The initial public offering from a tripartite point of view

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Abstract

Existing literature on initial public offerings of a firm's stock has taken into analysis only two parties of the issuer, the underwriter, the informed investor, either by neglecting one party for simplification or uniting it with another one as if they pursue the unique interests by forming a coalition. In point of fact, the issuer, the underwriter, the informed investors are separate entities. They have often conflicting objectives each other and act in a different and independent manner to seek for profits of their own by making the most of their information. The present article explicitly introduces into enquiry the aspect of the tripartite conflicting interests of initial public offerings and investigates the effects.

1 introduction

It is very much documented that in the initial public offering(hereafter IPO) of the firm there are wide spread underpricing phenomena. There have appeared many theoretical articles so far. Allen and Faulhaber(1989), Welch(1989) ascribed the underpricing to signaling by the issuing firm. In their setting, there is an issuing firm and an investor. The issuer knows the better about its prospective results and has superior information to the investor. The articles showed that in this situation a good firm was willing to convey signals of its hopeful prospects to distinguish himself from a bad one. Underpricing is the signal. Baron(1982), Baron and Holmstrom(1980) analyzed a situation in which there were an issuer and an underwriter. The underwriter is in charge of organizing the IPO. He has superior information to the issuer. Moreover, for the distribution of shares, he has efforts to make which are unobservable by the issuer. There are, thus, two incentive problems involved here. The authors investigated the issuer's optimal incentive contract in these circumstances. Benveniste and Spindt(1989), Benveniste and Wilhelm(1990) set up a model in which there were an issuer, an underwriter, informed investors, and uninformed investors. In this setting, the underwriter, who was delegated to organize the IPO, was assumed

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to maximize total proceeds of the IPO, therefore the issuer's profits. The authors investigated the characters of the contract. Finally, Biais, Bossaerts and Rochet(1998) investigated the issuer's optimal contract. In their model, there are an issuer, uninformed investors and a party which is a coalition of an underwriter and informed investors. They studied the characters of this optimal contract.

Although the approaches are very different each other among these articles, one common character of the models is that of the issuer, the underwriter and the informed investor, only two parties are considered as separate entities in analysis. Specifically, in Allen et al. and Welch the underwriter is absent. In Baron and Baron et al., the informed investor is unconsidered. In Benveniste and Spindt, Benveniste and Wilhelm, the underwriter maximizes total proceeds, in other words, the issuer's utility. This is, therefore, a model of the issuer-underwriter incorporate. In Biais et al., the underwriter is assumed to be allied with the informed investor.

In reality, the issuer, the underwriter, the informed investor are separate entities. They act each in a different and independent manner to pursue their own profits often conflicting one another. The objective of the present article is to explicitly bring this aspect into analysis.

Let us take a look at the main idea of the article. In the first place, this is a positive analysis, in which regard it is different from Baron, Baron et al., Biais et al., above. In order to introduce the feature of tripartite conflicts into analysis, the present article takes a point of view of contract delegation. The issuer is assumed rather unsophisticated;for the first time issuer has usually little expertise in financial affairs. He wants for various ability to organize the IPO, information gathering, information offering, advertising, pricing and so forth. Therefore, the issuing firm delegates the whole IPO procedure to an underwriter and pays the latter some commissions of a fixed rate per share price. The underwriter has as a seasoned financial institution ample knowledge of the financial market to collect and analyze information possessed by institutional investors and estimate the market valuation of shares to be issued. In full charge of the IPO procedure, the underwriter decides upon the quantity allocation and the price to maximize the profits of his own, not those of the issuer's nor the investor's, while trying to collect the investor's private information.

There are two distinguishing features in the present article. One is, as pointed out above, that account is taken of the difference of interests between the issuer, the underwriter, the informed investor. In the setting of the present article, those three parties pursue their own profits different from each other. The second feature is the introduction of the effects of the possibility that the underwriter buys part of shares himself for the purpose of making profits by reselling them after the IPO. Indeed, there are no definite legal restrictions to how IPO shares are allotted to subscribers by the underwriter. The IPO procedure, especially, the allocation and the pricing of shares is for the most part left to the discretion of the underwriter. The underwriter can adjust his own and subscribers' allocation of shares according to information collected among the subscribers in order to make the largest profits.

The underwriter can make profits through two channels. First, he earns commissions of a certain rate for the issue price. Therefore, he gains the more the higher he sets the price. In this respect, the underwriter is motivated to give a high price to the issue. On the other hand, he can make profits by purchasing part of the IPO shares and selling them on the market after the issue if the after-market price is higher than the issue price. In this regard, therefore, it is in the interest of the underwriter to set a low IPO price in order to make profits by reselling.

There is, as is seen, a trade-off of incentives on the part of the underwriter to set a high or low price. In the model of the present article, there are restrictions, that is, there is a fixed amount of shares to be issued and there are only an underwriter and an informed investor between whom shares are distributed. However, there is no uninformed investor considered. Even in this simple context, it is not obvious how the underwriter assigns and prices shares in consideration of the maximization of his profits.

