Unemployment and Inflation Regimes

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Abstract: In this paper we study 2-state Markov switching VAR models of monthly unemployment and inflation for three countries: Sweden, United Kingdom, and the United States. We find that such models seem to provide a better description of the data than single regime VARs and need fewer lags to account for serial correlation. To interpret the regimes the empirical results are compared with the predictions from a version of Rogoff's (1985) model of monetary policy. We find that both the theoretical and the empirical results suggest that an increase in central bank "conservativeness" can be associated with either a higher or a lower variance in unemployment. In the U. S. case we find that the variance of unemployment is lower in the low inflation regime than in the high inflation regime, while the Swedish case suggests that unemployment variability is higher in the low inflation regime. According to the theoretical model this may be explained by a higher labor supply elasticity in the U. S. than in Sweden.

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1. Introduction

Many macroeconomic models, and policy discussions, are based on the assumption that there is no long run relation between inflation and unemployment. The average rate of unemployment is assumed to be equal to some natural or equilibrium rate that is determined by demand and supply conditions on the labor market, and these conditions are believed to be at most temporarily affected by factors that determine inflation. Average inflation is assumed to be a monetary phenomenon, essentially arising from a too rapid growth of money supply in relation to demand. Since central banks can control the money supply (at least in the long run) they can determine the rate of inflation independently of labor market conditions.

The existence of some long run relation between inflation and unemployment cannot be excluded on theoretical grounds. It is possible to make quite realistic assumptions about central banks' (and/or labor unions') objective functions and behavior that imply that there is such a relation. The Barro–Gordon (1983) model implies that there could be a positive long run relation. If the natural rate of unemployment increases, then average inflation may go up if the central bank has incentives to try to lower real wages through surprise inflation. Looking at U. S. data from the 1960s through the 1990s, Ireland (1998) argues that this hypothesis cannot be rejected, i.e. the long run Phillips curve seems to have a

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positive slope. Regarding the short run Phillips curve, however, the idea that there is a negative relation between inflation and unemployment seems well established (see King and Watson, 1994).[1] It has also been stressed that if this is indeed the case, then one could also expect there to be a relation between the variances of inflation and unemployment (Taylor, 1994). Evidence that supports this hypothesis, also based on U. S. data from the 1960s through the 1990s, has been presented by Lee (1999).[2]

Furthermore, it has been argued that there may be a negative relation between the average level of inflation and the variance of unemployment. The reason is that a central bank with strong aversion against inflation may choose to accommodate negative supply shocks to a smaller extent, which will result in a high variance of production and employment (Rogoff, 1985). Using cross-country data, Alesina and Summers (1993) and Jonsson (1995) have failed to find such a relation between average inflation and the volatility of unemployment. This is in itself hardly surprising, since, there are theoretical models of monetary policy which do not have the same implication as Rogoff's (e.g. models by Walsh, 1995, Persson and Tabellini, 1993, and Svensson, 1997).

The use of cross–country data is based on the assumption that monetary policy regimes are quite stable over time within individual countries, and that differences in regimes across countries are large enough to make comparisons between countries a meaningful way to examine the relation between unemployment volatility and inflation. There do indeed seem to exist differences between countries, e.g. regarding the degree of central bank independence, that are quite persistent (see e.g. Alesina, 1988, and Cukierman, 1992). But it is also the case that there are frequent changes in monetary policy in individual countries that, although they are not always associated with formal changes in institutions, reflect changes in policy makers' preferences, e.g. regarding the relative benefits of inflation and employment stabilization. The inflation process in a particular country thus sometimes undergoes changes because of changes in monetary policy, and these may also alter the relations between inflation and unemployment.

In this paper we look at unemployment and inflation data from three countries: the U. S., the U. K., and Sweden. We estimate bivariate VAR models and examine if they appear to be stable over the sample or if it is possible to detect regime changes. Two-state Markov switching VAR models are estimated and analysed using the techniques suggested by e.g.

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[1] There are certain ambiguities here, since it is not obvious that the concepts "Phillips curve" and "long run" always mean the same thing. Haldane and Quah (1998) stress the importance of distinguishing the simple unconditional correlation between inflation and unemployment (what they call a Phillips curve) from a more complex aggregate supply relation. In Ireland (1998) long run means cointegration between unemployment and inflation (only), while there is negative "short run" relation conditioned on expected inflation and the natural rate of unemployment. In King and Watson (1994) and Haldane and Quah (1998) short run Phillips curves are correlations between filtered unemployment and inflation series.

[2] The hypothesis is rejected for the full sample, but not for the subsamples 1960–79 and 1980–97.

Hamilton (1990, 1994, 1996) and Warne (1999b).[3] In order to interpret the regimes we compare the empirical results with predictions from Rogoff's (1985) model of monetary policy. In particular, we are interested in if the regime switches seem to be associated with changes in monetary policy.

In Section 2 we recapitulate the relevant relations in a Rogoff-type model (a detailed derivation is presented in the Appendix) and show how this model may be compared with a VAR model. Section 3 contains both a description of the econometric models and a rather detailed discussion of the results for the U. S. data. Results for the other countries are more briefly presented in Section 4. Section 5 contains summary and conclusions.

2. Simple Models of Unemployment and Inflation

2.1. *A Version of Rogoff's Model*

The model by Rogoff (1985) can be used to derive the following relation between unemployment and unexpected changes in the price level:

$$u_t = n^s - n^u + \omega\alpha(n^d - n^u) + \left[\omega + \frac{1}{\alpha}\right](E_{t-1}p_t - p_t) - \frac{1}{\alpha}z_t, \tag{1}$$

where u is the unemployment rate, p the log of the price level, and z a technology shock with mean zero and variance σ_z^2. The average rate of unemployment increases if there is an increase in the intercept in the labor supply function, n^s, but it also goes up if there is an increase in the intercept in the labor demand function, n^d. The reason is that the nominal wage is predetermined and that wage setters (labor unions) choose a higher real wage if labor demand goes up. This effect dominates the initial demand effect and hence unemployment goes up. Wage setters want to stabilize employment around some desired level n^u and an increase in that level leads to lower wages and lower unemployment. α is capital's share of value added and equal to the inverse of the slope of the labor demand function; the higher is α, the less does the real wage affect labor demand and hence unemployment. For the same reason, the higher is α, the less sensitive is labor demand and unemployment to technology shocks. ω is the slope of the labor supply function and hence the elasticity of unemployment with respect to inflation surprises increases in ω.[4]

The central bank wants to stabilize inflation around the inflation target π^* and employment around the target n^*, which is assumed to be higher than the equilibrium level of

[3] See also Warne (1999a) and Jacobson, Lindh, and Warne (1998) for further presentation of the econometric model and other applications.

[4] The version of Rogoff's (1985) model that produces equations (1)–(3) is presented in the Appendix. In Rogoff's original model, wages are set so as to stabilize employment around the equilibrium level that would result if wages were perfectly flexible. This implies that the average rate of unemployment is zero, which is not desirable for our purposes.

employment that would arise if wages were perfectly flexible. The relative weight on inflation in the central bank's loss function is λ. The central bank takes the unemployment equation (1) and wages and inflation expectations as given, but the private sector has rational expectations about monetary policy. The equilibrium inflation rate $\pi_t = p_t - p_{t-1}$ can be characterized as follows:

$$\pi_t = \pi^* + \frac{n^* - n^u}{\alpha\lambda} - \frac{1}{1 + \alpha^2\lambda} z_t. \tag{2}$$

There is a positive inflation bias if the central bank strives for a higher employment rate than wage setters. Following Barro and Gordon (1983) and Rogoff (1985) we assume that $n^* > n^u$. The bias will however be lower, the higher is the weight on inflation in the central bank's objective function. Using (2) in (1) gives the following expression for unemployment in equilibrium:

$$u_t = n^s - n^u + \omega\alpha(n^d - n^u) + \frac{\omega - \alpha\lambda}{1 + \alpha^2\lambda} z_t. \tag{3}$$

Table 1 summarizes how the means and variances of inflation and unemployment are affected by changes in the parameters of the model.

