Monetary Stability and Liquidity Crises: The Role of the Lender of Last Resort[¤]

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Abstract

We study an economy where agents are subject to liquidity demand shocks, and banks arise endogenously to insure consumers against these shocks. In this environment we evaluate the desirability of a lender of last resort who can provide liquidity loans to banks in distress. In the absence of a lender of last resort, the economy has a unique, stationary equilibrium. The introduction of unlimited and costless lender of last resort services allows the economy to achieve a steady state allocation that is pareto optimal. However, this economy also displays a continuum of hyperin‡ationary equilibria. We then explore restrictions on the provision of lender of last resort services that rule out such monetary instability while preserving some of the e¢ciency obtained by unrestricted lender of last resort services. When the lender of last resort charges an interest rate on liquidity loans, the economy has a unique steady state equilibrium, and when the interest rate charged is high enough, no hyperin‡ationary equilibria arise. Finally, when the lender of last resort faces an upper bound on loanable funds, there is again a unique long-run equilibrium, and when the upper bound on loanable funds is small enough, hyperin‡ationary equilibria are ruled out.

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1. Introduction

As recently as 1961, Mundell argued that “it hardly appears within the realm of political feasibility that national currencies would ever be abandoned in favor of any other arrangement,” [Mundell (1961), p. 657]. Yet over the last decade, a single currency area was created in Europe, and several countries have installed a currency board. More recently, some economies have begun considering the possibility of adopting the U.S. dollar as legal tender, thereby eliminating the use of a national fiat currency altogether.

Why are countries implementing or contemplating such drastic measures? Obviously there are several advantages to joining a monetary union - even if unilaterally as is the case with “dollarization”. First, abandoning a domestic fiat currency is a device to commit to low and stable inflation. Second, it may reduce the costs and uncertainties associated with international transactions that involve currency conversion and are subject to the risk of exchange rate changes. Third, supporters of “dollarization”, especially in Latin-America, argue that it will eradicate excess volatility stemming from speculation against a domestic currency. But abolishing the use of a domestic fiat currency also entails considerable costs. These include the loss of monetary policy independence and the loss of seigniorage revenue. In addition, the lender of last resort loses the ability to provide liquidity by creating money in the event of a banking crisis.

In light of these recent developments, we think it is opportune to revisit the subject of the desirability of a national fiat money. Among the various topics mentioned above, we want to focus on the role of a domestic fiat money for the provision of liquidity to a banking system in distress, and we want to analyze the implications of such lender of last resort services for the stability of a national currency. These issues are at the heart of the recent debate on “dollarization”. As far back as 1873, Bagehot argued that “in opposition to what might be at first sight supposed, the best way for the bank or banks who have custody of the bank reserve to deal with a drain arising from internal discredit, is to lend freely” (1873, p.48). This view is widely accepted by contemporary scholars as reflected in Fisher who states “there is considerable agreement on the need for a domestic lender of last resort” (1999, p. 86). On the other hand, the existence of a lender of last resort has also been identified as a cause for excess volatility in emerging economies’ financial markets, and for the currency crises that have plagued these economies in the 1990s[1]. Thus, while there appears to be a consensus on the desirability of a lender of last resort, the provision of liquidity services by the central bank is also associated with the potential for currency instability.

In this paper, we take up these arguments and seek to answer two specific questions: (1) is it desirable to have a lender of last resort that is able to print fiat currency? (2) does the presence of such a lender of last resort affect the stability of a currency? To answer these questions, we construct a model with aggregate liquidity shocks that create a role for a lender of last resort. We show that having a lender of last resort that can print money and lend freely to the banking system at zero cost, allows the economy to completely overcome the liquidity shocks. Under this regime there is a stationary equilibrium that is pareto optimal. However, there is also a continuum of non-optimal hyperinflationary equilibria. Hence, while having an unrestricted lender of last resort allows the economy to attain an efficient allocation, it also opens the door to currency instability.

Having identified the lender of last resort as a potential source of monetary instability, we move on to a third question: (3) what measures could be implemented to eliminate the bad equilibria associated with unlimited and costless lender of last resort services, while retaining some of its "good" properties and without having to eliminate a national currency? We show that this may be achieved either by credibly committing to a sufficiently low cap on lending by the lender of last resort, or by charging a sufficiently high real interest rate on liquidity loans.

We examine a simple monetary endowment economy originally developed by Champ, Smith and Williamson (1996). It is a classic pure exchange, two-period-lived overlapping generations economy, where some agents are lenders and some are borrowers, such that there is a store of value role for money. Agents are assigned to either of two locations, and each period, a fraction of lenders is forced to relocate. Limited communication prevents claims on specific agents from being traded across locations and only money has value in exchange after relocation. As in Townsend (1987), Mitsui and Watanabe (1989), and Hornstein and Krusell (1990), this generates a transactions role for currency and allows money to be dominated in return by other assets. Moreover, in this set-up stochastic relocations act like the portfolio preference shocks in Diamond and Dybvig (1983) and banks will arise endogenously to insure consumers against such random liquidity shocks. Thus, banks write deposit contracts insuring lenders against the possibility of relocation, hold reserves,

[1] See, for example, Chang and Velasco (1998), Mishkin (1999), and Fisher (1999).

and provide intermediation between borrowers and lenders.

In this framework, we obtain the following results. In the absence of a lender of last resort, the economy displays a unique equilibrium. This equilibrium is stationary, with banks holding the same fraction of their portfolio in the form of reserves at all times. We show that there is a critical value of the relocation shock below which these precautionary reserves suffice to fully cover the demand for liquidity and to equalize the return on deposits for all agents. However, for realizations of the relocation shock above this critical value, banks face a "liquidity crisis". This corresponds to a situation of complete exhaustion of banks' cash reserves and since other bank assets are illiquid, this event precludes depositors from being fully insured. A wedge is driven between the returns earned by depositors who are subject to the relocation shock and those who are not. Hence for shocks above the critical value, the economy is not efficient.

When we allow for the unrestricted provision of lender of last resort services at zero cost, the set of equilibria is substantially different. In this case there exists a unique steady state equilibrium which is pareto optimal. Compared to the equilibrium for the benchmark case, banks hold a lower fraction of their portfolio as real balances. They contract a liquidity loan from the issuer of fiat money whenever they face a crisis and precautionary reserves are insufficient to cover the demand for liquidity. By doing so they are able to fully insure agents against random liquidity shocks in all states of the world. Relocated and non-relocated agents earn the same return under all circumstances and the economy is always efficient. However, in addition to the pareto optimal stationary equilibrium, the economy also displays a continuum of hyperinflationary equilibria which are not pareto optimal. In fact, in the presence of an unrestricted lender of last resort, the money supply moves to exactly match the stochastic movements in demand for liquidity, allowing the economy to completely overcome the stochastic relocation friction. This renders the economy equivalent to a standard, Samuelson-case economy with a constant money supply [see Gale (1973)]. Hence, having an unrestricted lender of last resort generates a pareto optimal steady state allocation, but also opens the door to currency instability.

We then explore whether certain restrictions on the provision of liquidity loans may allow the economy to preserve some of the desirable features of a world with unrestricted and costless lender of last resort services, without permitting the existence of hyperinflationary equilibria. First we study an economy with a lender of last resort that charges a real interest rate on liquidity loans to banks facing a crisis. When this interest rate is high enough, hyperinflationary equilibria are indeed ruled out, and the economy displays a unique equilibrium which is stationary. Finally, we study the case of a lender of last resort that faces an upper bound on loanable funds and show that this policy can also eliminate currency instability as long as the cap imposed on liquidity loans is small enough.

Before turning to the model, we want to point out some differences between this paper and the existing literature on the lender of last resort. Obviously our model has a lot in common with Diamond and Dybvig (1983). However, it is also different in several key aspects. Diamond and Dybvig consider a purely real economy while Champ, Smith and Williamson (1996) have argued that models of banking crises should be expanded to consider monetary factors. Thus we follow them in studying a world where money has a role, both as a store of value and in the completion of transactions. This allows us to examine the relationship between the terms of last resort lending and currency stability, the central question of our paper. Moreover, the banking panics studied in Diamond and Dybvig and the liquidity crises arising in our economy are of a very different nature. In Diamond and Dybvig, private information regarding the preferred timing of consumption, the presence of real assets that can be liquidated prematurely, and the imposition of a sequential service constraint, combine to create the potential for self-fulfilling panics. These panics can be ruled out by providing deposit insurance. In contrast, the liquidity crises in our model are the result of fundamentals, and it is the presence of the lender of last resort that permits self-fulfilling, hyperinflationary equilibria to arise.

We also want to underline that we have abstracted from the problems of moral hazard and "excessively risky" behavior associated with the presence of a lender of last resort. Several authors, including Solow (1982), Mishkin (1997) and Fisher (1999), have argued that these problems are crucial in understanding the potential for instability related to the provision of lender of last resort arrangements. In our model, currency instability may arise even when there is no scope for moral hazard and "overly risky" portfolio allocation.

The remainder of the paper proceeds as follows. Section 1 lays out the model environment. Section 2 describes equilibrium without a lender of last resort, while Section 3 presents the case of a central bank that provides unrestricted lender of last resort services at zero cost. Section 4 looks at a policy of charging interest on liquidity loans, while Section 5 describes the behavior of an economy with a lender of last resort that faces an upper bound on the amount it can lend to banks in distress. Concluding comments are offered in Section 7.

2. The Basic Model

2.1 The Environment

We examine a monetary endowment economy originally presented by Champ, Smith, and Williamson (1996). The economy consists of an infinite sequence of two period lived, overlapping generations, plus an initial old generation. There is a single, perishable consumption good. At each date $t = 0, 1, ...$, a continuum of agents with unit mass is born at each of two identical locations. Half of these agents are "lenders" and the remaining half are "borrowers". The former have endowments $(\omega_1, \omega_2) = (x, 0)$, while the latter's endowment vector is $(\omega_1, \omega_2) = (0, y)$. All consumers have preferences given by $u(c_1, c_2) = \ln(c_1) + \beta \ln(c_2)$. We assume that $\beta x > y$ holds, which implies that this is a "Samuelson case" economy (Gale, 1973) and hence there is a role for money as a store of value in this world. At $t = 0$ there is a continuum of old agents with unit mass in each location. Each of these agents is endowed with $M > 0$ units of fiat money, and there are no subsequent injections or withdrawals of currency.

In addition to a store of value role for money, spatial separation and limited communication will generate a transactions role for money in a way reminiscent of Townsend (1987), Mitsui and Watanabe (1989), and Hornstein and Krusell (1990). This will allow money to be dominated in rate of return by other assets. Indeed, at the beginning of each period, agents cannot move between or communicate across locations. Goods can never be transported between locations. Hence, goods and asset transactions occur autarkically within each location at the beginning of each period. After this trade is concluded at time t, a fraction π_t of young lenders in each location is forced to move to the other location. Limited communication prevents the cross-location exchange of privately issued liabilities. Currency, on the other hand, is universally recognizable and non-counterfeitable, and is therefore acceptable in inter-location exchange. Hence the relocation process acts like a stochastic cash-in-advance constraint for young lenders. The old-age consumption of a mover will be equal to the real value of the money that she takes with her to the new location. The relocation probability,

π_t, is a random variable in each period with support $[0,1]$, and is drawn from the twice continuously differentiable, strictly increasing distribution function F with associated density function f. The relocation probability is identically and independently distributed over time.

Stochastic relocations also act like shocks to portfolio preferences which have the same consequences as the "liquidity preference shocks" in Diamond and Dybvig (1983). Hence they motivate a role for banks to insure lenders against random liquidity needs. Banks take deposits, hold cash reserves, and intermediate lending. We now describe the behavior of borrowers, lenders, and banks.

2.2 Consumers

Borrowers, who never move, face a gross market interest rate of R_t. They choose their quantity of borrowing l_t to solve the problem

$$\max_{l_t} \ln(l_t) + \beta \ln(y - R_t l_t).$$

The solution to this problem is given by

$$l_t = \frac{y}{(1+\beta) R_t}. \tag{1}$$

Lenders face a more complicated problem. Given that they are confronted with random relocation, they deposit all of their savings in a bank, and the return they receive depends on whether or not they move and on what fraction of all young lenders move. Specifically, given that the relocation status is public information, they are promised a real return $r_t(\pi)$ if they do not move and $r_t^m(\pi)$ if they do move. Notice that, given our assumptions on the distribution of π, these return schedules depend only on the relocation probability at time t, and not on the history of realizations of relocation probabilities. Lenders then choose the amount they save and deposit, d_t, to maximize expected utility

$$\max_{d_t} \ln(x - d_t) + \beta \int_0^1 \pi \ln[r_t^m(\pi) d_t] f(\pi) d\pi + \beta \int_0^1 (1-\pi) \ln[r_t(\pi) d_t] f(\pi) d\pi.$$

The solution to this problem sets

$$d_t = d = \frac{\beta}{1+\beta} x. \tag{2}$$

Hence saving is independent of the distribution of the rates of return. This result clearly depends on the assumptions of log utility and no old-age income for lenders, which imply that the income and substitution effects of a change in the rate of return exactly offset each other.

