Moral Hazard and Limited Liability

Jacques Lawarrée
University of Washington and ECARE

and

Marc Van Audenrode
Université Laval

28 October 1999

Abstract:

Real world contracts limit the liabilities of agents by imposing constraints on their transfers or on their utilities. In an adverse selection model, Sappington (1983) has shown that the two constraints yield an equivalent problem for the principal. We show that this result does not hold in a moral hazard framework. More specifically, we show that restrictions on the utilities are stronger in the sense that they yield a lower level of effort from the agent. Moreover, given an optimal contract constrained by a limited liability on utility, one can always find a Pareto dominating contract constrained by a limited liability on transfers.

JEL Classification : D82
Keywords: Moral hazard, Limited Liability, Bankruptcy.

We thank Patrick Bolton, Mathias Dewatripont, and Fahad Khalil for helpful comments. Van Audenrode thanks Canada's SSHRC and Québec's FCAR for generous financial support.

Introduction

In situation of moral hazard, the results of a manager's actions are subject to uncertainty. One way to limit the potential consequences of this uncertainty is to buy insurance. Another way is simply to limit the extent of liability that economic agents are forced – or willing – to accept. Most countries have bankruptcy laws that allow people or corporations to limit their financial liability. Limited liability clauses are often included in private contracts. Bankers who extend loans to corporations or individuals have their liability limited to the amount of their investment. Firms accepting risky procurement contracts are often protected from excessive losses and are therefore guaranteed a certain level of profit.

The pervasiveness of limits to liability is particularly justified in situations of moral hazard. The fact that efforts, even exerted in good faith, can have random effects could deter many people from entering into contracts and probably be detrimental to economic activity.

Despite the multitude of sources and forms of limited liability, economists have classified them into two major categories. Those that guarantee a certain level of profit (or utility) and those that guarantee a certain level of transfers (or penalty). Bankruptcy laws place an upper limit to an agent's losses therefore guaranteeing him a certain level of (perhaps negative) profit. On the other hand, when buying a stock in a corporation, an investor's risk is limited to the amount invested (i.e., its transfer to the firm). Minimum wage laws also guarantee a certain level of transfer from an employer to the employee.

A limited liability constraint on transfers can in general be interpreted as resulting from limited financial resources; while the constraint on utility corresponds to a minimum level of well-being (Sappington 1983). The constraint on utility can also approximate a situation of extreme risk aversion beyond a certain level of losses. For instance, Stole (1994) argues that "it is plausible that a firms' behavior may be risk neutral over a moderate range, but risk averse for dramatic losses."

Despite these very intuitive differences in limiting liability, not much attention has been paid to the consequences of imposing one form of limited liability rather than the other. A reason might be that the two constraints are equivalent in most models. Indeed, as the now classical paper by Sappington (1983) shows, imposing a restriction on transfer payments to the agent is exactly equivalent to imposing a restriction on his utility in an adverse selection model.

In this paper, we show that this equivalence is not true in a moral hazard framework. Sappington's result relies on the one-to-one relationship between the agent's decision variable (e.g. effort) on the one hand and the commonly observed output and the contingent transfers on the other hand. In a moral hazard framework, there is no longer a one-to-one relationship between the agent's effort and his transfer. When the agent chooses a particular level of effort, his output and therefore his transfer are uncertain. Guaranteeing him a level of utility is no longer equivalent to guaranteeing a level of transfer.

We also develop a simple model of imperfect output observability which shows the same properties as the moral hazard model and confirms that the key element in breaking the equivalence between the two forms of limited liability constraints is the absence of a one-to-one relationship between effort and transfer.

A question naturally arises: which constraint should be used to limit the liability of the agent? Is a constraint “better” than the other? We show that a constraint on transfer Pareto dominates a constraint on the utility. In other words, given an optimal contract with a limited liability constraint on utility, it is possible to derive an optimal contract with the same constraint on transfer that makes both the principal and the agent better off.

In section 1, we will briefly present the intuition behind Sappington’s result of equivalence of the ways of imposing limited liability in an adverse selection framework. In section 2 we will show that this equivalence does not hold in a moral hazard model. In section 3, we will compare the optimal contracts under the two constraints. In section, we present a model of imperfect output observability.Finally, the consequences of our finding will be briefly discussed as concluding remarks.

