A NAIVE BIDDER IN A COMMON VALUE AUCTION[#]

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Abstract

We study a common value auction in which two bidders compete for an item the value of which is a function of three independent characteristics. Each bidder observes one of these characteristics, but one of them is "naive" in the sense that he does not realize the other bidder's signal contains useful information about the item's value. Therefore, this bidder bids as if this were an Independent Private Values auction. We show that the naive's bidder payoff exceeds that of his fully rational opponent for all symmetric unimodal signal distributions. We also show that naive bidding is persistent in the evolutionary sense.

Keywords: Bounded Rationality, Evolutionary Stability, Bidding.

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1. INTRODUCTION.

In many common value auctions bidders competing for an item differ in the type of expertise they have. Consider, for instance, two art dealers competing for a painting. Suppose that the value of the painting in the retail market is a function of the artistic quality of the painting, the quality of the material used to create the painting, and other factors, such as demand for paintings of this type and the state of the economy. Suppose further that one of the two dealers is an expert in judging the artistic quality of the painting while the other dealer is an expert in judging the quality of the materials used for the painting.

We consider an ascending oral auction in which the first of these bidders is “naive” in the sense that he does not infer any information about the value of the item from the bidding behavior of his opponent. Naive bidding is not necessarily due to an inability to “do the math” right. It could arise if the “naive” bidder erroneously believes that the other bidder is not an expert in judging the quality of the painting. It could also arise if the “naive” bidder believes (erroneously) that his competitor wants to purchase the item for private consumption on the basis of some characteristic that is not valued by the market. Finally, it could also arise if the “naive” bidder wants to purchase the painting for his own consumption on the basis of some characteristic that is not observed by his competitor. In all of these possibilities, the bidding of the other bidder is “noise” as far as the first bidder is concerned. In the last one of these possibilities, the “naive” bidder is not naive at all, he simply has a different preference structure than the other player: Even though he values the item on average as much as his competitor, his valuation does not depend on any attributes that are observed by his competitor. The second bidder is fully rational, i.e., he understands that he competes against a naive bidder and he formulates his strategy optimally. Each of the two bidders observes a component of the value of the item, which is referred to as that bidder’s signal. The actual value of the item equals the product of the two signals and other unobserved factors.

We show that, contrary to what is commonly believed, the payoff of the naive bidder exceeds, in expected terms, the payoff of the rational bidder when the signal distribution is symmetric and unimodal. When the distribution of signals is not symmetric and unimodal the rational bidder could, but is not guaranteed to, have higher expected payoff than the naive bidder.

Our findings have implications for the persistence of naive strategies in market environments. A frequently used assertion used to support the use of the optimization hypothesis and agent rationality in economic models is that suboptimal strategies and bounded rational agents will earn lower payoffs, on average, than rational agents employing optimal strategies. If selection of the strategies over time is a function of their relative profitability, then rational agents and optimal strategies will eventually predominate. The results of this paper suggest that, in the environment that we study, this is clearly not the case. Naive bidders can, and for symmetric unimodal distributions of signals will, earn higher payoffs when matched against a fully rational bidder. Therefore, naive bidding is likely to persist and, indeed, be more widespread than fully rational bidding even if it *were equally costly* to acquire the ability to employ either strategy. We leave the formal investigation of the distribution of bidder types in this bidding game for future work.

2. MODEL.

2.1. Preliminaries.

Consider two bidders competing for an object. The value of the object is common to both bidders and equals

$$V = \alpha \; S_1 \; S_2 \; \epsilon$$

where α is a constant, S_1 is a signal of the value received by bidder 1, S_2 is a signal of the value received by bidder 2, and ϵ is a random variable. The expected value of two signals and of the random variable ϵ are normalized, without loss of generality, to 1. Further, S_1, S_2, and ϵ are assumed to take non-negative values and be independent of each other. For notational simplicity, we assume that the two signals are identically distributed with a non-degenerate density $f(s)$ and associated continuous distribution function $F(s)$. Finally, we assume that $f(s)$ is positive in the interval $[S_{\min}, S_{\max}]$.