2.3 Banks

Banks take deposits, make loans, hold reserves and announce return schedules. We assume that any borrower can establish a bank and that banks behave competitively in the sense that they take the real return on assets as given. On the deposit side, we assume that banks behave as Nash competitors, which will lead them to choose deposit returns to maximize the expected utility of young lenders. The constraints that banks face in this maximization problem depend on what lender-of-last-resort services are available to them.

Below we consider four different scenarios. First, as a benchmark case, we consider a world without a lender of last resort. We then turn our attention to the economy with a government that provides unlimited lender of last resort services, at a zero nominal interest rate. Next, we analyze an economy with a lender of last resort that charges a positive interest rate when providing liquidity to banks. Finally, we examine the case of a lender of last resort constrained by an upper bound on the amount that it can lend to a bank in distress.

3. No Lender of Last Resort

In this section we discuss general equilibrium for an economy in which banks are unable to borrow from anyone other than lenders. We begin by describing the bank's problem for this benchmark case.

3.1 The Bank's Problem

A young lender will deposit her entire savings d with a bank. Per young depositor, the bank acquires an amount z_t of real balances, and makes loans with a real value $d - z_t$. The bank faces two constraints with respect to the return it promises to movers, r_t^m, and the return it promises to non-movers, r_t. First, relocated agents, of which there are π_t, must be given currency, since that is the only asset which will allow these agents to consume at $t+1$ in their new location. This is accomplished by using a fraction $\alpha_t(\pi)$ of the bank's holdings of cash reserves. Hence, if p_t denotes the general price level at time t; and the return to holding money between time t and $t+1$ is given by $\frac{p_t}{p_{t+1}}$;

$$\pi d r_t^m(\pi) \leq \alpha_t(\pi) z_t \frac{p_t}{p_{t+1}}$$

must hold. If we denote by $\gamma_t \equiv \frac{z_t}{d}$ the ratio of reserves to deposits, then we can rewrite this constraint as follows

$$\pi r_t^m(\pi) \leq \alpha_t(\pi)\gamma_t \frac{p_t}{p_{t+1}}. \tag{3}$$

Second, real payments to nonmovers, which occur at time $t+1$; can not exceed the value of the bank's remaining portfolio – remaining reserves plus loan repayments. Since loans earn the gross real rate of return R_t, this can be written as

$$(1-\pi) d r_t(\pi) \leq [1-\alpha_t(\pi)] z_t \frac{p_t}{p_{t+1}} + (d - z_t) R_t$$

or

$$(1-\pi) r_t(\pi) \leq [1-\alpha_t(\pi)]\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t. \tag{4}$$

Of course, $0 \leq \gamma_t \leq 1$ and $0 \leq \alpha_t(\pi) \leq 1$ must hold.

Since π is i.i.d., the bank's problem is the same in every period and hence the optimal functions r_t; r_t^m; and α_t are independent of the history of realizations of π. Moreover, since there is free entry in banking, and since banks behave as Nash competitors, they will maximize young lenders' utility, taking deposit demand, d, as given. Banks will earn no profits, and constraints (3) and (4) will hold with equality. Given (2), the bank's problem is then to choose $r(\pi)$ and $r^m(\pi)$ to

$$\max \ln\left(\frac{x}{1+\rho}\right) + \beta\int_0^1 \left\{\pi \ln\left[r_t^m(\pi)\frac{\bar{x}}{1+\rho}\right] + (1-\pi)\ln\left[r_t(\pi)\frac{\bar{x}}{1+\rho}\right]\right\} f(\pi)\, d\pi \tag{5}$$

subject to

$$r_t^m(\pi) = \frac{1}{\pi}\alpha_t(\pi)\gamma_t\frac{p_t}{p_{t+1}}$$

$$r_t(\pi) = \frac{1}{1-\pi}\left\{[1-\alpha_t(\pi)]\gamma_t\frac{p_t}{p_{t+1}} + (1-\gamma_t)R_t\right\}$$

$$0 \leq \gamma_t \leq 1$$

$$0 \leq \alpha_t(\pi) \leq 1.$$

Substituting in the first two constraints and dropping the constant terms yields the problem

$$\max_{\alpha_t(\pi),\,\gamma_t} \int_0^1 \pi \ln\left\{\frac{\alpha(\pi)\gamma_t}{\pi}\frac{p_t}{p_{t+1}}\right\} f(\pi)\,d\pi + \int_0^1 (1-\pi)\ln\left\{\frac{[1-\alpha(\pi)]\gamma_t}{(1-\pi)}\frac{p_t}{p_{t+1}} + \frac{(1-\gamma_t)}{(1-\pi)}R_t\right\} f(\pi)\,d\pi.$$

$$0 \le \gamma_t \le 1$$

$$0 \le \alpha_t(\pi) \le 1.$$

The function α_t, which is the fraction of bank reserves paid out to movers, is chosen after the realization of π, while the function γ_t, the fraction of reserves in the bank's asset portfolio, is chosen before the realization of π. Hence we can first pick the optimal value of α_t for fixed values of γ_t and π. That is, we can choose α_t to solve

$$\max_{0\le\alpha_t\le 1} \pi \ln\left[\frac{\alpha_t\gamma_t}{\pi}\frac{p_t}{p_{t+1}}\right] + (1-\pi)\ln\left[\frac{(1-\alpha_t)\gamma_t}{(1-\pi)}\frac{p_t}{p_{t+1}} + \frac{(1-\gamma_t)}{(1-\pi)}R_t\right]$$

The solution to this problem sets

$$\alpha_t(\pi) = \begin{cases} \pi\left(1 + \frac{1-\gamma_t}{\gamma_t}R_t\frac{p_{t+1}}{p_t}\right) \\ 1 \end{cases} \text{for } \pi \in \begin{cases} (0, \pi^*) \\ [\pi^*, 1) \end{cases}.$$

where

$$\pi^* = \frac{\gamma_t\frac{p_t}{p_{t+1}}}{\gamma_t\frac{p_t}{p_{t+1}} + (1-\gamma_t)R_t} \tag{6}$$

Hence for realizations of the relocation shock below the critical value π^*, the bank pays out only a fraction of its reserves to movers. However, when a relocation shock $\pi > \pi^*$ materializes, all reserves are paid out to movers, and repayments to non-movers are drawn from loan repayments only. In other words, the bank holds precautionary reserves. When the realization of the relocation shock is below the critical value π^*, these cash reserves are sufficient to equalize the returns across movers and non-movers. However, when the realization of the relocation shock is greater than the critical value π^*, the bank faces a "liquidity crisis". It pays out all its cash reserves to movers, while repayments to non-movers are drawn from loan repayments only. In a "liquidity crisis", the bank no longer equalizes the returns of movers and non-movers; movers receive a lower return.

This result follows from the trade-off between two forces. First, the return on holding cash balances is dominated by the return on making loans to borrowers. Therefore, everything else being equal, the bank would like to minimize on cash reserves. On the other hand, in a quest to maximize young lenders' utility, the bank strives to provide insurance by equalizing the returns between movers and non-movers for all realizations of π. To do so, it has to hold sufficient cash balances. On the average, the welfare gains from equalizing the returns to movers and non-movers exactly offset the cost implied by the return dominance of loans over cash reserves.

It then remains to determine the optimal value of γ_t. To do so, we substitute the optimal value of α_t into the bank's objective function so that the only remaining choice variable is γ_t. Doing so yields the problem

$$\max_{0 \le \gamma_t \le 1} \int_0^{\pi^*} \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t\right] f(\pi)\, d\pi + \int_{\pi^*}^{1} \left\{ \pi \ln\left[\frac{\gamma_t}{\pi} \frac{p_t}{p_{t+1}}\right] + (1-\pi) \ln\left[\frac{(1-\gamma_t)}{(1-\pi)} R_t\right] \right\} f(\pi)\, d\pi.$$

This formulation of the problem makes it clear that the return earned by both movers and non-movers will be the same when π is less than π^*, but will - in general - be different when π is greater than π^*. The first-order condition for this problem is

$$\frac{R_t - \frac{p_t}{p_{t+1}}}{\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t} F(\pi^*) = \frac{1}{\gamma_t} \int_{\pi^*}^{1} \pi f(\pi)\, d\pi - \frac{1}{1-\gamma_t} \int_{\pi^*}^{1} (1-\pi) f(\pi)\, d\pi. \quad (7)$$

This can be solved for[2]

$$\gamma_t = 1 - \int_{\pi^*}^{1} F(\pi)\, d\pi. \quad (8)$$

This completes the solution to the bank's problem when no lender of last resort services are provided. We now turn to an analysis of general equilibrium for this case.

3.2 Equilibrium

An equilibrium of this economy is characterized by the market clearing conditions for real balances and loans. Since the supply of real balances, z_t, is equal to $\frac{M}{p_t}$, while the demand for real balances

[2] To facilitate the refereeing process, we provide the solution method in referee's Appendix A.

is given by $\gamma_t d$; market clearing for real balances and (2) require that we have

$$\frac{M}{p_t} = \gamma_t \frac{\beta}{1+\beta} x.$$

Similarly, from (1), the demand for loans, l_t, is equal to $\frac{y}{(1+\beta)R_t}$; while the supply of loans is given by $(1-\gamma_t)\, d$. This yields the following market clearing condition for loans

$$\frac{y}{(1+\beta)\, R_t} = (1-\gamma_t)\, \frac{\beta}{1+\beta} x.$$

These equations imply that we have

$$\gamma_t \frac{p_t}{p_{t+1}} = \gamma_{t+1} \tag{9}$$

and

$$R_t (1-\gamma_t) = \frac{y}{x}. \tag{10}$$

Substituting equations (9) and (10) into the expression for π^* in equation (6) yields

$$\pi^* = \frac{\gamma_{t+1}}{\gamma_{t+1} + \frac{y}{x}},$$

which we can substitute into (8) to obtain the difference equation

$$\gamma_t = 1 - \int_{\frac{\gamma_{t+1}}{\gamma_{t+1}+\frac{y}{x}}}^{1} F(\pi)\, d\pi. \tag{11}$$

We can now formulate the following proposition.

Proposition 1 When there is no lender of last resort, the economy has a unique equilibrium. This equilibrium is stationary with $\gamma_t = \gamma_a$ for all t, and $\max\left\{E(\pi); 1 - \frac{y}{x}\right\} < \gamma_a < 1$.

The proof of Proposition 1 is presented in Appendix A.

Proposition 1 is illustrated in Figure 1. The proposition states that our economy displays a unique, stationary equilibrium in the absence of a lender of last resort. Banks always hold the same fraction γ_a of their deposits in the form of cash reserves.

Why is the stationary trajectory the only equilibrium in this economy? Suppose that the initial price level is higher than $p_a = \frac{M}{\gamma_a d}$. In other words, in Figure 1, the economy starts on the part of the law of motion that lies to the left hand side of γ_a. Then the initial supply of real balances is lower than $\frac{M}{p_a}$. For this to be an equilibrium, the demand for real balances has to be lower than $\gamma_a d$ as well. Hence the return on holding cash has to be lower than $\frac{p_a}{p_a} = 1$, and the price level has to rise. Since the nominal supply of money is constant, this results in a further decrease in the real supply of money, which calls for a commensurate reduction in the demand for real balances and in the return on reserves. Therefore, along a trajectory that starts to the left of γ_a, the supply of real balances decreases continuously as the price level rises. However, in the presence of spatial separation, limited communication and random relocation, agents have a strong preference for real balances. Thus as the return on reserves decreases to zero, the demand for real balances falls to $E(\pi)d$, *not* to zero. For the demand for real balances to decrease below $E(\pi)d$ requires negative returns on real balances, which is not consistent with equilibrium. Hence hyper-inflationary trajectories in which the supply and demand for real balances drop to zero can not be sustained in this economy, and the stationary path is the only equilibrium.

Notice that the equilibrium of this reference economy is not efficient since there are states of the world for which the returns between relocated and non-relocated lenders is not equalized. We now want to examine whether the provision of lender of last resort services will render the economy more efficient.

4. An Unrestricted Lender of Last Resort

We first analyze the case of an economy in which the government is willing to make one-period loans of currency at a zero nominal interest rate in any quantity that banks desire. After the realization of π, a bank determines the real amount $b \geq 0$ that it would like to borrow at time t – which will depend on the realization of π – and the government simply prints bp_t dollars and gives them to the bank. Next period, the bank must then return these dollars to the government, and they are destroyed.

