Optimal Taxation and In-Kind Redistribution*

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Abstract

Within a framework of optimal taxation *a la* Mirrlees, we add a new heterogeneity between the individuals who differ not only on their marginal productivity, but also on their initial endowment in a good which is interpreted as an aggregate consumption.

We consider other goods in the economy whose characteristics (impossibility of resale, impossibility of increasing on the private market a public supply) are those of the hospital care or education for example.

We show that one can justify the public supply of this good on an economic criterion of efficiency, in spite of an assumption of separability of work and consumption in the utility function of the individuals.

1 Introduction

In all the developed countries, there exists a large public provision of private goods, in particular for health care or education. Today, in Europe, there is a large debate on the efficiency of this policy.

Indeed, the economic theory teach us that transfers in nature are always (weakly) dominated by monetary transfers in a first-best environment. This public provision of private good is justified in the literature in two different ways: imperfections of taxation and political economy.

Political economy teaches us that even in a perfect competition framework in which one can decentralize all Pareto's optima and thus in which all public provision of private good is useless, it may be that the majority of voters is favorable with public provision of private good. Gouveia (1993) justifies this argument in a model more particularly inspired by health economics. Epple and Romano (1996b) give the same kind of justification without restricting themselves in a particular field.

This type of approach allows to explain why the government provides private goods which can be found on markets, as it is the case for the health care and in particular for hospital.

We stand in a different framework from this kind of justification which belongs to the field of positive economy or even of political science. We try to justify by a pure economic welfare argument the public provision of private good. We consider the optimal policy of a perfectly benevolent planner. In the literature this normative approach results from the theory of optimal taxation.

In a world of perfect information where taxes are lump-sump transfers, in-kind transfers are useless. Moreover even in a optimal taxation framework with asymmetry of information on marginal productivities, (see Mirrlees, 1971) as long as one makes the assumption that the utility is weakly separable between consumption goods and work, indirect taxation and in-kind transfers are also useless (Cremer Gahvari 1997).

There is a huge literature on this subject. Guesnerie and Roberts (1984) show that the intervention of the government on private markets, by imposing constraints of quantity (quotas), can increase the welfare when the individuals have private information on theirs individual characteristics. Later, Blackorby and Donaldson (1988) consider in-kind transfers within an adverse selection framework and they show that in-kind transfers improve welfare:

because of the absence of public information on the type of individuals, in-kind transfers allow to target the individuals. But there is no optimal taxation in their model. Besley and Coate (1991) focus on the redistributive capacity of the public provision in a model where the government observes neither the type, nor individual's income. In their model, the government can only use lump-sum transfers, and no other kind of taxation.

Munro (1991,1992) equips the government with the tool of optimal linear taxation, and shows that in a second-best environment, public provision can increase welfare. Boadway and Marchand (1995) show that this result is still true with optimal taxation. But in their model the government can not use indirect taxation. Their result does not hold anymore if we allow indirect taxation.

In this kind of model, the taxation is not optimal according to the government's knowledge and this is a crucial point. If the government is allowed to use all the taxation tools, in-kind transfers becomes useless.

Cremer and Gahvari (1997), justify public provision of private good in a traditional framework of optimal taxation. They show that the goods complementary to labor are good candidates for an intervention of the government, public provision allowing to slacken the self selection constrain of the more productive individual.

Blomquist and Christiansen (1995) show the same result with different assumptions. They consider a good which can not be supplemented in a private market when it provided by the government (Cremer Gahvari assume exactly the contrary). In their work they only consider direct taxation, but their result still holds if we introduce indirect taxation in their model. In a more recent contribution, (Blomquist and Christiansen, 1998), these same authors characterize the cases in which the government should authorize complementary purchases and the cases in which it needs to prohibit them.

In this two articles, the form of the individuals' utility function is important. Cremer and Gahvari (1997) and Blomquist and Christiansen (1995) assume that this utility function in non-separable between labor supply and consumption goods, e.g. demand functions depends on labor supply. This assumption is also crucial to justify in-kind transfers.

However according to our knowledge, it does not exist tempirical justification for this technical assumption. On the contrary, Browning and Meghir (1991) in an econometric study on labor supply show that the assumption of separability seems to be more realistic. Thus, within a framework with heterogeneity on only one parameter, public provision of a private good would be unjustifiable regards to economic welfare.

The assumption on heterogeneity has been made for technical reasons rather than to represent the world reality. Cremer Pestieau and Rochet (1998) show that in an economy where the individuals do not differ only on theirs marginal productivities, but also on their initial endowment, optimal taxation includes indirect taxation even if individuals have separable utility functions.

In this article we consider a (simplified) version of the Cremer Pestieau Rochet's model in which we introduce public provision of private good. In this model, the government does not observe an initial endowment, this can appear as a strong assumption, but it allows to integrate the limits face the government in matter of collect information. It makes a trade off between exhaustiveness of information and cost of collect (or rather checking).

It is necessary moreover than the quantity of the goods provided by the government (or more generally the regulator) cannot be neither resold, nor supplemented on the private market (see Blomquist Christiansen 1995). This is why we interpret our model rather in term of quality, the government provides a unit of a good of some quality, this quality may be different from those proposed on the market. This type of assumption could represent medical care as hospitalization, surgical operation or heavy treatment (rays by example). Indeed, it seems to us that it is difficult to supplement the "quantity" of hospital on a private market by perfect substitute.

An individual has the choice between a public hospital and a private one, but once the choice of the hospital is made, it's difficult to him to increase his "provision of care" on the market. In the same way these assumptions can model education.

In the section 2 we present the model, in the section 3 we show some properties of the optimal allocation without in-kind transfers, in the section 4 we show that in-kind transfers are useful, and we conclude in the section 5. All the proof are presented in appendix.

