Expertise or Experience: What Raises Pay?

Fabien Postel-Vinay
INRA-LEA*

Jean-Marc Robin
INRA-LEA and CREST-INSEE†

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Abstract

An equilibrium job search model with on-the-job-search is presented and solved, in which we allow firms to implement optimal wage posting strategies in the sense that they leave no rent to their employees and counter the offers received by their employees from competing firms. Cross-firm productivity dispersion arises endogenously in equilibrium. The model delivers a hump-shaped aggregate earnings distribution that reflects both firm- and worker-heterogeneity.

The model also generates plausible individual career paths on the basis of which it is estimated, using a French panel of wages over the period 1994-96.

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*Addresses: Laboratoire d'Economie Appliquée de l'INRA, Ecole Normale Supérieure, 48 bld Jourdan, 75014 Paris, France, e-mail: fpostel@java.ens.fr. Internet: www.inra.fr/Internet/Departements/ESR/UR/lea/fpostel.htm.

†Address: Laboratoire d'Economie Appliquée de l'INRA, Ecole Normale Supérieure, 48 bld Jourdan, 75014 Paris, France. E-mail: robin@java.ens.fr.

1 Theory

1.1 Basic assumptions

We consider a search-theoretic model of the labor market in the line of Burdett and Mortensen (1998) as extended by Postel-Vinay and Robin (1999). The model economy features a continuum of workers with exogenous and constant mass M facing a continuum of competitive firms with a mass normalized to 1 that produce one unique multi-purpose good.

Workers differ in their personal 'abilities' or 'skills'. The skill level is measured by a parameter ε that varies across workers. Firms also differ in the technologies that they operate. Technologies are indexed by a number p measuring their generic level of productivity. The output flow of a type ε worker operating a type p technology is simply given by εp.

Workers can either be employed or unemployed, and the aggregate unemployment rate is denoted by u. A type ε unemployed worker has a flow earning of εb, with b a positive constant[1], thought of as the sum of possible sources of income during unemployment. Those may consist *e.g.* of unemployment benefits and the worker's valuation of 'home production'. (Although unemployment benefits *per se* may not be systematically related to skills, the adopted simple specification makes sense under the 'home production' interpretation.)

Unemployed workers have to forgo the flow income εb when they find a job, and they sample job offers sequentially at a Poisson rate λ_0. All firms have the same probability of being sampled (random matching technology).[2] We also allow workers to search for a better job while employed, so firms make offers to employed workers as well. The arrival rate of offers to on-the-job searchers is λ_1. The pool of unemployed workers is regularly fueled by layoffs

[1] The fact that a worker's productivities "at home" and at work are both proportional to ε greatly simplifies the upcoming analysis. Although neutral regarding most of our results, this assumption has some particular implications that we shall point out along the analysis.

[2] This is a disputable assumption. One could oppose to this assumption the alternative assumption of 'balanced matching' as in Burdett and Vishwanath (1988) (the probability that a worker samples a given firm is proportional to its size), or the endogenous matching process of Robin and Roux (1998). Since we are mostly interested in the wage setting mechanism in this paper, we leave the analysis of more involved matching processes for future research.

that occur at the exogenous rate δ, and by a constant flow μM of newborn workers who begin their working life as unemployed. To keep population constant over time, we assume that every living worker, employed or not, faces a constant mortality rate μ. Although not essential, the birth-death process will be shown to be useful in the empirical applications. Agents discount the future at a common rate $\rho \geq 0$.

Finally, we make the following important three assumptions on the wage setting mechanism:

1. Firms can vary their wage offers according to the characteristics of the particular worker they meet;

2. Firms can (although they don't necessarily do) counter the offers received by their employees from competing firms;

3. Wage contracts are long-term contracts that can be renegotiated by mutual agreement only. The only way for an employer to break the contract against the employee's will is to fire him.

The first two assumptions are a departure from the standard Burdett and Mortensen (1998) model. Their implications are explored by Postel-Vinay and Robin (1999) in a model where workers are all equally productive, but differ in their opportunity cost of employment.

Assumption 3 is more standard and only ensures that a firm cannot unilaterally cancel a promotion obtained by one of its employees after having received an outside job offer, once the worker has eventually turned down that offer. It follows that wage cuts within the firm are not permitted.

1.2 Worker behavior

We first introduce some notation. Firms are heterogeneous with respect to the technologies that they operate, which are distributed over $\left[\underline{p}, \overline{p}\right]$ according to some continuous cdf Γ, which for the time being is taken as given, and will be derived endogenously later along the lines described in Postel-Vinay and Robin (1999). To save on space, we also define $\overline{\Gamma}(\cdot) = 1 - \Gamma(\cdot)$. The workers' 'personal ability' parameters ε are exogenously distributed among the total population of workers according to the cdf H over $[\underline{\varepsilon}, \overline{\varepsilon}]$, both positive numbers. Without loss of generality we normalize the distribution of worker heterogeneity and set the mean of ε, $\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon dH(\varepsilon)$, to 1. Workers are risk-neutral and discount the future at an exogenous and constant rate $\rho > 0$. They face a constant mortality rate μ. To keep the population constant, μ also designates the constant rate of natality, and newborn workers are assumed to draw their value of ε randomly in the distribution H.

The lifetime utility of an unemployed worker with competence ε (a worker of type ε, for short) is denoted by $V_0(\varepsilon)$, and that of the same worker when employed at a firm of type p and paid a wage w is $V(\varepsilon, w, p)$.

A type p firm is able to employ a type ε unemployed worker if the match is productive enough to at least compensate the worker for his forgone unemployment income, *i.e.* $\varepsilon p \geq \varepsilon b$. Therefore, the infimum of Γ's support has to be no less than b, for a firm less productive than b would never attract any worker. Whenever that condition is met, any type p firm will want to hire any type ε unemployed worker upon 'meeting' him on the search market. To this end, the type p firm optimally offers to the type ε unemployed worker the minimum wage $\phi_0(\varepsilon, p)$ that exactly compensates this worker for his opportunity cost of employment, which is defined by

$$V(\varepsilon, \phi_0(\varepsilon, p), p) = V_0(\varepsilon). \tag{1}$$

Because a given employed worker's future employment prospects depend on both the technological level of the firm he works at and his personal ability, the minimum wage at which a type ε unemployed worker is willing to work at a given type p firm depends on p and ε, as shown by equation (1).

When a given type p firm's employee receives an outside offer from a firm type p', the incumbent employer can either counter the offer or just stay passive. Throughout the analysis, we shall assume that outside offers are countered with a given probability θ. In case the offer is matched, denote by $\phi(\varepsilon, p, p')$ the optimal wage that the challenging firm p' wants to propose to a worker (of type ε) employed at a firm with technology p, and that the worker is willing to accept. With our assumption of constant returns to (efficient) labor, the best the firm of type p can do for its employee is to set his wage exactly equal to εp. The highest level of utility the worker can attain by staying at the type p firm is therefore $V(\varepsilon, \varepsilon p, p)$. Accordingly, he accepts to move to a potentially better match with a firm of type p' if the latter offers at least the wage $\phi(\varepsilon, p, p')$ defined by

$$V(\varepsilon, \phi(\varepsilon, p, p'), p') = V(\varepsilon, \varepsilon p, p). \tag{2}$$

Any less generous offer on the part of the type p' firm is successfully countered by the type p firm. It is naturally not always profitable for the type p' firm to offer the wage $\phi(\varepsilon, p, p')$. Specifically, it will become clear that if p' is less than p, then $\phi(\varepsilon, p, p') \geq \varepsilon p'$, in which case the type p' firm will never raise its offer up to this level. Rather, the worker will stay at his current firm, and be promoted to the wage $\phi(\varepsilon, p', p)$ that makes him indifferent between staying and working at the type p' firm.

