Unanimity versus Consensus Bargaining Games: Strategic Delegation and Social Optimality*

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Abstract

Decisions on common policies are often the outcome of a bargaining game played by members of a committee. This paper investigates whether policies are socially optimal when the members of a committee are delegates sent by different constituencies. It is shown that, in standard unanimity bargaining games, the incentive to strategically delegate the power to bargain induces a bias towards the status quo. Policies are therefore socially suboptimal. As an alternative, a bargaining game based on a consensus approach is presented. which overcomes the bias. Although the constituencies still strategically delegates, the chosen policies are socially optimal in this game.

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1 Introduction

Decisions on political and economic issues are usually very controversial. Agents are heterogeneous and prefer therefore different policies, but most policies are nevertheless binding for everyone. These inherent conflicts are often resolved by an indirect democratic system. Different constituencies send delegates into a committee, where the policy is chosen by means of a bargaining procedure. Such mechanisms are widely used in confederate systems such as the European Union (EU) and in international organization like the World Trade Organization (WTO), but also to some extent in federal states.

This paper investigates how bargaining procedures used in a committee affect the policy outcome and the choice of delegates made by the different constituencies (or different countries in an international context). Answering this question helps to relate observed policy decisions and frequently observable preferences of the delegates to the, normally unobservable, distribution of preferences within constituencies. This allows a welfare analysis of different bargaining procedures. Another aim of this paper is to develop socially optimal decision mechanisms for committees. This is especially important for the EU, where modifications in the decision procedures are discussed in connection with the envisaged enlargement.

We analyze two different bargaining games played in a committee, called unanimity bargaining and consensus bargaining. These games are preceded by a delegation stage in which each constituency chooses its delegate non-cooperatively. Both bargaining games allow for sidepayments that are valuable not only for the delegates but for every citizen in a constituency. Especially in the context of European policy making, this seems to be an appropriate assumption. In the EU, sidepayments exist either in form of direct monetary transfers (e.g. the cohesion fund) or in form of concessions on other policy issues. Hence, the sidepayments determined in the EU Council affect the utility of all European citizens.

The unanimity bargaining game, adopted from Hart and Mas-Colell (1996), implements the Nash bargaining solution. The equilibrium policy is thus jointly optimal from the viewpoint of the delegates. Nevertheless, the policy is socially suboptimal since, under unanimity bargaining, the composition of the committee itself is suboptimal. The constituencies choose individually rational socially 'wrong' delegates. The effect that a constituency delegates the power to bargain to an agent who prefers a policy different from the preferred policy of the constituency's majority is called strategic delegation. In equilibrium, each single delegate and the chosen policy are biased towards the status quo. This is because a delegate for whom the status quo is quite favorable has a strong bargaining position and can achieve positive sidepayments. Moreover, we will see that the bias towards the status quo policy is increasing in the number of players. Hence, in large committees one would observe almost no policy change, even if the status quo is very unfavourable for the majorities in all constituencies. These results can be interpreted as a warning against the use of unanimity rules in committees.

The consensus bargaining game implements a bargaining solution that has not been described in the existing literature on multilateral bargaining. The game aims at finding the consent of all delegates, but allows for the possibility of implementing a policy even in the case where some delegates oppose. There are three main reasons justifying a thorough investigation of this game. First, it captures certain features frequently attributed to decision processes in real world committees. There exists anecdotal evidence that in some institutions, most notably the Bundesbank Council in Germany and the Council of the EU[1], the committee members typically try to find a solution that is acceptable for every member. This happens despite the fact that the official rules require only a simple (or qualified) majority for a decision. To our best knowledge, this paper is a first attempt to model such practices. Second, the game is closely related to recent developments in the theory of non-cooperative foundations of cooperative game theory. The game presented in this paper is inspired by a game introduced by Hart and Mas-Colell (1996). Third, as we will see, consensus bargaining overcomes some of the major shortcomings of unanimity bargaining.

The basic idea behind consensus bargaining can be described as follows. At the beginning, a proposal has to be approved by all members of the committee. However, if a consensus cannot be reached, the delegate who induced the failure is excluded from future bargaining. Due to this feature the delegates are individually in a rather weak position, since being excluded is mostly disadvantageous in this game. We will see that strategic delegation occurs in consensus bargaining games as well. This indicates that strategic delegation might be a common feature for all committees that use bargaining procedures.

[1]For the EU, see Nugent (1995).

Hence, it is not (necessarily) a sign of political failure if committee members have different interests than the majority of the people they are representing. Such a situation is likely to be the outcome of rational behavior of all voters in their constituencies. Nevertheless, equilibrium policies are socially optimal in consensus bargaining games. Strategic delegation has therefore no negative welfare implications, which contrasts the case of unanimity bargaining. This is because in consensus bargaining game a delegate has a strong bargaining position (and can achieve positive sidepayments) if his preferred policy is close to the average. Therefore, we can conclude that consensus bargaining games within committees are very attractive mechanisms for (political and economic) decisions on common policies in systems consisting of heterogeneous constituencies.

The remainder of this paper is as follows. Section 2 discusses some of the related literature. Section 3 presents the underlying basic model. Section 4 analyzes the case of unanimity bargaining. It is shown how the incentives to delegate the power to bargain strategically leads to socially suboptimal policies. Section 5 presents the consensus bargaining game and analyzes how this game implements socially optimal policies. Finally, section 6 concludes the paper. Most of the proofs are contained in the appendix.

2 Related Literature

Strategic delegation, in the context of public policy decisions, has been investigated by some authors before. The papers in this area are built mainly on two country models. In the tax competition model of Persson and Tabellini (1992), strategic delegation offsets, at least partly, the inefficiency arising from the externality effects of national tax rates. These tax rates are set non-cooperatively. The paper of Seggendorf (1998) is more closely related to my analysis. In his paper, two delegates who represent two different countries bargain over one pure and two local public goods. In equilibrium, strategic delegation occurs and leads to inefficient provision of the pure public good. Unlike Seggendorf (1998), this paper considers n-player (unanimity) bargaining and analyzes policy decisions in a given institutional setting. Besley and Coate (1999) examine strategic delegation in a given institutional setting. However, their economy consists of only two districts. Each district selects one delegate and provides one local public good exhibiting cross border externalities. In case of a federation, the (distinct) levels of the local public good and the uniform tax rate are not determined by (unanimity) bargaining, but are instead chosen by one randomly selected delegate. However, for exogenous reasons, the decisive delegate acts not purely selfish but cares as well for the utility of the other delegate. In equilibrium, both districts delegate strategically and the policy outcome is not welfare maximizing.

A paper addressing strategic delegation in a n-person bargaining model within a given institutional framework is Chari, Jones and Marimon (1997). In their model, local representatives (members of the congress) and one president bargain over local public spending and (uniform) taxation. In equilibrium, strategic delegation occurs. Local representatives are fiscally more liberal and the president is fiscally less liberal than the local electorates. Depending on the importance of public goods different from taxation and the number of possible types for the president, the outcome might be efficient. The bargaining requires unanimity between the president and congress as a whole, but not within congress. In contrast to Chari, Jones and Marimon (1997) where the bargaining is essentially over local spending, giving utility only for the respective electorate, this paper addresses the case where the bargaining is over a common policy, i.e., a pure public good. Decisions on common policies differ from decisions on private goods since a policy is valid (and gives utility) to all members. Bargaining on private goods is essentially about dividing a pie, see, e.g., the pioneering work of Baron and Ferejohn (1989) on bargaining in legislatures. Bargaining on public goods, however, is simultaneously about the size and the division of a pie. Most importantly, the pie has its maximal size only if the common policy depends on the preferences of all members. Minimal winning coalitions, that occur in equilibrium in the model of Baron and Ferejohn, imply therefore a suboptimally small pie. Moreover, the preferences of all players are crucial in bargaining on a policy, whereas in Baron and Ferejohn, the outcome is solely determined by the rules of the game.[2]

3 The Model

The economy consists of n constituencies, each of them inhabited by a continuum of citizens with unit mass. For simplicity, I assume n to be odd. The set of constituencies

[2]A similar description can be found in a recent paper by Diermeier and Merlo (1998) on government formation in multiparty parliaments. In their paper parties simultaneously bargain on public and private goods (policies and perks).

is denoted by N. It is assumed that each constituency sends one delegate into a committee that decides on a one-dimensional common policy x and a vector of constituency specific sidepayments $\boldsymbol{\tau} = (\tau_1, ..., \tau_n)$. Evidently, the sidepayments must fulfil the condition $\sum_{i \in N} \tau_i = 0$. The policy x is binding for all citizen in the economy, whereas the sidepayments received or paid by a constituency are shared equally among its members. Each citizen j in constituency i is characterized by his preferred policy x_{ij}. This policy is given by $x_{ij} = c_j + x_i$. Here, c_j can be interpreted as an individual taste parameter and x_i as a measure for the general economic situation in constituency i. It is assumed that, in all constituencies, c_j has a distribution function $f(c_j)$ with zero mean and variance σ. Hence, the median and the mean type within each constituency coincide. This citizen is called principal i and has a preferred policy x_i.