It is noteworthy that this model does *not* predict that the variance of unemployment is unambiguously increasing in the central bank's inflation aversion parameter, λ. A low λ implies that technology shocks are allowed to have a large effect on the price level. This, in turn, implies that the real wage responds strongly to such shocks, which partly offsets the direct effects of such shocks on labor demand. In the extreme case of $\lambda = 0$, monetary policy stabilizes employment at n^u, but this does not stabilize unemployment, which is also affected by the effects of price surprises on labor supply.[5] At low values of $\lambda (< \omega/\alpha)$, an increase in the central bank's inflation aversion may thus lower the variance of unemployment.[6]

2.2. *The Empirical Model*

It is possible to interpret empirical evidence on inflation and unemployment using the model (2)–(3). For instance, Jonsson (1995) compares inflation rates in different countries and suggests that differences are due to "conservativeness" in Rogoff's sense, i.e. π^* and/or λ. Since there are no corresponding differences in terms of unemployment (mean or variance),

[5] Generally, the equilibrium level of employment is given by $n_t = n^u + \Big(\alpha\lambda / \big(1 + \alpha^2\lambda\big)\Big) z_t$.

[6] If we define a short run Phillips curve as

$$u_t = u_t^n + \phi\big(\pi_t - E_{t-1}\pi_t\big),$$

with the natural rate of unemployment being $u_t^n \equiv n^s - n^u + \omega\alpha(n^d - n^u)$, then equations (2) and (3) imply that $\phi = \alpha\lambda - \omega$. Accordingly, if $\lambda < \omega/\alpha$, then the slope of the short run Phillips curve is negative. Reversely, if the short run Phillips curve is negatively sloped, then a small increase in the central bank's inflation aversion will lower the variance of unemployment.

Table 1 suggests that it is more likely that the differences in average inflation rates are due to π^* than λ. Ireland (1998), in contrast, does find a positive long run (cointegration) relation between inflation and unemployment in the U. S. According to Rogoff's model, such a relation could not be due to changes in π^* or λ. It could be due to changes in n^u, which is consistent with Ireland's interpretation.

However, the model of inflation and unemployment given by (2) and (3) has some features which are inconsistent with other empirical facts. In the theoretical model fluctuations in inflation and unemployment around their means stem entirely from technology shocks, i.e. the two variables are perfectly correlated. The model can be made more realistic by assuming e.g. that there are stochastic changes in the wage setting, labor supply, or labor demand schedules, or in monetary policy (π^* and λ). Moreover, one may assume that there are control errors in monetary policy. Another problem with the model (which such assumptions do not automatically solve) is its counter factual prediction that the fluctuations in inflation and unemployment are serially uncorrelated (unless technology shocks are serially correlated).

The observed persistence in inflation is sometimes attributed to price stickiness and adaptive expectations (see e.g. Galí and Gertler, 1998). An additional source of persistence in inflation might be that monetary policy affects aggregate demand and output with a lag. It seems also likely that inflation persistence partly depends on central banks' preferences for employment stabilization and interest rate smoothing (see e.g. Svensson, 1999).

In principle, the persistence of unemployment could also be due to nominal rigidities and monetary policy. It seems at least equally plausible, however, that unemployment persistence depends on properties of labor supply and wage setting functions, i.e. some kind of real rigidities.[7]

In order to analyze the empirical relations between inflation and unemployment within a model which is quite simple, yet able to capture important stylized facts, King and Watson (1994) and Ireland (1998) use a bivariate vector autoregressive (VAR) model:

$$x_t = \delta + \sum_{j=1}^{k} A_j x_{t-j} + \varepsilon_t, \tag{4}$$

where $x_t = (\pi_t, u_t)$, k is the lag length, and ε_t is a linear combination of technology shocks and shocks to labor demand or supply, wage setting, and/or monetary policy.

[7] Theoretical and empirical models of this issue have been presented by Jacobson, Vredin, and Warne (1997), Hansen and Warne (1997), and Ireland (1998), among others. Galí and Gertler (1998) suggest that real rigidities may also give rise to inflation persistence.

On the basis of Rogoff's model, we may expect the parameters of the VAR to change if there are changes in e.g. monetary policy (π^* or λ) or wage setting behavior (n^u).[8] In this paper we therefore analyze both single regime VAR models like (4) and regime dependent models like

$$x_t = \delta_{s_t} + \sum_{j=1}^{k} A_{j,s_t} x_{t-j} + \varepsilon_t, \tag{5}$$

where s_t denotes an unobservable (discrete) regime variable, and $\varepsilon_t | s_t \sim N(0, \Omega_{s_t})$. For simplicity we assume that s_t follows an ergodic Markov process with switching probabilities $\Pr[s_t = j | s_{t-1} = i, x_{t-1}, x_{t-2}, \ldots] = p_{ij}$. The theoretical model in Section 2.1 suggests that the regime may change due to changes in $\Xi = (n^s, n^u, \kappa, \alpha, \omega, \lambda, \pi^*, n^*, \sigma_z^2)$, where κ is the capital stock.

3. Unemployment and Inflation Regimes in the U. S.

In this section we shall discuss specification results for U. S. monthly data on unemployment and inflation. In relation to the general specification in (5) we shall focus on four issues. First, for a VAR model with s_t constant, is there any evidence of cointegration between output and unemployment? If there is evidence of one unit root in x_t, then the linear combination between inflation and unemployment can be interpreted as a "long run Phillips curve", possibly consistent with the Barro–Gordon and Rogoff models, as suggested by Ireland (1998). Second, does the constant (or single) regime model appear to be well specified? To address this question we shall perform some common misspecification tests. Third, we shall consider estimation of a cointegration relation under the assumption that the VAR model is subject to switching regimes. Finally, we will check which changes in the set Ξ_t that are feasible explanations for the differences between the regimes.

3.1. *Single–Regime VAR Models*

The U. S. time series for the sample period 1959:1–1998:12 are portrayed in Figure 1. The inflation series is computed from the CPI (base year is 1967) for all urban consumers (U. S. city average, not seasonally adjusted) and is taken from the U. S. Department of Labor, Bureau of Labor Statistics. The series is in natural logarithms and measured as the monthly change in annual percent. The unemployment series is also taken from the Bureau of Labor Statistics and is measured as (100 times) the natural logarithm of the civilian labor force relative to the civilian employment (number of people). Both these labor market series are seasonally adjusted and are based on workers that are 16 years or older.

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[8] Ireland (1998) does not find any signs of parameter instability in his VAR model, although such evidence has earlier been reported by King and Watson (1994). The results from Lee's (1999) multivariate GARCH model also suggest that the parameters are unstable.

Figure 2 presents scatter plots of the data. The top panel contains the monthly inflation figures measured as a yearly inflation rate ($\pi_t = 12[p_t - p_{t-1}]$), while the bottom panel depicts yearly inflation ($\pi_t^{(12)} = p_t - p_{t-12}$). As expected, the monthly variation in inflation seems to be greater than the yearly. Moreover, both inflation measures as well as unemployment seem to have positively skewed distributions, where in particular large values for unemployment tend to coincide with small values for inflation. For other values, however, it's difficult to see any relation between the variables.

The results from testing for cointegration, i.e. a long run relation between these variables, are displayed in Table 2. The statistical model is a standard, single–regime VAR(k) with Gaussian errors, centered seasonal dummies, and the constant is restricted to the cointegration space; see e.g. Johansen (1995). The restrictions on the constant ensure that if there are unit roots in x_t, then the time series will not have a linear trend. According to the asymptotic distribution of the so called trace statistic (LR_{tr} in Panel A), there is evidence of one, but not two unit roots.[9]

In Panel B we report tests of the hypothesis that either unemployment or inflation is stationary, conditional on a single unit root. For lag orders between 6 and 12, all hypotheses are rejected at the 5 percent level and only in the case of inflation at shorter lags is there an indication that the series may be stationary at the 1 percent level. The point estimates of the cointegration vector when we normalize the relation on inflation are presented in Panel C of Table 2. For lag orders between 6 and 12 the coefficient on unemployment is negative and, in absolute terms, greater than unity. Hence, the cointegration analyses from the linear VAR models suggest that there is a positive long run relation between inflation and unemployment for the U. S. data.

This finding of a cointegration relation between (or, equivalently, a common stochastic trend in) inflation and unemployment should not be surprising. Although the estimate of the normalized cointegration relation, $\pi_t - \beta_u u_t$, yields a much larger slope of the "long run Phillips curve" than reported by Ireland (1998), an estimate of β_u in the interval $[1, 2]$ is not unreasonable in view of the theoretical model in Section 2.1. According to that model, a common stochastic trend may be due to a trend in wage setters' employment goal n^u.[10] In that case, $\beta_u = 1/(\alpha\lambda(1 + \omega\alpha))$ and the estimates of β_u are consistent with "realistic values" of the theoretical parameters.

