Strategic Information Revelation and Revenue Sharing in an R&D Race with Learning Labs*

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Abstract

Inherent to most research projects is the fact that researchers learn about their project during the course of it. Research investments result in signals on development costs. This paper studies how this fact influences firms' investments, and how revenue sharing can correct inefficient behavior. We compare efficient R&D investments, equilibrium investments when signals are public, and equilibrium investments for private signals. Furthermore, we show which equilibrium is played when firms strategically reveal information. The paper focuses on the trade-off between incentives to acquire and reveal information, and incentives to develop the innovation.

Keywords: R&D race, information acquisition, revelation, revenue sharing

JEL Codes: D82, D83, L23, O31, O32

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1 Introduction

Innovative activity has at least three basic properties. It is mostly done in a competitive environment. Firms compete to get an innovation first. Second, it is a dynamic activity. Research and Development (R&D) is a process for which we can distinguish several stages, at least there is a research stage, resulting in a raw prototype, and a development stage in which the prototype is transformed into a final product. Finally R&D is an uncertain activity. Not only is it uncertain when an innovation is going to occur, but also firms could be uncertain about the complexity of the project that they start working on. Only in the course of doing research firms learn whether their project is worthwhile proceeding. For marginal product improvements this learning effects can be ignored. But for more fundamental innovations and more experimental research these effects cannot be ignored. The fact that firms learn during the race, and the fact that they could learn from each other, creates new and interesting incentives to invest in R&D. This paper analyzes these incentives. Our analysis consists of three parts. First, we investigate how the fact that firms learn affects their incentives to invest in R&D. Second, we study how these incentives are affected under different regimes of appropriability of the innovation. And, finally, we analyze under what conditions firms will and will not learn from each other in equilibrium.

The simplest situation that captures the competitive, dynamic and informational aspects of innovative activity is the following. Two firms compete over two stages to get an innovation. In the first stage firms obtain an intermediate discovery, and learn about their R&D project. In the second stage firms decide how much to invest in developing the intermediate innovation, given the information acquired in stage 1. Firms compete to get the developed final product first. An early intermediate discovery in an R&D race can have two opposite effects on competition. We distinguish a strategic and an informational effect, and discuss them in the next two paragraphs.

In most literature on dynamic R&D competition the progress of one firm in their project discourages his rivals to invest in the innovation. Taking a lead in the race gives the leading firm a strategic advantage, e.g. see Grossman and Shapiro (1987), and Harris and Vickers (1987). This is a "strategic effect". If firms could credibly signal that they made an early intermediate discovery without revealing the contents of this discovery, they would always do so. The problem is that this revelation cannot happen credibly unless the contents of the discovery are revealed also. But revealing the contents of the discovery enables rivals to catch up in the race, which encourages further investments. Therefore a leading firm is only willing to obtain and reveal information about his progress in the race if he is sufficiently compensated for doing so. Compensation can happen by means of a licencing arrangement or an intermediate patent (see, e.g., Chang (1995), Green and Scotchmer (1995), Scotchmer and Green (1990) and Kabla (1997)) or grace period (see Goyal and De Laat, 1998). Thus there is a trade-off between the incentive to reveal information and leaving the informed firm an advantage in the race. The literature mostly focuses on this trade-off in R&D races.

For fundamental innovations[1] we see an effect that is opposite to the strategic effect. After an early intermediate success by one firm, rivals flock in and invest to obtain the final innovation first. This effect could be explained in the following setting, as in Choi (1991). Firms learn about the properties of the project while they work on it. These properties are universal for the industry. Favorable information for one firm is favorable also for his rivals. Then progressing in the race and disclosing this progress makes all firms more optimistic, and more willing to invest. This is an "informational effect". But when favorable information for one firm also encourages rivals to invest in the project, the firm might want to prevent its rivals from learning this information. There might be an incentive not to reveal any good news that firms learned.

The strategic effect gives firms an incentive to state that they made early intermediate discoveries, while they would keep intermediate successes secret under the informational effect. In practice these two effects interact, and this interaction determines the firms' incentives to invest in both stages of the race. In this paper we separate the acquisition of information from the acquisition of a leading position in the race. First, firms invest solely in acquiring information, and then invest in winning the race. Furthermore, we maximize the scope for firms to learn from their rival by assuming perfect positive correlation between the firms' projects. We thereby focus on the informational effect of intermediate discoveries and its subsequent problems of information revelation. This gives a sharper trade-off between incentives to reveal and acquire information.

This paper contributes in two important ways to the study of the trade-off between the informational and strategic effect. First, we study the effects of appropriability of revenues on the firms' incentives to invest in R&D. Most literature on R&D races focuses on the winner-takes-all race. This is, however, an extreme setting that needs not be realistic. We add more realism to the economic analysis by studying settings in which the winner does not take all. In particular, we assume that firms share a fixed portion of

[1] A classic example of this kind of innovation would be the 1986 breakthrough in cold superconductivity. For a description of the breakthrough by IBM, and its resulting race for even colder superconductivity, see Choi (1991).

their revenues. Revenue sharing introduces free-rider effects to the analysis. These free-rider effects interact in an interesting way with the informational and strategic effects.

The second main contribution of this paper is to endogenize firms' information. Information is endogenized in two directions. First, each firm invests in costly information acquisition. The incentives to invest depend on the appropriability of both the acquired information, and the innovation's revenues. When the acquired information is public, firms have a low incentive to invest in information acquisition, because they prefer to free ride on their rival's information acquisition investments. And when only part of the revenues from innovation are appropriated by a firm, both negative as well as positive externalities on research incentives exist between firms. The negative effect is due to the erosion of expected revenues from a firm's own information acquisition investments. This is a free-rider effect. The positive externality of revenue sharing is active when the firms' acquired information is public. The externality is caused by the fact that the information generated by one firm affects beliefs and consequently expected revenues of the firm's rival. Since part of these revenues spill over, firms have a bigger incentive to invest in information acquisition.

The second reason why the firms' information is endogenous is because firms can choose what information they reveal. That is, the revelation of information is not exogenous, but a strategic choice of the firms. When information is non-verifiable, firms never completely reveal their information, while there is an equilibrium in which they completely conceal information. This result holds for any way in which firms share revenues. These results are reversed for extreme revenue shares, however, when information is verifiable. Firms cannot credibly conceal any verifiable information, and will therefore fully disclose. For intermediate revenue shares there is no equilibrium in which firms completely reveal their information.

These two main contributions of the paper are discussed in more detail in the next sections.

Related literature: Papers by Hendricks and Kovenock (1989), Choi (1991), Malueg and Tsutsui (1997), and Cyert and Kumar (1998) study models in which firms learn about their project's characteristics while they invest in it. In their models the information obtained from research is publicly observable. Firms learn from each other's experience without cost. Information is incomplete, but symmetric. We show in this paper that firms have incentives to misrepresent their intermediate research results to affect competition in the development stage. Furthermore, we analyze how investments are affected by revenue sharing, and how they compare to the industry's efficient investments. We show that firms' expected profits can be increased by relaxing the "winner-takes-all" assumption.

Dewatripont *et al.* (1999) give sufficient conditions under which a manager's incentives for information acquisition investments are affected by an additional signal about his project. We perform a similar exercise for signals that are generated by a firm's rival. We extend the analysis by introducing competition both in information acquisition, as well as in the determination of firms' revenue.

Problems of strategic information revelation in R&D races are studied by Bhattacharya *et al.* (1990, 1992) and d'Aspremont *et al.* (1996, 1998) but in their models information is exogenous (and partly verifiable). Another model of endogenous information spillovers between competing firms is analyzed by Katsoulacos and Ulph (1998), and Ulph and Katsoulacos (1998). Their problem deals with information about the contents of the intermediate innovation, and not information about the costs of proceeding with the project. This puts more emphasis on the strategic effect of information revelation.

The effects for incentives of racing firms after the relaxation of the "winner-takes-all" patent scheme are studied in La Manna *et al.* (1989) and Denicolò (1996). In these papers the social optimality of full-scope patents is seriously questioned. We perform a similar exercise, but in an environment in which firms learn.

Problems in which information revelation occurs between competitors are problems of information sharing in oligopoly.[2] Novshek and Sonnenschein (1982), Fried (1984), and Creane (1995) study models in which firms acquire and reveal information before they compete.[3] However, in these models firms can commit *ex ante* whether to reveal information or not. This is a strong assumption that need not always be realistic. In fact, Ziv (1993) shows that the scope for information sharing is drastically reduced when firms cannot commit *ex ante* and information is non-verifiable. We follow the same modelling approach as in the paper by Ziv.

The paper is organized as follows. In the next section we describe the basic model. In section 3 the efficient investments that maximize total industry's profits are characterized. These investments serve as a benchmark. Section 4 analyzes the effects of introducing competition in this setting, while

[2]For a survey of the main results of information sharing in oligopoly, see Gal-Or (1986) and Raith (1996).

[3]Other papers, e.g. see Li *et al.* (1987), Hwang (1993, 1995) and Hauk and Hurkens (1998), study the incentives of competing firms to acquire information given that the acquired information remains private.

signals remain public information. This gives the equilibrium investments of competing firms that receive publicly observable signals about the project's complexity. In section 5 we analyze the equilibrium investments when firms have only private signals. The sixth section discusses what information is revealed, and what investments are chosen, when firms reveal information strategically. Section 7 discusses the assumptions on observability of research investments, and the last section concludes. All proofs are relegated to the Appendix.

2 The Model

We consider an industry in which two firms compete over two stages to obtain an innovation. Firms work on the same innovation but compete to get it first. Since firms work on the same innovation, we assume that their costs of investments are perfectly positively correlated. In the first stage firms acquire information about the costs of development investments, that should lead to the innovation. This is the research stage. In the second stage the firms actually try to develop the innovation. We call this stage the development stage. The firm that develops the innovation first, the winner, receives prize W. When both firms develop the innovation, each firm receives prize T. Naturally, we assume that $0 \leq T \leq \frac{1}{2}W$. At one extreme firms share the revenues from innovation equally, while at the other extreme, firms compete fiercely in the product market which leaves no rents for either of them. A firm that does not develop the innovation successfully receives no revenues. Define $\Delta \equiv W - T$ as the difference between the prizes of winning and tying. Note that our assumption on T implies that $\frac{1}{2}W \leq \Delta \leq W$.

At the beginning of the race firms do not know the complexity of the project they work on. Complexity directly affects the cost of investments in developing the innovation, and is summarized by the parameter θ. The project can either be easy, $\theta = \underline{\theta}$, or difficult, $\theta = \bar{\theta}$, to complete, with $0 < \underline{\theta} < \bar{\theta}$. When the project is easy (resp. difficult), it is easy (resp. difficult) for both firms. An easy project has low marginal cost of development. A difficult project is completed at high marginal cost. The probability of an easy (resp. difficult) project is p (resp. $1-p$), with $0 < p < 1$.

In the research stage firms find a prototype, and learn about the costs of development investment. Firm i does research by making an investment $R_i \in [0, 1]$. Research investments are not observable. Firm i expects research investment r_j from his rival firm j. Costs of research investments are quadratic in investments, $C(R_i) = \frac{\rho}{2}R_i^2$, with ρ high enough such that coordinating firms both invest in research. Learning is, however, not perfect.

After firms invest in research they receive a signal about the project's complexity. The quality of the signal depends on the investments in research. When the project is difficult, investments always lead to a bad signal, $t_i = \bar{t}$, for $i = 1, 2$. For an easy project firm i's signal depends on his research investment, R_i. Firm i receives a good signal, $t_i = \underline{t}$, with probability R_i, while the probability of a bad signal, $t_i = \bar{t}$, is $1 - R_i$, with $i = 1, 2$. Signals are independently distributed between firms given the project's complexity. The first-stage stochastic structure for firm i is depicted in Figure 1 below. The dashed lines represent firm i's information sets.