All citizens in the economy have quasi-linear utility functions with identical functional form. More specifically, the utility function of a citizen j in constituency i is given by

$$U_{ij} = -(x_{ij} - x)^2 + \tau_i \tag{1}$$

In stage 1 of the game each constituency chooses by simple majority voting one of its citizens as delegate for the committee. For convenience the preferred policy of delegate i, that characterizes him, is denoted as d_i and his utility function as U_{di}.[3] Due to the single peakedness of preferences it is basically principal i who chooses d_i. Then, in stage 2, the delegates bargain over the policy x and the sidepayments $\boldsymbol{\tau}$. If they do not reach an agreement, the exogenously given status quo policy x^c prevails and sidepayments are zero. For convenience, I refer to the payoffs the delegates achieve in equilibrium as the bargaining outcome $\boldsymbol{\psi} = (\psi_1, \psi_2, ..., \psi_n)$. Formally,

$$\psi_i(\mathbf{d}) = U_{di}(x^e, \tau_i^e) = -(d_i - x^e)^2 + \tau_i^e. \tag{2}$$

A superscript e denotes an equilibrium value. In the reduced game taking place in the first stage, we can, therefore, write the utility of a principal as

$$U_i(d_i, d_{-i}) \quad = \quad -[x_i - x^e(d_i, d_{-i})]^2 + \tau_i^e(d_i, d_{-i}) \tag{3}$$

[3]Note that in order to get a transferable utility (TU) game, it is necessary that the utility functions are linear (with slope normalized to 1) in the medium of the sidepayments, see e.g. Myerson (1991) or Diermeier and Merlo (1998) who use a similar setup.

$$= -\left[x_i - x^e\left(d_i, d_{-i}\right)\right]^2 + \left[d_i - x^e\left(d_i, d_{-i}\right)\right]^2 + \psi_i\left(d_i, d_{-i}\right)$$

$$= \psi_i\left(d_i, d_{-i}\right) - x_i^2 + d_i^2 + 2\left[x_i - d_i\right] x^e\left(d_i, d_{-i}\right) \tag{4}$$

An equilibrium of the games we are going to consider consists of the delegates $\mathbf{d}^e = \left(d_1^e, d_2^e, ..., d_n^e\right)$ chosen in the first stage, the policy x^e and the sidepayments $\boldsymbol{\tau}^e = \left(\tau_1^e, \tau_2^e, ..., \tau_n^e\right)$, both chosen in the second stage. As usual, the equilibria are found by backward induction. First the equilibrium values determined in the second stage are computed. These results are used to solve for the equilibrium positions of the delegates. The solution concept used for this reduced game taking place in the first stage is standard Nash equilibrium.

For evaluating the effects of strategic delegation I use the following definitions.

Definition 1 *Principal i strategically delegates if $d_i^e \neq x_i$. A bargaining mechanism leads to strategic delegation if there exists a vector of preferences of the principals such that in equilibrium*

$$\mathbf{d}^e = \left(d_1^e, ..., d_n^e\right) \neq \mathbf{x} = \left(x_1, ..., x_n\right).$$

Concerning the question of optimality, we use the definition that a bargaining game is socially optimal if the equilibrium policy is always welfare maximizing. For measuring welfare, a standard additive social welfare function (SWF) is used. For convenience, we add a constant $k = n\sigma$ to the SWF. Therefore, welfare W is given by

$$W = \left(\sum_{i\in N} \int \left(-\left(c_j + x_i - x\right)^2 + \tau_i\right) df\left(c_j\right)\right) + n\sigma \tag{5}$$

$$= n\sigma + \sum_{i\in N}\left[-\left(x_i - x\right)^2 + \tau_i - \int\left(c_j^2 - 2c_j\left(x_i - x\right)\right) df\left(c_j\right)\right] \tag{6}$$

$$= \sum_{i\in N} -\left(x_i - x\right)^2$$

Denote the welfare maximizing policy as x^*. A socially optimal bargaining mechanism is then defined as follows:

Definition 2 *A bargaining mechanism is socially optimal if the equilibrium policy is welfare maximizing for any possible vector of the principals' preferred policies* $\mathbf{x}$. *Formally,*

$$x^e = x^* = \arg\max_x W\left(x; x_1, ..., x_n, \boldsymbol{\tau}\right) \qquad s.t. \sum_{i\in N} \tau_i = 0 \qquad \forall\left(x_1, ..., x_n\right)$$

In our model, we have

$$x^* = \arg\max_x \sum_{i \in N} \left(- (x_i - x)^2 + \tau_i \right)$$
$$\Rightarrow \; x^* = \frac{1}{n} \sum_{i \in N} x_i = \bar{x}$$

Hence, a bargaining mechanism is socially optimal whenever its equilibrium policy equals the mean of the preferred policies of the principals. This is because the mean minimizes the squared deviations from the common policy. The sidepayments solely serve to redistribute payoffs. In the following sections I will determine and discuss the equilibrium values $\mathbf{d}^e$, $\boldsymbol{\tau}^e$ and x^e for the two different bargaining games outlined in the introduction.

4 Unanimity bargaining

In this section I analyze a unanimity bargaining game that includes a risk of breakdown. The bargaining outcome of the game corresponds to the n-person pure Nash bargaining solution. Since there are various games implementing this solution [e.g. Krishna and Serrano (1996) or Hart and Mas-Colell (1996), in the following abbreviated as HM], the results derived in this section hold for a larger class of bargaining games. The game presented in this section follows HM.

At the beginning of the bargaining game one player (delegate) is randomly chosen (with probability $\frac{1}{n}$) to make a proposal. A proposal consists of a pair $(x_p, \boldsymbol{\tau}_p)$. If every player accepts, the game ends. In this case, x_p gets implemented and the players make and receive sidepayments according to $\boldsymbol{\tau}_p$. However, if one player refuses the proposal, another randomly chosen player (possibly the same as before) can make a proposal with probability ρ. With probability $(1-\rho)$ the game ends and the status quo policy prevails. Denote the payoff player j receives from a proposal of player i as a_i^j and the average payoff proposed to player j as a^j. The payoff of player j, if the status quo prevails, is denoted as a_c^j. The sum of the payoffs induced by a proposal of player i is denoted as a_i. Finally, denote the maximum utility the players can achieve as

$$v(N) = \max_{(x,\boldsymbol{\tau})} \sum_{i \in N} \left(- (d_i - x)^2 + \tau_i \right) \quad s.t. \sum_{i \in N} \tau_i = 0 \tag{7}$$

As standard, in multi-player bargaining games, stationary subgame perfect equilibrium (SSPE) is used as solution concept.[4] The SSPE of this game is given by the following proposition.