On the other hand, when we turn to the specification analysis in Table 3, we find that all these models, to various degrees, appear to suffer from serial correlation and/or conditional heteroskedasticity for the residuals. In Panel A we report two serial correlation tests, a

[9] These results are not qualitatively affected by the exclusion of seasonal dummies in the VAR.

[10] This also seems to be consistent with Ireland's (1998) interpretation of a stochastic trend in the natural rate of unemployment.

system based Ljung–Box test and a system based *LM* test, and in Panel B, equation based ARCH tests; the column "# Unit Roots" refers to the number of unit roots that have been imposed on the system, e.g. zero unit roots is an unconstrained VAR(k) model for x_t.

The *LM* tests indicate that the VAR residuals are serially correlated for all lags orders at the 5 percent level, while the Ljung–Box tests suggest that a lag order of 8 may be sufficient to capture serial correlation in x_t. Moreover, the test results are only weakly influenced by a unit root restriction.

From Panel B we find evidence of kth order ARCH in both equations at the shorter lags and in the case of inflation also for the VAR(12) models. Hence, the standard, single–regime VAR model does not seem to be consistent with the U. S. data.

3.2. *Two–State Markov Switching VAR Models*

In this section we shall examine a VAR model with 2 regimes where the regime process, for simplicity, is assumed to follow an unobserved ergodic Markov chain. Visually inspecting the unemployment series suggests that the "jumps" may either be due to large shocks to a stochastic trend or to regime shifts (or both); see Figure 1:III. The finding of a unit root in the single regime VAR models for x_t may thus be spurious. On the other hand, if there are unit roots in x_t, there are several ways one can account for such a feature in an MS–VAR model.

Karlsen (1990) presents a sufficient condition for stationarity for a q-state MS–VAR(k); see also Holst, Lindgren, Holst, and Thuvesholmen (1994). Let $e_1 \geq \ldots \geq e_{8k^2} \geq 0$ be the ordered eigenvalues (measured as e.g. the modulus) of the matrix

$$A = \begin{bmatrix} (A_1 \otimes A_1)p_{11} & (A_1 \otimes A_1)p_{21} \\ (A_2 \otimes A_2)p_{12} & (A_2 \otimes A_2)p_{22} \end{bmatrix}, \tag{6}$$

where A_{s_t} is the $2p \times 2p$ matrix obtained from a VAR(1) stacking of equation (5), and $p_{ij} = \Pr[s_t = j | s_{t-1} = i]$. Karlsen's condition for stationarity states that x_t is second order stationary if $e_1 < 1$. Similarly, if $e_1 = 1$ and $e_2 < 1$, then x_t has exactly one unit root.

A straightforward approach to imposing a unit root on the system in equation (5) is to first express it in an "error correction" form:

$$\Delta x_t = \delta_{s_t} + \sum_{i=1}^{k-1} \Gamma_{i,s_t} \Delta x_{t-i} + \Pi_{s_t} x_{t-1} + \varepsilon_t, \tag{7}$$

where $\Gamma_{i,s_t} = -\sum_{j=i+1}^{k} A_{j,s_t}$, $\Pi_{s_t} = \sum_{j=1}^{k} A_{j,s_t} - I_2 = \alpha_{s_t}\beta'$, with β being a 2×1 vector with rank 1.[11] Second, this system can be stacked in VAR(1) form, with autoregressive matrix

[11] A special case of the error correction model in (7) is discussed by Krolzig (1996).

Γ_{s_t}, and a new A matrix can be defined as in (6), but with A_{s_t} replaced with Γ_{s_t}. If $e_1 < 1$ for the new A matrix, then Δx_t and $\beta' x_t$ are stationary processes.

Alternatively, the MS–VAR model for x_t in (5) can be rewritten as an MS–VAR model for $y_t = (S\Delta x_t, \beta' x_t)$, where (S, β') has rank 2 (for instance, $S = \beta'_{\perp}$), i.e.

$$y_t = \psi_{s_t} + \sum_{j=1}^{k} B_{j,s_t} y_{t-j} + \varphi_t, \tag{8}$$

where $\psi_{s_t} = B\delta_{s_t}$, $\varphi_t = B\varepsilon_t$, $B = (S, \beta')$, and B_{j,s_t} is a function of $(B, \Gamma_{j,s_t}, \Gamma_{j-1,s_t})$ for $k \geq 2$, while B_{1,s_t} depends on $(B, \Gamma_{1,s_t}, \alpha_{s_t})$. Stacking this system in VAR(1) form, with autoregressive matrix B_{s_t}, then y_t is stationary if $e_1 < 1$ for an A matrix based on B_{s_t} rather than A_{s_t}.

For the U. S. data we find that the largest eigenvalue for an MS–VAR(3) model for x_t is about .962 and for an MS–VAR(2) model .974, thus suggesting that x_t does indeed have a unit root.

Maximum likelihood estimation of the parameters in (7) can be achieved via the EM algorithm (see e.g. Hamilton, 1990, 1994). One difficulty, relative to a model that is linear conditional on the regime (such as (5)), is the nonlinear relation involving α_{s_t} and β. In this paper we use a grid search procedure, where the grid is determined by β. This means that estimation of (7) and (8) involves solving the same problem since both systems are linear conditional on β and on the regime. We shall therefore only examine the representation in (8).

Specifically, we let the coefficient on inflation be equal to unity and vary β_u. For each value of β_u in the grid, the free parameters defined by $(p_{ii}, \psi_i, B_{j,i}, \Sigma_i : i = 1, 2; j = 1, \ldots, k)$, where $\Sigma_{s_t} = B\Omega_{s_t} B'$, are estimated via the EM algorithm and the corresponding value of the log–likelihood function is computed. The value of β_u which achieves the largest log–likelihood value is then selected as the estimate of β_u.

The grid search results from estimating a 2–state MS–VAR(2) model of y_t are summarized in Figure 3. In addition to the value of the log–likelihood function we have also plotted the largest eigenvalue for (8); the log–likelihood values have therefore been scaled in Figure 3.[12] This procedure gives us an estimate of β_u equal to .039, while the value of $\ln L$ is equal to -922.63.[13]

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[12] The scaling function is simply:

$$s\Big(\ln L\big(\beta_u\big)\Big) = 1 + \Big(\ln L\big(\beta_u\big) - \max_{\beta_u \in [-2,3]} \ln L\big(\beta_u\big)\Big)/20,$$

where the grid is specified over the interval $[-2, 3]$.

[13] For the MS–VAR model which does not impose the unit root, i.e. the system in (5) with $k = 3$ and $a_{i2,3,s_t} = 0$ for $i = 1, 2$, the value of $\ln L$ is -914.66. Relative to the model in (8), this MS–VAR has 3 additional free parameters.

The estimate of the cointegration relation conditional on two regimes thus produces a much smaller slope of "the long run Phillips curve" than what comes out of the single regime models. The estimate of β_u is also much smaller than that obtained by Ireland (1998). This result may be interpreted in two ways:

(i) There is a common trend in inflation and unemployment due to n^u. In this case, $\beta_u = 1/(\alpha\lambda(1 + \alpha\omega))$ and its small value is due to a high value of λ, the central bank's inflation aversion.[14] In this case, λ is constant across regimes.

(ii) Inflation is stationary and the stochastic trend in unemployment is due to something other than n^u, e.g. z_t, κ, or n^s. It is possible that changes in regime are due to changes in λ.[15]

We favor the second explanation, and the rest of this section will present results that support this idea.

In Table 4 we present specification tests and some system properties for 3 MS–VAR models. System 1 is defined from (8) with $y_t = (\pi_t - .039u_t, \Delta u_t)$ and $k = 2$, System 2 uses $y_t = (\pi_t, \Delta u_t)$, i.e. assumes that inflation is stationary, whereas System 3 is given by (5) with $k = 3$. In terms of the equation-by-equation tests in Panel A[16] the three MS–VAR systems behave satisfactorily. The system tests give a similar picture thus suggesting that an MS–VAR model with 2 states and a low lag order is consistent with the data.

In Panel C we report some system properties of the three MS–VAR models. Systems 1 and 2 generally display the same behavior, suggesting that conditional on a unit root inflation is stationary, whereas System 3 differs primarily in terms of its high maximum eigenvalue (e_1 close to unity). Comparing these system properties to those of the linear VAR models (see Panel C in Table 2) we find that the information criteria are smaller for the MS–VAR models. This is often due to higher log-likelihood values as well as a lower dimension of the parameter vector. Given the better performance of the specification tests, these results support the view that for the U. S. data an MS–VAR model with 2-states and a low lag order is to be preferred over a single regime model with a higher lag order.

