) + \gamma_2 (x-G)\, dF(x)\right] \\ &= \frac{2}{3}\left[\gamma_1 M_1 + \gamma_2 M_2 + (1-q-\gamma_2 G)(1-F(G))\right]. \end{aligned}$$

where $M_1 = \int_0^k x dF(x)$ and $M_2 = \int_G^K x dF(x)$, whence by the budget constraint $M_1 + M_2 = \frac{3}{2}$.

The share of the vote of a candidate who redistributes according to F, when the one of his opponents redistributes according to the equilibrium distribution F^*, and the other offers the public good, is

$$\begin{aligned} S &= \frac{1}{3}(1-F(G)) + \frac{1}{3}\int_G^K 1-q+\gamma_2(x-G)\, dF(x) \\ &= \frac{1}{3}(1-F(G)) + \frac{1}{3}\left[\gamma_2 M_2 + (1-q-\gamma_2 G)(1-F(G))\right] \end{aligned}$$

Finally, the share of the vote of a candidate who redistributes according to F when the other two candidates offer the public good is

$$\frac{2}{3}(1-F(G)).$$

Thus, the expected share of the vote when redistributing according to F and the opponents follow the equilibrium strategy is

$$\begin{aligned} &\beta^2\left(\frac{2}{3}\right)(1-F(G)) + 2\beta(1-\beta)\left(\frac{1}{3}(1-F(G)) + \frac{1}{3}\left(\gamma_2 M_2 + (1-q-\gamma_2 G)(1-F(G))\right)\right) \\ &+(1-\beta)^2\left(\frac{2}{3}\left(\gamma_1 M_1 + \gamma_2 M_2 + (1-q-\gamma_2 G)(1-F(G))\right)\right) \end{aligned} \tag{15}$$

Expression (15) depends on F through $F(G), M_1$, and M_2. The candidate should not gain by changing $F(G)$, so the derivative of the above expression with respect to $F(G)$ should be zero

$$\beta^2\left(\frac{2}{3}\right) + 2\beta(1-\beta)\left(\frac{1}{3} + \frac{1}{3}(1-q-\gamma_2 G)\right) + (1-\beta)^2\frac{2}{3}(1-q-\gamma_2 G) = 0.$$

Furthermore, the candidate should not be able to improve his lot by changing the allocation of his expenditures between M_1 and M_2, given the constraint that $M_1 + M_2 = 3/2$. Thus, the derivative of expression (15) with respect to M_1 must be equal to the derivative with respect to M_2

$$(1-\beta)^2\frac{2}{3}\gamma_1 = (1-\beta)^2\frac{2}{3}\gamma_2 + 2\beta(1-\beta)\frac{1}{3}\gamma_2.$$

The last two equations complete the system (11).

The system that characterizes equilibrium can be solved. The solution is

$\gamma_1 = 3\frac{3-G+2\rho G-3\rho}{(G+3)(-3+G)^2}$

$\gamma_2 = -\frac{1}{-3+G}\rho$

$k = \frac{1}{3}G^2 - G + 3 - \frac{1}{3}\rho G^2 - \rho G$ $\blacksquare$

$\beta = \frac{2}{3}G - \frac{1}{3}\rho G - \rho$

$K = 3$

$q = \rho$ where ρ is a root of $(G+3)\,Z^2 + (-2G+3)\,Z - 3 + G$.

The following table reports a numerical calculation of the equilibrium values of all the variables.

G	β	k	K	γ_1	γ_2	q
1.3	2.9637×10^{-2}	1.175 2	3.0	.35401	.34351	.58397
1.4	8.1605×10^{-2}	1.060 9	3.0	.3952	.36295	.58072
1.5	.133 97	.95096	3.0	.44444	.3849	.57735
1.6	.18677	.8455	3.0	.50403	.40989	.57384
1.7	.24003	.74471	3.0	.57714	.43861	.5702
1.8	.29377	.64879	3.0	.66833	.47199	.56639
1.9	.34805	.55797	3.0	.78425	.51129	.56242
2.0	.4029	.47247	3.0	.93495	.55826	.55826
2.5	.68782	.13626	3.0	3.4207	1.0678	.53391
2.8	.87286	2.4352×10^{-2}	3.0	19.908	2.576	.51521
2.9	.93438	6.3822×10^{-3}	3.0	77. 402	5. 0794	. 50794

In this equilibrium with fragmented competition, the probability of provision of the public good is uniformly lower than in the case with equal treatment. The interpretation is that, when political rivals tailor their platforms to subgroups of the population, a politician has greater incentive to target promises narrowly to his own supporters. Promising the public good is therefore less profitable, relative to the case where all politicians appeal to a broader fraction of the electorate.

