Consensus and the Accuracy of Signals: Optimal Committee Design with Endogenous Information

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Abstract

We study a model of private production of information in collective decision making. Identical agents gather costly information, and then aggregate it through voting. Because information is a public good, it will be underprovided relative to the social optimum. A "good" voting rule must give incentives to acquire information, as well as aggregate information efficiently.

We show that a voting rule that requires a large plurality (in the extreme, unanimity) to upset the status quo can be optimal only if the information available to each agent is sufficiently accurate. This result is independent of the preferences of voters and of the cost of information.

This result is stronger than results on the inferiority of unanimous verdicts by Feddersen and Pesendorfer (1998), who can only distinguish between unanimous and non-unanimous rules.

1 Introduction

This paper is concerned with consensus in collective decision making. The idea that dissent may be a healthy feature of collective decision making is at least as old as Machiavelli, who wrote:

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> Those who condemn the quarrels between the nobles and the plebes seem to be cavilling at the very things that were the primary causes of Rome's retaining her freedom, and that they pay more attention to the noise and clamours resulting from such commotions than to what resulted from them, that is, to the good effects they produced
>
> (Niccolo' Machiavelli, cited from Moscovici and Doise (1994), p. 59)

The sociological literature recognizes a relationship between dissent and the degree to which decision makers identify with a committee: "protests and disputes [...] measure the degree to which each individual is attached to [a committee]." (Moscovici and Doise (1994) p. 59. See also the literature cited therein). The implication is that identification with the committee's goals generates dissent.

In this paper, we show the reverse causation link: that dissent encourages participation in the collective decision process. We define the level of consensus of a voting rule as the fraction of votes that is required to switch from a status quo to an alternative policy. In an environment where agents can acquire information, we show that tolerating dissent in collective decision making may be necessary to encourage agents to contribute to the decision process.

Specifically, in a model where first agents acquire information, and then they vote, we show that consensual voting rules attenuate agents' incentives to invest in information. The reason is the following. Take the most consensual voting rule, unanimity. The only circumstance where a decision maker has any use for his information is when the remaining $n-1$ decision makers vote to switch from the status quo. But this is an unlikely event since, due to noisy information, agents are likely to have different perceptions about the best policy. If information is costly and must be acquired before the voting stage, agents will not acquire it. Thus, the drawback of consensual decision making is that agents may not invest in information, because they expect that their vote will not influence the final decision.

The standard model of collective decisions under uncertainty postulates that decision makers are endowed with (costless) information (Young (1988), Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1997, 1998)). There are n agents with identical preferences. There are two alternatives, a status quo and an alternative policy. Agents prefer the status quo in state of the world 1, but not in state of the world 2. Agents are endowed with noisy binary signals about the state of the world, and aggregate their information through voting.

In this world, any voting rule can be optimal, depending on the agents' preferences. Characterize a voting rule by R, the minimum number of votes required to switch from the status quo. If agents are very concerned with mistakenly abandoning the status quo, the optimal value of R/n will be high. If instead their main concern is to be stuck with the status quo when the alternative opportunity is preferable, R/n will be low.[1] In contrast, in a world where information is endogenous, the optimal value of R/n can be bounded independently of the agents' preferences.

Moreover, the model with exogenous information yields implausible normative results on the question of committee size. In that model, increasing the number of agents unambiguously helps the decision process, as each agent contributes his private information. For instance, in the face of public concern about the unpredictability of jury verdicts,[2] the existing model prescribes a (presumably large) increase in the size of juries. Why are committees not expanded in size, if that increases the accuracy of their decision? Common sense suggests that if committees and juries were enlarged, committee members would become less responsible for the decision, and thereby less engaged in the collection of information. In fact, this argument is usually invoked to explain why the average voter is so poorly informed in large elections.

This points to a model where information is endogenous, i.e., acquired by committee members. In such a model, increasing the size of the committee need not increase the aggregate information of its members, because a member's incentives to acquire information decrease with committee size. This feature captures the previous argument for keeping committees small. For this model, we derive an interesting characterization of the optimal voting rule.

We consider the same environment as in the standard model described previously, except for the information structure. We analyze a game where: First, n agents are selected to become committee members; at this stage, agents have no private informa-

[1] To determine the optimal voting rule, consider a statistician who has the same utility function as the decision makers, and observes their signals directly. The optimal decision rule is characterized by an integer R; the statistician implements the alternative policy if at least R signals indicate state of the world 2. It is clear that the analogous voting rule, which switches from the status quo if at least R agents vote 'switch', is the optimal voting rule. Under this voting rule, it is a Nash equilibrium for agents to vote sincerely, i.e. to vote 'switch' if and only if their signal indicates state 2. This is also the best possible outcome given the information in the economy, because it implements the decision of a benevolent statistician/social planner.

[2] Concerning public mistrust of the accuracy of jury decisions see "Jury Trials: A Mistery," *The Economist*, January 27, 1996. The UK recently moved to eliminate jury trials in cases that deal with several middle-ranking crimes.

tion about which of the two alternatives is better. Then, committee members decide whether to incur a cost to learn a noisy signal about the state of the world. The signal is binary, and is accurate — i.e. reveals the true state — with probability p. Finally, the committee members participate in a voting game. The rules of the game are that the status quo is implemented unless R or more agents vote 'switch'. A mechanism designer, with preferences identical to the agents', picks R and n to maximize his expected utility from the decision.

We take the fraction R/n as our measure of consensus: if R/n is close to one, we say that the voting rule is *consensual*, because a high level of consensus is necessary to change the status quo. If R/n is close to $1/2$, we say that the rule is not consensual, because the alternative policy can be implemented against the vote of a large fraction of agents.

We show that, regardless of the preferences and of the cost of information, the optimal value of R/n can never be greater than p, the accuracy of the signal. Thus, consensual voting rules (such as unanimity) can be optimal only for high values of p. This statement is interesting because it does not depend on the preferences of agents or on the cost of information. Instead, the bound on R/n depends on the quality of information available to agents.

Consider the implications for optimal jury design. In contrast to the model with a finite number of exogenously informed jurors, where unanimity can be the optimal rule if the jurors' preferences are sufficiently averse to convicting the innocent, in our model this is not enough to justify unanimity as the optimal rule; it is also necessary that jurors be very confident in their information. Conversely, if we believe that criminal juries are designed to solve the problem described previously, observing unanimity as requirement for conviction yields a lower bound on the quality of information that is available to jurors.

The reason that consensual rules cannot be optimal is that pieces of information are complementary goods when the optimal rule is consensual. We show that when R/n is close to one, an agent's value for information is increasing in the number of informed agents. In view of this, it is profitable to increase the committee size to $n+2$, which requires adjusting the voting rule to $R+1$. This process continues until the ratio R/n is sufficiently close to $1/2$. When R/n is close to $1/2$, we show that pieces of information become substitutes, in the sense that the agents' value for information is decreasing in the number of informed agents.