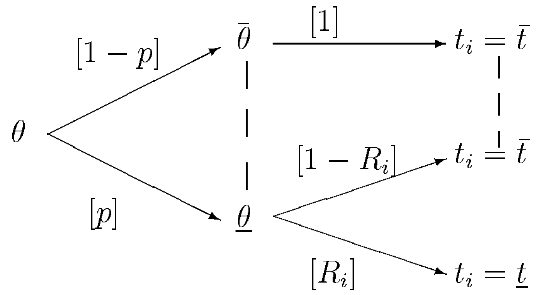

Figure 1: Firm i's research stage

We make different assumptions about the nature of the firms' signals. In the following two sections we assume that signals are public information, while in section 5 we assume that signals are private information to firms. Besides the fact that these cases are interesting by itself, they also enable us to analyze a richer model in which firms strategically choose how much information to reveal to their rival. We introduce this model in section 6 of this paper.

Whenever a firm receives a good signal, $t_i = \underline{t}$, it learns that both firms work on an easy project. Whenever both firms receive a bad signal, they are in one of the following situations. Either the project is difficult, or firms work on an easy project and were simply unlucky. The extent to which firms were unlucky under an easy project depends on firms' research investments, R. The more firms invested in research, the more pessimistic they get about the project's complexity.

In the second stage firms invest in the development of the innovation by spending $D_i \in [0, 1]$. Firm i's probability of making a final innovation is then D_i. An easy project, $\theta = \underline{\theta}$, has low marginal costs of development, while a difficult project has higher development costs, $\theta = \overline{\theta}$. In order to keep the model manageable, we assume that firm i's development cost is quadratic in development investment D_i, i.e. $c(D_i; \theta) = \frac{\theta}{2} D_i^2$, for $i = 1, 2$. Furthermore, we assume that $\underline{\theta} > 2\Delta$ to obtain interior solutions for firms' development investments.

We assume that firms are risk neutral. For firms' profits we define the following. Given development investments $D \equiv (D_1, D_2)$, firm i's development profits are:

$$\begin{aligned}\pi_i(D;\theta) &\equiv D_i D_j T + D_i(1-D_j)W - \frac{\theta}{2}D_i^2 \\ &= D_i(W - \Delta D_j) - \frac{\theta}{2}D_i^2.\end{aligned}$$

Then firm i's expected payoff is given by

$$\Pi_i(R, D) = E_\theta\{\pi_i(D;\theta)|R_i\} - \frac{\rho}{2}R_i^2.$$

We solve the game backwards, and focus on symmetric, pure-strategy Bayes perfect equilibria.

3 Benchmark: Efficient Investments

In this section we analyze the efficient outcome for the industry. This means that we calculate the research and development investments that maximize expected total industry's profits. We analyze this solution to understand firms' incentives when all relevant externalities are internalized.[4] We use the efficient outcome as a benchmark to study the effects of competition and private information on equilibrium strategies.

Given public signals, we calculate the efficient information acquisition and development choices by solving the model backwards. In the first subsection we find the efficient development investments, $\overline{D}$. In the second subsection we compute efficient information acquisition investments, $\overline{R}$, given efficient investments in the development stage.[5]

[4] Such a benchmark could be relevant for policy analysis when firms can fully appropriate the social value of their innovation.

[5] We assume that firms' research investments are unobservable in the efficient outcome, in the equilibrium with public signals, and in the equilibrium with private signals. Keeping research investments unobservable throughout the whole analysis enables us to focus on the effects of competition and private information on the firms' signals. It enables us to compare firms' investment in the different benchmarks. In fact, efficient investments for observable and unobservable research investments are identical, since all relevant externalities are internalized.

3.1 Efficient Development Investments

After the information acquisition stage there are two basic states of the world. Either there is at least one firm that received a good signal, $t \in \{(\underline{t},\underline{t}),(\underline{t},\overline{t}),(\overline{t},\underline{t})\}$, or both firms received a bad signal, $t=(\overline{t},\overline{t})$. In the first case both firms learn that their project is easy, $\theta=\underline{\theta}$, while in the latter case they cannot establish with certainty whether the project is easy or difficult. For both these states of the world we calculate the efficient development investments $\overline{D}$. In the industry's efficient outcome firm i chooses development investment D_i that maximizes expected total development profits, given the signals t, research investment R_i, and expected research investments r:

$$\max_{D_i \in [0,1]} E_\theta\{\pi_1(D;\theta)+\pi_2(D;\theta)|t;R_i\}, \text{ for } i=1,2.$$

Expectations are taken after observing the signals. Firm i's posterior belief of working on an easy project is $\mu_i=\mu(t;R_i,r_j)$. The expected cost of investment parameter is

$$\mu_i\underline{\theta}+(1-\mu_i)\overline{\theta}=E(\theta|t;R_i,r_j).$$

Total profit maximization leads to the first-order conditions for development investments:

$$W-2D_j\Delta=E(\theta|t;R_i,r_j)D_i, \text{ for } i,j=1,2, \text{ and } j\neq i.$$

When there is a firm that receives a good signal after the first stage, the firms know that the project is easy, i.e. $\mu=1$. Firms' first-order conditions for these beliefs give their optimal development investments, and development profits, for $i=1,2$:

$$\overline{D}_i(\underline{t})=\frac{W}{2\Delta+\underline{\theta}} \text{ and } \pi_i\left(\overline{D}(\underline{t});\underline{\theta}\right)=\frac{1}{2}\overline{D}_i(\underline{t})W.$$

After both firms received a bad signal, $t=(\overline{t},\overline{t})$, firms update their beliefs about the project's complexity by applying Bayes' rule: $\mu(\overline{t},\overline{t};R_i,r_j)=\frac{p(1-R_i)(1-r_j)}{p(1-R_i)(1-r_j)+1-p}$. Therefore expected costs after two bad signals is:

$$\begin{aligned} E(\theta|\overline{t},\overline{t};R_i) &= \underline{\theta}+\phi(R_i,r_j), \text{ with} \\ \phi(R_i,r_j) &\equiv \frac{(1-p)(\overline{\theta}-\underline{\theta})}{p(1-R_i)(1-r_j)+1-p}. \end{aligned}$$

If firms still receive bad signals even though they invested more in information acquisition, firms become more pessimistic about the complexity of the project. Firm i's expected development cost increases in firms' information acquisition investments. The optimal development investments and expected profits are:

$$\begin{aligned}\overline{D}_i(\bar{t},\bar{t};R_i) &= \frac{W}{\underline{\theta}+\phi(r)+2\Delta}\cdot\frac{\underline{\theta}+\phi(r)}{\underline{\theta}+\phi(R_i,r_j)} \text{ and} \\ \overline{\pi}_i(\bar{t},\bar{t};R_i) &\equiv E_\theta\left\{\pi_i\left(\overline{D}(\bar{t},\bar{t};R_i);\theta\right)\left|\bar{t},\bar{t};R_i\right.\right\} \\ &= \frac{1}{2}\overline{D}_i(\bar{t},\bar{t};R_i)W.\end{aligned}$$

Since in the efficient outcome expectations will be realized, $\overline{r}=\overline{R}$, firms' efficient investments will be symmetric along the optimizing path. Since $\phi(R)>0$, expected marginal cost, $\underline{\theta}+\phi(R)$, strictly exceeds marginal cost of investing in an easy project, $\underline{\theta}$. Along the optimizing path it is therefore efficient to invest less after observing $(\bar{t},\bar{t})$ than after observing a good signal, i.e. $\overline{D}_i(\underline{t})>\overline{D}_i(\bar{t},\bar{t};R_i)$ for $i=1,2$. Greater information acquisition efforts that result still in two bad signals make firms more pessimistic about the project's complexity, and expected costs increase. Therefore efficient investments should decrease. After partially differentiating the efficient development investments toward R_i, we establish this:

$$\frac{\partial\overline{D}_i(\bar{t},\bar{t};R)}{\partial R_i}=\frac{-p(1-r_j)\phi(R_i,r_j)\overline{D}_i(\bar{t},\bar{t};R_i)}{[p(1-R_i)(1-r_j)+1-p]\left(\underline{\theta}+\phi(R_i,r_j)\right)}\le 0,$$

with $i,j=1,2,i\neq j$.

We summarize our results in the following lemma.

Lemma 1 *The efficient development investments are such that, for* $i=1,2$*:*
(i) along the optimizing path, i.e. $\overline{r}_i=\overline{R}_i$*, investments after a good signal exceed those after two bad signals:* $\overline{D}_i(\underline{t})>\overline{D}_i(\bar{t},\bar{t};\overline{R}_i)$ *for all* $\overline{R}_i$*;*
(ii) investments after two bad signals decrease in information acquisition investments: $\frac{\partial\overline{D}_i(\bar{t},\bar{t};R_i)}{\partial R_i}\le 0$ *for all* R_i*.*

3.2 Efficient Research Investments

Given prior beliefs concerning the complexity of the R&D project and given efficient development investments, $\overline{D}$, firms choose research investments, R. Firm i's *ex ante* expected profit, given efficient development investments, is:

$$\begin{aligned}\Pi_i(R_i,\overline{D}) &= (1-p)\pi_i\left(\overline{D}(\bar{t},\bar{t}|R_i);\bar{\theta}\right)+p(1-R_i)(1-R_j)\pi_i\left(\overline{D}(\bar{t},\bar{t}|R_i);\underline{\theta}\right)+ \\ &\quad +p[1-(1-R_i)(1-R_j)]\overline{\pi}_i(\overline{D}(\underline{t});\underline{\theta})-\frac{\rho}{2}R_i^2.\end{aligned}$$

Efficient information acquisition investments $\overline{R}$ are determined by maximizing $\Pi_i(R_i, \overline{D}) + \Pi_j(R_j, \overline{D})$. When we calculate the first-order conditions, and let expectations on research investments be realized, $r = R$, we obtain:

$$\begin{aligned}\rho R_i &= p(1-R_j)\left(\sum_{\ell=1}^{2} \pi_\ell\left(\overline{D}(\underline{t});\underline{\theta}\right) - \sum_{\ell=1}^{2} \pi_\ell\left(\overline{D}(\bar{t},\bar{t}|R);\underline{\theta}\right)\right) \\ &= \frac{p(1-R_j)W^2\phi(R)^2}{(\underline{\theta}+2\Delta)(\underline{\theta}+\phi(R)+2\Delta)^2}, \text{ for } i,j = 1,2. \qquad (1)\end{aligned}$$

That is, marginal costs equal marginal revenues of information acquisition investments. Marginal costs are the direct cost of research investment, ρR_i. The marginal revenue of information gathering investment is the total profit gained from obtaining a good signal and finding out that the project is easy after investing a marginal amount more.

Observe that net marginal revenues in the right hand side of expression (1) are positive for all $R_j < 1$. Direct marginal costs are linear and increase monotonically from 0. Therefore efficient information acquisition investments are in the interior of the unit interval, i.e. $0 < \overline{R} < 1$.[6]

4 Race with Public Signals

In this section we calculate the equilibrium of the R&D race where signals t are publicly observable. We derive equilibrium investment decisions of noncooperative firms, and analyze how they relate to the efficient outcome. Research and development investment choices are now made under competition, while firms' information remains symmetric. Again, we solve the game backwards in pure strategy equilibrium.

4.1 Public Signal Development Investments

The qualitative properties of equilibrium development investments do not differ from those of efficient investments. Again development investments are high after a good signal and are generically decreasing in research investment. Quantitatively equilibrium investments differ from the efficient

[6] Unfortunately, the net revenue function need not always be concave for all R. Especially for big p ($p > \frac{2\Delta+\overline{\theta}}{4\Delta+\overline{\theta}+\underline{\theta}}$) net revenues are convex for small R. For big p and small costs of information acquisition, ρ, there can exist two local optima. We avoid nonconcavities by assuming that the costs of information acquisition investments ρ are big enough to guarantee a unique optimum.

ones. Competing firms do not internalize the negative effect of their investment on the expected revenue of their rival. Therefore firms overinvest in development, which is shown in the remainder of this subsection.