2 The model

In the economy there are only individuals (called individual) who consume and work, a benevolent regulator (the government), there are four goods: x, y_1, y_2 and L. Goods are produce by firms who exhibit constant return to scale.

L represents the individual's labor supply. The variable x represents the consumed quantity of a good that one can interpret as an aggregate consumption of all the goods of the economy except goods y_1 and y_2 which present some particular characteristics. It is supposed that goods x, y_1 and y_2 are trade on competitive markets. To simplify the notations and without loss of generality, we consider that all the firms have the same marginal cost, equal to one. All this goods are trade in perfect market, thus their prices are equal to their marginal cost, and wages are equal to the considered marginal productivity.

We consider individual who have the same utility function, separable[1] between labor supply and consumption good. Thus this utility function has the form:

$$U(y_1, y_2, x) - v(L).$$

We suppose that the functions U and v have the usual properties, namely: v is increasing convex and derivable; U is increasing in x, y_1 and y_2, quasi concave and derivable.

The individuals differ by their marginal productivity of labor ω_k ($k \in \{l, h\}$) and by an initial endowment d^j ($j \in \{l, h\}$) in good x. We interpret this initial endowment as the individual's initial wealth, (which does not come from labor). The marginal productivity of labor can be low or high ($\omega_l < \omega_h$) and the individual's initial endowment can be also low or high ($d^l < d^h$). The values of this two variables are private information of the individuals, they are not observed by the government.

Thus we have four types of individuals, those who have low productivity of labor and a low initial endowment, those who have a high productivity of labor and a high initial endowment, and two intermediate cases.

To differentiate the individuals we will adopt the following notation: the individual with low productivity and low endowment will be the individual

[1] Here we consider separable utility functions. The analyse is exactly the same if the utility functions are wealky separable.

1, the individual who has a high productivity and a high initial endowment will be individual 4, individual 2 will be this who has a low productivity but a high endowment and finally the individual remainder will be individual 3. These notations can be summarized in following table:

	d^l	d^h
ω_l	individual 1	individual 2
ω_h	individual 3	individual 4

To simplify the notation, we will use the following notation: the individual i, where $i \in \{1, 2, 3, 4\}$, characterizing by the marginal productivity ω_i and his initial endowment d^i.

Naturally we have the following relations:

$$d^1 = d^3 = d^l, \qquad d^2 = d^4 = d^h$$

$$\omega_1 = \omega_2 = \omega_l, \qquad \omega_3 = \omega_4 = \omega_h$$

The proportion of individual i in the economy is denoted π_i. We normalize the population to 1.

$$\sum \pi_i = 1.$$

The government does not observe the type of the individuals, he only observes anonymous transactions on markets. However, it observes incomes resulting from labor. Thus taxation tools available are of two kinds: income tax, and commodity tax. We consider only linear commodity taxes. The government observes only anonymous transactions, this implies that it lacks the required information to levy non-linear commodity taxes. The linearity of this taxes is imposed for informational reasons and does not result from ad hoc assumptions.

In order to study the optimal taxation characteristics, i.e. income tax paid by each type of individual and commodity taxes, we must characterize first Pareto-efficient allocations that are constrainted by self-selection constraints. To do this we derive the optimal "revelation mechanism" which here consists in a set of type-specific before-tax incomes Y^i and after-tax income C^i and a vector of commodity prices (p_x, p_1, p_2).

Therefore the behavior of an individual i (the two variables C and Y are given) can be modelled by the following program:

$$\begin{array}{cc} \max_{(y_1,y_2,x)} & U\left(y_1,y_2,x\right)-v\left(L\right) \\ s.t. & \begin{array}{c} p_1y_1+p_2y_2+p_xx=C^j+p_xd^i \\ Y^j=L\omega_i. \end{array} \end{array}$$

We deduce the indirect utility for this individual:

$$V\left(p_1,p_2,p_x,C^j+p_xd^i\right)+v\left(\frac{Y^j}{\omega_i}\right)$$

and his demand functions:

$$x^{ij}=x(p_1,p_2,p_x,C^j+p_xd^i)$$

$$y_k^{ij}=h(p_1,p_2,p_x,C^j+p_xd^i).$$

For more simplicity, we will use for the indirect utility functions the following notation:

$$V\left(p_1,p_2,p_x,C^j+p_xd^i\right)+v\left(\frac{Y^j}{\omega_i}\right)=V^{ij}$$

Remark 1 *Because of the particular form of utility functions (separability between labor supply and consumption goods), Demand functions do not depend on the marginal productivity.*

We suppose that the government has a rawlsian objective function: it maximizes the poorest individual utility function. This is a quite traditional assumption in the optimal tax literature.

We study only a polar case (the other polar case would be the utilitarian welfare function) but it makes sense, because want to focus on redistribution problems. Furthermore, we think that our results are robust and does not depends on a specific welfare function[2].

[2] If this function is convex and increasing.