The remainder of the article is organized as follows. In the next section, the parties involved of the game and the scenario are described. In section 3, the model is formally presented. In section 4, the characteristics of the solution are scrutinized. The section 5 is a conclusion.

2 scenario

There are a firm, an underwriter, an informed investor[1] The firm wants to sell its fixed amount of shares on the market for the first time. This firm or issuer is assumed to be unable to do it by itself; initial public offering requires marketing, allotting, pricing of shares to be issued and this demands a lot of expertise that the first time issuer does not usually have. Therefore, the issuer has to rely on someone possessing such expertise. Here is where the underwriter comes in.

The underwriter, which has great experiences and expertise as a seasoned financial institution, takes on the task of organizing the IPO. He markets, prices, distributes IPO shares to subscribers. In reality the syndicate of underwriters is often formed by several financial institutions. Here we assume though that there is only one underwriter.

The informed, often said regular, investor is a large investor like an investment bank, a broker, a securities firm which has great expertise on the financial market. Such an investor may well have some information on the post-offering market valuation of IPO equity. The underwriter gets in touch with this investor to seek for his information. In practice, there will be several informed investors. Here, it is assumed though, by putting together all possible informed investors, that there is only one informed investor.

The informational structure is assumed in a following way. The issuer reveals all his information to the underwriter in respect of the share value[2] The

[1] In this article, the presence of uninformed retail investors is neglected. Accordingly, the issue of their winner's curse will not be studied as in Rock(1986).

[2] Therefore, the aspect is disregarded concerning signaling by the issuer as in Allen and

underwriter reveals to the informed investor all information provided by the issuer and his own information. The informed investor has private information unobservable by the other parties. Therefore, in this model only the informed subscriber possesses private information.

The issuer of the present model is rather unsophisticated. The issuer delegates the whole IPO procedures to the underwriter. The issuer pays as a commission a fixed percentage of the price per share to the underwriter[3] Totally designated to organize the whole IPO process, the underwriter decides upon the quantity allocation and the price of the shares while seeking the informed subscriber's private information. It is irrelevant whether or not the underwriter reveals his information to the issuer as well as the subscriber, because of the whole delegation of the IPO procedures to the underwriter.

In fact, since the underwriter and the subscriber have some information, the issuer may as well try to make the underwriter reveal those two pieces of information by the construction of a commission scheme. However, in this article, we will not consider such a sophisticated issuer. The issuer here just agrees to pay a commission of some fixed rate for the price per share. This commission rate may be decided by some industry standard or mainly by the underwriter taking advantage of his superior financial knowledge.

Here is some justification for such an issuer. Usually, the first time issuer has not acquired so many experiences in financial affairs that he can deal with so complicated IPO procedures. Not having grown so big yet, such a firm has relatively poor financial expertise. First of all, it will be unable to collect information in want of acquaintances among financial institutions. Even if able to gather information, it cannot be expected to analyze the information with so little financial knowledge.

Besides, the prospective underwriter is usually a financial institution with which the issuer already has long-standing relationships. As the result, it has far more information on the issuer than is acquirable by other financial competitors. The underwriter has, therefore, a very strong in-competitive position in IPO business.

When it comes to the determination of a commission rate, it will be very difficult for an issuer in a situation described above to build an optimal commission schedule for information gathering dealing with a longstanding, much practised financial institution. The issuer modeled in this article is quite a reasonable approximation of the reality.

Let us return to the underwriter. The underwriter, as described above, organizes the IPO. What is distinctive in this article is that account is taken of the fact that the underwriter may also buy part of the shares himself as well as the informed investor. Therefore he distributes the shares between him and the informed subscriber. He makes profits through two channels; commissions

Faulhaber(1989), Welch(1989).

[3]Although the linear compensation scheme is assumed for the underwriter, in reality it may not be so because of the existence of the overallotement right and the warrant right granted to the underwriter. This issue will not be treated in this article. For warrants, see Barry, Muscarella, Vetsuypens(1991).

gained per share price and purchasing and reselling of shares. He maximizes his total profits, building a quantity-price scheme to make the informed subscriber to reveal his information.

3 model

In this section, we shall formally present the model. All the three parties are risk neutral, the issuer, the underwriter, the informed subscriber. The issuer plans to issue Q units of equity through the underwriter.

The issuer is assumed to convey to the underwriter all relevant information it has relative to the valuation of the shares. Accordingly, the underwriter is informationally advantaged and there is no issue of double asymmetric

information between the issuer and the underwriter.

The underwriter organizes the IPO. He sets the per share price p and the quantity allocations to himself and the informed investor q_0, q_1. The underwriter, for his part, has some information on the post-IPO stock value v_0, which is positive. It is known only by himself. Let us assume, however, that he reveals credibly to the informed subscriber his private information v_0 as well as all information furnished by the issuer during the road show in which the underwriter communicates with the informed investor.

The informed investor or subscriber has private information on the post-issue value of the stocks v_1, which none of the other agents know. v_1 takes a value in the interval of $[\underline{v}_1 \overline{v}_1] \subset R_+$. The agents other than the informed subscriber knows the distribution of v_1 while unaware of the realized value v_1. The distribution function of v_1 is $F(v_1)$, which is absolutely continuous and has the density $f(v_1)$ that never takes zero in the interval of its definition. Finally, the post-IPO price per share is $\sum_{i=0}^{1} v_i$.