To obtain some intuition for how pieces of information can be complements, consider the problem of a statistician who observes signals directly. Assume he has extreme preferences; he is very concerned with mistakenly overturning the status quo. Suppose the statistician is endowed with one signal. If he is sufficiently averse to mistakenly overturning the status quo, the optimal decision is to ignore that signal and always implement the status quo. Suppose now the statistician is endowed with two signals. Again, assuming sufficiently extreme preferences, the optimal rule is to ignore the two signals. At this point, signals are neither complements nor substitutes, because adding a second signal has not changed the marginal value of the first signal. However, as we add more signals there comes a point where the statistician wants to make use of these signals. At that point, signals are complement, because adding one more signal makes the statistician want to use all of the previous signals.

Section 2 presents the model. The analysis is conducted in Sections 3 and 4.

In Section 5 we consider the case where agents are heterogeneous in their disutility from mistakenly overturning the status quo. We show that our analysis extends verbatim to this case. This is because the mechanism designer would include only one type of agent in the optimal committee. The reason for this is strategic voting. Given any voting rule, if it is incentive compatible for one type of agent to reveal his information, then the other types are unwilling to reveal any information they might have.

In Section 6 we contrast our result on consensual rules with the result of Feddersen and Pesendorfer (1998) concerning unanimity. In Section 7 we relax the assumption that the mechanism designer only controls the size of the committee and the voting rule. We show that if we allow the mechanism designer to make arbitrarily large transfers, the designer can trivially achieve an arbitrarily precise decision.

1.1 Related Literature

Most of the literature on juries has ignored the issue of information acquisition. Young (1988) discusses majority rule as a means of aggregating private information. Austen-Smith and Banks (1996) point out that, in situations such as Young's, voters generally vote strategically even if they have identical preferences. Feddersen and Pesendorfer (1997) show that large numbers of voters voting strategically aggregate information perfectly under any voting rule R that, roughly, can be expressed as a nonzero fraction of n as n goes to infinity (for example, simple majority has $R = n/2$). Furthermore, Feddersen and Pesendorfer (1998) show that information aggregation cannot happen under unanimity rule. We discuss this result in Section 7. Chwe (1998) is concerned with minority representation. He considers the mechanism design problem of maximizing the utility of one type of juror (the majority), and gives sufficient conditions under which it is optimal to give some decision power to another type of juror (the minority). The majority wants to delegate some of its decision power if the minority's interests are sufficiently collinear with its own's. This is because the minority is endowed with information that is valuable to the majority. Therefore, when the majority is very uncertain about which alternative is best, it finds it beneficial to include the minority into the decision process.

Mukhopadaya (1998) analyzes a jury with endogenous information. He assumes majority rule, and concentrates on symmetric mixed strategy equilibria. He shows that, for certain parameter values, increasing the jury size may lead to a less accurate decision.

2 The Model

We use the terminology of jury models. There are two states of the world, I and G (Innocent and Guilty). There are two possible outcomes, conviction or acquittal.

There is an infinite supply of identical agents. Agents assign prior probability $P(I)$ and $P(G)$ to the two states of the world. Agents care about the outcome of the process: $-q$ is their cost of convicting the innocent, $-(1-q)$ is the cost of acquitting the guilty. Taking the right decision gives 0. Agents maximize expected utility.

Each agent can pay a cost c and acquire a signal $X \in \{i, g\}$. The conditional probabilities of signals are $P(X = i|I) = P(X = g|G) = p > 1/2$.[3]

A voting game of n jurors with rule R is a game where each juror announces Convict or Acquit, and if R or more jurors vote to convict, the defendant is convicted. A mechanism designer picks n and R.

We define a *mechanism* as the following game:

stage 1) The mechanism designer picks n jurors and sets the voting rule R.

stage 2) The jurors observe R and n. Each juror either gets informed, at a cost c, or not.

stage 3) Each juror votes, without oberving whether other jurors have acquired information in the previous stage. If R or more jurors vote Convict, the defendant is convicted.

The mechanism designer has valuation q and $1-q$ for type I and type II errors, just

[3] Information acquisition could be reinterpreted as information processing. In that case, the cost c captures the effort that each decision maker puts into updating his beliefs given the available information.

as agents. The mechanism designer incurs a per-juror cost λ that is lexicographic with respect to his expected utility. He maximizes his expected utility from the decision.

2.1 Discussion of the Model

We assume that, at the voting stage, jurors cannot share information. This prevents jurors from coordinating at the voting stage, which would allow them to implement the optimal outcome regardless of the voting rule. Although we allow for equilibria that are suboptimal in the voting stage, we will prove that optimality in the whole game prescribes optimality at the voting stage. Thus, in the optimal mechanism for the whole game, jurors cannot improve their lot by communicating at the voting stage. Therefore, the assumption that jurors cannot share information actually makes our results more interesting, but our results would not change if jurors could communicate at the voting stage.

The mechanism designer does not take into account the cost of information incurred by jurors. This introduces a difference in objective between the mechanism designer and jurors, in the spirit of principal-agent theory. This modeling choice is appropriate to model decisions which have an impact on a large number of people besides jurors; in that case, the information costs incurred by the jurors are negligible with respect to the benefits of an accurate decision.[4]

The only role of the lexicographic cost λ is to lead the mechanism designer to prefer the smallest among all juries that deliver a given expected utility. Thus, we rule out as suboptimal environments where some jurors do not get informed with probability one. However, the cost λ does not affect the choice among two juries that yield different expected utilities. In particular, λ does not prevent the mechanism designer from increasing the jury size whenever this enhances the accuracy of the decision.

2.2 The Statistical Rule $R^S(n)$

Fix the number of jurors at n, and assume that all jurors are informed. We say that a juror votes *sincerely* if his vote replicates his signal.

Definition 1 *A juror votes* sincerely *if he votes Convict if and only if he receives* $X = g$.

[4] This argument can be made formal by modeling a mechanism designer who maximizes social utility (including the juror's information cost), and then letting the number of people in society increase.

If all jurors vote sincerely, a juror's expected payoff from a rule R is

$$\Pi(R,n) = -qP(I)\sum_{x\geq R}\binom{n}{x}(1-p)^x p^{n-x} - (1-q)P(G)\sum_{x<R}\binom{n}{x}(1-p)^{n-x}p^x.$$

Suppose a statistician chooses the voting rule to maximize a juror's expected payoff. If jurors vote sincerely, this reduces to the statistical problem of constructing the most efficient test to discriminate between the alternatives Guilty and Innocent given n independent signals. Under the assumptions of our model, the most efficient test takes the form: "convict if and only if the sum of guilty signals is $R^S(n)$ or more." We call $R^S(n)$ the *statistical rule*.

Definition 2 *The rule* $R^S(n) \in \arg\max_R \Pi(R,n)$ *is called the* ***statistical rule****.*

Next, we characterize an important property of the statistical rule. Irrespective of primitives, $R^S(n)$ grows at half the speed of n. Thus, for n large enough, the ratio $R^S(n)/n$ approaches 1/2. In other words, the optimal voting rule converges to majority rule as the number of informed jurors grows large.

Lemma 1 *Suppose* $R^S(n) \neq 0, n+1$. *Then* $R^S(n+2) = R^S(n) + 1$.