This framework captures a number of different bidding environments. One such environment is the competition of two dealers for an item which they intend to resell. The signals of the bidders represent different facets of the item. That is, each bidder is an expert in evaluating a different determinant of the item's value. In our model, these signals are assumed to be independent of each other and identically distributed, i.e., the two characteristics that describe the item are not correlated and of equal importance in determining the variability of the item's value. The actual value of the item to the winner also depends on factors that are not observable to either of the two bidders, such as idiosyncratic consumer tastes for the item, economic conditions at the time that the item is actually sold, and randomness in the realized price due to the thinness in the market for this item. Another such environment is the competition between two firms for oil leases. Both firms evaluate the oil bearing capacity of the track but drill in different places, thus obtaining independent signals of the tracks value. The value will be super-additive in the signals (as modeled here) if there are any returns to scale in extraction.

Bidder 1 is assumed to be "naive" in the sense that he does not infer any information about the value of the item from the bidding behavior of his opponent. The naive bidder could be a bidder who believes that the other bidder is not an expert in judging the quality of the item and instead observes a signal that is pure noise. Alternatively, the naive bidder could be a bidder who believes that his competitor wants to purchase the item on the basis of some characteristic that is not valued by the market. Under both of these beliefs, the bidding behavior of the other bidder is pure "noise."[1] Therefore, the naive bidder does not condition his evaluation of the item's worth on the bidding behavior of his opponent. In effect, this bidder believes that the second facet of the object's value is not observed by anyone and that the value of the item to his competitor is a random variable, V_2, independent of S_1, S_2, and ϵ. Therefore, for bidder 1, the process that describes the value of the item to himself can be described as

$$V_1 = \alpha \; S_1 \; \eta$$

[1] More loosely, one could also think of the naive bidder as being a bidder who has not thought about the problem and, therefore, has not formulated any beliefs.

where η is a random variable with mean equal to 1 and no correlation with V_2.[2]

The second bidder is fully rational, i.e., he understands that he competes against a naive bidder and he formulates his strategy optimally.[3]

Both bidders are risk neutral and compete in an oral ascending auction with no reserve. We model the bidding process by a "thermometer" auction. Alternatively, one could model the bidding as a 2nd price sealed-bid auction, as the two formats are equivalent in this particular setting.

2.2. Bidder Strategies.

Based on his beliefs, the naive bidder has a dominant strategy to stay in the auction until the price reaches his reservation value. Since he is risk neutral, his reservation value, R_1, equals the expected value of the object conditional on his signal, that is,

$$R_1 = \alpha \; S_1$$

The rational bidder will stay in the auction as long as the price is such that, if he were to win at that price, he would reap positive expected surplus. Given the naive bidder's strategy, winning the object at a price P implies that the signal of bidder 1 was

$$S_1 = \frac{P}{\alpha}.$$

Therefore, immediately upon winning the item at a price equal to P the rational bidder would calculate its expected value to equal:

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[2] Notice that the above assumptions imply that bidder 1 believes that he is competing in a privately known values auction.

[3] We note, parenthetically, that this model could also arise if both bidders are rational but have the following asymmetric preference structures. Both bidders have the same average value for the item. However, bidder 1 does not care about the facet of the object that is observed by Bidder 2 and wants to purchase the item for his own consumption. The preferences of bidder 2 are as described in the text.

$$E[V \mid S_1 = P/\alpha] = \alpha \; S_2 \; \frac{P}{\alpha}$$
$$= S_2 \; P$$

Notice that when the signal of bidder 2 is less than 1, he would regret winning the item regardless of the realized price. On the contrary, when his signal is greater than 1 the expected value of the item, upon winning it, exceeds its price regardless of the price that he wins it at. Hence, an optimal strategy of the rational bidder is to immediately quit the auction if he observes a below average signal and stay in the auction until he wins if he observes an above average signal.