Now, in case the offer from p' is not matched by p, the challenging firm can attract the worker by merely compensating him for the utility he gets from his current wage, say w, i.e. $V(\varepsilon, w, p)$. The optimal wage offer from p' in this case is denoted by $\psi(\varepsilon, p, p', w)$ and is defined by:

$$V(\varepsilon, \psi(\varepsilon, p, p', w), p') = V(\varepsilon, w, p). \tag{3}$$

Again here, it is not always feasible for p' to offer $\psi(\varepsilon, p, p', w)$. Clearly, if $w \geq \phi(\varepsilon, p', p)$, then the incumbent employer already does better for the worker than what he can at best get by moving to the type p' firm. The worker therefore never moves in such cases.

The next step is to define the value functions $V_0(\cdot)$ and $V(\cdot)$. Since offers accrue to unemployed workers at rate λ_0, $V_0(\varepsilon)$ solves the following Bellman equation:

$$(\rho + \mu + \lambda_0) \cdot V_0(\varepsilon) = \varepsilon b + \lambda_0 \cdot E_p \{V(\varepsilon, \phi_0(\varepsilon, p), p)\}.$$

Using definition (1) to replace $V(\varepsilon, \phi_0(\varepsilon, p), p)$ by $V_0(\varepsilon)$ in the latter equation then shows that:

$$V_0(\varepsilon) = \frac{\varepsilon b}{\rho + \mu}. \tag{4}$$

We thus find that an unemployed worker's expected lifetime utility depends on his personal ability ε only through the amount of output he produces when engaged in home production, εb. This naturally results from the fact that their first employer is able to appropriate the entire surplus generated by the match until the worker gets his first outside offer. The only income the employer originally has to compensate the worker for is εb.

Now turning to employed workers, consider a type ε worker employed at a type p firm and receiving a wage $w \leq \varepsilon p$. This worker is hit by outside offers from competing firms at rate λ_1. If the offer stems from a firm with technology p' such that $\phi(\varepsilon, p', p) \leq w$, then the challenging firm is obviously less attractive to the worker than his current employer since it cannot even offer him his current wage. The worker thus rejects the offer and continues his current employment relationship at an unchanged wage rate. Now if the offer stems from a type p' firm such that $w < \phi(\varepsilon, p', p) \leq \varepsilon p$, then either the offer is matched by p, in which case the challenging firm p' will not be able to attract the worker but the incumbent employer will have to grant the worker a raise—up to $\phi(\varepsilon, p', p)$—to retain him from accepting the other firm's offer, or the offer is not matched and the worker moves to p' for a wage equal to $\psi(\varepsilon, p, p', w)$. The first situation arises with probability θ and leaves the worker with a lifetime utility of $V(\varepsilon, \varepsilon p', p')$, while the second one arises with probability $(1-\theta)$ and leaves the worker's utility unchanged at $V(\varepsilon, w, p)$. Finally, if the offer originates from a firm equipped with a better technology than p, then the worker eventually accepts the outside offer and goes working at the type p' firm for a wage $\phi(\varepsilon, p, p')$ and a utility $V(\varepsilon, \varepsilon p, p)$ if p counters the offer (probability θ), and for a wage $\psi(\varepsilon, p, p', w)$ and an unchanged utility of $V(\varepsilon, w, p)$ if p doesn't counter the offer (probability $1-\theta$).

For a given worker type ε and a given productivity p, define the threshold productivity $q(\varepsilon, w, p)$ by $\phi(\varepsilon, q(\varepsilon, w, p), p) = w$, so that $\phi(\varepsilon, p', p) \leq w$ if $p' \leq q(\varepsilon, w, p)$. Contacts with firms with a technology less productive than $q(\varepsilon, w, p)$ end up not causing any wage increase because the current employer (with a technology yielding productivity p) can outbid such a challenging firm by offering a wage *lower* than w. Since in addition layoffs and deaths still occur at respective rates δ and μ, we may now write the Bellman equation solved by the value function $V(\varepsilon, w, p)$:

$$\begin{aligned}\left[\rho + \delta + \mu + \lambda_1 \overline{\Gamma}(q(\varepsilon, w, p))\right] \cdot V(\varepsilon, w, p) = w \\ + \lambda_1 \theta \cdot [\Gamma(p) - \Gamma(q(\varepsilon, w, p))] \cdot E_{p'}\{V(\varepsilon, \varepsilon p', p') \,|\, q(\varepsilon, w, p) \leq p' \leq p\} \\ + \lambda_1 \theta \cdot \overline{\Gamma}(p) \cdot V(\varepsilon, \varepsilon p, p) + \delta V_0(\varepsilon). \quad (5)\end{aligned}$$

In case the offer from the challenging firm of type $p'$is matched by the incumbent employer of type p, then if $q(\varepsilon, w, p) < p' \leq p$, the incumbent firm can keep its employee but must promote him to the wage $\phi(\varepsilon, p', p)$ such that $V(\varepsilon, \phi(\varepsilon, p', p), p) = V(\varepsilon, \varepsilon p', p')$. If $p' > p$ then the firm of type p' wins the competition and hires the type ε worker at the wage $\phi(\varepsilon, p, p')$ defined by (2): $V(\varepsilon, \phi(\varepsilon, p, p'), p') = V(\varepsilon, \varepsilon p, p)$. In case the offer is not countered, then the worker moves to p' whenever $p' > q(\varepsilon, w, p)$ but his lifetime utility remains unchanged.

Imposing $w = \varepsilon p$ in the latter relationship, we easily get:

$$V(\varepsilon, \varepsilon p, p) = \frac{\varepsilon p + \delta V_0(\varepsilon)}{\rho + \delta + \mu}. \quad (6)$$

Plugging this back into (5), replacing the expectation term by its expression and integrating by parts, we finally get a definition for $V(\cdot)$:

$$(\rho + \delta + \mu) \cdot V(\varepsilon, w, p) = w + \delta V_0(\varepsilon) + \frac{\lambda_1 \theta \varepsilon}{\rho + \delta + \mu} \cdot \int_{q(\varepsilon, w, p)}^{p} \overline{\Gamma}(x)\, dx. \quad (7)$$

We can now derive expressions of the reservation wages $\phi_0(\cdot)$, $\phi(\cdot)$ and $\psi(\cdot)$, as well as the threshold technological index $q(\cdot)$. We begin with the latter for a given productivity p and a given worker type ε. Using (6) and (7) together with the fact that, by definition, $V(\varepsilon, w, p) = V(\varepsilon, \varepsilon q(\varepsilon, w, p), q(\varepsilon, w, p))$, we get an implicit definition of $q(\varepsilon, w, p)$:

$$q(\varepsilon, w, p) - \frac{\lambda_1 \theta}{\rho + \delta + \mu} \cdot \int_{q(\varepsilon, w, p)}^{p} \overline{\Gamma}(x)\, dx = \frac{w}{\varepsilon}. \tag{8}$$

Note that, as intuition suggests, (8) shows that $q(\varepsilon, \varepsilon p, p) = p$. Now consider a pair of technologies $p \leq p'$. Substituting $\phi(\varepsilon, p, p')$ for w in (8), using the fact that $q(\varepsilon, \phi(\varepsilon, p, p'), p') = p$, and rearranging terms, we get:

$$\phi(\varepsilon, p, p') = \varepsilon \cdot \left(p - \frac{\lambda_1 \theta}{\rho + \delta + \mu} \cdot \int_{p}^{p'} \overline{\Gamma}(x)\, dx \right). \tag{9}$$

The last expression brings about some comments. First off, a type ε worker employed at a firm with productivity p, and who receives an alternative offer from a firm of productivity of p' (not necessarily greater than p) will be promoted but never to a wage greater than εp. The worker's current productivity therefore imposes an upper bound on his next promotion and his potential productivity in the challenging firm p' imposes an upper bound on the second next promotion. Therefore, the employee's reservation wage $\phi(\varepsilon, p, p')$ increases with p and falls with p', because to some extent workers are willing to trade a smaller share of the total rent today for a larger share tomorrow. It is thus more difficult to draw a worker out of a more productive firm, and equivalently workers are more easily willing to work at more productive firms.