Proof: The rule $R^S(n)$ maximizes the payoff $\Pi(R,n)$, hence

$$\begin{cases} \Pi(R+1,n) - \Pi(R,n) \geq 0 \text{ for } R < R^S(n) \\ \Pi(R+1,n) - \Pi(R,n) \leq 0 \text{ for } R \geq R^S(n) \end{cases}$$

Compute

$$\begin{aligned} & \Pi(R+1,n) - \Pi(R,n) \qquad (1) \\ = \; & qP(I)\binom{n}{R}(1-p)^R p^{n-R} - (1-q)P(G)\binom{n}{R}(1-p)^{n-R}p^R \\ = \; & \binom{n}{R}(1-p)^{n-R}p^R\left[qP(I)\left(\frac{1-p}{p}\right)^{2R-n} - (1-q)P(G)\right]. \end{aligned}$$

We are interested in the sign of this expression. Define

$$h(2R-n) = qP(I)\left(\frac{1-p}{p}\right)^{2R-n} - (1-q)P(G).$$

The sign of $\Pi(R+1,n) - \Pi(R,n)$ is equal to the sign of $h(2R-n)$. Noticing that $h(2(R+1)-(n+2)) = h(2R-n)$ yields the result. ∎

If n is too small, it may be the case that $R^S(n)$ equals 0 or $n+1$. However, given any specification of $P(I)$ and q, there is a value $\underline{n}$ large enough such that $R^S(\underline{n}) \neq 0, \underline{n}+1$. For values of n larger than $\underline{n}$, $R^S(n)$ grows at half the speed of n.

For the purpose of interpretation we now designate one of the two outcomes as the status quo. The status quo is the outcome that the statistician implements unless more than half of the signals are against it.

Definition 3 *We say that the status quo is Acquittal if $R^S(n) > n/2$. Otherwise, we say that the status quo is Conviction.*

A nice property of this definition is that, which outcome is the status quo depends on $P(I)$ and q, but not on n. In fact, Acquittal is the status quo if and only if $qP(I) > (1-q)P(G)$ (this is easily seen from expression (1) above).

3 Equilibrium With n Informed Jurors

Throughout this section we fix the number of jurors at n, and assume that all jurors are informed. We solve for the equilibrium of that voting game. In the next sections we discuss information acquisition.

Consider the voting game with n informed jurors and rule $R^S(n)$. If jurors $2...n$ vote sincerely, juror 1 prefers to vote sincerely, because his incentives coincide with the statistician's. Thus, sincere voting is an equilibrium at rule $R^S(n)$. When the rule is different from $R^S(n)$, is there an equilibrium where everybody votes sincerely? The answer is in the negative, as shown in Austen-Smith and Banks (1996).

The intuition for this result is that, when R is different from $R^S(n)$ the statistician and the jurors want to move R towards $R^S(n)$. Suppose for instance that $R < R^S(n)$, so with sincere voting convictions are more frequent than they optimally should be. Then a juror will try to rectify this by always voting Acquit, regardless of his signal.

Proposition 1 *(Austen-Smith and Banks (1996)). Assume there are n informed jurors. If $R = R^S(n)$ there is an equilibrium in which all jurors votes sincerely. If $R \neq R^S(n)$ there is no equilibrium in which all jurors votes sincerely.*

Proof: See Austen-Smith and Banks (1996). ∎

The previous result has interesting implications for a juror's willingness to reveal his information: if the voting rule differs from the optimal one, a juror prefers not to reveal his information, assuming all other jurors reveal their information. Thus, suboptimal voting rules cause some information to be wasted. To understand how this happens at equilibrium, it is necessary to understand the equilibrium under a suboptimal voting rule.

We wish to characterize the most efficient pure strategy equilibrium.[5] When $R \neq R^S(n)$, at the most efficient equilibrium a fraction n_1 of the jurors ignore their information and use their vote to correct the voting rule, and the remaining $n - n_1$ jurors vote sincerely. Thus, at the most efficient pure strategy equilibrium, some of the jurors are ignoring their information.

Proposition 2 *Assume there are n informed jurors. If $R < R^S(n)$, at the most efficient pure strategy equilibrium, a number $n_1 \leq n$ of jurors vote Acquit regardless of their signal, and the remaining $n - n_1$ jurors vote sincerely. If $R > R^S(n)$, at the most efficient pure strategy equilibrium, a number $n_1 \leq n$ of jurors vote Convict regardless of their signal, and the remaining $n - n_1$ jurors vote sincerely.*

Outline of Proof: Here we outline the structure of the proof using an example. The formal argument is presented in the Appendix.

Suppose $n = 13$. Let $q = P(I) = 1/2$; then the statistical rule says to convict if and only if more than half the jurors have a guilty signal, so $R^S(13) = 7$. Suppose that $R = 9$, hence R is suboptimal. Let Juror 13 deviate from sincere voting and vote Convict regardless of his signal. Is this enough to restore the incentives of other jurors to vote sincerely? No, because now to convict we need eight out of the first twelve jurors to vote Convict, while $R^S(12) = 7$. So, let also Juror 12 deviate to always voting Convict. Now, to convict we need seven out of the first eleven jurors to vote Convict, which is still not the statistical rule. So, it is not an equilibrium for the remaining eleven jurors to vote sincerely. This procedure continues until we reach the point where Jurors 10 to 13 always vote Convict, and Jurors 1 to 9 vote sincerely. To convict we need at least five out of the first nine "sincere voters" to vote Convict, which is the statistical rule for nine jurors. Hence, it is a best response for Jurors 1 to 9 to vote sincerely at this equilibrium. What about Jurors 10 to 13? If one of them

[5] There are many other Nash equilibria, for instance, those where no juror is ever pivotal.

deviates to voting sincerely, then he is voting sincerely under a rule which is not the statistical one for the number of players, so he is better off always voting Convict. ∎

Thus, the efficiency of a jury with a suboptimal voting rule is equal to the efficiency of a smaller jury endowed with the optimal voting rule.

As an aside, the example just presented allows to replicate the result of Feddersen and Pesendorfer (1997) in pure strategy equilibria. Feddersen and Pesendorfer study symmetric (mixed strategy) equilibria in large juries where jurors are exogenously informed. They show that any rule R that can be expressed as $R = k \cdot n$ for some $k \in (0, 1)$ aggregates information prefectly as n goes to infinity. Take our example, and let n and R grow so as to keep $R = (9/13)\, n$. Reasoning as in the above example, there is an equilibrium where a fraction s of the jurors vote sincerely (Jurors 1 to 9 in the example), and the remaining $(1 - s)$ of the jurors vote to rectify the suboptimal rule (Jurors 10 to 13 in the example, who vote Convict regardless of their signal). The outcome is equivalent to that of a smaller jury of size sn which decides by simple majority rule. As n converges to infinity, such a jury aggregates information perfectly.

4 Equilibrium With Information Acquisition

In this section we look for the optimal voting rule with information acquisition. In designing the optimal rule, we allow the designer to choose the number of jurors n.