Observe that the inefficiency resulting from coordination requires more than two candidates; thus, this is a different phenomenon from the one that emerged in our analysis of ideology (Section 5), which is present even with just two candidates. Furthermore, in the equilibrium of Section 5 partisan voters expect lower transfers (ex ante) than swing voters; in contrast, voters' expected transfers are the same in the equilibrium of Theorems 5 and 6.

7 Implications

7.1 Contrast with winner-take-all with two candidates

In a winner-take-all system, candidates maximize the probability of winning the election and not the expected vote share. This would be the case in a system with a strong majoritarian bias, where most of the powers of policy setting are delegated to the party with a majority of votes, and where minority parties do not have much influence. A winner-take-all system of electoral incentives also prevails in the direct election of a premier, where the candidate with the highest number of votes receives all the spoils of office.

Duverger's Law suggests that majoritarian systems should be associated with only two competing parties. The argument is that voters should coordinate on the two candidates most likely to be elected, and avoid wasting their vote on other candidates. A corollary of Duverger's Law, largely borne out in reality, is that proportional systems have a larger number of effective political parties. We compare the results in Section 4 to the equilibrium in a two-candidates election with a winner-take-all rule.

Austen-Smith and Banks (1988) compare the equilibrium in a proportional system with three candidates with the equilibrium of a winner-take-all system with two candidates. They analyze proportional representation in a three-candidate model of spatial competition that integrates the electoral and legislative processes. Under plurality rule, both candidates adopt the policy preferred by the median voter. Under proportional representation the equilibrium electoral platforms are symmetrically distributed around the median. The government is formed between the party that adopts the median position (which receives the fewest votes) and one of the other parties. The policy outcome that emerges from the legislative process is a compromise between the platforms of these two parties and is different from the median voter's preferred policy. The possibility of using their analysis for a discussion of efficiency is limited by their use of the spatial model.

In our model, the comparison rests on the following theorem, proved in Lizzeri and Persico (1998).

Theorem 7 *Suppose* $1 < G < 2$, *and* $N = 2$. *Under the winner-take-all system in the unique equilibrium both candidates offer the public good with probability* $1/2$ *for* $G \in (1, 2)$. *Candidates offer the public good with probability* 0 *if* $G < 1$ *and with probability one if* $G > 2$.

Comparing this equilibrium with the analysis in Section 4 shows that the proportional system is generally more inefficient than a winner-take-all system. The probability of public good provision is larger in the winner-take-all system relative to the equilibrium with equal treatment in the proportional system, unless $N = 2$ and $G < 3/2$, or $N = 3$ and G is between 1.963 and 2. (The equilibrium with fragmented competition of Section 6 is always less efficient than the winner-take-all system). This suggests that two-candidate elections lead to more efficient outcomes.

If we believe Duverger's Law, this observation would be an argument in favor of winner-take-all systems, which increase efficiency by reducing the number of competing parties.

7.2 The Size of Government (PRELIMINARY)

We define the size of government as the total amount of taxes levied. Recall that we can think of our model as applying to an economy where voters have an endowment of one unit of money, and candidates need to tax voters if they want to provide the public good or to offer transfers to other voters. Thus, the size of government is maximal if the public good is provided since voters are taxed the full amount. If a candidate provides transfers, we can compute the size of taxes from the equilibrium distribution of transfers F^* by observing that the support of F^* is a translation of the support of the equilibrium distribution if we incorporate taxes. Thus, the size of government in that case is given by $1 - \int_0^1 x dF^*(x)$.

The effect of the number of candidates on the size of government is affected by two contrasting forces. First, as we saw in Section 4, as the number of candidates increases, the probability that the public good is provided decreases. Thus, this effect is in the direction of lower size of government as the number of candidates increases. Second, the distribution of transfers changes. It can be shown that in the event that transfers are offered, the total size of taxes increases with the number of candidates. It is easy to see that the first effect is going to dominate when the value of the public good is large whereas the second is going to dominate when the value of the public good is close to one.

