Competition and Reputation[*]

Ramon Marimon, Juan Pablo Nicolini, Pedro Teles[†]

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Abstract

We analyze the interaction of two disciplinary mechanisms: competition and reputation. We first study a dynamic model of monopolistic competition with experienced goods. If consumers' beliefs satisfy a weak regularity condition, then there is a unique sustainable equilibrium with quality goods being produced and the price has a mark-up which is either the full information monopolistic mark-up or, if this is not sustainable (e.g., when goods are very close substitutes), the rate of time preference, that acts as a reputation constraint. A variation of the model allows us to study the private provision of currencies. In particular, we inquire whether competition between profit maximizing currency issuers would drive inflation rates to the efficient outcome, as suggested prominently by Hayek. Without commitment, the efficient outcome with deflation -as implied by the Friedman rule- cannot be sustained. However, if currencies are close substitutes (and beliefs are regular) the equilibrium inflation rate is zero.

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[†]**R. Marimon**, European University Institute, Universitat Pompeu Fabra, NBER and CEPR; **J.P. Nicolini**, Universidad Di Tella and Universitat Pompeu Fabra; **P. Teles**, Banco de Portugal and Universidade Catolica Portuguesa.

1 Introduction

Can currency be efficiently provided by competitive markets? As we show in this paper, such a *macro* question has a very similar *micro* formulation: can experienced goods (i.e., goods where quality is observed after being purchased) of high quality be provided by competitive markets? In this paper, we study a dynamic monopolistic equilibrium model to provide an answer to the *micro* question. A variation of the model allow us to answer the original *macro* question. Both questions address the more general theme of how the reputation and the competitive mechanisms interact. This is the central theme of the paper.

In monetary theory, the standard *laissez-faire* view –as, for example, has been expressed by Hayek– is based on a "Bertrand competition" argument, according to which competition will drive the price (return) of money to its marginal cost. Given the real rate of interest, and abstracting from legal restrictions or acceptability issues, the demand for a given currency depends on its real return. Therefore, a competitive issuer will try to undercut competitors with a lower inflation rate. If the marginal cost of producing currency is zero, this process of competition should make the inflation rate equal to the symmetric of the real rate of interest. That is, an "optimal monetary policy" of zero nominal interest rates should be the result of competition.

There is, however, one major flaw in this "Bertrand competition" argument, when applied to fiat money. When suppliers of currency cannot commit to their future actions, then competition may lose its bite. The reason being that, while currencies compete on their promised rates of return, once agents hold a particular currency there may be an incentive for the issuer to inflate the price of goods in terms of this currency, reducing, in this way, the outstanding liabilities. This problem, notice, is at the root of the time inconsistency problem in monetary theory. Current currency portfolios have been pre-specified, while there is full flexibility to choose tomorrow's portfolios. Currencies compete for tomorrow portfolios. Thus, Bertrand competition drives *promised* rates of return to the efficient level. But those promises might not be credible. That is, the currency supplier has an incentive to default on the promise made while competing with other currencies. In other words, when the choices are sequential, currencies are no longer perfect substitutes; in a sense, they are not substitutes at all!

In the industrial organization theory, competition in experienced good markets has similar properties. "Bertrand competition" can only affect market prices, but not the qualities which are observed *ex-post*. Firms have an incentive to "fly-by-night" providing low quality products. This problem, notice, is at the root of the "market for lemons" problem in industrial organization theory. In summary, in a monopolistic competition context, even if goods are close substitutes, competition does not discipline firms.

In a dynamic economy, however, firms are concerned for their future market position and such reputation mechanism may be enough to discipline firms to provide high quality goods. Similarly, the reputation mechanism may resolve the time inconsistency problem in the supply of money: concern for the future circulation of money may disincentive currency issuers to create unexpected inflations. Nevertheless, reputational concerns exist as long as firms, monopolistic firms in one contest and currency suppliers in the other, expect high enough future profits as to refrain from capturing the short-term profits. Therefore, competition, driving down profits, may enhance efficiency, but may also destroy the disciplinary properties of the reputation mechanism. The analysis of this trade-off is the central contribution of this paper.

In studying monopolistic competition we first consider the case of perfect observability where, as the well understood theory predicts, equilibrium is uniquely determined by the degree of substitution and, as goods become closer substitutes, equilibria become more efficient, achieving Pareto efficiency in the limiting case of perfect substitution. However, when quality is only observed with a lag the mark-up must be bounded away from zero as to guarantee enough future profits for reputation to play a role. The rate of time preferences -as indicator of the observability lag- defines the lower bound on mark-ups. More precisely, we first show that any price covering such mark-up could be sustained as a symmetric stationary *sequential equilibrium* (and as a *sustainable equilibrium).* This also means that competition does not play any role since, for example, any arbitrary large price can be an equilibrium price, sustained by beliefs that price deviations, from such a high price, signal low quality. That is, arbitrariness of beliefs results in arbitrary price equilibria.