As is the case in reality, the issuer is supposed to pay as a commission a fixed percentage of the price per share to the underwriter. Specifically, the issuer agrees to pay a commission of rate a for the price per share, thus ap per share when the price is p. Since the issuer entrusts completely the IPO to the underwriter, it is irrelevant whether or not the underwriter's information is revealed to the issuer. We disregard completely the question of the optimal commission schedule from the issuer's point of view.

The underwriter, as already seen, has two channels through which to make profits; commissions pQ and gains by the reselling after the IPO $(\sum_{i=0}^{1} v_i - p)q_0$. He buys shares when finding it in his interest. Let us put the upper limit to the amount of shares he can buy, that is, $0 \leq q_0 = (Q - q_1) \leq tQ$ where $0 < t < 1$. Even if there is no definite legal restriction to the quantity the underwriter can buy, he may have a budgetary constraint or take account of the possibility that he may impair future business by putting in jeopardy relationships with informed institutional investors if he does not allot any shares to them. This observation justifies the imposition of the limit on the underwriter's shares to be alloted. While unaware of the private information of the informed subscriber, he can make a mechanism to make this subscriber reveal the information. Let us concentrate on the direct mechanism (Myerson(1982)); specifically, the underwriter proposes to the informed subscriber the map

$$(q_1(v_1), p(v_1)) : [\underline{v}_1, \overline{v}_1] \mapsto [(1-t)Q, Q] \times R.$$

$q_1(v_1)$ is a quantity alloted to the informed subscriber and $p(v_1)$ is the per share price.

Indeed, q_1 and p must be also dependent on v_0. Let us recall, however, the underwriter reveals v_0 to the informed subscriber. In that event, regarding v_0 as fixed, we can neglect the dependence of q_1 and p on v_0. From now on, we think of v_0 as fixed.

The underwriter is unaware of the true value of the informed subscriber's information. Therefore, the latter does not necessarily choose the contract of v_1. If the informed subscriber with information v_1 chooses the contract of $\tilde{v}_1$, his expected profit is

$$u_1(v_1, \tilde{v}_1) := ((v_0 + v_1) - p(\tilde{v}_1))\, q_1(\tilde{v}_1).$$

If the informed subscriber with information v_1 selects the contract of his true information, his expected profit is

$$u_1(v_1) := \left(\sum_{i=0}^{1} v_i - p(v_1) \right) q_1(v_1). \tag{1}$$

As is mentioned above, these two equations also depend upon v_0 but this is known to the informed subscriber as well. Therefore, we can ignore it by thinking it fixed.

Unable to force the informed subscriber to divulge his information, the underwriter has to make a contract which induces him to betray his information at will. We thus define the implementable contract;

Definition. *The contract $(q_1(v_1), p(v_1))$ is implementable if and only if*

$$u_1(v_1) = \max_{\tilde{v}_1} u_1(v_1, \tilde{v}_1)$$

As is usually done, we turn the implementable contract into the manageable form. The implementable contract is characterized by the following incentive compatibility condition.

Lemma (incentive compatibility). *If the contract $(q_1(v_1), p(v_1))$ is implementable, the following two conditions are satisfied;*

$$q_1(v_1) \textit{is non-decreasing}, \tag{2}$$
$$q_1(v_1) = \dot{u}_1(v_1) \; a.e.. \tag{3}$$

Conversely, the implementable contract $(q_1(v_1), p(v_1))$ *can be constructed, if there are* $q_1(v_1) and u_1(v_1)$*, which satisfies 2 and 3, by putting*

$$p(v_1) = \sum_{i=0}^{1} v_i - \frac{u_1(v_1)}{q_1(v_1)}. \tag{4}$$

Proof. See Rochet(1985). □

It is not enough that the underwriter manages to get the informed investor to tell the truth. The latter has no obligation to participate in the IPO. The condition has to be put which ensures that the informed investor gains no less in participating in than abstaining from the IPO. Let this participation condition be

$$c \leq u_1(v_1) \text{,where } c \text{ is a positive constant.}$$

Usually, it would be natural to put this condition as $r \leq \frac{\left(\sum_{i=0}^{1} v_i - p(v_1)\right)}{p}$ where r is an yield rate of other financial products but for simplification this article adopts a simpler condition. Indeed, this participation condition can be transformed into a more manageable form, considering that u_1 is non-decreasing by the incentive compatibility condition, 2 and 3, into

$$c \leq u_1\left(\underline{v}_1\right). \tag{5}$$

We have to put another participation condition which this time applies to the issuer, who will relinquish the IPO if the issue price is too unfavorable to him. This condition would be natural to put as something like $d \leq p$, which indicates that the issue price is higher than some value. However, to keep the model manageable, we express the participation constraint in the following way;

$$u_1\left(v_1\right) \leq d \text{,where } d \text{ is a positive constant.}$$

With the same argument as for 5, the following condition is sufficient;

$$u_1\left(\overline{v}_1\right) \leq d \text{,where } d \text{ is a positive constant.} \tag{6}$$

It is assumed as a matter of course that $c < d$.