1. Limited Liability in an adverse selection model

Sappington (1983) considers a standard adverse selection model and studies the effect on the optimal contract of two different limited liability constraints: one which is imposed upon the transfers and the other on the utility. Sappington proves that both formulations lead to the same optimal contract.

To provide some intuition for that result, consider the following adverse selection model. Suppose a principal hires an agent to perform a task and offers him a contract to regulate their relationship. The agent's effort ($e \in \Re^{+}$) together with a productivity parameter ($\theta \in \Re$) determines the output $X=\alpha(\theta, e)$. The function $\alpha(\theta,e)$ satisfies the conditions $\alpha'_e>0$, $\alpha'_\theta>0$, $\alpha''_{ee}\leq 0$, $\alpha''_{e\theta}\geq 0$, $\alpha(\theta,0)=\theta$. [1] The productivity parameter is of known density ($\theta \in [\theta^-, \theta^+]$) but its realization is private information of the agent. Effort is costly for the agent, and this cost is represented by $\varphi(e)$. The function $\varphi(e)$ satisfies the conditions $\varphi'_e>0$, $\varphi''_{ee}>0$, $\varphi(0)=0$, $lim_{e\to 0}\ \varphi'_e=0$, $lim_{e\to\infty}\ \varphi'_e=\infty$.[2] Assume that the principal freely and perfectly observes X, while he cannot observe either θ or e. The transfer from the principal to the agent is represented by $t(X)$ (with $t(X) \in \nabla$). The utility of the agent is assumed to be separable in income and utility and can then be represented by $U=t-\varphi(e)$. The principal's objective is to maximize his expected profit $\Pi=E[\alpha(\theta,e)-t]$. Assume also that the agent knows his type (θ) when signing the contract.

---

[1] α'_e denotes the first partial derivative with respect to the first argument, α_θ denotes the first partial derivative with respect to the second argument, α''_{ee} denotes the second partial derivative and $\alpha''_{e\theta}$ denotes the cross-partial derivative.

[2] φ'_e denotes the first derivative and φ''_{ee} denotes the second derivative of $\varphi(e)$.

Suppose that a limited liability constraint is imposed on the contract. Following Sappington (1983), we use two formulations. In formulation LL_t, the limited liability constraint requires the transfers to be above some level L,

$$(LL_t) \qquad t(X) \geq L, \quad \forall X$$

while formulation LL_u requires that the utility level of the agent be above the same level,

$$(LL_u) \qquad U = t(X) - \varphi(e) \geq L, \quad \forall X$$

regardless of the level of output required from the agent. In formulation LL_u, we interpret limited liability as an ex ante constraint on contracts. The fact that the agent's level of utility and effort are unobserved does not limit the enforceability of this constraint: all it does is forbid the principal and the agent from entering into contracts which could potentially result in losses for the truthful agent.

Sappington established that, in an adverse selection model, both formulations (with the same L) will lead to exactly identical optimal contracts.[3] The intuition for that result goes as follows. If the relevant constraint to the contract is (LL_t), the agent can always guarantee himself a level of utility L by exerting no effort. In that case $U = t \geq L$. Similarly, if the contract requires (LL_u), it must always give a transfer t such that $t \geq L, \ \forall X$, otherwise, the agent will never exert any effort. Clearly the maximization of the principal's objective function under the constraint of (LL_t) or (LL_u) is equivalent, since both constraints imply each other.

2. Limited Liability in a moral hazard model

We now consider the impact of the two forms of limited liability on the optimal contract in a moral hazard problem, using a simple model similar to the one presented in Kreps (1990). Assume that the principal hires an agent to perform a task and offers him a contract to regulate their relationship. The agent actions can always result in two outcomes: success or failure. In case of success, the principal collects an output X_S; while in case of failure he will only collect X_F, $X_S > X_F$.

By exerting effort, the agent can increase the probability that his actions will result in success. We note $r(e)$ the probability of success, where e is the effort exerted by the agent. In addition, we assume that $r(0)=0$; $r'_e \geq 0$; $r''_{ee} \leq 0$ and $\lim_{e\to\infty} r(e) \leq 1$.[4] The agent is assumed to be risk neutral and to have a utility function represented by $U = t - \varphi(e)$, where t is the transfer received from the principal and $\varphi(e)$ represents the cost of effort, defined in section 1.