7.3 Other models of ideology

Lindbeck and Weibull's (1987) treatment of redistributive politics cannot logically be separated from the role of ideology. This is because ideology is essential for the existence of the pure strategy equilibria that they focus on; this requires making assumptions on ideology that are necessary for technical reasons. An advantage of our model is that the analysis of ideology can be logically separated from the basic model of redistributive politics. Furthermore, if we introduce a public good in the Lindbeck-Weibull model, there is an efficient equilibrium if groups are homogeneous (see the appendix for a proof of this). The reason for this is the following. The equilibrium outcome of the Lindbeck-Weibull model involves groups with the same characteristics receiving the same promises. Thus, the tradeoff between targetability and efficiency disappears when groups are homogeneous. It seems to us that this negates an important aspect of redistributive politics since the incentives to target subgroups of the populations to obtain their votes does not seem to depend exclusively on differences between groups.

8 Conclusion

We have provided a model of N-candidate competition where candidates choose whether to offer to provide a public good or to target transfers to sub-groups of the population. We have shown that in equilibrium the probability that the public good is provided decreases with the number of candidates. Then we discussed the role of ideology in voters' preferences and showed that the larger the group of pre-committed voters, the less efficient the political process in terms of provision of public goods. Finally, we discussed an equilibrium with fragmented competition where the probability of provision of the public good is uniformly lower than in the case with equal treatment. The interpretation is that, when political rivals tailor their platforms to subgroups of the population, a politician has greater incentive to target promises narrowly to his own supporters. Promising the public good is therefore less profitable, relative to the case where all politicians appeal to a broader fraction of the electorate.

9 Appendix

9.1 Proof of Theorem 4.

Proof: Since partisan voters get no money in equilibrium, we now focus on swing voters. By the same argument as in the proof of Theorem 2, equilibrium requires that if a candidate offers transfers, he cannot do better than by choosing F^*. This requires that $H^*(x) = x(1-\varphi)/N$ on $[0,w] \cup [G, N/(1-\varphi)]$.

The probability of winning a vote with an offer of x in $[0,w]$ is

$$H^*(x) = [(1-\beta) F^*(x)]^{N-1}.$$

Equating to $x(1-\varphi)/N$ and solving for F^* yields

$$F^*(x) = \frac{1}{1-\beta}\left(\frac{x(1-\varphi)}{N}\right)^{\frac{1}{N-1}} \text{ for } x \in (0,w). \tag{16}$$

The probability of winning a vote with an offer of x in $(G, N/(1-\varphi)]$ is

$$\begin{aligned} H^*(x) &= \sum_{j=0}^{N-1} \binom{N-1}{j} ((1-\beta) F^*(x))^j \beta^{N-1-j} \\ &= [(1-\beta) F^*(x) + \beta]^{N-1}. \end{aligned}$$

Equating to $x(1-\varphi)/N$ and solving for F^* yields

$$F^*(x) = \frac{1}{1-\beta}\left[\left(\frac{x(1-\varphi)}{N}\right)^{\frac{1}{N-1}} - \beta\right] \text{ for } x \in (G, N/(1-\varphi)). \tag{17}$$

To complete the characterization of F^* we look for conditions to pin down β and w. The first condition is given by the continuity of F^*, which requires $F^*(w) = F^*(G)$. Substituting from (16) and (17), we get

$$\beta = \left(\frac{G(1-\varphi)}{N}\right)^{\frac{1}{N-1}} - \left(\frac{w(1-\varphi)}{N}\right)^{\frac{1}{N-1}}. \tag{18}$$

The second condition is given by the budget constraint, i.e., $\int_0^w xf(x)\,dx + \int_G^{\frac{N}{(1-\varphi)}} xf(x)\,dx = 1/(1-\varphi)$. To compute the budget constraint, observe that, on $[0,w] \cup [G, N/(1-\varphi)]$,

$$f^*(x) = \frac{1}{1-\beta}\left(\frac{(1-\varphi)}{N}\right)^{\frac{1}{N-1}} \frac{1}{N-1} x^{\frac{2-N}{N-1}},$$

and therefore

$$\int_a^b xf^*(x)\,dx = \frac{1}{1-\beta}\left(\frac{(1-\varphi)}{N}\right)^{\frac{1}{N-1}} \frac{1}{N-1}\frac{x^{\frac{N}{N-1}}}{N}(N-1)\Bigg|_a^b = \frac{(1-\varphi)^{\frac{1}{N-1}}}{1-\beta}\left(\frac{x}{N}\right)^{\frac{N}{N-1}}\Bigg|_a^b.$$

