The Objective of an Imperfectly Competitive Firm and Constrained Pareto Efficiency

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Abstract

We consider a simple model of a firm acting strategically on behalf of its shareholders. The price normalization problem arising in general equilibrium models of imperfect competition is overcome by using the concept of real wealth maximization. This concept is based on shareholders' aggregate demand and does not involve any comparison of utility profiles that shareholders can possibly obtain. In this paper we explore the efficiency properties of real wealth maximizing strategies for the group of shareholders.

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1 Introduction

In this paper we consider a simple model of a firm acting strategically on behalf of its owners. The firm influences relative prices in the economy either by its production decision or directly as a price setter. We assume that the firm is owned by a large number (ideally, a continuum) of small shareholders who take prices and wealth as given when choosing their consumption plans. If a firm is engaged in imperfect competition, the choice of a strategy does not only affect the wealth of its shareholders, but also the prices shareholders face as consumers on the market. Since demand patterns and shareholdings differ across individuals, different shareholders would like their firm to pursue different objectives. A similar problem is encountered in economies with incomplete markets, where shareholders disagree because of their idiosyncratic insurance needs. Thus, in case of imperfect competition as well as in case of incomplete markets, a social choice problem arises that does not have an obvious solution [see, in particular, Arrow (1950)].

In the literature this social choice problem is often simply assumed away. In the field of industrial organization it is nearly always taken for granted that shareholders only consume and own goods the prices of which do not depend on the action of their firm. Similarly, in the finance literature dealing with incomplete markets one encounters the assumption that shareholders want the firm to increase their wealth today independently of any consequences for the random consumption tomorrow.

In many models of imperfect competition firms are assumed to maximize profits. However, it is well-known from the literature that this objective is ill-defined unless particular, strong assumptions are made, see e.g. Gabszewicz and Vial (1972) and H. Dierker and Grodal (1986). Since the price level remains undetermined, profits are normalized by using one of the commodities as numéraire or, more generally, by applying some price normalization rule. But different price normalizations entail profit functions, which are in general not related to each other by monotone transformations. Hence, maximization of profits in different normalizations amounts to firms pursuing different objectives.

To overcome the price normalization problem E. Dierker and Grodal (1999) propose the concept of a strategy maximizing the real wealth of the firm's shareholders. Given the strategies of all other firms the strategy $\hat{\sigma}$ maximizes shareholders' real wealth if it is undominated in the following sense: There does not exist another strategy σ such that the aggregate demand of all owners at $\hat{\sigma}$ is in the interior of their aggregate budget set corresponding to σ.

Real wealth maximization is based on the aggregate demand of the firm's owners and on profits. Moreover, it is independent of any a priori chosen price normalization. Although it depends on relative prices only one can think of shareholders' aggregate demand as yielding endogenously a yardstick to compare profits, since in a real wealth maximum it is impossible for the shareholders to obtain more than 100 percent of their present aggregate demand by varying $\hat{\sigma}$.

In an important contribution to the theory of incomplete markets, Drèze (1974) defines the goal of a firm by using Pareto comparisons accompanied by redistribution. Given the production decisions of all other firms, a production decision of the firm under consideration leads to an allocation of goods among its shareholders. In general, these allocations will not be Pareto comparable. Therefore, Drèze proposes the following test that a production decision has to pass: It must be impossible to choose another production plan together with a redistribution scheme for the group of shareholders such that all shareholders become better off if they keep their portfolio fixed. Clearly, since markets are incomplete, the redistribution must only involve the good available to the group of shareholders at the present date $t = 0$.

We are now going to formulate the analogue of the Drèze criterion for the case of imperfect competition. Consider a specific firm and assume that the strategies of all other firms are given. The strategy σ of the firm under consideration gives rise to a price system which, together with the profits $\Pi(\sigma)$, determines the budget set of each shareholder of the firm. Thus, each strategy σ induces an allocation of goods among the shareholders of the firm under consideration. As in case of incomplete markets, these allocations will in general not be Pareto ranked. We say that the strategy σ of a firm dominates the strategy σ', if the aggregate demand of the shareholders of this firm at σ can be redistributed in such a way that every shareholder becomes better off. In analogy to the Drèze criterion we propose the following test that a strategy $\bar{\sigma}$ has to pass: There is no strategy σ that dominates $\bar{\sigma}$. [1] In this case $\bar{\sigma}$ is called undominated or constrained efficient.

Observe that the original Drèze criterion for firms operating in a system of incomplete markets requires the firm to know the distribution of shareholders' preferences or at least, as shown by Drèze (1974) and Geanakoplos et al. (1990), the joint distribution of shares and utility gradients at the optimal consumption bundles. Similarly, the analogue of the Drèze criterion for oligopolistic firms is based on Pareto comparisons and cannot be stated without reference to preferences. In contrast, a firm maximizing shareholders' real wealth only needs to know their aggregate demand function in addition to their aggregate budget set (which is determined by profits). Obviously, the informational requirements are

[1]Notice that in the incomplete market case the test only involves redistribution of the good available at $t = 0$, but in its analogue just described all goods are redistributed.

much less demanding here and one is led to ask how both concepts are related to each other.

In the paper we first show that any constrained efficient strategy maximizes shareholders' real wealth if the firm's profit function is concave. Since real wealth maximization is defined without reference to utility functions, the more interesting question is whether a real wealth maximizing strategy is constrained efficient. The answer is no: We provide an example with two real wealth maximizing strategies each of which dominates the other one. Moreover, in the example there does not exist any constrained efficient strategy. In other words, constrained efficiency is an ambitious goal that can very well be out of reach although real wealth maximizing strategies do exist.

What is the reason for the nonexistence of constrained efficient strategies? Given a strategy profile, every shareholder faces a budget set and his wealth consists of the value of his initial endowments and his profit income. This fact restricts the set of allocations that can be reached, since there are allocations that can only be obtained if wealth is distributed in a different way. Therefore, one has to focus on the role of redistribution.

In our setting of imperfect competition, the notion of domination underlying the concept of constrained efficiency involves redistribution of endowments among shareholders. On the other hand, no redistribution among the shareholders of a firm is made at a real wealth maximum or at any other allocation that can be reached by some strategy choice. Thus, the hypothetical planner who freely redistributes goods among shareholders in order to obtain a Pareto superior allocation performs a task that the shareholders themselves cannot do.[2]

In economies with complete markets, perfect competition, private ownership of resources and firms and a convex aggregate production set, this point can be disregarded: A Walras equilibrium exists under rather mild assumptions and every Walras equilibrium is, according to the first welfare theorem, Pareto efficient. In other words, a planner who can freely redistribute everything when he chooses an allocation cannot do better than the agents do without a redistribution of wealth. However, Guesnerie (1975) has shown that in an environment with nonconvex production sets there can be a conflict between efficiency and distribution. Due to the nonconvexity of the production set Walras equilibria need not exist.