Let us see that this form of the participation condition will be justified. When 6 is satisfied, we have from the incentive compatibility condition of the lemma

$$d \geq u_1\left(\overline{v}_1\right) \geq u_1\left(v_1\right). \tag{7}$$

Then by 1

$$
\begin{aligned}
p\left(v_{1}\right) &\geq \sum_{0}^{1} v_{i}-\frac{d}{q_{1}\left(v_{1}\right)} \\
&\geq\left(v_{0}+\underline{v}_{1}\right)-\frac{d}{q_{1}\left(v_{1}\right)} \\
&\geq\left(v_{0}+\underline{v}_{1}\right)-\frac{d}{(1-t) Q} .
\end{aligned}
$$

The last inequality shows that the participation condition 6 leads to an inferior limit of the price which is a natural condition to participation of the issuer. We assume quite naturally that d satisfies

$$
0<\left(v_{0}+\underline{v_{1}}\right)-\frac{d}{(1-t) Q} . \tag{8}
$$

Then it is ensured that the issue price is always bounded from below by a positive number. This condition can be interpreted yet another way. It can be rewritten as $\frac{d}{(1-t) Q}<\left(v_{0}+\underline{v_{1}}\right)$. Let us recall d is a superior limit of the underwriter's utility and $(1-t) Q \leq q_{1} \leq Q$. Then $\frac{d}{(1-t) Q}$ is the maximum possible utility per share. Therefore, 8 can be interpreted in a way that the maximum possible utility per share is less than the minimum share value. In another phrase, the maximum per share utility can be effectuated even when the minimum per share value is realized.

We assume yet another condition;

$$
\overline{v}_{1}-\underline{v}_{1}<\frac{d-c}{(1-t) Q} . \tag{9}
$$

Let us rewrite it as $\left(v_{0}+\overline{v}_{1}\right)-\left(v_{0}+\underline{v}_{1}\right)<\frac{d-c}{(1-t) Q}$. The left side is obviously the difference of the highest and the lowest value of the shares. On the other hand, $d-c$ is a difference of the superior and the inferior limit of the informed investor's utility. Then $\frac{d-c}{(1-t) Q}$ is the largest difference per share of those limits since $(1-t) Q \leq q_{1} \leq Q$. Considering this, 9 can be interpreted as meaning that the largest possible difference of the share value is smaller than the greatest difference of the investor's per share profits.

Finally, it must be ensured that 8 and refe;c d can hold at the same time. If these conditions are satisfied, it must be that

$$
\overline{v}_{1}-\underline{v}_{1}+\frac{c}{(1-t) Q}<\frac{d}{(1-t) Q}<v_{0}+\underline{v}_{1} .
$$

We assume that it holds.

We are now ready for the statement of the underwriter's maximization problem. He maximizes his expected profit under the incentive constraints and the participation constraints, that is,

$$\begin{aligned}
\max_{q_1,\, p,\, u_1} & \int_{\underline{v}_1}^{\overline{v}_1} (ap(v_1)Q + (\sum_{i=0}^{1} v_i - p(v_1))(Q - q_1(v_1)))f(v_1)dv_1 \\
& \text{s. t.} \\
& 2,\ 3,\ 5,\ 6, \\
& (1-t)Q \le q_1(v_1) \le Q.
\end{aligned}$$

We use optimal control to solve the maximization problem. It is possible by virtue of the incentive compatibility lemma. We first find the optimal q_1 and u_1, and then retrieve p by 4.

Let us proceed now to the optimal control setting of the problem. The tactics is to make q_1 and u_1 state variables and introduce a control variable. First, making use of 1 to eliminate p, we can write the underwriter's expected profit as

$$\int_{\underline{v}_1}^{\overline{v}_1} (aQ\sum_{i=0}^{1} v_i - (a-1)\frac{u_1}{q_1}Q - u_1)f(v_1)dv_1. \tag{10}$$

We can also transform $(1-t)Q \le q_1 \le Q$ by the monotonicity of q_1 (see the lemma) into

$$(1-t)Q \le q_1\left(\underline{v}_1\right), \quad q_1\left(\overline{v}_1\right) \le Q. \tag{11}$$

Finally, with regard to 2, introducing a control variable, we put

$$z := \dot{q}_1, \quad z \ge 0. \tag{12}$$

We are at length able to formalize the maximization problem as that of optimal control.

$$\max_{z,q_1,u_1} \int_{\underline{v}_1}^{\overline{v}_1} (aQ\sum_{i=0}^{1} v_i - (a-1)\frac{u_1}{q_1}Q - u_1) f(v_1) dv_1 \tag{13}$$

s. t.