Proposition 1 *The first proposal in the unique stationary subgame perfect equilibrium is accepted and is characterized by*

$$\begin{aligned}
i)\quad a_i &= v(N) \\
ii)\quad a_i^j &= \rho a^j + (1-\rho)\, a_c^j \\
&= \frac{\rho}{n}\left[v(N) - \sum_{j\in N} a_c^j\right] + a_c^i \\
iii)\quad a_i^i &= v(N) - \sum_{j\neq i} a_i^j \\
&= \frac{n-(n-1)\rho}{n}\left[v(N) - \sum_{j\in N} a_c^j\right] + a_c^i \\
iv)\quad a^i &= \frac{1}{n}\left[v(N) - \sum_{j\in N} a_c^j\right] + a_c^i \\
v)\quad \textit{if } \rho &\to 1 \textit{ then } a_i^j \to a^j \quad \forall i,j \\
vi)\quad x_{pi} &= \frac{1}{n}\sum_{j\in N} d_j \\
vii)\quad \tau_{pi}^j &= a_i^j + (d_j - x_{pi})^2
\end{aligned}$$

Proof. See appendix. The proof of *i*) to *v*) is almost identical to HM. Note that *iv*) is the definition of the pure n-player Nash solution in the transferable utility case, see for example Myerson (1991). ■

Proposition 1 implies that we can write the bargaining outcome, for $\rho \to 1$, as

$$\begin{aligned}
\psi_i &= \frac{1}{n}\sum_{j\in N}\left[U_{d_j}(x^e) - U_{d_j}(x^c)\right] + U_{d_i}(x^c) \\
&= \frac{1}{n}\sum_{j\in N}\left[-d_j^2 - x^{e^2} + 2d_j x^e + d_j^2 + x^{c^2} - 2d_j x^c\right] - d_i^2 - x^{c^2} + 2d_i x^c \\
&= -x^{e^2} + x^e \frac{2}{n}\sum_{j\in N} d_j - x^c \frac{2}{n}\sum_{j\in N} d_j - d_i^2 + 2d_i x^c
\end{aligned}$$

[4] Multiplayer bargaining games usually exhibit multiple subgame perfect equilibria, see, among others, Osborne and Rubinstein (1990). A justification for restricting attention to SSPE can be found in Baron and Ferejohn (1989).

$$
\begin{aligned}
&= x^{e^2} - d_i^2 - 2x^c x^e + 2d_i x^c \\
&= -\frac{n^2-1}{n^2} d_i^2 + \frac{1}{n^2}\sum_{j\neq i} d_j^2 + \frac{2}{n^2} d_i \sum_{j\neq i} d_j + \frac{1}{n^2}\sum_{j\neq i}\sum_{k\neq i,j} d_j d_k \\
&\quad +2\frac{n-1}{n} d_i x^c - \frac{2}{n} x^c \sum_{j\neq i} d_j \qquad (8)
\end{aligned}
$$

Using (8), we can solve for the equilibrium choice of the delegates. It should be emphasized that the equilibrium values of the preferred policies of the delegates and the equilibrium policy are independent of ρ. Hence, concentrating on $\rho \to 1$ is without loss of generality with respect to $\mathbf{d}^e$ and x^e. This is because in stage 1, when the principals decide on the delegates, it is unknown who will be the first proposer in the bargaining in stage 2. Thus, the principals base their choice on the expected value of the bargaining game, which is given by *iv*). Since this value is independent of ρ, $\mathbf{d}^e$ is independent of ρ. Moreover, we see from *vi*) that in this case, the equilibrium policy is independent of ρ as well. Nevertheless, we see from *ii*) and *vii*) that the equilibrium value of the sidepayments depends on ρ. The calculation of $\boldsymbol{\tau}^e$ becomes quite cumbersome in the general case $0 \leq \rho \leq 1$. To avoid complicated expressions for $\boldsymbol{\tau}^e$, I concentrate on $\rho \to 1$. However, it should be taken into account that we loose generality only with respect to the equilibrium sidepayments, but not with respect to the equilibrium positions of the delegates and the equilibrium policy.

If the equilibrium policy is given by $x^e = \frac{1}{n}\sum d_i$ and the bargaining outcome is given by (8), we can write the utility function of the principals (4) as

$$
\begin{aligned}
U_i &= \left(-\frac{n^2-1}{n^2} + \frac{n-1}{n}\right) d_i^2 + 2d_i \sum_{j\neq i} d_j \left(\frac{1}{n^2} - \frac{1}{n}\right) \\
&\quad + \frac{2}{n} d_i x_i + 2\frac{n-1}{n} d_i x^c + h\left(x_i, x^c, d_{-i}\right)
\end{aligned}
$$

Here, $h\left(x_i, x^c, d_{-i}\right)$ collects all the terms in the utility function U_i that do not depend on the principal's choice variable d_i. Hence, the FOC for the maximization problems of the principals in stage 1 are given by[5]

$$
\begin{aligned}
2d_i\left(-\frac{n^2-1}{n^2} + \frac{n-1}{n}\right) + 2\sum_{j\neq i} d_j \left(\frac{1}{n^2} - \frac{1}{n}\right) + \frac{2}{n} x_i + 2\frac{n-1}{n} x^c &= 0 \\
\Rightarrow -\frac{2n-1}{n} d_i - \frac{n-1}{n} \sum_{j\neq i} d_j + x_i + (n-1)\, x^c &= 0 \qquad (9)
\end{aligned}
$$

[5] Also the second order conditions are fulfilled.

With these results, the equilibrium for the two-stage game is given by the following proposition:

Proposition 2 *If the bargaining game among the delegates exhibits a risk of a breakdown and allows for sidepayments, the equilibrium of the delegation game is*

$$\begin{aligned} d_i^e &= \frac{n^2-n+1}{n^2}x_i - \frac{n-1}{n^2}\sum_{j\neq i} x_j + \frac{n-1}{n}x^c \qquad \forall i \\ &= x_i - \frac{n-1}{n}\bar{x} + \frac{n-1}{n}x^c \\ x^e &= \frac{1}{n^2}\sum_{i\in N} x_i + \frac{n-1}{n}x^c = \frac{\bar{x}}{n} + \frac{n-1}{n}x^c \\ \tau_i^e &= 2\frac{1}{n}\left[(\bar{x}-x_i)(\bar{x}-x^c)\right] \qquad \forall i \end{aligned}$$

Hence, strategic delegation occurs in equilibrium and the equilibrium policy is suboptimal.

Proof. See appendix ■

Remark 1 *The limiting behavior of the equilibrium values if the number of constituencies n tends to infinity is given by*

$$\begin{aligned} \lim_{n\to\infty} d_i^e &= x_i - \bar{x} + x^c \\ \lim_{n\to\infty} x^e &= x^c \\ \lim_{n\to\infty} \tau_i^e &= 0 \end{aligned}$$

Therefore, the equilibrium policy is always biased towards the status quo policy x^c and the bias increases with the number of players. Hence, if there exists a risk of breakdown, we should observe only small policy changes in large committees using unanimity rule. This holds even if all principals want drastic changes. Moreover, we see from proposition 2 that the sidepayments are positive whenever the individually preferred policy x_i and the status quo policy x^c are both smaller or both larger than the jointly optimal policy $\bar{x}$.

To get some intuition for proposition 2, it is helpful to explicitly consider the two incentives for strategic delegation, that are called policy incentive and sidepayment incentive For this, rewrite the FOC of the principals as

$$\begin{aligned} \frac{\partial U_i}{\partial d_i} &= -\frac{\partial (x_i-x)^2}{\partial d_i} + \frac{\partial \tau_i}{\partial d_i} = 0 \qquad (10) \\ &= \frac{2}{n}\left(x_i - \frac{1}{n}d_i - \frac{n-1}{n}\bar{d}_{-i}\right) - 2\left(\frac{n-1}{n^2}\left(d_i + (n-2)\,\bar{d}_{-i} - nx^c\right)\right) = 0 \end{aligned}$$

In (10) , the average position of the delegates chosen by the other principals is denoted as $\bar{d}_{-i} = \frac{1}{n-1}\sum_{j\neq i} d_j$. The first term in (10) describes the incentive to achieve the most favorable policy (policy incentive). The first term is maximized if

$$(d_i - x_i) = (n-1)\left(x_i - \bar{d}_{-i}\right) \tag{11}$$

If this were the only incentive, a principal would send a delegate with $d_i > x_i$ ($d_i < x_i$) in case his preferred policy is higher (lower) than the average position of the other delegates. This incentive is mean preserving in the sense that rearranging and averaging the equations (22) for all principals gives $\frac{1}{n}\sum_{j\in N} d_j = \bar{x}$. The second term in (10) describes the sidepayment incentive. Maximizing sidepayments results in

$$d_i = \frac{n}{2}x^c - \frac{n-2}{2}\bar{d}_{-i}$$

Solving these n equations resulted in $d_i = x^c$. The reason is straightforward. A delegate for whom the status quo policy is favorable has a credible threat to block an agreement and receives therefore sidepayments from those who dislike the status quo. In equilibrium, the principals take both incentives into account. Since the policy incentive is mean preserving while the sidepayment incentive leads to status quo, the equilibrium policy is a convex combination between the optimal policy $\bar{x}$ and the status quo policy x^c. If n increases, the policy incentive becomes relatively weaker and the outcome therefore closer to the status quo.