After observing the signals, t, and given expected rival's research investment, r_j, firm i updates his beliefs $(\mu(t; R_i, r_j), 1-\mu(t; R_i, r_j))$, and chooses development investments that maximize his expected profit. This gives first-order conditions

$$(W - \Delta D_j) = E(\theta|t, R_i)D_i, \text{ for } i,j = 1,2, \text{ and } j \neq i,$$

with $E(\theta|t; R_i, r_j) = \mu(t; R_i, r_j)\underline{\theta} + (1-\mu(t; R_i, r_j))\overline{\theta}$.

When at least one of the firms observes a good signal, firms learn that their project is easy. Firm i's posterior belief is $\mu(\underline{t}, .) = 1$, and his first-order condition gives his reaction function for development investments. Firms' reaction functions slope downward. When firm j invests more in development, it becomes less likely that firm i will be the winner of the race, which depresses his expected prize, and his incentive to invest. Both firms' reaction functions together determine the symmetric equilibrium research investments and profits:

$$\widehat{D}_i(\underline{t}) = \frac{W}{\underline{\theta}+\Delta} \text{ and } \pi_i\left(\widehat{D}(\underline{t}); \underline{\theta}\right) = \frac{1}{2}\underline{\theta}\widehat{D}_i(\underline{t})^2, \text{ for } i = 1,2.$$

Whenever both firms receive a bad signal, they remain uncertain about the true state of the project. Depending on the information acquisition investments, each firm updates his beliefs about the project's complexity, and forms beliefs $\mu(\overline{t}, \overline{t}; R_i, r_j) = \frac{p(1-R_i)(1-r_j)}{p(1-R_i)(1-r_j)+1-p}$. From both firms' reaction functions we derive equilibrium investments and expected profits:

$$\begin{aligned}
\widehat{D}_i(\overline{t}, \overline{t}; R_i) &= \frac{W}{\underline{\theta}+\phi(r)+\Delta} \cdot \frac{\underline{\theta}+\phi(r)}{\underline{\theta}+\phi(R_i, r_j)} \text{ and} \\
\widehat{\pi}_i(\overline{t}, \overline{t}; R_i) &\equiv E_\theta\left\{\pi_i\left((\widehat{D}(\overline{t}, \overline{t}; R_i); \theta\right) \middle| \overline{t}, \overline{t}; R_i\right\} \\
&= \frac{1}{2}(\underline{\theta}+\phi(R_i, r_j))\widehat{D}_i(R_i)^2.
\end{aligned}$$

Along the equilibrium path expectations about rival's research investments are realized, i.e. $\widehat{r}_i = \widehat{R}_i$ for $i = 1,2$, and we obtain that $\widehat{D}_i(\overline{t}, \overline{t}; \widehat{R}_i) < \widehat{D}_i(\underline{t})$ for all $\widehat{R}_i$.

Suppose that firms' research investments only result in bad signals. The greater a firm's information acquisition investments, R_i, the higher his expected costs of development investments, and the more cautious the development investments. Pessimistic firms invest less than optimistic ones. Therefore equilibrium development investments decrease in research investments, given bad signals:

$$\frac{\partial \widehat{D}_i(\bar{t},\bar{t};R_i)}{\partial R_i} = \frac{-p(1-r_j)\phi(R_i,r_j)\widehat{D}_i(\bar{t},\bar{t};R_i)}{(p(1-R_i)(1-r_j)+1-p)\left(\underline{\theta}+\phi(R_i,r_j)\right)} \leq 0. \tag{2}$$

These findings are qualitatively identical to those summarized in lemma 1 for efficient development investments.

Given a signal combination t, firms overinvest compared to the efficient investments: $\widehat{D}_i(t;R_i) > \overline{D}_i(t;R_i)$. This is due to the fact that competing firms do not internalize the negative effect of their own development investments on their rival's expected revenues. Firm i's investment D_i marginally decreases firm j's revenue with $D_j\Delta$. Therefore firms invest more aggressively than would be efficient for them. This is a common observation in the literature on R&D races. Competition leads to overinvestments, which is stated in the following proposition.

Lemma 2 *For the game with public signals firms overinvest in equilibrium:* $\widehat{D}_i(t;R_i) > \overline{D}_i(t;R_i)$ *for all* t, R, *and* r, *with* $i=1,2$. *All qualitative properties of lemma 1 hold true for* $\widehat{D}_i(.)$ *too.*

4.2 Public Signal Research Investments

Working backwards, we calculate the equilibrium information acquisition investments given the equilibrium development investments. Firm i chooses R_i that maximizes expected profit $\Pi_i(R,\widehat{D})$, given equilibrium development investments, $\widehat{D}$. Profit maximization leads to the following research equilibrium condition:

$$\begin{aligned}\rho R_i &= p(1-R_j)\left\{\pi_i\left(\widehat{D}(\underline{t});\underline{\theta}\right) - \pi_i\left(\widehat{D}(\bar{t},\bar{t}|R_i);\underline{\theta}\right)\right\} + \\ &\quad +[p(1-R_i)(1-R_j)+1-p]\frac{\partial \widehat{\pi}_i(\bar{t},\bar{t};R_i)}{\partial R_i}.\end{aligned} \tag{3}$$

Notice that marginal revenues contain two informational effects now. The first effect captures the marginal increase in revenue after more research results in jumping from a bad to a good signal. However, when bad signals persist despite increased research, then the increase in research leads to growing pessimism and lower expected revenues. The second effect captures this loss in expected revenues due to firms' growing pessimism after persistence of bad news. Firms internalize the second effect in the efficient outcome.

In equilibrium firms' expectations about research investments are realized, i.e. $\widehat{r}_i = \widehat{R}_i$. Therefore we can rewrite the equilibrium condition for $\widehat{R}_i$ to:

$$\begin{aligned}\rho R_i &= p(1-R_j)\frac{1}{2}\underline{\theta}\left\{\widehat{D}_i(\underline{t})^2 - \widehat{D}_i(\overline{t},\overline{t};R_i)^2\right\} + \\ &\quad +[p(1-R_i)(1-R_j)+1-p](\underline{\theta}+\phi(R))\widehat{D}_i(\overline{t},\overline{t};R_i)\frac{\partial \widehat{D}_i(\overline{t},\overline{t};R_i)}{\partial R_i} \\ &= \max\left\{0, \widehat{MR}(R)\right\},\end{aligned}$$

with

$$\widehat{MR}(R) \equiv \frac{p(1-R_j)W^2\phi(R)\left(\frac{1}{2}\underline{\theta}\phi(R) - (\underline{\theta}+\Delta)\Delta\right)}{(\underline{\theta}+\Delta)^2(\underline{\theta}+\phi(R)+\Delta)^2}. \tag{4}$$

When we focus on symmetric equilibria ($\widehat{R}_1 = \widehat{R}_2$), we can show that firms underinvest in research. Since signals are public, and firms can learn from each others' signals, they have an incentive to free-ride on their rival's information acquisition investments.

Proposition 1 *Symmetric equilibrium research investments in the race with public signals do not exceed the efficient investments:* $\overline{R}_i \geq \widehat{R}_i$ *for* $i = 1, 2$. *For interior equilibrium and efficient research investments the inequality is strict.*

It is efficient to invest more than $\widehat{R}_i$, because the efficient investments internalize the positive externality of informational spillovers among firms. If firm i's research leads to a good signal, this improves both his own and his rival's expected profit. On top of that each firm takes into account that the decrease in his equilibrium development investments from higher research investments after bad news improves his rival's development profits. Internalizing these two effects results in higher research investments.

It would be an interesting exercise to characterize firms' expected equilibrium development investments given their equilibrium research investments, and characterize expected equilibrium profits. This would shed more light on the interaction between research and development. Such an exercise awaits future research.

In these two subsections we saw that firms would improve their profits if they could find a way to both lower development investments $\widehat{D}$, and increase research investments $\widehat{R}$. In the next subsection we will show that revenue sharing provides firms with a way to achieve this.

4.3 Effects of Revenue Sharing

So far we assumed that in the race the winner takes all. This is, however, only an extreme way of distributing revenues from the innovation among firms. In general the loser of the race gets a share, σ, of the revenues. In US sports tournaments revenue sharing is used to decrease firms' overinvestments in talent. Cook and Frank (1995) observe the following:

> "*Revenue sharing — the practice whereby team owners pool and share gate and television revenues with each other — is another common device for limiting expenditures. Because fans strongly prefer to watch winning teams, there is a strong link between a team's winning percentage and the amount of television and gate revenues the team generates. Without revenue sharing, owners thus face powerful incentives to bid for star players, coaches, scouts, and other inputs that make winning more likely. Revenue sharing weakens these incentives and thus helps to restrain player salaries and other key costs.*" [Frank and Cook (1995), pp 169]

In the race for a patent revenue sharing should have the same desirable effect on development overinvestments. But in the race for a patent revenue sharing has an effect on the firms' incentives to invest in development but also on incentives to invest in information acquisition. In what direction these effects point, is studied here. We argue that revenue sharing introduces a free-rider incentive in the development stage which depresses development investments. For research revenue sharing introduces the following effect. When research investments result in a good signal, then this increases the expected revenue of a firm's rival too. Since part of the revenues are shared, this gives each firm a bigger incentive to invest in research. An effect in the opposite direction results from the fact that in absolute terms firms' development investments are lower than in the "winner-takes-all" race, which would reduce the incentive to invest in research. However, initially both investments change in the right direction. This is the main point made in the remainder of this subsection.

Observe that for $\sigma = 0$, we are in the "winner-take-all" race, and for $\sigma = \frac{1}{2}$ firms share the prize equally. Such a share in the revenue affects firms' incentives to invest. In this subsection we characterize the revenue share that brings firms' equilibrium investments closer to the efficient investments. In the following paragraphs we solve the game backwards for any revenue share $\sigma \in [0, 1]$.

■ Given revenue share σ and cost parameter θ development profits are:

$$\begin{aligned}\pi_i(D;\theta|\sigma) &= D_iD_jT + D_i(1-D_j)(1-\sigma)W + (1-D_i)D_j\sigma W - \frac{1}{2}\theta D_i^2 \\ &= \pi_i(D;\theta) + \sigma\left(D_j - D_i\right)W.\end{aligned}$$

This changes first-order conditions into:

$$(1-\sigma)W - D_j\Delta = E(\theta|t,R_i)D_i,$$

with $E(\theta|t,R_i)$ the expected costs, depending on first-stage signals and research investment. Note that marginal expected revenues are reduced with σW from introducing revenue sharing, while marginal costs remain the same. Therefore, equilibrium development investments decrease in the revenue share σ. The marginal effect of firm i's development investment on firm j's expected profits is now $\sigma W - D_j\Delta$. Hence the negative externality $-D_j\Delta$ of the "winner-takes-all" race is reduced by σW. Sharing revenues makes firms less aggressive competitors, because their profits are more interdependent. Development investments and profits are

$$\begin{aligned}\widehat{D}_i(\sigma) &= (1-\sigma)\widehat{D}_i, \text{ and} \\ \pi_i(\widehat{D}(\sigma);\theta^E|\sigma) &= (1-\sigma)^2\widehat{\pi}_i(t;R_i) + (1-\sigma)\sigma\widehat{D}_jW.\end{aligned}$$

for $i = 1, 2$. Notice that equilibrium development investments range from 0, in the "loser-takes-all" race ($\sigma = 1$), to $\widehat{D}_i$, in the "winner-takes-all" race ($\sigma = 0$). In the "equal-sharing" race, $\sigma = \frac{1}{2}$, firms underinvest in development. From the first-order conditions $\frac{1}{2}W - \widehat{D}_j(\frac{1}{2})\Delta = \theta^E\widehat{D}_i(\frac{1}{2})$ and $\frac{1}{2}W - \overline{D}_j\Delta = \frac{1}{2}\theta^E\overline{D}_i$, and the symmetry of equilibrium it follows immediately that $\widehat{D}_i(\frac{1}{2}) < \overline{D}_i$.

These results are summarized in the following proposition.