$$\dot{q}_1 = z \quad \text{a.e.,} \tag{14}$$

$$\dot{u}_1 = q_1 \quad \text{a.e.,} \tag{15}$$

$$c \leq u_1 (\underline{v}_1), \tag{16}$$

$$u_1 (\overline{v}_1) \leq d, \tag{17}$$

$$(1-t)Q \leq q_1 (\underline{v}_1), \tag{18}$$

$$q_1 (\overline{v}_1) \leq Q, \tag{19}$$

$$0 \leq z. \tag{20}$$

Theorem. *The optimal contract is characterized as follows;*
if $a > t$,

$$q_1 = (1-t) Q, \tag{21}$$

$$u_1 = (1-t)Q (v_1 - \underline{v}_1) + c, \tag{22}$$

$$p = v_0 + \underline{v}_1 - \frac{c}{(1-t)Q)}, \tag{23}$$

and if $a = t$,

$$q_1 = (1-t) Q, \tag{24}$$

$$u_1 = (1-t)Q (v_1 - \underline{v}_1) + u_1(\underline{v}_1), \tag{25}$$

$$p = v_0 + \underline{v}_1 - \frac{u_1(\underline{v}_1)}{(1-t)Q)}, \tag{26}$$

where $u_1(\underline{v}_1)$ *is undetermined,* (27)

and if $a \leq t$,

$$q_1 = (1-t) Q, \tag{28}$$

$$u_1 = -(1-t)Q (\overline{v}_1 - v_1) + d, \tag{29}$$

$$p = v_0 + \overline{v}_1 - \frac{d}{(1-t)Q)}. \tag{30}$$

Proof. We apply the necessary condition of optimal control to solve the problem. Let us first set the Hamiltonian. Setting λ_0, λ_1, λ_2 as multipliers, we have

$$H (u_1, q_1, z, \lambda) = \lambda_0 \left(aQ\sum_{i=0}^{1} v_i - (a-1) \frac{u_1}{q_1}Q - u_1 \right) f + \lambda_1 q_1 + \lambda_2 z \tag{31}$$

where $\lambda := (\lambda_0, \lambda_1, \lambda_2)$.

We examine the two cases $\lambda_0 = 0, 1$ separately.

impossibility of $\lambda_0 = 0$.
Let us suppose$\lambda_0 = 0$ and write necessary conditions in the first place.

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial u_1} = 0 \qquad \text{a.e.,} \tag{32}$$
$$\dot{\lambda}_2 = -\frac{\partial H}{\partial q_1} = -\lambda_1 \qquad \text{a.e..} \tag{33}$$

As end point conditions, we have

$$\lambda_1(\underline{v}_1) \leq 0, \qquad \lambda_1(\underline{v}_1)(u_1(\underline{v}_1) - c) = 0, \tag{34}$$
$$\lambda_2(\underline{v}_1) \leq 0, \qquad \lambda_2(\underline{v}_1)(q_1(\underline{v}_1) - (1-t)Q) = 0, \tag{35}$$
$$\lambda_1(\overline{v}_1) \leq 0, \qquad \lambda_1(\overline{v}_1)(u_1(\overline{v}_1) - d) = 0, \tag{36}$$
$$\lambda_2(\overline{v}_1) \leq 0, \qquad \lambda_2(\overline{v}_1)(q_1(\overline{v}_1) - Q) = 0, \tag{37}$$

In addition, z has to maximize $H(u_1, q_1, z, \lambda)$ with optimal u_1 and q_1 when it is viewed as a function of z. Hence

$$\lambda_2 \leq 0$$

and

$$z \begin{cases} = 0 & \text{if } \lambda_2 < 0, \\ \text{is undetermined} & \text{if } \lambda_2 = 0. \end{cases} \tag{38}$$

From 32, λ_1 is constant and besides due to 34, 36, λ_1 is a non-positive constant.

We will see that λ_1 is in point of fact negative. To see that, let us suppose $\lambda_1 = 0$. Then $\lambda_2 < 0$ by 33 and $(\lambda_0, \lambda_1, \lambda_2) \neq 0$. It follows that $z = 0$ from 38. We see that q_1 is constant from 14. On the other hand, since $\lambda_2 < 0$, it must be that $q_1(\underline{v}_1) = (1-t)Q$ and $q_1(\overline{v}_1) = Q$ by the end point conditions , 35 and 37. This is a contradiction to q_1 being constant. Therefore, $\lambda_1 \neq 0$.

Let us proceed now with λ_1 negative. First, let us recall $\lambda_2 \leq 0$ from the existence of z maximizing the Hamiltonian. Then, since $\dot{\lambda}_2 = -\lambda_1 > 0$, at least

$$\lambda_2 < 0 \quad \text{in} \, [\underline{v}_1, \overline{v}_1),$$

which leads to

$$z = 0 \quad \text{in} \, [\underline{v}_1, \overline{v}_1).$$

Thus from 14, seeing that $\overline{v}_1$ is negligible,

$$q_1 = (1-t)Q \quad \text{in} \left[\underline{v}_1, \overline{v}_1\right]. \tag{39}$$

Then from the terminal condition 37,

$$\lambda_2 \left(\overline{v}_1\right) = 0.$$

Besides we have from 33

$$\lambda_2 = -\lambda_1 \left(v_1 - \underline{v}_1\right) + \lambda_2 \left(\underline{v}_1\right). \tag{40}$$

Therefore from this equation

$$\lambda_2 \left(\overline{v}_1\right) = 0 = -\lambda_1 \left(\overline{v}_1 - \underline{v}_1\right) + \lambda_2 \left(\underline{v}_1\right).$$