This in turn has two crucial implications. The first one is that workers may be willing to accept wage cuts, even though they are not threatened of losing their job. Consider for instance the top rank type ε worker in a type p firm, who earns exactly $w = \varepsilon p$, and assume this worker gets an offer from a firm of type $p' \geq p$ which p counters. This worker is thus willing to work at the type p' firm for any wage above $\phi(\varepsilon, p, p')$, which is *strictly less than* *his current wage* εp, according to equation (9). The second key implication of ϕ's properties is that senior workers are predicted by the model to be on average less mobile than junior workers for a given level of personal skills ε. To see this, note that a type ε worker making w in a type p firm is 'upgraded' (*i.e.* either promoted or hired by a better firm) when he receives an offer from a type p' firm such that either $p' \leq p$ and $w \leq \phi(\varepsilon, p', p)$, in which case he gets a raise, or $p' > p$, in which case he goes to the firm of type p'. This makes workers with long tenures, who on average have received more offers and therefore get higher wages in better firms, less likely to receive an attractive offer that would result in an upgrade.

Finally consider a pair of technologies p and p' such that $p' \geq q(\varepsilon, w, p)$. From (3) and (7), one can get an expression for $\psi(\varepsilon, p, p', w)$:

$$\psi(\varepsilon, p, p', w) = w - \frac{\lambda_1 \theta}{\rho + \delta + \mu} \cdot \varepsilon \cdot \int_p^{p'} \overline{\Gamma}(x)\, dx \tag{10}$$

$$= w - \varepsilon p + \phi(\varepsilon, p, p'). \tag{11}$$

The mobility wage $\psi(\cdot)$ for unmatched offers has the same properties as $\phi(\cdot)$: it increases with p, decreases with p' and it can be greater or smaller than w, depending on the relative productivities p and p'. It naturally increases with the current wage w.

We now turn to the unemployed workers' reservation wages $\phi_0(\cdot)$, which are defined by the equality (1). Replacing w by $\phi_0(\varepsilon, p)$ in (5) and noticing that $q(\varepsilon, \phi_0(\varepsilon, p), p) = \varepsilon b$,[3] we get for any given ε:

$$\phi_0(\varepsilon, p) = \phi(\varepsilon, b, p) = \varepsilon \cdot \left(b - \frac{\lambda_1 \theta}{\rho + \delta + \mu} \cdot \int_b^p \overline{\Gamma}(x)\, dx \right). \tag{12}$$

Again, this calls for some comments. First, we see that unemployed workers of all types are prepared to work for a wage $\phi_0(\cdot)$ that is *less* than the opportunity cost of employment

[3] This is shown by the definitions of $q(\cdot)$ and $\phi_0(\cdot)$:

$$V_0(\varepsilon) = V(\varepsilon, \phi_0(\varepsilon, p), p) = V(\varepsilon, \varepsilon q(\cdot), q(\cdot)),$$

which implies from (4) and (6) that $q(\varepsilon, \phi_0(\varepsilon, p), p) = \varepsilon b$.

εb. This is because being employed means not only earning a wage, but also getting better employment prospects. Second, as we naturally expected, $\phi_0(\cdot)$ turns out to be a *decreasing* function of p. Since better matches yield better future job opportunities, they are more attractive to workers and take advantage of this feature by offering lower wages. Third, the reservation wage does not depend on the arrival rate of offers λ_0. In standard search theory, reservations wages do depend on λ_0, because the wage offers are not necessarily equal to the reservation wage. A longer search duration may thus increase the value of the eventually accepted job. Here, this does not happen: Firms always pay the reservation wage to workers; Therefore, there is no gain to expect from rejecting an offer and waiting for the following one.

1.3 Worker flows

Let us denote by $L(\varepsilon, w, p)$ the number of type ε employees at a type p firm earning a wage less than or equal to w, by $L(\varepsilon, p)$ the total number of type ε employees at a type p firm and by $L(p)$ the total workforce of such a firm. It is shown in appendix A that:

$$L(\varepsilon, p) = \frac{\lambda_0 u M}{N} \cdot \frac{\delta + \mu + \lambda_1 \theta}{\left[\delta + \mu + \lambda_1 \theta \overline{\Gamma}(p)\right]^2} \cdot \mathbf{1}(p \geq b) \cdot h(\varepsilon), \tag{13}$$

where $\mathbf{1}(\cdot)$ is the indicator function, and that

$$L(\varepsilon, w, p) = L(\varepsilon, q(\varepsilon, w, p)). \tag{14}$$

Consequently, replacing $L(p) = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} L(\varepsilon, p)\, d\varepsilon$ into (13), we get:

$$L(\varepsilon, p) = h(\varepsilon) \cdot L(p)$$

with

$$L(p) = \frac{L(\varepsilon, p)}{h(\varepsilon)} = \frac{\lambda_0 u M}{N} \cdot \frac{\delta + \mu + \lambda_1 \theta}{\left[\delta + \mu + \lambda_1 \theta \overline{\Gamma}(p)\right]^2} \cdot \mathbf{1}(p \geq b). \tag{15}$$

Note for future use that $L(p)$ increases with p.

Under the current model's assumptions, the distribution of individual heterogeneity within the firms is independent of their types. More productive firms simply hire more workers. This stems from the assumption that the productivity of a match (ε, p) is εp. Nothing thus prevents the formation of highly dissimilar pairs (low ε, high p, or low p, high ε). A might seem more realistic to restrict match formation by imposng worker selection to some extent, which could be done simply e.g. by assuming that, to operate a technology p, a minimal value of ε is required. Yet, this would be an *ad hoc* way of generating assortative mating. Matching theory suggests that assortative mating only rules out additive substitutability. The more fundamental reason why we do not obtain assortative mating here is the assumption of constant returns to labor. A firm has no incentive to select workers upon their quality because it is optimal to hire *anyone* yielding positive profits, since hiring today in no way hampers future recruitment. Extending the current model to allow for diminishing returns to labor is a very promising line of research but clearly out of scope given the additional complexity it implies.

Finally equating aggregate labor market flows pins down the rate of unemployment:

$$u = \frac{\delta + \mu}{\delta + \mu + \lambda_0}. \tag{16}$$

The steady-state assumption has allowed us to compute the various stocks and flows of workers. The model is now completely solved given a particular distribution Γ of productivities. Taking Γ as exogenous may be a sensible assumption in a short-run perspective. However, to the extent that a firm's productivity follows from its investment choices, it certainly should be made endogenous in the longer run.

1.4 Rent sharing

The lowest paid type ε worker in a type p firm is one that has just been hired, therefore earning $\phi_0(\varepsilon, p)$, while the highest-paid type ε worker in that firm earns his marginal productivity εp. Having thus defined the support of the within-firm earnings distribution of type ε

workers for any type p firm, we can readily derive the value of the current operating surplus for such a firm:

$$\pi(p) = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left(\int_{\phi_0(\varepsilon,p)}^{\varepsilon p} (\varepsilon p - w) \cdot L(\varepsilon, dw, p) + (\varepsilon p - \phi_0(\varepsilon, p)) \cdot L(\varepsilon, \phi_0(\varepsilon, p), p) \right) d\varepsilon,$$

where $\int_A^B g(w) L(\varepsilon, dw, p)$ denotes the Stieljes integral of $g(w)$ with respect to the measure $L(\varepsilon, dw, p)$ over the interval $(A, B]$. Note that this measure has a mass point at $w = \phi_0(\varepsilon, p)$ (see Appendix A).