[2]Drèze's characterization of the objective of a firm is intimately related to Shapley's extension of the value to NTU games. In both cases the solution concept involves a tool that is not available to the agents. This hypothetical tool is used to formulate the following postulate: The outcome should be such that no improvement would be possible even if the tool were available. Clearly, if this condition is fulfilled, there is no need to employ the tool and the purely hypothetical character of the tool is immaterial.

Thus, firms are required to sell at marginal costs rather than to maximize profits. Guesnerie (1975) gives an example in which no efficient marginal cost pricing equilibrium exists given the distribution of ownership and discusses extensively the role convexity plays for the separation of efficiency and issues of distribution.

In this paper we show that a similar problem arises in models of imperfect competition. In E. Dierker, H. Dierker, and Grodal (1999) incomplete markets are considered and a robust example of an economy with a unique Drèze equilibrium is constructed in such a way that the equilibrium is not constrained efficient. As Guesnerie's setting, these models exhibit an intrinsic nonconvexity. Drèze (1974) already pointed out that the set of feasible allocations in models with incomplete markets with firms necessarily becomes nonconvex. Also, in models of imperfect competition the aggregate budget set of the group of shareholders of a firm is nonconvex.

This paper deals with imperfect competition and we face the question of whether there are conditions which imply that real wealth maximizing strategies are constrained efficient. It should be no surprise that a convexity assumption is helpful. We require a certain aggregate preferred set of the shareholders to be convex in order to show that every real wealth maximum is constrained efficient.

The paper is organized as follows. Section 2 introduces the model. In Section 3 and Section 4 real wealth maximization and constrained efficiency are defined, respectively. Section 5 contains the example. In Section 6 conditions are stated such that real wealth maximization entails constrained efficiency. In Section 7 it is shown that a certain convexity assumption is independent of which commodity bundle is used to normalize prices and measure wealth. In Section 8 the connection between uniqueness of real wealth maximizing strategies and the convexity of the aggregate preferred set is investigated.

2 Model and Basic Notation

It suffices to consider an economy with two commodities and one price setting monopolist who produces good 1 using good 0, the numéraire, as input. The analysis will be essentially the same as that of a price setting firm in an oligopolistic market, if the prices of its competitors are given. For simplicity, we assume that the firm has fixed unit costs c. The strategy P of the firm is the decision to offer one unit of the product in exchange for P units of the numéraire. If profit or wealth is measured in terms of the numéraire, we use the subscript N. Profits obtained at prices $(1, P)$ are denoted $\Pi_N(P)$.

The consumers are denoted by $I = \{1, \cdots, m\}$. Consumer $i \in I$ has shares $\vartheta^i \geq 0$ in the firm. We assume that the firm has a large set $\mathcal{J} = \{i \in I \mid \vartheta^i > 0\}$ of owners and that all consumers, owners as well as non-owners, take their budget sets as given. Suppose for simplicity that the consumption set of every consumer equals $\mathbb{R}^2_+$ and that no consumer has initial endowments of the product, i.e. consumer i has the initial endowment $e^i = (e^i_0, 0)$ where $e^i_0 > 0$.

Since Pareto comparisons are made, we assume that every consumer i has a continuous demand function d^i that is generated by a strictly convex, monotone, and continuous preference relation $\succ^i$, which can conveniently be represented by the C^2 utility function U^i. The demand function d^i is homogeneous of degree 0 and satisfies the budget identity $(1, P)d^i(1, P, W^i_N) = W^i_N$. The wealth of consumer i at prices $(1, P)$ is described by the continuous function $W^i_N(P) = e^i_0 + \vartheta^i \Pi_N(P)$ and $W_N(P) = \sum_{i\in\mathcal{I}} W^i_N(P)$ denotes the aggregate wealth of the shareholders.

We assume throughout that profit expectations are correct, i.e. the demand based on consumers' wealth expectations generates precisely the expected profits, if the monopolist satisfies the demand for its product. That is to say, profits fulfill the equation $\Pi_N(P) = (P - c)d_1(P)$, where $d_1(P) = \sum^m_{i=1} d^i_1(1, P, e^i_0 + \vartheta^i \Pi_N(P))$ is the total demand of all consumers for good 1 if prices are $(1, P)$ and profits are $\Pi_N(P)$.We assume throughout that Π_N is a continuous function. Let $D^i(P) = d^i(1, P, e^i_0 + \vartheta^i \Pi_N(P))$ denote shareholder i's demand corresponding to strategy P. Shareholders' aggregate demand is $D(P) = \sum_{i\in\mathcal{I}} D^i(P)$. We assume that $\Pi_N(P)$ attains its maximum in the interior of the set of strategies $\mathcal{P} = [c, \bar{c}]$ and that $W_N(P) \geq 0$ for all $P \in \mathcal{P}$. Moreover, the demand of the non-owners for the firm's product is supposed to be positive if $P = c$.

Since we only analyze commodity assignments to the group $\mathcal{I}$ of shareholders, we call such assignments allocations for short. For every strategy $P \in \mathcal{P}$, there exists exactly one allocation, namely the allocation $(D^i(P))_{i\in\mathcal{I}}$. An allocation is attainable iff it can be implemented by a strategy choice of the firm.

Definition . *The allocation $(x^i)_{i\in\mathcal{I}}$ is* attainable *iff there exists $P \in \mathcal{P}$ such that $x^i = D^i(P)$ for all $i \in \mathcal{I}$.*

Observe that no sidepayments occur in the definition of an attainable allocation. As we have already mentioned, all agents, shareholders as well as non-shareholders, buy the firm's product at market prices. Thus, consumers are treated as anonymous. Nobody knows which characteristics any other, particular person possesses. This fact is in accordance with the Walrasian tradition, since Walras equilibria depend only on the distribution of agents' characteristics. In the present framework no additional information is assumed. Therefore, sidepayments cannot be carried out.

3 Real Wealth Maximization

Each strategy P defines the budget line

$$BL(P) = \{(x_0, x_1) \in \mathbb{R}^2 \mid x_0 + Px_1 = W_N(P)\} \tag{1}$$

and the corresponding budget set

$$AB(P) = \{(x_0, x_1) \in \mathbb{R}^2_+ \mid (1, P)(x_0, x_1) \leq W_N(P)\} \tag{2}$$

of the group of owners. Their aggregate budget set is $AB = \bigcup_{P \in \mathcal{P}} AB(P)$. Notice that AB is compact, since $\mathcal{P}$ is compact and W_N is continuous. Since $\mathbb{R}^2_+ \setminus AB(P)$ is convex for every P and $AB = \mathbb{R}^2_+ \setminus \bigcap_{P \in \mathcal{P}} \left(\mathbb{R}^2_+ \setminus AB(P)\right)$, the aggregate budget set is the complement of a convex set. The North-East boundary of AB is called the *aggregate budget curve* ABC. More precisely,

$$ABC = \{x \in AB \mid \nexists\, z \gg 0 \text{ such that } x + z \in AB\}. \tag{3}$$

We define the objective of the monopolist without making a priori assumptions on the demand behavior of the owners. Consider two different strategies P_1, P_2 and the corresponding aggregate budget sets $AB(P_1)$ and $AB(P_2)$. First, we look at the extreme case, in which $AB(P_1)$ is strictly contained in $AB(P_2)$. Let $x \geq 0, x \neq 0$, be any commodity bundle. Clearly, the number of units of the bundle x which the owners can afford, if the firm chooses the strategy P_2, is strictly larger than the number of units they can buy if the firm chooses the strategy P_1. Whatever bundle the firm uses to evaluate the real wealth of the owners, their aggregate wealth is larger at P_2 than at P_1. We assume that a real wealth maximizing firm choosing between P_1 and P_2 will select P_2, although it may very well be that some owners, due to distributional effects, prefer the strategy P_1 to P_2.