4.1 The Bank's Problem

With such borrowing, the bank's constraints become

$$\pi d r_t^m(\pi) = \alpha_t(\pi) z_t \frac{p_t}{p_{t+1}} + b_t(\pi) \frac{p_t}{p_{t+1}};$$

and

$$(1-\pi)\, d r_t(\pi) = [1-\alpha_t(\pi)]\, z_t \frac{p_t}{p_{t+1}} + (d - z_t) R_t - b_t(\pi)\frac{p_t}{p_{t+1}}.$$

Using our earlier notion for the reserve deposit ratio, $\gamma_t \equiv \frac{z_t}{d}$, and denoting by $\delta_t \equiv \frac{b_t}{d}$ the liquidity loan to deposit ratio, these constraints can be expressed as

$$\pi r_t^m(\pi) = \alpha_t(\pi)\gamma_t \frac{p_t}{p_{t+1}} + \delta_t(\pi)\frac{p_t}{p_{t+1}}, \tag{12}$$

and

$$(1-\pi) r_t(\pi) = [1-\alpha_t(\pi)]\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t - \delta_t(\pi)\frac{p_t}{p_{t+1}}. \tag{13}$$

After substituting (12) and (13) into the bank's objective function (5), and dropping the constant terms, we obtain

$$\max_{\alpha_t(\pi),\, \delta_t(\pi),\, \gamma_t} \int_0^1 \pi \ln\left\{\left[\frac{\alpha_t(\pi)\gamma_t}{\pi} + \frac{\delta_t(\pi)}{\pi}\right]\frac{p_t}{p_{t+1}}\right\} f(\pi)\, d\pi + \int_0^1 (1-\pi)\ln\left\{\frac{[1-\alpha_t(\pi)]\gamma_t}{(1-\pi)}\frac{p_t}{p_{t+1}} + \frac{(1-\gamma_t)}{(1-\pi)}R_t - \frac{\delta_t(\pi)}{(1-\pi)}\frac{p_t}{p_{t+1}}\right\} f(\pi)\, d\pi.$$

subject to

$$0 \le \gamma_t \le 1$$
$$0 \le \alpha_t(\pi) \le 1$$
$$\delta_t(\pi) \ge 0.$$

Both the fraction of bank reserves paid out to movers, α_t, and the real value of the liquidity loan, δ_t, are chosen after the realization of π, while the function γ_t, the fraction of reserves in the bank's asset portfolio, is chosen before the realization of π. Hence we can first solve for the optimal values of α_t and δ_t, while keeping γ_t and π fixed. That is, we can choose α_t and δ_t to solve

$$\max_{\alpha_t,\delta_t}\; \pi \ln\left\{\left[\frac{\alpha_t\gamma_t}{\pi} + \frac{\delta_t}{\pi}\right]\frac{p_t}{p_{t+1}}\right\} + (1-\pi)\ln\left\{\frac{(1-\alpha_t)\gamma_t}{(1-\pi)}\frac{p_t}{p_{t+1}} + \frac{(1-\gamma_t)}{(1-\pi)}R_t - \frac{\delta_t}{(1-\pi)}\frac{p_t}{p_{t+1}}\right\}$$

subject to

$$0 \le \alpha_t \le 1$$
$$\delta_t \ge 0.$$

The solution to this problem sets

$$\alpha_t(\pi) = \begin{cases} \pi\left(1 + \frac{1-\gamma_t}{\gamma_t} R_t \frac{p_{t+1}}{p_t}\right) \\ 1 \end{cases} \quad \text{and } \delta_t(\pi) = \begin{cases} 0 \\ \gamma_t\left[\pi\left(1 + \frac{1-\gamma_t}{\gamma_t} R_t \frac{p_{t+1}}{p_t}\right) - 1\right] \end{cases}$$

$$\text{for } \pi \in \begin{Bmatrix} [0, \pi^*) \\ [\pi^*, 1] \end{Bmatrix},$$

where π^* continues to be given by (6).

For realizations of the relocation shock below the critical value π^*, the bank pays out only a fraction of its reserves to movers. When the relocation shock is larger than π^*, the bank takes a liquidity loan from the lender of last resort, and pays out all its reserves, plus the liquidity it obtains from the loan, to movers. At the beginning of next period, non-movers are paid what is left from loan repayments after the bank has reimbursed the lender of last resort.

We now proceed to solve for the optimal value of γ_t. To do so, we substitute the optimal values of α_t and δ_t into the bank's objective function so that the only remaining variable to be determined is γ_t. We thus obtain the problem

$$\max_{0 \le \gamma_t \le 1} \int_0^{\pi^*} \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t\right] f(\pi)\, d\pi + \int_{\pi^*}^1 \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t\right] f(\pi)\, d\pi$$
$$= \int_0^1 \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t\right] f(\pi)\, d\pi.$$

Clearly, then, the introduction of an unrestricted lender of last resort allows the bank to offer complete insurance to lenders: Both movers and non-movers receive the average return. In order to maximize this average return, the optimal choice of reserve-deposit ratio γ must be given by

$$\gamma_t = \begin{Bmatrix} 0 \\ \in (0,1) \\ 1 \end{Bmatrix} \text{ if } R_t \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \frac{p_t}{p_{t+1}}.$$

4.2 Equilibrium

The market-clearing equations are the same as in the previous section, hence equations (9) and (10) continue to hold. Moreover, in equilibrium, we cannot have $\gamma_t = 0$, since then movers would have zero old-age consumption. We cannot have $\gamma_t = 1$ either, since then borrows would have zero young-period consumption. Hence, in equilibrium, the pricing relationship

$$R_t = \frac{p_t}{p_{t+1}} \tag{14}$$

must obtain. After substituting (14) into (9) and (10), the market clearing conditions simplify into the law of motion for γ;

$$\gamma_{t+1} = \frac{y}{x} \frac{\gamma_t}{1 - \gamma_t}. \tag{15}$$

We can now state the following proposition.

Proposition 2 When there is an unlimited lender of last resort, the economy displays a unique steady state equilibrium, $\gamma_b = 1 - \frac{y}{x} \in (0; \gamma_a)$. There is a stationary equilibrium path for $\gamma_0 = \gamma_b$, and a continuum of hyperinflationary equilibrium paths for $\gamma_0 \in (0; \gamma_b)$.

The proof of Proposition 2 is presented in Appendix B.

The results of Proposition 2 are illustrated in Figure 2. The proposition states that the economy with unlimited lender of last resort services displays a unique steady state, for which the reserve-deposit ratio is smaller than in the case without a lender of last resort. Moreover, there is continuum of hyperinflationary equilibria.

Why can hyperinflationary equilibria be sustained in this economy? Suppose again that the initial price level is higher than $p_b = \frac{M}{\gamma_b d}$. That is, in Figure 2, the economy starts on the part of the law of motion that lies to the left hand side of γ_b. This implies a trajectory along which the supply of real balances net of liquidity loans, $\gamma_t d = \frac{M}{p_t}$, decreases continuously as the price level rises, and falls to zero as the price level reaches infinity. However, now the bank takes out a liquidity loan, δ_t, every time its reserves, γ_t, fall short of its liquidity needs. In the limit, the expected real value of these liquidity loans is $\lim_{\gamma_t \to 0} E(\delta_t) = \lim_{\gamma_t \to 0} \int_{\pi^*}^{1} \gamma_t \left[\pi \left(1 + \frac{1-\gamma_t}{\gamma_t} R_t \frac{p_{t+1}}{p_t} \right) - 1 \right] f(\pi)\, d\pi = \int_0^1 \pi f(\pi)\, d\pi = E(\pi)$. Hence, as the return on cash balances falls to zero, the expected total supply of money goes to $E(\pi)\, d$, and this exactly equals the expected demand for real balances at zero return. Notice that in the limit, the real interest rate on liquidity loans is zero, while the market interest rate on loans to borrowers is $\frac{y}{x}$. Thus, in an economy with an unrestricted lender of last resort, hyperinflationary trajectories can be sustained in equilibrium. Indeed, in such an economy, an expectation of inflation leads to a decrease in the holdings of reserves, which is offset – when necessary – by the provision of a liquidity loan. This provision of liquidity by the lender of last resort, in turn, confirms the expectation of inflation. Consequently, hyperinflationary equilibria arise as a self-fulfilling prophecy.

From the preceding discussion, it follows that the provision of unlimited and costless lender of last resort services allows the economy to completely overcome the stochastic relocation friction. Note that in the steady state, p_t is constant even though the money supply is being changed every period as banks borrow from the government. The money supply is being moved to exactly match the stochastic movements in demand, leaving the value of money unchanged. In fact, the law of motion (15) is identical to the one that would obtain if there were no relocations in this economy. It is well known that the steady state is pareto optimal in this case, but that the hyper-inflationary equilibria are not[3].

Therefore, we can conclude that the introduction of an unlimited lender of last resort may allow the economy to attain a pareto optimal equilibrium. However, the presence of an unlimited lender of last resort also introduces an indeterminacy of equilibrium and the possibility hyper-inflationary equilibria which are not pareto efficient.

We now want to analyze whether the imposition of certain restrictions on the provision of lender of last resort services would allow to improve on the efficiency of the reference economy, while avoiding indeterminacies and hyper-inflationary equilibria. We first turn our attention to the case of a lender of last resort who charges interest on liquidity loans.

5. A Lender of Last Resort who Charges Interest

We now examine the case where the government continues to make one-period loans of currency in any quantity that banks desire. However, banks are now required to pay interest on liquidity loans. After the realization of π, a bank determines the real amount $b \geq 0$ that it would like to borrow at time t – which will depend on the realization of π – and the government prints bp_t dollars for the bank. Next period, the bank must return the dollars received in the lender of last resort operation. These dollars are destroyed such that the money supply remains unaffected. In addition, the bank must repay the lender of last resort br^d in goods, $r^d \geq 0$. Agents derive no utility from the revenue that the government earns from these interest payments.

[3] This follows from Proposition 5.6 in Balasko and Shell (1980). See also Champ, Smith and Williamson (1996, p. 838).

5.1 The Bank's Problem

Under this arrangement, the bank's constraints become

$$\pi r_t^m(\pi) = \alpha_t(\pi)\gamma_t \frac{p_t}{p_{t+1}} + \delta_t(\pi)\frac{p_t}{p_{t+1}}; \tag{16}$$

and

$$(1-\pi) r_t(\pi) = [1-\alpha_t(\pi)]\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t - \delta_t(\pi)\left(\frac{p_t}{p_{t+1}} + r^d\right): \tag{17}$$

Substituting (16) and (17) into the bank's objective function (5), and dropping the constant terms yields the problem

$$\max_{\alpha_t(\pi);\,\delta_t(\pi);\,\gamma_t} \int_0^1 \pi \ln\left\{\left(\frac{\alpha(\pi)\gamma_t}{\pi} + \frac{\delta_t(\pi)}{\pi}\right)\frac{p_t}{p_{t+1}}\right\} f(\pi)\,d\pi + \int_0^1 (1-\pi)\ln\left\{\frac{[1-\alpha(\pi)]\gamma_t}{(1-\pi)}\frac{p_t}{p_{t+1}} + \frac{(1-\gamma_t)}{(1-\pi)}R_t - \frac{\delta_t(\pi)}{(1-\pi)}\left(\frac{p_t}{p_{t+1}} + r^d\right)\right\} f(\pi)\,d\pi:$$

subject to

$$0 \le \gamma_t \le 1$$
$$0 \le \alpha_t(\pi) \le 1$$
$$\delta_t(\pi) \ge 0:$$

Again, given the timing of the bank's decisions, we can first solve for the optimal values of α_t and δ_t; while keeping γ_t and π fixed: That is, we can choose α_t and δ_t to solve

$$\max_{\alpha_t;b_t}; \ \pi \ln\left\{\left(\frac{\alpha_t\gamma_t}{\pi} + \frac{\delta_t}{\pi}\right)\frac{p_t}{p_{t+1}}\right\} + (1-\pi)\ln\left\{\frac{(1-\alpha_t)\gamma_t}{(1-\pi)}\frac{p_t}{p_{t+1}} + \frac{(1-\gamma_t)}{(1-\pi)}R_t - \frac{\delta_t}{(1-\pi)}\left(\frac{p_t}{p_{t+1}} + r^d\right)\right\}$$

subject to

$$0 \le \alpha_t \le 1$$
$$\delta_t \ge 0:$$

The solution to this problem sets

$$\alpha_t(\pi) = \begin{Bmatrix} \pi\left(1 + \frac{1-\gamma_t}{\gamma_t} R_t \frac{p_{t+1}}{p_t}\right) \\ 1 \\ 1 \end{Bmatrix} \text{ and } \delta_t(\pi) = \begin{Bmatrix} 0 \\ 0 \\ \gamma_t\left[\pi\left(1 + \frac{1-\gamma_t}{\gamma_t}\frac{R_t}{\frac{p_t}{p_{t+1}} + r^d}\right) - 1\right] \end{Bmatrix}$$

$$\text{for } \pi \in \begin{Bmatrix} [0, \pi^*) \\ [\pi^*, \pi^{**}] \\ (\pi^{**}, 1] \end{Bmatrix}.$$

where π^* continues to be given by (6), while

$$\pi^{**} = \frac{\gamma_t\left(\frac{p_t}{p_{t+1}} + r^d\right)}{\gamma_t\left(\frac{p_t}{p_{t+1}} + r^d\right) + (1-\gamma_t) R_t}. \tag{18}$$

Hence for realizations of the relocation shock below the critical value π^*, the bank pays out only a fraction of its reserves to movers. When a relocation shock $\pi \in [\pi^*, \pi^{**}]$ materializes, all reserves are paid out to movers, but the bank does not resort to a liquidity loan. Finally, when the relocation shock is larger than π^{**}; the bank pays out all its reserves to movers, and in addition, obtains a liquidity loan from the lender of last resort. For a given level of reserves γ, the return to the bank of holding money that it already has is $\frac{p_t}{p_{t+1}}$, while the cost of acquiring more money is $\left(\frac{p_t}{p_{t+1}} + r^d\right)$ per unit. This difference generates a kink in the bank's opportunity set, which in turn generates the range of inaction $[\pi^*, \pi^{**}]$.