3 Optimal taxation

Because of the incentives constraints, the individual 1 is necessarily the worst off in our setting. Thus the government maximizes his utility function subject to incentives constraints.

$$\max V\left(p_1, p_2, p_x, C^1 + d^l\right) + v\left(\frac{Y^1}{\omega_l}\right)$$

Subject to :

$$\begin{array}{lr}
V\left(p_1,p_2,p_x,C^1+p_xd^l\right)+v\left(\frac{Y^1}{\omega_l}\right)\geq V\left(p_1,p_2,p_x,C^2+p_xd^l\right)+v\left(\frac{Y^2}{\omega_l}\right) & (1.2)\\
V\left(p_1,p_2,p_x,C^1+p_xd^l\right)+v\left(\frac{Y^1}{\omega_l}\right)\geq V\left(p_1,p_2,p_x,C^3+p_xd^l\right)+v\left(\frac{Y^3}{\omega_l}\right) & (1.3)\\
V\left(p_1,p_2,p_x,C^1+p_xd^l\right)+v\left(\frac{Y^1}{\omega_l}\right)\geq V\left(p_1,p_2,p_x,C^4+p_xd^l\right)+v\left(\frac{Y^4}{\omega_l}\right) & (1.4)\\
V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_l}\right)\geq V\left(p_1,p_2,p_x,C^1+p_xd^h\right)+v\left(\frac{Y^1}{\omega_l}\right) & (2.1)\\
V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_l}\right)\geq V\left(p_1,p_2,p_x,C^3+p_xd^h\right)+v\left(\frac{Y^3}{\omega_l}\right) & (2.3)\\
V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_l}\right)\geq V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_l}\right) & (2.4)\\
V\left(p_1,p_2,p_x,C^3+p_xd^l\right)+v\left(\frac{Y^3}{\omega_h}\right)\geq V\left(p_1,p_2,p_x,C^1+p_xd^l\right)+v\left(\frac{Y^1}{\omega_h}\right) & (3.1)\\
V\left(p_1,p_2,p_x,C^3+p_xd^l\right)+v\left(\frac{Y^3}{\omega_h}\right)\geq V\left(p_1,p_2,p_x,C^2+p_xd^l\right)+v\left(\frac{Y^2}{\omega_h}\right) & (3.2)\\
V\left(p_1,p_2,p_x,C^3+p_xd^l\right)+v\left(\frac{Y^3}{\omega_h}\right)\geq V\left(p_1,p_2,p_x,C^4+p_xd^l\right)+v\left(\frac{Y^4}{\omega_h}\right) & (3.4)\\
V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_h}\right)\geq V\left(p_1,p_2,p_x,C^1+p_xd^h\right)+v\left(\frac{Y^1}{\omega_h}\right) & (4.1)\\
V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_h}\right)\geq V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_h}\right) & (4.2)\\
V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_h}\right)\geq V\left(p_1,p_2,p_x,C^3+p_xd^h\right)+v\left(\frac{Y^3}{\omega_h}\right) & (4.3)\\
\sum_{i=1}^4 \pi_i\left[\left(Y^i-C^i\right)+\sum_{k=1,2}\left(p_k-1\right)y_k^{ii}+\left(p_x-1\right)\left(x^i-d^i\right)\right]\geq G & (BC)
\end{array}$$

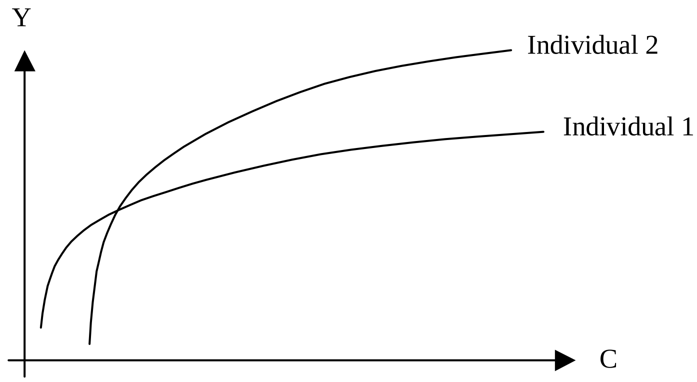
Unfortunately this program does not allow to obtain a explicit simple solution.

First, contrary to the case of unidimensional heterogeneity, we cannot determine which self-selection constraints are binding and which are not.

Moreover the Spence-Mirrlees condition is not verified for individuals 1 and 4. Let's consider two different individual indifference curves in the plan (C, Y), for example individuals 1 and 2. In each dot of the plan, we can determine whose individual has the highest slope of the indifference curve, and this ranking is true for each point of the plan. Using the utility functions properties, one can easily show that:

$$\forall\left(C,Y\right), \qquad \frac{\frac{\partial V\left(p_1,p_2,p_x,C+d^l\right)}{\partial C}}{\frac{\partial v\left(\frac{Y}{\omega_l}\right)}{\partial Y}} > \frac{\frac{\partial V\left(p_1,p_2,p_x,C+d^h\right)}{\partial C}}{\frac{\partial v\left(\frac{Y}{\omega_l}\right)}{\partial Y}}.$$

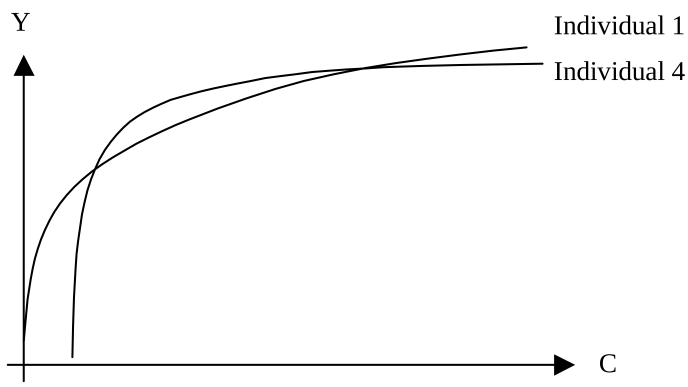
This implies that indifference curves of individual 1 and 2 cross only once.



This property is also true for individuals 1 and 3, 2 and 3, 2 and 4, 4 and 3. But it is not true anymore for individual 1 and 4 for them there is no simple relation between there respective value of the slopes of their indifference curve. We have:

$$\frac{\partial V\left(p_1, p_2, p_x, C + d^l\right)}{\partial C} > \frac{\partial V\left(p_1, p_2, p_x, C + d^h\right)}{\partial C} \text{ et } \frac{\partial v\left(\frac{Y}{\omega_l}\right)}{\partial Y} > \frac{\partial v\left(\frac{Y}{\omega_h}\right)}{\partial Y}.$$

Thus we cannot exclude the case where indifference curves of individuals 1 and 4 would cross twice.