In general, the budget sets corresponding to different strategies of the firm will not be ordered by inclusion. Hence the ordering of the budget sets according to the number of units of the bundle x which can be bought out of owners' aggregate wealth depends on the choice of the reference bundle x. However, when the firm considers a strategy P, it is assumed to know the composition $x(P) = D(P)/\|D(P)\|$ of owners' aggregate demand at P. In our opinion it is natural for the firm to use $x(P)$ as the reference bundle.

Notice that, in general, owners do not agree on the strategy choice of their firm. Shareholder i wants the firm to maximize $U^i(D^i(P))$. Since owners differ with regard to shares, endowments, and preferences, they want the firm to pursue different goals. As a consequence, there will be a continuum of strategies that cannot be Pareto ranked. Pareto comparisons of attainable states cannot provide us with a useful definition of the goal of the firm.

The same holds true for profit maximization unless very strong assumptions are made. The maximization of profits Π_N measured in terms of the numéraire is justified provided shareholders only own and consume the numéraire. Moreover, different ways to normalize prices and measure profits lead to different profit functions and hence different maxima. If there is no clear, a priori specified connection between some commodity basket used to define profits and the owners' desires, the maximization of a profit function cannot be used as an objective of the firm acting on behalf of its owners. In E. Dierker and Grodal (1999) the following relation is used to introduce an objective of the firm.

Definition . *Owners' real wealth at* $P_1 \in \mathcal{P}$ *can be increased by the strategy* $P_2 \in \mathcal{P}$*, in symbols* $P_1 \prec_{rw} P_2$*, iff* $(1, P_2)D(P_1) < W_N(P_2)$ *.*

The objective of the firm is to choose a strategy $\hat{P}$ such that there is no other strategy P which increases owners' real wealth. That is to say, there is no other strategy P with the property that the aggregate demand $D(\hat{P})$ is in the interior of the budget set $AB(P)$. If such a strategy P would exist, the group of owners could buy more units of the bundle $D(\hat{P})$, if the firm would choose the strategy P instead of $\hat{P}$.

Definition . *The strategy* $\hat{P} \in \mathcal{P}$ maximizes shareholders' real wealth*, if there does not exist a strategy* $P \in \mathcal{P}$ *such that* $(1, P)D(\hat{P}) < W_N(P)$*, that is to say, if* $D(\hat{P}) \in ABC$*.*

Strategies maximizing shareholders' real wealth need not exist, since the relation $\prec_{rw}$ need neither be acyclic nor convex. In E. Dierker and Grodal (1998) conditions on the aggregate demand are given which imply that $\prec_{rw}$ is acyclic. Also, they show that convexity of $\prec_{rw}$ obtains if the profit function is concave. In either case, a real wealth maximum exists.

Observe that the concept of real wealth maximization is a natural generalization of profit maximization in the perfectly competitive case with complete markets. Moreover, it generalizes the standard approach in industrial organization, in which it is (implicitly) assumed that shareholders only own and consume the numéraire commodity.

The first order condition for real wealth maximization states that shareholders' marginal wealth equals shareholders' aggregate demand for the product. [3] Since we assume the value of the initial endowment, which is of the form $(e_0^i, 0)$, to be independent of P, we know that marginal wealth equals marginal profits.

Remark 1. *Assume that there exists* $P \in \mathcal{P}$ *such that* $c \prec_{rw} P$*. If* Π_N *is* C^1 *and* $\hat{P} \in \mathcal{P}$ *maximizes shareholders' real wealth, then*

$$W_N'(\hat{P}) = \Pi_N'(\hat{P}) = D_1(\hat{P}) \ . \tag{4}$$

[3] For a proof see E. Dierker and Grodal (1998).

It is worth noticing that the first order condition for real wealth maximization characterizes the following geometrical object. To each strategy $P \in \mathcal{P}$ there corresponds the aggregate budget line $BL(P)$ defined in (1). The 1-parameter family $\{BL(P)\}_{P \in \mathcal{P}}$ forms a smooth 2-dimensional manifold denoted $\mathcal{L}$, since the derivative of the mapping $(x_0, x_1, P) \mapsto x_0 + Px_1 - W_N(P)$ does not vanish. $\mathcal{L}$ is embedded in $\mathbb{R}^2 \times \mathcal{P}$. Intuitively, the budget lines in $BL(P) \in \mathbb{R}^2$ correspond to the strategies P and the embedding lists the names of these strategies explicitly. Now, project $\mathcal{L}$ to the commodity space $\mathbb{R}^2$, i.e. drop the names of the strategies. The *envelope* of the family of budget lines is defined as the set of critical values of the projection of $\mathcal{L}$ into the commodity space $\mathbb{R}^2$. It is characterized by the condition $\frac{\partial}{\partial P}(x_0 + Px_1 - W_N(P)) = 0$, that is to say $W_N'(P) = x_1$, together with the budget equation. Thus, the envelope is given by

$$Env = \{(x_0, x_1) \in \mathbb{R}^2 \mid \exists P \in \mathcal{P} \text{ with } x_0 + Px_1 = W_N(P) \text{ and } W_N'(P) = x_1\}.$$

Remark 2. *The strategy $\bar{P} \in \mathcal{P}$ satisfies the first order condition $W_N'(\bar{P}) = \Pi_N'(\hat{P}) = D_1(\bar{P})$ for real wealth maximization iff $D(\bar{P})$ lies in Env.*

4 Constrained Efficiency

In the present setting constrained efficiency refers to Pareto comparison among the shareholders only. The constraint is due to the fact that we analyze a second best problem. The firm wants to extract wealth from the non-owners. However, if the firm raises its price for that purpose, then the owners themselves also have to pay more, since they must buy the firm's product at market prices.

If the firm chooses P, then the group of shareholder obtains the profit $\Pi_N(P)$ as well as the commodity bundle $D(P)$. Observe that the way the profit is raised cannot be separated from the way it is spent, since the choice of the strategy P determines both, the profit income and the consumption of every shareholder. The definition of constrained efficiency must take this link into account.