5 Consensus bargaining

In this section I assume that the delegates use a consensus approach to find an agreement. As in most democratic institutions, an agreement requires the support of at least a simple majority of the delegates. However, the first proposal made in the bargaining game needs to be approved by all delegates in order to be implemented. This feature serves to ensure that no constituency is neglected in the final decision. In case a proposal is refused, the proposer is excluded from the bargaining procedure. This is because the proposer is hold responsible for the failure since he tried to enforce a non consensual policy. The construction of the games guarantees the committee's ability to act at any point in time. The bargaining game is a modification of the game introduced by HM. Its exact definition is as follows.

Definition 3 *In each round, there is a set of active players $S \subseteq N$, a set of inactive players $N \backslash S$ and one randomly (with probability $\frac{1}{s}$, where $s = |S|$) chosen proposer $i \in S$. In the first round we have $S = N$. Player i proposes $(x, \boldsymbol{\tau}_S)$. Here, x is the policy and $\boldsymbol{\tau}_S$ is a vector of sidepayments restricted to the active players. The sidepayments fulfill $\sum_{i \in S} \tau_i = 0$. The players in $S \backslash i$ respond sequentially to the proposal.*[6] *If all players in S accept the proposal, the game ends. The policy x gets implemented and is binding for all (active and inactive) players and the active players receive and make sidepayments according to the proposal. If one player in S rejects the proposal, the game moves to the next round. With probability ρ, the sets S and $N \backslash S$ remain the same and with probability $(1 - \rho)$ the proposer "drops out". As long as $S \backslash i$ still represents the majority of all players (i.e. if $s > \frac{n+1}{2}$), $S \backslash i$ becomes the set of active players and one of them is chosen as the proposer. However, if $S \backslash i$ has no majority anymore, the set of active player becomes $(N \backslash S) \cup i$. The game continues until an agreement is reached.*

This game differs from the one of Hart and Mas-Colell by imposing that the set of active players must consist of a majority of players. In HM, this set might reduce to a single player. In our model, the possibility that the set of active players had no majority anymore (and therefore no new policy could be implemented) would imply that the status quo policy affects the equilibrium. Hence, the equilibrium would not be optimal. For that reason, the game in definition 5 is more promising since it avoids a dependence from the status quo. Besides this technical argument, the assumption that the active players have always a majority seems quite appropriate for bargaining in committees. If in a committee a minimum winning coalition (i.e., in our case, a coalition with $s = \frac{n+1}{2}$) would break up, there are other players around that are willing to form a coalition with one of the members of the old coalition in order to influence the final decision in their interest. The game implies that the underlying minimal requirement for implementing a policy is the simple majority rule. We could easily alter this requirement such that a qualified majority (e.g. 2/3 of the delegates) is needed.[7] However, since simple majority is the most widespread rule, it seems reasonable to restrict attention to it.

[6] In order to facilitate exposition, I abuse notation by using $S \backslash i$ instead of $S \backslash \{i\}$.

[7] Evidently, the unanimity rule has to be excluded.

First, I introduce some notation that closely follows HM. I denote the maximum utility a majority coalition can achieve without the consent of the other players as

$$v(S) = \max_{(x,\tau_S)} \sum_{i\in S} \left(-(d_i - x)^2 + \tau_i \right) \quad s.t. \sum_{i \in S} \tau_i = 0 \tag{12}$$

Denote the policy that is jointly optimal for a coalition as x_S^*. Formally, we have

$$\begin{aligned} x_S^* &= \arg\max_x \sum_{i\in S} \left(-(d_i - x)^2 + \tau_i \right) \\ &\Rightarrow \ x_S^* = \frac{1}{s} \sum_{i \in S} d_i \end{aligned} \tag{13}$$

Hence, (12) can be written as

$$\begin{aligned} v(S) &= \sum_{i\in S} \left(- \left(d_i - \frac{1}{s} d_i - \frac{1}{s} \sum_{j \in S, j \neq i} d_j \right)^2 + \tau_i \right) \\ &= -\frac{s-1}{s} \sum_{i \in S} d_i^2 + \frac{2}{s} \sum_{i\in S} \sum_{\substack{j \neq i \\ j \in S}} d_i d_j \quad \forall S \mid s \geq \frac{n+1}{2} \end{aligned} \tag{14}$$

Note that $v(S)$ can be seen as a coalitional (or characteristic) function restricted to the set of majority coalitions. Denote the policy that player i proposes, if the set of active players is S, as $x_{S,i}$ and the proposed sidepayments as $\tau_{S,i}$. The payoff of player $j \in S$ arising from the proposal of player i is denoted as $a_{S,i}^j$. Correspondingly, $a_{S,i}$ is the vector of payoffs arising to active players from the proposals of player i if the set of active players is S. The proposals have to be feasible, i.e. $\sum_j a_{S,i}^j \leq v(S)$. The average payoff of the proposals to player j, if S is the set of active players, is denoted as $a_S^j = \frac{1}{s}\sum_{i\in S} a_{S,i}^j$. Finally, the payoff a player j, who belongs to the set S of inactive players, receives from a proposal made by an active player $i \in N \backslash S$, is defined as

$$\begin{aligned} a_S^j &= -\left(d_j - x_{N\backslash S,i} \right)^2 \qquad \forall S \ni j, s < \frac{n+1}{2} \\ &= -d_j^2 - \frac{1}{(n-s)^2} \sum_{i \in N\backslash S} d_i^2 + \frac{2}{n-s} d_j \sum_{i \in N \backslash S} d_i - \frac{2}{(n-s)^2} \sum_i \sum_{\substack{k \neq i \\ k \in N\backslash S}} d_i d_k \end{aligned} \tag{15}$$

As in the previous paper, I use the concept of stationary subgame perfect equilibrium to solve the bargaining game. Moreover, I assume that proposer and responders break ties in favor of an early termination of the game. The stationary subgame perfect equilibrium of the bargaining game is characterized by the following proposition

Proposition 3 *In the unique stationary subgame perfect equilibrium (SSPE) of the bargaining game of definition 5, all proposals are accepted and are characterized by the following expressions that hold for all $i, j \in S$,* $\quad s \geq \frac{n+1}{2}$

$$
\begin{aligned}
i) \quad a_{S,i} &= v(S) \\
ii) \quad a^j_{S,i} &= \rho a^j_S + (1-\rho)\, a^j_{S\backslash i} \\
iii) \quad a^i_{S,i} &\geq \rho a^i_S + (1-\rho)\, a^i_{S\backslash i} \\
iv) \quad a^i_S &= \sum_{T\subseteq S, T\ni i, |T|>\frac{n+1}{2}} \frac{(t-1)!\,(s-t)!}{s!}\left[v(T) - v(T\backslash i)\right] \\
&\quad + \sum_{T\subseteq S, T\ni i, |T|=\frac{n+1}{2}} \frac{(t-1)!\,(s-t)!}{s!}\left[v(T) - \sum_{j\in T, j\neq i} a^j_{T\backslash i} + \sum_{j\in T, j\neq i} a^i_{T\backslash j}\right] \\
v) \quad \text{if } \rho &\to 1 \text{ then } a^j_{S,i} \to a^j_S \quad \forall i,j\in S. \\
vi) \quad x_{S,i} &= \frac{1}{s}\sum_{j\in S} d_j \\
vii) \quad \tau^j_{S,i} &= a^j_{S,i} + (d_j - x_{S,i})^2
\end{aligned}
$$

Proof. See appendix, *i*) and *ii*) are identical to HM, the proof of *iv*) follows the same argument as in HM, the proof of *v*) is identical to HM. *vi*) follows directly from *i*), *vii*) follows from the definition of sidepayments. ■

For ρ converging to 1, the bargaining is thus a quadratic-linear function in the d_i. The following lemma shows how this function can be written in a more convenient form.