We now proceed to solve for the optimal value of γ_t. To do so, we substitute the optimal values of α_t and δ_t into the bank's objective function so that the only remaining variable to be determined is γ_t. We thus obtain the problem

$$\max_{0 \le \gamma_t \le 1} \int_0^{\pi^*} \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t\right] f(\pi)\, d\pi + \int_{\pi^*}^{\pi^{**}} \left\{ \pi \ln\left[\frac{\gamma_t}{\pi}\frac{p_t}{p_{t+1}}\right] + (1-\pi)\ln\left[\frac{(1-\gamma_t)}{(1-\pi)} R_t\right] \right\} f(\pi)\, d\pi + \int_{\pi^{**}}^{1} \begin{Bmatrix} \pi \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)\frac{R_t}{\frac{p_t}{p_{t+1}} + r^d}\frac{p_t}{p_{t+1}}\right] + \\ (1-\pi)\ln\left[\gamma_t\left(\frac{p_t}{p_{t+1}} + r^d\right) + (1-\gamma_t) R_t\right] \end{Bmatrix} f(\pi)\, d\pi.$$

This formulation of the problem makes it clear that the return earned by both movers and non-movers will be the same when π is less than π^*; but will - in general - be different when π is greater than π^*. The first-order condition for this problem is

$$\frac{R_t - \frac{p_t}{p_{t+1}}}{\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t} F(\pi^*) + \frac{R_t - \left(\frac{p_t}{p_{t+1}} + r^d\right)}{\gamma_t \left(\frac{p_t}{p_{t+1}} + r^d\right) + (1-\gamma_t) R_t} [1 - F(\pi^{**})]$$
$$= \frac{1}{\gamma_t} \int_{\pi^*}^{\pi^{**}} \pi f(\pi)\, d\pi - \frac{1}{1-\gamma_t} \int_{\pi^*}^{\pi^{**}} (1-\pi) f(\pi)\, d\pi; \tag{19}$$

which can be solved for[4]

$$\gamma_t = \pi^{**} - \int_{\pi^*}^{\pi^{**}} F(\pi)\, d\pi. \tag{20}$$

This completes the solution to the bank's problem in the presence of a lender of last resort who charges interest. We now turn to an analysis of general equilibrium for this case.

5.2 Equilibrium

Again, the market-clearing equations are the same as in the benchmark case, hence equations (9) and (10) continue to hold. Substituting equations (9) and (10) into the expression for π^* in equation (6) and the expression for π^{**} in equation (18) yields

$$\pi^* = \frac{\gamma_{t+1}}{\gamma_{t+1} + \frac{y}{x}}; \tag{21}$$

$$\pi^{**} = \frac{\gamma_{t+1} + r^d \gamma_t}{\gamma_{t+1} + r^d \gamma_t + \frac{y}{x}}. \tag{22}$$

Using these solutions for π^* and π^{**} in equation (20), we obtain the law of motion for γ_t;

$$\gamma_t = \frac{\gamma_{t+1} + r^d \gamma_t}{\gamma_{t+1} + r^d \gamma_t + \frac{y}{x}} - \int_{\frac{\gamma_{t+1}}{\gamma_{t+1} + \frac{y}{x}}}^{\frac{\gamma_{t+1} + r^d \gamma_t}{\gamma_{t+1} + r^d \gamma_t + \frac{y}{x}}} F(\pi)\, d\pi. \tag{23}$$

We can now state the following proposition.

Proposition 3 When there is a lender of last resort who charges an interest rate $r^d \geq 0$ on liquidity loans, the economy displays a unique steady state equilibrium $\gamma_c \in [\gamma_b, \gamma_a]$. When $r^d \leq \frac{y}{x}$; there is a stationary equilibrium path for $\gamma_0 = \gamma_c$, and a continuum of hyper-inflationary equilibrium paths for $\gamma_0 \in (0, \gamma_c)$. When $r^d > \frac{y}{x}$; the economy displays a unique equilibrium, which is stationary with $\gamma_t = \gamma_c$ for all t.

[4] To facilitate the refereeing process, we provide the solution method in referee's Appendix B.

The proof of Proposition 3 is presented in Appendix C.

Proposition 3 is illustrated in Figure 3. The proposition states a very important result. When the interest rate on liquidity loans charged by the government is smaller than $r^{d*} = \frac{y}{x}$, our economy continues to display hyperinflationary equilibria. However, when the interest rate charged by the lender of last resort is greater than r^{d*}, the indeterminacy of equilibrium is eliminated, and our economy displays a unique, stationary equilibrium.

What is the intuition for this result? Why can hyperinflationary equilibria be sustained when the interest rate charged by the lender of last resort is below the critical rate r^{d*}? To see that, suppose the economy follows a hyperinflationary trajectory, starting with an initial price level higher than $p_c = \frac{M}{\gamma_c d}$. That is, in Figure 3, the economy starts on the part of the law of motion that lies to the left hand side of γ_c. Along such a trajectory, γ_t approaches zero, and the expected real value of the liquidity loan approaches $\lim_{\gamma_t \to 0} E(\delta_t)d = \lim_{\gamma_t \to 0} \int_{\pi^{**}}^{1} \gamma_t d\, \pi \left[\left(1 + \frac{1-\gamma_t}{\gamma_t} \frac{R_t}{\frac{p_t}{p_{t+1}} + r^d}\right) - 1\right] f(\pi)\, d\pi = \int_0^1 d \frac{y}{x r^d} \pi f(\pi)\, d\pi$. Clearly, when $r^d > \frac{y}{x} = r^{d*}$, the expected real value of liquidity loans is – in the limit – smaller than the expected demand for real balances at zero return, $E(\pi)d$. Hence, an excess demand for real balances exists even as their return drops to zero, and therefore hyperinflationary trajectories can not be sustained. On the other hand, when $r^d < \frac{y}{x} = r^{d*}$; the demand for real balances can always be satisfied by liquidity loans provided by the lender of last resort, and hyperinflationary equilibria may arise. Notice that in this case, as the return on cash balances falls to zero, all deposits in the economy are lent to borrowers at $R_t = \frac{y}{x}$ while the lender of last resort provides liquidity every period at $r^d < \frac{y}{x}$.

6. A Lender of Last Resort with an Upper Bound on Loanable Funds

We now consider the case of a lender of last resort who faces an upper bound $c \in (0, 1)$ on the real value of liquidity per unit deposited that it can lend to a bank in distress.

6.1 The Bank's Problem

For this case, the bank's constraints continue to be given by (12) and (13). Substituting these constraints into the bank's objective function (5), dropping the constant terms, and taking into account the upper bound on loanable funds yields

$$\max_{\alpha_t(\pi);\,\delta_t(\pi);\,\gamma_t} \int_0^1 \pi \ln\left\{\left[\frac{\alpha(\pi)\gamma_t}{\pi} + \frac{\delta_t(\pi)}{\pi}\right]\frac{p_t}{p_{t+1}}\right\} f(\pi)\,d\pi + \int_0^1 (1-\pi)\ln\left\{\frac{[1-\alpha(\pi)]\gamma_t}{(1-\pi)}\frac{p_t}{p_{t+1}} + \frac{(1-\gamma_t)}{(1-\pi)}R_t - \frac{\delta_t(\pi)}{(1-\pi)}\frac{p_t}{p_{t+1}}\right\} f(\pi)\,d\pi.$$

$$0 \le \gamma_t \le 1$$

$$0 \le \alpha_t(\pi) \le 1$$

$$0 \le \delta_t \le c.$$

Clearly, the upper bound on loanable funds can only be binding for some states if its value is smaller than the value of the loan a bank would take for $\pi = 1$ in the presence of an unlimited lender of last resort. Hence, the above program differs from the one in Section 4 only if $c < (1-\gamma_t)\frac{R_t}{\frac{p_t}{p_{t+1}}} \equiv \hat{c}.$ We will henceforth assume that this condition is satisfied.

As before, we can first solve for the optimal values of α_t and δ_t, while keeping γ_t and π fixed. So we can choose α_t and δ_t to solve

$$\max_{\alpha_t,\delta_t}\ \pi \ln\left\{\left[\frac{\alpha_t\gamma_t}{\pi} + \frac{\delta_t}{\pi}\right]\frac{p_t}{p_{t+1}}\right\} + (1-\pi)\ln\left\{\frac{(1-\alpha_t)\gamma_t}{(1-\pi)}\frac{p_t}{p_{t+1}} + \frac{(1-\gamma_t)}{(1-\pi)}R_t - \frac{\delta_t}{(1-\pi)}\frac{p_t}{p_{t+1}}\right\}$$

subject to

$$0 \le \alpha_t \le 1$$

$$0 \le \delta_t \le c.$$

The solution to this problem sets

$$\alpha_t(\pi) = \begin{Bmatrix} \pi\left(1 + \frac{1-\gamma_t}{\gamma_t}R_t\frac{p_{t+1}}{p_t}\right) \\ 1 \\ 1 \end{Bmatrix} \text{ and } \delta_t(\pi) = \begin{Bmatrix} 0 \\ \gamma_t\left[\pi\left(1 + \frac{1-\gamma_t}{\gamma_t}R_t\frac{p_{t+1}}{p_t}\right) - 1\right] \\ c \end{Bmatrix}$$

$$\text{for } \pi \in \begin{Bmatrix} [0;\pi^*) \\ [\pi^*;\pi^{**}] \\ (\pi^{**};1] \end{Bmatrix};$$

where π^* continues to be given by (6), while

$$\pi^{**} = \frac{(\gamma_t + c)\frac{p_t}{p_{t+1}}}{\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)R_t} \tag{24}$$

Hence for realizations of the relocation shock below the critical value π^*, the bank pays out only a fraction of its reserves to movers. When a relocation shock $\pi \in [\pi^*; \pi^{**}]$ materializes, all reserves are paid out to movers, and the bank obtains a liquidity loan $b(\pi) < cd$. Finally, when the relocation shock is larger than π^{**}; the bank pays out all its reserves to movers, and in addition, takes the maximum loan cd it can get from the lender of last resort. Notice that $\pi^{**} = 1$ when $c = \bar{c}$; which confirms that the program laid out above is only correct for $c < \bar{c}$.