The concept of constrained efficiency is based on the following thought experiment. Suppose $\bar{P}$ has been chosen and is compared with the alternative P. Clearly, if P is implemented, the group of shareholders receives the bundle $D(P)$ and the profit $\Pi_N(P)$ contains the part $(P - c)D(P)$ derived from this bundle. In order to keep this relation intact, $D(P)$ is kept fixed in the thought experiment. Assume that the group of shareholders could, after having gotten $D(P)$, redistribute this bundle in order to compensate the losers of the move from $\bar{P}$ to P and that the outcome of the redistribution would be their final consumption. Then they would certainly not move from the original strategy $\bar{P}$ to the alternative P, if they could not even obtain a Pareto improvement for themselves in this hypothetical situation.

We are now going to define constrained efficiency in a more formal way.

Definition . *The strategy* $P_1 \in \mathcal{P}$ *is* dominated *by the strategy* $P_2 \in \mathcal{P}$ *iff there exist bundles* $(x^i)_{i\in\mathcal{I}}$ *such that* $\sum_{i\in\mathcal{I}} x^i = D(P_2)$ *and* $D^i(P_1) \prec^i x^i$ *for all* $i \in \mathcal{I}$.

A strategy $\bar{P}$ is undominated if there is no $P \in \mathcal{P}$ such that $D(P)$ can be distributed among the shareholders in a way which makes them better off than at $\bar{P}$. An undominated strategy $\bar{P}$ and the corresponding allocation $D^i(\bar{P})$ are called constrained efficient.

Definition . *The strategy* $\bar{P}$ *and the corresponding allocation* $(D^i(\bar{P}))_{i\in\mathcal{I}}$ *are* constrained efficient *iff there does not exist a strategy* $P \in \mathcal{P}$ *dominating* $\bar{P}$.

We derive the first order condition for constrained efficiency.

Remark 3. *Let shareholders' aggregate demand function* D *and the profit function* Π_N *be* C^1. *Assume that the allocation* $(D^i(\bar{P}))_{i\in\mathcal{I}}$ *is constrained efficient. Then the strategy* $\bar{P}$ *satisfies the first order condition*

$$(1, \bar{P}) \cdot (D_0'(\bar{P}), D_1'(\bar{P})) = 0\,. \tag{5}$$

Condition (5) is equivalent to the first order condition for real wealth maximization, i.e. $D_1(\bar{P}) = W_N'(\bar{P})$.

Proof. Let $\bar{P} \in \mathcal{P}$ and assume that the allocation $(D^i(\bar{P}))_{i\in\mathcal{I}}$ is constrained efficient. Assume by way of contradiction that $(1, \bar{P}) \cdot (D_0'(\bar{P}), D_1'(\bar{P})) \neq 0$. Without loss of generality let $(1, \bar{P}) \cdot (D_0'(\bar{P}), D_1'(\bar{P})) > 0$. (If this expression is negative, consider strategies $P < \bar{P}$.) Then $(1, \bar{P})(D(P) - D(\bar{P})) > 0$ for $P > \bar{P}$ and $|P - \bar{P}|$ sufficiently small. Since all preferences are strictly convex and the utility functions are C^2, there exists $\varepsilon > 0$ such that for any shareholder i the following condition holds. If $u \in \mathbb{R}^2$, $\|u\| < \varepsilon$, $(1, \bar{P}) \cdot u > 0$, and $D^i(\bar{P}) + u \in \mathbb{R}^2_+$, then $D^i(\bar{P}) + u \succ^i D^i(\bar{P})$ [see, e.g., Magill and Quinzii (1996), p. 359]. Since D is continuous, there exists $\delta > 0$ such that $\|D(P) - D(\bar{P})\| < \epsilon$ for $|P - \bar{P}| < \delta$. Now let $u^i = \vartheta^i(D(P) - D(\bar{P}))$. For $|P - \bar{P}|$ sufficiently small we obtain $D^i(\bar{P}) + \vartheta^i(D^i(P) - D(\bar{P})) \succ^i D^i(\bar{P})$ for all owners i. However, as $\sum_{i\in\mathcal{I}}(D^i(\bar{P}) + \vartheta^i(D(P) - D(\bar{P}))) = D(P)$, this contradicts the fact that $(D^i(\bar{P}))_{i\in\mathcal{I}}$ is constrained efficient. Hence, $(1, \bar{P}) \cdot (D_0'(\bar{P}), D_1'(\bar{P})) = 0$.

To see that the first order condition for constrained efficiency coincides with the first order condition for real wealth maximization we differentiate the budget equation $(1, P) \cdot D(P) = W_N(P)$ and obtain $(1, P) \cdot (D_0'(P), D_1'(P)) + D_1(P) = W_N'(P)$ for all $P \in \mathcal{P}$. Hence, $(1, P) \cdot (D_0'(P), D_1'(P)) = 0$ iff $D_1(P) = W_N'(P)$. □

Under the assumption that the profit function Π_N is concave E. Dierker and Grodal (1998) show that a real wealth maximum obtains whenever the first order condition for real wealth maximization is satisfied. Thus, Remark 3 implies:

Proposition 1. *Let* D *and* Π_N *be* C^1 *and* Π_N *concave. Assume that the strategy* $\bar{P}$ *is constrained efficient. Then the strategy* $\bar{P}$ *maximizes shareholders' real wealth.*

5 Mutual Domination of Real Wealth Maxima

The following example shows that real wealth maxima need not be constrained efficient. In the example there are two strategies that maximize the real wealth of the shareholders, but each of them dominates the other one. Moreover, the example shows that constrained efficient allocations need not exist.

The example is constructed as follows. The aggregate demand function g of all non-owners is taken as linear. The group of owners of the firm can "almost" be represented by one agent. Clearly, if there is only one representative owner, a constrained efficient strategy must exist, since the utility of the representative owner takes its maximum when the strategy P varies over the compact interval $\mathcal{P}$. However, the example shows that just adding a small, even quasilinear, owner can entail that constrained efficient allocations cease to exist and, in particular, that real wealth maximizing strategies can dominate each other.

There are two owners of the firm, a very large one with a CES utility function and a small one with a quasilinear utility function. The weights are calibrated such that the two real wealth maxima yield approximately the same utility for both owners.

In the example the profit function of the firm is not concave. Hence, the aggregate budget curve ABC has a kink. We have chosen the non-concave profit function, in order to avoid complicated demand functions of the non-owners. The example could easily be turned into one with a concave profit function. Consider the concavification $\tilde{\Pi}_N$ of the profit function in the example and define the demand function g for the non-owners as $g(P) = (\tilde{\Pi}_N - (P-1)\tilde{D}_1(P))/(P-1)$, where $\tilde{D}_1$ is the aggregate demand of the shareholders when they get the profit $\tilde{\Pi}_N$.

There is one firm with constant unit costs $c = 1$. The demand function of the non-owners is given by

$$g(1, P) = -P + 1000.$$

There are two (types of) owners with initial endowments $e^1 = (1000, 0)$ and $e^2 = (542, 0)$. They have the following CES utility function and a quasilinear utility function, respectively.

$$\begin{aligned} U^1(x_0, x_1) &= x_0^{\frac{10}{11}} + (21x_1)^{\frac{10}{11}} \\ U^2(x_0, x_1) &= x_0 + 144x_1^{\frac{1}{2}}. \end{aligned}$$

The (large) CES shareholder owns the fraction 0.999 of the firm and the (small) quasilinear shareholder the fraction 0.001.