Proposition 2 *For the race with public signals and revenue share σ the following holds.*
(i) Equilibrium development investments decrease in the revenue share: $\frac{\partial\widehat{D}_i(\sigma)}{\partial\sigma} < 0$ for all σ. (ii) Firms underinvest in the "equal-sharing" development equilibrium: $\widehat{D}_i(t,R_i|\frac{1}{2}) \leq \overline{D}_i(t,R_i)$ for all t, R_i, r.

■ Now we calculate equilibrium research investments given equilibrium development investments. To derive first-order conditions, it is useful to recall how equilibrium development profits with revenue sharing relate to those without revenue sharing. This gives the following first-order conditions for firm i's research.

$$\begin{aligned}\rho R_i &= (1-\sigma)^2 p(1-R_j)\left\{\pi_i\left(\widehat{D}(\underline{t});\underline{\theta}\right) - \pi_i\left(\widehat{D}(\bar{t},\bar{t};R_i);\underline{\theta}\right)\right\} + \\ &+(1-\sigma)^2[p(1-R_i)(1-R_j)+1-p]\frac{\partial\widehat{\pi}_i(\bar{t},\bar{t};R_i)}{\partial R_i} + \\ &+\sigma(1-\sigma)p(1-R_j)W\left\{\widehat{D}_j(\underline{t}) - \widehat{D}_j(\bar{t},\bar{t};r_j)\right\}\end{aligned}$$

Both firms' first-order conditions determine the research equilibrium investments. The first two terms of firm i's marginal revenues trade off similar informational effects as in the previous subsection. They are his "winner-takes-all" marginal revenues, as in expression (3), where we correct for the fact that the firm can only keep $(1-\sigma)$ of his generated revenue, and that firms' incentives to invest in development are reduced with factor $(1-\sigma)$. The last term is exactly the change in revenue that firm i expects to receive from his rival after making him more optimistic by providing the industry with a good signal. Remember that share σ of firm j's development revenues spill over to firm i, while equilibrium development investments are reduced with factor $(1-\sigma)$.

In equilibrium expectations are realized. The equilibrium research investments $\widehat{R}(\sigma)$ are then the solution to:

$$\rho R_i = \max\left\{0,(1-\sigma)\left((1-\sigma)\widehat{MR}(R)+\sigma\widehat{MQ}(R)\right)\right\}, \tag{5}$$

with $\widehat{MR}(R)$ as defined in (4), and

$$\widehat{MQ}(R) \equiv p(1-R_j)W\left\{\widehat{D}_j(\underline{t}) - \widehat{D}_j(\bar{t},\bar{t};r_j)\right\} = \frac{p(1-R_j)W^2\phi(R)}{(\underline{\theta}+\Delta)(\underline{\theta}+\phi(R)+\Delta)}. \tag{6}$$

Due to the model's symmetry, firms' research investments are symmetric. In the remainder of this subsection we characterize the equilibrium research investments, $\widehat{R}(\sigma)$, by deriving how investment depends on the industry's prize share σ.

We show that research investments do not decrease after introducing a sufficiently small revenue share $\sigma > 0$. Increasing the revenue share has two conflicting effects. At the one hand it internalizes a fraction of the positive informational externalities from research, which increases firms' incentives to invest in research. However, when the revenue share is increased this shrinks the development investments, and consequently firms' revenues of research. The equilibrium research investment trades off internalizing informational externalities of research against free-rider effects in development. This is summarized in the following proposition.

Proposition 3 *For the race with public signals in which firms make positive research investments in the "winner-takes-all" race, $\widehat{R}_i(0) > 0$, there is a revenue share $\widehat{\sigma} \in (0, \frac{1}{2})$ such that equilibrium research investments are increasing for all $\sigma < \widehat{\sigma}$ and decreasing for all $\sigma > \widehat{\sigma}$.*

A direct consequence of this proposition is that total profits are increased by introducing a (small) positive revenue share in the "winner-takes-all" race. On the one hand, the introduction of development spillovers give firms a smaller incentive to invest in development. On the other hand, these development spillovers initially stimulate the creation of informative public signals, i.e. they increase the incentive to invest in research. Since both overinvestments in development and underinvestments in research are reduced, total profits are increased.

5 Race with Private Signals

In the previous section we assumed that the firms' signals are public. However such an assumption need not be realistic. In this section we make the assumption that information is private to the firms and cannot be revealed to rivals. We derive the equilibrium investment levels and compare them with those of firms with public signals.

5.1 Private Signal Development Investments

When signals are private information to firms, firms can condition their development investments on their own signal only. A good signal received by one firm does not imply that both firms become optimistic about development costs. It is possible that the other firm is unlucky and receives a bad signal. Therefore the expected rival to a firm with a good private signal is less aggressive than the rival to a firm with a good public signal. This makes equilibrium development investments of a good private signal firm exceed those of a good public signal firm. When both firms receive a bad private signal, a firm faces the following trade-off. On the one hand a firm with only one bad signal is more optimistic about development costs, because he does not pool his information with his rival. However, on the other hand, the firm expects a more aggressive rival compared to the race with public signals. The first effect encourages, while the second effect discourages development investments. We show that the informational effect dominates the strategic effect along the equilibrium path. This is done in the remainder of this subsection.

Given private signals, and firm i expects his rival's information acquisition investments are r_j, his reaction functions are the following:

$$\begin{aligned} \underline{\theta} D_i^*(\underline{t}) &= (1-\sigma)W - \left(r_j D_j^*(\underline{t}) + (1-r_j) D_j^*(\overline{t})\right)\Delta, \\ \left(\underline{\theta} + \varphi(R_i)\right) D_i^*(\overline{t}; R_i) &= (1-\sigma)W - \left(P(R_i) D_j^*(\underline{t}) + [1-P(R_i)] D_j^*(\overline{t})\right)\Delta, \\ \text{with } \varphi(R_i) &= \frac{(1-p)(\overline{\theta}-\underline{\theta})}{p(1-R_i)+1-p} \text{ and } P(R_i) = \frac{p(1-R_i) r_j}{p(1-R_i)+1-p}. \end{aligned}$$

The equilibrium development investments of a firm with a bad signal, $D_i^*(\overline{t}; R_i)$, depends on his investments in information acquisition, R_i. If the firm keeps receiving a bad signal despite the fact that he invested more in information acquisition, he becomes more pessimistic about the complexity of the project. The firm's growing pessimism has two effects. First, the firm expects higher development costs. This decreases his development investments. Second, he attaches a stronger belief to the contingency that his rival also receives a bad signal, i.e. $P(R_i)$ decreases. A rival with a bad signal is a weaker competitor, which encourages the firm's development investments. These two effects are captured in the following expression.

$$\frac{\partial D_i^*(\overline{t}; R_i)}{\partial R_i} = \frac{-p\varphi(R_i) D_i^*(\overline{t}; R_i)}{\left(p(1-r_j)+1-p\right)\left(\underline{\theta}+\varphi(R_i)\right)} + \frac{p(1-p) r_j \left(D_j^*(\underline{t}) - D_j^*(\overline{t}; r_j)\right)\Delta}{\left(p(1-r_j)+1-p\right)^2 \left(\underline{\theta}+\varphi(R_i)\right)}.$$

Along the equilibrium path, where expectations are realized and symmetric, i.e. $r_i^* = R_i^* = R$ for $i = 1, 2$, the direct effect outweighs the indirect effect. Development investments of a firm with bad news decreases in his research investments. Expected equilibrium development profits given $t_i = \underline{t}$ and $t_i = \overline{t}$ are respectively:

$$\begin{aligned} \pi_i^*(\underline{t}) &= \frac{1}{2}\underline{\theta} D_i^*(\underline{t})^2 + \sigma\left(r_j D_j^*(\underline{t}) + (1-r_j) D_j^*(\overline{t})\right) W \text{ and} \\ \pi_i^*(\overline{t}; R_i) &= \frac{1}{2}\left(\underline{\theta}+\varphi(R_i)\right) D_i^*(\overline{t})^2 + \sigma\left(P(R_i) D_j^*(\underline{t}) + (1-P(R_i)) D_j^*(\overline{t})\right) W. \end{aligned}$$

Along the equilibrium path, where firms invest equal amounts in information acquisition and expectations concerning investments are fulfilled, $r_i = R_i = R$, development investments are the following:

$$\begin{aligned} D_i^*(\underline{t}; R) &= \frac{(1-\sigma)\left(\underline{\theta}+\varphi(R)+(R-P(R))\Delta\right) W}{\left(\underline{\theta}+R\Delta\right)\left(\underline{\theta}+\varphi(R)+(1-P(R))\Delta\right) - (1-R)P(R)\Delta^2}, \\ D_i^*(\overline{t}; R) &= \frac{(1-\sigma)\left(\underline{\theta}+(R-P(R))\Delta\right) W}{\left(\underline{\theta}+R\Delta\right)\left(\underline{\theta}+\varphi(R)+(1-P(R))\Delta\right) - (1-R)P(R)\Delta^2}. \end{aligned}$$

It is immediate that $D_i^*(\underline{t};R) > D_i^*(\bar{t};R)$. A firm with a bad signal is more reluctant to invest in the development of the intermediate innovation than a firm with a good signal.

Since a rival with a bad signal invests less than one with a good signal, a $\underline{t}$-firm expects his rival to invest less aggressively than with public signals. This means that a firm with a good private signal invests more in development than a firm with a good public signal, provided that information acquisition investments are symmetric ($R_1 = R_2$) and expectations are realized: $D_i^*(\underline{t};R) \geq \widehat{D}_i(\underline{t})$ for all R.

Now consider the situation in which nature chose two bad signals, $(\bar{t},\bar{t})$. For each firm there are two effects when we turn from a public to a private bad signal. First, the firm becomes more optimistic about his costs. Since he conditions his beliefs only on his own bad signal, expected cost is lower. This drives the firm's investments up. Second, he expects higher investments from his rival. This decreases the firm's investment incentives. The direct cost effect outweighs the indirect effect of rival's expected investments, when information acquisition investments are symmetric and expectations are realized. Therefore a firm with a private bad signal invests more in development than a firm with bad public signal: $D_i^*(\bar{t};R) > \widehat{D}_i(\bar{t},\bar{t};R)$ for all R.

In the situation in which there is one firm who receives a good signal while the other receives a bad signal, equilibrium investments for firms with private signals are lower than investments with publicly observable signals. That is, $D_i^*(\underline{t}) + D_i^*(\bar{t}) - 2\widehat{D}_i(\underline{t},\bar{t}) < 0$. Again, this holds provided that $R_1 = R_2$, and that expectations are fulfilled.

We summarize the findings of this subsection in the following proposition.

Proposition 4 *In the race with private signals where expected research investments are symmetric and realized, with $r_i^* = R_i^* = R < 1$, equilibrium development investments are such that, for $i = 1, 2$:*
(i) $D_i^*(\underline{t};R) > D_i^*(\bar{t};R)$, and $\frac{\partial D^*(\bar{t};R)}{\partial R_i} < 0$,
(ii) $D_i^*(\underline{t};R) > \widehat{D}_i(\underline{t})$ and $D_i^*(\bar{t};R) > \widehat{D}_i(\bar{t},\bar{t};R)$.

5.2 Private Signal Research Investments

In the first stage of the race firms invest in information acquisition. The information that each firm acquires remains private information for that firm. The free-rider incentive in research that exists in the race with public signals is no longer present. Firms will therefore invest more in research when the winner takes all. When firms share revenues, it is not clear in which direction equilibrium research investments change. We have seen that for public signals there are two free-rider effects. First there is the direct effect, that a firm's own revenues are negatively affected because also the firm's rival learns from his research. But, second, the rival's learning has a positive indirect effect through revenue sharing. We substantiate these observations in the remainder of this subsection.