Thus we have no hope to compute explicitly the optimal allocations, this why we only show some general properties and check if in-kind could improve welfare. The first thing to show is that there constraint of budget of government is binding with the optimum: it is useless to raise taxes which do not finance anything.

Lemma 1 *Any optimal taxation is such that constraint (BC) is binding.*

Proof See Appendix 1.

Now, the problem is to determine which incentives constraints are binding and those who are not. As we saw higher, it is very difficult to answer to this question in a very general framework. But by using the fact that the budget constraint is binding and that the welfare function is the rawlsian function, we can obtain a certain number of results. We can establish the following lemma:

Lemma 2 *For individual* 2, 3 *and* 4, *at least one of their incentives constraint is binding.*

Proof See Appendix.

This lemma allows us to limit considerably the incentives constraints study. Among all the combinations which remain possible, it is interesting to show that some are also impossible.

Lemma 3 *An allocation, such that:*

$$V\left(p_1, p_2, p_x, C^2 + p_x d^h\right) + v\left(\tfrac{Y^2}{\omega_l}\right) > V\left(p_1, p_2, p_x, C^1 + p_x d^h\right) + v\left(\tfrac{Y^1}{\omega_l}\right) \quad (2.1)$$
$$V\left(p_1, p_2, p_x, C^2 + p_x d^h\right) + v\left(\tfrac{Y^2}{\omega_l}\right) > V\left(p_1, p_2, p_x, C^3 + p_x d^h\right) + v\left(\tfrac{Y^3}{\omega_l}\right) \quad (2.3)$$
$$V\left(p_1, p_2, p_x, C^2 + p_x d^h\right) + v\left(\tfrac{Y^2}{\omega_l}\right) \geq V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\tfrac{Y^4}{\omega_l}\right) \quad (2.4)$$
$$V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\tfrac{Y^4}{\omega_h}\right) > V\left(p_1, p_2, p_x, C^1 + p_x d^h\right) + v\left(\tfrac{Y^1}{\omega_h}\right) \quad (4.1)$$
$$V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\tfrac{Y^4}{\omega_h}\right) \geq V\left(p_1, p_2, p_x, C^2 + p_x d^h\right) + v\left(\tfrac{Y^2}{\omega_h}\right) \quad (4.2)$$
$$V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\tfrac{Y^4}{\omega_h}\right) > V\left(p_1, p_2, p_x, C^3 + p_x d^h\right) + v\left(\tfrac{Y^3}{\omega_h}\right) \quad (4.3)$$

is not optimal.

It is useless to continue further in the optimal taxation study to show that public provision of goods y_1 and y_2 can improve the social welfare.

4 In-Kind Redistribution

To have a efficient public provision of private goods, a necessary condition is that the individuals cannot resell these goods. Indeed, the welfare increase is obtained by change in incentives constraints. A individual who wants to mimic another will be constrained by the quality provided by the government, if this quality is different from quality (or quantity) which this individual would have selected on private market. Thus, the dissimulation behavior is less attractive compared to a situation without public provision. Thanks to the public provision, the government is less severely constrained when it defines its optimal policy, which enables him to improve the level of social welfare. But if the "mimicker" can resell the publicly provided good all the benefic effect disappears. In the same way, when the quality publicly provided is lower than the quality wished by the mimicker, if he can increase the quality on private market, then the effect of public provision is cancelled.

In a traditional model of optimal taxation with utility nonseparable, Cremer and Gahvari (1997) show that the goods complementary to labor are goods candidates for public provision when individuals can increase their provision on a market. The reason is that the individual with high marginal productivity who imitates an individual with low productivity will work less (to obtain the same income) and thus will wish a lower quantity than the quantity provided by the government.

By considering a good which cannot be supplemented on a private market, Blomquist and Christiansen (1995) show that the public provision improves the social welfare if the "mimicker" has different preferences from the mimicked individual i.e. he wants a different amount of leisure (because of the non separability of the utility functions).

Since the program of optimal taxation does not allow us to have explicit solutions, it would be delicate to check an optimal quality of private good provided by the government or to check its optimal tarification.

We are going only to show that public provision can increase social welfare. We will use the same kind of proof as Blomquist Christiansen (1995), it acts to show that by using the public provision of a private good one can slacken one of the constraints of self-selection which was saturated. The difficulty is that in our model we have multiple incentive constraints and we cannot say precisely which one is binding and which is not, whereas in the usual literature there are only two incentives constraints and the only one is binding. But thanks to the preceding lemmas, we have some ideas on which constraints are binding.

Let's consider the binding incentive constraint (ij), (Thanks to precedent lemmas we know that at least one of the incentives constraint is binding), we thus have:

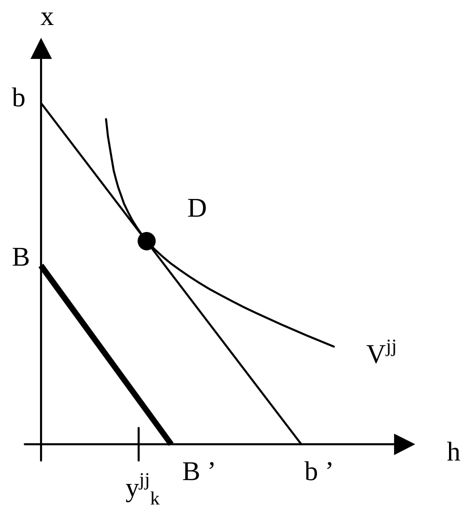
$$V\left(p_1, p_2, p_x, C^i + p_x d^i\right) + v\left(\frac{Y^i}{\omega_i}\right) = V\left(p_1, p_2, p_x, C^j + p_x d^i\right) + v\left(\frac{Y^j}{\omega_i}\right). \qquad (ij)$$

Which can be denoted by:

$$V^{ii} = V^{ij}$$

This constraint is written in term of indirect utilities, we will denote y_1^{ij} and y_2^{ij} the qualities chosen by individual i, and V^{ij} his utility when he says to be individual j and so is subjected to taxation (C^j, Y^j) corresponding with the individual j's before-tax income and after-tax income.