The intuition for this is as follows: the naive bidder computes the expected value of the item "as if" the signal of other bidder is equal to its mean, i.e., equals 1. When the signal of bidder 2 is less than one, the naive bidder overestimates the value of the item and therefore would be willing to push its price above its expected value. Realizing this, the rational bidder will have no reason to enter the bidding. When bidder 2 gets an above average signal, i.e., $S_2 > 1$, the naive bidder will underestimate the value of the item and therefore would drop out before its price reaches its expected value. Realizing this, the rational bidder will stay in the auction until he wins the item.

Of course, this is not, in general, the only optimal strategy of bidder 2. Observe that the lowest possible price at which the naive bidder will drop out is $P_{\min} = \alpha \; S_{\min}$. This is a positive number unless $S_{\min} = 0$. Therefore, bidder 1 can bid the price up to $P_{\min}$ with impunity: He is at no risk of winning. Doing so will result in no benefit to him compared to the strategy of dropping out at the start of the auction. In fact, if he incurs even infinitesimal costs from bidding he will strictly prefer to drop out at the very start. On the other hand, if he is aggressive, in the sense that he attaches a tiny value to making his opponent pay a higher price, he will stay in the auction until the price reaches $P_{\min}$ (and will drop out at that point). In the main body of the paper we consider the equilibrium in which he the rational bidder drops out at a price equal to zero when he gets a below average signal. In the Appendix we consider the other extreme at which the rational bidder drops out at a price equal to $P_{\min} = \alpha \; S_{\min} \geq 0$. Similar results are obtained in either case.

3. RESULTS.

3.1. Payoff of the Naive Bidder.

Notice that if the rational bidder's signal is less than 1 and the naive bidder's signal is equal to S_1, the payoff of the rational bidder, $\Pi_2^{cc}(S_1, S_2)$, is zero, since he will drop out of the auction, while the payoff of the naive bidder conditional on S_1 and S_2, $\Pi_1^{cc}(S_1, S_2)$, equals

$$\Pi_1^{cc}(S_1, S_2) = \alpha\ S_1\ S_2 .$$

This follows from the fact that, in the absence of a reserve, the naive bidder will win the item at a price equal to zero. The naive bidder wins the item when its value is lower than average but he pays a zero price to get it. The expected payoff of the naive bidder with a signal S_1, $\Pi_1^c(S_1)$, is given by

$$\Pi_1^c(S_1) = \int_0^1 f(S_2)\ \alpha\ S_1\ S_2\ dS_2$$

Integrating over the distribution of bidder 1's signals yields the *ex ante* payoff of the naive bidder, Π_1,

$$\begin{aligned}\Pi_1 &= \int_0^{S_{max}} \int_0^1 f(S_2)\ \alpha\ S_1\ S_2\ dS_2\ f(S_1)\ dS_1 \\ &= \int_0^{S_{max}} Pr[S_2{<}1]\ E[S_2\,|\,S_2{<}1]\ \alpha\ f(S_1)\ S_1\ dS_1 \\ &= \alpha\ Pr[S_2{<}1]\ E[S_2\,|\,S_2{<}1] \int_0^{S_{max}} f(S_1)\ S_1\ dS_1\end{aligned}$$

Given that the expected value of a bidder's signal equals 1, the ex ante payoff of the naive bidder can be written as

$$\Pi_1 = \alpha\ Pr[S_2{<}1]\ E[S_2\,|\,S_2{<}1] .$$

Note that this payoff is positive since every term is positive and less than the expected value of the item, α, since the last two terms are both less than 1.