First-order conditions for equilibrium research investments R^* are the following:

$$\begin{aligned}
\rho R_i &= p\frac{1}{2}\underline{\theta}\left[D_i^*(\underline{t})^2 - D_i^*(\bar{t};R_i)^2\right] + \\
&\quad + [p(1-R_i)+1-p]\left(\underline{\theta}+\varphi(R_i)\right)D_i^*(\bar{t};R_i)\frac{\partial D_i^*(\bar{t};R_i)}{\partial R_i} \\
&= p\left[\frac{1}{2}\underline{\theta}D_i^*(\underline{t})^2 - \left(\frac{1}{2}\underline{\theta}+\varphi(R_i)\right)D_i^*(\bar{t};R_i)^2\right] + \\
&\quad + \frac{p(1-p)R_j D_i^*(\bar{t})\left(D_j^*(\underline{t}) - D_j^*(\bar{t};R_j)\right)\Delta}{(p(1-R_j)+1-p)}.
\end{aligned}$$

If we focus on symmetric equilibria, we get the following:

$$\rho R = \frac{\frac{1}{2}p\underline{\theta}\varphi(R)^2(1-\sigma)^2W^2}{\left((\underline{\theta}+R\Delta)\left(\underline{\theta}+\varphi(R)+(1-P(R))\Delta\right)-(1-R)P(R)\Delta^2\right)^2}. \tag{7}$$

A marginal increase in firm i's research investments that gives firm i a good signal does not directly affect firm j's investments, because the firm's signal is private information. Therefore the revenue that firm i receives from his rival, through revenue share σ, is no longer affected by his research investments. The incentive to invest in research for firm i now only depends on the appropriability of his research investments, which is the share of his own revenue that the firm keeps, i.e. $1-\sigma$. The more revenue spills over to the rival, the less valuable his own research becomes for the firm. Therefore we observe that each firm's equilibrium research investment decreases in the revenue share σ.

Lemma 3 *In the race with private signals, for all revenue shares $\sigma \in [0,1]$ firms' research investments decrease in the revenue share: $\frac{\partial R_i^*(\sigma)}{\partial \sigma} < 0$ for $i = 1,2$.*

In the "winner-takes-all" race equilibrium research investments in acquiring private signals exceed those of the equilibrium with public signals. Since firms can no longer free ride on their rival's investments and signals, they have an incentive to invest more in information acquisition. This is stated in the following proposition.

Proposition 5 *In the "winner-takes-all " race ($\sigma = 0$) firms invest in equilibrium more in acquiring private than public signals: $R_i^*(0) \geq \widehat{R}_i(0)$. This holds with strict inequality whenever firms choose interior equilibrium research investments.*

More insight in the interaction between research and development could be gained from the characterization of firms' expected equilibrium development investments given equilibrium research investments, and their equilibrium profits. An overall comparison between equilibrium development investments and equilibrium profits for public and private signals would close the analysis. This exercise awaits future research.

6 Strategic Revelation

In this section we extend the game by adding an information revelation stage. After firms invested in research and received their private signal, firms choose what message to send to their rival. After firms received each other's message, they invest in development. Firms have an incentive to manipulate their information in order to alter their rival's beliefs, and consequently change competition in the development stage in their favor. Firms in a "winner-takes-all" race have an incentive to make their rival as pessimistic as possible to discourage rival's development investments. For high revenue shares firms have an incentive to make their rival as optimistic as possible. An optimistic rival invests relatively much, and the revealing firm can take a free ride on the revenue generated by those investments. The extent to which firms can actually manipulate rival's beliefs and investments and the direction in which this happens is the main topic of this section. Typically we are also interested in learning in what direction firms want to shift rival's investments for intermediate revenue shares.

We make two distinct informational assumptions. First we analyze what information is revealed when firms' information is non-verifiable. We do this in the next subsection. How firms' incentives and possibilities to reveal information are affected when information is costlessly verifiable is studied in the second subsection.

6.1 Non-verifiable Information

In this subsection we assume that firms cannot verify the truthfulness of their rival's messages. This makes it costless for firms to lie about their signal. Since lying is for free and there is always a firm with incentives to lie, firms never fully reveal their signals. We establish this in this subsection. Our contribution is here to show that even for intermediate revenue shares firms' incentives are not aligned. Naturally, a firm's rival is aware of the strategic nature of the firm's messages, and will be less willing to rely on the firm's information. In fact we can show that there always is an equilibrium in which no information is revealed. That is, investments for the race in private signals are equilibrium investments in this situation.

Since information is non-verifiable, firms can make any statement about their information they like. Formally, after each firm received his private signal, firms simultaneously choose their revelation rules $(\tau_i(\underline{t}, R_i), \tau_i(\bar{t}, R_i))$, with $\tau_i(t_i, R_i) \in \{\underline{t}, \bar{t}\}$, and reveal information $\widetilde{\tau}_i \in \{\tau_i(t_i, R_i) | t_i = \underline{t}, \bar{t}$ and $0 \leq R_i \leq 1\}$ accordingly. Information is not verifiable for firms. For example, revelation rule $(\tau_i(\underline{t}, R_i), \tau_i(\bar{t}, R_i)) \equiv (\underline{t}, \bar{t})$ gives full revelation, while rules $(\tau_i(\underline{t}, R_i), \tau_i(\bar{t}, R_i)) \equiv (\bar{t}, \bar{t})$ and $(\underline{t}, \underline{t})$ do not reveal any information to the rival firm. After messages are sent, firms simultaneously invest in development.

A natural first step of analysis is to see whether firms voluntarily reveal all their information in equilibrium. This would give us investments of the race with public signals. First consider the "winner-takes-all" race. In this race each firm has an incentive to make his rival invest as little as possible. If it is expected that a firm fully reveals his information, then this firm has an incentive to always send bad news. That is, he always states $t = \bar{t}$. The rival believes this is truthfully revealed information, and becomes pessimistic. The pessimistic rival invests little in development of the prototype, which increases the expected profit of the sender of bad news. Second, consider the "equal-sharing" race where firms believe that their rival fully reveals information. In an "equal-sharing" race each firm has an incentive to make his rival's investments as big as possible in order to take a free ride on those investments. Then a firm has an incentive to always send good news. The firm's rival believes that $\underline{t}$ was observed, and becomes optimistic about the costs of investment. The rival's investments increase, and the sender of good news takes a free ride on these high investments. Similar incentives to under- or overstate information exist for other revenue shares. And full disclosure does never happen in equilibrium, as is stated in the following proposition.

Proposition 6 *For all revenue shares $\sigma \in [0, 1]$, there does not exist an equilibrium of the game with strategic revelation of non-verifiable information in which signals are completely revealed.*

The polar case of complete revelation is no revelation of any information. No revelation of information can always be sustained as an equilibrium. Given that the statements of firms contain no information whatsoever, firms ignore them. Since statements are ignored, neither truthful nor false statements affect rival's investments. Therefore firms are indifferent between all statements, and it is optimal to choose the non-revealing rule that is consistent with equilibrium beliefs. This is stated in the following lemma.

Lemma 4 *There is an equilibrium of the game with strategic revelation of non-verifiable information in which no information is revealed for any revenue share* $\sigma \in [0,1]$.

This result is similar to that of Ziv (1993), and is standard for models with non-verifiable signals. The paper by Ziv focuses on the incentives of Cournot duopolists to understate costs of producing homogeneous products, In our analysis we consider a situation in which revenue sharing affects firms' incentives. And we show that irrespective of how firms share the revenue from innovation, they never reveal their information. Depending on how much of the revenue is shared between firms, firms have an incentive to give less (low σ), more (high σ), or both less and more (intermediate σ) favorable information to the rivals.

It would be interesting to see whether there are revenue shares for which revelation of some information will be chosen in equilibrium. This question awaits future research.

6.2 Verifiable Information

In the previous subsection we assumed that firms can costlessly misrepresent their private signal. Therefore credible revelation of information is not possible in equilibrium. A natural question to ask is how the results are affected when information is costlessly verifiable. The only choice that a firm with verifiable information has, is to either disclose his information or conceal it. For low (resp. high) revenue shares firms have an incentive to disclose only bad (resp. good) news. A firm's rival anticipates this and knows that a concealing firm's cost signal is low (resp. high). This evaporates a firm's possibilities to effectively conceal information. However for intermediate revenue shares complete disclosure is not an equilibrium strategy. For intermediate shares both the high- and low-cost type of firms have an incentive to conceal, and can therefore credibly do so. This is shown in the remainder of this subsection.

The seminal paper by Okuno-Fujiwara *et al.* (1990) gives sufficient conditions on firms' strategic interaction and information under which an equilibrium with full disclosure of private information with sceptical inferences exists. For our R&D race neither sufficient condition 4c nor 4d from Okuno-Fujiwara *et al.* (1990) are met. Assumption 4c (resp. 4d) states that as a firm's signal increases, his reaction curve shifts out (resp. in) while his rival's reaction function shifts in (resp. out) or stays the same.

In our model firms' signals, and expected profits, are correlated. Therefore firm i's marginal expected development profit is non-increasing both in its own and its rival's signal. The negative relationship between a firm's disclosure and his own marginal profit is a strategic effect. After disclosing verifiable good news, a firm discloses to be an aggressive development investor. The negative relationship between a firm's signal and his rival's marginal profit is caused by the informational effect of disclosure. Disclosure of good news by one firm makes the other firm more optimistic which shifts out his development reaction function.

The violation of the sufficient conditions for complete revelation raises the question whether the "unraveling" result still goes through. Okuno-Fujiwara *et al.* discuss a common value example in which neither condition 4c nor 4d is satisfied, but full disclosure is still established. The result is obtained here because the strategic effect dominates the informational effect. In our model the informational effect dominates the strategic effect, and we obtain a similar result for extreme revenue shares.

Proposition 7 *When firms' signals are costlessly verifiable after revelation, then there are revenue shares $\overline{\sigma}$ and $\underline{\sigma}$, with $0 < \overline{\sigma} < \underline{\sigma} < 1$, such that:*
(i) for $\sigma \leq \overline{\sigma}$ firms fully disclose in equilibrium with skeptical inferences,
(ii) for $\overline{\sigma} < \sigma < \underline{\sigma}$ no inferences support full disclosure in equilibrium,
(iii) for $\sigma > \underline{\sigma}$ firms fully disclose in equilibrium with skeptical inferences.

Note that skeptical inferences of (i) and (iii) are not identical. For revenue shares $\sigma \leq \overline{\sigma}$ firms have an incentive to conceal good news, while they have an incentive to disclose bad news. Therefore firms infer that a concealing firm received signal $\underline{t}$ under (i). These beliefs make strategic concealment of information unprofitable. For revenue shares that exceed $\underline{\sigma}$ firms have an incentive to conceal only bad news. Hence for (iii) firms rationally infer that a concealing rival has signal $\overline{t}$, which establishes full revelation.

For extreme revenue shares the verifiability of firms' information enables a firm to unravel his rival's private information, as in Grossman (1981) and Milgrom (1982). Such a result is the opposite of our results on revelation in the previous subsection. For non-verifiable signals firms cannot credibly reveal any information, while for verifiable signals firms cannot credibly conceal information from their rival.

Under (ii) both firms have an incentive to misrepresent their information, and full disclosure is not chosen in equilibrium. For intermediate revenue shares different effects dominate for different firm types. A firm who received a bad signal has an incentive to conceal since it makes his rival more optimistic about the costs of investment. The rival will invest more in development, and the high-signal firm can take a free ride on his rival's higher expected revenue. A firm with a good signal has an incentive to conceal information, and discourage his rival in the development stage. For the good-signal firm the informational effect outweighs the free-rider effect. A similar result is found in a different setting by Hendricks and Kovenock (1989).

The results of this section indicate that the assumption of publicly observable signals, as in Choi (1991) and Malueg and Tsutsui (1997), need not always hold. When the assumption is relaxed and signals can be costlessly misrepresented, complete revelation no longer happens in equilibrium. For costlessly verifiable signals public signals appear to be a proper assumption. The amount of information that can and will be shared among firms crucially depends on the verifiability of this information.

7 Observable Research Investments

In this section we discuss how results depend on the non-observability of research investments. We illustrate the effect of publicly observable research investments by looking at the case in which signals are public.