From this situation, we will show that if the government provides y_1^{jj} and y_2^{jj}, he can improve social welfare. Let's suppose that the government provides the quality y_k^{jj} to all individuals who declare to be of the j, and he finances this provision by an increase in the tax on the income of an amount p_k. The line of budget moves from (bb) towards (BB) because of tax increase, but the quality is then provided for free, the point D remains feasible. For the individual j nothing changes: he can consume the bundle that he consumed before the public intervention, and because the prices did not change, it is still his preferred bundle.

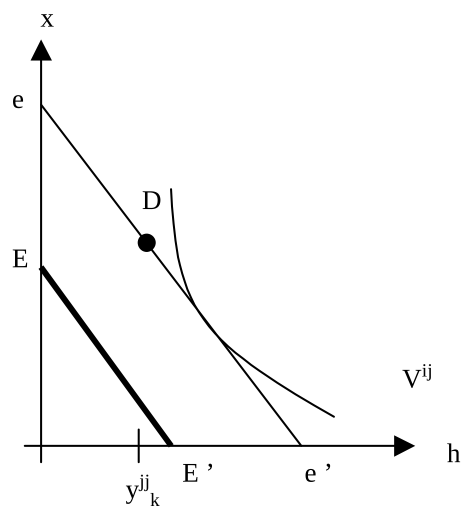


For the individual i who choose the bundle designed for him, nothing changes: either prices and taxes are the same for him. The government budget constraint remains balanced, there is just a substitution between commodity taxes and income tax for the individual j. On the other hand, one will show that because of the public provision, the constraint (ij) is not binding any more.

- if $y_k^{jj} \neq y_k^{ij}$ then the utility of the individual i , when he imitates the individual j decreases: either he continues to choose the quality that he wishes and his utility decreases because he pays more taxes, or he consumes quality y_k^{jj}, but then he doesn't have his preferred quality, and his utility decreases:

$$V_P^{ij} < V^{ij}.$$

(Where V_P^{ij} denotes the utility of the individual i which imitates the individual j when there is public provision). We had $V^{ii} = V^{ij}$ (self-selection constraint binding without public provision), and we have $V_P^{ii} = V^{ii}$ (individual i's utility is not affected by the public provision). Thus $V_P^{ij} < V_P^{ii}$.



Individual i budget constraint when he imitates j moves from (ee) towards (EE) and F remains feasible. But quality y_k^{jj} provided by the government is not his preferred quality, his utility decreases. The constraint (ij) which was binding without public provision is not anymore when the government provides directly the quality y_k^{jj} while adjusting taxation.

- if $y_k^{jj} \neq y_k^{ij}$, then utility of individual i when it imitates the individual j remains unchanged, the self-selection constraint remain binding, public provision is useless.

However using remark 1, we can easily determine the cases where $y_k^{jj} \neq y_k^{ij}$, and where $y_k^{jj} = y_k^{ij}$:

- If $d_i = d_j$, then $y_k^{jj} = y_k^{ij}$. Indeed, individual j and individual i who chooses (C^j, Y^j) to imitate j have the same after-tax income C^j and the same initial endowment. They are different only by their quantity of labor. Because utilities are separable, the quantity of work does not have any importance in the consumption choices, then $y_k^{jj} = y_k^{ij}$.

- If $d_i \neq d_j$, then $y_k^{jj} \neq y_k^{ij}$. the difference between initial endowments implies different incomes (and it is the relevant difference between individual j and individual i when he imitates j, since the differences in labor supply have effect on consumption choices) and thus consumption will be different. In particular we have $y_k^{jj} \neq y_k^{ij}$, i.e. the individual j and individual i who imitates j wish different qualities.

By using this type of reasoning, we show that the government can always improve individual 1's utility by providing publicly the right qualities.

Let us note that the other self selection constraints are still verified since nor utility of type i nor utility of the type j change if the government provides the quality chosen by the individual j and compenses this provision by the right taxation increase.

The social welfare increase does not come directly from the public provision, but from the adjustment of the taxation which is possible thanks to this public supply, making more costly the dissimulation strategy.

Proposition 1*Public provision of goods y_1 and y_2 is always optimal.*

Proof see appendix 2.

We have considered two goods which cannot be supplement on the market. It is why we interpret our model in term of quality rather than in term of quantity, each individual consumes one units of goods y_1 and y_2, but the quality could be different. If we consider than qualities of goods y_1 and y_2 are given, but individual can choose different amount, we can obtain the same kind of results. To do this we need some assumptions:

- Individual can supplement the public provision, but they cannot resale this provision.
- There exists in the economy at least two goods y_1 and y_2 such that[3] at the optimal taxation:

$$\begin{cases} y_1^{11} > y_1^{41} \\ y_1^{33} > y_1^{43} \end{cases} \text{or} \begin{cases} y_1^{11} > y_1^{41} \\ y_2^{33} > y_2^{43} \end{cases} \text{or} \begin{cases} y_2^{11} > y_2^{41} \\ y_1^{33} > y_1^{43} \end{cases}$$

Proposition 1'*Public provision of goods y_1 and y_2 is always optimal.*

Proof We can apply the proof of the proposition 1.