Shareholders have rational expectations. An easy computation yields that the profit function is given by

$$\Pi_N(P) = (P-1)\frac{-P + 1000 + \frac{(0.5\cdot 144)^2}{P^2} + \frac{21^{10}\cdot 10^3}{P(21^{10}+P^{10})}}{1-(P-1)\frac{21^{10}\cdot 0.999}{P(21^{10}+P^{10})}}.$$

The demand function of the first and second owner are

$$\begin{aligned} D^1(P) &= \left(\frac{P^{10}(10^3 + 0.999\cdot\Pi_N(P))}{21^{10}+P^{10}}, \frac{21^{10}(10^3 + 0.999\cdot\Pi_N(P))}{P(21^{10}+P^{10})}\right) \\ D^2(P) &= \left(542 + 0.001\cdot\Pi_N(P) - P\frac{(0.5\cdot 144)^2}{P^2}, \frac{(0.5\cdot 144)^2}{P^2}\right), \end{aligned}$$

respectively. Total demand of the shareholders equals $D(P) = D^1(P) + D^2(P)$.

A calculation yields that there are three strategies which satisfy the first order condition $\Pi'_N(P) = D_1(P)$ for real wealth maximization, namely

$$P_A \approx 12.94, \qquad P_B \approx 500.48, \qquad P_C \approx 26.45.$$

However, as the profit function Π_N is not concave, the first order condition is not sufficient. A direct investigation shows that the two strategies $P_A \approx 12.94$ and $P_B \approx 500.48$ are real wealth maximizing strategies, whereas P_C is not. For instance, $D(P_C)$ lies in the interior of the budget set $AB(P_B)$ associated with P_B.

The aggregate budget curve ABC has a kink, since the profit function does not coincide with its concavification [cf. E. Dierker and Grodal (1999), Section 3]. However, the profit function Π_N and its concavification coincide at P_A and P_B. Thus, real wealth is also maximized at P_A and P_B, if the concavification of Π_N rather than Π_N is used as profit function.

We want to show that the strategy $P_A \approx 12.94$ and the strategy $P_B \approx 500.48$ are dominated. Indeed, we show that any of the real wealth maximizing strategies is dominated by the other one. First we calculate the utility levels of the two owners when the firm chooses the strategy P_B and obtain

$$U^1(D^1(P_B)) \approx 80840.74 \qquad \text{and} \qquad U^2(D^2(P_B)) \approx 801.87.$$

In order to show that the strategy P_A dominates the strategy P_B we calculate the aggregate demand at P_A and get

$$D(P_A) \approx (1496.08,\ 11828.65).$$

Now let

$$x^2 = (0.2, 31) \qquad \text{and} \qquad x^1 = D(P_A) - x^2$$

and obtain the corresponding utility levels of the owners

$$U^1(x^1) \approx 80872.80 \qquad \text{and} \qquad U^2(x^2) \approx 801.96.$$

Hence, we have distributed the aggregate demand at P_A such that both owners are better off, i.e. P_A dominates P_B.

Similarly, the utility levels at the strategy P_A are

$$U^1(D^1(P_A)) \approx 80734.10 \qquad \text{and} \qquad U^2(D^2(P_A)) \approx 1095.69$$

and the aggregate demand at the strategy P_B is

$$D(P_B) \approx (251042.23, 0.021).$$

Now we let

$$x^2 = (1095.70, 0) \qquad \text{and} \qquad x^1 = D(P_B) - x^2$$

and obtain the utility levels

$$U^1(x^1) \approx 80748.83 \qquad \text{and} \qquad U^2(x^2) = 1095.70\,.$$

We see that the strategy P_B dominates P_A. Thus, in the example none of the real wealth maximizing strategies leads to a constrained efficient allocation. Moreover, each of the two real wealth maximizing strategies dominates the other one.

If we concavify the economy described above then P_A and P_B are again real wealth maximizing strategies each of which dominates the other one.

Remark 4. *In the example no attainable allocation is constrained efficient.*

Proof. According to Remark 2 the first order condition $\Pi'_N(P) = D_1(P)$ for real wealth maximization holds at any constrained efficient allocation. The only strategies satisfying this condition are $P_A, P_B,$ and P_C. Since P_A dominates P_B and vice versa, the only remaining candidate is P_C. However, the utility levels at $(D^1(P_C), D^2(P_C))$ are

$$U^1(D^1(P_C)) \approx 11282.17 \qquad \text{and} \qquad U^2(D^2(P_C)) \approx 765.40\,.$$

Both owners prefer the bundles they get at P_A or at P_B. Thus, the strategy P_C is dominated by P_A and by P_B. □

Proposition 2. *There are robust examples in which no constrained efficient strategy exists. It can happen that two real wealth maximizing strategies dominate each other.*

6 When are Real Wealth Maxima Constrained Efficient?

In this section we state a condition under which real wealth maxima are constrained efficient. It is clear from the example that concavity of Π_N is not sufficient. The condition we impose requires the aggregate preferred set in the strategy-wealth space to be convex.

Consider a shareholder $i \in \mathcal{I}$ and define his *indirect utility function* $u^i : \mathcal{P} \times \mathbb{R}_+ \to \mathbb{R}$ by

$$u^i(P, \tau) = U^i(d^i(1, P, e^i_0 + \vartheta^i \Pi_N(P) + \tau))\,. \tag{6}$$

$u^i(P, \tau)$ is the utility agent i obtains if the firm chooses the relative price P and i gets a sidepayment τ in terms of the numéraire. Shareholder $i's$ marginal willingness to pay for an infinitesimal change of the relative price $P \in \mathcal{P}$ of the firm's product is defined by

$$MW^i(P, 0) = \frac{\partial_P u^i(P, 0)}{\partial_\tau u^i(P, 0)}\,. \tag{7}$$

Lemma . *Let W_N be concave and $\hat{P} \in \operatorname{int} \mathcal{P}$. Then $\hat{P}$ maximizes owners' real wealth iff $\sum_{i \in \mathcal{I}} MW^i(\hat{P}, 0) = 0$.*

Proof. By differentiation of u^i we obtain

$$\begin{aligned} \partial_P u^i(P, 0) &= \partial_P \tilde{v}^i(1, P, e^i_0 + \vartheta^i \Pi_N(P)) \\ &+ \partial_W \tilde{v}^i(1, P, e^i_0 + \vartheta^i \Pi_N(P)) \cdot \partial_P(e^i_0 + \vartheta^i \Pi_N(P)), \end{aligned}$$

where $\tilde{v}^i$ is shareholder $i's$ ordinary indirect utility function. Roy's identity yields

$$MW^i(P, 0) = -D^i_1(P) + \vartheta^i \Pi'_N(P)\,.$$

By summation,

$$\sum_{i \in \mathcal{I}} MW^i(P, 0) = \Pi'_N(P) - D_1(P) = W'_N(P) - D_1(P)\,. \tag{8}$$

Since Π_N is concave, the strategy $\hat{P}$ maximizes the real wealth iff $D_1(\hat{P}) = W'_N(\hat{P})$ and the Lemma obtains [see also E. Dierker and Grodal (1999), Section 3.2]. □

Consider the N-normalization in which prices take the form $(1, P)$ and let E^i_N be shareholder $i's$ associated expenditure function. More precisely, $E^i_N(1, P, \hat{U}^i) = \inf\{(1, P) \cdot x \mid U^i(x) \geq \hat{U}^i\}$. The expenditure function E^i_N is concave in P. Now we strengthen the concavity assumption on Π_N by relating it to the concavity of $\sum E^i_N$. We assume that $\Pi_N(\cdot) - \sum_{i \in \mathcal{I}} E^i_N(1, \cdot, \hat{U}^i)$ is concave. In the next section we show that this assumption does not depend on the use of the N-normalization. That is to say, the concavity assumption is independent of which bundle is used to normalize prices and measure wealth [see Corollary 1].