Integrating by parts, we change the latter expression into:

$$\pi(p) = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \int_{\phi_0(\varepsilon,p)}^{\varepsilon p} L(\varepsilon, w, p)\, dw\, d\varepsilon.$$

Using (14) and the change of variables $x = q(\varepsilon, w, p)$, we get a new expression for $\pi(p)$,[4]

$$\pi(p) = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \int_b^p \varepsilon L(\varepsilon, x) \cdot \varepsilon \left(1 + \frac{\lambda_1 \theta \overline{\Gamma}(x)}{\rho + \delta + \mu}\right) dx\, d\varepsilon \tag{17}$$

$$= \frac{\lambda_0 u M}{N} \cdot \int_b^p \frac{\delta + \mu + \lambda_1 \theta}{\left[\delta + \mu + \lambda_1 \theta \overline{\Gamma}(x)\right]^2} \cdot \left(1 + \frac{\lambda_1 \theta \overline{\Gamma}(x)}{\rho + \delta + \mu}\right) dx, \tag{18}$$

in which we have used (13) and since, by convention, $\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon h(\varepsilon)\, d\varepsilon = 1$. Note that the current operating surplus $\pi(p)$ is thus an increasing and continuous function.

A parameter of interest is the following: on an average how much a worker can hope to be paid when employed? Individual earnings processes being stationary and ergodic, one can compute this number by evaluating the mean wage of a cross-section of many employees:

$$E_G w \equiv \int_{\underline{p}}^{\overline{p}} \left[p - \frac{\pi(p)}{L(p)}\right] \frac{L(p)\gamma(p)}{\overline{L}} dp$$

where $\overline{L}$ is the average firm size $(1-u)M/N$ and $\frac{L(p)\gamma(p)}{\overline{L}}$ is the density of firm types in a large cross-section of workers (each firm p is weighted by how many workers it employs).

[4]Since

$$\frac{\partial q(\varepsilon, w, p)}{\partial w} = \frac{1}{\varepsilon\left[1 + \frac{\lambda_1 \theta}{\rho + \delta + \mu} \overline{\Gamma}(q(\varepsilon, w, p))\right]}.$$

After some straightforward (but cumbersome) algebra, one finally obtains:

$$E_G w = b + \frac{\rho}{\rho + \delta + \mu} \int_{\underline{p}}^{\overline{p}} \frac{(\delta + \mu + \lambda_1 \theta)(\delta + \mu) \overline{\Gamma}(p)^2 \, dp}{\left[\delta + \mu + \lambda_1 \theta \overline{\Gamma}(p)\right]^2}, \tag{19}$$

which shows that when ρ gets small, $E_G w$ approaches b. Firms exploit the long-sightedness of workers to make them give initially ($\phi_0(\varepsilon, p) < b$) all the gains they can later expect from firm competition for employees. A result similar to Diamond's (1971) monopsony wage therefore emerges: when firms can fully take advantage of their monopsony power on unemployed workers, they can recover a large part of the rent they are constrained to yield to workers because of on-the-job search. Our model thus fills the gap between Diamond's equilibrium search with no search on the job (and Albrecht and Axell's (1984) extensionto heterogenous workers and firms), and Burdett and Mortensen's (1989) wage posting model with on-the-job search.

1.5 Equilibrium productivity dispersion

We can now endogenize the productivity parameter p along the lines of Postel-Vinay and Robin (1999). Equilibrium productivity dispersion arises from the firms' dispersed investment choices. Assume firms are *ex ante* identical and endowed with a technology exhibiting constant returns to labor and decreasing returns to capital. More specifically, the output per efficiency unit of labor of a firm with a capital stock of k is $p = k^\alpha$, with $0 < \alpha < 1$.[5]

With this specification, and taking the user cost of capital as an exogenous constant r, the final level of profit made by a type p employer is $\Pi(p) = \pi(p) - rp^{1/\alpha}$. In equilibrium, all firms must make the same (maximal) profit, say Π^*. The following thus holds in equilibrium:

$$\begin{cases} \pi(p) - rp^{1/\alpha} \equiv \Pi^* & \text{for all } p \in \text{supp}(\Gamma), \\ \pi(p) - rp^{1/\alpha} < \Pi^* & \text{otherwise.} \end{cases}$$

[5] This specification is borrowed from Acemoglu and Shimer (1997). Also, Robin and Roux (1998) estimate the coefficients of a Cobb-Douglas production function $K^\alpha \cdot L^\beta$ on firm data and find that β is roughly equal to one while α is much smaller, somewhere between 0 and .1 depending on the particular sector considered.

Since $\pi(p) - rp^{1/\alpha}$ is a constant on the support of Γ, and since π is differentiable, it is therefore true that:

$$\pi'(p) = \frac{r}{\alpha} \cdot p^{(1-\alpha)/\alpha}, \tag{20}$$

which is equivalent to:

$$\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} L(\varepsilon, p) \cdot \left[1 + \frac{\lambda_1 \theta \overline{\Gamma}(p)}{\rho + \delta + \mu}\right] d\varepsilon = \frac{r}{\alpha} \cdot p^{(1-\alpha)/\alpha},$$

that is to say, with (13):

$$\frac{\lambda_0 u M}{N} \cdot \frac{\delta + \mu + \lambda_1 \theta}{\left[\delta + \mu + \lambda_1 \theta \overline{\Gamma}(p)\right]^2} \cdot \frac{\rho + \delta + \mu + \lambda_1 \theta \overline{\Gamma}(p)}{\rho + \delta + \mu} = \frac{r}{\alpha} \cdot p^{(1-\alpha)/\alpha}$$

This is a quadratic equation in $\overline{\Gamma}(p)$ which can easily be solved:

$$\begin{aligned} \delta + \mu + \lambda_1 \theta \overline{\Gamma}(p) &= \frac{\frac{\lambda_0 u M}{N}(\delta + \mu + \lambda_1 \theta)}{2(\rho + \delta + \mu)\frac{r}{\alpha} \cdot p^{(1-\alpha)/\alpha}} \left[1 + \sqrt{1 + 4\rho \frac{(\rho + \delta + \mu)\frac{r}{\alpha} \cdot p^{(1-\alpha)/\alpha}}{\frac{\lambda_0 u M}{N}(\delta + \mu + \lambda_1 \theta)}}\right] \\ &= \frac{2\rho}{\sqrt{1 + 4\rho \frac{(\rho + \delta + \mu)\frac{r}{\alpha}}{\frac{\lambda_0 u M}{N}(\delta + \mu + \lambda_1 \theta)} p^{(1-\alpha)/\alpha}} - 1}. \end{aligned} \tag{21}$$

The bounds of Γ's support solve $\overline{\Gamma}(\underline{p}) = 1$ and $\overline{\Gamma}(\overline{p}) = 0$ respectively.[6]

Note that this discussion is empirically useful because it provides a natural specification of the distribution ofthe firm heterogeneity parameter p. It shows that when $\overline{\Gamma}(p)$ has the form in (21) then firms' current operating surplus is

$$\begin{aligned} \pi(p) &= rp^{1/\alpha} + \pi(\underline{p}) - r\underline{p}^{1/\alpha} \\ &= rp^{1/\alpha} + \frac{r}{\alpha}\underline{p}^{(1-\alpha)/\alpha}\left[(1-\alpha)\underline{p} - b\right] \end{aligned}$$

[6]Since the beginning, we have been taking for granted that the support of γ has the form $[\underline{p}, \overline{p}]$, *i.e.* is an interval. The above analysis confirms this conjecture. Also, we have been working under the assumption that $\Gamma(\cdot)$ had no mass points. In Postel-Vinay and Robin (1999, proposition 1), we show that it is indeed true in equilibrium. The reason why this is the case is easy to spell out. If a mass of firms choose the same p then there exists a mass of employed workers who can be attracted by a firm with a greater productivity. Consequently, the current operating surplus increases faster right after p than just before. Since $rp^{1/\alpha}$ is continuously differentiable, it is therefore impossible that p be an optimal response of any firm to all other firms behaving according to a distribution of productivities exhibiting a mass point at p. It thus follows that the equilibrium distribution of productivities is continuous, hence Γ and L are differentiable.

and the the wage bill of a firm of type p is

$$\begin{aligned} c(p) &= pL(p) - \pi(p) \\ &= pL(p) - rp^{1/\alpha} - \frac{r}{\alpha}\underline{p}^{(1-\alpha)/\alpha}\left[(1-\alpha)\underline{p} - b\right] \end{aligned}$$

which provides a particularly simple and attractive natural specification of mean wage per firm for empirical analysis. Specification (21) can be adopted without assuming that p results from capital choice and *ex ante* profit maximisation.

