The Optimal Provision of Products with Income E¤ects

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Abstract

Discrete choice models have been used to describe imperfect competition between ...rms selling horizontally di¤erentiated products. In all theoretical models, the indirect utility function is assumed to be linear in income so that there is no income e¤ect. We consider here a situation in which income enters nonlinearly into the indirect utility function. We propose a correct (hicksian) measure of consumer surplus based on a willingness to pay principle. In order to grantee the existence of a price equilibrium, match values are assumed logconcavilly distributed. Using a correct measure of welfare, we extent the results of Anderson, de Palma and Nesterov to the case where income e¤ects are involved. We proof that under these general assumptions, overentry prevails. Our ...ndings, which extend the conventional discrete choice oligopoly approach provide various guidelines for empirical research.

1 Introduction

Market performance could be evaluated according to various indexes capturing competitiveness, cost e¢ciency, quality and variety of the goods o¤ered. Over the last decades, the production costs have decreased enormously all over the world with a tendency towards lower sunk costs and as a consequence more variety o¤ered to the consumers (This trend may be somewhat reduced at the moment given the trends towards merging and acquisition). Since variety is costly but at the same time bene...cial to the consumers, one is thus led to wonder whether or not variety o¤ered to the customers is too large.

We consider here a di¤erentiated product market in which ...rms compete strategically. Since each ...rm has some market power, equilibrium price is higher than marginal cost and the two fundamental theorems of welfare economics break down. This imperfection induces at the same time some potential biases in the quality o¤ered and in the variety o¤ered. For example, consider the case of two ...rms competing in quality on the [0,1] quality line. The equilibrium involves these two ...rms located at the center. For such equilibrium solution, there is a duplication of the resources since the welfare would be the same if there were only one ...rm. In this case, the market provides too much product variety. Now, if the number of ...rms is set to 2, and assuming inelastic individual demand, the planner will locate the two ...rms on the ...rst and the third quartiles. The market equilibrium provides also the wrong quality and there is not enough quality di¤erentiation.

We restrict this analysis to the biases associated to over- or underprovision of product variety and set all the product qualities exogenously. Since we assume decreasing average cost, the market will be served at the minimum cost with a single ...rm. However, since the products are di¤erentiated, the customers would prefer, ceteris paribus, the largest number of product (...rms). There is an obvious trade-o¤ between e¢ciency and bene...ts from product di¤erentiation. To study this problem we will adopt here the Chamberlin demand system, which assumes symmetry: the product have the same quality and are horizontally di¤erentiated (Chamberlin (1933)). This type of approach has been exploited by Spence (1976) and later on by Dixit and Stiglitz (1977) and Deneckere and Rothschild (1992) to address the problem of optimum product variety. The trade-o¤ identi...ed by Spence is as follows. When a new ...rm decides to enter into the market, it does not take into account the fact that it will tend to erode the pro...t of the other ...rms. This "business stealing" e¤ect, per se, induces too many ...rms in the market. On the other hand, the ...rm, which decides to enter into a market, generates some surplus for the consumers. Since ...rms are unable to discriminate perfectly, they will only be able to appropriate a part of the surplus they generate. As a consequence, this second e¤ect, per se, the "non appropriability of consumer surplus" leads to not enough variety. Market equilibrium is the outcome of these two opposing forces. In the pure symmetric location game, Salop (1979) has shown that there is far excessive entry in the market. This is because each …rm competes with only two neighbors; as a consequence competition is local and reduced, which contributes to explain that the entry is exacerbated.

On the other extreme, we have non-localized competition à la Chamberlin. In this case each …rm competes with all other …rms. Spence (1976) and Dixit and Stiglitz (1977) have considered these models. We focus here our attention to a class of models, the discrete choice models, which have become increasingly popular over the last decade. For the most well known model, the logit, Anderson and de Palma (1992a) have shown that there is excessive entry although the amount of overentry is limited (at most one). Discrete choice models have become very popular tools in industrial organization, because they are ‡exible tool (see Anderson, de Palma and Thisse (1992)), and they are embedded with very strong econometric properties (see McFadden (1981)) and because they allow for existence of a Nash equilibrium under mild assumptions. Existence of a Bertrand-Nash equilibrium has been shown under the hypothesis that the taste distribution is log-concave (Caplin and Nalebuf (1991)). Interestingly, the intuition suggested by the logit model goes through for a class of discrete choice models that retain the log-concavity of the taste distribution (Anderson, de Palma and Nesterov (1995)). Their model allows for inelastic as well as elastic demand and provides various well-known models (the probit model, the CES model, and the linear model) as special cases. Using speci…c examples, Anderson et al. (1995) have shown that the amount for over-entry could be substantial and as large as 10% of the optimal number of …rms. Berry, Levinsohn and Pakes (1995) and Goldberg (1995) have used these models for empirically analyzing equilibrium in the car industry.

The discussion concerning over/under-entry for discrete choice oligopoly models has failed till now to incorporate income e¤ects. McFadden (1996, 1997) provided the …rst extensions of discrete choice model to treat explicitly income e¤ects. Unfortunately, the results derived by McFadden are based on numerical simulations and could not be used in the theoretical framework we wish to use. Later on, de Palma and Kilani (1999) derive welfare formula (based on the idea of compensating variations) for a version of the logit model taking into account income e¤ects. In this logit model with income e¤ects, both the demand and the welfare depend on income.