As usual in voting games, given any voting rule there are many equilibria. We focus on equilibria where agents play pure strategies. This allows us to restrict attention to equilibria where all jurors get informed. Indeed, if there were jurors who do not get informed at some equilibrium, the mechanism designer could offer another mechanism to only those jurors who get informed, replicating the original mechanism and saving on the cost λ.

Once we restrict attention to rules where all n jurors get informed, it is easy to characterize the optimal rule for given n. For all jurors to be willing to acquire information in the first stage, they all must be voting sincerely in the second stage. By Proposition 1, the only rule under which jurors vote sincerely is the statistical rule $R^S(n)$. Hence, given any number n, the only mechanism that stands a chance of inducing information acquisition on the part of the n jurors is the statistical rule $R^S(n)$.[6] This is established in Part 1 of the following Theorem.

[6] Of course, given n it may be that c is too high and therefore it is not an equilibrium for all jurors to acquire information even under the statistical rule $R^S(n)$.

Part 2 can approximately be stated as: "the optimal fraction of votes required to convict belongs to the interval $(1-p,p)$." In other words, the optimal voting rule can never be closer than $1-p$ to unanimity or veto power. Using the definition of status quo, we can rephrase this as follows. If the status quo is Acquittal, then the optimal voting rule can't require more than pn "convict" votes to convict; if the status quo is Conviction, then the optimal voting rule can't require more than pn "acquit" votes to acquit.

Theorem 1 *Assume agents play a pure strategy equilibrium. Given $c>0, P(I), p, q$, denote with n^* the optimal number of jurors and with R^* the optimal voting rule. Suppose $n^*>0$. Then*

1. $R^* = R^S(n^*)$.
2. n^* *is such that* $(1-p)\,p < \frac{n^*+1}{n^*}\frac{R^S(n^*)}{n^*+1}\left[1-\frac{R^S(n^*)}{n^*+1}\right]$.

Proof: **Part 1**: At the equilibrium of the optimal mechanism, all jurors acquire information. Indeed, take an equilibrium of a mechanism where some agents do not get informed with some probability. Because we are at a pure strategy equilibrium, these agents do not get informed with probability one. Then the equilibrium of the voting game is as described in Proposition 2. Thus, one can construct another mechanism which is equivalent in terms of outcomes, where the role of the uninformed agents is played by the mechanism itself, and all agents who get informed play the same strategy as before. This new mechanism gives the same expected utility to the mechanism designer, and saves on the lexicographic cost λ. Thus, the original mechanism is dominated.

If all jurors acquire information, it must be that they expect to make use of this information in the voting stage. Thus, jurors must be voting sincerely at equilibrium. Then Proposition 1 yields the result.

Part 2: Since c is positive, the optimal value n^* is chosen so that the marginal incentive for jurors to stay informed is greater than c at n^*, and smaller at n^*+2. Consider a putative equilibrium where all n jurors are informed and $R=R^S(n)$. By Proposition 1, an informed juror votes sincerely and nets a payoff (gross of information cost) of $\Pi(R,n)$. An uninformed juror can vote Convict,in which case the payoff is $\Pi(R-1,n-1)$; or Acquit, in which case his payoff is $\Pi(R,n-1)$. Therefore, a juror's marginal incentive to stay informed is

$$MISI(R,n) = \min\left\{\Pi(R,n)-\Pi(R-1,n-1), \Pi(R,n)-\Pi(R,n-1)\right\}.$$

At the optimal n, $MISI(R^S(n+2), n+2) - MISI(R^S(n), n)$ must be negative, for otherwise it would be possible to increase n to $n+2$ while maintaining the jurors' incentives to acquire information. From Lemma 6 in Appendix A.2, $MISI(R^S(n+2), n+2) - MISI(R^S(n), n)$ is negative if and only if

$$(1-p)\,p < \frac{R^S(n)\left[n+1-R^S(n)\right]}{(n+1)\,n}.$$

Rearranging the left hand side yields the result. ∎

If jurors were able to exchange information at the voting stage, Theorem 1 Part 1 would be trivial, since jurors would always implement the optimal (statistical) voting rule. Instead, the result is interesting because we assume that jurors cannot communicate at the voting stage. Under this assumption, one may suspect that the mechanism designer might ameliorate the incentives to acquire information by committing to a voting rule that is suboptimal given the equilibrium number of informed jurors. However, Part 1 of the previous theorem guarantees that this is not the case, because choosing a suboptimal rule causes some jurors to forego information acquisition.

Part 2 of Theorem 1 is a necessary condition for optimality, that links the voting rule to the quality of information that is available to jurors. This relationship can be used to obtain restrictions on the minimum quality of information that can justify a given voting rule as optimal. Suppose we observe a requirement of unanimity among thirteen jurors for conviction, and we believe this to be an optimal mechanism. Then, Part 2 yields that the quality of information p must be such that

$$(1-p)\,p < \frac{13+1}{13}\frac{13}{13+1}\left[1-\frac{13}{13+1}\right] = 1/14,$$

so $p > 0.92258$. The relationship in Part 2 could potentially be tested empirically. The number of members of a committee and the voting rule are observable. The quality of information p is not observable, but it can be recovered from the dispersion in the votes (remember that at equilibrium we have sincere voting). In fact, in our simple model the variance of votes is $np(1-p)$.

The condition in Part 2 requires no restriction on the parameters c, q and $P(I)$, provided that n^* is positive. This is surprising because, for fixed number of informed jurors, adjusting the relative cost of type I and II errors (choosing q large enough) guarantees that unanimity is the optimal rule. Why is this not true when information is endogenous? The reason is that, when the voting rule is consensual, the incentives to acquire information increase as the number of jurors increases. In other words, informed jurors are a complementary good.

It is surprising that the value of information can be increasing in the number of jurors. To understand the incentives to acquire information, observe that they are proportional to the probability of being pivotal. Given a voting rule R, jurors are pivotal only when there are exactly $R-1$ guilty signals out of n realizations. The probabilty of exactly $R-1$ guilty signals decreases as n increases, because any particular realization of signals becomes unlikely as n grows. This suggests that the probability of being pivotal must decrease (**effect 1**). However, notice that R itself moves with n **(effect 2)**; if the optimal rule with n jurors is R, when there are $n+2$ jurors the optimal rule becomes $R+1$ (see Lemma 1). Now, assume that unanimity rule is optimal with n jurors. As we increase the size of the jury to $n+2$ and we correspondingly adjust the voting rule, the ratio $Rule/number$ moves closer to $1/2$, and hence to p, if p is close to $1/2$. This increases the chance that jurors are pivotal, because percentages of guilty signals close to p are more likely (happen more often) than percentages close to 1.[7] Thus, effect 2 works towards increasing the probability of being pivotal. Part 2 of Theorem 1 shows that effect 2 dominates effect 1. Thus, an increase in the number of jurors increases the incentives of all jurors to acquire information.

We have focussed on pure strategy equilibria of the mechanism. What would happen if we relaxed this assumption? Intuitively, allowing for mixed strategies does not help making jurors more pivotal, but rather the opposite, as mixed strategies introduce noise in the system. Thus, mixed strategies are unlikely to help motivate agents to acquire information. To make this point, we now present an example where we look at symmetric mixed strategies equilibria, and we show that these equilibria are very inefficient for large n.