QED

This result is similar to the Rochet (1991)'s result. He proves than if probability of illness and marginal productivities are negatively correlated, then public provision of health insurance is optimal. But in our model, contrary to the Rochet's one, we have markets: in the Rochet's model, the public provision is the unique provision.

The good y_1 and y_2 must be, inferior goods (at least locally), this in an important differences from Cremer Gavahri's model. Then find that government should provide goods which are complement with labor supply. Because of the separability, it cannot be the case in our model. In some sens, our result is more intuitive.

Remark 2 *It is necessary for the regulator to provide two different goods. Indeed it exist several cases for which the provision of only one good is not sufficient. See appendix 3 for an example.*

[3] The notation we use is the folowing: y_k^{ij} is the quatity of good y_k chosen by the individual i when he mimickes the individual j.

Ours results are not due to a particular form of the utility function U. Indeed even if we consider a function U separable (weakly or not) between goods y_1 and y_2, the demand remains function of the initial endowments since they are share of individual i's income. Of course, we do not solve the complex problem of the optimal quality of the public provision, nor the problem of its tariffing. We obtain a simple result, in particular, we show that the assumption of non separability between labor supply and consumption good in the utility function is not essential if the government does not perfectly observe individual wealth.

5 Conclusion

In this work we show in a simple way that public provision of a private god can increase the welfare of economy, and the medical care (such as hospitalization) or education are goods which all present characteristics of good candidates for this public intervention because one cannot resell or increase there quantity/quality by going on a private market.

Up to now this result was shown in the litterature within a framework of optimal taxation a la Mirrlees making the assumption that utility functions were not separable in labor supply, this assumption was necessary for the justification of the public provision.

We show that if the assumption is made that the government does not observe perfectly individuals' wealth, while preserving the traditional assumption that the government observes perfectly labor incomes, then one can justify the public intervention with separable utilities in work, which seem to be the most realistic assumption taking into account econometric work on the subject (Browning and Meghir (1991)). To show the result, we face the usual problem in the presence of double heterogeneity of individuals.

Future research could consist in obtain result on optimal taxation and optimal publicly provided quality/quantity by simulation, using simple utility functions. One could also check with empirical datas which kind of good should be publicly provided

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Appendix 1

For simplicity, we will use the following notation for indirect utility functions.

$$V^{ij} = V\left(p_1, p_2, p_x, C^j + p_x d^i\right) + v\left(\frac{Y^j}{\omega_i}\right)$$

Lemma 1 all optimal taxation is such that the budget constraint is binding.

Proof Straightforward.

Lemma 2 *for all individual* 2, 3 *et* 4, *At least one of their self-selection constraint is binding.*

Proof We will prove this lemma for the individual 4. the proof is the same for individual 2 and 3.

Let's suppose that at the optimum, none of the individual 4's incentives constraint is binding:

$$V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\frac{Y^4}{\omega_h}\right) > V\left(p_1, p_2, p_x, C^1 + p_x d^h\right) + v\left(\frac{Y^1}{\omega_h}\right) \quad (41)$$
$$V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\frac{Y^4}{\omega_h}\right) > V\left(p_1, p_2, p_x, C^2 + p_x d^h\right) + v\left(\frac{Y^2}{\omega_h}\right) \quad (42)$$
$$V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\frac{Y^4}{\omega_h}\right) > V\left(p_1, p_2, p_x, C^3 + p_x d^h\right) + v\left(\frac{Y^3}{\omega_h}\right) \quad (43)$$

Then, using the continuity of the utility functions, we can increase the individual 4's before-tax income, such that the incentives constraints remains verified. Because of the separability between labor and consumption, the amounts of consumption don't change.

The budget constraint, which was banlanced, becomes in excess, thus, using lemma 1, we can increase the social welfare.

this proof is based on the rawlsian criterion. Even if the individual 4 is worse off, it doesn't matter: his utility function doesn't have any importance regard to social welfare.

QED

Lemma 3

$$\left\{ \begin{array}{l} V\left(p_1,p_2,p_x,C^3+p_xd^h\right)+v\left(\frac{Y^3}{\omega_l}\right)=V\left(p_1,p_2,p_x,C^1+p_xd^h\right)+v\left(\frac{Y^1}{\omega_l}\right) \\ V\left(p_1,p_2,p_x,C^3+p_xd^h\right)+v\left(\frac{Y^3}{\omega_h}\right)=V\left(p_1,p_2,p_x,C^1+p_xd^h\right)+v\left(\frac{Y^1}{\omega_h}\right) \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} C^1=C^3 \\ Y^1=Y^3. \end{array} \right.$$

Proof We substract the first equality from the second and we remark that $v\left(\frac{Y}{\omega_l}\right)-v\left(\frac{Y}{\omega_h}\right)$ is a monotonic function of Y.