We provide in this paper a formulation of conventional discrete choice model when there are income e¤ect. We assume that the tastes are log-concavilly distributed and show that there exists in this case a symmetric equilibrium. We consider two cases: one where demand is inelastic and one where it is elastic. For these two models, we solve for symmetric price equilibrium. We show that the existence results derived by Caplin and Nalebu¤ (1991) could be readily extended to the case where there are income e¤ects. Finally, we provide the main result of the paper by showing that in the two cases, elastic and inelastic demand, over-entry is the norm. These results extend the scope of the results of Anderson, de Palma and Nesterov (1995).

The paper is organized as follows. In section 2, a model with unitary demand allowing for income e¤ects is analyzed. We proof that under assumptions of logconcavity and with decreasing marginal utility of income, a symmetric price equilibrium exists. We also proof uniqueness of the price equilibrium under logit assumptions. We extent this framework in section 3 where we assume that individual demand is elastic and the existence of a symmetric equilibrium is proved. In section 4, we introduce an aggregate measure of consumer surplus based on the willingness to pay principle. Using this measure, we proof that market equilibrium yields too many …rms compared with a second-best optimum. Finally, concluding remarks are provided in section 5.

2 Unitary demand

There is a continuum of consumers of mass 1 with identical incomes, $y > 0$, facing a choice among n mutually exclusive products indexed by i, and sold at price p_i, $i = 1...n$. Consumer preferences are modelled within the discrete choice framework (see McFadden (1981)). The conditional indirect utility for a consumer indexed by k from buying product i is:

$$u_i^k = v_i + e_i^k; \quad i = 1...n;$$

where v_i, which depends on price p_i and income y is the systematic part of the indirect utility and e_i^k is a match value between consumer k and product i.

We make the following assumption about the systematic part, v_i of the indirect utility function:

A1: The systematic part of the (conditional) indirect utility is:

$$v_i = g(z_i);\ i = 1...n; \tag{1}$$

where $g(.)$ is twice differentiable, concave and strictly increasing on R_+, and where $z_i = y - p_i$.

The formulation (1) generalizes assumption A1 made in Anderson et al. (1995), where the conditional indirect utility is taken to be linear in income and price. In their formulation $g(z) = z$ and the marginal utility of income is constant. Note that since the price and income enter the conditional indirect utility function as $y - p$, the individual (conditional) demand, d_i, is according to the Roy's identity, inelastic and unitary: $d_i = -[(\partial v_i/\partial p_i)/(\partial v_i/\partial y)] = 1$.

We consider in this study the Chamberlinian set up and restrict our attention to symmetric equilibria. That is, we assume that the individual are statistically identical and that the firms are identical. As a consequence (see assumption A2 in Anderson et al. (1995)) the match value satisfy assumption A2:

A2: The density of types is $\prod_{i=1} f(e_i)$, with $\int_a^b f(x)dx = 1$, and $F(X) = \int_a^X f(x)dx$.

The interval $(a; b)$, where a and b can be either finite or infinite, is the support of the distribution of the match values.

The demand for good i corresponds to the mass of consumers who derive the largest utility by purchasing good i. Therefore, the demand function for good i, denoted by D_i, is:

$$D_i = \int_a^b f(x) \prod_{j \neq i} F(v_i - v_j + x)dx;\ i = 1...n; \tag{2}$$

where v_i are given by (1).

We assume that each firm has the same constant marginal cost set for now to zero with no loss of generality[1], and incurs the same fixed cost $K > 0$. The profit of firm i, π_i, is given by:

$$\Pi_i = p_i D_i - K;\ i = 1...n: \tag{3}$$

[1]Since the profit function is $\Pi_i = (p_i - c)D_i$, if $c > 0$, we can consider the following change in variable: $P_i = p_i - c$ and $Y = y - c$.

Existence of an equilibrium depends crucially on the distribution of the match value. Caplin and Nalebuff (1991) introduce the idea of logconcavity as a key tool to prove the existence of a Bertrand-Nash equilibrium. Recall that the function $f(x)$ is logconcace if $\ln[f(x)]$ is concave. We assume:

A3: $f(x)$ is logconcave and twice differentiable on (a, b).

For an interior equilibrium, the following price first-order conditions:

$$\frac{\partial \Pi_i}{\partial p_i} = D_i \left(1 + p_i \frac{\partial \ln D_i}{\partial p_i} \right) = 0; \quad i = 1 \ldots n; \tag{4}$$

must be fulfilled. Differentiating the demand function (2) with respect to p_i yields:

$$\frac{\partial D_i}{\partial p_i} = -g'(z_i) \sum_{j \neq i} \Omega_{ij}; \quad i = 1 \ldots n; \tag{5}$$

where:

$$\Omega_{ij} = \int_a^b f(x) f(v_i - v_j + x) \prod_{k \neq i, j} F(v_i - v_k + x)\, dx; \quad j \neq i; \quad i = 1 \ldots n:$$

For the symmetric candidate equilibrium, we obtain:

$$\left(\sum_{j \neq i} \Omega_{ij} \right) / n = n(n-1) \int_a^b f^2(x) F^{n-2}(x)\, dx \equiv \Omega(n): \tag{6}$$

Substituting (6) into (5) and using equation (4) leads to the following implicit equation for the price candidate equilibrium:

$$p^* = \frac{1}{g'(y - p^*)\, \Omega(n)}: \tag{7}$$

Its existence is stated in the following proposition.