We can now determine the bank's optimal portfolio in the presence of a constrained lender of last resort. To do so, we substitute the optimal values of α_t and b_t into the bank's objective function. This yields the problem

$$\begin{aligned}
\max_{0 \le \gamma_t \le 1} \quad & \int_0^{\pi^*} \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)R_t\right] f(\pi)\,d\pi + \int_{\pi^*}^{\pi^{**}} \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)R_t\right] f(\pi)\,d\pi + \\
& \int_{\pi^{**}}^{1} \left\{ \pi \ln\left[\left(\frac{\gamma_t}{\pi} + \frac{c}{\pi}\right)\frac{p_t}{p_{t+1}}\right] + (1-\pi)\ln\left[\frac{(1-\gamma_t)}{1-\pi}R_t - \frac{c}{(1-\pi)}\frac{p_t}{p_{t+1}}\right]\right\} f(\pi)\,d\pi \\
= \quad & \int_0^{\pi^{**}} \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)R_t\right] f(\pi)\,d\pi + \\
& \int_{\pi^{**}}^{1} \left\{ \pi \ln\left[\left(\frac{\gamma_t}{\pi} + \frac{c}{\pi}\right)\frac{p_t}{p_{t+1}}\right] + (1-\pi)\ln\left[\frac{(1-\gamma_t)}{1-\pi}R_t - \frac{c}{(1-\pi)}\frac{p_t}{p_{t+1}}\right]\right\} f(\pi)\,d\pi.
\end{aligned}$$

Hence the return earned by both movers and non-movers will be the same when π is less than π^{**}; but will - in general - be different when π is greater than π^{**}. The first-order condition for this problem is

$$\frac{\frac{p_t}{p_{t+1}} - R_t}{\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)R_t} \int_0^{\pi^{**}} f(\pi)d\pi + \frac{1}{\gamma_t + c}\int_{\pi^{**}}^{1} \pi f(\pi)d\pi = \frac{R_t}{(1-\gamma_t)R_t - c\frac{p_t}{p_{t+1}}}\int_{\pi^{**}}^{1} (1-\pi) f(\pi)d\pi. \tag{25}$$

which can be solved for[5]

$$\gamma_t = 1 - \frac{1 - \pi^{**} + c}{1 - \pi^{**}} \int_{\pi^{**}}^{1} F(\pi)d\pi. \tag{26}$$

[5] To facilitate the refereeing process, we provide the solution method in referee's Appendix C.

We are now ready to state the general equilibrium conditions for this case.

6.2 Equilibrium

The market-clearing conditions (9) and (10) continue to hold. Substituting these equations into the expression for π^{**} in equation (24) yields

$$\pi^{**} = \frac{\gamma_{t+1} + \frac{\gamma_{t+1}}{\gamma_t} c}{\gamma_{t+1} + \frac{y}{x}}.$$

Using the solution for π^{**} in equation (26), we obtain the law of motion for γ_t when $c < \bar{c}$,

$$\gamma_t = 1 - \frac{\frac{y}{x} - c\left(\frac{\gamma_{t+1}}{\gamma_t} - \gamma_{t+1} - \frac{y}{x}\right)}{\frac{y}{x} - c\frac{\gamma_{t+1}}{\gamma_t}} \int_{\frac{\gamma_{t+1} + \frac{\gamma_{t+1}}{\gamma_t} c}{\gamma_{t+1} + \frac{y}{x}}}^{1} F(\pi)\, d\pi. \tag{27}$$

For $c > \bar{c}$, the law of motion of this economy is given by (15). We are now ready to state the following proposition.

Proposition 4 When there is a lender of last resort with an upper bound on loanable funds, $c \geq 0$, the economy displays a unique steady state equilibrium $\gamma_d \in [\gamma_b, \gamma_a]$. When $c < \frac{y}{x}$, the economy displays a unique equilibrium, which is stationary with $\gamma_t = \gamma_d$ for all t. When $c \geq \frac{y}{x}$ there is a stationary equilibrium path for $\gamma_0 = \gamma_d = \gamma_b$, and a continuum of hyper-inflationary equilibrium paths for $\gamma_0 \in (0, \gamma_d)$.

The proof of Proposition 4 is presented in Appendix D.

For the case with $c < \frac{y}{x}$, Proposition 4 is illustrated in Figure 5. When $c \geq \frac{y}{x}$, the economy is exactly like the economy with an unrestricted lender of last resort which is depicted in Figure 2.

The proposition states a very important result. When the upper bound placed on liquidity loans is smaller than $c^* = \frac{y}{x}$, the economy displays a unique, stationary equilibrium. However, when that upper bound is larger than c^*, hyperinflationary equilibria can arise, and the economy displays an indeterminacy of equilibrium.

(Need to add intuition for this result.)

7. Conclusions

We have studied a simple endowment economy in which spatial separation, limited communication and random relocation create a role for money which may be dominated in rate of return. Banks arise endogenously in this world to insure agents against the liquidity shocks implied by random relocation. This benchmark economy displays a unique stationary equilibrium which is not efficient.

When we introduce a lender of last resort into this economy providing costless and unlimited liquidity loans to banks in distress, the stationary equilibrium is pareto optimal. However, the presence of an unrestricted lender of last resort is also associated with the existence of a continuum of hyperinflationary equilibria that are not pareto efficient. Thus, while allowing the economy to achieve a pareto optimal stationary equilibrium, the introduction of an unrestricted lender of last resort also makes the economy vulnerable to currency instability.

We then show that these hyperinflationary equilibria disappear when the lender of last resort charges an interest rate on liquidity loans which is sufficiently high or when the central bank credibly commits to a cap on liquidity loans which is sufficiently low.

In our analysis we have abstracted from several issues that figure prominently in contemporary discussions of the desirability and optimal design of lender of last resort services. First, we set up the model in such a way that the provision of liquidity loans does not affect the government's intertemporal budget constraint. Yet the fiscal cost of bank bailouts is a primary concern in the design of lender of last resort arrangements. Second, it is often argued that the explicit or implicit access to liquidity loans provides banks with an incentive to take on "excessive" risk in its asset portfolio. Clearly, our model is not equipped to study this issue. Third, in many emerging economies countries, and certainly in those that are contemplating to adopt the U.S. dollar as legal tender, a large fraction of banks' liabilities and assets is denominated in foreign currency. Hence, the provision of lender of last resort services, because of its effect on the money supply and thus on the exchange rate, may affect the real value of that part of the portfolio which is denominated in foreign exchange. This should be taken into account when designing lender of last resort arrangements. We plan to address all these important questions in future research.

APPENDIX A: PROOF OF PROPOSITION 1

We cannot solve (11) explicitly for γ_{t+1} as a function of γ_t; but we can derive some properties of the implicit function. First note that when $\gamma_{t+1} = 0$, which is its minimum value, γ_t is given by $\gamma_t = 1 - \int_0^1 F(\pi)\,d\pi = \int_0^1 \pi f(\pi)\,d\pi = E(\pi)$. Thus the implicit function is not defined for values of γ_t below the expected value of π. Next, when $\gamma_{t+1} = 1$, which is its maximum value, γ_t is strictly below one. Since the implicit function is continuous, there exists at least one steady state for γ. Moreover, at a steady state, (11) implies

$$\gamma = 1 - \int_{\frac{\gamma}{\gamma + \frac{y}{\bar{x}}}}^{1} F(\pi)\,d\pi.$$

We know $F < 1$ always holds; so any steady state, γ, must satisfy

$$\gamma > 1 - \int_{\frac{\gamma}{\gamma + \frac{y}{\bar{x}}}}^{1} d\pi = \frac{\gamma}{\gamma + \frac{y}{\bar{x}}}.$$

Hence for any steady state

$$\gamma > 1 - \frac{y}{\bar{x}} \tag{28}$$

must hold.

For the slope of (11), we can use Leibnitz' integral rule to obtain

$$\frac{d\gamma_t}{d\gamma_{t+1}} = F\left(\frac{\gamma_{t+1}}{\gamma_{t+1} + \frac{y}{\bar{x}}}\right) \frac{\frac{y}{\bar{x}}}{\left(\gamma_{t+1} + \frac{y}{\bar{x}}\right)^2} > 0.$$

Thus γ_t as a function of γ_{t+1} is always increasing and hence is invertible. The inverse function is the law of motion for γ_t, and it is also strictly increasing. By the inverse function rule we obtain the slope of the law of motion,

$$\frac{d\gamma_{t+1}}{d\gamma_t} = \frac{\left(\gamma_{t+1} + \frac{y}{\bar{x}}\right)^2}{\frac{y}{\bar{x}}} \frac{1}{F\left(\frac{\gamma_{t+1}}{\gamma_{t+1} + \frac{y}{\bar{x}}}\right)} > 0. \tag{29}$$

We can now evaluate the derivative of the law of motion, (29), at any steady state. Given (28) and taking into account that the economy is a Samuelson case economy, which implies that $y < \bar{x}$, the first term of (29) is greater than one for any steady state. The second term is always greater than or equal to one, so for any steady state the derivative itself must be greater than one. The law of motion for γ_t must therefore cross the 45° line from below at every steady state. This implies that there is exactly one steady state, which we will denote by γ_a and that this steady state is in the open interval $(E(\pi), 1)$. The steady state is globally unstable, and all nonstationary trajectories eventually leave the feasible region. Hence the steady state is the only equilibrium of this economy. ■

APPENDIX B: PROOF OF PROPOSITION 2

The law of motion (15) starts at the origin, and is increasing and convex. The slope of (15) at its origin is $\frac{y}{x} < 1$; and hence there is a unique steady state, the value of which is given by $\gamma_b = 1 - \frac{y}{x} > 0$. From Proposition 1, we know that $1 - \frac{y}{x} < \gamma_a$, therefore $\gamma_b < \gamma_a$. The slope of (15) evaluated at the steady state is $\frac{x}{y} > 1$. Hence the unique steady state is unstable. Each $\gamma_0 > \gamma_b$ generates a trajectory that leaves the feasible region and hence cannot be an equilibrium. Each $\gamma_0 \in (0, \gamma_b]$ is associated with a trajectory that remains within the feasible region, and hence there is a continuum of equilibrium paths. For $\gamma_0 = \gamma_b$, the equilibrium path is stationary. For $\gamma_0 \in (0, \gamma_b)$, $\lim_{t\to\infty} \gamma_t = \lim_{t\to\infty} z_t = 0$; hence $\lim_{t\to\infty} p_t = \infty$ and the equilibrium trajectories display hyperinflation. ■

APPENDIX C: PROOF OF PROPOSITION 3

We cannot solve (23) explicitly for γ_{t+1} as a function of γ_t; but we can nevertheless derive some properties of the law of motion for γ_t. First note that when $\gamma_{t+1} = 1$, γ_t is strictly below one. Moreover, when $\gamma_{t+1} = 0$, (23) implies

$$\int_0^{\frac{r^d \gamma_t}{r^d \gamma_t + \frac{y}{x}}} F(\pi)\, d\pi = \gamma_t \left(\frac{r^d}{r^d \gamma_t + \frac{y}{x}} - 1 \right). \tag{30}$$

Clearly, $\gamma_t = 0$ satisfies (30), so the law of motion starts at the origin. Since (23) is continuous, we can conclude that there exists at least one steady state.

However, $\gamma_t = 0$ may not be the only value for which $\gamma_{t+1} = 0$. Indeed, when $r^d > \frac{y}{x}$ then there will exists at least one other $\gamma_t \in (0, 1)$ for which $\gamma_{t+1} = 0$.

Further, at any steady state, γ, (23) simplifies to

$$\gamma - \frac{\left(1 + r^d\right)\gamma}{(1 + r^d)\gamma + \frac{y}{x}} = -\int_{\frac{\gamma}{\gamma + \frac{y}{x}}}^{\frac{(1+r^d)\gamma}{(1+r^d)\gamma + \frac{y}{x}}} F(\pi)\, d\pi.$$

Notice that this implies that for $r^d = 0; \gamma = 1 - \frac{y}{x} = \gamma_b$: Moreover, we know $F < 1$ always holds: Thus, for $r^d > 0$, the steady state level of γ must satisfy

$$\gamma - \frac{\left(1+r^d\right)\gamma}{(1+r^d)\gamma + \frac{y}{x}} > - \int_{\frac{\gamma}{\gamma+\frac{y}{x}}}^{\frac{(1+r^d)\gamma}{(1+r^d)\gamma+\frac{y}{x}}} d\pi = \frac{\gamma}{\gamma + \frac{y}{x}} - \frac{\left(1+r^d\right)\gamma}{(1+r^d)\gamma + \frac{y}{x}}:$$

Hence, at any steady state, (28) is satisfied.

Next, it can be shown that

$$\frac{d\gamma_{t+1}}{d\gamma_t} = \frac{1 - \frac{r^d \frac{y}{x}}{\left(\gamma_{t+1}+r^d\gamma_t+\frac{y}{x}\right)^2}\left[1 - F\left(\frac{\gamma_{t+1}+r^d\gamma_t}{\gamma_{t+1}+r^d\gamma_t+\frac{y}{x}}\right)\right]}{\frac{\frac{y}{x}}{\left(\gamma_{t+1}+r^d\gamma_t+\frac{y}{x}\right)^2}\left[1 - F\left(\frac{\gamma_{t+1}+r^d\gamma_t}{\gamma_{t+1}+r^d\gamma_t+\frac{y}{x}}\right)\right] + \frac{\frac{y}{x}}{\left(\gamma_{t+1}+\frac{y}{x}\right)^2} F\left(\frac{\gamma_{t+1}}{\gamma_{t+1}+\frac{y}{x}}\right)}: \tag{31}$$

Clearly, the law of motion is increasing everywhere if $r^d \leq \frac{y}{x}$: However, when $r^d > \frac{y}{x}$; that is no longer the case.