The problem for the principal is to solve the following program:

$$\text{Max } r(e)\, X_S + [1-r(e)]\, X_F - r(e)\, t_S - [1-r(e)]\, t_F$$

s.t.

$$e \in \arg\max\, r(e)\, t_S + [1-r(e)]\, t_F - \varphi(e) \qquad \text{(IC)}$$

$$r(e)\, t_S + [1-r(e)]\, t_F - \varphi(e) \geq 0 \qquad \text{(IR)}$$

---

[3] See Sappington (1983), Appendix A for a formal proof.

[4] r'_e denotes the first derivative with respect to effort and r''_{ee} denotes the second derivative.

where t_S and t_F are the transfers paid, respectively, in case of success or failure. (IC) is the incentive constraint which insures that the agent maximizes his utility; and (IR) is the rationality constraint which guarantees a positive value to the contract for the agent. Since our problem is concave, we can implement the first order approach. (IC) then becomes:

$$r'_e\, t_S - r'_e\, t_F = \varphi'_e \qquad \text{(IC)}$$

2.1 Optimal contract without Limited Liability

The optimal contract for this simple problem when no limited liability is imposed is well known and simple since our agent is risk neutral. In this model, the optimal level of effort to be exerted by the agent is given by:

$$r'_e\, (X_S\text{-}X_F) = \varphi'_e \qquad (1)$$

This is the first best level of effort. It is also immediate to show that in this setting,

$$t_S\text{-}t_F = X_S\text{-}X_F \qquad (2)$$

which implies that the agent becomes the residual claimant and supports all the risk associated with failure, a result known in the literature as "selling the firm to the agent."

2.2 A Constraint on transfers

Assume that the limited liability constraint the principal faces is of the (LL_t) type, i.e., a minimum level on transfers. The problem for the principal becomes:

$$\text{Max } r(e)\, X_S + [1\text{-}r(e)]\, X_F - r(e)\, t_S - [1\text{-}r(e)]\, t_F$$

subject to

$$r(e)\, t_S + [1-r(e)]\, t_F - \varphi(e) \geq 0 \qquad \text{(IR)}$$

$$r'_e\, t_S + r'_e\, t_F = \varphi' \qquad \text{(IC)}$$

$$t_S \geq L, \quad t_F \geq L \qquad (\text{LL}_t)$$

where (LL_t) is the relevant form of the limited liability constraint, which guarantees that all transfers will be above the required minimum.[5] We assume that L is large enough so that LL_t is binding. We will solve the problem assuming that (IR) is not binding. It can be verified ex-post that this assumption is justified. Since IC implies that $t_S \geq t_F$ [6], the relevant limited liability constraint is $t_F \geq L$. Solving the optimization problem gives the solution:

$$r_e'(X_S - X_F) = \varphi'_e - \frac{r(e)}{r_e'}\left[r_{ee}''\left(\frac{\varphi_{ee}''}{r_e'}\right) - \varphi_{ee}''\right] \qquad (3)$$

Call e_t the optimal level of effort defined by (3).

2.3 A Constraint on Utility

Now assume that, instead of taking the form of a constraint on transfers, the limited liability takes the form of a constraint on the utility of the agent (LL_u). The problem remains identical to the one described in the preceding section, except that now

[5] For example, Innes (1990), Banerjee and Besley (1990), Schmidt (1997) and Kim (1997) examine moral hazard models with this type of limited liability constraint.

[6] Indeed, (IC) implies that $t_S - t_F = \frac{\varphi'}{r'} \geq 0$.

the relevant limited liability constraint will be $t_F - \varphi(e) \geq L$.[7] Solving the optimization problem leads to the solution:

$$r_e'(X_S - X_F) = 2\varphi'_e - \frac{r(e)}{r_e'}\left[r_{ee}''\left(\frac{\varphi_{ee}''}{r_e'}\right) - \varphi_{ee}'' \right] \qquad (4)$$

Call e_u the optimal level of effort defined by (4).

3. Comparisons of the Two Results

The inspection of the two results clearly shows that the two constraints do not lead to the same optimal level of effort: the optimal effort under LL_u (e_u) will always be lower than under LL_t (e_t).

PROPOSITION 1: *The optimal level of effort is always lower under LL_u than under LL_t.*

PROOF: Immediate, comparing (3) and (4).