We then constraint beliefs to satisfy two weak regularity conditions which are consistent with most learning procedures. In other words, we constraint rational expectations equilibria to be *learnable* in the sense of being supported by beliefs that satisfy minimum regularity conditions, such as a weak forms of continuity and monotonicity. We introduce this way, the concept of *regular sequential equilibrium*, which is distinct from other existing refinements and can be of interest in other applications. In contrast with sequential equilibria, there is a unique stationary regular sequential equilibrium, in which competition plays a crucial role. In particular, the mark-up is either the mark-up of the monopolistic competition with perfect observability or, if goods are too close substitutes as such a price not being sustainable, the -lower bound-rate of time preferences.

Our model of currency competition is one where goods are supplied in perfectly competitive markets, but consumers, who would like to consume all goods, must pay for each good with a specific currency, that is, they face multiple (a continuum) of cash-in-advance constraints (as in Woodford 1990). Each currency is supplied by a profit maximizer provider, therefore currency competition place the role of monopolistic competition in the original model. The parallel with the case of perfect observability is the case with full commitment. Currency competition achieves the efficient (Friedman rule) monetary equilibrium if currencies become perfect substitute, as Hayek had envisioned. However, the parallel with the case of unobservable quality is the case without full commitment. Both cases share the property that competition is on announced prices (interest rates), while consumers base their demands on expectations on whether such announcements will be realized. The equivalent of the lower bound on mark-ups is a non-negative inflation condition, which must be guaranteed for reputation to play a role. As in the monopolistic competition model, there is a large set of sequential equilibria, which shrinks to a unique equilibrium when beliefs satisfy regularity conditions. In particular, the unique stationary *regular sequential equilibrium* is characterized by an inflation that it is either zero or the inflation of the full commitment equilibrium, if such inflation rate is positive. It follows, for example, that the efficient (Friedman rule) monetary equilibrium can not be sustained as Hayek had envisioned. It also follows, however, that, with rent maximizing currency providers, competition enhances efficiency, contradicting some of the arguments of those advocating that "money is a natural monopoly".

Although we exploit the similarities between experienced goods and monies, such analogy should not be pushed too far. For example, there is a "coordination problem" associated with the "acceptance of fiat money," which is more similar to "coordination problems" associated with some products, such as "fashion goods". In contrast with the industrial organization literature, in monetary theory often the argument for "money as a natural monopoly" has been made based on the existence of such a "coordination problem". In this paper we will not address such endogeneity problems. We will, however, take into account another differential feature of fiat money: the fact that if people do not expect that a currency will be held in the future, then their current demand for such a currency is zero. In this regard money is more like a non-perishable durable good that requires good maintenance for life. That is, it is enough that you do not expect to find service (of quality) in the future for you not to buy -say, a car of a certain maker- even if current quality (both of the car and the service) is the appropriate one.

Our work is related to different strands of literature. With respect to the industrial organization literature on experience goods, our work is closely related to Shapiro (1983)[1]. He considers a similar model of monopolistic competition in which consumers' expectations regarding quality follow an ad-hoc exogenous process. He does not study the trade-offs between "competition and reputation". In contrast, we consider rational expectations about quality and, as we have said, our central theme is the study these trade-offs.

The issue of currency competition has been the subject of an extensive academic debate. This debate has seen many supporters of free competition making an exception when it comes to money (as Friedman, 1960), while advocates of free currency competition (notably, Hayek1974 and 1978, and Rockoff, 1975) have been somewhat isolated . Although, the recent reappraisal of the self-regulating properties of free banking[2], has raised new interest on the study of currency competition. Woodford (1990) studies currency competition in a model similar to the one studied here. He is interested in studying the stability of exchange rates and does not consider reputational issues.

The problem of the *time-inconsistency of monetary* policies has been extensively studied (see, for example, Chang (1998), Chari & Kehoe (1990), Ireland (1994), Stokey, 1991), but with the partial exception of Taub (1985), the "currency competition" argument has not been considered[3]. Taub (1985) studies two commitment regimes: one with full commitment with non-stationary ("time-inconsistent") policies, and another in which polices are constrained to be "time-consistent" (stationary). He shows that in the commitment case,

[1] See also Tirole (1988) for an introductory account.

[2] See, for example, Calomiris and Kahn (1996), Dowd (1992), King (1983), Rolnick and Weber (1983), Selgin (1987), Selgin & White (1987), Vaubel (1985), and, more generally, White (1993) . See also Schuler (1992), for an account of historical episodes of free banking, and Hayek (1974,1978), Dowd (1992) and White (1993) for a broad perspective on the literature on free banking.