Proposition 1 Under assumptions A1-A3 and provided that income is sufficiently high, a unique symmetric Bertrand-Nash price equilibrium, $p^* = p^*(n)$, given by (7) exists. Moreover, p^* is nonincreasing in n.

Proof. The price first-order condition evaluated at a symmetric solution is equivalent to:

$$\psi(p;n) \equiv p g'(y - p)\,\Omega(n) = 1. \qquad (8)$$

We have $\psi(0;n) = 0$ and $\psi(p;n)$ is strictly increasing in p since $\partial\psi/\partial p = [g'(p) - p g''(p)]\,\Omega(n) > 0$. Note also that $\Omega(n)$ (and therefore $\psi(p;n)$) is nondecreasing in n (see Anderson et al. (1995), Proposition 1). Assuming that y is large enough, i.e. for $y \geq [\Omega(2)\, g'(0)]^{-1}$, it follows that $\psi(y;n) \geq 1$.[2] Therefore, there exists a unique solution p^* to equation (8) given by (7), with $p^* \in [0;y]$. The price equilibrium is nonincreasing in n since $\partial\psi/\partial n \geq 0$ and $\partial\psi/\partial p > 0$.

Since $g(.)$ is strictly increasing and concave (assumption A1) and since the match values are logconcavilly distributed (assumptions A2 and A3), Proposition 4 in Caplin and Nalebuff (1991) guarentee that the profit function (3) is logconcave (and hence quasi-concave) in own price. As a consequence the symmetric candidate equilibrium is a Bertrand Nash equilibium. ■

Note that with a linear specification, $g(z) = z$, the price equilibrium reduces to:

$$p^* = \frac{1}{\Omega(n)},$$

which is independent of consumer income. In that case, the welfare function reduces to $W(n) = y + n\int_a^b x f(x) F^{n-1}(x)dx - nK$. The welfare analysis is more intricate for a non-linear specification of $g(.)$ and is relegated to Section 3.

An important particular case of the demand function occurs when the match value are distributed according to the double exponential distribution, with cumulative density function given by:

$$F(x) = \exp\left[-\exp\left(-\frac{x}{\mu}\right)\right],\quad x \in (-\infty; +\infty),$$

where $\mu > 0$ is a scale parameter. In that case, demand given by (2) reduces to the logit specification (see Anderson, de Palma and Thisse (1992)):

$$D_i = \frac{e^{v_i/\mu}}{\sum_{j=1}^{n} e^{v_j/\mu}}, \quad i = 1...n. \qquad (9)$$

[2] If $g'(0) = +\infty$, a price equilibrium exists for any $y > 0$.

With the double exponential specification, it can be shown that $-(n) = (n-1)/\mu n$ (see equation (6)) so that the price equilibrium solves:

$$p^* = \frac{\mu n}{(n-1)\, g'(y-p^*)}:$$

Let us consider two particular cases:

- With double-exponentially distributed match values and $g(z) = z$, we obtain the standard logit model introduced by Anderson and de Palma (1988) to study oligopoly competition with differentiated products. The (unique) price equilibrium is therefore $p^* = \mu n/(n-1)$, $n \geq 2$.
- With double-exponentially distributed match values and with a logarithmic specification, $g(z) = \ln z$, the unique price equilibrium is explicitly given by:

  $$p^* = \frac{\mu n}{(n-1) + \mu n} y;$$

  for $n \geq 1$.[3] In this case, the equilibrium price depends linearly on consumer income. The monopolistic competition limit prices remains bounded away from zero since $\lim_{n \to \infty} p^* = \mu y/(\mu + 1)$. For very large product differentiation we have $\lim_{\mu \to \infty} p^* = y$.[4]

Caplin and Nalebuff (1991) have provided results on the uniqueness of the price equilibrium with logconcave density of preferences for the duopoly model (see Proposition 6 in Caplin and Nalebuff (1991)). No general results have been derived for logconcave distributions. The uniqueness of the price equilibrium is however guaranteed for the standard logit model with no income effects (see Anderson and de Palma (1988)).[5] We extend below this result when there are income effects:

Proposition 2 Under assumption A1-A2 and with double exponentially distributed match values, the symmetric price equilibrium is unique.

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[3] The monopoly price is $p^*(1) = y$.

[4] This new model has not the undesirable property of the logit model: $\lim_{\mu \to \infty} p^* = \infty$.

[5] Another proof is available in Milgrom and Roberts Milgrom and Roberts (1990) which also show that the price game has a log-supermodularity property.