Thus the equality:

$$v\left(\frac{Y^3}{\omega_l}\right)-v\left(\frac{Y^3}{\omega_h}\right)=v\left(\frac{Y^1}{\omega_l}\right)-v\left(\frac{Y^1}{\omega_h}\right)$$

implies

$$Y^3=Y^1.$$

$V\left(p_1,p_2,p_x,C+p_xd^h\right)$ is also a monotonic function of C, thus we have the equality:

$$C^3=C^1.$$

QED

Lemma 4

$$\left\{ \begin{array}{l} V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_l}\right)=V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_l}\right) \\ V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_h}\right)=V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_h}\right) \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} C^2=C^4 \\ Y^2=Y^4. \end{array} \right.$$

Proof: see lemma 3

QED

Lemma 5 *A situation such that:*

$$V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\tfrac{Y^2}{\omega_l}\right) > V\left(p_1,p_2,p_x,C^1+p_xd^h\right)+v\left(\tfrac{Y^1}{\omega_l}\right) \quad (2.1)$$
$$V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\tfrac{Y^2}{\omega_l}\right) > V\left(p_1,p_2,p_x,C^3+p_xd^h\right)+v\left(\tfrac{Y^3}{\omega_l}\right) \quad (2.3)$$
$$V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\tfrac{Y^2}{\omega_l}\right) \geq V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\tfrac{Y^4}{\omega_l}\right) \quad (2.4)$$
$$V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\tfrac{Y^4}{\omega_h}\right) > V\left(p_1,p_2,p_x,C^1+p_xd^h\right)+v\left(\tfrac{Y^1}{\omega_h}\right) \quad (4.1)$$
$$V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\tfrac{Y^4}{\omega_h}\right) \geq V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\tfrac{Y^2}{\omega_h}\right) \quad (4.2)$$
$$V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\tfrac{Y^4}{\omega_h}\right) > V\left(p_1,p_2,p_x,C^3+p_xd^h\right)+v\left(\tfrac{Y^3}{\omega_h}\right) \quad (4.3)$$

is impossible.

Proof: one can increase Y^4 and Y^2 such that constraint (2.1), (2.3), (4.1) and (43) are verified,

all functions are continuous. This before-tax income transformation can be done in a way such that constraint (2.4) and (4.2) are still verified.

- In a situation such that:

$$\begin{aligned}V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_l}\right) &> V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_l}\right)\\ V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_h}\right) &\geq V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_h}\right)\end{aligned}$$

  or

$$\begin{aligned}V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_l}\right) &\geq V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_l}\right)\\ V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_h}\right) &> V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_h}\right)\end{aligned}$$

  proof is straightforward.

- If we have:

$$\begin{aligned}V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_l}\right) &= V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_l}\right)\\ V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_h}\right) &= V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_h}\right).\end{aligned}$$

Thank lemma 4, we have:

$$\begin{aligned} C^2 &= C^4 \\ Y^2 &= Y^4. \end{aligned}$$

decreasing C^2 individual 2 and 4's before-tax income, self-selection constraints (2.4) et (4.2) are still verified.

QED

Appendix 2

Proposition 1*Public provision of goods y_1 and y_2 is always optimal.*

Proof We must consider different cases.

We will use letter j et i to denote individual 4 and 2, j denotes individual 2 or individual 4, i denotes the individual who is not denoted by the letter j.

In the same way, we will use s et t to denote individuals 1 et 3.

- **Case 1**

$$V\left(p_1, p_2, p_x, C^j + p_x d^j\right) + v\left(\frac{Y^j}{\omega_j}\right) = V\left(p_1, p_2, p_x, C^s + p_x d^j\right) + v\left(\frac{Y^s}{\omega_j}\right) \qquad (j.s)$$
$$V\left(p_1, p_2, p_x, C^j + p_x d^j\right) + v\left(\frac{Y^j}{\omega_j}\right) > V\left(p_1, p_2, p_x, C^i + p_x d^j\right) + v\left(\frac{Y^i}{\omega_j}\right) \qquad (j.i)$$
$$V\left(p_1, p_2, p_x, C^j + p_x d^j\right) + v\left(\frac{Y^j}{\omega_j}\right) > V\left(p_1, p_2, p_x, C^t + p_x d^j\right) + v\left(\frac{Y^t}{\omega_j}\right) \qquad (j.t)$$

Providing freely the quality y_k^{ss} (here the government can provide y_1 or y_2 it doesn't matter) and increasing the taxes paid by individual s to compensate this expense as indicated in section 2, individual s's utility doesn't change, and individual j is worse off when he mimicks individual s. Thus equation $(j.s)$ is not binding anymore. One can increase Y^j such that all the individual j's incentives constraints are verified. All others incentives constraints are still verified and individual s's utility doesn't change. The budget constraint becomes in excess and, thanks to the lemma 1, we can increase individual s's utility function, i.e.. increase social welfare.

- **Case 2**

$$V\left(p_1, p_2, p_x, C^2 + p_x d^h\right) + v\left(\frac{Y^2}{\omega_l}\right) \geq V\left(p_1, p_2, p_x, C^s + p_x d^h\right) + v\left(\frac{Y^s}{\omega_l}\right) \qquad (2.s)$$
$$V\left(p_1, p_2, p_x, C^2 + p_x d^h\right) + v\left(\frac{Y^2}{\omega_l}\right) = V\left(p_1, p_2, p_x, C^t + p_x d^h\right) + v\left(\frac{Y^t}{\omega_l}\right) \qquad (2.t)$$
$$V\left(p_1, p_2, p_x, C^2 + p_x d^h\right) + v\left(\frac{Y^2}{\omega_l}\right) \geq V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\frac{Y^4}{\omega_l}\right) \qquad (2.4)$$
$$V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\frac{Y^4}{\omega_h}\right) = V\left(p_1, p_2, p_x, C^s + p_x d^h\right) + v\left(\frac{Y^s}{\omega_h}\right) \qquad (4.s)$$
$$V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\frac{Y^4}{\omega_h}\right) \geq V\left(p_1, p_2, p_x, C^2 + p_x d^h\right) + v\left(\frac{Y^2}{\omega_h}\right) \qquad (4.2)$$
$$V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\frac{Y^4}{\omega_h}\right) \geq V\left(p_1, p_2, p_x, C^t + p_x d^h\right) + v\left(\frac{Y^t}{\omega_h}\right) \qquad (4.t)$$

As in case 1, we must decrease individual 4 and 2's utilities when they mimick individual 1 or 3. To do this, it is sufficient that the government provides to individuals who declare to be individual 1 the good y_1^{11} and to individuals who declare to be individual 3 the good y_2^{33} and in the two cases increases taxes as indicates in section 2. Individuals 1 and 3 remain unchanged, and we can increase individuals 2 and 4 before-tax income such that their self-selection constraints remain verified. The budget constraint becomes in excess, and one can increase social welfare.