Lemma 1 *The bargaining solution can be written in the form*

$$
\begin{aligned}
\phi_i &= \sum_{S\ni i, s>\frac{n+1}{2}} \frac{(s-1)!\,(n-s)!}{n!}\left[v(S) - v(S\backslash i)\right] \\
&\quad + \sum_{S\ni i, s=\frac{n+1}{2}} \frac{(s-1)!\,(n-s)!}{n!}\left[v(S) - \sum_{,j\neq i} a^j_{S\backslash i} + \sum_{,j\neq i} a^i_{S\backslash j}\right] \\
&= -\alpha_n d_i^2 + \frac{\alpha_n}{n-1} 2d_i \sum_{j\neq i} d_j + \beta_n \sum_{j\neq i} d_j^2 + \gamma_n \sum_{j\neq i}\sum_{k\neq i,j} d_i d_k \qquad (16) \\
\textit{where } \alpha_n &= \frac{1}{n}\left[\frac{n-1}{2}\left(1 - \frac{4}{(n+1)^2}\right) + \sum_{m=\frac{n-1}{2}}^{n-1} \frac{m}{m+1}\right] \\
\beta_n &= \frac{\alpha_n}{n-1} - \frac{1}{n} \\
\gamma_n &= \frac{1}{n(n-2)} - \frac{2\alpha_n}{(n-1)(n-2)}
\end{aligned}
$$

Proof. See appendix. The proof involves only simple algebra. The appendix contains also a table giving some values for α_n. ■

With this result we can solve the first stage of the game where the principals decide which delegate is to be sent. It should be emphasized that, as in the previous section, the equilibrium values of the preferred policies of the delegates and the equilibrium policy are independent of ρ. Using proposition 3 and (16), we can write the utility function of the principals (4) as

$$\begin{aligned} U_i &= \left(-\alpha_n + \frac{n-2}{n}\right) d_i^2 + 2d_i \sum_{j\neq i} d_j \left(\frac{\alpha_n}{n-1} - \frac{1}{n}\right) \\ &\quad + \frac{2}{n} d_i x_i + f\left(x_i, x^c, d_{-i}\right) \end{aligned}$$

Hence, the FOC for the maximization problems of the principals in stage 1 are given by[8]

$$\left(\frac{n-2}{n} - \alpha_n\right) d_i + \left(\frac{\alpha_n}{n-1} - \frac{1}{n}\right) \sum_{j\neq i} d_j + \frac{1}{n} x_i = 0 \tag{17}$$

Solving this system leads to the following proposition that characterizes the equilibrium in the two stage game.

Proposition 4 *If the bargaining game between the delegates follows definition 3, the unique equilibrium of the delegation game is given by*

$$\begin{aligned} d_i^e &= \frac{1-\alpha_n'}{1-n\alpha_n'} x_i - \frac{\alpha_n'}{1-n\alpha_n'} \sum_{j\neq i} x_j \qquad \forall i \\ &= \frac{1}{1-n\alpha_n'} x_i - \frac{n\alpha_n'}{1-n\alpha_n'} \bar{x} \qquad (18) \\ \textit{where } \alpha_n' &= \frac{1}{2} + \frac{2}{(n+1)^2} - \frac{1}{n-1} \sum_{m=\frac{n-1}{2}}^{n-1} \frac{m}{m+1} \\ x^e &= \frac{1}{n} \sum_{i\in N} x_i = \bar{x} \qquad (19) \\ \tau_i^e &= \frac{1}{(1-n\alpha_n')\, n^2} \left[-(n-1)\, x_i^2 + 2x_i \sum_{j\neq i} x_j \right. \\ &\quad \left. + \sum_{j\neq i} x_j^2 - \frac{4}{n-2} \sum_{j\neq i} \sum_{k\neq i,j} x_j x_k \right] \qquad \forall i \qquad (20) \\ &= \frac{n-1}{(1-n\alpha_n')\, n^2} \left[x_i \left(\bar{x}_{-i} - x_i\right) + \frac{2}{n-1} \sum_{j\neq i} x_j \left(\frac{x_j + x_i}{2} - \bar{x}_{-ij}\right) \right] \end{aligned}$$

[8] Also, the second order conditions are fulfilled.

Hence, the bargaining game described in definition 1 leads to strategic delegation, but is socially optimal.

Proof. See appendix. Table 4 in the appendix gives explicit values of α'_n and $n\alpha'_n$. ■

As an illustration, the following table presents the preferred policies of the delegates sent in equilibrium for some values of n.

Table 1: d_i^e

n	d_i^e
3	$\frac{8}{7}x_i - \frac{1}{7}\bar{x}$
5	$\frac{144}{143}x_i - \frac{1}{143}\bar{x}$
7	$\frac{1440}{1529}x_i + \frac{89}{1529}\bar{x}$
9	$\frac{11200}{12389}x_i + \frac{1189}{12389}\bar{x}$

For investigating the limit behavior of the equilibrium, the following lemma is useful.

Lemma 2 $\frac{2n}{(n+1)^2} - \frac{11n^2+5n}{35n^2+26n+3} < n\alpha'_n < \frac{2n}{(n+1)^2} - \frac{7n+1}{24n+8}$

$\lim_{n\to\infty} n\alpha'_n = 0.3069$

Proof. See appendix. ■

Remark 2 *If n tends to infinity, the equilibrium values approach*

$$\begin{aligned}
\lim_{n\to\infty} d_i^e &= 0.765x_i + 0.235\bar{x} \\
\lim_{n\to\infty} x^e &= \frac{1}{n}\sum x_i \\
\lim_{n\to\infty} \tau_i^e &= 0
\end{aligned}$$

We see that the equilibrium position of delegate i is a convex combination between the preferred policy of his principal x_i and the socially optimal policy $\bar{x}$. The weight of x_i is decreasing in n. However, since this weight is always strictly higher than $\frac{3}{4}$, we can say that the individually preferred policies are the predominant factors for choosing the delegates. Most notably, the effects of strategic delegation across the principals cancel out so that the equilibrium policy is socially optimal. The equilibrium sidepayments look quite complicated even for the case $\rho \to 1$. Loosely speaking, the sidepayments are positive for principals (and delegates) with intermediate preferred policies and negative for those with extreme positions.

To grasp intuition of the optimality result, consider the two different incentives for strategic delegation explicitly. For that, rewrite the FOC of the principals as

$$\begin{aligned}\frac{\partial U_i}{\partial d_i} &= -\frac{\partial\left(x_i - x\right)^2}{\partial d_i} + \frac{\partial \tau_i}{\partial d_i} = 0 \qquad (21)\\ &= \frac{2}{n}\left(x_i - \frac{1}{n}d_i - \frac{n-1}{n}\bar{d}_{-i}\right) - 2\left(\alpha_n - \left(\frac{n-1}{n}\right)^2\right)\left(d_i - \bar{d}_{-i}\right) = 0\end{aligned}$$

The first term in (21) describes the incentive to achieve the most favorable policy (policy incentive). This incentive has the same form as in the previous paper. The term is maximized if

$$\left(d_i - x_i\right) = (n-1)\left(x_i - \bar{d}_{-i}\right) \qquad (22)$$

If this were the only incentive, a principal would send a delegate with $d_i > x_i$ ($d_i < x_i$) in case his preferred policy is higher (lower) than the average position of the other delegates. This incentive is mean preserving in the sense that rearranging and averaging the equations (22) for all principals gives $\frac{1}{n}\sum_{j\in N} d_j = \bar{x}$. The second term in (21) describes the sidepayment incentive. Since $\left(\alpha_n - \left(\frac{n-1}{n}\right)^2\right) > 0$, the sidepayments are decreasing in the distance between the position of a delegate and the mean position of the other delegates.[9] This is because a delegate with an average preferred policy has a rather strong bargaining position since he is harmed less by being excluded. The second term is maximized if $d_i = \bar{d}_{-i}$. This incentive alone would lead to identical positions of all delegates and is therefore (weakly) mean preserving. Hence, the chosen policy is optimal in equilibrium, where both incentives are at work. A remaining question is which incentives dominates. If we look at equation (18) , we get

$$\begin{aligned} d_i &> x_i \\ \Rightarrow \frac{1}{1-n\alpha_n'}x_i - \frac{n\alpha_n'}{1-n\alpha_n'}\bar{x} &> x_i \\ \Leftrightarrow \alpha_n'\left(x_i - \bar{x}\right) &> 0\end{aligned}$$

Hence, the first incentive leading to more extreme delegates dominates if $\alpha_n' > 0$. In this game, this holds for $n = 3$ and $n = 5$. While, for $n \geq 7$, the second incentive dominates.