Proof. Using a change in variables $v_i = g(y - p_i)$, profit functions can be written: $\pi_i = [y - g^{-1}(v_i)]\, D_i$ (we set $K = 0$, w.l.o.g). The logarithm of the profit function: $\ln \pi_i = \ln[y - g^{-1}(v_i)] + \ln D_i$ is strictly concave since:

$$\frac{\partial^2 \ln \pi_i}{\partial v_i^2} = \frac{\partial^2 \ln[y - g^{-1}(v_i)]}{\partial v_i^2} + \frac{\partial^2 \ln D_i}{\partial v_i^2} < 0;$$

where the inequality holds since $g(.)$ is concave and D_i is logconcave with respect to the vector $(v_1 \ldots v_n)$. Moreover we have the following relation:[6]

$$\frac{\partial^2 \ln \pi_i}{\partial v_i \partial v_j} = \frac{\partial^2 \ln D_i}{\partial v_i \partial v_j} > 0; \quad j \neq i.$$

Now, the demand function (9) satisfy the following identity:

$$\frac{\partial^2 \ln D_i}{\partial v_i^2} = -\sum_{j \neq i} \frac{\partial^2 \ln D_i}{\partial v_i \partial v_j}.$$

Therefore the uniqueness of a price equilibrium is guarenteed by the diagonal dominant property (cf. Friedman (1977)):

$$\left|\frac{\partial^2 \ln \pi_i}{\partial v_i^2}\right| > \sum_{j \neq i} \left|\frac{\partial^2 \ln \pi_i}{\partial v_i \partial v_j}\right|.$$

■

3 Elastic demand

In this section, the framework is enlarged to allow for elastic demand. In Anderson et al. (1995), elastic demand is modelled by assuming that: $v_i = y + v(p_i)$, where $v(.)$ is twice continuously differentiable, convex and stricly decreasing over $(0; \infty)$. With this specification, a model with elastic individual (conditional) demand is obtained. This framework is further extended in Anderson et al. (1995) to the case where the marginal utility of income is not constant but remains independent on price.[7] However, their extension

[6] For the logit demand model given by (9), we have: $\left(\partial^2 \ln D_i / \partial v_i \partial v_j\right) = (-1/\mu)(\partial D_i / \partial v_j) > 0;\ j \neq i$.

[7] With this assumption, results provided by Caplin and Nalebuff cannot be used directly. However, to proof the existence of a price equilibrium while still using results of Caplin and Nalebuff, it suffices to operate a change in variable and use v_i instead of p_i as strategic variable. In this case, it is straightforward to show that profit function of Firm i is quasi-concave in v_i.

leaves away income effects since demand functions remain independent on income.

We provide here a generalization of these frameworks where the marginal utility of income is non linear and depends on price. In this case, the conditional demand is elastic. Assumption A1 is replaced by:

A1': The systematic part of the (conditional) indirect utility is:

$$v_i = g\left(y + v\left(p_i\right)\right); \; i = 1...n; \tag{10}$$

where $g(.)$ is twice differentiable, concave and strictly increasing on $\mathbb{R}_+$ and $v(.)$ is twice continuously differentiable, convex and strictly decreasing on $\mathbb{R}_+$ with $\lim_{p\to\infty} v(p) = -\infty$.

Using the Roy's identity, the conditional demand for good i is:

$$d(p_i) \equiv -\frac{\partial v_i / \partial p_i}{\partial v_i / \partial y} = -v'(p_i):$$

The demand function for good i is therefore: $d(p_i)\,D_i$, where D_i is given by (2). Let c the constant marginal cost. The profit of firm i is now:

$$\Pi_i = \pi(p_i)\,D_i - K;$$

where $\pi(p) = (p - c)\,d(p)$ is the net revenue pe consumer for a firm charging p.

We need to impose some conditions on the conditional demand $d(.)$ (and hence on $v(.)$) which guarantees the logconcavity of the profit function. One way is to assume that $d(.)$ is logconcave in p. But since this condition is not satisfied for many standard specifications (CES, ...), we select according to Caplin and Nalebuff (1991), the weaker assumption that $d(.)$ is logconcave with respect to $\ln p$. It is equivalent to assume that the elasticity of the conditional demand $d(.)$, defined by $\varepsilon(p) = -pd'(p)/d(p)$, is nondecreasing in p.

A4: The elasticity $\varepsilon(p)$ of the conditional demand function is nondecreasing for all p.

A weaker sufficient condition to obtain quasi-concave profits is provided in Anderson et al. (1995) where they define:

$$\eta(p) \equiv -\frac{(p-c)\,v''(p)}{v'(p)} = \frac{p-c}{p}\varepsilon(p);$$

which is the elasticity of the net revenue per consumer (with respect to the markup). They assume that $\eta(.)$ is nondecreasing for all $p > c$ such that $\eta(p) \leq 1$. However, assumption A4 made here seems more intuitive and implies assumption A4 made in Anderson et al. (1995). Indeed, we have the following lemma:

Lemma 3 Under assumption A4, $\eta(.)$ is increasing in p for all $p > c$. Moreover, firms must charge a price within $[c, \tilde{p})$ where $\tilde{p}$ is such that $\eta(p) < 1$ for $p < \tilde{p}$ and $\eta(p) \geq 1$ for $p \geq \tilde{p}$. Within $[c, \tilde{p})$, net profit per consumer $\pi(p)$ is strictly increasing.