Using this, we can rewrite λ_2 as

$$\lambda_2(v_1) = -\lambda_1(v_1 - \underline{v}_1) + \lambda_1(\overline{v}_1 - \underline{v}_1) = -\lambda_1(v_1 - \overline{v}_1).$$

Let us find u_1. Seeing that, from 34 and 36,

$$u_1 \left(\underline{v}_1\right) = c \quad \text{and} \quad u_1 \left(\overline{v}_1\right) = d,$$

we have by 15

$$u_1 = (1-t)Q \left(v_1 - \underline{v}_1\right) + u_1 \left(\underline{v}_1\right) = (1-t)Q \left(v_1 - \underline{v}_1\right) + c. \tag{41}$$

At the same time, u_1 must satisfy $u_1 \left(\overline{v}_1\right) = d$. As the result, it must be that

$$d = (1-t)Q \left(\overline{v}_1 - \underline{v}_1\right) + c. \tag{42}$$

Altogether, only when this condition is satisfied, there is a candidate of optimal solutions in case of $\lambda_0 = 0$. However, the assumption 9 does not admit of this condition. Hence the impossibility of $\lambda_0 = 0$.

case of $\lambda_0 = 1$.

We have seen that we can posit $\lambda_0 = 1$. From now onwards we suppose so. First, as above, let us write necessary conditions.

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial u_1} = ((a-1)Q\frac{1}{q_1} + 1)f \qquad \text{a.e.,} \tag{43}$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial q_1} = \frac{(1-a)u_1 Q f}{q_1^2} - \lambda_1 \qquad \text{a.e.,} \tag{44}$$

and as end conditions, 34, 35, 36, 37.

As is in the previous analysis, there must be z maximizing the Hamiltonian. Therefore, here too obtains 3 and 38.

We split the analysis into three cases $a > t$, $a \leq t$.

case of $a > t$.

From 43, we see that $\dot{\lambda}_1$ is strictly increasing with respect to q_1 and that

$$((a-1)Q\frac{1}{(1-t)Q}+1)f = \frac{a-t}{1-t}f > 0 \quad \text{a.e..}$$

Therefore,

$$\dot{\lambda}_1 > 0 \quad \text{a.e..}$$

From this and seeing that the end values of λ_1 are non-positive, it follows that

$$\lambda_1 \begin{cases} < 0 & \text{in } [\underline{v}_1, \underline{v}_1) \\ \leq 0 & \text{at } \overline{v}_1. \end{cases}$$

Moreover, from this first inequality and the transversality condition 34, we deduce

$$u_1(\underline{v}_1) = c.$$

Let us turn to λ_2. From the value of λ_1 deduced above and 44, it is easy to see that $\dot{\lambda}_2 > 0$. Therefore, we have

$$\lambda_2 \begin{cases} < 0 & \text{in } [\underline{v}_1, \underline{v}_1) \\ \leq 0 & \text{at } \overline{v}_1. \end{cases}$$

The first inequality also leads to $q_1(\underline{v}_1) = (1-t)Q$ by the transversality condition.

Given $lambda_2$ as above, we can find z in the same way as 3, 38;

$$z \begin{cases} = 0 & \text{in } [\underline{v}_1, \underline{v}_1) \\ \geq 0 & \text{at } \overline{v}_1. \end{cases}$$

Therefore, from 14,

$$q_1 = (1-t)Q. \tag{45}$$

Moreover,

$$u_1 = (1-t)Q(v_1 - \underline{v}_1) + u_1(\underline{v}_1) = (1-t)Q(v_1 - \underline{v}_1) + c. \tag{46}$$

Then, from 4

$$p = \sum_{i=0}^{1} v_i - \frac{(1-t)Q(v-\underline{v}_1)+c}{(1-t)Q} = v_0 + \underline{v}_1 - \frac{c}{(1-t)Q}. \quad (47)$$

case of $a \leq t$. Let us suppose $a \leq t$. First, we prove that

$$\lambda_1 \leq 0. \quad (48)$$

Proof. We shall prove $\lambda_1 \leq 0$ by the reductio ad absurdum. Let us suppose there exists v_1' such that $\lambda_1(v_1') > 0$. Then there is in the neighborhood of v_1' such v_1 that $v_1 \neq \underline{v}_1$ and $\lambda_1(v_1) > 0$ and that there exists $\dot{\lambda}_1$ at v_1, because λ_1 is absolutely continuous. Now we can suppose $\lambda_1(v_1) > 0$. Then there is in $[\underline{v}_1, v_1]$ a non-negligible set S at which point $\dot{\lambda}_1$ exists (this is obvious again by absolute continuity) and $\dot{\lambda}_1 > 0$. For if there is not, $\dot{\lambda}_1 \leq 0$ a. e. in $[\underline{v}_1, v_1]$ and

$$\lambda_1(v_1) = \int_{\underline{v}_1}^{v_1} \dot{\lambda}_1(s)ds + \lambda_1(\underline{v}_1) \leq 0.$$