In moral hazard problems, where the relationship between effort and output is random and limited liability is imposed through a minimum payment, the effort level required from the agent is set at a suboptimal level (relative to first best). This level is however higher than the one that would be required had the limited liability constraint been introduced by imposing a minimum level of utility.

---

[7] For example, Brander and Spencer (1989), Macho-Stadler and Perez-Castillo (1997, p. 65), and Stole (1994) examine moral hazard models with this type of limited liability constraint.

The intuition behind the result is simple: when limited liability is imposed through a minimum payment, the principal can increase effort simply by raising t_S. Increasing t_S alone will preserve both the LL_t constraint and the IC. To increase effort, the principal will have to pay the high transfer in r% of the cases.

On the other hand, when limited liability is imposed as a minimum level of utility, the principal has to increase both t_S and t_F to elicit effort from the agent. Indeed, the LL_u constraint shows that an increase in effort has to lead to an increase in t_F, in order to protect the agent from losses resulting from failure despite having exerted the optimal effort. This increase will in turn lead to an increase in t_S in order to meet the incentive requirements. By contrast to the previous case, the principal always has to increase transfers (100% of the cases) to elicit a higher effort.

It is interesting to contrast our result with Sappington's result. The main reason why Sappington's intuition does not carry in a moral hazard model is that the concept of agent's utility is different whether it is measured before the output is realized (expected utility) or after (effective utility). When the principal guarantees $t_S \geq L$ and $t_F \geq L$, he also ensures that the agent will receive an expected utility greater than L. The agent can always choose to exert no effort (which induces failure) and therefore receive at least L. However, if the agent exerts an effort, he cannot be sure that his effective utility will be above L. Indeed, the contract could specify that an output corresponding to a failure should be paid no more than L. However, a failure could be due to "bad luck." An agent who has effectively exerted a positive effort would receive a utility level below L.

In figure 1, we illustrate the optimal level of effort in a case where $r(e)=e/(1+e)$ and $\varphi(e)=e^2/2$. The graph shows that the limit on utility constraint appears to be a much more stringent requirement leading to a much lower optimal effort.