[9] See also table 4 in the appendix.

6 Conclusion

This paper investigated the incentives for strategic delegation and its effects in two political bargaining games. A standard unanimity bargaining game was contrasted with a consensus bargaining game that captured important features of some real world bargaining processes and that proved to be an attractive decision mechanism for many political and economic institutions. Since we observed strategic delegation in both games, this phenomenon seems to be a common feature in this kind of political decision mechanisms. Discrepancies between the political interests of electorates (characterized by their median voter) and their representatives are not necessarily related to political failure within an electorate. They can rather be the result of rational behavior in a well functioning (local) political system.

We have seen that two different motives induce the principals (local median voters) to delegate strategically. The motive to move the chosen policy towards the individually preferred one is identical in both games. The motive to maximize sidepayments, however, differs crucially between the two games. In the case of unanimity bargaining this incentive drives the delegates and therefore the equilibrium policy towards the status quo. Under consensus bargaining it leads to a more homogenous committee and guarantees socially optimal policies. Since institutions following the consensus approach implement an optimal compromise between the heterogeneous interests of the citizens, we should expect these institutions to be more viable in the long run.

The policy recommendations based on this analysis are straightforward. Whenever the underlying basic model is applicable, new committees should introduce consensus bargaining and those applying the unanimity rule should switch to it. However, distributional conflicts might endanger such welfare improving reforms because principals with preferred policies close to the status quo fare relatively well in the unanimity case. In case of the EU, two reasons could improve the support for a beneficial institutional reform. First, the current economic and political convergence increases the homogeneity of the principals' preferences. Second, the planned enlargement increases the number of delegates. As this paper has shown, both arguments lead to larger benefits of consensus bargaining.

A Proofs

Proof of proposition 1:

i) $a_i = v(N)$. Suppose $a_i < v(N)$ and all responders accept a_i^j. Then the proposer could propose $\tilde{a}_i$ with

$$\tilde{a}_i^j = a_i^j \quad \forall j, \quad \tilde{a}_i^i = v(N) - \sum_{j \neq i} a_i^j > a_{i,r}^i$$

This proposal would be accepted as well by all responders and would give the proposer a higher utility.

ii) first part: $a_i^j = \rho a^j + (1-\rho) a_c^j$. This proposal gives to all responders their continuation value and makes them thus indifferent between accepting and refusing. Any proposal $a_i^j < \rho a^j + (1-\rho) a_c^j$ would be refused, any proposal $a_i^j > \rho a^j + (1-\rho) a_c^j$ gives the proposer a lower payoff.

iii) first part: $a_i^i = v(N) - \sum_{j \neq i} a_i^j$ follows from *i)* and *ii)*.

iv) $a^i = \frac{1}{n}\left[v(N) - \sum_{j \in N} a_c^j\right] + a_c^i$. By the first parts of *ii)* and *iii)* and the definition of a^i we have

$$\begin{aligned} na^i &= a_i^i + \sum_{j \neq i} a_j^i \\ &= v(N) - \rho\left[v(N) - a^i\right] - (1-\rho)\sum_{j \neq i} a_c^j + (n-1)\left[\rho a^i + (1-\rho) a_c^i\right] \\ &= (1-\rho)\left[v(N) - \sum_{k \in N} a_c^k\right] + n\rho a^i + n(1-\rho) a_c^i \\ \Rightarrow a^i &= \frac{1}{n}\left[v(N) - \sum_{j \in N} a_c^j\right] + a_c^i \end{aligned}$$

Inserting *iv)* in the first parts of *ii)* and *iii)* proofs the second parts of *ii)* and *iii)*.

v) follows from *ii)*.

vi) follows from *i)*.

vii) follows from *ii)*, *iii)* and the definition of sidepayments. ■

Proof of proposition 2:

Proof. Write (9) as

$$\begin{aligned} B \cdot \mathbf{d} &= \mathbf{x} + \frac{n(n-1)}{2n-1}\mathbf{x}^c \\ \text{where } b_{ii} &= \frac{n}{2n-1},\ b_{ij} = \frac{n-1}{2n-1} \end{aligned}$$

Inverting B gives then

$$d_i = \frac{n^2-n+1}{n^2}x_i - \frac{n-1}{n^2}\sum_{j\neq i} x_j + \frac{n-1}{n}x^c$$

For the chosen policy we have thus

$$\begin{aligned} x^e &= \frac{1}{n}\sum_{i\in N} d_i = \frac{1}{n}\sum_{i\in N}\left(\frac{(n^2-n+1)-(n-1)^2}{n^2}x_i\right) + \frac{n-1}{n}x^c \\ &= \frac{1}{n^2}\sum_{i\in N} x_i + \frac{n-1}{n}x^c \end{aligned}$$

For the equilibrium sidepayments we get

$$\begin{aligned} \tau_i^e &= \psi_i + (d_i - x^e)^2 \\ &= x^{e^2} - d_i^2 - 2x^c x^e + 2d_i x^c + x^{e^2} + d_i^2 - 2d_i x^e \\ &= 2\left[\left(\frac{1}{n}\sum_{i\in N} x_i - x_i\right)\left(\frac{1}{n^2}\sum_{i\in N} x_i - \frac{x^c}{n}\right)\right] \end{aligned}$$

■

Proof of proposition 3

i) $a_{S,i} = v(S)$. Suppose $a_{S,i} < v(S)$ and all responder accept $a_{S,i}^j$. Then the proposer could propose $\tilde{a}_{S,i}$ with

$$\tilde{a}_{S,i}^j = a_{S,i}^j \quad \forall j, \quad \tilde{a}_{S,i}^i = v(S) - \sum_{j\neq i} a_{S,i}^j > a_{S,i}^i$$

This proposal would be accepted as well by all responder and would give the proposer a higher utility.

ii) $a_{S,i}^j = \rho a_S^j + (1-\rho)\, a_{S\backslash i}^j$. This proposal gives to all responder their continuation value and makes them thus indifferent between accepting and refusing. Any proposal $a_{S,i}^j < \rho a_S^j + (1-\rho)\, a_{S\backslash i}^j$ would be refused and any proposal $a_{S,i}^j > \rho a_S^j + (1-\rho)\, a_{S\backslash i}^j$ gives a lower pay-off for the proposer.

iii) $a^i_{S,i} \geq \rho a^i_S + (1-\rho)\, a^i_{S\backslash i}$. This states that a proposer receives at least his continuation value. If *iii)* would not hold, the proposer would be better off by making a proposal that would be refused by another player.[10]

First, I look at coalitions where $s = |S| > \frac{n+1}{2}$. Using *i)*, *ii)* and $\sum_{j\neq i} a^j_{S\backslash i} = v(S\backslash i)$ we can write *iii)* as

$$\begin{aligned} v(S) - \sum_{j\neq i} \rho a^j_S + (1-\rho)\, a^j_{S\backslash i} &\geq \rho a^i_S + (1-\rho)\, a^i_{S\backslash i} \\ \Rightarrow v(S) - v(S\backslash i) &\geq a^i_{S\backslash i} \end{aligned}$$

Inserting (14) and (15) gives

$$\begin{aligned} & -\frac{s-1}{s} d_i^2 - \left(\frac{s-1}{s} - \frac{s-2}{s-1}\right) \sum_{j\neq i} d_j^2 + \frac{2}{s} d_i \sum_{j\neq i} d_j + \left(\frac{2}{s} - \frac{2}{s-1}\right) \sum_{j\neq i} \sum_{k\neq i,j} d_j d_k \\ \geq\ & -d_i^2 + \frac{1}{s(s-1)^2} \sum_{j\neq i} d_j^2 + \frac{2}{s(s-1)} d_i \sum_{j\neq i} d_j + \frac{2}{s(s-1)^2} \sum_{j\neq i} \sum_{k\neq i,j} d_j d_k \\ \Rightarrow\ & \frac{1}{s}\left(d_i - \frac{1}{s-1} \sum_{j\neq i} d_j\right)^2 \geq 0. \end{aligned}$$