Thus if we take $t \in S$, $\dot{\lambda}_1(t) = ((a-1)Q\frac{1}{q_1(t)} + 1)f(t) > 0$. It follows that $q_1(t) > (1-a)Q$, which leads to $q_1(v_1) > (1-a)Q$ due to the monotonicity of q_1. Therefore,

$$\dot{\lambda}_1(v_1) = ((a-1)Q\frac{1}{q_1(v_1)} + 1)f(v_1) > 0$$

.
Again, by the monotonicity of q_1, it is true in fact that $\dot{\lambda}_1 > 0$ a.e. in $[t, \overline{v}_1]$. Then recalling $\lambda_1(v_1) > 0$, we have for $x \geq v_1$

$$\lambda_1(x) = \int_{v_1}^{x} \dot{\lambda}_1(s)ds + \lambda_1(v_1) > 0.$$

Therefore,

$$\lambda_1(\overline{v}_1) = \int_{v_1}^{\overline{v}_1} \dot{\lambda}_1(s)ds + \lambda_1(v_1) > 0.$$

This is a contradiction to $\lambda_1(\overline{v_1}) \leq 0$. Consequently, we have proved $\lambda_1 \leq 0$. □

Now we can see as before that

$$\lambda_2 \begin{cases} < 0 & \text{in } [\underline{v}_1, \underline{v}_1) \\ \leq 0 & \text{at } \overline{v}_1, \end{cases}$$

,which leads from the transversality condition to

$$q_1\left(\underline{v}_1\right) = (1-t)Q.$$

Then by the maximization of the Hamiltonian, it is deduced that

$$z \begin{cases} = 0 & \text{in } [\underline{v}_1, \underline{v}_1) \\ \geq 0 & \text{at } \overline{v}_1. \end{cases}$$

Therefore, from 38

$$q_1 = (1-t)Q. \tag{49}$$

Let us split now the analysis into two cases $a = 1$ and $a < t$.

case of $a = t$.

Now that we have known q_1, we obtain from 43

$$\dot{\lambda}_1 = \frac{a-t}{1-t} f = 0.$$

Therefore $\lambda_1 = \alpha \leq 0$, where α is a constant. It turns out in fact that $\alpha = 0$. Suppose $\alpha < 0$. Then by the transversality condition $u_1\left(\underline{v}_1\right) = c$ and $u_1\left(\overline{v}_1\right) = d$. Since we know q_1, we have

$$u_1 = (1-t)Q(v_1 - \underline{v}_1) + u_1(\underline{v}_1) = (1-t)Q(v_1 - \underline{v}_1) + c.$$

It must be at the same time from the terminal condition that

$$u_1(\overline{v}_1) = (1-t)Q(\overline{v}_1 - \underline{v}_1) + c = d$$

It is impossible by the assumption 9. It has been proved that $\alpha = 0$.

Let us suppose so. u_1 can be written as above as

$$u_1 = (1-t)Q(v_1 - \underline{v}_1) + u_1(\underline{v}_1).$$

However, since $\lambda_1 = 0$, $u_1(\underline{v}_1)$ can not be determined from the transversality condition. Indeed if q_1 and u_1 are put into the maximand of the optimal control problem, it is seen that $u_1(\underline{v}_1)$ is irrelevant to the maximization of the objective function. Therefore

case of $a < t$.

In this case, it is seen from 43 and q_1 that

$$\dot{\lambda}_1 < 0.$$

Therefore $\lambda_1(\overline{v}_1) < 0$ and from the transversality condition,

$$u_1(\overline{v}_1) = d.$$

Now we have

$$u_1 = -(1-t)Q(\overline{v}_1 - v_1) + d. \tag{50}$$

and

$$p = v_0 + \overline{v}_1 - \frac{d}{(1-t)Q}. \tag{51}$$

□

4 characterization of the solution

From the theorem, several propositions can be derived immediately.

Proposition 1. *Whatever the relation between a and t, the underwriter always buys himself his maximum amount of the shares,* $(1-t)Q$.

In general, the underwriter is in a dilemma. If he sets the price high, he gains by way of commissions a but has to pay more for the shares he retains for himself for the purpose of reselling them after the IPO. If the price is set low, he makes less profits by commissions but gains more by reselling later the shares he has bought at the low price. Intuitionally, the underwriter will decide to set a high price to gain by the commission rate when this is enough high and refrain from buying many shares himself. On the other hand, in case of the low commission rate, he will choose to make profits by setting a low price even though to his disadvantage in commission earnings and buying shares himself and reselling them after the IPO.

However, contrary to this intuition, the proposition states that despite the existence of such strategic choices, the underwriter does not change how to distribute the shares according as the commission rate(thus the relations between a and t)changes; He always keeps for himself up to the limit.

Let us see this inflexibility of the quantity allocation from another point of view.

Proposition 2. *Given a commission rate(that is, with a and t fixed) the underwriter does not change the allocation pattern of the shares according as the realized value of the subscriber's information changes, namely hold back as many as possible and giving the rest to the subscriber.*

Therefore, even if the share value $\sum_{i=0}^{1} v_i$ varies, the share allocation is unchanged. This quantity inflexibility translates into that of the price.