Proof. Since $\eta'(p)$ has the same sign as: $[c\varepsilon(p)/p] + (p - c)\varepsilon'(p)$, it is positive for all $p > c$.

The profit derivative with respect to price is: $\partial\Pi/\partial p = \pi'(p)D + \pi(p)(\partial D/\partial p)$, where the firm subscript is dropped for the sake of convenience. Therefore, a firm must necessarily charge a price such that $\pi'(p) > 0$. Define the price $\tilde{p}$ such that $\pi'(p) > 0$ if $p \in [c, \tilde{p})$ and $\pi'(p) \leq 0$ if $p \geq \tilde{p}$ which is well defined since $\pi'(p) = d(p)[1 - \eta(p)]$ and the term into brackets is strictly decreasing in p. Therefore, within $[c, \tilde{p})$ (admissible prices), $\eta(p) < 1$ and hence $\pi(p)$ is strictly increasing. ■

The existence of a symmetric solution to the price game is provided in:

Proposition 4 Under assumptions A1'-A4 and provided that income is sufficiently high, a unique symmetric Bertrand-Nash price equilibrium, $p^* = p^*(n)$, exists. It is the unique solution of:

$$p^* - c = \frac{1 - \eta^*}{d(p^*)\, g'(z^*)\, \phi(n)}, \tag{11}$$

where $\eta^* = \eta(p^*)$ and $z^* = y + v(p^*)$. Moreover, $p^*(n)$ is nonincreasing in n.

Proof. The conditional demand for good i, $d(p_i)$, is logconcave in $\ln p_i$ (see A4). Since $f(.)$ is logconcave, we know from Caplin and Nalebuff (1991) (cf. Proposition 12) that the profit function is quasi-concave with respect to p_i, where profits are strictly positive.

The price first-order condition evaluated at a symmetric solution leads to:

$$\psi(p, n) \equiv \eta(p) + \pi(p)\, g'(y + v(p))\, \phi(n) = 1. \tag{12}$$

We have $\partial\psi/\partial p > 0$ within $[c;\bar{p})$ and $\psi(c;n) = 0$. To prove the existence of a solution, we have to show that $\lim_{p\to\bar{p}}\psi(p;n) > 1$. Three cases must be distinguished:

- If $\bar{p}$ is finite, necessarily $\eta(\bar{p}) = \infty$ and hence $\lim_{p\to\bar{p}}\psi(p;n) > 1$. It is therefore sufficient to consider any positive income such that $y + v(\bar{p}) \geq 0$.

- If $\bar{p} = \infty$ with $\lim_{p\to\infty}\eta(p) = 1-\varepsilon$ with $\varepsilon \in]0;1[$. Note first that $\pi(p)$ is unbounded. Indeed, $(\ln\pi(p))' \geq (\ln(p-c)^{\varepsilon})'$. Let $p_0 > c$, we have: $\pi(p) \geq (p-c)^{\varepsilon}\pi(p_0)/(p_0-c)^{\varepsilon}$ for $p \geq p_0$. It follows that $\lim_{p\to\infty}\pi(p) = \infty$.[8] . Hence, for y large enough, define $p(y)$ as the unique solution to $y + v(p(y)) = 0$ which exists since $\lim_{p\to\infty} v(p) = -\infty$. Function $p(.)$ is strictly increasing and $\lim_{y\to\infty} p(y) = \infty$. Therefore, there exists y^b such $\pi\left(p\left(y^b\right)\right) = [g'(0) - (2)]^{-1}$. Now, for each y such that $y \geq y^b$, we have $\psi(p(y);n) > 1$ and hence equation (12) has an admissible solution.

- If $\bar{p} = \infty$ with $\lim_{p\to\infty}\eta(p) = 1$, we have $\psi(p(y);n) = \eta(p(y)) + \pi(p(y)) - (n)g'(0)$. Since it is strictly increasing in y and tends towards a limit greater than 1, there exists $\bar{y^b}$ such that $\psi\left(p\left(\bar{y^b}\right);n\right) = 1$. Therefore, for any $y \geq \bar{y^b}$, equation (12) has an admissible solution.

Finally, since $\partial\psi/\partial p > 0$ and $\partial\psi/\partial n \geq 0$, it follows that $p^*(n)$ is non-increasing in n. ■

4 Welfare measure and optimum

This section contains a measure of consumer surplus (CS) which will be used later on to evaluate the market long-run outcome. Following McFadden (1997, 1998), we proceed by computing first a (monetary) measure of the individual consumer's willingness to pay (WTP). Second, we aggregate this measures to obtain the mean WTP which will be used as a CS variation measure.

First, we compute the WTP at the individual level. Since individuals have specific match values, each individual will have, a priori, a specific WTP for

[8] See also Anderson et al. (1995), footnote 16.

a given change. Then, we compute the WTP for a population of consumers described by its distribution of tastes (aggregation over the population).