3.3. *Regime Properties of Inflation and Unemployment in the U. S.*

In this section we will first consider the robustness of the estimated regimes over small changes in the preselected parameters. Second, the estimated first and second moments

[14] In principle, it may also be due to high values of α and/or ω, but this seems less likely.

[15] Ireland (1998) reports that inflation is indeed a borderline case and may very well be stationary. His theoretical model does not, however, allow for the possibility of different stochastic trends in inflation and unemployment. The reason is that the sources behind the trend in unemployment are not modeled.

[16] See Hamilton (1996) for details on the setup of the three hypotheses for the F-versions of the conditional scores test due to Newey (1985), Tauchen (1985), and White (1987).

conditional on the regime are presented, and, finally, we compare these to the effects of small changes in the parameters of the economic model.

The estimated smooth probabilities, i.e. $\Pr[s_t = 1|x_T, x_{T-1}, \dots x_1; \hat{\theta}]$, are displayed for 4 models in Figure 4. In Graph I the model is given by (5) with 3 lags and zero restrictions on the 3rd lag for the parameters on unemployment; Graph II gives the estimated state 1 probabilities for a 2 lag version of (5) with zero restrictions on the 2nd lag of unemployment; Graph III contains the estimates for a 2 lag model of the type in equation (8) with inflation stationary while unemployment has a unit root (i.e. this model is the same as the model in Graph I but with a unit root restriction); and finally Graph IV presents the estimates for a 2 lag version of (8) with inflation and unemployment cointegrating with the coefficient on unemployment equal to 1.84 (the Johansen ML estimate from the VAR(12) model).

The four models give very similar estimates. The major difference is the period between late 1975 and the end of 1979. Here the model estimates in Graphs I and III suggest that the regime process remains in Regime 1, whereas the estimates in Graphs II and IV prefer Regime 2. From a statistical point of view, the models that yield the plots in Graphs I and III are to be preferred.[17] Comparing these estimates to the case when $\beta_u = .039$ (the grid estimate), we find that the smooth probabilities are virtually the same as those for the $\beta_u = 0$ case. The maximum posterior estimates of the regime process are taken from the $\beta_u = 0$ model, and these regime estimates are displayed in Figure 1, where the shaded areas represent Regime 2.[18]

In Table 5 we present the estimated unconditional (Panel A) and conditional (Panel B) moments of y_t systems under the grid estimate of β_u and under the assumption that inflation is stationary while unemployment has a unit root. The conditional moments refer to conditioning on the current state only, e.g. the conditional mean is $E[y_t|s_t]$; analytical formulas and the estimation of such moments is examined by Warne (1999b). From Panel B it can be seen that inflation (or the cointegrating relationship) tends to be higher on average in Regime 1 and also more volatile than during Regime 2. Similarly, unemployment is typically rising in Regime 1 and falling during Regime 2.[19] Hence, Regime 1 (Regime 2) can be characterized as a high (low) inflation, rising (falling) unemployment regime with large (small) variances.

The effects on the first and second moments of inflation and unemployment from changing the theoretical parameters in the model in Section 2.1 were presented in Table 1. Since

[17] There are signs of model misspecification for the 2 lag model of x_t, while the model with $\beta_u = 0$ has a much higher log-likelihood value than the model with $\beta_u = 1.84$.

[18] The maximum posterior estimate for s_t is defined by

$$\hat{s}_t = \arg\max_{i=\{1,2\}} \Pr\Big[s_t = i \Big| x_T, x_{T-1}, \dots, x_1; \hat{\theta}\Big], \qquad t = p+1, \dots, T.$$

[19] The standard errors are computed using the delta method with numerical partial derivatives.

we have modeled the U. S. unemployment series as nonstationary (while unemployment growth is taken as stationary), the theoretical model needs to be respecified to account for such nonstationarity. In the Appendix we present two plausible hypotheses. First, there is a (common) stochastic trend in the wage setters' and central bank's employment targets. Second, there is a stochastic trend in technology. In the latter case, inflation is always stationary, while the former case implies that inflation is stationary when $n_t^* - n_t^u$ is stationary.

In Table 6 we give the effects on the mean and the variance of inflation and unemployment growth from changes in the wage elasticity of labor supply (ω) and the central bank's weight on inflation (λ). The theoretical predictions from changes in these parameters are quite similar under both types of nonstationarity (cf. Panels A and B in Table 6). In particular, if the central bank's weight on inflation (λ) is higher in Regime 2 than in Regime 1, shifting from Regime 1 to Regime 2 results in a lower mean and variance of inflation as we have found for the U. S. data. According to both theoretical models, average unemployment growth will, however, not be affected. Since the mean rate of change in Regime 1 is not significantly different from zero, it still seems quite likely that it is differences in λ that separate the two regimes. Specifically, an increase in λ may raise or lower the variance of unemployment growth depending on how large λ is in relation to ω and α (capital's share of value added). The properties of the MS–VAR model are thus consistent with the prediction of the theoretical model if λ is not too large. Hence, the theoretical model can explain the empirical patterns in Table 5.

In Figure 5 we have plotted unemployment against monthly and yearly inflation, respectively, for the 10 subsamples determined by the maximum posterior estimate of the regime process. In most cases, the plots suggest a horizontal Phillips curve within a subsample although there are some weak tendencies of a negative relation.

3.4. *Conclusions about the U. S. Data*

Our model of unemployment and inflation in the U. S. shares certain similarities with, but is in important ways different from, the model by e.g. Ireland (1998) and King and Watson (1994). Ireland suggests that there is a stochastic trend in the natural level of unemployment that, through monetary policy, is translated into a stochastic trend in inflation. U. S. inflation rose (fell) before (after) 1980 because of the changes in the natural rate of unemployment, but monetary policy has been stable over time. In contrast, our results suggest that the stochastic trend in unemployment does not influence inflation, and that a short run relation between unemployment and inflation has varied because of changes in monetary policy. The period 1973–83 was a high inflation regime because the Fed put relatively more weight on employment during this period. Analogously, 1991–98 has been a low inflation regime because the weight on employment has been relatively low.

Our analysis does not suggest that negative supply shocks are unimportant for inflation. But in the theoretical model we propose, and we believe also in reality, such shocks do not affect the *average* level of inflation. If there is strong persistence in inflation, single negative supply shocks may of course have lasting effects on inflation, but this only raises the question why there is such persistence in inflation. Many theoretical models suggest that inflation persistence will increase if the central bank's objective function puts a larger weight on employment stabilization (e.g., Rogoff, 1985, Svensson, 1999). This is also our explanation of why inflation stays high for extended periods of time.[20]

Orphanides (1999) shows that the Fed overestimated the potential level of output during the 1970's. In the version of Rogoff's (1985) model that we use, this may be interpreted as an increase in the central bank's employment target n^*, and will indeed give rise to a larger inflation bias whenever $n^* > n^u$ (wage setters' employment target). However, this should affect neither the variance of inflation nor the variance of unemployment, while empirically we do find that high inflation regimes are associated with more volatility. Such a relation is easier to understand if we allow for changes in the central bank's inflation aversion.

Ireland (1998) proposes that inflation and unemployment are both nonstationary (although inflation is a borderline case) and cointegrated. King and Watson (1994) also suggest that both series are nonstationary, but do not find them to be cointegrated. They stress that the links between inflation and unemployment are unstable over time, and indicate that there are recurrent changes in regime. Conditional on the regimes, we find that inflation and unemployment may be cointegrated, but in that case the "long run Phillips curve" is almost horizontal and we may as well treat inflation as stationary.

King and Watson emphasize that a distinguishing feature of the 1970–92 period is persistence in the effects of shocks. Our results are consistent with the view that this is mainly due to the Fed's weight on employment being relatively high during 1971–83.

[20] Ireland notes that his (Barro–Gordon) model cannot explain the persistence in inflation.

Appendix

We present a version of Rogoff's (1985) model. The presentation follows Rogoff closely, and where notation is obvious we leave out detailed explanations.