Proposition 3. *Given a commission rate a and t, the price is insensitive to the share value change.*

The price and the quantity are related by 4. Obviously, the rigid quantity allocation leads to the inflexible price.

This price schedule is rather anomalous. It is totally independent of the realized share value v_1. As is seen from the theorem, it only depends on the outside opportunity values c and d and the boundary points of the interval of v_1, $\underline{v}_1$ and $\overline{v}_1$. This price schedule, accordingly, indeed reflects outside opportunities and to some degree the distribution of the possible value of the private information, but not in the least the realized private information value.

Proposition 4. *Underpricing persists in both cases $a > t$, $a < t$. It is more serious in the latter case.*

Proof. The expected price of the share value is $v_0 + Ev_1$. Let us denote it as m. In case of $a < t$, $m - p = Ev_1 - \overline{v}_1 + \frac{d}{(1-t)Q}$ and when $a > t$, $m - p = Ev_1 - \underline{v}_1 + \frac{c}{(1-t)Q}$. Therefore, it is obvious from 9. □

Let us first notice the difference of the prices between the two case $a > t$ and $a < t$. The two prices are different although the quantity allocations are identical. What does this difference result from? In fact, as is seen from the theorem or the proof, it comes from which end condition is binding between the initial and terminal condition, that is, $u_(\underline{v}_1) \geq c$ and $u_1(\overline{v}_1) \leq d$.

As is explained above, when the commission rate is high enough, the underwriter elects to earn more with the commissions by setting the price high, therefore siding with the issuer; hence the initial condition is binding, that is, $u_(\underline{v}_1) = c$ in case of $a > t$.

On the other hand, with a low commission rate, the underwriter is willing to make profits rather by the purchase and reselling of the shares; therefore by setting a low price. He takes sides with the subscriber with the result that the terminal condition is binding, $u_1(\overline{v}_1) = d$, in case of $a \leq t$. Therefore, in this case, the price is restrained low because of the underwriter's ability to buy shares.

In the real IPO, it is usual that the commission rate a is much lower than the maximum possible share of the underwriter t. Therefore it may be that the underwriter has strong incentives to set the price low because being able to buy himself, he looks after the same interests as the subscriber.

Let us turn to the underpricing now. In the model of the present article are three sources of the underpricing. The first is the subscriber's private information. It requires some costs to get him to participate in the IPO and reveal his private information.

The second is the underwriter's own interests in buying shares himself and making profits by selling. As has been seen, it leads to the quantity and price rigidness to hinder the rent extraction from the subscriber.

The third is a possibility of the underwriter's collusion, as it were, with the subscriber described above. It has been seen that when the commission rate is enough small with respect to his maximum possible share amount(i.e. $a < t$), the underwriter elects to make more profits by the buying and reselling of shares. He shares interests with the subscriber and sides with him to set the price low. This is the reason why in case of $a < t$ there is greater underpricing. As is mentioned above, this is a usual case in the real world. Therefore, it is worthwhile to consider whether the underwriter's ability to buy shares himself does not give him strong incentives to set a low price in an actual IPO process.

5 conclusion

This article has analyzed the circumstances of the IPO in which the three parties,namely the issuer, the underwriter, the informed subscriber pursue each their own interests which may be conflicting.

The premise of the model has been that the issuer lacks know-how relative to the issue of the shares. He does not possess enough financial expertise to be able to analyze market reactions to or market evaluation of the IPO. On the other hand, the underwriter has ample knowledge of the financial market to collect and analyze information possessed by institutional investors and estimate the market valuation of shares to be issued. Therefore the issuer relies on the underwriter for the issuing with a quite simple remuneration scheme for the latter, that is, a linear commission.

In these circumstances, the underwriter decides on the share allocation. He can buy part of the shares himself while allotting the rest to the informed subscriber. There are two sources of gains for the underwriter; commission gains and profits of purchase and reselling of shares. This article has adopted a rather simple setting in which there are only two parties buying shares, the underwriter and the informed subscriber but not an uninformed investor. Even in this simple context, it is not so obvious how the underwriter assigns shares and decides upon the price, allowing for commission earnings and profits of reselling shares.

The interaction of these two effects has been investigated. Intuition asserts that when commission fees are high enough, the underwriter allocates shares in such a way that he can rather make profits through the commission by setting the high price and in the opposite case he sets the price as low as possible to make gains by way of selling retained shares on the market after the IPO.

It has been demonstrated in the present article that contrary to this intuition, the underwriter does not change the pattern of share allocation according to the commission rate. In addition, given a commission rate, the underwriter does not change allocations nor the price whatever the subscriber's private information. Therefore the price reflects no realized value of the shares in our setting.

Finally, although not changing the quantity allocation in response to the variation of the commission rate, the underwriter does change the price. When the commission rate is low enough, the underwriter sets a higher price than in case of a high commission rate, and that keeping the same quantity allocation. To no surprise, there is larger underpricing with a low commission rate. Indeed, in this case, the underwriter shares the same interest as the subscriber; buying shares at a low price. This result gives some insight into the actual IPO, where there may be some factors of underpricing caused by this incentive of the underwriter's.

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