Proposition 3. *Let shareholders' demand functions D^i, their expenditure functions E^i_N, and also the profit function Π_N be C^2 in the variable P. Consider any real wealth maximizing strategy $\hat{P} \in \operatorname{int} \mathcal{P}$. Let $\hat{U}^i = U^i(D^i(\hat{P}))$ and assume that $\Pi_N(\cdot) - \sum_{i\in\mathcal{I}} E^i_N(1,\cdot,\hat{U}^i)$ is concave. Then $\hat{P}$ is constrained efficient.*

Before we give the proof we want to point out the following facts:

i) Since $E^i_N(1,\cdot,\hat{U}^i)$ and $\Pi_N(\cdot) - \sum_{i\in\mathcal{I}} E^i_N(1,\cdot,\hat{U}^i)$ are concave, $\Pi_N(\cdot)$ must be concave.

ii) For each agent i the domain

$$C^i = \{(P,\tau) \in \mathcal{P} \times \mathbb{R} \mid e^i_0 + \vartheta^i \Pi_N(P) + \tau \geq 0\}$$

of the indirect utility function is convex.

Let $\hat{P} \in \mathcal{P}$ be a real wealth maximizing strategy and $\hat{U}^i = U^i(D^i(\hat{P}))$. The associated *aggregate preferred set* of the shareholders is defined as

$$\hat{A} = \{(P, \sum_{i\in\mathcal{I}} \tau^i) \in \mathcal{P} \times \mathbb{R} \mid (P,\tau^i) \in C^i \text{ and } u^i(P,\tau^i) \geq \hat{U}^i \text{ for each } i \in \mathcal{I}\}\,.$$

Remark 5. *Under the assumptions of Proposition 3 the aggregate preferred set $\hat{A}$ is convex.*

Proof. First we show that $\hat{A}$ is convex and then we complete the proof of Proposition 3. Given $\hat{P}$, the function $\tau^i : \mathcal{P} \to \mathbb{R}$ describes the compensation shareholder i needs in order to stay at the utility level $\hat{U}^i$, i.e. $u^i(P,\tau^i(P))$ is uniquely defined by the equation $u^i(P,\tau^i(P)) = \hat{U}^i = u^i(\hat{P},0)$. Observe that $\hat{A}$ is the epigraph of the function $\sum_{i\in\mathcal{I}} \tau^i$ and that $\hat{A}$ is convex iff $\sum_{i\in\mathcal{I}} \tau^i$ is a convex function. To show that $\sum_{i\in\mathcal{I}} \tau^i$ is convex, notice that $\tau^i(P) = E^i_N(1,P,\hat{U}^i) - e^i_0 - \vartheta^i \Pi_N(P)$. Moreover, τ^i is C^2 as $E^i_N(1,\cdot,\hat{U}^i)$ and Π_N are C^2. Hence,

$$\frac{d^2}{d^2P} \sum_{i\in\mathcal{I}} \tau^i(P) = \sum_{i\in\mathcal{I}} \frac{\partial^2}{\partial^2 P} E^i_N(1,P,\hat{U}^i) - \Pi''_N(P) \geq 0$$

by assumption and the convexity of $\hat{A}$ obtains.

Clearly, $(\hat{P},0)$ lies on the boundary of $\hat{A}$ and the line tangent to $\hat{A}$ at $(\hat{P},0)$ has the slope $\partial_P(\sum \tau^i(\hat{P}))$. As $u^i(P,\tau^i(P)) = \hat{U}^i$ for all P we obtain $\partial_P u^i(P,\tau^i(P) + \partial_W u^i(P,\tau^i(P)\partial_P \tau^i(P) = 0$. Thus, $MW^i(P,\tau(P)) = \partial_P u^i(P,\tau(P)/\partial_W u^i(P,\tau(P))$ equals $-\partial_P \tau^i(P)$. By the Lemma, $\sum_{i\in\mathcal{I}} MW^i(\hat{P},0) = 0$, i.e. $\partial_P(\sum \tau^i(\hat{P})) = 0$. Consequently, the vector $(0,1)$ is normal to $\hat{A}$ at the boundary point $(\hat{P},0)$. Since $\hat{A}$ is convex any point $(P,\tau) \in \hat{A}$ must have its second component $\tau \geq 0$.

Now assume that $\hat{P}$ can be dominated by the strategy $P \in \mathcal{P}$. Then there exists $(x^i)_{i\in\mathcal{I}}$ such that $\sum x^i = D(P)$ and $x^i \succ^i D^i(\hat{P})$ for all $i \in \mathcal{I}$. Now let

$\tau^i = (1, P)x^i - (\vartheta^i \Pi_N(P) + (1, P)e^i)$. Hence $u^i(P, \tau^i) \geq U^i(x^i) > \hat{U}^i$ for all $i \in \mathcal{I}$. By continuity of U^i and hence u^i there exists $\nu > 0$ such that $u^i(P, \tau^i - \nu) > \hat{U}^i$ for all $i \in \mathcal{I}$. Consequently, $(P, \sum_{i\in\mathcal{I}}(\tau^i - \nu)) \in \hat{A}$ and hence $\sum_{i\in\mathcal{I}}(\tau^i - \nu) \geq 0$. However, from the definition of τ^i and the facts that $\sum_{i\in\mathcal{I}} x^i = D(P)$ and $(1, P)D(P) = \Pi_N(P) + \sum_{i\in\mathcal{I}}(1, P)e^i$ we get that $\sum_{i\in\mathcal{I}} \tau^i = (1, P)D(P) - (\Pi_N(P) + (1, P)\sum_{i\in\mathcal{I}} e^i) = 0$. As $\nu > 0$ we obtain a contradiction. $\square$

In Proposition 3 we assume $\Pi_N(\cdot) - \sum_{i\in\mathcal{I}} E^i_N(1, \cdot, \hat{U}^i)$ to be concave. As can be seen from the proof, this assumption is equivalent to the convexity of the aggregate preferred set $\hat{A}$ in the strategy-wealth space. Notice that the concavity of the function $\Pi_N(\cdot) - \sum_{i\in\mathcal{I}} E^i_N(1, \cdot, \hat{U}^i)$ is implied by the corresponding assumption on the individual level. Clearly, the assumption that $\vartheta^i \Pi_N(\cdot) - E^i_N(1, \cdot, \hat{U}^i)$ is concave for each $i \in I$ is stronger than the aggregate one. Especially for agents who own a small fraction of the firm it may be unreasonable to assume that the concavity of the profit function offsets the convexity of the function $-E^i_N(1, \cdot, \hat{U}^i)$. Usually, convexity of the aggregate preferred set is obtained for free in large, atomless economies [compare Hildenbrand (1974)]. However, in the present setting the individually preferred sets are not added in the usual way. In order to obtain the convexity of the aggregate preferred set $\hat{A}$ in an atomless economy we need to have an argument entailing that $T(P) = \int_{\mathcal{I}} \tau^\lambda(P) d\lambda$ is a convex function although the individual τ^λ are not convex. We are not aware of such an argument.