Example: Consider a jury with n jurors, $c>0$, and $P(I)=q=1/2$. Suppose the voting rule is simple majority. First, suppose all jurors are informed. Then, it is an equilibrium of the voting stage for all jurors to vote sincerely. As n grows, at this equilibrium the right decision is implemented with probability one. However, the probability that jurors are pivotal goes to zero, as it becomes extremely unlikely that exactly $n/2$ jurors vote convict.

Now, consider the information acquisition stage. For large n, it cannot be an equilibrium for all jurors to get informed, as a juror anticipates that he will almost

[7] This is because it is unlikely that *all* jurors get a guilty signal, even if the defendant is guilty; in fact, if the defendant is guilty the most likely configuration is to have pn guilty signals out of n jurors.

never be pivotal. At the symmetric equilibrium, in the information acquisition stage jurors randomize over whether to acquire information. At the voting stage, informed jurors vote sincerely and uninformed jurors randomize between voting convict and acquit. The randomization at the voting stage reduces the chance that the informed jurors are pivotal, and so decreases the incentive to acquire information in the first stage. For n large enough, the only symmetric equilibrium is for all jurors to not acquire information and then randomize at the voting stage. At this equilibrium, no information is acquired or aggregated. In contrast, at the pure strategy equilibrium some information is acquired for c low enough. ∎

5 Heterogeneous Jurors and Minority Representation

The previous analysis has assumed homogenous agents. In this section we show that our results are robust to introducing heterogeneity. We consider a model where there are two types of jurors, indexed by their disutility from convicting the innocent and cost of acquiring information. We show that the optimal jury composition only includes jurors of one type. Therefore, the analysis of the previous section applies verbatim.

The model is as before, except now there are two types of agents, 1 and 2, with $q_1 < q_2$ and cost of information acquisition c_1 and c_2. There is an infinite supply of agents of each type. The mechanism designer now maximizes a convex combination of the utility functions of the two type of agents. Thus, the mechanism designer has a disutility of convicting the innocent $q_\alpha = \alpha q_1 + (1-\alpha) q_2$.

We assume that the type of each agent is observable by all, and that the mechanism designer can pick agents of each type. The assumption that types are observable is appropriate to discuss issues such as minority representation and quotas. Such issues are relevant precisely because types are observable.

We start by considering the equilibrium in a jury with n informed jurors. Let n_1 and n_2 denote the number of jurors of each type, so that $n = n_1 + n_2$. Denote with $\Pi_1(n), \Pi_2(n)$, the expected payoffs of jurors of each type if all jurors vote sincerely, and with $\Pi_\alpha(n)$ the mechanism designer's payoff. For $j = 1, 2, \alpha$, let $R_j^S(n)$ denote the rule that maximizes $\Pi_j(n)$.

Clearly, $R_1^S(n) \leq R_\alpha^S(n) \leq R_2^S(n)$. If all jurors voted sincerely, the mechanism designer would choose rule $R_\alpha^S(n)$. However, this is typically not feasible. If jurors are sufficiently heterogeneous, $R_1^S(n) \neq R_2^S(n)$, and so by Proposition 1 there is no rule under which both types of jurors vote sincerely.

We now define a measure of heterogeneity.

Definition 4 *Assume $q_1 < q_2$. Then $H(q_1, q_2) = \frac{(1-q_1)q_2}{(1-q_2)q_1}$ is the **coefficient of heterogeneity**.*

Since $q_1 < q_2$, we have $H(q_1, q_2) \geq 1$, with equality holding if and only if $q_1 = q_2$. If $q_1 \to 0$, or $q_2 \to 1$, then $H(q_1, q_2) \to \infty$. A value of $H(q_1, q_2)$ close to 1 indicates that the preferences of types 1 and 2 are collinear.

The next Lemma gives a lower bound for $H(q_1, q_2)$ such that, if $H(q_1, q_2)$ is larger than this bound, there is no rule under which both types of jurors vote sincerely.

Lemma 2 *Let $n_1, n_2 > 0$, and assume all jurors are informed. If $H(q_1, q_2) > \left(\frac{p}{1-p}\right)^2$ there is no voting rule under which both types of jurors vote sincerely .*

Proof: Fix $n = n_1 + n_2$, and assume voters of type j vote sincerely. Then, in view of Proposition 1, the only rule under which jurors of type k vote sincerely is R_k^S. This rule solves

$$\begin{cases} \Pi_k(R, n) - \Pi_k(R-1, n) \geq 0 \text{ for } R \leq R_k^S(n) \\ \Pi_k(R, n) - \Pi_k(R-1, n) < 0 \text{ for } R > R_k^S(n) \end{cases}$$

Using expression (3) in the Appendix, this translates into

$$q_k P(I) \left(\frac{1-p}{p}\right)^{2R-n-2} - (1-q_k) P(G) \geq 0 \text{ if and only if } R \leq R_k^S(n)$$

or, equivalently,

$$R \leq \frac{1}{2}\left\{n + 2 + \frac{1}{\log\left(\frac{1-p}{p}\right)} \log\left(\frac{(1-q_k) P(G)}{q_k P(I)}\right)\right\} \text{ if and only if } R \leq R_k^S(n).$$

So, R_k^S is the larger integer smaller than $\frac{1}{2}\left\{n + 2 + \frac{1}{\log\left(\frac{1-p}{p}\right)} \log\left(\frac{(1-q_k)P(G)}{q_k P(I)}\right)\right\}$. Since $q_1 \neq q_2$, in general $R_1^S(n) \neq R_2^S(n)$. A sufficient condition for $R_1^S(n) \neq R_2^S(n)$ is that

$$\left| \frac{1}{2}\left\{n + 2 + \frac{1}{\log\left(\frac{1-p}{p}\right)} \log\left(\frac{(1-q_1) P(G)}{q_1 P(I)}\right)\right\} \right.$$
$$\left. -\frac{1}{2}\left\{n + 2 + \frac{1}{\log\left(\frac{1-p}{p}\right)} \log\left(\frac{(1-q_2) P(G)}{q_2 P(I)}\right)\right\} \right| > 1$$

which simplifies to

$$\frac{(1-q_1)\,q_2}{(1-q_2)\,q_1} > \left(\frac{p}{1-p}\right)^2 .$$

∎

Thus, if $H(q_1, q_2) > \left(\frac{p}{1-p}\right)^2$, jurors of at least one type vote uninformatively under any voting rule. By the same logic as in Section 4, their role can more cheaply be fulfilled by the voting rule. Thus, the principal gains something by restricting to one type of juror. The gain for the principal comes from the cost λ: the principal saves on participation fees by restricting access to those jurors who would not get informed. This discussion is summarized as follows:

Fact: *In the optimal mechanism, the principal does not lose anything, and sometimes gains something, by restricting to one type of juror.*

6 On Feddersen and Pesendorfer (1998)

In a model with n exogenously informed jurors, Feddersen and Pesendorfer (1998) show that unanimity can "never be optimal." More formally, Feddersen and Pesendorfer show that unanimity is a suboptimal rule as we fix $P(I)$ and q, and let n go to infinity. It is interesting to contrast their result to ours. To do this, first we present a pure strategy version of their result in an example.