Using this equation, the budget constraint can be expressed as

$$\frac{(1-\varphi)^{\frac{1}{N-1}}}{1-\beta}\left[\left(\frac{w}{N}\right)^{\frac{N}{N-1}}+\frac{1}{(1-\varphi)^{\frac{N}{N-1}}}-\left(\frac{G}{N}\right)^{\frac{N}{N-1}}\right]=\frac{1}{(1-\varphi)},$$

or

$$\beta=\left(\frac{G(1-\varphi)}{N}\right)^{\frac{N}{N-1}}-\left(\frac{w(1-\varphi)}{N}\right)^{\frac{N}{N-1}}. \tag{19}$$

Equations (18) and (19) form a system of two equations in the unknowns w and β.

It is possible to solve for w. Indeed, from (18) and (19), w must solve

$$\left(\frac{G(1-\varphi)}{N}\right)^{\frac{1}{N-1}}-\left(\frac{w(1-\varphi)}{N}\right)^{\frac{1}{N-1}}=\left(\frac{G(1-\varphi)}{N}\right)^{\frac{N}{N-1}}-\left(\frac{w(1-\varphi)}{N}\right)^{\frac{N}{N-1}},$$

or

$$\left(\frac{G(1-\varphi)}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{G(1-\varphi)}{N}\right)=\left(\frac{w(1-\varphi)}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{w(1-\varphi)}{N}\right). \tag{20}$$

The function $h(z)=\left(\frac{z}{N}\right)^{\frac{1}{N-1}}\left(1-\frac{z}{N}\right)$ is single-peaked on $[0,\infty)$, has a maximum at $z=1$, and has value zero at $z=0$ and $z=N$. Because h is single peaked, equation (20) only has two solutions: one is $w=G$; the other is $w^*(G)=h^{-1}(G)$, where h^{-1} denote the inverse of h on the interval $[0,1]$. Only the second solution can be part of an equilibrium, since the second solution requires that β equals zero, and this is impossible when $1<G(1-\varphi)<N$. Solving for β is accomplished by substituting $w^*(G)$ into equation (18), to obtain $\beta^*(G)$. Observe that $w^*(G)$ is decreasing in G since h is increasing on $(0,1)$; therefore, $\beta^*(G)=(G(1-\varphi)/N)^{\frac{1}{N-1}}-((1-\varphi)w^*(G)/N)^{\frac{1}{N-1}}$ is increasing in G.

To verify that the pair $\beta^*(G)$ and $w^*(G)$ indeed forms an equilibrium, we need to check that the expected vote share from offering the public good, S_G, is equal to $1/N$,

$$S_G=\sum_{j=0}^{N-1}\frac{1}{j+1}\binom{N-1}{j}(1-\beta)^{N-1-j}\beta^j(F^*(G))^{N-1-j}=\frac{1}{N}.$$

Using the fact that $\sum_{j=0}^{N-1}\frac{1}{j+1}\binom{N-1}{j}q^{N-1-j}p^j=\left[(q+p)^N-q^N\right]/pN$, we rewrite the above equation as

$$\frac{\left[(1-\beta)F^*(G)+\beta\right]^N-\left[(1-\beta)F^*(G)\right]^N}{\beta N}=\frac{1}{N}.$$

Since $F^*(G)=F^*(w)$ we can rewrite this as

$$\frac{\left[(1-\beta)F^*(G)+\beta\right]^N-\left[(1-\beta)F^*(w)\right]^N}{\beta N}=\frac{1}{N},$$

and after substituting from (16) and (17), we obtain

$$\frac{\left[\frac{G(1-\varphi)}{N}\right]^{\frac{N}{N-1}} - \left[\frac{w(1-\varphi)}{N}\right]^{\frac{N}{N-1}}}{\beta N} = \frac{1}{N}.$$

This equation is the same as equation (19). Thus, when $\beta = \beta^* (G), w = w^* (G)$, it is indeed the case that $S_G = 1/N$. This shows that the pair $\beta^* (G), w^* (G)$ forms an equilibrium.