Changes in one firm's publicly observable research investments affect both firms' beliefs. When firms keep receiving bad news after a firm increases his research investments, this has two conflicting effects. First the usual effect is that the investing firm decreases his development investment, because he becomes more pessimistic. However, also the firm's rival becomes more pessimistic, and contracts his development investments. This spillover effect gives the firm a bigger incentive to invest in development. Therefore increases in observable research investments make development investments after bad signals decrease less steeply than unobservable research investments. This has two consequences for equilibrium research investments. First the spillover effect makes a firm's own expected revenues of observable research investments bigger than those of unobservable investments. Therefore equilibrium observable research investments exceed equilibrium unobservable investments in the "winner-takes-all" race. The second observation is that the spillover effect decreases rivals' expected revenue. Therefore the marginal benefit of revenue sharing is reduced, which makes observable equilibrium research investments smaller than unobservable investments in the "equal-sharing" race. A more detailed discussion of these effects is given in the remainder of this section.

With observable research investments equilibrium development investments D^o depend on the research investments of both firms, R:

$$\begin{aligned} D_i^o(\underline{t}) &= \frac{(1-\sigma)W}{\underline{\theta}+\Delta}, \\ D_i^o(\bar{t},\bar{t};R) &= \frac{(1-\sigma)W}{\underline{\theta}+\phi(R)+\Delta}, \end{aligned}$$

and

$$\frac{\partial D_i^o(\bar{t},\bar{t};R)}{\partial R_i} = \frac{-p(1-R_j)\phi(R)D_i^o(\bar{t},\bar{t};R)}{[p(1-R_i)(1-R_j)+1-p]\left(\underline{\theta}+\phi(R)+\Delta\right)} \leq 0,$$

for $i=1,2$.When we compare this expression with that in equation (2) where expectations concerning research are realized, $r_j = R_j$ for $j=1,2$, we note the following:

$$\frac{\partial \widehat{D}_i(\bar{t},\bar{t};R)}{\partial R_i} < \frac{\partial D_i^o(\bar{t},\bar{t};R)}{\partial R_i} \leq 0.$$

When research investments are observable, firm i's equilibrium development investments are less sensitive to unilateral investment changes. This is caused by the following spillover effect. When firm i's research investment is observable, and firm i increases research investments while the signals remain $(\bar{t},\bar{t})$, not only firm i, but also firm j becomes more pessimistic. Firm j therefore decreases his development investments. Since development investments are strategic substitutes, this countervails firm i's direct decrease in development investments. This spillover effect reduces the direct effect of firm i's own growing pessimism.

The equilibrium profits, given cost of investment θ and equilibrium development investments D^o, is:

$$\pi_i(D^o(\sigma);\theta) = (1-\sigma)\left((1-\sigma)\frac{1}{2}\theta(D_i^o)^2 + \sigma W D_j^o\right).$$

The equilibrium condition for research investments, R^o, becomes:

$$\rho R_i = (1-\sigma)\left((1-\sigma)MR^o(R) + \sigma MQ^o(R)\right),$$

with

$$
\begin{aligned}
MR^o(R) &= p(1-R_j)\frac{1}{2}\underline{\theta}\left(D_i^o(\underline{t})^2 - D_i^o(\overline{t},\overline{t};R)^2\right) + \\
&\quad +[p(1-R_i)(1-R_j)+1-p](\underline{\theta}+\phi(R))D_i^o(\overline{t},\overline{t};R)\frac{\partial D_i^o(\overline{t},\overline{t};R)}{\partial R_i} \\
&= \frac{p(1-R_j)[\phi(R)]^2W^2\left(\underline{\theta}\phi(R)+(\underline{\theta}+\Delta)(\underline{\theta}-2\Delta)\right)}{2(\underline{\theta}+\Delta)^2[\underline{\theta}+\phi(R)+\Delta]^3}, \\
MQ^o(R) &= p(1-R_j)W\left(D_j^o(\underline{t}) - D_j^o(\overline{t},\overline{t};R)\right) + \\
&\quad +[p(1-R_i)(1-R_j)+1-p]W\frac{\partial D_j^o(\overline{t},\overline{t};R)}{\partial R_i} \\
&= \frac{p(1-R_j)W^2[\phi(R)]^2}{(\underline{\theta}+\Delta)(\underline{\theta}+\phi(R)+\Delta)^2}.
\end{aligned}
$$

When we compare these expressions with expressions (4) and (6) we observe the following. Along the equilibrium path, when expectations are realized, $\widehat{D}_i = D_i^o$. Since observable changes in research investment affect own equilibrium development investments less than unobservable changes, we have $\widehat{MR}(R) < MR^o(R)$, for all R. It is therefore immediate that for the "winner-takes-all" race ($\sigma = 0$) observable equilibrium research investments are greater than unobservable equilibrium research investments for any cost of research investment ρ: $R_i^o(0) > \widehat{R}_i(0)$. But since observable changes in research investment do affect rival's equilibrium development investments, $\frac{\partial D_j^o(\overline{t},\overline{t};R)}{\partial R_i} < 0$, we have $\widehat{MQ}(R) > MQ^o(R)$, for all R. It is easily verified that in the "winner-takes-all" race firms with observable research investments still underinvest in research: $R_i^o(0) < \overline{R}_i$, for $i = 1, 2$.

For the "equal-sharing" race ($\sigma = \frac{1}{2}$) the effect of observable research investments on a firm's own as well as his rival's development investments are present. These effects point in opposing directions. Since the effect of observable research on own development investments is an indirect effect, while the effect on the rival's development investments is direct, the latter dominates the former in the "equal-sharing" race. This is reflected in the fact that for all R: $\widehat{MR}(R) + \widehat{MQ}(R) > MR^o(R) + MQ^o(R)$.[7] Therefore unobservable research investments exceed observable ones in equilibrium: $\widehat{R}_i(\frac{1}{2}) > R_i^o(\frac{1}{2})$, for $i = 1, 2$.

These results are summarized in the following proposition:

[7]It is easily verified that:

$$
[\widehat{MR}(R) + \widehat{MQ}(R)] - [MR^o(R) + MQ^o(R)] = \frac{p(1-R_j)W^2\phi(R)(\underline{\theta}+\phi(R))}{(\underline{\theta}+\phi(R)+\Delta)^3} > 0.
$$

Proposition 8 *Consider the race with public signals, and take $r_i = R_i$ for $i = 1, 2$. With observable research investments equilibrium is such that:*
(i.a) development investments do not differ: $\widehat{D}_i(t; R) = D_i^o(t; R)$, for all t, R,
(i.b) development investments fall more steeply in unobservable research investments than in observable ones: $\frac{\partial \widehat{D}_i(\bar{t},\bar{t};R)}{\partial R_i} < \frac{\partial D_i^o(\bar{t},\bar{t};R)}{\partial R_i} \leq 0$,
(ii) observable equilibrium research investments exceed unobservable ones in the "winner-takes-all" race, $R_i^o(0) \geq \widehat{R}_i(0)$, while the reverse holds in the "equal-sharing" race, $R_i^o(\frac{1}{2}) \leq \widehat{R}_i(\frac{1}{2})$. Where strict inequalities hold for interior equilibrium research investments.

We conclude that observable research investments create an effect on the way both firms respond to changes in research investments. The direction in which this effect points depends on the direction in which spillovers between firms point. In a setting where Cournot competitors acquire private information on their demand intercept, Hauk and Hurkens (1998) show that observable research investments exceed unobservable investments in equilibrium. The analysis of this subsection suggests that this conclusion is sensitive to their assumption on the kind of information that is acquired. Our analysis could be interpreted as one of Cournot competitors who acquire public information about the "slope of demand", and give different equilibrium results.[8]

8 Conclusion

In this paper we studied investment incentives of firms who learn about the R&D project they work on, while they invest in it. We showed that revenue sharing could be a way to correct incentive distortions among competing firms. Firms that share revenues are no longer competing in a "winner-take-all" race. In an R&D race with public signals both research underinvestment and the development overinvestments are initially reduced by revenue sharing. This suggests that firms would be better off if they would share revenues from innovation.

Not only the extent to which firms share revenues, but also the observability of intermediate research results affects investment incentives substantially. We have shown that the verifiability of intermediate research results is crucial in determining what kind of investments are actually made in equilibrium. No information is credibly revealed for any share of revenues when

[8]Malueg and Tsutsui (1996) show that incentives to commit to sharing exogenous information change too when they move from information about unknown demand intercept to information about unknown slope. (Firms have a bigger incentive to share information about unknown slope.)

information is non-verifiable. In that case revealing no information is an equilibrium strategy. Therefore the research and development investments of the race with private signals are equilibrium investments for a race in which non-verifiable intermediate information is created. When firms' private information is verifiable, the "unraveling result" ensures for extreme revenue shares full disclosure of research results, and gives the public signal equilibrium investments. For intermediate revenue shares verifiable information will not be disclosed. Naturally both research and development investments are affected by the amount of information that can be shared among the firms.

Although we made a substantial first step in the analysis of learning effects in R&D races, there remain some open questions. It would be interesting to study the overall effect of the interaction between research and development investments by characterizing the expected equilibrium development investments given equilibrium research investments. One of the goals of this research project would be to make an overall comparison between expected profit levels under public and private signals. It would improve the paper if we could prove statements on partial information revelation in equilibrium. Also comparative statics would improve our understanding of the results. These, and other extensions of the analysis await future research.

9 Appendix

In this Appendix we prove the main propositions of this paper. The first subsection proves the main propositions on equilibrium investments for public signals. Subsection 2 proves the main proposition for a race with private signals. In subsection 3 we prove the lemmas and propositions concerning strategic information revelation.

9.1 Proofs for Public Signal Race

In this subsection we prove propositions 1 and 3.

9.1.1 Proof of Proposition 1 ($\widehat{R} > \overline{R}$)

First we show that marginal revenues of research investments in the optimum are strictly larger than those in the public signal race:

$$\frac{p(1-R_j)W^2\phi(R)^2}{(\underline{\theta}+2\Delta)(\underline{\theta}+\phi(R)+2\Delta)^2} > \frac{p(1-R_j)W^2\phi(R)\left(\frac{1}{2}\underline{\theta}\phi(R)-\Delta(\underline{\theta}+\Delta)\right)}{(\underline{\theta}+\Delta)^2(\underline{\theta}+\phi(R)+\Delta)^2},$$

which certainly holds whenever

$$\frac{1}{(\underline{\theta}+2\Delta)(\underline{\theta}+\phi(R)+2\Delta)^2} > \frac{\frac{1}{2}\underline{\theta}}{(\underline{\theta}+\Delta)^2(\underline{\theta}+\phi(R)+\Delta)^2} \Leftrightarrow$$

$$\begin{aligned}(\underline{\theta}+\Delta)^2(\underline{\theta}+\phi(R)+\Delta)^2 > \frac{1}{2}\underline{\theta}(\underline{\theta}+2\Delta)(\underline{\theta}+\phi(R)+2\Delta)^2 \quad \Leftrightarrow \\ \phi(R)^2(2\Delta^2+2\underline{\theta}\Delta+\underline{\theta}^2)+2\phi(R)(2\Delta^3+2\underline{\theta}\Delta^2+2\underline{\theta}^2\Delta+\underline{\theta}^3)+ \\ +(2\Delta^4+2\Delta\underline{\theta}^3+\underline{\theta}^4)>0,\end{aligned}$$

which obviously holds. Since marginal costs are identical, this gives underinvestments in research by competing firms.