Consider the most efficient pure strategy equilibrium of a voting game with $n > 2$, $q = P(I) = 1/2$, and voting rule equal to unanimity. In view of Proposition 1, simple majority is the only rule under which jurors are willing to vote truthfully. Then, given unanimity rule, only Jurors 1 and 2 can vote informatively at equilibrium; they decide the outcome according to "simple majority between the two of them." Jurors 3 through n always vote Convict.[8]

The efficiency of this jury with unanimity is equal to the efficiency of a two-member jury with simple majority rule. An increase in n only has the effect of increasing the number of those jurors who vote uninformatively, and thus has no informational benefit.

[8] This is an application of Proposition 2. To see that it is a best response for jurors $3 \ldots n$ to always vote Convict, consider the change in outcome that would result from juror 3 voting sincerely. The only case where the outcome changes is when he is pivotal. If juror 3 is pivotal it means that jurors 1 and 2 are voting Convict, i.e., both have received a g signal. In this case, juror 3 prefers to vote Convict regardless of his signal.

The result in F-P (1998) is therefore a result on the *asymptotic* inefficiency of *some* suboptimal mechanisms (unanimous rules). Given $P(I)$ and q, there are many more mechanisms which are suboptimal for given n, but are efficient asymptotically (when n goes to infinity). Among those are almost-unanimous rules such as $R = 0.99 \cdot n$. Feddersen and Pesendorfer accept these rules as "good," because asymptotically efficient.[9] In contrast, we are able to rule out such rules (if $p < 0.99$). This is because our result characterizes optimal (not only asymptotically optimal!) mechanisms.

Furthermore, notice that it is crucial for their result that the number of jurors converges to infinity while all other primitives are fixed. If we reverse the order of taking limits, we find that for any finite size of jury it is possible to find sufficiently extreme preferences such that unanimity is the optimal rule. As we have seen, this is not possible in our model unless the quality of information is very high. This is not a criticism of Feddersen and Pesendorfer, but rather a remark on the model with exogenously informed jurors, where jury size is an exogenous parameter.

7 Monetary Transfers

Until now, we have examined a problem where the mechanism designer's only instruments are the size of the jury and the voting rule. Implicitly, we have forbidden monetary transfers.[10] This is appropriate to model many situations where monetary incentives are not allowed. We now show that there is a theoretical reason to restrict to that case: the full information outcome can trivially be attained if monetary transfers are allowed.

Consider any number $n > 0$ of jurors; jurors may acquire signals $x_i \in \{i, g\}$, and they report $\widehat{x}_1 \ldots \widehat{x}_n$. Let $\widehat{x}_i \in \{0, 1\}$, with the convention that 0 means innocent and 1 means guilty. The mechanism designer wants to implement some decision rule $d(x_1, \ldots, x_n) \in \{\text{Convict, Acquit}\}$. The mechanism designer can set up a transfer scheme $T_1(\widehat{x}_1, \ldots, \widehat{x}_n), \ldots, T_n(\widehat{x}_1, \ldots, \widehat{x}_n)$.

Timing:

1) The mechanism designer picks n jurors, and sets the decision rule d and the transfer scheme $T_1, \ldots T_n$.

[9] As a side remark, this way of evaluating mechanism is quite "discontinuous," in some intuitive sense, in the space of mechanism.

[10] Except perhaps as lump sums refunds for participating to the committee meeting. Such transfers are just a rescaling of the juror's utility and have no implications for incentives.

2) Jurors observe n, d, and $T_1 ... T_n$. Then, jurors choose whether to acquire information at cost c.

3) Jurors report $(\widehat{x}_1, \dots \widehat{x}_n)$

4) The mechanism designer operates transfers $T_1(\widehat{x}_1, \dots, \widehat{x}_n), \dots, T_n(\widehat{x}_1, \dots, \widehat{x}_n)$ and implements $d(\widehat{x}_1, \dots, \widehat{x}_n)$.

By choosing the transfer scheme T appropriately, the mechanism designer can induce the jurors to report their true signals, regardless of the specification of d.

To simplify the analysis, assume $P(I) = 1/2$. Let n be even. Set

$$T_i(\widehat{x}_1, \dots, \widehat{x}_n) = \begin{cases} K & \text{if } sgn\left\{\widehat{x}_i - \frac{1}{2}\right\} = sgn\left\{\left(\sum_{j \neq i} \widehat{x}_j\right) - \frac{n-1}{2}\right\} \\ -K & \text{otherwise} \end{cases}$$

Thus, an agent gets a transfer of K if more than half the reports of the other agents agree with his report, and $-K$ otherwise. Under the assumption that the other jurors report truthfully, a juror with signal g deems it more likely that the true state is G and thus more likely that the majority of jurors reports 1. This gives him and incentive to report 1, i.e. report truthfully, to maximize the chance of receiving K. Setting K large ensures that the percentage of a juror's payoff that depends on d is small; that is, the juror's decision is guided solely by the concern of earning a large transfer. This shows that, for K large enough, reporting truthfully is an equilibrium. This procedure allows to extract the signals from the jurors, regardless of the policy d that is implemented.

For K large enough, all jurors will want to acquire information in stage 2, since the voting stage is very risky. Thus, this procedure induces all jurors to acquire information and then report it truthfully. Since this can be done for any n, the mechanism designer can reach an arbitrarily good decision by calling up large numbers of jurors.

Finally, we can modify this game so that the expected transfer is zero. This is done by imposing a tax on the jurors in stage 1, of an amount equal to the expected revenue that they expect to make from the transfer.

8 Conclusion

We have analyzed the design of institutions for collective decisions, such as management teams, committees, and juries. We have posed questions such as: Under what circumstances should collective decision making rely on majority decisions, and when should it require more stringent measures of consensus, such as supermajorities or unanimity? How large should teams or juries be?[11]

We have explored the answers to these questions within a model of private production of information. In our model, agents privately gather costly information, and then aggregate it to produce a collective decision. Because information is a public good, it will be underprovided relative to the social optimum. Our problem is therefore one of team effort, where the mechanism designer can modify the production function to achieve higher effort.

We have characterized the voting mechanism that produces the most informed decision. We have obtained a necessary condition for the voting rule to be "consensual," i.e. close to unanimity or veto power: the optimal voting rule cannot be closer to being consensual than the quality of information is close to being perfect. Although a consensual rule ensures that an alternative that carries a heavy risk is chosen only if all comittee members' information is uniformly favorable, we have shown that consensual rules don't give the right incentives to acquire information. Our analysis suggests that consensus-based decision making is suboptimal in environments where individual information is coarse, precisely those environments where collective decision making is thought to be most beneficial.

This result yields a prediction on the minimal quality of information that is consistent with unanimous voting rules being optimal. This suggests that the relationship between the voting rule and the quality of information could be tested. For this relationship to be testable, an important implicit assumption is that the aggregation of information actually happens in the formal voting phase, and not unobservably in some informal discussion prior to the vote. This is a strong assumption, given the strong empirical support for the importance of pre-vote communication in juries. From a theoretical viewpoint, however, this objection is not particularly relevant in our model, since information is aggregated optimally at equilibrium, and pre-voting communication could not improve on the outcome.