2 Structural estimation

2.1 Description of the DAS data

The "Déclarations Annuelles des Salaires" dataset is a large collection of matched employer-employee information collected by the Income Division of the French Statistical Institute INSEE (*Institut National de la Statistique et des Etudes Economiques — Division des Revenus*). The data are based on a mandatory employer report of the gross earnings of each salaried employee of the private sector subject to French payroll taxes. Our analysis sample covers all individuals employed in French enterprises who were born in October of even-numbered years. Our extract runs from 1995 through 1997, our last available survey. We have deliberately selected a much shorter period than is available because we want to find out whether it is possible to estimate our structural model on an homogeneous period of the business cycle. Moreover, firm size is only available from 1995 onward. Each observation corresponds to a unique enterprise-individual-year combination. The observation includes an identifier that corresponds to the employee and an identifier that corresponds to the establishment. For each observation, we have information on the number of days during the calendar year the individual worked at the establishment, as well as the full-time/part-time/intermittent/at-home work-status of the employee. Each observation also includes, in addition to the variables listed above, the sex, month, year and place of birth, occupation, total net nominal earnings during the year and annualized gross nominal earnings during the year for the individual, as well as the location (*département*) and industry of the employing establishment.

The sample provides individual wage bills reported by the employers on a yearly basis. We know for example that worker i was employed by the firm-establishment j during d days in 1995 within a time interval beginning this day of 1995 and ending that day of 1995. A trajectory featuring an employer change may be such that the end of one employment spell does not coincide with the beginning of the next one, and a worker may also leave the panel before the end of the recording period. There is no way of knowing the status of the worker during such periods not covered by a wage statement. He/she may have permanently or temporarily quit participating, or be unemployed, or have found a job in the Public Sector, or have started up his/her own business. In the estimation, we shall interpret temporary attrition as resulting from layoffs (with instantaneous probability δdt) and permanent attrition as resulting either from layoffs or from retirements (with instantaneous probability μdt).

We may have several wages recorded for the same individual in the same firm-establishment if the worker stays employed by one firm for more than one year. Unfortunately, there is no way to know exactly at which moment he/she received a wage increase if the daily wage reported one year is greater than the one reported the year before. Finally, it frequently happens that real wages decrease from one year to the next even if the worker has not changed employers. This may reflect fluctuations of bonuses with the firm's activity since there is no way of separating contractual wages from bonuses which in some cases may be a non negligible share of salaries. Wage changes may also reflect occupation changes within the same establishment and compensating differentials. These wage fluctuations could be captured in an *ad hoc* way by a pure idiosyncratic shock. We preferred to estimate the sructural model as it was laid out in the preceding sections at the price of a lack of fit because our main goal here is precisely to evaluate the capacity of the structural model to reproduce the main features of the wage dynamics. Incorporating productivity fluctuations in the model is certainly not a straightforward extension of the model as we know that it could generate job destruction as in Mortensen and Pissarides (1994) for example.

2.2 Parametric specification

To keep things as simple as possible, we shall estimate the model under the assumption that the psychological discount rate ρ is nil and that the distribution of firm productivities satisfies the equilibrium characterization (21). Under those assumptions, the model predicts that:

- the productivity distribution is Pareto: for all $p \in [\underline{p}, \overline{p}]$,

$$\delta + \mu + \lambda_1 \theta \overline{\Gamma}(p) = \frac{\lambda_0 u M}{N} \cdot \frac{\delta + \mu + \lambda_1 \theta}{(\delta + \mu) \frac{r}{\alpha} \cdot p^{(1-\alpha)/\alpha}}$$

  or again, introducing $\kappa_1 = \lambda_1 / (\delta + \mu)$ and $\overline{L} = (1-u)M/N$ the mean labor force per active firm,

$$1 + \kappa_1 \theta \overline{\Gamma}(p) = \overline{L}(1 + \kappa_1 \theta) \frac{\alpha}{r} p^{1-1/\alpha},$$

  since in a steady state equilibrium $\lambda_0 u = (\delta + \mu)(1-u)$;

- the support of firm productivities is $[\underline{p}, \overline{p}]$ with

$$\begin{aligned} \Gamma(\underline{p}) &= 0 \Longleftrightarrow \underline{p} = \left(\frac{\alpha \overline{L}}{r}\right)^{\frac{\alpha}{1-\alpha}}, \\ \Gamma(\overline{p}) &= 1 \Longleftrightarrow \overline{p} = \left(\frac{\alpha \overline{L}(1 + \kappa_1 \theta)}{r}\right)^{\frac{\alpha}{1-\alpha}}; \end{aligned}$$

- the labor force of a firm of type p is

$$\begin{aligned} L(p) &= \frac{\lambda_0 u M}{N} \cdot \frac{\delta + \mu + \lambda_1 \theta}{\left[\delta + \mu + \lambda_1 \theta \overline{\Gamma}(p)\right]^2} \\ &= \overline{L} \cdot \frac{1 + \kappa_1 \theta}{\left[1 + \kappa_1 \theta \overline{\Gamma}(p)\right]^2} \\ &= \frac{1}{\overline{L}(1 + \kappa_1 \theta)} \left(\frac{r}{\alpha}\right)^2 \cdot p^{2(1-\alpha)/\alpha}; \end{aligned}$$

- firms' profit then becomes:

$$\pi(p) = rp^{1/\alpha} + \overline{L}\left[(1-\alpha)\underline{p} - b\right];$$

- the distribution of firm sizes in the population of active firms, given that $\overline{\Gamma}(p)$ is uniformly distributed over $[0,1]$, has density

$$f(\ell) = \frac{\sqrt{\overline{L}\,(1+\kappa_1\theta)}}{\kappa_1\theta} \cdot \frac{1}{2\ell^{3/2}},$$

  with support $[\underline{\ell}, \overline{\ell}]$ where

$$\begin{aligned} \underline{\ell} &= L(\underline{p}) = \frac{\overline{L}}{1+\kappa_1\theta}, \\ \overline{\ell} &= L(\overline{p}) = \overline{L}\,(1+\kappa_1\theta); \end{aligned}$$

- the distribution $H(\varepsilon)$ of worker types ε within each firm is independent of the employer type p and can be specified as one wants, up to the assumed normalization of its mean;

- the wage offered to a formerly unemployed worker of type ε by a type-p firm is

$$\begin{aligned} \phi_0(\varepsilon, p) &= \varepsilon\left[b - \kappa_1\theta \int_b^p \overline{\Gamma}(x)\,dx\right] \\ &= \varepsilon\left[b(1+\kappa_1\theta) - \kappa_1\theta\underline{p} - \kappa_1\theta \int_{\underline{p}}^p \overline{\Gamma}(x)\,dx\right]; \end{aligned}$$

- the wage resulting from the competition between a type-p firm and a type-p' firm ($p' > p$) is:

$$\begin{aligned} \phi(\varepsilon, p, p') &= \varepsilon\left[p - \kappa_1\theta \int_p^{p'} \overline{\Gamma}(x)\,dx\right] \\ &= \varepsilon\left[p' - \int_p^{p'} \left[1 + \kappa_1\theta\overline{\Gamma}(x)\right]\,dx\right] \\ &= \varepsilon\left[p' - \overline{L}\,(1+\kappa_1\theta)\,\frac{\alpha}{r} \cdot \frac{(p')^{2-1/\alpha} - p^{2-1/\alpha}}{2-1/\alpha}\right]; \end{aligned}$$

- the wage a worker can get in a type-p' firm when the worker is paid w by a type-p firm which decides not to match the alternative offer is:

$$\begin{aligned} \psi\left(\varepsilon, p, p', w\right) &= \varepsilon\left[\frac{w}{\varepsilon} - \kappa_1 \theta \int_p^{p'} \overline{\Gamma}(x)\ dx\right] \\ &= w - \varepsilon p + \phi\left(\varepsilon, p, p'\right). \end{aligned}$$

2.3 Estimation

The complicated nature of the data (namely that only annual wages are observed) and the fact that the distributions of firm sizes and productivities have bounded supports[7], rules out maximum likelihood as a potential candidate for the estimation of the model. Moreover parameters strongly interact with each other. One should therefore be cautious not to estimate simultaneously all parameters without being assured that each parameter is separately identified by a specific data source. Since here efficiency is not an issue (we have a potentially infinite source of DADS data), if we can estimate one parameter independently of the others, we do it.