- Remain cases such that:

$$V\left(p_1, p_2, p_x, C^2 + p_x d^h\right) + v\left(\frac{Y^2}{\omega_l}\right) > V\left(p_1, p_2, p_x, C^1 + p_x d^h\right) + v\left(\frac{Y^1}{\omega_l}\right) \quad (2.1)$$
$$V\left(p_1, p_2, p_x, C^2 + p_x d^h\right) + v\left(\frac{Y^2}{\omega_l}\right) > V\left(p_1, p_2, p_x, C^3 + p_x d^h\right) + v\left(\frac{Y^3}{\omega_l}\right) \quad (2.3)$$
$$V\left(p_1, p_2, p_x, C^2 + p_x d^h\right) + v\left(\frac{Y^2}{\omega_l}\right) \geq V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\frac{Y^4}{\omega_l}\right) \quad (2.4)$$
$$V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\frac{Y^4}{\omega_h}\right) > V\left(p_1, p_2, p_x, C^1 + p_x d^h\right) + v\left(\frac{Y^1}{\omega_h}\right) \quad (4.1)$$
$$V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\frac{Y^4}{\omega_h}\right) \geq V\left(p_1, p_2, p_x, C^2 + p_x d^h\right) + v\left(\frac{Y^2}{\omega_h}\right) \quad (4.2)$$
$$V\left(p_1, p_2, p_x, C^4 + p_x d^h\right) + v\left(\frac{Y^4}{\omega_h}\right) > V\left(p_1, p_2, p_x, C^3 + p_x d^h\right) + v\left(\frac{Y^3}{\omega_h}\right) \quad (4.3)$$

Lemma 2 and lemma 3 show that these situations are impossible.

QED

Appendix 3

It is necessary for the regulator to provide two different goods. Indeed it exist several cases for which the provision of only one good is not sufficient. For example. if the optimal imposition leads to a situation such as:

$$V\left(p_1,p_2,p_x,C^1+p_xd^l\right)+v\left(\frac{Y^1}{\omega_l}\right)>V\left(p_1,p_2,p_x,C^2+p_xd^l\right)+v\left(\frac{Y^2}{\omega_l}\right) \quad (1.2)$$
$$V\left(p_1,p_2,p_x,C^1+p_xd^l\right)+v\left(\frac{Y^1}{\omega_l}\right)>V\left(p_1,p_2,p_x,C^3+p_xd^l\right)+v\left(\frac{Y^3}{\omega_l}\right) \quad (1.3)$$
$$V\left(p_1,p_2,p_x,C^1+p_xd^l\right)+v\left(\frac{Y^1}{\omega_l}\right)>V\left(p_1,p_2,p_x,C^4+p_xd^l\right)+v\left(\frac{Y^4}{\omega_l}\right) \quad (1.4)$$
$$V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_l}\right)>V\left(p_1,p_2,p_x,C^1+p_xd^h\right)+v\left(\frac{Y^1}{\omega_l}\right) \quad (2.1)$$
$$V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_l}\right)=V\left(p_1,p_2,p_x,C^3+p_xd^h\right)+v\left(\frac{Y^3}{\omega_l}\right) \quad (2.3)$$
$$V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_l}\right)=V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_l}\right) \quad (2.4)$$
$$V\left(p_1,p_2,p_x,C^3+p_xd^l\right)+v\left(\frac{Y^3}{\omega_h}\right)=V\left(p_1,p_2,p_x,C^1+p_xd^l\right)+v\left(\frac{Y^1}{\omega_h}\right) \quad (3.1)$$
$$V\left(p_1,p_2,p_x,C^3+p_xd^l\right)+v\left(\frac{Y^3}{\omega_h}\right)>V\left(p_1,p_2,p_x,C^2+p_xd^l\right)+v\left(\frac{Y^2}{\omega_h}\right) \quad (3.2)$$
$$V\left(p_1,p_2,p_x,C^3+p_xd^l\right)+v\left(\frac{Y^3}{\omega_h}\right)>V\left(p_1,p_2,p_x,C^4+p_xd^l\right)+v\left(\frac{Y^4}{\omega_h}\right) \quad (3.4)$$
$$V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_h}\right)=V\left(p_1,p_2,p_x,C^1+p_xd^h\right)+v\left(\frac{Y^1}{\omega_h}\right) \quad (4.1)$$
$$V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_h}\right)=V\left(p_1,p_2,p_x,C^2+p_xd^h\right)+v\left(\frac{Y^2}{\omega_h}\right) \quad (4.2)$$
$$V\left(p_1,p_2,p_x,C^4+p_xd^h\right)+v\left(\frac{Y^4}{\omega_h}\right)>V\left(p_1,p_2,p_x,C^3+p_xd^h\right)+v\left(\frac{Y^3}{\omega_h}\right). \quad (4.3)$$

If the government provides only one good, he cannot increase the individual 1*'s welfare. Let's consider that the government provides* y_1^{11} *to all individuals who declares being individual* 1*. Individual* 4*'s utility when he declares being individual* 1 *decreases. To follow our reasoning, we should to increase individual* 4*'s before-tax income, to do this in our example, we should increase individual* 2*'s before-tax income. And because of equation* (2.3) *we should also increase individual* 3*'s before-tax income, so we should increase individual* 1*'s before tax income, which decrease his utility.*