Next I, look at coalitions with $s = \frac{n+1}{2}$. We can write *iii)* as

$$\begin{aligned} v(S) - \sum_{j\neq i} \rho a^j_S + (1-\rho)\, a^j_{S\backslash i} &\geq \rho a^i_S + (1-\rho)\, a^i_{S\backslash i} \\ v(S) &\geq \sum_{j\neq i} a^j_{S\backslash i} + a^i_{S\backslash i} \end{aligned}$$

Inserting (14) and (15) results in

$$\begin{aligned} & -\frac{s-1}{s} d_i^2 - \frac{s-1}{s} \sum_{j\neq i} d_j^2 + \frac{2}{s} d_i \sum_{j\neq i} d_j + \frac{2}{s} \sum_{j\neq i} \sum_{k\neq i,j} d_j d_k \\ \geq\ & - \sum_{j\neq i, j\in S} \left(d_j - \frac{1}{s} d_i - \frac{1}{s} \sum_{l\in N\backslash S} d_l\right)^2 - \left(\frac{s-1}{s} d_i - \frac{1}{s} \sum_{l\in N\backslash S} d_l\right)^2 \end{aligned} \tag{23}$$

We can add a constant to every position of the delegates without changing the utility functions. I consider the transformed positions of the delegates $\tilde{d}_i = d_i + c \quad \forall i$ where $\sum_{l\in N\backslash S} \tilde{d}_l = 0$. Hence (23) can be written as

$$-\frac{s-1}{s} \tilde{d}_i^2 - \frac{s-1}{s} \sum_{j\neq i} \tilde{d}_j^2 + \frac{2}{s} \tilde{d}_i \sum_{j\neq i} \tilde{d}_j + \frac{2}{s} \sum_{j\neq i} \sum_{k\neq i,j} \tilde{d}_j \tilde{d}_k$$

[10]This condition corresponds to the non-negativity property in the SPE in HM. However, since in our game the pay-off of a player that drops out cannot be normalized to zero (since it depends on the set of remaining active player), the proof of *iii)* differs from the one in HM.

$$
\begin{aligned}
&\geq \quad -\sum_{j\neq i}\tilde{d}_j^2-\frac{s-1}{s^2}\tilde{d}_i^2+\frac{2}{s}\tilde{d}_i\sum_{j\neq i}\tilde{d}_j-\left(\frac{s-1}{s}\right)^2\sum_{j\neq i}\tilde{d}_j^2 \\
&\Rightarrow \quad \frac{1}{s}\left(\sum_{j\neq i}\tilde{d}_j\right)^2+\left(\frac{s-1}{s}\right)^2\sum_{j\neq i}\tilde{d}_j^2\geq 0
\end{aligned}
$$

$$
\begin{aligned}
iv)\quad a_T^i &= \sum_{S\subseteq T,S\ni i,s>\frac{n+1}{2}}\frac{(s-1)!\,(n-s)!}{n!}\left[v(S)-v(S\backslash i)\right] \\
&+\sum_{S\subseteq T,S\ni i,s=\frac{n+1}{2}}\frac{(s-1)!\,(n-s)!}{n!}\left[v(S)-\sum_{j\in S,j\neq i}a_{S\backslash i}^j+\sum_{j\in S,j\neq i}a_{S\backslash j}^i\right]
\end{aligned}
$$

By $i)$ and $ii)$ we have

$$
\begin{aligned}
na_N^i &= a_{N,i}^i+\sum_{j\neq i}a_{N,j}^i=v(N)-\sum_{j\neq i}a_{N,i}^j+\sum_{j\neq i}a_{N,j}^i \\
&= v(N)-\sum_{j\neq i}\left(\rho a_N^j+(1-\rho)\,a_{N\backslash i}^j\right)+\sum_{j\neq i}\left(\rho a_N^i+(1-\rho)\,a_{N\backslash j}^i\right) \\
&= v(N)-\sum_{j\neq i}\rho a_N^j+(n-1)\,\rho a_N^i+\sum_{j\neq i}(1-\rho)\,a_{N\backslash i}^j+\sum_{j\neq i}(1-\rho)\,a_{N\backslash j}^i \\
n(1-\rho)\,a_N^i &= (1-\rho)\left[v(N)-v(N\backslash i)\right]+\sum_{j\neq i}(1-\rho)\,a_{N\backslash j}^i \\
a_N^i &= \frac{1}{n}\left[v(N)-v(N\backslash i)+\sum_{j\neq i}a_{N\backslash j}^i\right]
\end{aligned}
$$

By the same argument, we can compute a_S^i for all coalitions in which i is not necessary to form a majority as

$$
a_S^i=\frac{1}{s}\left[v(S)-v(S\backslash i)+\sum_{j\neq i,j\in S}a_{S\backslash j}^i\right]\qquad \forall S\ni i\mid s>\frac{n+1}{2} \tag{24}
$$

If we look at coalitions $s=\frac{n+1}{2}$, the same argument results in

$$
a_S^i=\frac{1}{s}\left[v(S)-\sum_{j\neq i}a_{S\backslash i}^j+\sum_{j\neq i}a_{S\backslash j}^i\right]\qquad \forall S\ni i\mid s=\frac{n+1}{2} \tag{25}
$$

Since $S\backslash i$ and $S\backslash j$ are minority coalitions, the values for $a_{S\backslash i}^j$ and $a_{S\backslash j}^i$ are those defined as in (15). Recursively inserting of (25)and (24) then proves $iv)$.

$v)$ follows directly from $i)$ and $ii)$.

$vi)$ follows directly from $i)$.

$vii)$ follows from the definition of sidepayments. ■

Proof of lemma 1

First I determine α_n. For that we have to only consider terms where d_i occurs. For coalitions of equal size, we have

$$\begin{aligned}
&\sum_{S\ni i,|S|=s} \frac{(s-1)!\,(n-s)!}{n!}\left[v(S)-v(S\backslash i)\right] \\
&= \frac{1}{n}\left[-\frac{s-1}{s}d_i^2-\frac{2}{s}\frac{s-1}{n-1}d_i\sum_{j\in S,j\neq i} d_j\right]+f(s,d_{-i})
\end{aligned}$$

For coalitions of size $s=\frac{n+1}{2}$, we have to consider the terms $a^j_{S\backslash i}$ and $a^i_{S\backslash j}$ as well, as defined in (15). We have

$$\begin{aligned}
&\sum_{S\ni i,s=\frac{n+1}{2}} \frac{(s-1)!\,(n-s)!}{n!}\left[-\sum_{j\in S,j\neq i} a^j_{S\backslash i}+\sum_{j\in S,j\neq i} a^i_{S\backslash j}\right] \\
=\;& \sum_{S\ni i,s=\frac{n+1}{2}} \frac{(s-1)!\,(n-s)!}{n!}\left[\frac{-(s^2-1)(s-1)}{s^2}d_i^2+\frac{(s^2-1)}{s^2}2d_i\sum_{j\in S,j\neq i} d_j\right] \\
=\;& \frac{n+1}{2}\left[1-\frac{4}{(n+1)^2}\right]d_i^2+\left[1-\frac{4}{(n+1)^2}\right]d_i\sum_{j\in S,j\neq i} d_j
\end{aligned}$$

Combining these expressions then results in

$$\alpha_n=\frac{1}{n}\left[\frac{n-1}{2}\left(1-\frac{4}{(n+1)^2}\right)+\sum_{m=\frac{n-1}{2}}^{n-1}\frac{m}{m+1}\right]$$

To determine β_n and γ_n, note that by proposition 3

$$\begin{aligned}
\sum\phi_i &= v(N) \\
(-\alpha_n+(n-1)\beta_n)\sum d_i^2 & \quad +\left(\frac{4\alpha_n}{n-1}+2(n-2)\gamma_n\right) \\
&= -\frac{n-1}{n}\sum d_i^2+\frac{2}{n}\sum_i\sum_{j\neq i} d_i d_j \\
\Rightarrow \beta_n &= \frac{\alpha_n}{n-1}-\frac{1}{n} \\
\gamma_n &= \frac{1}{n(n-2)}-\frac{2\alpha_n}{(n-1)(n-2)}
\end{aligned}$$