Uniqueness follows from the discussion in Section 3 together with the uniqueness of the solutions of β and w. ∎

9.2 "Lindbeck-Weibull" Model with Homogeneous Voters (PRELIMINARY)

There is a continuum of identical voters. There are two goods, money and a public good. Each voter has an endowment of one unit of money. The public good can only be produced by using all the money in the economy. The public good is perceived as G units of money by each voter. Voters' preferences are represented by a utility function U, which is assumed to be strictly increasing and concave.

There are three candidates, indexed by i. Candidates make binding promises to each voter. A candidate can offer to provide the public good (to all voters); alternatively, he can offer different taxes and transfers to different voters. Each voter votes for the candidate who promises her the greatest utility. Voters have no a priori preference for either candidate.

Voters also care about ideology. For each voter v, let y_i denote the realization of a random variable Y_i with distribution H. Suppose voter v with ideological preference (y_1, y_2, y_3) is promised consumption (c_1, c_2, c_3). Then this voter will vote for candidate 1 if

$$U(c_1) + y_1 > \max \{U(c_2) + y_2, U(c_3) + y_3\}$$

The probability that this voter votes for candidate 1 is

$$\Pr (U(c_1) + Y_1 > \max \{U(c_2) + Y_2, U(c_3) + Y_3\}).$$

Let c_i^v denote the consumption offered to voter v by candidate i, and Y_i^v denote the ideological preference of voter v for candidate i. Assume that Y_i^v and $Y_i^{v'}$ are i.i.d. for $v \neq v'$. Candidate 1 maximizes his share of the vote,

$$\int_V \Pr (U(c_1^v) + Y_1^v > \max \{U(c_2^v) + Y_2^v, U(c_3^v) + Y_3^v\})\, dv.$$

Proposition 8 *If U is sufficiently concave, there is an efficient equilibrium in pure strategies where the public good is provided if $G > 1$.*

Proof: Suppose candidates 2 and 3 promise the public good. Let us consider the benefits for candidate 1 of offering money. Candidate 1's problem is

$$\begin{aligned}
&\max_{\{c_1^v\}} \int_V \Pr\left(U\left(c_1^v\right) + Y_1^v > \max\left\{U\left(G\right) + Y_2^v, U\left(G\right) + Y_3^v\right\}\right) dv \\
&\text{s.t. } \int c_1^v = 1. \\
&= \max_{\{c_1^v\}} \int_V \Pr\left(U\left(c_1^v\right) - U\left(G\right) > -Y_1^v + \max\left\{Y_2^v, Y_3^v\right\}\right) dv \\
&\text{s.t. } \int c_1^v = 1. \\
&= \max_{\{c_1^v\}} \int_V \widetilde{H}\left(U\left(c_1^v\right) - U\left(G\right)\right) dv \\
&\text{s.t. } \int c_1^v = 1.
\end{aligned}$$

where $\widetilde{H}$ denotes the c.d.f. of the random variable $\max\left\{Y_2^v, Y_3^v\right\} - Y_1^v$. If U is sufficiently concave, the first-order conditions characterize optimality for the schedule $\{c_1^v\}$. Since the first-order conditions do not depend on v, the optimal schedule gives the same amount to each voter, $c_1^v = 1$ for all v. If $G > 1$ candidate one is better off promising the public good. ∎

We now show that it is possible to construct examples where there is an equilibrium where the public good is underprovided in the electoral equilibrium.

Example

Suppose Y_i is uniform on $[-10, 10]$. Suppose $U\left(c\right) = 100c$. There is a $\hat{G} > 1$ such that for all $G < \widehat{G}$ the following is an equilibrium: No candidate offers to provide the public good. Candidate 1 offers 3/2 to voters from 5 to 1 o'clock and zero to others, candidate 2 offers 3/2 to voters from 9 to 5 o'clock and zero to others, and candidate 3 gives 3/2 to voters from 1 to 9 o'clock and zero to others.

Proof: Similar logic to Theorem 5. Details to be finished. ∎

10 References

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