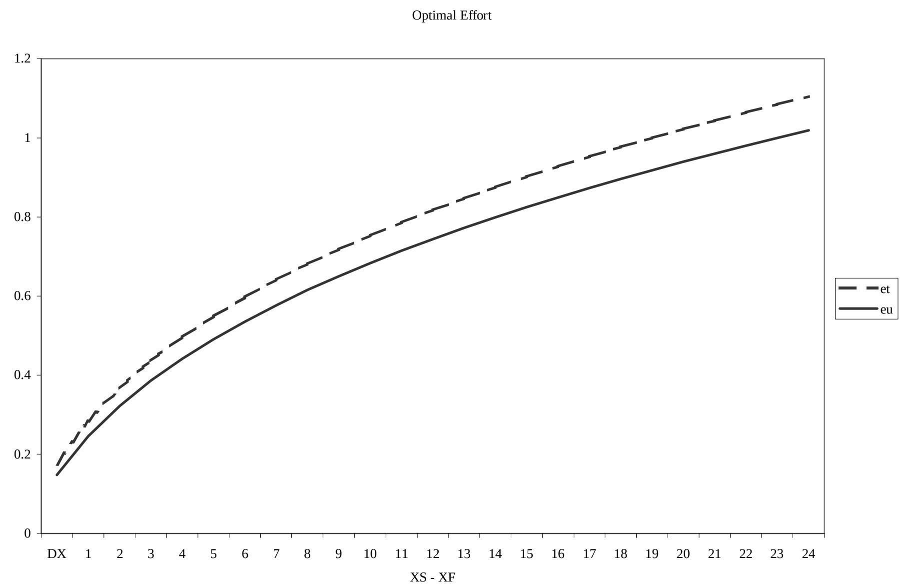

Figure 1a: Optimal Effort Under Two Forms of Limited Liability

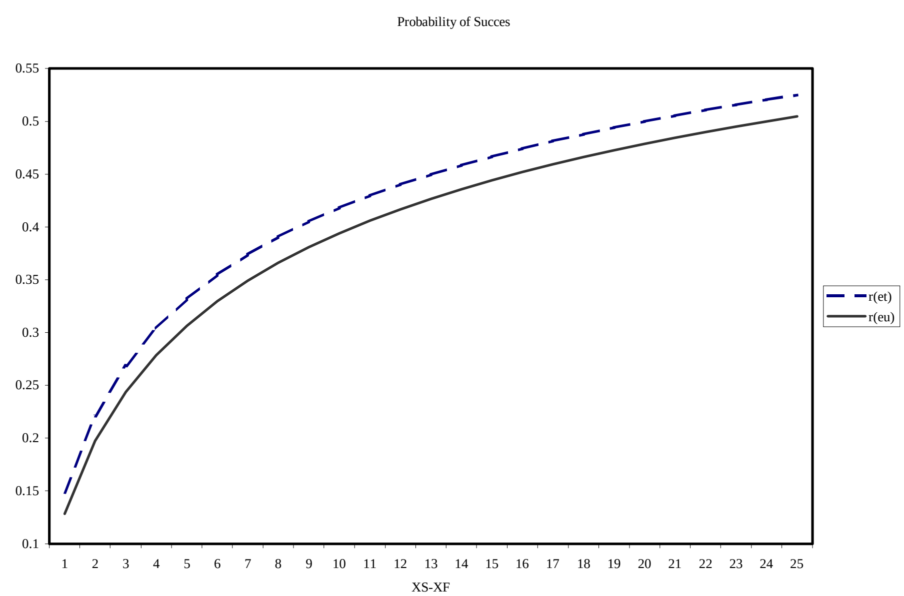

Figure 1b: Probability of Success Under Two Forms of Limited Liability

We can characterize further the two resulting contracts. First note is that it is always possible to identify two distinct levels of liability, L_t and L_u, such that the relevant agency problems give rise to the same level of expected utility for the agent when L_t is the relevant liability level under constraint LL_t and L_u is the relevant liability level under constraint LL_u. See lemmas 1 and 2 in appendix.

We can now formulate an interesting result emerging from the optimal contract: for a given level of expected utility of the agent, the principal will always receive a higher expected profit by offering the agent a liability limit on transfers (LL_t) rather than one on the utility (LL_u).

The following lemma is useful in demonstrating this result.

LEMMA 3: *Given* $E(U_t) = E(U_u)$, $E(\Pi_t) > E(\Pi_u)$ *if*

$$[r(e_t)-r(e_u)](X_S - X_F) > \varphi(e_t)-\varphi(e_u) \tag{5}$$

PROOF: See appendix.

Therefore, as long as condition (5) holds, a contract offering a limited liability of the type LL_t will yield higher profits for the principal for any given level of expected utility for the agent than a contract offering a limited liability of the type LL_u and generating the same level of expected utility for the agent.

PROPOSITION 2: *Optimal contracts with limited liability on transfers Pareto dominate optimal contracts with limited liability on utility.*

PROOF: See appendix.

This result follows from proposition I. As LL_u generates a larger distortion in effort, it forces the optimal contract farther away from the first best level than LL_t. The resulting additional loss in output for the principal is not compensated by the reduction in effort cost for the agent. This is sufficient to guarantee that inequality (5) is respected. Therefore, given lemma 3, there is an incentive compatible contract under LL_t which Pareto dominates the best incentive compatible contract under LL_u.

The intuition behind proposition 2 is illustrated in figure 2. On the graph, e_t and e_u represent the optimal level of effort given LL_t and LL_u respectively.