3.2. Payoff of the Rational Bidder.

Notice that if the rational bidder's signal is $S_2 > 1$ and the naive bidder's signal is S_1, the payoff of the naive bidder is zero, since he fails to win the auction, while the payoff of the rational bidder, conditional on S_1 and S_2 equals

$$\begin{aligned}\Pi_2^{cc}(S_1, S_2) &= \alpha\ S_1\ S_2 - P \\ &= \alpha\ S_1\ S_2 - \alpha\ S_1 \\ &= \alpha\ S_1\ (S_2 - 1)\end{aligned}$$

Notice that the higher the signal of the naive bidder, the higher the conditional payoff of the rational bidder. This is because the price raises slower than the value of the object when $S_2 > 2$, as the naive bidder implicitly sets $S_2 = 1$ when computing the value of the object. Also notice that the rational bidder wins the item when it is relatively valuable.

The expected payoff of the rational bidder with signal S_2, $\Pi_2^c(S_2)$, then equals

$$\Pi_2^c(S_2) = \int_0^{S_{\max}} f(S_1)\ \alpha\ S_1\ (S_2 - 1)\ dS_1$$

This integrates to

$$\Pi_2^c(S_2) = \alpha\ (S_2 - 1)$$

since the expected value of S_1 is equal to 1. The *ex ante* payoff of the rational bidder, Π_2, i.e., his payoff before he obtains any information on the item, is obtained by integrating his conditional profit over the distribution of his signals for signals that are higher than 1. This yields

$$\begin{aligned}\Pi_2 &= \int_1^{S_{\max}} f(S_2)\ \alpha\ (S_2 - 1)\ dS_2 \\ &= \alpha\ Pr[S_2 > 1]\ E[S_2 - 1 \mid S_2 > 1]\end{aligned}$$

Notice that $\Pi_2 > 0$ since every term in the expression above is positive.

3.3. Comparison of Payoffs.

Which type of bidder has higher *ex ante* profits? It turns out, as we show below, that the payoffs of the two bidders can not be unambiguously ranked: under some parametrizations the naive bidder earns a higher payoff than the rational bidder, for other parametrizations the reverse is true. However, much insight (and some interesting results) can be obtained by considering the signal distributions that are symmetric with respect to their mean.

In particular, when the signal distributions are symmetric, the expected payoffs of either bidder can be written as a function of a single conditional expectation of the signals. This conditional expectation is a measure of signal dispersion. As Proposition 1 shows below, the profits of each bidder are monotonic in this expectation: the rational bidder's payoff increases with signal dispersion whereas the naive bidder's payoff decreases with signal dispersion.

PROPOSITION 1. Consider signal distributions that are symmetric. Then, a sufficient information for the expected payoff of either bidder is the signal dispersion as measured by $z \equiv 1 - E[S \mid S<1]$, i.e., the difference between the mean and the expected value of the signal conditional on it being lower than the mean. The rational bidder's payoff increases with the dispersion of the signal while the naive bidder's payoff decreases with the dispersion of the signal.

Proof. When the signal distribution is symmetric the probability that the rational bidder's signal will exceed the mean equals the probability that it will be lower than the mean, i.e.,

$$Pr[S_2>1] = Pr[S_2<1] = \frac{1}{2}.$$

Let

$$E[S_2 \mid S_2<1] = 1 - z$$

where $z \in [0,1]$ is a measure of the dispersion of the signal distribution. Then,

$$E[S_2 \,|\, S_2] \;=\; 1 \;+\; z\,.$$

Therefore, the *ex ante* profits of the two bidders become

$$\Pi_1 \;=\; \frac{\alpha}{2}\;(1 \;-\; z)$$

and

$$\Pi_2 \;=\; \frac{\alpha}{2}\;(1 \;+\; z \;-\; 1)$$

$$\;=\; \frac{\alpha}{2}\; z$$

Note that the rational bidder's payoff increases with the dispersion of the signal while the naive bidder's payoff decreases with the dispersion of the signal. **#**

The naive bidder's payoff decreases with the dispersion of the signal because (i) in the event that he wins, he pays of price of zero, and (ii) he wins when the signal of his opponent, i.e., one of the components of the item's value, is below average. Therefore, an increase in the signal dispersion reduces the expected value of the item in the events that he wins without reducing the price that he pays for it. Conversely, an increase in the signal dispersion increases the payoff of the rational bidder because (i) the expected price that he pays equals α and is independent of his signal but (ii) he wins when his signal is above average. Therefore, signal dispersion does not increase his expected payment but increases the expected value of the item in the events that he wins.