Our results are conceptually different than what can be obtained in a model with exogenously informed jurors. In such a model, Feddersen and Pesendorfer (1997, 1998) show that as the jury size increases, unanimity is inefficient, but rules requiring 99 percent of the votes to convict are efficient (in the limit). In contrast, we are able

[11] See Haleblian and Finkelstein (1993) for a review of the theoretical and empirical literature on team size. Concerning jury size, the US Supreme Court holds that juries of more or less than twelve jurors are consitutional, but there is a minimum of six jurors for nonpetty criminal cases. Today, over thirty US states allow juries of less than twelve members. See Abramson (1995), p. 180, 181. For a review of the debate on unanimity in criminal juries, see Abramson (1995), Chapter 5.

to exclude such rules as suboptimal, so long as the jurors' information is less that 99 percent accurate.

We have shown that our results are robust to introducing heterogeneity of agents, because the optimal mechanism only comprises one kind of agent. This is because, given a certain committee composition and voting rule, if one type of agent is willing to vote informatively then the other kind is not. From the viewpoint of the mechanism designer, the only contribution of agents is to report private information, so the role of committee members who do not vote informatively can cheaply be fulfilled by the mechanism itself. Incidentally, this finding seems at odds with the cross-sectional ideal that informs jury selection in the US since 1968, when Congress expressed the defendants' right to a jury "selected at random from a fair cross-section of the population."[12] Instead, our analysis emphasizes the advantage of homogeneous juries. Thus, our findings may support the practice of peremptory challenge whereby parties can eliminate prospective jurors likely to sympathize with the other side, and the traditional (pre-1968) practices of jury selection in the US which relied on notable members of a community for the pool of jurors.

[12] "The Jury Selection and Service Act," 28 U.S.C., secs. 1861-69. For a discussion of jury selection and the cross-sectional ideal see Abramson (1995), pp. 99-142.

A Proofs

A.1 Proof of Proposition 2

Proof: Let us construct the most efficient pure strategy equilibrium. Suppose $R^S(n) > 1 + n/2$ (the case where $R^S(n) < 1 + n/2$ is treated in exactly the same way). Remember, from Lemma 1, that $R^S(n)$ is increasing at a speed of $n/2$.

Case A: $R < R^S(n)$.

Start with $n_1 = 0$, and progressively increase n_1 until one of the two conditions below is satsified.

Condition 1: $R^S(n - n_1) = n - n_1 + 1$. In this case, the best pure strategy equilibrium is for everyone to vote Acquit regardless of their signal.

Condition 2: $R^S(n - n_1) = R$. In this case, there is an equilibrium where jurors $1...n_1$ vote Acquit regardless of their signal, and jurors $n_1 + 1...n$ vote sincerely. The outcome at this strategy profile is equivalent to the outcome of a game with $n - n_1$ jurors who vote sincerely. Thus, by Proposition 1 it is a best response for jurors $n_1 + 1...n$ to vote sincerely. Next, consider a deviation by a juror among $1...n_1$. Suppose he deviates to voting sincerely; then this is equivalent to a voting game with $n - n_1 + 1$ jurors and rule R which, by construction of n_1, is smaller than $R^S(n - n_1 + 1)$. Since

$$\Pi(R, n) - \Pi(R, n-1) = -\frac{R}{n}\left[\Pi(R+1, n) - \Pi(R, n)\right], \tag{2}$$

whenever R is smaller than the optimal rule $R^S(n - n_1 + 1)$, we have $\Pi(R, n - n_1 + 1) < \Pi(R, n - n_1)$, i.e. the payoff from voting sincerely is smaller than the payoff from always voting Acquit. This shows that it is not profitable for jurors $1...n_1$ to deviate to voting sincerely. A fortiori, it is not profitable to deviate to always voting Convict. Thus, it is a best response to always vote Acquit.

Case B: $R > R^S(n)$.

This case is treated symmetrically to Case A, except that here jurors who ignore their information will vote Convict. Notice that in Case B the case treated in Condition 1 cannot happen.

Finally, this is the most efficient pure strategy equilibrium because it is the equilibrium where the greatest number of jurors vote sincerely. ∎

A.2 Proof of Lemma 6

We break the proof into a series of lemmas.

The marginal incentive to stay informed is

$$MISI(R,n) = \min\left\{\Pi\left(R,n\right) - \Pi\left(R-1,n-1\right), \Pi\left(R,n\right) - \Pi\left(R,n-1\right)\right\}.$$

Lemma 3 $MISI(R,n) = \min\left\{-\frac{R}{n}\left[\Pi(R+1,n) - \Pi\left(R,n\right)\right], \frac{n-R+1}{n}\left[\Pi\left(R,n\right) - \Pi\left(R-1,n\right)\right]\right\}$

Proof: Let us start by observing that

$$\begin{aligned} & \Pi(R+1,n) - \Pi\left(R,n\right) \\ = \; & qP\left(I\right)\binom{n}{R}\left(1-p\right)^{R}p^{n-R} - \left(1-q\right)P\left(G\right)\binom{n}{R}\left(1-p\right)^{n-R}p^{R} \\ = \; & \binom{n}{R}\left(1-p\right)^{n-R}p^{R}\left[qP\left(I\right)\left(\frac{1-p}{p}\right)^{2R-n} - \left(1-q\right)P\left(G\right)\right], \end{aligned}$$

whence

$$\Pi(R,n) - \Pi\left(R-1,n\right) = \binom{n}{R-1}\left(1-p\right)^{n-R+1}p^{R-1}\left[qP\left(I\right)\left(\frac{1-p}{p}\right)^{2R-n-2} - \left(1-q\right)P\left(G\right)\right] \tag{3}$$

and

$$\begin{aligned} & \Pi(R,n-1) - \Pi\left(R-1,n-1\right) \\ = \; & \binom{n-1}{R-1}\left(1-p\right)^{n-R}p^{R-1}\left[qP\left(I\right)\left(\frac{1-p}{p}\right)^{2R-n-1} - \left(1-q\right)P\left(G\right)\right]. \end{aligned} \tag{4}$$

1) Derivation of the term $\Pi\left(R,n\right) - \Pi\left(R,n-1\right)$

The term $\Pi\left(R,n+1\right) - \Pi\left(R,n\right)$ represents the change in expected utility from adding an additional juror, as we keep fixed the rule R. The event in which the new juror changes the outcome is the event where $R-1$ out of the first n jurors vote convict. So,

$$\begin{aligned} & \Pi\left(R,n+1\right) - \Pi\left(R,n\right) \\ = \; & P\left(I\right)\Pr\left\{\sharp \text{ of } g\text{'s over } n = R-1|I\right\}\left(-q\right)\left(1-p\right) + \\ & +P\left(G\right)\Pr\left\{\sharp \text{ of } g\text{'s over } n = R-1|G\right\}\left(1-q\right)\left(p\right) \\ = \; & P\left(I\right)\binom{n}{R-1}\left(1-p\right)^{R-1}p^{n-R+1}\left(-q\right)\left(1-p\right) + \\ & +P\left(G\right)\binom{n}{R-1}\left(1-p\right)^{n-R+1}p^{R-1}\left(1-q\right)\left(p\right) \\ = \; & \binom{n}{R-1}\left(1-p\right)^{n-R+1}p^{R}\left[\left(1-q\right)P\left(G\right) - qP\left(I\right)\left(\frac{1-p}{p}\right)^{2R-n-1}\right]. \end{aligned}$$