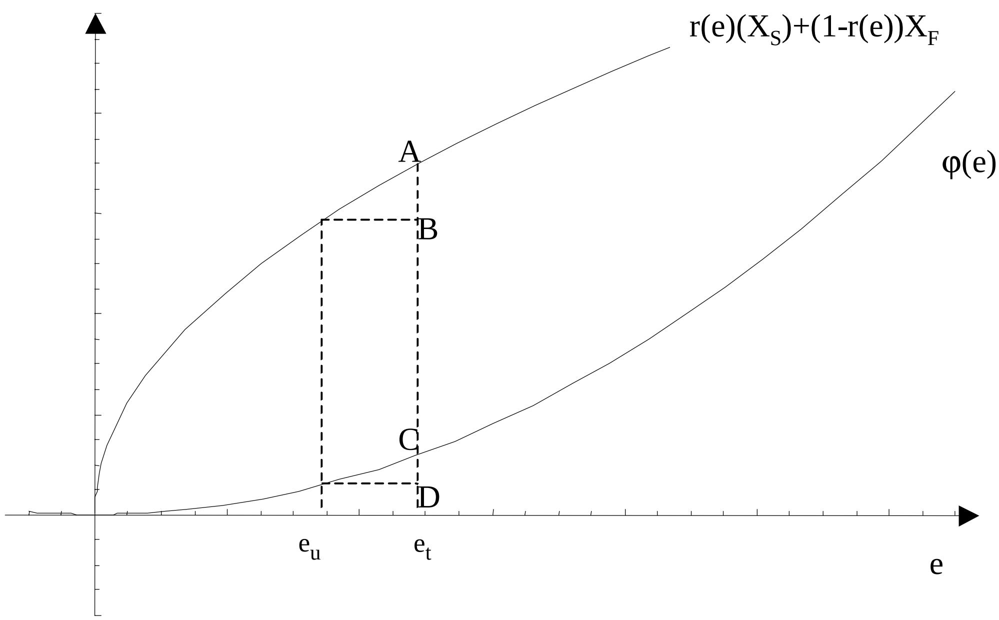

Figure 2: Optimal effort under LL_u and LL_t

Condition (5) requires that the segment (A-B) be larger than the segment (C-D). The first order conditions (equation (3)) requires that e_A and e_B must be in the area where the slope of the expected production $[r(e_t)(X_S-X_F)]$ is greater than the slope of the effort function $[\varphi_e(e_t)]$. Then, it is clear that (5) is true.

Proposition 2 has important consequences for the design of optimal contracts under limited liability. Suppose a government wishes to protect a defense contractor from bankruptcy. This could lead the government to offer a contract constrained by a limited liability on profit (utility)(see Stole (1994) for example). For any such contract, Proposition 2 tells us that there exists an alternative contract derived with limited liability on transfers that guarantees both the principal and the contractor a higher level of utility.

The reason why our result differs from Sappington's result is that, in a moral hazard model, when the agent chooses a level of effort, he does not know which transfer he will receive because the output is uncertain. In a standard adverse selection, the agent's choice variable (whether it is the effort or another variable) has a one-to-one relationship with the output observed by the principal.

Therefore, our result would also apply to an adverse selection model as long as the one-to-one relationship does not exist. An example would be a standard adverse selection model with a random shock on the output. The model becomes a hybrid mix of adverse selection and moral hazard as in Picard (1987). Another example would be a standard adverse selection model with imperfect output observability (Lawarrée-Van Audenrode (1996)). Such models are, however, complex and relatively uncommon in the literature. To focus on our point while keeping the analysis simple, we present below a model of imperfect output observability where the adverse selection parameter has been removed. We show that our propositions 1 and 2 apply.

4. Limited Liability when Output is Imperfectly Observed

Consider a model where, in equilibrium, as in our moral hazard case, the agent has only two levels of effort. He can either work hard (e=e*) or he can shirk (e=0). Imperfect output observability means that, when he works hard, the agent could be accused of shirking. Precisely, we assume that the agent working hard will be recognized as such with probability *r* (*r*>1/2) and will be said to have shirked with probability *(1-r)*. Symmetrically, an agent shirking will be detected with probability *r*. The problem for the principal will be to induce the agent to work hard while meeting its limited liability constraint. The difference between this model and the previous one is that here the principal's objective function embodies the *true* output and not the one observed by the principal.

If the limited liability constraint he is faced with is of the (LL_t) type, i.e. a minimum level on transfers, the problem for the principal is the following:

$$\max \alpha(e) - tr - (1-r)\tilde{t}$$

subject to:

$$rt + (1-r)\tilde{t} - \varphi(e) \geq \tilde{t}r - (1-r)t \qquad \text{(IC)}$$

$$rt + (1-r)\tilde{t} - \varphi(e) \geq 0 \qquad \text{(IR)}$$

$$t \geq L, \ \ \tilde{t} \geq L \qquad (LL_t)$$

where $\tilde{t}$ is the transfer paid to the agent found shirking and t is the transfer paid otherwise. Since, in this model too, $t \geq \tilde{t}$, the relevant limited liability constraint is $\tilde{t} \geq L$. Solving the optimization problem gives the solution:

$$\varphi'(.)\frac{r}{2r-1} = \alpha'_e(.)$$

When a limited liability constraint takes the form of a constraint on the utility of the agent (LL_u), the relevant limited liability constraint will be $\tilde{t} - \varphi(e) \geq L$. Solving the optimization problem leads to the solution:

$$\varphi'(.)\frac{3r-1}{2r-1} = \alpha'_e(.)$$

The inspection of the two results clearly shows that the two constraints do not lead to the same optimal level of effort. Our proposition 1 still applies. The proof is provided in the appendix.