9.1.2 Proof of Proposition 3 ($\partial\widehat{R}/\partial\sigma$)

For positive "winner-takes-all" research investments, i.e. $\widehat{MR}(R) > 0$, we apply the implicit function theorem to first-order condition (5) to derive that

$$\frac{\partial\widehat{R}_i(\sigma)}{\partial\sigma} = \left.\frac{(1-2\sigma)\left(\widehat{MQ}(R)-\widehat{MR}(R)\right)-\widehat{MR}(R)}{(1-\sigma)\left((1-\sigma)\widehat{MR}'(R)+\sigma\widehat{MQ}'(R)\right)-\rho}\right|_{R=\widehat{R}(\sigma)},$$

Note that for an interior solution $\widehat{R}_i(\sigma)$, the second-order condition gives a non-negative denominator. The numerator is linear in prize share σ. For $\sigma = 0$ we obtain

$$\frac{\partial\widehat{R}_i(0)}{\partial\sigma} = \left.\frac{\widehat{MQ}(R)-2\widehat{MR}(R)}{\widehat{MR}'(R)-\rho}\right|_{R=\widehat{R}} > 0,$$

since $\widehat{MQ}(R) > 2\widehat{MR}(R)$ for all R, and therefore also for $\widehat{R}$. For $\sigma = \frac{1}{2}$ we get

$$\frac{\partial\widehat{R}_i(\frac{1}{2})}{\partial\sigma} = \left.\frac{-\widehat{MR}(R)}{\frac{1}{4}\left(\widehat{MQ}'(R)+\widehat{MQ}'(R)\right)-\rho}\right|_{R=\widehat{R}(\frac{1}{2})} < 0,$$

because $\widehat{MR}(R) > 0$. From the linearity of the numerator we deduce that there always is a $\widehat{\sigma} \in (0, \frac{1}{2})$ such that $\frac{\partial\widehat{R}_i(\widehat{\sigma})}{\partial\sigma} = 0$.

9.2 Proofs for Private Signal Race

In this subsection of the Appendix we prove proposition 5 and 4.

9.2.1 Proof of Proposition 4 ($D_i^*(.)$)

In part (i) the first inequality is obvious, while the second, for $r_i = R_i = R$ and $i = 1, 2$, reduces to:

$$\frac{\partial D_i^*(\bar{t};R)}{\partial R_i} = \frac{-p(1-pR)\varphi(R_i)D_i^*(\bar{t};R) + p(1-p)R\Delta\left(D_i^*(\underline{t}) - D_i^*(\bar{t};R)\right)}{(1-pR)^2\left(\underline{\theta} + \varphi(R_i)\right)},$$

where the numerator is proportional to

$$\begin{aligned} & -(1-pR)\varphi(R)[\underline{\theta} + \varphi(R) + (R - P(R))\Delta] + (1-p)R\varphi(R)\Delta \\ = \; & -(1-pR)\varphi(R)[\underline{\theta} + \varphi(R)], \end{aligned}$$

which is negative for $R < 1$.

For part (ii) it suffices to observe that:

$$D_i^*(\underline{t};R) - \widehat{D}_i(\underline{t}) = \frac{(1-\sigma)(1-R)\Delta\varphi(R)W}{[(\underline{\theta} + R\Delta)(\underline{\theta} + \varphi(R) + (1-P(R))\Delta) - (1-R)P(R)\Delta^2](\underline{\theta} + \Delta)},$$

and

$$D_i^*(\bar{t};R) - \widehat{D}_i(\bar{t},\bar{t};R) = \frac{(1-\sigma)\underline{\theta}[\phi(R) - \varphi(R)]W}{[(\underline{\theta} + R\Delta)(\underline{\theta} + \varphi(R) + (1-P(R))\Delta) - (1-R)P(R)\Delta^2](\underline{\theta} + \phi(R) + \Delta)},$$

which obviously exceeds zero for $R < 1$. This completes the proof.

9.2.2 Proof of Proposition 5 ($R_i^*(0) \geq \widehat{R}_i(0)$)

In this proof we compare marginal research revenues for public signals with those for private signals. Naturally, from (4) we obtain:

$$\begin{aligned} \widehat{MR}(R) & < \frac{\frac{1}{2}p\underline{\theta}\phi(R)^2W^2}{(\underline{\theta} + \Delta)^2(\underline{\theta} + \phi(R) + \Delta)^2} \\ & = \frac{\frac{1}{2}p\underline{\theta}(1-p)^2(\overline{\theta} - \underline{\theta})^2W^2}{[p(1-R)^2 + 1 - p]^2(\underline{\theta} + \Delta)^2(\underline{\theta} + \phi(R) + \Delta)^2}. \end{aligned} \tag{8}$$

For $\sigma = 0$ (7) marginal research revenues for private signals reduce to:

$$\begin{aligned} MR^*(R) &\equiv \frac{\frac{1}{2}p\underline{\theta}\varphi(R)^2 W^2}{\left((\underline{\theta}+R\Delta)\left(\underline{\theta}+\varphi(R)+(1-P(R))\Delta\right)-(1-R)P(R)\Delta^2\right)^2} \\ &= \frac{\frac{1}{2}p\underline{\theta}(1-p)^2(\overline{\theta}-\underline{\theta})^2 W^2}{(1-pR)^2\left((\underline{\theta}+R\Delta)\left(\underline{\theta}+\varphi(R)+(1-P(R))\Delta\right)-(1-R)P(R)\Delta^2\right)^2} \end{aligned} \quad (9)$$

When we compare denominators of (8) and (9), we obtain:

$$\begin{aligned} & [p(1-R)^2+1-p](\underline{\theta}+\Delta)(\underline{\theta}+\phi(R)+\Delta)+ \\ & -(1-pR)\left((\underline{\theta}+R\Delta)\left(\underline{\theta}+\varphi(R)+(1-P(R))\Delta\right)-(1-R)P(R)\Delta^2\right) \\ = \; & (E(\theta)+\Delta)[2\underline{\theta}+(1+R)\Delta]-pR(3-R)(\underline{\theta}+\Delta)^2. \end{aligned}$$

Since this expression is linear and decreasing in p it suffices to evaluate it for $p = 1$. For $p = 1$ the expression reduces to:

$$(1-R)(\underline{\theta}+\Delta)[\underline{\theta}+(1-R)(\underline{\theta}+\Delta)] \geq 0.$$

This implies that $\widehat{MR}(R) < MR^*(R)$ for all $R < 1$, which completes the proof.

9.3 Proofs for Strategic Revelation

In this subsection we prove lemma 6, propositions 4 and 7.

9.3.1 Proof of Proposition 6 (No Complete Revelation)

Suppose complete revelation does happen in equilibrium. Then equilibrium beliefs are such that any statement is believed. Firm j's equilibrium investments would be $\widehat{D}_j(\underline{t}) = \frac{(1-\sigma)W}{\underline{\theta}+\Delta}$ and $\widehat{D}_j(\bar{t},\bar{t}) = \frac{(1-\sigma)W}{\underline{\theta}+\phi(r)+\Delta} \cdot \frac{\underline{\theta}+\phi(r)}{\underline{\theta}+\phi(R_i,r_j)}$, respectively. Suppose that firm j completely reveals his information, and that he received signal $t_j = \bar{t}$ from nature. Then if firm i received signal $\underline{t}$ and reveals it, firms invest $\widehat{D}(\underline{t})$, and firm i has expected profit:

$$\pi_i(\underline{t}|\underline{t}) = (1-\sigma)W^2 \frac{(1-\sigma)\frac{1}{2}\underline{\theta}+\sigma(\underline{\theta}+\Delta)}{(\underline{\theta}+\Delta)^2}.$$

If firm i states $\bar{t}$ instead, this makes firm j invest $\widehat{D}_j(\bar{t},\bar{t})$. Firm i's optimal response to $\widehat{D}_j(\bar{t},\bar{t})$ is $D_i = \frac{(1-\sigma)W(\underline{\theta}+\phi(r))}{(\underline{\theta}+\phi(r)+\Delta)\underline{\theta}}$. Firm i's profit from overstating his signal is

$$\pi_i(\bar{t}|\underline{t}) = (1-\sigma)W^2 \frac{(1-\sigma)\frac{1}{2}(\underline{\theta}+\phi(r))^2+\sigma\underline{\theta}(\underline{\theta}+\phi(r)+\Delta)}{\underline{\theta}(\underline{\theta}+\phi(r)+\Delta)^2}.$$

The difference in profit between overstating and truth-telling is

$$\pi_i(\bar{t}|\underline{t}) - \pi_i(\underline{t}|\underline{t}) = \frac{(1-\sigma)W^2}{\underline{\theta}(\underline{\theta}+\Delta)^2(\underline{\theta}+\phi(r)+\Delta)^2}\left((1-\sigma)a + \sigma(-b)\right),$$

with

$$\begin{aligned} a &\equiv \frac{1}{2}\left([(\underline{\theta}+\phi(r))(\underline{\theta}+\Delta)]^2 - [\underline{\theta}(\underline{\theta}+\phi(r)+\Delta)]^2\right), \\ b &\equiv \underline{\theta}(\underline{\theta}+\Delta)(\underline{\theta}+\phi(r)+\Delta)\phi(r). \end{aligned}$$

Hence, there is a $\underline{\sigma} \in (0,1)$ such that $\pi_i(\bar{t}|\underline{t}) > \pi_i(\underline{t}|\underline{t})$ iff $\sigma \leq \underline{\sigma}$. Similar for a $\bar{t}$-firm i, stating $\underline{t}$ (resp. $\bar{t}$) makes $\bar{t}$-firm j choose $\widehat{D}_j(\underline{t})$ (resp. $\widehat{D}_j(\bar{t},\bar{t})$). Firm i's optimal response to this investment is $D_i = \frac{(1-\sigma)W\underline{\theta}}{(\underline{\theta}+\Delta)(\underline{\theta}+\phi(R_i,r_j))}$ (resp. $\widehat{D}_i(\bar{t},\bar{t})$). Firm i's profit for understating his signal is

$$\pi_i(\underline{t}|\bar{t}) = (1-\sigma)W^2\frac{(1-\sigma)\frac{1}{2}\underline{\theta}^2 + \sigma(\underline{\theta}+\Delta)(\underline{\theta}+\phi(R_i,r_j))}{(\underline{\theta}+\Delta)^2(\underline{\theta}+\phi(R_i,r_j))},$$

while truth-telling gives him

$$\pi_i(\bar{t}|\bar{t}) = (1-\sigma)W^2\frac{(1-\sigma)\frac{1}{2}(\underline{\theta}+\phi(r))^2 + \sigma(\underline{\theta}+\phi(r)+\Delta)(\underline{\theta}+\phi(R_i,r_j))}{(\underline{\theta}+\phi(r)+\Delta)^2(\underline{\theta}+\phi(R_i,r_j))}.$$

The difference in profit between understating and truth-telling is

$$\pi_i(\underline{t}|\bar{t}) - \pi_i(\bar{t}|\bar{t}) = \frac{(1-\sigma)W^2}{(\underline{\theta}+\Delta)^2(\underline{\theta}+\phi(r)+\Delta)^2(\underline{\theta}+\phi(R_i,r_j))}\left((1-\sigma)(-A) + \sigma B\right),$$

with

$$\begin{aligned} A &\equiv a, \text{ and} \\ B &= (\underline{\theta}+\phi(R_i,r_j))(\underline{\theta}+\Delta)(\underline{\theta}+\phi(r)+\Delta)\phi(r). \end{aligned}$$

Hence, there is a $\bar{\sigma} \in (0,1)$ such that $\pi_i(\underline{t}|\bar{t}) \geq \pi_i(\bar{t}|\bar{t})$ whenever $\sigma \geq \bar{\sigma}$. It is straightforward that $b \leq B$. This implies that $\bar{\sigma} \leq \underline{\sigma}$, and, thus, is deviating from complete revelation profitable for all $\sigma \in [0,1]$. This completes the proof.

9.3.2 Proof of Lemma 4 (No Revelation)

Observe that when firms never update their beliefs, each firm is indifferent between all revelation rules, i.e. $\pi_i(\tau_i(t_i),\tau_j) = \pi_i(\tau_i'(t_i),\tau_j) = E_\theta\{\pi_i(D^*;\theta)|t_i;R_i\}$ for all τ_i, τ_i' and τ_j. No revelation, e.g. $\widehat{\tau}_i(t_i) = \underline{t}$ for $i = 1,2$, is therefore weakly preferred by firms, which is consistent with beliefs. By stating high costs, no type of firm i can obtain higher profits, since beliefs are not updated.