We focus here on symmetric situations where prices are identical, since the price equilibrium is symmetric. Consider an initial situation involving n goods sold at price p^0. At the final state, there are $n + 1$ goods priced at p^1.

We compute the compensating variation (CV) at the individual level, which is the maximum amount that a given consumer with income y and specific match values would be willing to pay for a change from the initial situation $(p^0; n)$ to the final situation $(p^1; n + 1)$. Denote $v^0 = v(p^0)$ and $v^1 = v(p^1)$. The aggregate measure of CS variation is described in the following proposition:

Proposition 5 Under assumptions A1'-A3, the aggregate CS variation due to a change from $(p^0; n)$ to $(p^1; n + 1)$ is:

$$\Delta(CS)_{n \to n+1} = \left(v^1 - v^0\right) + \int_a^b \int_a^x \phi(z^0; x_n; x) f_n(x_n)\, dx_n f(x)\, dx; \quad (13)$$

where $f_n(\cdot) = nF^{n-1}(\cdot) f(\cdot)$ and:

$$\phi(z; x_n; x) = z - g^{-1}(g(z) + x_n - x)\,. \quad (14)$$

Proof. The CV denoted by cs^k for a consumer of type k solves the following equation:

$$g\left(z^0\right) + \tilde{e}_n^k = g\left(z^1 - cs^k\right) + \max\left(\tilde{e}_n^k; e_{n+1}^k\right); \quad (15)$$

where $z^0 = y + v^0$; $z^1 = y + v^1$ and $\tilde{e}_n^k = \max_{i=1...n} e_i^k$ which are distributed according to the density function $f_n(\cdot)$.

Note that the CV depends on z^0, z^1, $\tilde{e}_n^k$, and e_{n+1}^k. Solving (15) with respect to cs^k leads to:

$$cs^k = \left(v^1 - v^0\right) + \phi\left(z^0; \tilde{e}_n^k; e_{n+1}^k\right); \quad (16)$$

where ϕ is given by (14). Now, integrating this CV measures given by (16) over the population, we obtain the mean CV variation given by (13). ■

Note that the first term $(v^1 - v^0)$ in (13) is the CS variation due to the change in price, the remaining term being the CS variation due to the increase in variety. Let us consider some particular cases.

- With a standard linear specification with $g(z) = z$, the aggregate CS variation given by (13) reduces to:

$$\Delta(CS)_{n\to n+1} = \left(v^1 - v^0\right) + \int_a^b \int_a^x (x - x_n) f_n(x_n)\, dx_n f(x)\, dx. \quad (17)$$

  Using an integration by parts for the integral in the right hand side of (17), we obtain:

$$\int_a^b \int_a^x (x - x_n) f_n(x_n)\, dx_n f(x)\, dx = \int_a^b F^n(x)(1 - F(x))\, dx. \quad (18)$$

  The CS variation is therfore equivalent to the difference between the expected utility levels. With double-exponentially distributed match values, the CS variation reduces to:

$$\Delta(CS)_{n\to n+1} = \left(v^1 - v^0\right) + \mu\left[\ln(n+1) - \ln n\right];$$

  which could also be obtained from the standard logsum formula (cf. McFadden (1981)).

- Consider the logarithmic specification $g(z) = \ln z$. In that case, expression (13) reduces to

$$\Delta(CS)_{n\to n+1} = \left(v^1 - v^0\right) + z^0 \int_a^b \int_a^x \left(1 - e^{x_n - x}\right) f_n(x_n)\, dx_n f(x)\, dx.$$

  For the double exponential distribution, de Palma and Kilani (1999) have shown that this expression can be simplified as:

$$\Delta(CS)_{n\to n+1} = \left(v^1 - v^0\right) + z^0 \left(1 - \int_0^1 \frac{t^\mu}{n+t}\, dt\right).$$

The social welfare is taken as the sum of consumer surplus plus profits, where profit variation is:

$$\Delta\Pi_{n\to n+1} = \pi\left(p^1\right) - \pi\left(p^0\right) - K.$$

Therefore, the expression for the welfare variation is $\Delta W_{n\to n+1} = \Delta(CS)_{n\to n+1} + \Delta\Pi_{n\to n+1}$.

With unitary demand and $g(z) = z$, the incremental welfare is, using (17) and (18):

$$\Delta W_{n \to n+1} = \int_a^b F^n(x)(1 - F(x))\,dx - K;$$

which coincide with formula (7) used in Anderson et al. (1995).[9] It is worth to note that this expression does not depend on the final price and the price variation is a pure transfer between consumers and firms. In that particular case, the first-best and the zero-profit constrained second-best optimum coincide.

We now describe the market equilibrium under free-entry. Revenue per firm is, using (11):

$$R^*(n) \equiv \frac{\pi(p^*)}{n} = \frac{1 - \eta_n^*}{g'(z^*)\,n\,\Omega(n)};$$

where $\eta_n^* = \eta(p^*(n))$.

>From lemma 3 and proposition 4, $R^*(n)$ is strictly decreasing in n. Since it tends towards zero as n goes to infinity, the free-entry equilibrium is uniquely defined by condition:

$$R^*(n^e) \geq K > R^*(n^e + 1);$$

provided that the market is served $R^*(\hat{p}) \geq K$.