Indeed, for $\gamma_{t+1} = \gamma_t = 0$, we obtain

$$\frac{d\gamma_{t+1}}{d\gamma_t}\Big|_{\gamma_{t+1}=\gamma_t=0} = \frac{y}{x} - r^d:$$

Hence

$$\frac{d\gamma_{t+1}}{d\gamma_t}\Big|_{\gamma_{t+1}=\gamma_t=0} \begin{cases} \in \left[0; \frac{y}{x} < 1\right] \\ < 0 \end{cases} \text{if } r^d \begin{cases} \leq \\ > \end{cases} \frac{y}{x}:$$

Moreover, using (31), (21) and (22), it is clear that at any steady state $\gamma > 0$, $\frac{d\gamma_{t+1}}{d\gamma_t}\big|_{\gamma_{t+1}=\gamma_t=\gamma} > 1$ iff

$$\frac{\left(1+r^d\right)\frac{y}{x}}{\left[(1+r^d)\gamma + \frac{y}{x}\right]^2}\left[1 - F(\pi^{**})\right] + \frac{\frac{y}{x}}{\left(\gamma + \frac{y}{x}\right)^2} F(\pi^*) < 1 \tag{32}$$

holds. Given that (28) holds at any steady state, (32) is equivalent to

$$\frac{\left(1+r^d\right)\gamma}{\left[(1+r^d)\gamma + \frac{y}{x}\right]}\left[1 - F(\pi^{**})\right] + \left(\frac{\gamma}{\gamma + \frac{y}{x}}\right) F(\pi^*) < \frac{\gamma}{\frac{y}{x}};$$

which, using (21) and (22), can be expressed as

$$\pi^{**}\left[1 - F(\pi^{**})\right] + \pi^* F(\pi^*) < \frac{\gamma}{\frac{y}{x}}:$$

But from Referee's Appendix B, equation (38), we have

$$\pi^{**}[1 - F(\pi^{**})] + \pi^{*}F(\pi^{*}) = \gamma - \int_{\pi^{*}}^{\pi^{**}} \pi f(\pi)\, d\pi < \gamma < \frac{y}{x}.$$

Hence $\frac{d\gamma_{t+1}}{d\gamma_t}\big|_{\gamma_{t+1}=\gamma_t=\gamma} > 1$ always holds, and the law of motion must cross the 45° line from below at every steady state. This implies that there is exactly one steady state, which we will denote by γ_c. From (28) and Proposition 2, it is clear that $\gamma_c > \gamma_b$ for $r^d > 0$, while $\gamma_c = \gamma_b$ for $r^d = 0$

Moreover, $\gamma_c < \gamma_a$ also holds. Indeed, it can be shown that

$$\frac{d\gamma_c}{dr^d} = \frac{\gamma_c \frac{y}{x}\left[1 - F\left(\frac{(1+r^d)\gamma}{(1+r^d)\gamma + \frac{y}{x}}\right)\right]}{\left[1 - F\left(\frac{\gamma}{\gamma+\frac{y}{x}}\right)\frac{\frac{y}{x}}{\left(\gamma+\frac{y}{x}\right)^2}\right]\left[(1+r^d)\gamma + \frac{y}{x}\right]^2 + (1+r^d)\frac{y}{x}\left[1 - F\left(\frac{(1+r^d)\gamma}{(1+r^d)\gamma + \frac{y}{x}}\right)\right]}.$$

Clearly, given (28), $\frac{d\gamma_c}{dr^d} > 0$. Moreover, for $r^d \to \infty$, which is the case without a lender of last resort, $\frac{\gamma_{t+1}+r^d\gamma_t}{\gamma_{t+1}+r^d\gamma_t+\frac{y}{x}} \to 1$; and hence (23) collapses to (11). Hence for $r^d \to \infty$; $\gamma_c \to \gamma_a$ holds, while $\gamma_c < \gamma_a$ for $r^d < \infty$.

Given that the law of motion crosses the 45° line from below at any steady state, γ_c is globally unstable. When $r^d \le \frac{y}{x}$, each $\gamma_0 \in (0, \gamma_c]$ is associated with a trajectory that remains within the feasible region. Hence there is a continuum of equilibrium paths. For $\gamma_0 = \gamma_c$, the equilibrium path is stationary. For $\gamma_0 \in (0, \gamma_c)$, $\lim_{t\to\infty} \gamma_t = \lim_{t\to\infty} m_t = 0$; hence $\lim_{t\to\infty} p_t = \infty$; and the equilibrium trajectories display hyperinflation. On the other hand, when $r^d > \frac{y}{x}$, all nonstationary trajectories eventually leave the feasible region. Hence the steady state is the only equilibrium. ■

APPENDIX D: PROOF OF PROPOSITION 4

Again, it is impossible to solve (42) explicitly for γ_{t+1} as a function of γ_t. However, we can derive certain properties of the law of motion for γ_t. First note that in equilibrium, $\tilde{c} = \frac{y}{x}\frac{\gamma_t}{\gamma_{t+1}}$ holds. Hence for a steady state equilibrium to exists, it has to be the case that $c < \frac{y}{x}$. [For $c > \frac{y}{x}$, the case of the lender of last resort with an upper bound on loanable funds reduces to the case of the unlimited lender of last resort.]

Next, when $\gamma_{t+1} = 0$, which is its minimum value, γ_t is given by $\gamma_t = 1 - [1 + c]\int_0^1 F(\pi)d\pi = E(\pi) - c[1 - E(\pi)]$. Thus the implicit function is not defined for values of $\gamma_t < E(\pi) - c[1 - E(\pi)]$. For the uniform distribution, $E(\pi) - c[1 - E(\pi)] = 0.5(1 - c) > 0$. [Moreover, it is easy to verify that γ_t is strictly below one when $\gamma_{t+1} = 1$]. Since the implicit function is continuous, there exists at least one steady state for γ.

Further, at a steady state, (42) simplifies to

$$\gamma = 1 - \frac{\frac{y}{x}[1+c]\int_{\frac{\gamma+c}{\gamma+\frac{y}{x}}}^{1} F(\pi)d\pi}{\frac{y}{x} - c\left[1 - \int_{\frac{\gamma+c}{\gamma+\frac{y}{x}}}^{1} F(\pi)d\pi\right]}. \qquad (33)$$

Notice that (33) implies that $\gamma \to \gamma_a$ for $c \to 0$. Also, it is clear from (33) that

$$\lim_{c \to \frac{y}{x}} \gamma = \frac{\frac{y}{x}\left[1+\frac{y}{x}\right]}{\gamma + 2\frac{y}{x}},$$

which is solved by $\gamma = 1 - \frac{y}{x} = \gamma_b$. Hence at any steady state, $\gamma \in (\gamma_b, \gamma_a)$ must hold.

In addition, note that the law of motion (42) can be written as

$$\gamma_t = 1 - \frac{1-\pi^{**}+c}{1-\pi^{**}}\int_{\pi^{**}}^{1} F(\pi)d\pi. \qquad (34)$$

Differentiating (34) with respect to γ_t yields

$$\frac{\partial \pi^{**}}{\partial \gamma_t} = \frac{1-\pi^{**}}{(1-\pi^{**}+c)F(\pi^{**}) - \frac{c}{(1-\pi^{**})}\int_{\pi^{**}}^{1} F(\pi)d\pi}.$$

To determine the sign of $\frac{\partial \pi^{**}}{\partial \gamma_t}$, note that $\int_{\pi^{**}}^{1} F(\pi)d\pi < 1 - \pi^{**}$ since $F(\pi) < 1$. Hence we have $(1-\pi^{**}+c)F(\pi^{**}) - \frac{c}{(1-\pi^{**})}\int_{\pi^{**}}^{1} F(\pi)d\pi > (1-\pi^{**}+c)F(\pi^{**}) - c$. Moreover, for the case of a uniform distribution, $F(\pi^{**}) = \pi^{**}$. Hence $(1-\pi^{**}+c)F(\pi^{**}) - \frac{c}{(1-\pi^{**})}\int_{\pi^{**}}^{1} F(\pi)d\pi > (1-\pi^{**})(\pi^{**} - c)$. In addition, it follows from (24) that $\pi^{**} = \frac{\gamma+c}{\gamma+\frac{y}{x}}$ in any steady state. Hence, for $c < \frac{y}{x} < 1$, it is the case that $(\pi^{**} - c) > 0$ in steady state. Therefore, $(1-\pi^{**}+c)F(\pi^{**}) - \frac{c}{(1-\pi^{**})}\int_{\pi^{**}}^{1} F(\pi)d\pi > (1-\pi^{**})(\pi^{**} - c) > 0$ and $\frac{\partial \pi^{**}}{\partial \gamma_t} > 0$ as well.

Since F is strictly increasing, clearly $\int_{\pi^{**}}^{1} F(\pi)d\pi > F(\pi^{**})(1-\pi^{**})$ holds. Therefore, given that $\frac{\partial\pi^{**}}{\partial\gamma_t} > 0$ for a uniform distribution, it is also the case that

$$\frac{\partial\pi^{**}}{\partial\gamma_t} > \frac{1-\pi^{**}}{(1-\pi^{**}+c)F(\pi^{**}) - \frac{c}{(1-\pi^{**})}F(\pi^{**})(1-\pi^{**})} = \frac{1}{\pi^{**}} > 1$$

holds. Moreover, differentiating (24) with respect to γ_t, yields

$$\frac{\partial\pi^{**}}{\partial\gamma_t} = \frac{-\frac{c}{\gamma_t^2}\left(1+\frac{\beta y}{x\gamma_{t+1}}\right) + \frac{\beta y}{x\gamma_{t+1}^2}\left[1+\frac{c}{\gamma_t}\right]\frac{\partial\gamma_{t+1}}{\partial\gamma_t}}{\left(1+\frac{\beta y}{x\gamma_{t+1}}\right)^2}; \qquad (35)$$

which establishes a relationship between $\frac{\partial\gamma_{t+1}}{\partial\gamma_t}$ and $\frac{\partial\pi^{**}}{\partial\gamma_t}$. Indeed, evaluating (35) at any steady state, we obtain

$$\frac{\partial\pi^{**}}{\partial\gamma_t} = \frac{-c\left(\gamma+\frac{\beta y}{x}\right)+\frac{\beta y}{x}(\gamma+c)\frac{\partial\gamma_{t+1}}{\partial\gamma_t}}{\gamma\left(\gamma+\frac{\beta y}{x}\right)^2}.$$

Since $\frac{\partial\pi^{**}}{\partial\gamma_t} > 1$, this implies $\frac{\partial\gamma_{t+1}}{\partial\gamma_t} > \frac{(\gamma+\frac{\beta y}{x})}{\frac{\beta y}{x}}\left[\frac{\gamma(\gamma+\frac{\beta y}{x})+c}{\gamma+c}\right]$. Given that $\gamma \geq 1-\frac{\beta y}{x}$ or $\gamma+\frac{\beta y}{x} \geq 1$ at any steady state, we know that $\frac{\partial\gamma_{t+1}}{\partial\gamma_t} > \frac{1}{\frac{\beta y}{x}}\left[\frac{\gamma+c}{\gamma+c}\right] = \frac{x}{\beta y} > 1$. For a uniform distribution, the law of motion for γ_t must therefore cross the 45° line from below at every steady state. This implies that there is exactly one steady state, which we will denote by γ_d and that this steady state is in the open interval (γ_b, γ_a). The steady state is globally unstable, and all nonstationary trajectories eventually leave the feasible region. Hence the steady state is the only equilibrium in this economy. ■

REFEREE’S APPENDIX A: DERIVATION OF EQUATION ((8))

Using (6), the first-order condition (7) can be written as

$$\pi^* F(\pi^*) + \int_{\pi^*}^{1} \pi f(\pi)\, d\pi = \frac{p_{t+1}}{p_t} R_t \pi^* F(\pi^*) + \frac{\gamma_t}{1-\gamma_t} \int_{\pi^*}^{1} (1-\pi) f(\pi)\, d\pi,$$

or

$$\pi^* F(\pi^*) + \int_{\pi^*}^{1} \pi f(\pi)\, d\pi = \frac{p_{t+1}}{p_t} R_t \pi^* F(\pi^*) + \frac{\gamma_t}{1-\gamma_t} \left\{ [1-F(\pi^*)] - \int_{\pi^*}^{1} \pi f(\pi)\, d\pi \right\}.$$