The proof that proposition 2 also applies is similar to the one provided for the moral ahazard model and is available from the authors. This exercise shows that the crucial element that differentiate Sappington’s result from our result is the presence or not of a one-to-one relationship between effort and output.

Conclusions:

We have shown that under moral hazard and other frameworks where there is not a one-to-one relationship between effort and output, modeling a limited liability constraint as a lower limit on transfers is not equivalent to modeling it as a lower limit on utility. Moreover, contracts with limits on transfers Pareto dominate contracts with limits on utility. This has important implications on the design of optimal contracts. Limited liability restrictions guaranteeing a certain level of utility to the agent are often imposed upon the parties – like the case of bankruptcy laws for example. We have shown these contracts to be inefficient for instance when moral hazard is present.

Note that this raises the question why the limited liability constraint was imposed in the first place. In the introduction we mentioned Sappington's explanation: to guarantee a minimum level of well-being. An alternative explanation is the extreme level of risk aversion. For instance, if the defense contactor goes bankrupt, national security would be jeopardized. In such a case, the relevant limited liability constraint is always a constraint on the utility.

While many legal restrictions can easily be classified as constraint on utility or transfer, others are more difficult to categorize. Minimum wage laws, for instance, are perfect examples of a lower limit on transfers to be paid to an agent in execution of a work contract. Yet, by many aspects, the U.S. minimum wage law also guarantees a certain level of utility to some workers. This is achieved by imposing minimums to piece-rate wages and commissions, by regulating pay for overtime work as well as working conditions, and by prohibiting the employer from imposing obligations on their employees (for example forcing them to buy uniforms) which would bring their compensation below the minimum required (Murphy, 1987).

Bibliography:

Brander, James and Barbara Spencer, [1989], "Moral Hazard and Limited Liability: Implications for the Theory of the Firm," *International Economic Review*, vol. 30, No.4, pp. 833-849.

Innes, Robert, [1990], "Limited Liability and Incentive Contracting with Ex-ante Action Choices," *Journal of Economic Theory*, vol. 52, pp. 45-67.

Kim Son Ku, [1997], "Limited Liability and Bonus Contracts," *Journal of Economics and Management Strategy*, vol.6, No.4, Winter, pp. 899-913.

Kreps, David, [1990], *A Course in Microeconomic Theory*, Oxford University Press.

Macho-Stadler Inés and David Perez-Castrillo, [1997], *An Introduction to the Economics of Information*, Oxford University Press.

Murphy, Betty, [1987], "A Guide to Wage and Hour regulation," *Corporate Practice Series*, Washington, D. C.

Sappington, David, [1983], "Limited Liability Contracts between Principal and Agent," *Journal of Economic Theory*, vol. 29, pp. 1-21.

Schmidt, Klaus, [1997], "Managerial Incentives and Product Market Competition," *Review of Economic Studies*, vol.64(2), April, pp.191-213..

Appendix A: moral hazard model

LEMMA 1: *Given L_t, the following value of L_u gives the same level of expected utility to the agent:*

$$L_u = L_t + \frac{r(e_t)}{r'_{e_t}}\varphi'_{e_t} - \varphi(e_t) - \frac{r(e_u)}{r'_{e_u}}\varphi'_{e_u}$$

PROOF:

At the optimum, the expected utility of the agent under LL_t is given by:

$$E(U_t) = L_t + \frac{r(e_t)}{r'_{e_t}}\varphi'_{e_t} - \varphi(e_t)$$

while under LL_u it is

$$E(U_u) = L_u + \frac{r(e_u)}{r'_{e_u}}\varphi'_{e_u}$$

The expected utility of the agent will be identical in both problems if

$$L_t - L_u = \frac{r(e_u)}{r'_{e_u}}\varphi'_{e_u} - \frac{r(e_t)}{r'_{e_t}}\varphi'_{e_t} + \varphi(e_t) \qquad \text{(A1)}$$