Production is determined by a Cobb–Douglas function

$$y_t = \alpha\kappa + (1-\alpha)n_t + z_t, \tag{A.1}$$

where κ is the fixed capital stock and technology, z_t, follows the process

$$z_t = \rho z_{t-1} + \varepsilon_{z,t}, \tag{A.2}$$

with $\varepsilon_{z,t}$ being white noise with mean zero and variance σ_z^2. Profit maximization, taking p_t and w_t as given, yields the labor demand function

$$n_t^D = n^d - \frac{1}{\alpha}(w_t - p_t) + \frac{1}{\alpha}z_t, \tag{A.3}$$

where $n^d = \kappa + \ln(1-\alpha)/\alpha$. The (notional) labor supply function is assumed to be given by

$$n_t^S = n^s + \omega(w_t - p_t). \tag{A.4}$$

However, the wage is set at w_t^f in period $t-1$, and labor is supplied infinitely elastically at that wage in period t. Hence, employment in period t is given by

$$n_t = n^d - \frac{1}{\alpha}(w_t^f - p_t) + \frac{1}{\alpha}z_t. \tag{A.5}$$

The wage is set in order to minimize $E_{t-1}(n_t - n_{t-1}^u)^2$, where n_{t-1}^u is the wage setters' employment target in period $t-1$ for period t. The nominal wage for period t is therefore

$$w_t^f = E_{t-1}p_t + \alpha(n^d - n_{t-1}^u) + \rho z_{t-1}. \tag{A.6}$$

The central bank's objective function is given by

$$\Lambda_t = (n_t - n_t^*)^2 + \lambda(\pi_t - \pi^*)^2, \tag{A.7}$$

where n_t^* is the central bank's employment target for period t. Minimizing Λ_t with respect to p_t, using (A.5), gives

$$p_t = \frac{\dfrac{w_t^f}{\alpha^2} + \dfrac{1}{\alpha}\left(n_t^* - n^d - \dfrac{z_t}{\alpha}\right) + \lambda(p_{t-1} + \pi^*)}{\lambda + \dfrac{1}{\alpha^2}}. \tag{A.8}$$

Rational expectations, (A.6) and (A.8) imply

$$E_{t-1}p_t = p_{t-1} + \pi^* + \frac{1}{\alpha\lambda}(E_{t-1}n_t^* - n_{t-1}^u). \tag{A.9}$$

Using (A.6) and (A.9) in (A.8) and defining $\pi_t = p_t - p_{t-1}$ then yields

$$\pi_t = \pi^* - \frac{1}{1+\alpha^2\lambda}\varepsilon_{z,t} + \frac{1}{\alpha\lambda(\lambda+\alpha^{-2})}\left(\lambda n_t^* + \frac{1}{\alpha^2}E_{t-1}n_t^*\right) - \frac{1}{\alpha\lambda}n_{t-1}^u. \tag{A.10}$$

Defining unemployment as $u_t = n_t^S - n_t$ (noting that $w_t = w_t^f$) and using (A.6) we get

$$u_t = n^s - n_{t-1}^u + \omega\alpha(n^d - n_{t-1}^u) + \left[\omega + \frac{1}{\alpha}\right](E_{t-1}p_t - p_t) + \omega\rho z_{t-1} - \frac{1}{\alpha}\varepsilon_{z,t}. \tag{A.11}$$

Inserting (A.9) and (A.10) into (A.11) gives

$$u_t = n^s - n_{t-1}^u + \omega\alpha(n^d - n_{t-1}^u) - \frac{1+\alpha\omega}{1+\alpha^2\lambda}(n_t^* - E_{t-1}n_t^*) + \omega\rho z_{t-1} + \frac{\omega-\alpha\lambda}{1+\alpha^2\lambda}\varepsilon_{z,t}. \tag{A.12}$$

The components in unemployment are due to employment being determined by

$$n_t = n_{t-1}^u + \frac{1}{1+\alpha^2\lambda}(n_t^* - E_{t-1}n_t^*) + \frac{\alpha\lambda}{1+\alpha^2\lambda}\varepsilon_{z,t}, \tag{A.13}$$

while labor supply is

$$n_t^S = n^s + \omega\alpha(n^d - n_{t-1}^u) - \frac{\alpha\omega}{1+\alpha^2\lambda}(n_t^* - E_{t-1}n_t^*) + \omega\rho z_{t-1} + \frac{\omega}{1+\alpha^2\lambda}\varepsilon_{z,t}. \tag{A.14}$$

The assumptions $n_t^* = n^*$, $n_t^u = n^u$, and $\rho = 0$ yield the inflation and unemployment relations in equations (2) and (3).

A Stochastic Trend in the Employment Targets

Suppose that the employment targets evolve according to the process

$$n_t^u = n_{t-1}^u + \varepsilon_{u,t}, \tag{A.15}$$

$$n_t^* = n^* + \gamma n_{t-1}^u, \tag{A.16}$$

where $\varepsilon_{u,t}$ is white noise with mean zero and variance σ_u^2. In addition, suppose that $\rho = 0$. From (A.10) we find that

$$\pi_t = \pi^* + \frac{1}{\alpha\lambda}n^* + \frac{\gamma-1}{\alpha\lambda}n_{t-1}^u - \frac{1}{1+\alpha^2\lambda}z_t,$$

while (A.12) provides us with

$$u_t = n^s + \omega\alpha n^d - (1+\alpha\omega)n_{t-1}^u + \frac{\omega-\alpha\lambda}{1+\alpha^2\lambda}z_t.$$

Hence, unemployment is nonstationary and driven by the stochastic trend in wage setters' employment target, while the linear combination $\pi_t - \beta_u u_t$ is stationary with

$$\beta_u = \frac{1-\gamma}{\alpha\lambda(1+\alpha\omega)}.$$

We thus find that inflation is stationary if (and only if) $\gamma = 1$. Moreover, the sign of the slope of the long run Phillips curve depends entirely on how big γ is.

Moreover, if we define a short run Phillips curve according to

$$u_t = u_t^n + \phi(\pi_t - E_{t-1}\pi_t), \tag{A.17}$$

with the natural rate of unemployment being given by

$$u_t^n \equiv n^s + \omega\alpha n^d - (1 + \alpha\omega)n_{t-1}^u$$

then the slope of the short run Phillips curve is given by $\phi = \alpha\lambda - \omega$. The model thus exhibits a negatively sloped curve if $\lambda < \omega/\alpha$.

A Stochastic Trend in Technology

Suppose instead that $n_t^u = n^u$, $n_t^* = n^*$, while $\rho = 1$. From (A.10) we now have that

$$\pi_t = \pi^* + \frac{1}{\alpha\lambda}\left(n^* - n^u\right) - \frac{1}{1 + \alpha^2\lambda}\varepsilon_{z,t},$$

while (A.12) gives us

$$u_t = n^s - n^u + \omega\alpha(n^d - n^u) + \omega z_{t-1} + \frac{\omega - \alpha\lambda}{1 + \alpha^2\lambda}\varepsilon_{z,t}.$$

Inflation is thus stationary, while unemployment is nonstationary and driven by the stochastic trend in technology. Accordingly, $\beta_u = 0$.

In this case, there are two ways we can define the natural rate of unemployment for (A.17). With

$$u_t^n \equiv n^s - n^u + (n^d - n^u) + \omega z_{t-1},$$

we find that $\phi = \alpha\lambda - \omega$. If instead we let

$$u_t^n \equiv n^s - n^u + (n^d - n^u) + \omega z_t,$$

then $\phi = \alpha\lambda(1 + \alpha\omega)$. Hence, the latter definition leads to a positively sloped short run Phillips curve, while the former definition is consistent with both a negative and a positive slope parameter.

TABLE 1: Effects on the mean and the variance of inflation and unemployment from changes in the theoretical parameters.

		Inflation		Unemployment	
Parameter	Interpretation	Mean	Variance	Mean	Variance
π^*	inflation target	+	0	0	0
n^*	central bank's employment target	+	0	0	0
n^u	wage setters' employment target	–	0	–	0
λ	central bank's weight on inflation	–	–	0	?
α	capital's share of value added	–	–	?	?
n^s	labor supply	0	0	+	0
κ	capital stock	0	0	+	0
ω	wage elasticity of labor supply	0	0	?	?
σ_z^2	variance of supply shock	0	+	0	+

NOTES: If n^d is greater (less) than n^u, then the mean of unemployment is increasing (decreasing) in ω (cf. equation (3)). Similarly, the mean of unemployment is increasing (decreasing) in α if $n^d + 1/(1-\alpha)$ is greater (less) than n^u. The variance of unemployment is equal to $V(u_t) = \left(\omega - \alpha\lambda\right)^2 \sigma_z^2 / \left(1 + \alpha^2\lambda\right)^2$. This variance is increasing (decreasing) in ω if λ is less (greater) than ω/α; it is increasing in α if $\lambda \in \left(\omega/\alpha, 2(\omega/\alpha) + 1/\alpha^2\right)$ and decreasing in α if $\lambda < \omega/\alpha$ or $\lambda > 2(\omega/\alpha) + 1/\alpha^2$; and it is increasing (decreasing) in λ if λ is greater (less) than ω/α.