[3] A shortcoming that has not gone unnoticed (see, for example, Hellwig (1985)).

the Friedman rule emerges as the competitive outcome, while in the "time-consistent" case the outcome is inefficient and, as a result, he argues in favor of the "natural monopoly" argument. While we have the same result when there is full commitment, our analysis of the "non-commitment case" differs substantially, showing how, and when, competition enhances efficiency. Finally, the paper which is closest to this one, is our, Marimon, Nicolini and Teles (1998), companion paper on the effects on (the unique) monetary policy of competition from electronic money, and other currency substitutes.

2 A model of monopolistic competition with experienced goods

Our model is a version of the model of monopolistic competition of Dixit and Stiglitz (1977), with experienced goods. Consider an economy with a large number of identical households that gain utility from services and leisure. The utility function of the representative household is

$$\sum_{t=0}^{\infty} \beta^t \left[U(y_t) - \alpha n_t \right], \tag{1}$$

where U is increasing and concave and, without loss of generality, $U(0) = 0$, α is a positive constant, n_t is work effort and y_t is an index of services

$$y_t = \left[\int_0^1 \left(y(i)_t q(i)_t \right)^{1/\mu} di \right]^{\mu},$$

with $\mu \geq 1$. $y(i)_t$ is the consumption of good $i \in [0, 1]$. Each of the goods can be provided with variable quality, $q(i)_t = 0$ or 1.

Time must be devoted to the production of services, according to the linear technology

$$y(i)_t q(i)_t = n(i)_t,$$

Total effort per capita is

$$n_t = \int_0^1 n(i)_t di.$$

We assume that there is a single monopolist that produces each good.

Producers have, at any time, the option of producing "fake" units of the consumption good that are costless to produce. A key assumption for the characterization of the equilibria is whether consumers can distinguish the good quality goods from the bad quality ones before they buy them. We proceed to characterize the equilibrium when the services obtained with the consumption of the goods are observed before they are purchased.

2.1 Monopolistic competition with perfect observability

If the quality of the good is public information, there exists a unique equilibrium in this model economy with monopolistic competitive firms. Each firm sets the price equal to a constant mark up over the unitary marginal cost.

In each period t, the representative household chooses the number of units of each good i to purchase, $y(i)_t$, as well as work effort, n_t, in order to maximize utility, (1), subject to

$$\sum_{t=0}^{\infty} Q_t \left[\int_0^1 \left(p(i)_t y(i)_t - \Pi(i)_t \right) di - n_t \right] \leq 0,$$

where $\Pi(i)_t$ are the per-capita profits of firm i, $p(i)_t$ is the price of goods in units of labor time, and Q_t is the price of labor at time t, in units of labor at time zero. The demand functions for goods will be given by

$$U'(y_t) y_t^{\frac{\mu-1}{\mu}} \left(y(i)_t q(i)_t \right)^{\frac{1-\mu}{\mu}} q(i)_t = \alpha p(i)_t, \tag{2}$$

for all i and t. When $q(i)_t = 0$, then $y(i)_t = 0$.

When instead $q(i)_t = 1$, all i, the price of the composite good y_t is

$$p_t \equiv \left[\int_0^1 p(i)_t^{1/1-\mu} di \right]^{1-\mu} = \frac{U'(y_t)}{\alpha}$$

The demand functions for services of each of the goods, (2), can then be written as

$$y(i)_t = y_t \left[\frac{p(i)_t}{p_t} \right]^{\frac{\mu}{1-\mu}} \tag{3}$$

The monopolist of product i chooses the quality and the price to maximize profits

$$\sum_{t=0}^{\infty} \beta^t \left(p(i)_t y(i)_t - q(i)_t y(i)_t \right). \tag{4}$$

Since with $q(i)_t = 0$, $y(i)_t = 0$, and profits will be zero, then the firms will provide the high quality, $q(i)_t = 1$. They choose the prices to maximize profits (4) subject to the demand functions (3). This is a static problem. As the demand function has constant price elasticity, the optimal price per unit of service of each good will be

$$p(i)_t = \mu. \tag{5}$$

The market clearing condition

$$\int_0^1 q(i)_t y(i)_t di = n_t$$

must hold in equilibrium. The unique equilibrium will be characterized by a price which will be constant over time and across goods

$$\overline{p} = \mu, \tag{6}$$

as equation (5) shows. Therefore, the quantity of services of the goods, $y_t = \overline{y}$, will be constant and will satisfy the following condition

$$U'(\overline{y}) = \alpha\mu \tag{7}$$

The value of the parameter μ determines the substitutability of the goods. The closer is μ to one, the higher is the degree of substitutability. Note that when μ is in fact one, the mark-up goes to zero and the equilibrium is a perfectly competitive one. On the other hand, as μ gets larger, so do the mark-ups. Note that we are not allowing for free entry, so profits will indeed be positive except in the limiting case in which $\mu = 1$.