LEMMA 2: *Given L_t, the following value of L_u gives the same level of expected utility to the principal:*

$$L_u = L_t + (X_S - X_F)[r(e_t) - r(e_u)] + \frac{r(e_t)}{r'_{e_t}}\varphi'_{e_t} - \frac{r(e_u)}{r'_{e_u}}\varphi'_{e_u}$$

PROOF:

At the optimum, the expected profit of the principal under LL_t is given by:

$$E(\Pi_t) = r(e_t)(X_S - X_F) + X_F - L_t - \frac{r(e_t)}{r'_{e_t}}\varphi'_{e_t} \qquad \text{(A2)}$$

while under LL_u it is:

$$E(\Pi_u) = r(e_u)(X_S - X_F) + X_F - L_u - \varphi(e_u) - \frac{r(e_u)}{r'_{e_u}}\varphi'_{e_u} \qquad \text{(A3)}$$

Therefore the principal's expected profit will be identical under LL_t and LL_u if (A2)=(A3), i.e., if

$$L_u = L_t + (X_S - X_F)[r(e_t) - r(e_u)] + \frac{r(e_t)}{r'_{e_t}}\varphi'_{e_t} - \frac{r(e_u)}{r'_{e_u}}\varphi'_{e_u} - \varphi(e_u)$$

PROOF OF LEMMA 3:

At the optimum, the principal's expected profit under LL_t will be larger than LL_u if

$$r(e_t)(X_S - X_F) + X_F - L_t - \frac{r(e_t)}{r'_{e_t}}\varphi'_{e_t} > r(e_u)(X_S - X_F) + X_F - L_u - \varphi(e_u) - \frac{r(e_u)}{r'_{e_u}}\varphi'_{e_u} \quad \text{(A4)}$$

Imposing the constraint $E(U_t) = E(U_u)$, i.e., substituting (A1) into (A4) and rearranging terms yields $[r(e_t)-r(e_u)](X_S - X_F) > \varphi(e_t)-\varphi(e_u)$.

PROOF OF PROPOSITION 2:

Using a second-order Taylor expansion around e_t, we get:

$$r(e_u) - r(e_t) \approx r'_{e_t}(e_u - e_t) + r''_{e_t e_t}\frac{(e_u - e_t)^2}{2} \qquad \text{(A5)}$$

Similarly,

$$\varphi(e_u) - \varphi(e_t) \approx \varphi'_{e_t}(e_u - e_t) + \varphi''_{e_t e_t}\frac{(e_u - e_t)^2}{2} \qquad \text{(A6)}$$

Therefore, (5) will be true if

$$(X_S - X_F)\left[r'_{e_t} + r''_{e_t e_t}\frac{(e_u - e_t)}{2}\right] > \varphi'_{e_t} + \varphi''_{e_t e_t}\frac{(e_u - e_t)}{2} \qquad \text{(A7)}$$

It is straightforward to verify that (A7) is true given (i) $r''_{e_t e_t} < 0$ and $\varphi''_{e_t e_t} > 0$, (ii) $(e_u - e_t) < 0$ by Proposition 1 and (iii) $r'_{e_t} > \varphi'_{e_t}$ from inspection of (3).

Notice that, given Lemma 2, the following "dual" relationship could also be derived: given $E(\Pi_t) = E(\Pi_u)$, $E(U_t) > E(U_u)$.

Appendix B: imperfect output observability model

PROOF OF PROPOSITION 1: (by contradiction):

Assume $e_B > e_A$. Since $\varphi_{ee} > 0$ and $\alpha_{ee} < 0$, this implies

$$\varphi_e(e_B) > \varphi_e(e_A) \qquad \text{(A1)}$$

and

$$\alpha_e(e_B) < \alpha(e_A) \qquad \text{(A2)}$$

Since $r > 1/2$, we also have $\frac{r}{2r-1} < \frac{3r-1}{2r-1}$ (A3)

Then (A1) and (A3) imply $\varphi_e(e_A)\frac{r}{2r-1} < \varphi_e(e_B)\frac{3r-1}{2r-1}$ (A4a)

which, given (2) and (3) implies $\alpha_e(e_A) = \varphi_e(e_A)\frac{r}{2r-1} < \varphi_e(e_B)\frac{3r-1}{2r-1} = \alpha_e(e_B)$ (A4b)

and this contradicts (A2).

QED.