We proceed as follows: first we estimate the in-to and out-of sample transition parameters δ, μ and λ_0 from a cohort of workers initially employed and whom we follow until they either change employer or quit the survey. Second, we estimate the remaining parameters from a cross-section of individual observations of wages and employer sizes. Finally, we tune these estimates using a simulated method of moments based on dynamic moments such as the covariance between two consecutive wages or between current wages and present and lagged employer sizes. Note that each estimation algorithm was first tested and verified working using Monte Carlo simulation.

[7] With bounds which cannot be directly estimated. The model is certainly not very good at predicting minimum and maximum firm sizes. In the data the minimum firm size is one and the inverse mean size is likely not a good estimate of $1 + \kappa_1 \theta$!

2.3.1 Consistent estimation of in-to and out-of sample transition parameters δ, μ and λ_0

The recording period starts at time 0 (namely January 1st, 1995) and ends at time T (namely December 31st, 1997). All the N sampled individuals are employed at the beginning of the observation period. Define d_{i1} as the length of time idividual i spends employed (by one or several employers) before exiting recorded employment (toward unemployment, inactivity, selfemployment or government jobs, whatsoever) or until the end of the recording period. If individual i stays employed without interruption let $d_{i2} = 0$. Otherwise, let d_{i2} be the length of time spent out of the sample before re-entry or until the end of the recording period. We shall estimate δ, μ and λ_0 by maximizing the likelihood of the N observations (d_{i1}, d_{i2}).[8]

Employees face two independent risks that may make them leave the employment activities covered by the DADS data: lay-off, occuring at rate δ, and attrition (i.e. inactivity, selfemployment or government jobs), at rate μ. The attrition risk also applies to unemployed workers who can yet re-enter the sample by finding a new job at rate λ_0. Let (E_i, U_i, D_i) be a triple of independent random variables following three exponential distributions with parameters δ, μ and λ_0. We have

$$d_{i1} \stackrel{d}{=} \min(E_i, D_i, T)$$

where $\stackrel{d}{=}$ means "‘equal in distribution", and

$$\begin{aligned} d_{i2} &\stackrel{d}{=} T - d_{i1} && \text{if } E_i + U_i > \min(D_i, T) \\ &\stackrel{d}{=} U_i && \text{if } E_i + U_i \leq \min(D_i, T) \end{aligned}$$

or again:

$$\begin{aligned} \begin{pmatrix} d_{i1} \\ d_{i2} \end{pmatrix} &\stackrel{d}{=} \begin{pmatrix} \min(D_i, T) \\ T - \min(D_i, T) \end{pmatrix} && \text{if } E_i \geq \min(D_i, T) \\ &\stackrel{d}{=} \begin{pmatrix} E_i \\ T - E_i \end{pmatrix} && \text{if } E_i + U_i > \min(D_i, T) > E_i \\ &\stackrel{d}{=} \begin{pmatrix} E_i \\ U_i \end{pmatrix} && \text{if } E_i + U_i \leq \min(D_i, T). \end{aligned}$$

[8] Note that we could use more in- and out-of-sample spells than the first two.

Note that it is also true that $d_{i1} = \min(\min(E_i, D_i), T)$ where $\min(E_i, D_i)$ is a random variable following an exponential distribution with parameter $\delta + \mu$. In particular, $Ed_{i1} = \frac{1-\exp(-(\delta+\mu)T)}{\delta+\mu}$, rendering possible a consistent estimation of $\delta + \mu$ by a method of moments.

We have

$$\Pr\{\min(D_i, T) = d_{i1} \text{ and } E_i \geq \min(D_i, T)\} = \mathbf{1}(d_{i1} = T) \cdot e^{-\mu T} e^{-\delta T} + \mathbf{1}(d_{i1} < T) \cdot \mu e^{-\mu d_{i1}} e^{-\delta d_{i1}},$$

and given $E_i = d_{i1} < T$,

$$\begin{aligned}
\Pr\{d_{i1} &< \min(D_i, T) < d_{i1} + U_i\} \\
&= \Pr\{U_i > T - d_{i1} \text{ and } D_i > T\} + \Pr\{U_i > D_i - d_{i1} \text{ and } d_{i1} < D_i \leq T\} \\
&= e^{-\mu T} e^{-\lambda_0 (T - d_{i1})} + \int_{d_{i1}}^{T} \mu e^{-\mu x} e^{-\lambda_0 (x - d_{i1})} dx \\
&= e^{-\mu T} e^{-\lambda_0 (T - d_{i1})} + \mu \frac{e^{-\mu d_{i1}} - e^{-\mu T} e^{\lambda_0 (T - d_{i1})}}{\mu + \lambda_0} \\
&= \left[\lambda_0 e^{-(\lambda_0 + \mu) d_{i2}} + \mu\right] \frac{e^{-\mu d_{i1}}}{\lambda_0 + \mu}.
\end{aligned}$$

The likelihood of one observation (d_{i1}, d_{i2}) then follows as

$$\begin{aligned}
\ell(d_{i1}, d_{i2}) &= \mathbf{1}(d_{i1} + d_{i2} < T) \cdot \delta e^{-(\delta+\mu) d_{i1}} \cdot \lambda_0 e^{-(\lambda_0 + \mu) d_{i2}} \\
+\mathbf{1}(d_{i1} &< T, d_{i2} = T - d_{i1}) \cdot \left[\frac{\delta}{\lambda_0 + \mu}\left(\lambda_0 e^{-(\lambda_0+\mu) d_{i2}} + \mu\right) + \mu\right] \cdot e^{-(\delta+\mu) d_{i1}} \\
+\mathbf{1}(d_{i1} &= T, d_{i2} = 0) \cdot e^{-(\delta+\mu)T}.
\end{aligned}$$

2.3.2 Estimation of the remaining parameters from a cross-section of wages/firm size data

Given a cross-section of individual wages and employer's sizes, one can obtain consistent estimates of $\overline{L}$, $\kappa_1 \theta$, α and r in the following way. Contrary to the previous case, we cannot easily resort to maximum likelihood since the support of these cross-sectional distributions depend on the parameters of interest. We therefore use moment-based estimation methods. A consistent estimate of $\overline{L}$ and $\kappa_1 \theta$ is thus straightforward to obtain from the distribution of employer sizes, using the first two moments of this distribution. Remember that the estimation sample is a sample of workers, not firms: each observation of firm size has therefore to be weighted by the number of times it is replicated in the sample which is precisely proportional to the firm size. The cross-sectional firm size mean should therefore converge to