■

Table for different values of α_n and α'_n

Table 2

n	α_n	$\alpha_n - \left(\frac{n-1}{n}\right)^2$	α'_n	$n\alpha'_n$
3	$\frac{23}{36} \cong 0.639$	$\frac{7}{36} \cong 0.194$	$\frac{1}{24} \cong 0.042$	$\frac{1}{8} \cong 0.125$
5	$\frac{719}{900} \cong 0.799$	$\frac{143}{900} \cong 0.159$	$\frac{1}{720} \cong 0.001$	$\frac{1}{144} \cong 0.007$
7	$\frac{10169}{11760} \cong 0.865$	$\frac{1529}{11760} \cong 0.130$	$-\frac{89}{10080} \cong -0.009$	$-\frac{89}{1440} \cong -0.062$
9	$\frac{101989}{113400} \cong 0.899$	$\frac{12389}{113400} \cong 0.109$	$-\frac{1189}{100800} \cong -0.012$	$-\frac{1189}{11200} \cong -0.106$
∞	1	0	0	-0.307

Proof of lemma 2:

First, note that for $p, q > 0$, $p < q$ the condition

$$\begin{aligned} \frac{1}{2}\left(\frac{q-p}{q-p+1} + \frac{q+p}{q+p+1}\right) &< \frac{q}{q+1} \\ \Longleftrightarrow \left(q^2 + q - p^2\right)(q+1) &< q\,(q-p+1)\,(q+p+1) \\ \Longleftrightarrow p^2 &> 0 \end{aligned} \tag{26}$$

always holds. Next, I develop a lower bound for $n\alpha'_n$. Inequality (26) implies that

$$\begin{aligned} \frac{n}{n-1}\sum_{m=\frac{n-1}{2}}^{n-1}\frac{m}{m+1} &< \frac{n}{n-1}\frac{n+1}{2}\left[\frac{\frac{3}{4}(n-1)}{\frac{3}{4}(n-1)+1}\right] = \frac{3n\,(n+1)}{2\,(3n+1)} \\ n\alpha'_n &= \frac{n}{2} + \frac{2n}{(n+1)^2} - \frac{n}{n-1}\sum_{m=\frac{n-1}{2}}^{n-1}\frac{m}{m+1} \\ &> \frac{2n}{(n+1)^2} + \frac{n}{2}\left(1 - \frac{3\,(n+1)}{3n+1}\right) = \frac{2n}{(n+1)^2} - \frac{n}{3n+1} \end{aligned}$$

By the same argument, we have

$$\begin{aligned} \frac{n}{n-1}\sum_{m=\frac{n-1}{2}}^{n-1}\frac{m}{m+1} &< \frac{n}{n-1}\frac{n+1}{4}\left[\frac{\frac{5}{8}(n-1)}{\frac{5}{8}(n-1)+1} + \frac{\frac{7}{8}(n-1)}{\frac{7}{8}(n-1)+1}\right] \\ \frac{n}{n-1}\sum_{m=\frac{n-1}{2}}^{n-1}\frac{m}{m+1} &< \frac{n\,(n+1)}{4}\left(\frac{5}{5n+3} + \frac{7}{7n+1}\right) \\ n\alpha'_n &> \frac{2n}{(n+1)^2} + \frac{n}{2} - \frac{70n^3 + 96n^2 + 26n}{140n^2 + 104n + 12} \\ &= \frac{2n}{(n+1)^2} - \frac{11n^2 + 5n}{35n^2 + 26n + 3} \end{aligned}$$

Now, I determine an upper bound for $n\alpha'_n$. Inequality (26) also implies that

$$\frac{n}{n-1}\sum_{m=\frac{n-1}{2}}^{n-1}\frac{m}{m+1} > \frac{n}{n-1}\frac{n+1}{2}\left[\frac{\frac{n-1}{2}}{\frac{n-1}{2}+1} + \frac{n-1}{n}\right]\frac{1}{2} = \frac{2n+1}{4}$$

$$n\alpha_n' \quad < \quad \frac{2n}{(n+1)^2} - \frac{1}{4}$$

The same argument implies that

$$\frac{n}{n-1}\sum_{m=\frac{n-1}{2}}^{n-1}\frac{m}{m+1} \quad > \quad \frac{n}{n-1}\frac{n+1}{8}\left[\frac{\frac{n-1}{2}}{\frac{n-1}{2}+1}+2\frac{\frac{3}{4}(n-1)}{\frac{3}{4}(n-1)+1}+\frac{n-1}{n}\right]$$

$$\frac{n}{n-1}\sum_{m=\frac{n-1}{2}}^{n-1}\frac{m}{m+1} \quad > \quad \frac{12n^2+11n+1}{24n+8}$$

$$n\alpha_n' \quad < \quad \frac{2n}{(n+1)^2}+\frac{n}{2}-\frac{12n^2+11n+1}{24n+8}$$

$$n\alpha_n' \quad < \quad \frac{2n}{(n+1)^2}-\frac{7n+1}{24n+8}$$

This proves lemma 2. Note that lemma 2 implies

$$-\frac{11}{35} < \lim_{n\to\infty} n\alpha_n' < -\frac{7}{24}$$

Moreover, we can make the lower and upper bound more precise by applying the same arguments The following table shows how a series of upper and lower bounds for $\lim n\alpha_n'$ emerges.

Table 3: Lower and upper bounds for $\lim_{n\to\infty} n\alpha_n'$

step	lower bound		upper bound	
1	-0.33	$-\frac{1}{3}$	-0.25	$-\frac{1}{4}$
2	$-0.\,31429$	$-\frac{11}{35}$	$-0.\,29167$	$-\frac{7}{24}$
3	$-0.\,30878$	$-\frac{1987}{6435}$	$-0.\,30298$	$-\frac{509}{1680}$
4	-0.30734	$-\frac{96986\,24987}{3\,15567\,20475}$	-0.30588	$-\frac{88181}{288288}$
5	$-0.\,30697$		$-0.\,30661$	
6	-30688		-0.30679	
7			-0.30684	

■

Proof of proposition 4

Proof. Write (17) as

$$A \cdot d \quad = \quad x$$

$$\text{where } a_{ii} \quad = \quad n\alpha_n - n + 2,\ a_{ij} = 1 - \frac{n}{n-1}\alpha_n$$

Inverting A and defining $\alpha'_n = 1 - \frac{n}{n-1}\alpha_n$ result in

$$\begin{aligned} d_i^e &= \frac{1-\alpha'_n}{1-n\alpha'_n} x_i - \frac{\alpha'_n}{1-n\alpha'_n} \sum_{j \neq i} x_j \qquad (27) \\ x^e &= \frac{1}{n}\sum d_i = \frac{1}{n}\sum_{i\in N}\left(\frac{1-\alpha'_n}{1-n\alpha'_n} x_i - \frac{\alpha'_n}{1-n\alpha'_n}\sum_{j\neq i} x_j\right) \\ &= \frac{1}{n}\sum_{i\in N}\left(\frac{1-\alpha'_n}{1-n\alpha'_n} - (n-1)\frac{\alpha'_n}{1-n\alpha'_n}\right) x_i \\ &= \frac{1}{n}\sum_{i\in N} x_i \end{aligned}$$

For determining the sidepayments, note that we can write them as

$$\tau_i = \frac{\alpha_n n^2 - n^2 + 2n - 1}{n^2}\left[-d_i^2 + 2d_i \frac{\sum_{j\neq i} d_j}{n-1} + \frac{\sum_{j\neq i} d_j^2}{n-1} - 2\frac{2\sum_{j\neq i}\sum_{k\neq i,j} d_j d_k}{(n-1)(n-2)}\right]$$

Inserting the equilibrium position of the delegates, using $\alpha'_n = 1 - \frac{n}{n-1}\alpha_n$ and some algebra then gives

$$\tau_i^e = \frac{1}{(1-n\alpha'_n)\, n^2}\left(-(n-1)\, x_i^2 + x_i \sum_{j\neq i} x_j + \sum_{j\neq i} x_j^2 - \frac{2}{n-2}\sum_{j\neq i}\sum_{k\neq i,j} x_j x_k\right) \blacksquare \qquad (28)$$

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