TABLE 2: Cointegration analysis for bivariate VAR(k) models of inflation and unemployment for the U. S., 1959:1–1998:12

(A) *Cointegration Tests*

# lags	# Unit Roots	Eigenvalue	LR_{tr}	p-value
6	2	.0493	28.71	.00
	1	.0099	4.72	.32
8	2	.0409	25.63	.01
	1	.0125	5.92	.20
10	2	.0572	32.34	.00
	1	.0099	4.67	.32
12	2	.0575	30.96	.00
	1	.0069	3.24	.54

(B) *Testing for Stationarity*

	u_t		π_t	
# lags	LR	p-value	LR	p-value
6	15.04	.00	6.48	.01
8	8.56	.00	7.35	.01
10	13.22	.00	14.07	.00
12	15.24	.00	14.99	.00

(C) *Estimates of* $\pi_t - \beta_u u_t$

# lags	β_u	$\ln L$	AIC	BIC	LIL
6	1.22	−943.58	4.10	4.74	4.20
8	1.71	−924.50	4.07	4.88	4.20
10	1.91	−907.34	4.05	5.05	4.21
12	1.84	−889.93	4.03	5.20	4.21

TABLE 3: Testing for serial correlation and ARCH for the U. S. in a linear VAR(k) model, 1959:1–1998:12

(A) *Serial Correlation Tests*

# lags	# Unit Roots	Ljung-Box Test	p-value	*LM* Test	p-value
6	0	527.32	.01	18.81	.00
	1	526.18	.01	18.60	.00
8	0	481.44	.08	19.48	.00
	1	482.53	.09	18.75	.00
10	0	485.36	.03	20.84	.00
	1	486.99	.03	20.09	.00
12	0	460.10	.09	12.02	.02
	1	459.91	.10	11.72	.02

NOTES: The Ljung-Box test concerns the first 118 autocorrelations, while the *LM* statistic concerns serial correlation at the 12th lag for the residuals.

(B) *Testing for ARCH*

		u_t–equation		π_t–equation	
# lags	# Unit Roots	ARCH(k)	p-value	ARCH(k)	p-value
6	0	20.49	.00	44.35	.00
	1	20.62	.00	42.51	.00
8	0	16.48	.04	33.47	.00
	1	16.30	.04	32.64	.00
10	0	22.04	.01	26.43	.00
	1	21.06	.02	26.09	.00
12	0	17.82	.12	25.32	.01
	1	18.60	.10	25.48	.01

TABLE 4: Specification based on conditional scores in 2-state MS–VAR(k) systems for the U. S., 1959:1–1998:12

(A) *Equation-by-equation Tests*

Hypothesis	System 1 ($k = 2$) $\pi_t - .039u_t$	System 1 ($k = 2$) Δu_t	System 2 ($k = 2$) π_t	System 2 ($k = 2$) Δu_t	System 3 ($k = 3$) π_t	System 3 ($k = 3$) u_t
Autocorrelation	.74	.61	.78	.62	.71	1.39
p-value	.56	.65	.54	.65	.59	.24
ARCH	.80	1.22	.84	1.12	1.34	.46
p-value	.53	.34	.50	.35	.25	.76
Markov	.25	.38	.26	.37	.27	1.78
p-value	.91	.82	.91	.83	.89	.13

(B) *System Tests*

Hypothesis	System 1 ($\beta_u = 0.039$)	System 2 ($\beta_u = 0$)	System 3 (π_t, u_t)
Autocorrelation	.63	.64	.92
p-value	.86	.86	.55
ARCH	.96	.95	.79
p-value	.54	.56	.81
Markov	.30	.31	1.35
p-value	.94	.93	.23

(C) *System Properties*

	System 1 ($\beta_u = 0.039$)	System 2 ($\beta_u = 0$)	System 3 (π_t, u_t)
$\ln L(\hat{\theta})$	−922.63	−922.65	−910.01
AIC	3.98	3.98	3.97
BIC	4.58	4.58	4.66
LIL	4.07	4.07	4.08
e_1	.65	.65	.96
$\hat{\pi}_1$	.42	.42	.45
$\hat{\sigma}_{\pi_1}$	.15	.15	.16

TABLE 5: Estimated unconditional and conditional means and covariances for inflation and unemployment in the U. S., 1959:1–1998:12

(A) *Unconditional Moments*

System	Variable	Mean	Variance	Covariance
1	$\pi_t - .039u_t$	3.85	12.38	.06
		(.60)	(3.44)	(.06)
	Δu_t	−.01	.04	
		(.02)	(.01)	
2	π_t	4.09	12.39	.06
		(.60)	(3.44)	(.06)
	Δu_t	−.01	.04	
		(.02)	(.01)	

(B) *Conditional Moments*

System	Variable	Mean	Variance	Covariance
Regime 1				
1	$\pi_t - .039u_t$	5.31	20.74	.05
		(1.01)	(4.48)	(.13)
	Δu_t	.02	.06	
		(.08)	(.01)	
2	π_t	5.58	20.65	.05
		(1.01)	(4.47)	(.13)
	Δu_t	.02	.06	
		(.03)	(.01)	
Regime 2				
1	$\pi_t - .039u_t$	2.80	3.72	.00
		(.20)	(.43)	(.02)
	Δu_t	−.03	.02	
		(.01)	(.00)	
2	π_t	3.02	3.73	.00
		(.20)	(.43)	(.02)
	Δu_t	−.03	.02	
		(.01)	(.00)	

TABLE 6: Effects on the mean and the variance of inflation and unemployment growth from changes in ω and λ.

(A) *Stochastic Trend in Employment Targets*

Moment	Wage Elasticity (ω)	Inflation Weight (λ)
$E(\pi_t)$	0	$\frac{-n^*}{\alpha\lambda^2}$
$V(\pi_t)$	0	$\frac{-2\alpha^2\sigma_z^2}{(1+\alpha^2\lambda)^3}$
$E(\Delta u_t)$	0	0
$V(\Delta u_t)$	$2\alpha(1+\alpha\omega)\sigma_u^2 + \frac{4(\omega-\alpha\lambda)\sigma_z^2}{(1+\alpha^2\lambda)^2}$	$\frac{4\alpha(1+\alpha\omega)(\alpha\lambda-\omega)\sigma_z^2}{(1+\alpha^2\lambda)^3}$

(B) *Stochastic Trend in Technology*

Moment	Wage Elasticity (ω)	Inflation Weight (λ)
$E(\pi_t)$	0	$\frac{-(n^*-n^u)}{\alpha\lambda^2}$
$V(\pi_t)$	0	$\frac{-2\alpha^2\sigma_z^2}{(1+\alpha^2\lambda)^3}$
$E(\Delta u_t)$	0	0
$V(\Delta u_t)$	$\frac{2[(1+\alpha^4\lambda^2)\omega-(1-\alpha^2\lambda)\alpha\lambda]\sigma_z^2}{(1+\alpha^2\lambda)^2}$	$\frac{2\alpha(1+\alpha\omega)[(2+\alpha\omega)\alpha\lambda-\omega]\sigma_z^2}{(1+\alpha^2\lambda)^3}$

NOTES: Panel A is based on the assumptions in equations (A.15) and (A.16) with $\gamma = 1$ and, for simplicity, $\rho = 0$. The former implies that $\beta_u = 0$ so that inflation is stationary. Now,

$$\pi_t = \pi^* + \frac{1}{\alpha\lambda}n^* - \frac{1}{1+\alpha^2\lambda}\varepsilon_{z,t},$$

while

$$\Delta u_t = -\left(1+\alpha\omega\right)\varepsilon_{u,t-1} + \frac{\omega-\alpha\lambda}{1+\alpha^2\lambda}\Delta\varepsilon_{z,t}.$$