$$\begin{aligned} E(L) &= \int_{\underline{p}}^{\overline{p}} L(p) \cdot \frac{L(p)\gamma(p)}{\overline{L}} dp \\ &= \overline{L} \frac{(1+\kappa_1\theta)^2}{3\kappa_1\theta} \int_{\underline{p}}^{\overline{p}} \frac{3\kappa_1\theta\gamma(p)dp}{\left[1+\kappa_1\theta\overline{\Gamma}(p)\right]^4} \\ &= \overline{L} \frac{(1+\kappa_1\theta)^2}{3\kappa_1\theta} \left[1 - \frac{1}{(1+\kappa_1\theta)^3}\right] \\ &= \overline{L} \frac{3+3\kappa_1\theta+(\kappa_1\theta)^2}{3(1+\kappa_1\theta)}, \end{aligned}$$

where $\frac{L(p)\gamma(p)}{\overline{L}}$ is the density of firm types in the sample of employed workers; the second-order moment converges to

$$\begin{aligned} E\left(L^2\right) &= \int_{\underline{p}}^{\overline{p}} L(p)^2 \cdot \frac{L(p)\gamma(p)}{\overline{L}} dp \\ &= \overline{L}^2 \frac{(1+\kappa_1\theta)^3}{5\kappa_1\theta} \left[1 - \frac{1}{(1+\kappa_1\theta)^5}\right] \\ &= \overline{L}^2 \frac{[5+10\kappa_1\theta+10(\kappa_1\theta)^2+5(\kappa_1\theta)^3+(\kappa_1\theta)^4]}{5(1+\kappa_1\theta)^2} \end{aligned}$$

and the variance to

$$\begin{aligned} V(L) &= E(L^2) - E(L)^2 \\ &= \overline{L}^2 (\kappa_1\theta)^2 \frac{3+3\kappa_1\theta+\frac{4}{5}(\kappa_1\theta)^2}{9(1+\kappa_1\theta)^2}. \end{aligned}$$

It is also possible to obtain a consistent estimate of b from the wage mean (equation (19)), and an estimate of r and α by regressing earnings on firm sizes. The mean wage per firm of type p is indeed:

$$\begin{aligned} E(w|p) &= p - \frac{\pi(p)}{L(p)} \\ &= p - r\frac{p^{1/\alpha}}{L(p)} - \frac{\overline{L}\left[(1-\alpha)\underline{p} - b\right]}{L(p)} \end{aligned}$$

with

$$p = \left[\overline{L} \left(1 + \kappa_1 \theta\right) \left(\frac{\alpha}{r}\right)^2 \cdot L(p) \right]^{\frac{\alpha}{2(1-\alpha)}}.$$

Practically, we estimate r and α by non linear least squares:

$$(\widehat{r}, \widehat{\alpha}) = \arg \min_{(r,\alpha,\pi^*)} \sum_i \left[p(r, \alpha; \ell_i) - w_i - r \frac{p(r, \alpha; \ell_i)^{1/\alpha}}{\ell_i} - \frac{\pi^*}{\ell_i} \right]^2.$$

TO BE CONTINUED.

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Appendix

A Derivation of $L(\varepsilon, w, p)$

The workers of type ε paid less than w by firms of type p leave this category either because they are laid off—which occurs at rate δ—or because they die—which occurs at rate μ—or finally because they receive an offer which grants them a wage increase or induces them to move to a different firm. From previous paragraphs, we see that only those type ε workers who receive an offer from a firm no less productive than $q(\varepsilon, w, p) \leq p$ will either see their wage raised above w, or leave their type p employer. They receive those offers at rate $\lambda_1 \overline{\Gamma}(q(\varepsilon, w, p))$, and a proportion θ of those offers are countered. The total outflow of such workers from p is therefore equal to:

$$\left\{\delta + \mu + \lambda_1 \theta \overline{\Gamma}(q(\varepsilon, w, p))\right\} \cdot L(\varepsilon, w, p) + \lambda_1 (1-\theta) \cdot \left(\int_{\phi_0(\varepsilon,p)}^{w} \overline{\Gamma}(q(\varepsilon, x, p)) \cdot L(\varepsilon, dx, p) + L(\varepsilon, \phi_0(\varepsilon, p), p) \right). \quad (22)$$

The first term in the above equation corresponds to layoffs, deaths and workers having been hit by countered outside offers, while the second term corresponds to workers leaving because they have received unmatched offers.[9]

On the inflow side, workers entering the category (ε, w, p) come from three sources. Either they are hired away from a firm operating a technology less productive than $q(\varepsilon, w, p)$ that has matched the offer made by the type p firm at hand, or they are hired away from a firm of any type after an unmatched offer from p, or finally they come out of unemployment. Let $L(\varepsilon, p) = L(\varepsilon, \varepsilon p, p)$ denote the total number of type ε employees at a type p firm (the highest paid worker of type ε in such a firm earns exactly $w = \varepsilon p$), and let u be the total rate of unemployment. Summing the three sources, we get the total inflow into the category $L(\varepsilon, w, p)$:

$$\frac{\lambda_0 u M}{N} \cdot h(\varepsilon) \cdot \mathbf{1}(w \geq \phi_0(\varepsilon, p)) + \lambda_1 \theta \cdot \int_{\underline{p}}^{q(\varepsilon,w,p)} L(\varepsilon, x)\, d\Gamma(x) + \lambda_1 (1-\theta) \cdot \int_{\underline{p}}^{\overline{p}} L(\varepsilon, \psi(\varepsilon, p, x, w), x)\, d\Gamma(x). \quad (23)$$

The first term counts the number of type ε unemployed workers hired each period. The indicator function $\mathbf{1}(w \geq \phi_0(\varepsilon, p))$ is needed here because no type ε unemployed worker would accept a wage w striclty less than $\phi_0(\varepsilon, p)$, and all would accept any wage $w \geq \phi_0(\varepsilon, p)$. Note for future use that this term can take the equivalent form $\mathbf{1}(q(\varepsilon, w, p) \geq b)$. The second term counts the workers lost by firms of all types less than $q(\varepsilon, w, p)$ to the type p firm after they have unsuccessfully countered its offer. Finally, the last term counts the workers lost to the type p firm by firms of all types when the type p firm's offer has not been matched. Consider indeed a worker who is currently employed

[9]This is implicitely assuming that $w \geq \phi_0(\varepsilon, p)$, since $\phi_0(\varepsilon, p)$ is the smallest wage a firm of type p can pay to a worker of type ε. Therefore, as will become clear below, $L(\varepsilon, w, p) = 0$ for all $w < \phi_0(\varepsilon, p)$. Also note that, as will become clear below as well, there is a mass of type ε workers employed at the wage $\phi_0(\varepsilon, p)$. Finally, since $q(\varepsilon, \phi_0(\varepsilon, p), p) = b$, $\overline{\Gamma}(q(\varepsilon, \phi_0(\varepsilon, p), p)) = \overline{\Gamma}(b) = 1$.

by a firm of type x at a wage w'. Any firm with productivity p contacting this worker may hire him at the wage $\psi(\varepsilon, x, p, w')$ if his current employer does not respond to poaching (with probability $1-\theta$). Now

$$\begin{aligned}\psi\left(\varepsilon, x, p, w'\right) \quad &\leq \quad w \Longleftrightarrow w' - \frac{\lambda_1 \theta}{\rho+\delta+\mu} \cdot \varepsilon \cdot \int_x^p \overline{\Gamma}(x)\, dx \leq w \\ &\Longleftrightarrow \quad w' \leq w + \frac{\lambda_1 \theta}{\rho+\delta+\mu} \cdot \varepsilon \cdot \int_x^p \overline{\Gamma}(x)\, dx \\ &\Longleftrightarrow \quad w' \leq \psi(\varepsilon, p, x, w).\end{aligned}$$

It thus follows that a firm p hires at a wage lower than w any worker paid less than $\psi(\varepsilon, p, x, w)$ in an firm x which does not counter the offer.

The stationarity of $L(\varepsilon, w, p)$ implies that (22) must equal (23), which yields a complex differential equation solves by $L(\varepsilon, w, p)$. This equation is in fact much simpler than it looks, as we shall now see.