Since $\pi(p)$ is strictly increasing (lemma 3), profit per firm is lower than $\pi(\hat{p})/n - K$. Henceforth, define $\hat{n} \equiv \pi(\hat{p})/K$ as a bound to the number of firms which can make profits. For $n \leq \hat{n}$, define $\bar{p} = \bar{p}(n)$ as the unique price such that the zero-profit constraint is filled:

$$\pi(\bar{p}(n)) = nK; \tag{19}$$

which is well-defined since $\pi(p)$ raises continuously from 0 to $\hat{n}K$ within $[c; \hat{p})$.

The variation of welfare along the zero-profit constraint is (cf. (13)):

$$\Delta W^z_{n \to n+1} = v(\bar{p}(n+1)) - v(\bar{p}(n)) + \bar{T}(n);$$

[9] Anderson et al (1995) have used the expected maximimum utility as a measure of consumer benefit since in their model, the indirect utility is additive linear in income.

where:

$$\overline{T}(n) = \int_a^b \int_a^x \phi(\bar{z}_n; x_n; x) f_n(x_n)\, dx_n\, f(x)\, dx; \tag{20}$$

and where ϕ is given by (14) and $\bar{z}_n = y + v(\bar{p}(n))$.

Using a similar reasoning as in Anderson et al. (1995), we show that if it is socially optimal to have $n+1$ firms in the market, then n firms will have nonnegative profits at equilibrium. Formally, we proof that for any positive K:

$$\Delta W^z_{n \to n+1} \geq 0 \Rightarrow R^*(n) \geq K. \tag{21}$$

This condition implies that the equilibrium never entails underentry. This result is proved in the following theorem which provides a generalization to theorem 2 in Anderson et al. (1995):

Theorem 6 Under assumptions A1'-A4, the market will not underprovide diversity relative to the zero-profit constrained optimum.

Proof. Condition (21) leads to the following implication:

$$\overline{T}(n) \geq v(\bar{p}(n)) - v(\bar{p}(n+1)) \Rightarrow R^*(n) \geq K. \tag{22}$$

Deriving the zero-profit constraint given by (19), we obtain:

$$\frac{dv(\bar{p})}{dn} = v'(\bar{p})\frac{d\bar{p}}{dn} = \frac{-K}{1-\eta(\bar{p})}.$$

Using the theorem of the mean, there exists $\bar{n} \in [n; n+1]$ such that:

$$v(\bar{p}(n)) - v(\bar{p}(n+1)) = \frac{K}{1-\eta_{\bar{n}}};$$

where $\eta_{\bar{n}} = \eta(\bar{p}(\bar{n}))$. Hence, condition (22) implies:

$$\overline{T}(n) \geq \frac{K}{1-\eta_{\bar{n}}} \Rightarrow \frac{1}{g'(z^*_n)\, n\, -(n)} \geq \frac{K}{1-\eta^*_n}; \tag{23}$$

where $\overline{T}(n)$ is given by (20) and $z^*_n = y + v(p^*(n))$.

>From the theorem of the mean and given that $g(.)$ is strictly increasing and concave, we have the following inequality for $x_n \quad x$:

$$\phi(\bar{z}_n; x_n; x) \quad \frac{x - x_n}{g'(\bar{z}_n)};$$

which yields:

$$\bar{T}(n) \quad \frac{1}{g'(\bar{z}_n)} \int_a^b F^n(x)\,(1 - F(x))\,dx.$$

Therefore:

$$\bar{T}(n) \geq \frac{g'(\bar{z}_n)\,K}{1 - \eta_{\bar{n}}} \Rightarrow \int_a^b F^n(x)\,[1 - F(x)]\,dx \geq \frac{g'(\bar{z}_n)\,K}{1 - \eta_{\bar{n}}}. \qquad (24)$$

Implication (26) in Anderson et al. (1995) states that, for all positive K:

$$\int_a^b F^n(x)\,[1 - F(x)]\,dx \geq \frac{K}{1 - \eta_{\bar{n}}} \Rightarrow \frac{1}{n\Omega(n)} \geq \frac{K}{1 - \eta^*_n}.$$

In particular, we have:

$$\int_a^b F^n(x)\,[1 - F(x)]\,dx \geq \frac{g'(\bar{z}_n)\,K}{1 - \eta_{\bar{n}}} \Rightarrow \frac{1}{n\Omega(n)} \geq \frac{g'(\bar{z}_n)\,K}{1 - \eta^*_n}; \qquad (25)$$

Hence, from (24) and (25), we obtain:

$$\bar{T}(n) \geq \frac{K}{1 - \eta_{\bar{n}}} \Rightarrow \frac{1}{n\Omega(n)} \geq \frac{g'(\bar{z}_n)\,K}{1 - \eta_n}.$$

Now, since $z^*_n > \bar{z}_n$ (obviously, $p^*(n) < \bar{p}(n)$) and $g(\cdot)$ is concave, we obtain (23). ■

5 Concluding remarks

We have introduced in this paper a discrete choice oligopoly model with income effects. Two versions of the model have been proposed. In the first, each consumer purchases one unit of the good, in line with the discrete choice framework. In the extended model, consumers are allowed to purchase a variable amount of one of the variants. The demand depends on income even if, in the proposed formulation with variable individual demand, the conditional quantity purchased in independent of income. We were not able to derive the representative consumer model for this formulation. This was possible in the special case where taste are distributed according to a double exponential distribution (see for details de Palma and Kilani (1999)). Therefore we based our analysis on the compensating variation formula that was computed for each individual embedded with speci…c tastes and then sumed up for all the individuals in the population. Under those hypotheses, our main result is that the market always provide too many products (excessive variety) according to the second-best (zero-pro…t constrained) optimum.