The proof of Proposition 3 does not only show that real wealth maxima are constrained efficient. Actually the following stronger statement is derived: The firm cannot choose another strategy and a reallocation of the total wealth the group of shareholders gets at that strategy such that all shareholders can obtain a preferred bundle on the market.

7 Linear Structures on Strategies and Wealth

According to Proposition 3 the constrained efficiency of real wealth maxima depends on the concavity of the profit function and on the convexity of the aggregate preferred set $\hat{A}$ in the product of the strategy space and wealth space. However, when we speak about concavity of the profit function and convexity of the preferred set we need linear structures on strategies and on wealth. In order to introduce such linear structures we have expressed prices and wealth in units of commodity 0. Instead of using a particular good as numéraire one could have taken any commodity bundle $x = (x_0, x_1)$, $x_0 > 0$. This leads to the question of whether Proposition 3 and other statements involving the concavity of the profit function are invariant with respect to the choice of the bundle x. Indeed, we want to show that the concavity of the profit function and the convexity of the aggregate preferred set are invariant with respect to the choice of the bundle x.

Prices normalized with respect to x are denoted (π_0, π_1), i.e. (π_0, π_1) satisfies $\pi_0 x_0 + \pi_1 x_1 = 1$. Thus π_1 denotes the output price in terms of the bundle x and the corresponding input price is $\pi_0 = (1 - \pi_1 x_1)/x_0$. If the bundle x is used to measure wealth, then W_x denotes the maximal number of units of the bundle x affordable at prices (π_0, π_1). We shall now show the invariance of a convexity property with respect to the choice of the bundle x, which is used to normalize prices and express wealth. [4]

Consider any $A_N \subset \mathbb{R}_+ \times \mathbb{R}$. As before the subscript N indicates that we use good 0 as numéraire. The first component of an element of $(P, W_N) \in A_N$ corresponds to the price system $(1, P)$ and the second measures the wealth in terms of the bundle $(1, 0)$. Now replace $(1, 0)$ by an alternative bundle $x = (x_0, x_1)$, $x_0 > 0$,. Then the $\mathbb{R}_+ \times \mathbb{R}$ and hence A_N are transformed as follows:

$$(P, W_N) \mapsto t_x(P, W_N) = \left(\frac{1}{x_0 + P x_1} \right) (P, W_N) \,.$$

Indeed, the price system corresponding to $(1, P)$ in the x-normalization is $(1/(x_0 + Px_1), P/(x_0 + Px_1))$. Hence, the first coordinate of $t_x(P, W_N)$ equals $\pi_1 = P/(x_0 + Px_1)$. Moreover, the wealth W_N is the number of units of the bundle $(1, 0)$ which can be bought at the price system $(1, P)$. At the price system (π_0, π_1) the corresponding wealth is $(W_N, 0)(\pi_0, \pi_1) = \pi_0 W_N$. The number of units of the bundle x given the wealth $\pi_0 W_N$ equals $W_x = W_N \pi_0/(\pi_0 x_0 + \pi_1 x_1) = W_N/(x_0 + Px_1)$. Thus, the second coordinate of $t_x(P, W_N)$ is $W_N/(x_0 + Px_1)$.

Proposition 4. *If prices are normalized with respect to an arbitrary consumption bundle $x \in \mathbb{R}^2_+ \setminus \{0\}$, then the set $A_x = t_x(A_N)$ is (strictly) convex if and only if A_N is.*

Proof. Consider any points (P, W_N) and (P', W'_N) in A_N. An easy calculation shows that we have, for any $\delta \in [0, 1]$,

$$t_x(\delta(P, W_N) + (1 - \delta)(P', W'_N)) = \lambda t_x(P, W_N) + (1 - \lambda) t_x(P', W'_N) \,,$$

where $\lambda = \delta(x_0 + Px_1)/(x_0 + (\delta P + (1 - \delta)P')x_1))$. Observe that λ, considered as a function of δ, maps $[0, 1]$ on to $[0, 1]$. Hence A_x is (strictly) convex iff A_N is. □

Clearly, the profit function depends on which bundle x is used to normalize prices and express wealth. and the concavity of a function depends in general on the linear structure. Let Π_x denote the profit as function of the output price, if the bundle x has been used to normalize prices and measure wealth. We want to show that the profit function Π_x is concave for any bundle x iff Π_N is.

[4] This follows from observations in E. Dierker and Grodal (1999). Here we shall give a shorter and more direct argument.

At the price system (π_0, π_1) the firm obtains the profit $\Pi_N(\pi_1/\pi_0)$ in terms of good 0, which corresponds to the value $(\pi_0, \pi_1)(\Pi_N(\pi_1/\pi_0), 0)$. This profit enables the shareholders to buy $\Pi_x(\pi_1) = \pi_0 \Pi_N(\pi_1/\pi_0)$ units of the bundle x. Substitution yields

$$\Pi_x(\pi_1) = \frac{1 - \pi_1 x_1}{x_0} \, \Pi_N(\frac{\pi_1}{(1 - \pi_1 x_1)/x_0}) \, ,$$

where π_1 lies in the range of the transformed prices and hence $\pi_1 < 1/x_1$. Hence, we obtain that $t_x(P, \Pi_N(P)) = (\pi_1, \Pi_x(\pi_1))$ for all $P \in \mathbb{R}_+$. Clearly, Π_x is concave if and only if $\{\pi_1, r) \mid r \leq \Pi_x(\pi_1)\}$ is convex. Notice that, in the above argument, the profit function can be replaced by the wealth as a function of the output price. Thus, we obtain

Corollary . *If prices are normalized with respect to an arbitrary consumption bundle $x \in \mathbb{R}^2_+ \setminus \{0\}$, then the profit function Π_x is (strictly) concave if and only if Π_N is. A similar statement holds true if the profit is replaced by shareholders' aggregate wealth as a function of the output price.*

8 Uniqueness of Real Wealth Maxima and Constrained Efficiency

In order to derive the constrained efficiency of a real wealth maximum we used the assumption that $\Pi_N(\cdot) - \sum_{i \in \mathcal{I}} E^i_N(1, \cdot, \hat{U}^i)$ is concave. Now we are going to show that this assumption is intimately related to the uniqueness of a real wealth maximum.