So

$$
\begin{aligned}
& \Pi\left(R,n\right)-\Pi\left(R,n-1\right) \qquad (5)\\
= & \binom{n-1}{R-1}\left(1-p\right)^{n-R}p^{R}\left[\left(1-q\right)P\left(G\right)-qP\left(I\right)\left(\frac{1-p}{p}\right)^{2R-n}\right]\\
= & -\frac{R}{n}\left[\Pi(R+1,n)-\Pi\left(R,n\right)\right]
\end{aligned}
$$

2) Derivation of the term $\Pi\left(R,n\right)-\Pi\left(R-1,n-1\right)$

We have

$$\Pi\left(R,n\right)-\Pi\left(R-1,n-1\right)=\left[\Pi\left(R,n-1\right)-\Pi\left(R-1,n-1\right)\right]+\left[\Pi\left(R,n\right)-\Pi\left(R,n-1\right)\right]$$

Substituting from eq. (4) and (5),

$$
\begin{aligned}
& \Pi\left(R,n\right)-\Pi\left(R-1,n-1\right)\\
= & \binom{n-1}{R-1}\left(1-p\right)^{n-R}p^{R-1}\left[qP\left(I\right)\left(\frac{1-p}{p}\right)^{2R-n-1}-\left(1-q\right)P\left(G\right)\right]\\
& +\binom{n-1}{R-1}\left(1-p\right)^{n-R}p^{R}\left[\left(1-q\right)P\left(G\right)-qP\left(I\right)\left(\frac{1-p}{p}\right)^{2R-n}\right]\\
= & \binom{n-1}{R-1}\left(1-p\right)^{n-R+1}p^{R-1}\left[qP\left(I\right)\left(\frac{1-p}{p}\right)^{2R-n-2}-\left(1-q\right)P\left(G\right)\right]\\
= & \frac{n-R+1}{n}\left[\Pi\left(R,n\right)-\Pi\left(R-1,n\right)\right].
\end{aligned}
$$

∎

Lemma 4 $\left[\Pi\left(R+1,n+2\right)-\Pi\left(R,n+1\right)\right]-\left[\Pi\left(R,n\right)-\Pi\left(R-1,n-1\right)\right]$
$=\frac{n-R+1}{n}\left[\Pi\left(R,n\right)-\Pi\left(R-1,n\right)\right]\left[\frac{(n+1)n}{(n-R+1)R}\left(1-p\right)p-1\right].$

Proof:

$$
\begin{aligned}
& \left[\Pi\left(R+1,n+2\right)-\Pi\left(R,n+1\right)\right]-\left[\Pi\left(R,n\right)-\Pi\left(R-1,n-1\right)\right]\\
= & \frac{n-R+2}{n+2}\left[\Pi\left(R+1,n+2\right)-\Pi\left(R,n+2\right)\right]-\frac{n-R+1}{n}\left[\Pi\left(R,n\right)-\Pi\left(R-1,n\right)\right]\\
= & \frac{n-R+2}{n+2}\binom{n+2}{R}\left(1-p\right)^{n-R+2}p^{R}\left[qP\left(I\right)\left(\frac{1-p}{p}\right)^{2R-n-2}-\left(1-q\right)P\left(G\right)\right]
\end{aligned}
$$

$$-\frac{n-R+1}{n}\binom{n}{R-1}(1-p)^{n-R+1}p^{R-1}\left[qP(I)\left(\frac{1-p}{p}\right)^{2R-n-2}-(1-q)P(G)\right]$$
$$= \frac{n-R+1}{n}\left[\Pi(R,n)-\Pi(R-1,n)\right]\left[\frac{(n+1)n}{(n-R+1)R}(1-p)p-1\right].$$

∎

Lemma 5 $[\Pi(R+1,n+2)-\Pi(R+1,n+1)]-[\Pi(R,n)-\Pi(R,n-1)]$
$= -\frac{R}{n}[\Pi(R+1,n)-\Pi(R,n)]\left[\frac{(n+1)n}{(n-R+1)R}(1-p)p-1\right].$

Proof:

$$\begin{aligned}
&[\Pi(R+1,n+2)-\Pi(R+1,n+1)]-[\Pi(R,n)-\Pi(R,n-1)]\\
=\;&\binom{n+1}{R}(1-p)^{n-R+1}p^{R+1}\left[(1-q)P(G)-qP(I)\left(\frac{1-p}{p}\right)^{2R-n}\right]\\
&-\binom{n-1}{R-1}(1-p)^{n-R}p^{R}\left[(1-q)P(G)-qP(I)\left(\frac{1-p}{p}\right)^{2R-n}\right]\\
=\;&-\frac{R}{n}[\Pi(R+1,n)-\Pi(R,n)]\left[\frac{(n+1)n}{(n-R+1)R}(1-p)p-1\right].
\end{aligned}$$

∎

Lemma 6 *Suppose* $0<R^S(n)<n+1$*. Then* $MISI(R^S(n+2),n+2)-MISI(R^S(n),n)$ *is negative if and only if*

$$(1-p)p<\frac{R^S(n)\left[n+1-R^S(n)\right]}{(n+1)n}.$$

Proof: From the previous lemmas, we have that

$$\begin{aligned}
&[\Pi(R+1,n+2)-\Pi(R,n+1)]-[\Pi(R,n)-\Pi(R-1,n-1)]\\
=\;&\frac{n-R+1}{n}[\Pi(R,n)-\Pi(R-1,n)]\left[\frac{(n+1)n}{(n-R+1)R}(1-p)p-1\right]
\end{aligned}$$

and

$$\begin{aligned}
&[\Pi(R+1,n+2)-\Pi(R+1,n+1)]-[\Pi(R,n)-\Pi(R,n-1)]\\
=\;&-\frac{R}{n}[\Pi(R+1,n)-\Pi(R,n)]\left[\frac{(n+1)n}{(n-R+1)R}(1-p)p-1\right].
\end{aligned}$$

Let us evaluate the above quantities at $R = R^S(n)$. Notice that if $0 < R^S(n) < n$ then $R^S(n+2) = R^S(n) + 1$. Thus, if both the above quantites are positive when $R = R^S(n)$, then $MISI(R^S(n+2), n+2) - MISI(R^S(n), n)$ is positive too. In view of the definition of $R^S(n)$ the above quantities have the sign of

$$\frac{(n+1)\,n}{[n - R^S(n) + 1]\,R^S(n)}(1-p)\,p - 1.$$

The result then follows immediately. ∎

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