If we add $\frac{\gamma_t}{1-\gamma_t}\pi^* F(\pi^*)$ to both sides, we have

$$\pi^* F(\pi^*) + \frac{\gamma_t}{1-\gamma_t}\pi^* F(\pi^*) + \int_{\pi^*}^{1} \pi f(\pi)\, d\pi =$$
$$\left(\frac{p_{t+1}}{p_t} R_t + \frac{\gamma_t}{1-\gamma_t} \right) \pi^* F(\pi^*) + \frac{\gamma_t}{1-\gamma_t} \left\{ [1-F(\pi^*)] - \int_{\pi^*}^{1} \pi f(\pi)\, d\pi \right\};$$

which reduces to

$$\pi^* F(\pi^*) + \int_{\pi^*}^{1} \pi f(\pi)\, d\pi = \left[(1-\gamma_t) \frac{p_{t+1}}{p_t} R_t + \gamma_t \right] \pi^* F(\pi^*) + \gamma_t - \gamma_t F(\pi^*).$$

Making use of (6) again, we obtain

$$\pi^* F(\pi^*) + \int_{\pi^*}^{1} \pi f(\pi)\, d\pi = \gamma_t F(\pi^*) + \gamma_t - \gamma_t F(\pi^*) = \gamma_t \tag{36}$$

This can be written in another form by noting that

$$\frac{d}{dx}[xF(x)] = F(x) + x f(x), \tag{37}$$

which allows us to write

$$\begin{aligned} \int_{\pi^*}^{1} \pi f(\pi)\, d\pi &= \int_{\pi^*}^{1} \left\{ \frac{d}{d\pi}[\pi F(\pi)] - F(\pi) \right\} d\pi \\ &= \pi F(\pi)\big|_{\pi^*}^{1} - \int_{\pi^*}^{1} F(\pi)\, d\pi \\ &= 1 - \pi^* F(\pi^*) - \int_{\pi^*}^{1} F(\pi)\, d\pi. \end{aligned}$$

This demonstrates that

$$1-\int_{\pi^*}^{1} F(\pi)\,d\pi = \pi^* F(\pi^*) + \int_{\pi^*}^{1} \pi f(\pi)\,d\pi.$$

Substituting this into equation (36) yields the solution presented in equation (8).

REFEREE'S APPENDIX B: DERIVATION OF EQUATION (20)

Using (6) and (18) in the first-order condition (19), we obtain

$$\pi^*F(\pi^*) + \pi^{**}[1-F(\pi^{**})] + \int_{\pi^*}^{\pi^{**}} \pi f(\pi)\,d\pi =$$
$$\frac{p_{t+1}}{p_t}R_t\pi^*F(\pi^*) + \frac{R_t}{\frac{p_t}{p_{t+1}}+r^d}\pi^{**}[1-F(\pi^{**})] + \frac{\gamma_t}{1-\gamma_t}\int_{\pi^*}^{\pi^{**}}(1-\pi)f(\pi)\,d\pi;$$

or

$$\pi^*F(\pi^*) + \pi^{**}[1-F(\pi^{**})] + \int_{\pi^*}^{\pi^{**}} \pi f(\pi)\,d\pi =$$
$$\frac{p_{t+1}}{p_t}R_t\pi^*F(\pi^*) + \frac{R_t}{\frac{p_t}{p_{t+1}}+r^d}\pi^{**}[1-F(\pi^{**})] + \frac{\gamma_t}{1-\gamma_t}\left\{[F(\pi^{**})-F(\pi^*)] - \int_{\pi^*}^{\pi^{**}}\pi f(\pi)\,d\pi\right\}.$$

If we add $\frac{\gamma_t}{1-\gamma_t}\pi^*F(\pi^*) + \frac{\gamma_t}{1-\gamma_t}\pi^{**}[1-F(\pi^{**})]$ to both sides, we have

$$\pi^*F(\pi^*) + \pi^{**}[1-F(\pi^{**})] + \int_{\pi^*}^{\pi^{**}} \pi f(\pi)\,d\pi + \frac{\gamma_t}{1-\gamma_t}\pi^*F(\pi^*) + \frac{\gamma_t}{1-\gamma_t}\pi^{**}[1-F(\pi^{**})] =$$
$$\left(\frac{p_{t+1}}{p_t}R_t + \frac{\gamma_t}{1-\gamma_t}\right)\pi^*F(\pi^*) + \left(\frac{R_t}{\frac{p_t}{p_{t+1}}+r^d} + \frac{\gamma_t}{1-\gamma_t}\right)\pi^{**}[1-F(\pi^{**})] +$$
$$\frac{\gamma_t}{1-\gamma_t}\left\{[F(\pi^{**})-F(\pi^*)] - \int_{\pi^*}^{\pi^{**}}\pi f(\pi)\,d\pi\right\}.$$

This reduces to

$$\pi^*F(\pi^*) + \pi^{**}[1-F(\pi^{**})] + \int_{\pi^*}^{\pi^{**}} \pi f(\pi)\,d\pi =$$
$$\left[(1-\gamma_t)\frac{p_{t+1}}{p_t}R_t + \gamma_t\right]\pi^*F(\pi^*) + \left[(1-\gamma_t)\frac{R_t}{\frac{p_t}{p_{t+1}}+r^d} + \gamma_t\right]\pi^{**}[1-F(\pi^{**})] + \gamma_t[F(\pi^{**})-F(\pi^*)]$$

Making use of (6) and (18) again, we obtain

$$\pi^*F(\pi^*) + \pi^{**}[1-F(\pi^{**})] + \int_{\pi^*}^{\pi^{**}} \pi f(\pi)\,d\pi$$

$$= \gamma_t F(\pi^*) + \gamma_t [1 - F(\pi^{**})] + \gamma_t F(\pi^{**}) - \gamma_t F(\pi^*)$$

Therefore, the solution to the problem is

$$\gamma_t = \pi^* F(\pi^*) + \pi^{**}[1 - F(\pi^{**})] + \int_{\pi^*}^{\pi^{**}} \pi f(\pi)\, d\pi. \tag{38}$$

Using (37), note that

$$\begin{aligned} \int_{\pi^*}^{\pi^{**}} \pi f(\pi)\, d\pi &= \int_{\pi^*}^{\pi^{**}} \left\{ \frac{d}{d\pi}[\pi F(\pi)] - F(\pi) \right\} d\pi \\ &= \pi F(\pi) \Big|_{\pi^*}^{\pi^{**}} - \int_{\pi^*}^{\pi^{**}} F(\pi)\, d\pi \\ &= \pi^{**} F(\pi^{**}) - \pi^* F(\pi^*) - \int_{\pi^*}^{\pi^{**}} F(\pi)\, d\pi. \end{aligned}$$

Therefore, it is the case that

$$-\int_{\pi^*}^{\pi^{**}} F(\pi)\, d\pi = \pi^* F(\pi^*) - \pi^{**} F(\pi^{**}) + \int_{\pi^*}^{\pi^{**}} \pi f(\pi)\, d\pi.$$

Substituting this into equation (38) yields the solution presented in equation (20).

REFEREE'S APPENDIX C: DERIVATION OF EQUATION (26)

Using the definition of π^* given by (6), the first order condition (25) can be rewritten as:

$$\left(1 - R_t \frac{p_{t+1}}{p_t}\right) \frac{\pi^*}{\gamma_t} F(\pi^{**}) + \frac{1}{\gamma_t + c} \int_{\pi^{**}}^{1} \pi f(\pi) d\pi = \frac{R_t}{(1 - \gamma_t) R_t - c \frac{p_t}{p_{t+1}}} \int_{\pi^{**}}^{1} (1 - \pi) f(\pi) d\pi. \tag{39}$$

Now, note that (6) and (24) imply that

$$\frac{1}{\gamma_t + c} = \frac{\pi^*}{\pi^{**}} \frac{1}{\gamma_t}$$

and

$$\frac{R_t}{(1 - \gamma_t) R_t - c \frac{p_t}{p_{t+1}}} = \frac{\pi^*}{1 - \pi^{**}} R_t \frac{p_{t+1}}{p_t} \frac{1}{\gamma_t}.$$

This allows us to write (39) as

$$\left(1 - R_t \frac{p_{t+1}}{p_t}\right) \pi^{**} F(\pi^{**}) + \int_{\pi^{**}}^{1} \pi f(\pi) d\pi = \frac{\pi^{**}}{1 - \pi^{**}} R_t \frac{p_{t+1}}{p_t} \int_{\pi^{**}}^{1} (1 - \pi) f(\pi) d\pi. \tag{40}$$

Using (37), note that

$$
\begin{aligned}
\int_{\pi^{**}}^{1} \pi f(\pi)\, d\pi &= \int_{\pi^{**}}^{1} \left\{ \frac{d}{d\pi}\left[\pi F(\pi)\right] - F(\pi) \right\} d\pi \\
&= \pi F(\pi)\, \big|_{\pi^{**}}^{1} - \int_{\pi^{**}}^{1} F(\pi)\, d\pi \\
&= 1 - \pi^{**} F(\pi^{**}) - \int_{\pi^{**}}^{1} F(\pi) d\pi .
\end{aligned}
$$

Hence, (40) reduces to

$$
1 - \int_{\pi^{**}}^{1} F(\pi) d\pi = \frac{\pi^{**}}{1 - \pi^{**}} R_t \frac{p_{t+1}}{p_t} \int_{\pi^{**}}^{1} F(\pi) d\pi . \tag{41}
$$

Multiplying (41) by $(1 - \pi^{**})$ and rearranging gives

$$
1 - \pi^{**} = \left(1 - \pi^{**} + \pi^{**} R_t \frac{p_{t+1}}{p_t} \right) \int_{\pi^{**}}^{1} F(\pi) d\pi . \tag{42}
$$

But (24) implies that

$$
\pi^{**} R_t \frac{p_{t+1}}{p_t} = \frac{\gamma_t (1 - \pi^{**}) + c}{(1 - \gamma_t)} .
$$

Therefore, equation (42) becomes

$$
1 - \pi^{**} = \left[1 - \pi^{**} + \frac{\gamma_t (1 - \pi^{**}) + c}{(1 - \gamma_t)} \right] \int_{\pi^{**}}^{1} F(\pi) d\pi . \tag{43}
$$

Multiplying (43) $(1 - \gamma_t)$ and rearranging terms yields

$$
1 - \gamma_t = \left[\frac{(1 - \gamma_t)(1 - \pi^{**}) + \gamma_t (1 - \pi^{**}) + c}{(1 - \pi^{**})} \right] \int_{\pi^{**}}^{1} F(\pi) d\pi .
$$

Clearly, this reduces to (26).

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Figure 1:
Timing of Events Within a Period