Assume that the profit function and shareholders' demand functions are C^2, that Π_N is concave and takes its maximum in the interior of the strategy space $\mathcal{P} = [c, \bar{c}]$, and that $\Pi'_N(c) > D_1(c)$. Thus, $\sum_{i \in \mathcal{I}} MW^i(c, 0)$ is positive. Furthermore, we only have to consider strategies to the left of the strategy P_{max} that maximizes Π_N for the following reason. If $P > P_{max}$, then $\Pi'_N(P) < 0$ and a slightly lower price is favorable for all shareholders, since they do not only pay less, but also obtain higher profit incomes.

From the Lemma in Section 6 we know that $\sum_{i \in \mathcal{I}} MW^i(P, 0) = \Pi'_N(P) - D_1(P)$ vanishes at $P = \hat{P}$ if and only if $\hat{P}$ maximizes shareholders' real wealth. A natural condition for uniqueness states that the aggregate willingness of all shareholders to pay for an increase of P is monotonically declining, more precisely,

$$\frac{\partial}{\partial P} \sum_{i \in \mathcal{I}} MW^i(P, 0) = \Pi''_N(P) - D'_1(P) < 0 \qquad (9)$$

for $c < P < P_{max}$.

Let us compare this condition to the condition for constrained efficiency, i.e. to

$$\Pi_N''(P) - \sum_{i\in\mathfrak{I}} \frac{\partial^2}{\partial P^2} E_N^i(1,P,\hat{U}^i) = \Pi_N''(P) - \sum_{i\in\mathfrak{I}} \frac{\partial^2}{\partial P^2} h_1^i(1,P,\hat{U}^i) < 0\,, \qquad (10)$$

where h_1^i denotes shareholder i's Hicksian demand for good 1. Assume that all shareholders have quasilinear utility functions U^i. Then their Hicksian and their Walrasian demand for the firm's product coincide due to the absence of income effects and we obtain:

Remark 6. *In case of quasilinear preferences, condition* (9) *for uniqueness and condition* (10) *for constraint efficiency are equivalent .*

In general, however, these conditions differ. We assume that the firm's product is a normal good for all shareholders, i.e. $\partial_W d_1^i(1,P,W^i) \geq 0$ for all $P,(W^i)_{i\in\mathfrak{I}}$ and all i. The uniqueness condition (9) states $\Pi_N''(P) < D_1'(P)$. However,

$$\begin{aligned} D_1'(P) &= \sum_{i\in\mathfrak{I}} \partial_P d_1^i(1,P,\vartheta^i\Pi_N(P)+e_0^i) && (11)\\ &+ \sum_{i\in\mathfrak{I}} \partial_W d_1^i(1,P,\vartheta^i\Pi_N(P)+e_0^i)\cdot\vartheta^i\Pi_N'(P)\,. && (12)\end{aligned}$$

$D_1'(P)$ is at least as large as $\sum_{i\in\mathfrak{I}}(\partial_P d_1^i(1,P,\vartheta^i\Pi_N(P)+e_0^i)$, since (12) is non-negative. On the other hand, look at the efficiency condition (10) which requires $\Pi_N''(P)$ to be less than

$$\begin{aligned} \sum_{i\in\mathfrak{I}} \partial_P h_1^i(1,P,\hat{U}^i) &= \sum_{i\in\mathfrak{I}} \partial_P d_1^i(1,P,E_N^i(1,P,\hat{U}^i)) && (13)\\ &+ \sum_{i\in\mathfrak{I}} \partial_W d_1^i(1,P,E_N^i(1,P,\hat{U}^i))\cdot d_1^i(1,P,E_N^i(1,P,\hat{U}^i)). && (14)\end{aligned}$$

$\sum_{i\in\mathfrak{I}} \partial_P h_1^i(1,P,\hat{U}^i)$ is at least as large as $\sum_{i\in\mathfrak{I}} \partial_P d_1^i(1,P,E_N^i(1,P,\hat{U}^i))$, since (14) is non-negative. In general it is not possible to compare the size of the terms (12) and (14). Hence, to obtain a condition that implies uniqueness as well as constrained efficiency we have to disregard both "good" terms (12) and (14). Thus, we obtain

Remark 7. *Assume that the profit function and shareholders' demand functions are C^2, that the firm's product is a normal good, and that*

$$\Pi_N''(P) < \sum_{i\in\mathfrak{I}} \partial_P d_1^i(1,P,W^i) \qquad (15)$$

for all price-wealth combinations $(P,(W^i)_{i\in\mathfrak{I}})$ that are generated by a strategy $P\in]c,P_{max}[$. Then there exists a unique real wealth maximizing strategy $\hat{P}$.

Moreover, consider the compensated demand functions associated with the shareholders' utility profile $(\hat{U}^i)_{i\in\mathfrak{I}}$ *obtained at* $\hat{P}$ *and assume that* (15) *holds for all price-wealth combinations generated in this way by a strategy* $P \in]c, P_{max}[$. *Then* $\hat{P}$ *is constrained efficient.*

References

Arrow, K. : A Difficulty in the Concept of Social Welfare, Journal of Political Economy 58, 328-346 (1950)

Dierker, E., B. Grodal : Modelling Policy Issues in a World of Imperfect Competition, Scandinavian Journal of Economics 100, 153-179, 1998

Dierker, E., B. Grodal : The Price Normalization Problem in Imperfect Competition and the Objective of the Firm, Economic Theory 14, 257-284, 1999

Dierker, E., H. Dierker, B. Grodal : Incomplete Markets and the Firm, Discussion paper 99-03, Institute of Economics, University of Copenhagen, (1999)

Dierker, H., B. Grodal : Non-Existence of Cournot-Walras Equilibrium in a General Equilibrium Model with Two Oligopolists, in W. Hildenbrand and A. Mas-Colell eds., Contributions to Mathematical Economics, North Holland, 167-185 (1986)

Drèze, J.H. , Investment under Private Ownership: Optimality, Equilibrium and Stability, in Allocation Under Uncertainty; Equilibrium and Optimality, J.H. Drèze ed., New York: Wiley, 129-165 (1974)

Gabszewicz, J., J.P. Vial : Oligopoly "à la Cournot" in a General Equilibrium Analysis, Journal of Economic Theory 4, 381-400 (1972)

Geanakoplos, J., J. Drèze, M. Magill, M. Quinzii : Generic Inefficiency of Stock Market Equilibrium When Markets Are Incomplete, Journal of Mathematical Economics 19, 1990, 113-51 (1990)

Guesnerie, R. : Pareto Optimality in Non-convex Economies, Econometrica 43, 1-31 (1975)

Hildenbrand, W.: Core and Equilibria of Large Economies, Princeton: Princeton University Press (1974)

Magill, M., M. Quinzii : Theory of Incomplete Markets. Cambridge, London: MIT Press (1996)