The partial derivatives in Panel B are derived under the assumptions $n_t^u = n^u$, $n_t^* = n^*$, while $\rho = 1$. Here, $\beta_u = 0$ with

$$\pi_t = \pi^* + \frac{1}{\alpha\lambda}\left(n^* - n^u\right) - \frac{1}{1+\alpha^2\lambda}\varepsilon_{z,t},$$

and

$$\Delta u_t = \frac{\omega-\alpha\lambda}{1+\alpha^2\lambda}\varepsilon_{z,t} + \frac{\alpha\lambda\left(1+\alpha\omega\right)}{1+\alpha^2\lambda}\varepsilon_{z,t-1}.$$

FIGURE 1: Inflation and unemployment series for the U. S. in levels and first differences, 1959:1–1998:12

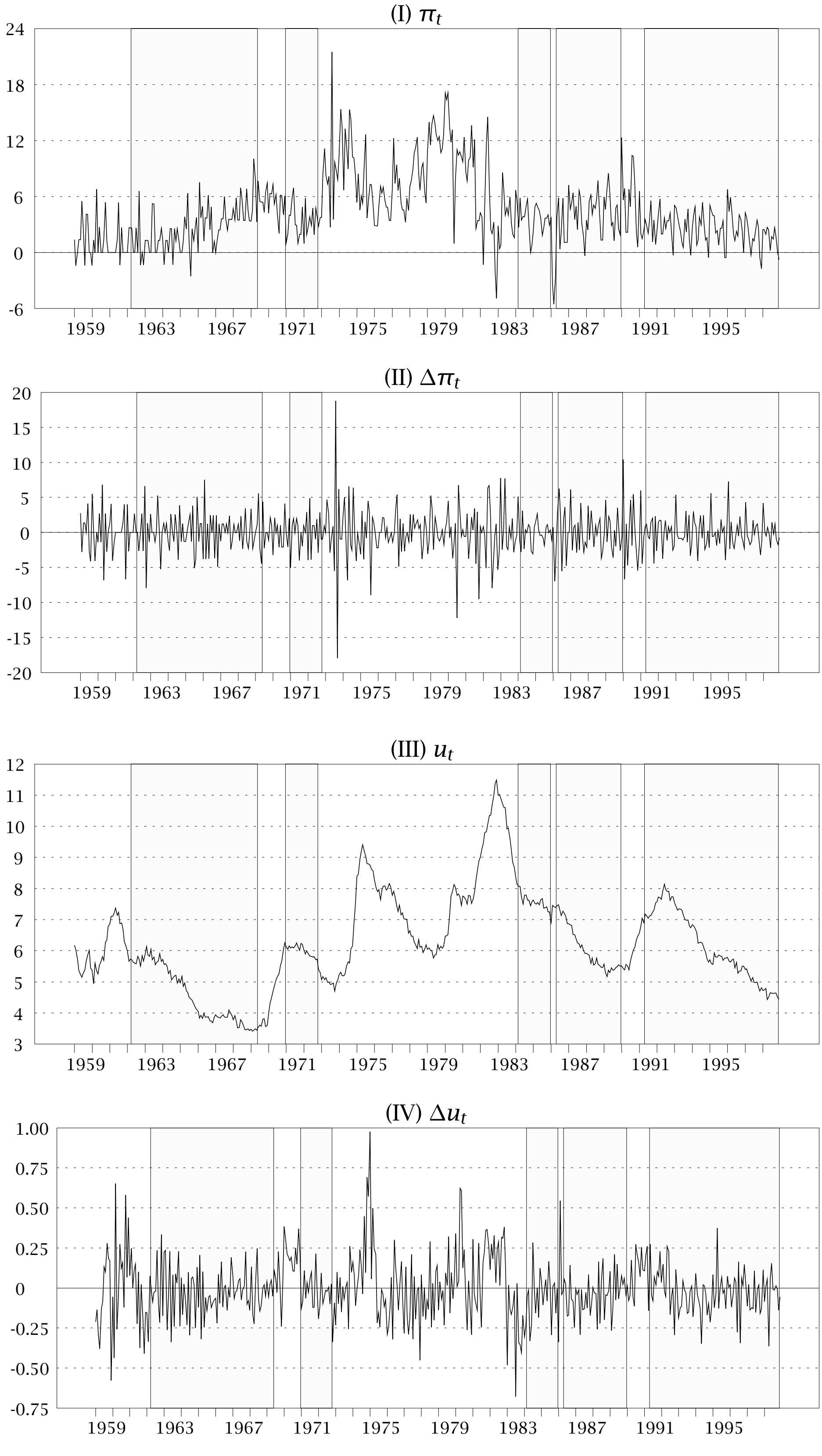
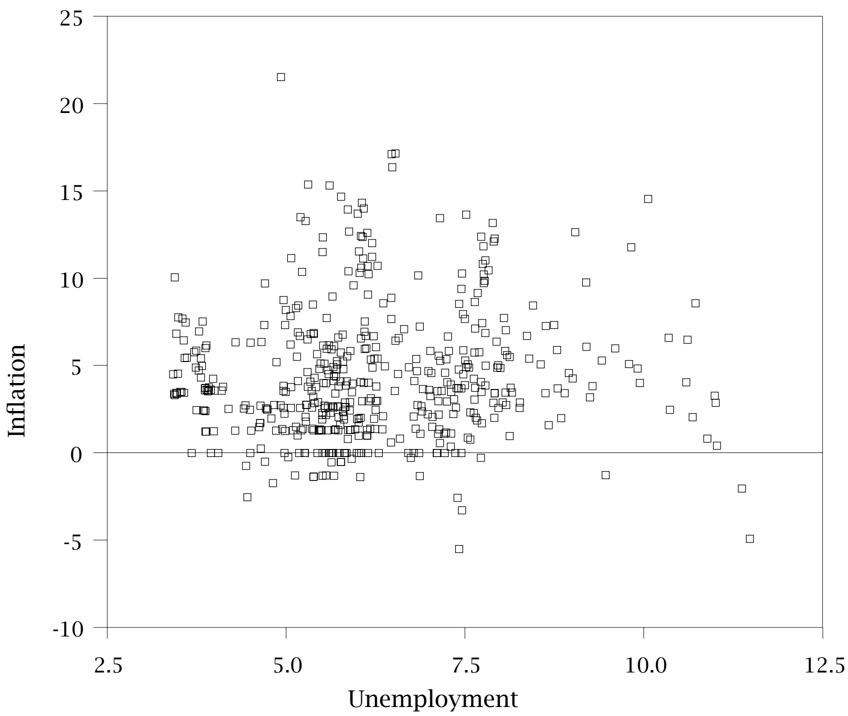


FIGURE 2: Unemployment and inflation in the U. S. for the sample 1959:1–1998:12

(I) *Monthly inflation*



(II) *Yearly inflation*

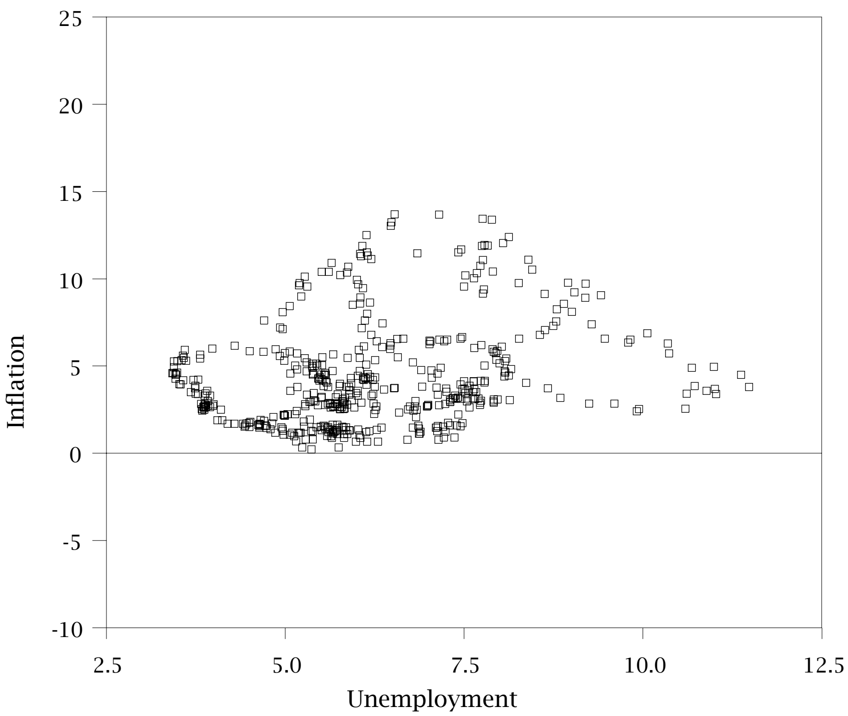


FIGURE 3: The scaled log-likelihood function (solid line) and the estimated maximum eigenvalue (dashed line) for 2–state MS–VAR(2) systems for the U. S., 1959:1–1998:12