First remark that integrating by parts and using the change of variables $z = q(\varepsilon, x, p)$ in (22) allows to rewrite (22) as:

$$\left\{\delta+\mu+\lambda_1 \overline{\Gamma}(q(\varepsilon, w, p))\right\} \cdot L(\varepsilon, w, p) + \lambda_1 (1-\theta) \cdot \int_{\underline{p}}^{q(\varepsilon, w, p)} L(\varepsilon, \phi(\varepsilon, z, p), p)\, d\Gamma(z) \tag{24}$$

Second, note from (10) that, for all $x \leq q(\varepsilon, w, p)$, $\psi(\varepsilon, p, x, w) \geq \varepsilon x$.[10] This implies that for all $x \leq q(\varepsilon, w, p)$, $L(\varepsilon, \psi(\varepsilon, p, x, w), x) = L(\varepsilon, x)$. This allows to rewrite (23) as:

$$\begin{aligned}&\frac{\lambda_0 u M}{N} \cdot h(\varepsilon) \cdot \mathbf{1}(q(\varepsilon, w, p) \geq b) + \lambda_1 \cdot \int_{\underline{p}}^{q(\varepsilon, w, p)} L(\varepsilon, x)\, d\Gamma(x) \\ &\qquad + \lambda_1 (1-\theta) \cdot \int_{q(\varepsilon, w, p)}^{\overline{p}} L(\varepsilon, \psi(\varepsilon, p, x, w), x)\, d\Gamma(x).\end{aligned} \tag{25}$$

Next, we may also notice from (8), (9) and (10), that for any (ε, w, p, p'):

$$\begin{aligned}\phi\left(\varepsilon, q(\varepsilon, w, p), p'\right) &= \overbrace{\varepsilon q(\varepsilon, w, p) - \frac{\lambda_1 \theta \varepsilon}{\rho+\delta+\mu} \cdot \int_{q(\varepsilon, w, p)}^{p} \overline{\Gamma}(x)\, dx}^{=w \text{ from (8)}} - \frac{\lambda_1 \theta \varepsilon}{\rho+\delta+\mu} \cdot \int_p^{p'} \overline{\Gamma}(x)\, dx \\ &= \psi\left(\varepsilon, p, p', w\right).\end{aligned}$$

That is: $\psi(\varepsilon, p, p', w)$ is the wage paid by firm p' to a worker ε yielding the same expected income flow as being paid w at firm p, which equivalent to being paid $q(\varepsilon, w, p)/\varepsilon$ at firm $q(\varepsilon, w, p)$, which in turn is the same as being paid $\phi(\varepsilon, q(\varepsilon, w, p), p')$ at firm p'.

[10] This can be seen by noticing that $\psi(\varepsilon, p, x, w) = w + \varepsilon x - \phi(\varepsilon, x, p)$, which equals εx for $x = q(\varepsilon, w, p)$.

(25) can thus be rewritten as:

$$\frac{\lambda_0 u M}{N} \cdot h(\varepsilon) \cdot \mathbf{1}\left(q(\varepsilon, w, p) \geq b\right) + \lambda_1 \cdot \int_{\underline{p}}^{q(\varepsilon,w,p)} L(\varepsilon, x)\ d\Gamma(x)$$
$$+ \lambda_1 (1-\theta) \cdot \int_{q(\varepsilon,w,p)}^{\overline{p}} L\left(\varepsilon, \phi\left(\varepsilon, q(\varepsilon, w, p), x\right), x\right)\ d\Gamma(x). \quad (26)$$

Now looking at a the specific case where $w = \varepsilon p$ is the maximal wage that firm p could pay to a worker ε, we get an equation defining $L(\varepsilon, \varepsilon p, p) = L(\varepsilon, p)$. If we notice that $q(\varepsilon, \varepsilon p, p) = p$ and that $\psi(\varepsilon, p, x, \varepsilon p) = \phi(\varepsilon, p, x)$, equating (24) and (26) yields:

$$\left\{\delta + \mu + \lambda_1 \overline{\Gamma}(p)\right\} \cdot L(\varepsilon, p) + \lambda_1 (1-\theta) \cdot \int_{\underline{p}}^{p} L\left(\varepsilon, \phi(\varepsilon, x, p), p\right)\ d\Gamma(x)$$
$$= \frac{\lambda_0 u M}{N} \cdot h(\varepsilon) \cdot \mathbf{1}(p \geq b) + \lambda_1 \cdot \int_{\underline{p}}^{p} L(\varepsilon, x)\ d\Gamma(x) + \lambda_1 (1-\theta) \cdot \int_{p}^{\overline{p}} L\left(\varepsilon, \phi(\varepsilon, p, x), x\right)\ d\Gamma(x).$$

Comparing the two sides of this last equation with (24) and (26) makes clear the following important result (equation (14) in the main text):

$$\text{for all } (\varepsilon, w \leq \varepsilon p, p),\ L(\varepsilon, w, p) = L\left(\varepsilon, q(\varepsilon, w, p)\right),$$

since those two numbers solve the same equation. It follows that

$$L\left(\varepsilon, \phi(\varepsilon, x, p), p\right) = L(\varepsilon, x) \quad \text{and} \quad L\left(\varepsilon, \phi\left(\varepsilon, q(\varepsilon, w, p), x\right), x\right) = L\left(\varepsilon, q(\varepsilon, w, p)\right) = L(\varepsilon, w, p).$$

Substituting the last two equalities in (24) and (26) respectively, and equating the two yields a final expression for the flow balance equation at the firm level:

$$\left\{\delta + \mu + \lambda_1 \theta \overline{\Gamma}\left(q(\varepsilon, w, p)\right)\right\} \cdot L(\varepsilon, w, p) = \frac{\lambda_0 u M}{N} \cdot h(\varepsilon) \cdot \mathbf{1}\left(q(\varepsilon, w, p) \geq b\right) + \lambda_1 \theta \cdot \int_{\underline{p}}^{q(\varepsilon,w,p)} L(\varepsilon, x)\ d\Gamma(x).$$

Again looking at the case $w = \varepsilon p$, we get:

$$\frac{\partial}{\partial p}\left[\left\{\delta + \mu + \lambda_1 \theta \overline{\Gamma}(p)\right\} \cdot \int_{\underline{p}}^{p} L(\varepsilon, x)\ d\Gamma(x)\right] = \frac{\lambda_0 u M}{N} \cdot h(\varepsilon) \cdot \mathbf{1}(p \geq b) \cdot \gamma(p),$$

which solves as:

$$\int_{\underline{p}}^{p} L(\varepsilon, x)\ d\Gamma(x) = \frac{\lambda_0 u M}{N} \cdot h(\varepsilon) \cdot \frac{\Gamma(p) \cdot \mathbf{1}(p \geq b)}{\delta + \mu + \lambda_1 \theta \overline{\Gamma}(p)},$$

hence (13):

$$L(\varepsilon, p) = \frac{\lambda_0 u M}{N} \cdot \frac{\delta + \mu + \lambda_1 \theta}{\left[\delta + \mu + \lambda_1 \theta \overline{\Gamma}(p)\right]^2} \cdot \mathbf{1}(p \geq b) \cdot h(\varepsilon).$$

A final remark is in order here. As can easily be shown either by directly equating (24) and (26) in the case $w = \phi_0(\varepsilon, p)$, or by applying the result (14) to that same case, there is a mass of lowest-paid type ε workers in each firm, given by:

$$L(\varepsilon, \phi_0(\varepsilon, p), p) = \frac{\lambda_0 u M}{N} \cdot \frac{h(\varepsilon)}{\delta + \mu + \lambda_1 \theta}.$$

Interestingly, this number is independent of the firm's type. This clearly results from the fact that all firms are productive enough to at least compensate all unemployed workers for their forgone productivity "at home", b. This remarkably simple result is partly due to our specification of output flows and unemployment income. Amending those specifications would make matters slightly more complex, for some firms might become unattractive to some unemployed workers. For the treatment of this case, see Postel-Vinay and Robin (1999).