9.3.3 Proof of Proposition 7 (Verifiable Information)

Since information is verifiable, a firm can only choose to either disclose or conceal his signal, $\tau_i(t_i) \in \{t_i, \varnothing\}$. If only one type of firm chooses to conceal his signal, his rival can infer his information perfectly. We therefore only need to distinguish between strategies of full disclosure and full concealment. We take $\overline{\sigma}$, $\underline{\sigma}$, and $\pi_i(.|.)$ as in the proof to proposition 6, and characterize part (i), (ii) and (iii), respectively.

(i) Take $\sigma \leq \overline{\sigma}$. Suppose that firm j discloses his information: $\tau_j(t_j) = t_j$ for $t_j \in \{\underline{t}, \overline{t}\}$. In that case firm i's disclosure rule can only affect the equilibrium outcome when firm j discloses $\overline{t}$. Firm i's expected profit from disclosing private signals $\underline{t}$ and $\overline{t}$ is then $\pi_i(\underline{t}|\underline{t})$ and $\pi_i(\overline{t}|\overline{t})$, respectively. Suppose that firm i deviates from complete revelation and conceals his signal. After concealment firm j updates his beliefs skeptically, and believes that $t_i = \underline{t}$ with probability 1, i.e. $\pi_i(\varnothing|t_i) \equiv \pi_i(\underline{t}|t_i)$. Consequently he invests $\widehat{D}_j(\underline{t})$ in development. This leaves firm i indifferent between disclosing and concealing when $t_i = \underline{t}$. When firm i has private signal $\overline{t}$, he prefers to disclose his signal, since $\pi_i(\overline{t}|\overline{t}) \geq \pi_i(\underline{t}|\overline{t})$ iff $\sigma \leq \overline{\sigma}$. Hence sceptical beliefs are consistent with firm's incentives, and firms' disclosure strategies are optimal given beliefs.

(ii) Take $\overline{\sigma} < \sigma < \underline{\sigma}$, and suppose that firm j discloses his information. Firm j's development investments can only be affected by firm i's disclosure decision when firm j receives a bad signal, $t_j = \overline{t}$. We consider this case. After firm i's concealment, $\widetilde{\tau}_i = \varnothing$, firm j assigns probability μ to the contingency that firm i received a good signal, $t_i = \underline{t}$, with $0 \leq \mu \leq 1$. Firm j's expected costs of development after concealment are $\underline{\theta} + (1-\mu)\phi(r_i, R_j)$. The first-order condition for firm j's investments is the following:

$$(\underline{\theta} + (1-\mu)\phi(r_i, R_j))D_j(\varnothing; R_j) = (1-\sigma)W - \left(\mu D_i(\underline{t}) + (1-\mu)D_i(\overline{t})\right)\Delta.$$

Firm i's first-order conditions remain unchanged. Given firm j's belief, we obtain the following equilibrium investments:

$$\begin{aligned}
D_j^{\mu}(\varnothing; r_j) &= \frac{(1-\sigma)W\left[\underline{\theta}(\underline{\theta}+\phi(r)) - (\underline{\theta}+\mu\phi(r))\Delta\right]}{\underline{\theta}(\underline{\theta}+\phi(r))(\underline{\theta}+(1-\mu)\phi(r)) - (\underline{\theta}+\mu\phi(r))\Delta^2} \\
D_i^{\mu}(\underline{t}) &= \frac{(1-\sigma)W(\underline{\theta}+\phi(r))(\underline{\theta}+(1-\mu)\phi(r)-\Delta)}{\underline{\theta}(\underline{\theta}+\phi(r))(\underline{\theta}+(1-\mu)\phi(r)) - (\underline{\theta}+\mu\phi(r))\Delta^2} \\
D_i^{\mu}(\overline{t}; R_i) &= \frac{(1-\sigma)W\underline{\theta}(\underline{\theta}+(1-\mu)\phi(r)-\Delta)}{\underline{\theta}(\underline{\theta}+\phi(r))(\underline{\theta}+(1-\mu)\phi(r)) - (\underline{\theta}+\mu\phi(r))\Delta^2} \cdot \frac{(\underline{\theta}+\phi(r))}{(\underline{\theta}+\phi(R_i, r_j))}.
\end{aligned}$$

Firm i's expected equilibrium profits are:

$$\begin{aligned}
\pi_i^\mu(\varnothing|\underline{t}) &= \frac{1}{2}\underline{\theta} D_i^\mu(\underline{t})^2 + \sigma W D_j^\mu(\varnothing; r_j) \\
\pi_i^\mu(\varnothing|\overline{t}) &= \frac{1}{2}(\underline{\theta} + \phi(R_i, r_j)) D_i^\mu(\overline{t}; R_i)^2 + \sigma W D_j^\mu(\varnothing; r_j)
\end{aligned}$$

Note that for belief $\mu = 0$ firm i strictly prefers to conceal $\underline{t}$, since $\pi_i^0(\varnothing|\underline{t}) = \pi_i(\overline{t}|\underline{t}) > \pi_i(\underline{t}|\underline{t})$ for $\sigma < \underline{\sigma}$. We can therefore rule out belief $\mu = 0$ as supporting a full disclosure equilibrium. Belief $\mu = 1$ can be ruled out too, because firm i prefers to conceal a bad signal given this belief, i.e. $\pi_i^1(\varnothing|\overline{t}) = \pi_i(\underline{t}|\overline{t}) > \pi_i(\overline{t}|\overline{t})$ for $\sigma > \overline{\sigma}$. For beliefs strictly between 0 and 1 there is a critical value $\underline{\sigma}^\mu$ (resp. $\overline{\sigma}^\mu$) such that disclosing $\underline{t}$ (resp. $\overline{t}$) is profitable for firm i whenever $\sigma \geq \underline{\sigma}^\mu$ (resp. $\sigma \leq \overline{\sigma}^\mu$). The critical values are defined as follows:

$$\begin{aligned}
\underline{\sigma}^\mu &= \frac{\frac{1}{2}\underline{\theta}\left(d_i^\mu(\underline{t})^2 - \widehat{d}_i(\underline{t})^2\right)}{\frac{1}{2}\underline{\theta}\left(d_i^\mu(\underline{t})^2 - \widehat{d}_i(\underline{t})^2\right) - \left(d_j^\mu(\varnothing; r_j) - \widehat{d}_j(\underline{t})\right)}, \text{ and} \\
\overline{\sigma}^\mu &= \frac{-\frac{1}{2}\left(\underline{\theta} + \phi(R_i, r_j)\right)\left(d_i^\mu(\overline{t}; R_i)^2 - \widehat{d}_i(\overline{t}; R_i)^2\right)}{-\frac{1}{2}\left(\underline{\theta} + \phi(R_i, r_j)\right)\left(d_i^\mu(\overline{t}; R_i)^2 - \widehat{d}_i(\overline{t}; R_i)^2\right) + \left(d_j^\mu(\varnothing; r_j) - \widehat{d}_j(\overline{t}; r_j)\right)},
\end{aligned}$$

where $d_\ell(.) \equiv D_\ell(.)/(1-\sigma)W$, with $\ell = i, j$. For prize share σ full disclosure is an equilibrium strategy given belief μ, whenever belief μ is such that $\underline{\sigma}^\mu \leq \sigma \leq \overline{\sigma}^\mu$. First we verify that both $\underline{\sigma}^\mu$ and $\overline{\sigma}^\mu$are monotonically decreasing in belief μ for $0 < \mu < 1$:

$$\begin{aligned}
\frac{\partial \underline{\sigma}^\mu}{\partial \mu} &= \frac{\underline{\theta}\left[\frac{1}{2}\frac{\partial d_j^\mu(\varnothing; r_j)}{\partial \mu}\left(d_i^\mu(\underline{t})^2 - \widehat{d}_i(\underline{t})^2\right) - d_i^\mu(\underline{t})\frac{\partial d_i^\mu(\underline{t})}{\partial \mu}\left(d_j^\mu(\varnothing; r_j) - \widehat{d}_j(\underline{t})\right)\right]}{\left[\frac{1}{2}\underline{\theta}\left(d_i^\mu(\underline{t})^2 - \widehat{d}_i(\underline{t})^2\right) - \left(d_j^\mu(\varnothing; r_j) - \widehat{d}_j(\underline{t})\right)\right]^2} \\
&= \frac{\frac{-(1-\mu)^2\frac{1}{2}\underline{\theta}^2\phi(r)^3\Delta^2(\underline{\theta}+\phi(r))(\underline{\theta}+\phi(r)-\Delta)^3(\underline{\theta}-\Delta)}{(\underline{\theta}+\Delta)^2[\underline{\theta}(\underline{\theta}+\phi(r))(\underline{\theta}+(1-\mu)\phi(r))-(\underline{\theta}+\mu\phi(r))\Delta^2]^4}}{\left[\frac{1}{2}\underline{\theta}\left(d_i^\mu(\underline{t})^2 - \widehat{d}_i(\underline{t})^2\right) - \left(d_j^\mu(\varnothing; r_j) - \widehat{d}_j(\underline{t})\right)\right]^2} < 0, \text{ and} \\
\frac{\partial \overline{\sigma}^\mu}{\partial \mu} &= \frac{(\underline{\theta} + \phi(R_i, r_j))\left[\begin{array}{c} \frac{1}{2}\frac{\partial d_j^\mu(\varnothing; r_j)}{\partial \mu}\left(d_i^\mu(\overline{t}; R_i)^2 - \widehat{d}_i(\overline{t}; R_i)^2\right) + \\ -d_i^\mu(\overline{t}; R_i)\frac{\partial d_i^\mu(\overline{t}; R_i)}{\partial \mu}\left(d_j^\mu(\varnothing; r_j) - \widehat{d}_j(\overline{t}; r_j)\right) \end{array}\right]}{\left[\frac{1}{2}\underline{\theta}\left(d_i^\mu(\overline{t}; R_i)^2 - \widehat{d}_i(\overline{t}; R_i)^2\right) - \left(d_j^\mu(\varnothing; r_j) - \widehat{d}_j(\overline{t}; R_i)\right)\right]^2} \\
&= \frac{\frac{-\mu^2\frac{1}{2}\underline{\theta}\phi(r)^3\Delta^2(\underline{\theta}+\phi(r))(\underline{\theta}+\phi(r)-\Delta)(\underline{\theta}-\Delta)^3}{(\underline{\theta}+\phi(r)+\Delta)^2[\underline{\theta}(\underline{\theta}+\phi(r))(\underline{\theta}+(1-\mu)\phi(r))-(\underline{\theta}+\mu\phi(r))\Delta^2]^4} \cdot \frac{(\underline{\theta}+\phi(r))^2}{(\underline{\theta}+\phi(R_i,r_j))}}{\left[\frac{1}{2}\underline{\theta}\left(d_i^\mu(\overline{t}; R_i)^2 - \widehat{d}_i(\overline{t}; R_i)^2\right) - \left(d_j^\mu(\varnothing; r_j) - \widehat{d}_j(\overline{t}; R_i)\right)\right]^2} < 0.
\end{aligned}$$

Furthermore, it is easily verified that

$$\lim_{\mu\uparrow 1}\underline{\sigma}^{\mu} = \frac{\Delta}{\underline{\theta}+2\Delta} > \frac{\Delta}{\underline{\theta}+\phi(r)+2\Delta} = \lim_{\mu\downarrow 0}\overline{\sigma}^{\mu}.$$

In combination with monotonicity this implies that $\underline{\sigma}^{\mu} > \overline{\sigma}^{\mu}$ for all $0 < \mu < 1$. Therefore there is no belief μ such that full disclosure is chosen in equilibrium. **(iii)** For $\sigma \geq \underline{\sigma}$ we have a similar argument as in (i). Sceptical beliefs after concealment are to believe that your rival has a "bad" signal, i.e. $\pi_i(\varnothing|t_i) \equiv \pi_i(\overline{t}|t_i)$. This leaves firm i with a bad signal indifferent between disclosing and concealing. Firm i with a good signal is worse off by concealing his signal, since $\pi_i(\overline{t}|\underline{t}) \leq \pi_i(\underline{t}|\underline{t})$ iff $\sigma \geq \underline{\sigma}$. This completes the proof.

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