One implication of this result is that an ad-valorem tax or a unit tax could increase the welfare in the market since such taxes tend to reduce producer pro…ts (even if producer prices could be over-shifted) and therefore reduce product variety. Such reduction could be, according to our main result, welfare improving. One major limitation of our approach is that we only consider single product …rms. Without income e¤ects and with double-exponentially distributed tastes, Anderson and de Palma (1992b) analyzed multiple product …rms and have shown that the market over-provide products, while the number of product o¤ered by each …rm is too small. It remains to be shown how those results, true for the multinomial logit model, could be extended for more general discrete choice models with income e¤ects. A second major limitation of our approach is that …rms are assumed to be symmetric and supply products with the same vertical quality. Indeed, Anderson and de Palma (1999) have shown that the market could under or over provide variety when qualities di¤er. Moreover, other biases are to be considered since typically, the low quality goods tend to produce too much variety while the large quality goods tend to produce to little. This result is less clear when income a¤ect consumer choices and when consumers are embedded with different incomes. The distribution of income is likely to preserve the existence proof, so that this research avenue should be pursued in order to obtain useful results to shed new light on empirical applications.

References

[1] Anderson, S.P., and A. de Palma (1988): "Spatial price Discrimination with Heterogenous Products," Review of Economic Studies, 55, pp. 573-592.

[2] Anderson, S.P., and A. de Palma (1992a): "The Logit as a Model of Product Di¤erentiation," Oxford Economics Papers, 44, pp. 51-67.

[3] Anderson, S.P., and A. de Palma. (1992b): "Multiproduct Firms: A Nested Logit Approach," Journal of Industrial Economics, 40, pp. 261-276.

[4] Anderson, S., A. de Palma, and J.-F. Thisse (1992): Discrete Choice Theory of Product Di¤erentiation. MIT Press, Cambridge, MA.

[5] Anderson, S.P., A. de Palma, and Y. Nesterov (1995): "Oligopolistic Competition and the Optimal Provision of Products," Econometrica, 63, pp. 1281-1301.

[6] Berry, S.T., J. Levinsohn, and A. Pakes (1995): "Automobile Prices in Market Equilibrium," Econometrica, 63, pp. 841-890.

[7] Caplin, A., and B. Nalebu¤ (1991): "Aggregation and Imperfect Competition: On the Existence of Equilibrium," Econometrica, 59, pp. 25-59.

[8] Chamberlin, E. (1933): The Theory of Monopolistic Competition. Harvard University Press, Cambridge, MA.

[9] Deneckere, R., and M. Rothschild (1992): "Monopolistic Competition and Preference Diversity," Review of Economic Studies, 59, pp. 361-373.

[10] De Palma, A., and K. Kilani (1999): "Discrete Choice Models with Income E¤ects," mimeo, Université de Cergy-Pontoise.

[11] Dixit, A., and J. E. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," American Economic Review, 67, pp. 297-308.

[12] Friedman, J. (1977): Oligopoly and the Theory of Games. North-Holland, Amsterdam.

[13] Goldberg, P.K. (1995): "Product Di¤erentiation and Oligopoly in International Markets : The Case of the U.S. Automobile Industry," Econometrica, 63, pp. 891-951.

[14] McFadden, D. (1981): "Econometrics Models of Probabilistic Choice," in: Structural Analysis of Discrete Data with Econometric Applications (C. Manski and D. McFadden, eds.). MIT Press, Cambridge, MA, pp. 198-272.

[15] McFadden, D. (1997): "Measuring Willingness-to-pay in Discrete Choice Models," in: Essays in the Honor of John Chipman (J. Moore and R. Hartman, eds.).

[16] McFadden, D. (1998): "Measuring Willingness-to-Pay for transportation improvements," in: Theoretical Foundations of Travel Choice Modelling, Pergamon Press, Gartling, T. and T. Laitila, and K. Westin, eds., pp. 339-364.

[17] Milgrom, P., and J. Roberts (1990): "Rationalizability, Learning and Equilibrium in Games with Strategic Complementarities," Econometrica, 58, pp. 1255-1277.

[18] Salop, S. (1979): "Monopolistic Competition with Outside Goods," Bell Journal of Economics, 10, pp. 141-156.

[19] Spence, M. (1976): "Product Selection, Fixed Costs, and Monopolistic Competition," Review of Economic Studies, 43, pp. 217-235.