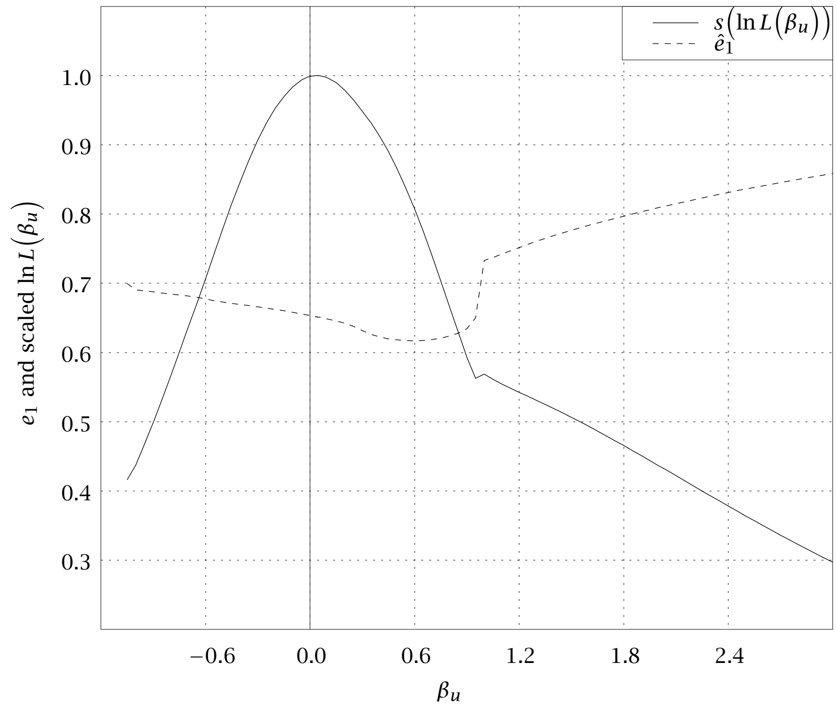


FIGURE 4: Estimated smooth probabilities for 2–state MS–VAR(k) model for the U. S., 1959:1–1998:12

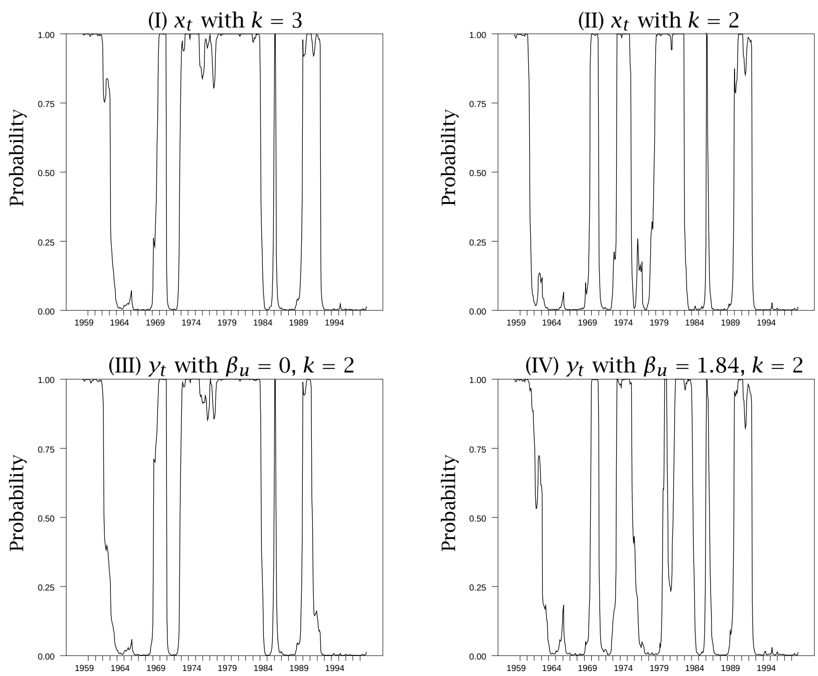
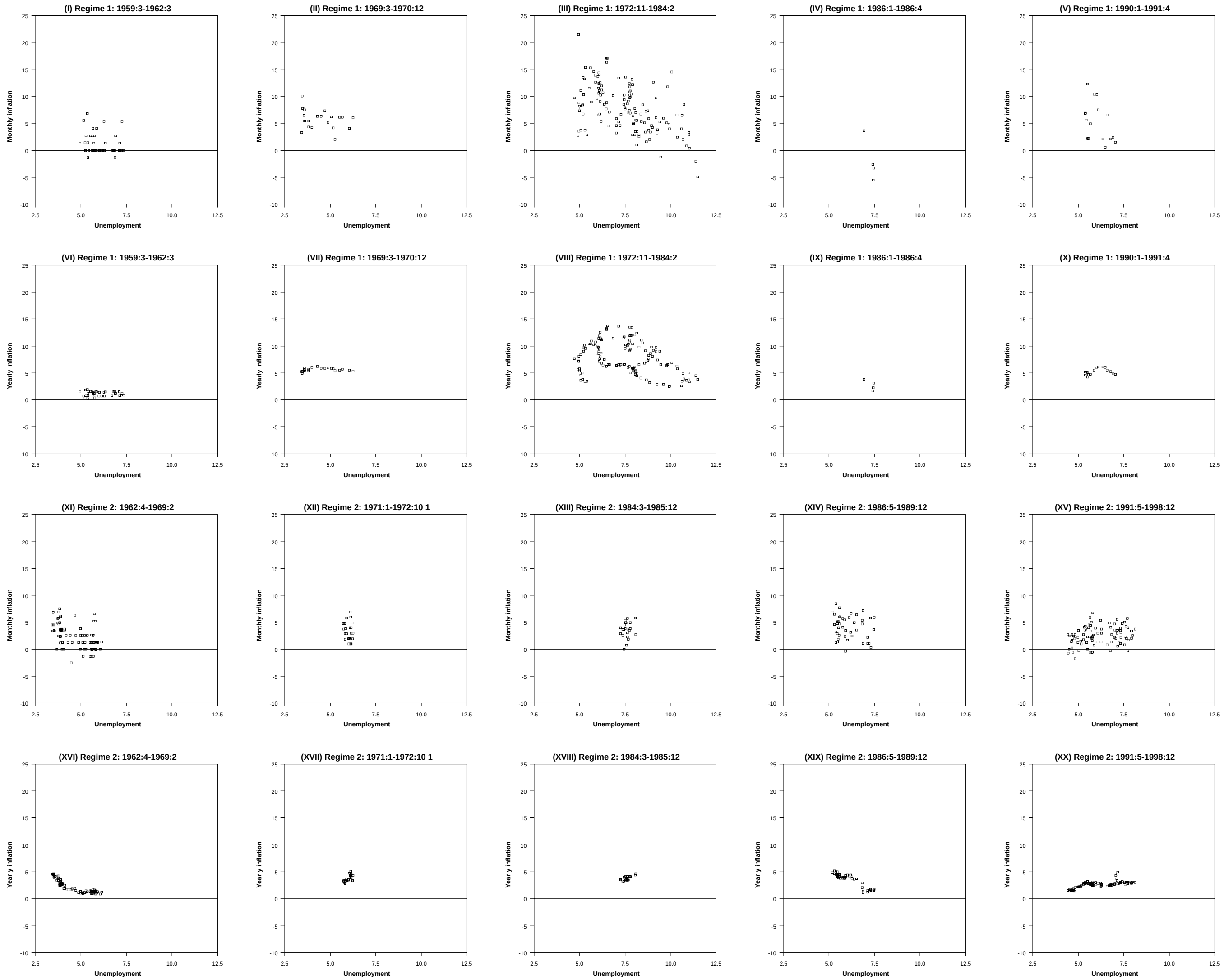


FIGURE 5: Unemployment and monthly/yearly inflation in the U. S. for the estimated Regime 1 and Regime 2 periods.



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