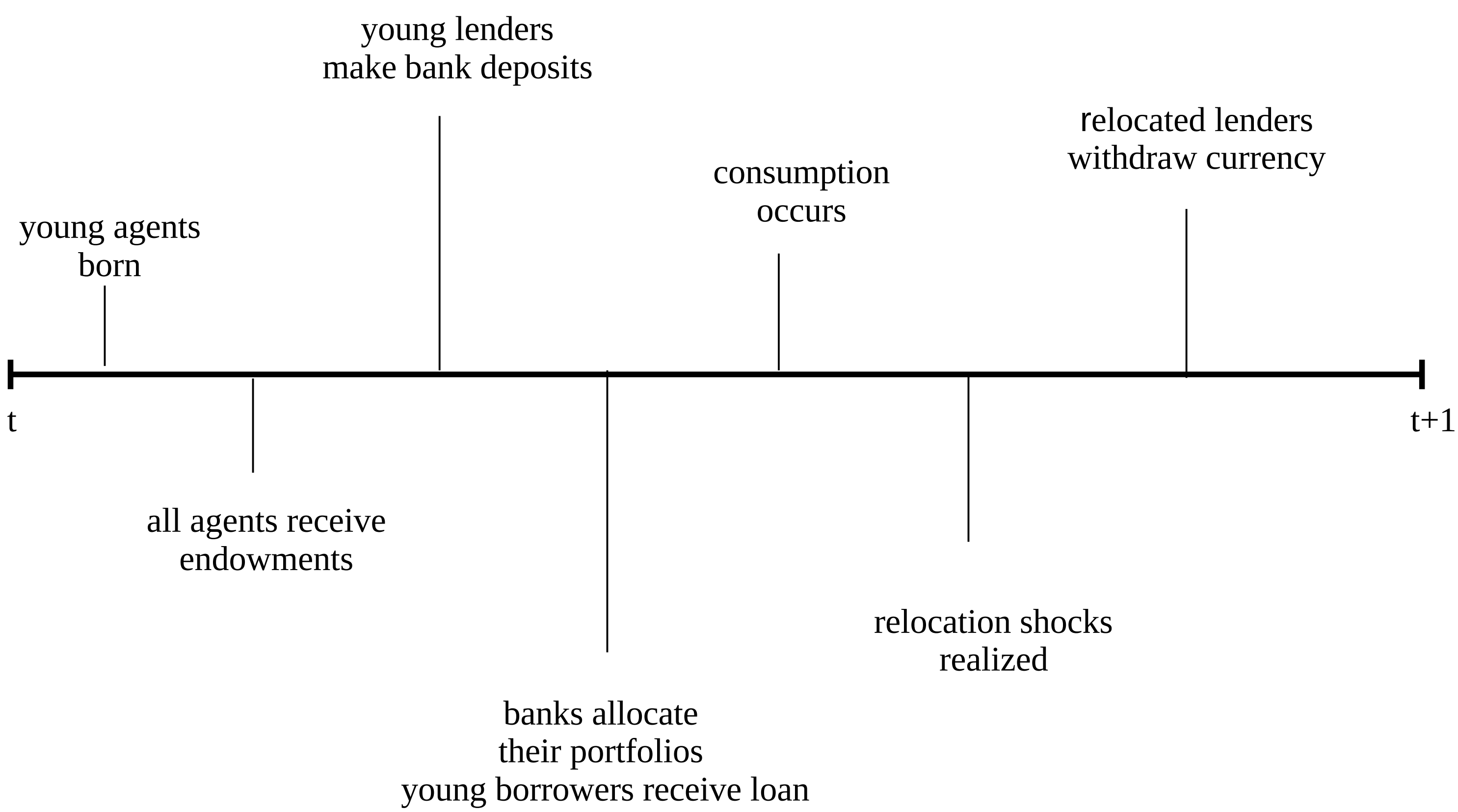

Figure 2: No Lender of Last Resort

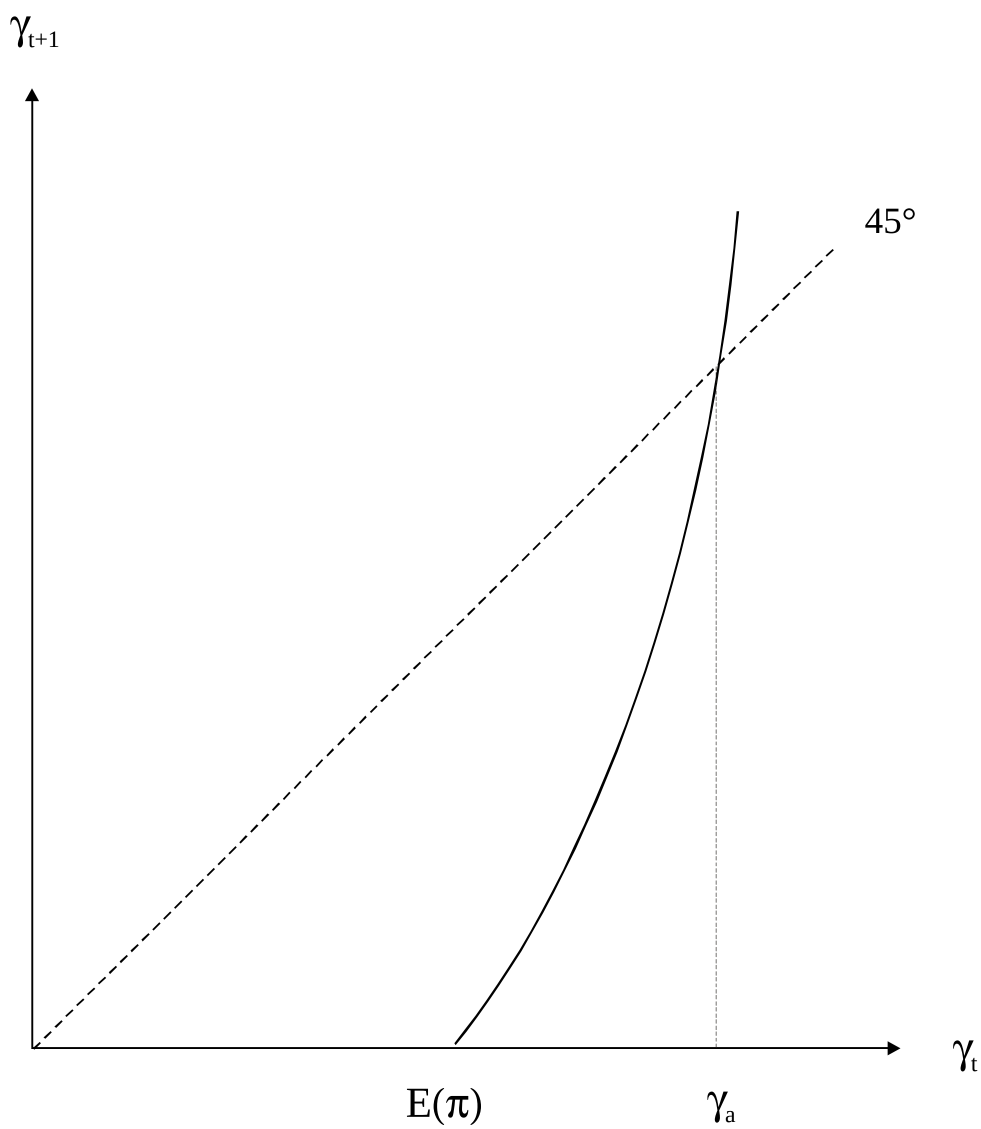

Figure 3: An Unrestricted Lender of Last Resort

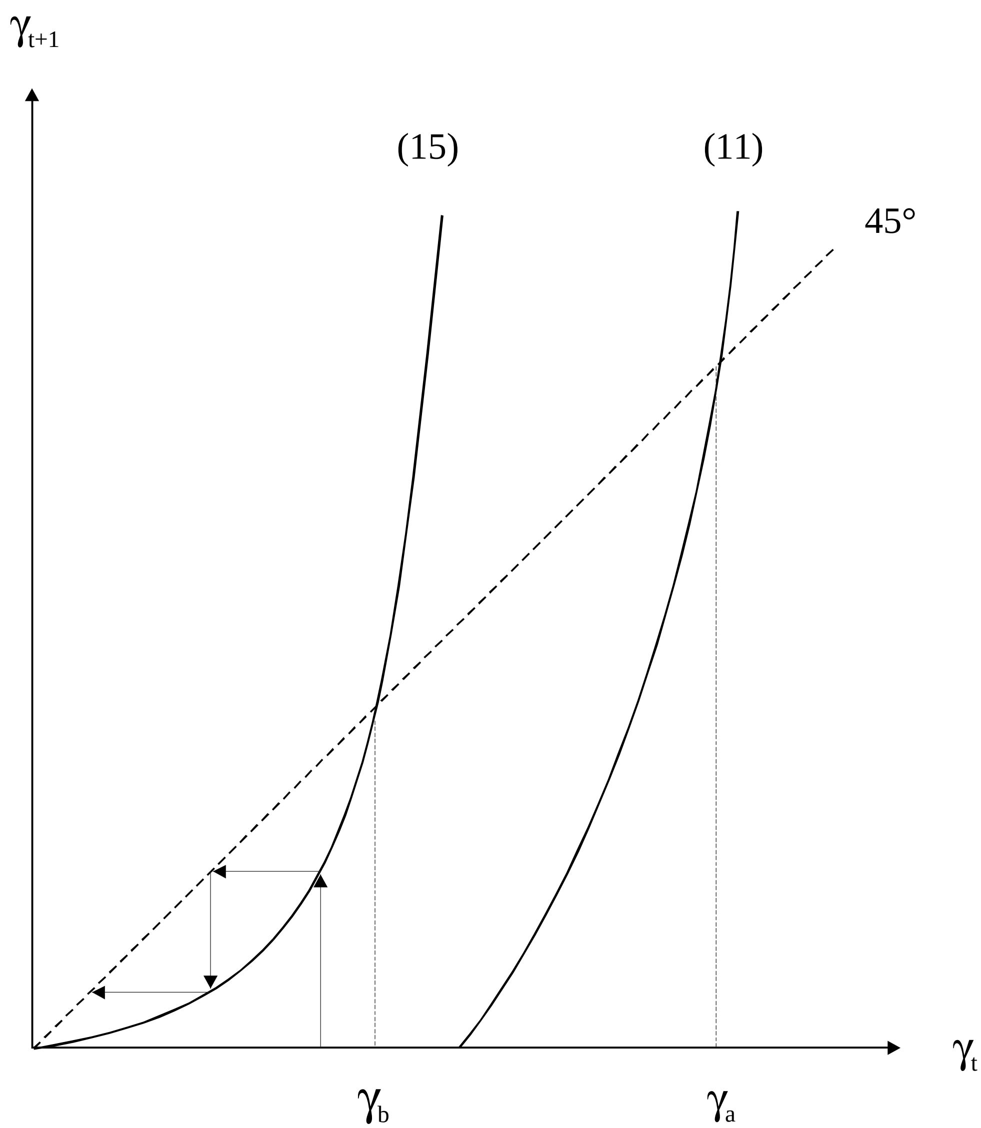

Figure 4: A Lender of Last Resort

Who Charges Interest, r^d **6** $y/\beta x$

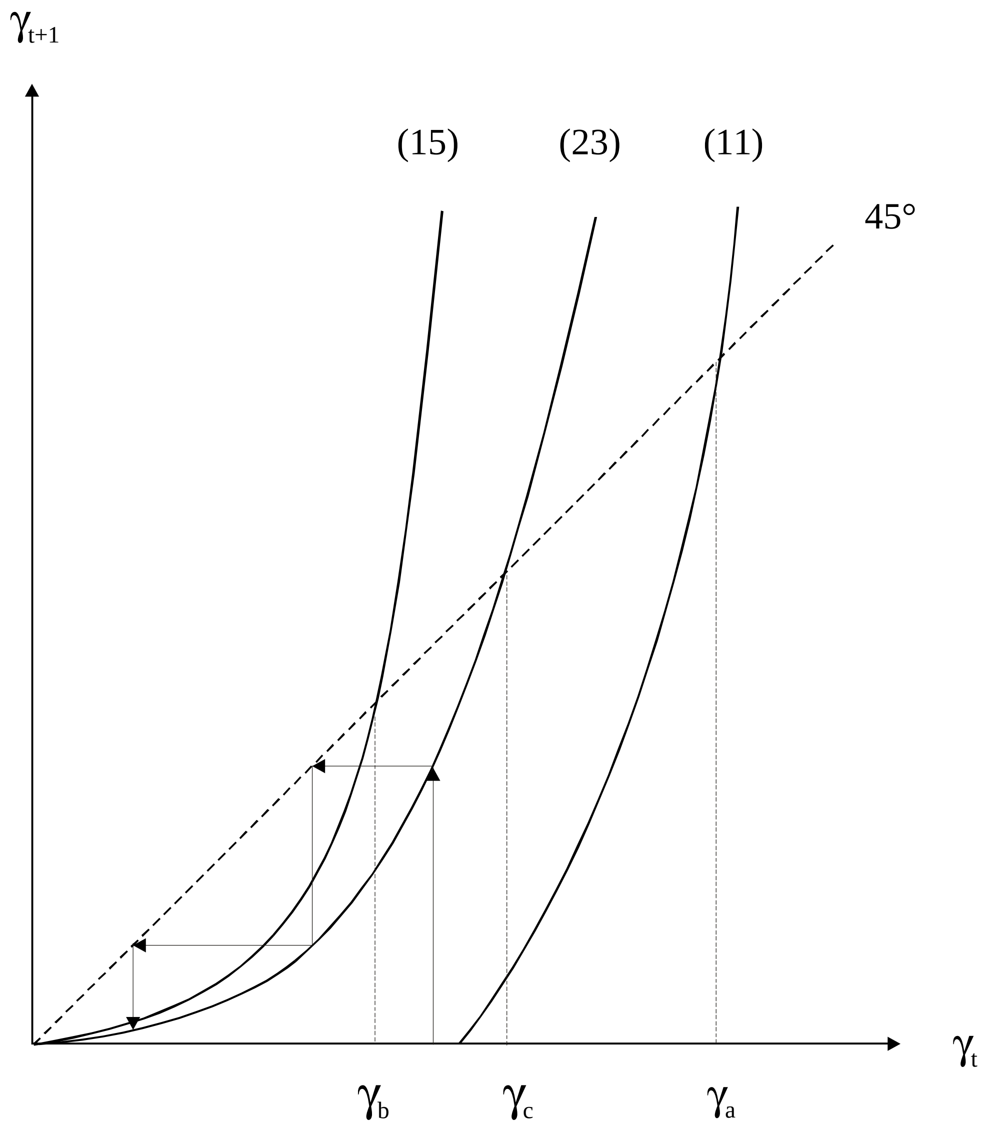

Figure 5: A Lender of Last Resort
Who Charges Interest, r^d **1** $y/\beta x$