Since for symmetric signal distributions the profit response of the two bidders is monotonic to signal dispersion but exhibits the opposite response, there may exist some critical level of signal dispersion such that for lower levels of dispersion the naive bidder's payoff exceeds that of the fully rational bidder while for higher levels of dispersion the converse is true. In principle, this critical dispersion level could be zero which would imply that the fully rational bidder has higher expected profits for all symmetric signal distributions. However, Corollary 1 below shows that this critical dispersion level is bounded away from zero.

COROLLARY 1. When the signal distribution is symmetric the expected payoff of the naive bidder exceeds that of the rational bidder if $E[S] < 2\,E[S\,|\,S<1]$, i.e. if the expected value of the signal is less than two times the expected value of signal conditional on it being lower than the mean.

Proof. Using the expressions for bidder payoffs obtained in the Proof of Proposition 1 above, we can see that the naive bidder has a higher payoff than the rational bidder when

$$\frac{\alpha}{2}\,(1 - z) > \frac{\alpha}{2}\,z \quad \Rightarrow$$

$$1 - z > z \quad \Rightarrow$$

$$z < \frac{1}{2}.$$

Since $z \equiv 1 - E[S\,|\,S<1]$ the above implies

$$1 - E[S\,|\,S<1] < \frac{1}{2} \quad \Rightarrow$$

$$E[S\,|\,S<1] > \frac{1}{2} \quad \Rightarrow$$

$$2\,E[S\,|\,S<1] > E[S]$$

where the last step follows from the fact that $E[S] = 1$. #

Notice that expected value of the item is equal to α regardless of signal dispersion. Corollary 2 below states that a change in signal dispersion does not affect the distribution of the surplus between the seller and the two bidders. Therefore, a change in dispersion does not affect the size of the "pie" the two bidders split between them, but only their shares of that pie.

COROLLARY 2. When the signal distribution is symmetric the seller's expected revenue is independent of signal dispersion and equals half the expected value of the item.

Proof. Using the profits of the two bidders, as derived in the Proof of Proposition 1 above, we can show that the sum of the bidders' profits, Π, is independent of the distribution and equals

$$\Pi = \frac{\alpha}{2}(1 - z + z)$$

$$= \frac{\alpha}{2}$$

In expected terms, the two bidders split with the seller the surplus of the auction regardless of the distribution of the signals. Since seller revenue and buyer surplus add up to the value of the item, the expected revenue of the seller equals $\alpha/2$. Therefore, a change in the signal dispersion changes the division of the surplus between the two bidders with affecting seller revenue. #

A much stronger result with regards to relative bidder profits can be obtained if we further reduce the distribution of signals to be unimodal. As Corollary 3 states below, for this important class of signal distributions the expected payoff of the naive bidder will exceed that of the rational bidder.

COROLLARY 3. When the signal distribution is symmetric unimodal the expected payoff of the naive bidder is higher than that of the rational bidder.

Proof. Consider a symmetric unimodal signal distribution, f(s), and denote the expectation of S conditional on it being lower than the mean by $E_f[S \mid S < 1]$. Note that the probability mass of f(s) can be spread evenly over its support to yield a uniform distribution with the same support. For this, corresponding, uniform distribution, denote the expectation of S, conditional on it being lower than the mean, by $E_u^f[S \mid S < 1]$.

It is clear that

$$E_u^f[\, S \mid S < 1] \;<\; E_f[\, S \mid S < 1] \qquad \Rightarrow$$

$$1 \;-\; E_u^f[\, S \mid S < 1] \;\;>\;\; 1 \;-\; E_f[\, S \mid S < 1].$$

Observe that for all uniform distributions with mean equal to 1 and support exclusively on the non-negative numbers

$$E[\, S \mid S < 1] \;\geq\; \frac{1}{2}.$$

Then, from Corollary 1 and the inequality above, it follows that when the signal distribution is symmetric unimodal the expected payoff of the naive bidder is higher than that of the rational bidder. **#**

3.4. Example With Non-Symmetric Signal Distributions.

The results of the previous section were all derived under the assumption that the signal distribution is symmetric. However, it is easy to see that the profits of the “naive” bidder will exceed, in expected terms, those of the fully rational bidder for all distributions with sufficiently low dispersion.

4. THE EVOLUTIONARY STABILITY OF "NAIVE" BIDDING.

The fact that a naive bidder obtains higher payoff than a fully rational competitor when the distribution of signals is symmetric and unimodal suggests that naive bidding may persist even in the long run. Indeed, this turns out to be the case, when persistence is defined as a non-zero proportion of types in an evolutionary equilibrium. In particular, one can show that when (i) the distribution belongs to a number of different sub-classes of symmetric unimodal distributions, (ii) pairs of bidders are matched randomly from a population of bidder types which includes a proportion q of naive bidders, and (iii) the proportion of the bidder type that has higher expected profits increases. Then there is a unique proportion $q_{eq} \in (0, 1)$ of naive bidders such that the expected profits of a naive bidder equal the expected profits of a rational bidder, that is, there is a unique interior proportion of naive types that constitutes an evolutionary equilibrium. Furthermore, populations that consist of one type only (i.e, either of all rational or of all naive bidders) are not stable, i.e, they are not robust to mutations. Further, we believe that this result holds more generally for all symmetric unimodal distributions (and not only for the sub-classes for which it has been demonstrated) and we believe are near a general proof.

5. CONCLUDING REMARKS.

It is generally thought that non-sophisticated bidding in common value auctions will result in lower profits. Bidders that are not properly taking into consideration the fact that other bidders behavior is informative about the value of an item to themselves and who are ignoring any possible asymmetries in the quality of information will endo up overpaying and winning items of lower quality. In the simple model considered in this paper we show that a mildly naive bidder will indeed win items of lower quality than his fully rational competitor. However, he ends up getting them at bargain prices, while his fully rational opponent pays much higher prices for the items he wins. As a result, the expected profits of the naive bidder exceed the expected profits of the fully rational bidder for a wide class of signal distributions.

The intuition for these results can be summarized as follows: When the rational bidder observes a low signal of the item's value he realizes that the naive bidder will overestimate the value of the item *regardless of his signal*. The overestimate arises because the naive bidder estimates the value of the item using the *expected* value of his competitor's signal. Therefore, the rational bidder can only win the item by overpaying for it and he optimally chooses not to compete. As a consequence, the naive bidder wins the item at the lowest possible price. In contrast, when the rational bidder observes a high signal he realizes that the naive bidder will underestimate the value of the item regardless of his signal. Therefore, the naive bidder will drop out at a price that is lower than the item's value and, as a result, the rational bidder optimally chooses to stay in the auction until he wins. However, in competing for the item, the naive bidder raises the price the rational bidder has to pay.

The results of this paper may appear, at first sight, paradoxical. How can a "boundedly rational bidder earn a higher payoff when competing with a bidder with a fully rational bidder? After all, the fully rational bidder can always imitate a boundedly rational one. The key insight is that the naive bidder enjoys an advantage in terms of commitment: nature has picked his type to be naive. Given that the rational bidder is faced with a naive opponent he does better by bidding rationally rather than naively. Were a rational bidder to face another rational bidder, he would have preferred to have been naive. Unfortunately (for the rational bidder) nature gives the naive bidder a first mover advantage.