Thus, there exists a unique equilibrium that is closer to the efficient outcome, the closer is the parameter μ to one. Indeed only when $\mu = 1$, the marginal rate of substitution equals the marginal rate of transformation. The increased substitutability between goods increases competition and increased competition implies an outcome closer to the efficient one. This models thus illustrates in a very clear way the nice properties of competition.

An alternative way to model imperfect competition is to assume that goods are perfect substitutes in the utility but production requires fixed entry costs, as in Salop ()'s circular-city-model. The lower the fixed costs, the stronger is competition and the lower the equilibrium mark-ups. Thus, there is a clear connection between lower values of μ in Dixit-Stiglitz and lower fixed costs in Salop. In fact, the same results go through in both models. However, the analogue monetary model in the Dixit-Stiglitz version is much nicer, so that the modelling choice was natural.

2.2 Monopolistic competition with unobservable quality

We now assume that, as with many durable goods, consumers can observe the quality of the good -or service- only after purchasing it. This feature modifies the model above in very important ways. In particular, note that each firm now faces a "time inconsistency problem". As is clear from the expression for profits (4), in each period t, once the consumers have paid the price of the good, $p(i)_t$, under the expectation that the good is of high quality, $q(i)_t = 1$, it is optimal to provide no services, $q(i)_t = 0$, and save the costs of production, as long as this does not affect future expectations[4]. Of course, the firms will refrain from doing so, if this action can affect future demand, since after observing low quality the consumers might choose $y(i)_{t+j} = 0$, $j \geq 1$. In this section, we develop a model of reputation to analyze this problem.

Let $p(i)_t$ denote the price set by firm i in period t for a good or service. Then, if the firm produces with quality $q(i)_t$, the price per unit of service is $p^q(i)_t = p(i)_t/q(i)_t$. Let $\lambda_{i,t}(p(i)_t) = \Pr\{p^q(i)_t = p(i)_t\}$,i.e., the probability that firm i sets the price per unit of quality $p^q(i)_t$ equal to $p(i)_t$. So, if $p(i)_t < \infty$, then $\lambda_{i,t}(p(i)_t)$ is the probability that quality is high. Let h_t be the information available to a firm at the moment of making period t decisions. That is, $h_0 = \{\emptyset\}$ and, for $t > 0$, $h_t = \{h_{t-1}, p(i)_{t-1}, p^q(i)_{t-1} \text{ for all } i\}$. A strategy for firm i, is a $\sigma_i^f = \left\{\sigma_{i,t}^f\right\}$, where, $\sigma_{i,t}^f(h_t) = (p(i)_t, \lambda_{i,t}(p(i)_t))$.

The representative household simply decides how much to work and to purchase of every service, i.e., $(n_t, y(i)_t, \text{for all } i)$, given the available information, which in period t is $\{h_t, p(i)_t, \text{ for all } i\}$, and the expectations on the quality of the service. An allocation rule is a $\sigma = \{\sigma_t\}$, where, $\sigma_t(h_t, p(i)_t) = (n_t, y(i)_t, \text{for all } i)$. Let $\upsilon_t^i(h_t, p(i)_t)$ denote the belief that, given history h_t and price $p(i)_t$ the price of good i per unit of quality, $p^q(i)_t$, is equal to the observed price, $p(i)_t$. That is, for $p(i)_t < \infty$, $\upsilon_t^i(h_t, p(i)_t)$ is the assessed probability that quality is high, $q(i)_t = 1$. Notice that we implicitly assume that beliefs about firm i do not depend on other firms' prices. Consumers' beliefs are *consistent* with firms' actions if for every (t, h_t), and price $p(i)_t = \sigma_{i,t}^{f,1}(h_t)$, $\upsilon_t^i(h_t, p(i)_t) = \lambda_{i,t}(p(i)_t)$[5].

[4] This feature has not been unnoticed in the Industrial Organization literature (see Shapiro 1983). However, to the best of our knowledge, the problem has not been analyzed in the context of fully rational agents.

[5] As in Kreps and Wilson (1982)' *Sequential Equilibrium* and as in *Perfect (Extended)*

A "*Sequential Monopolistic Competitive Equilibrium*" (SMCE) consists of $\left((\sigma, \upsilon), (\sigma_i^f)\right)$ such that, for every (t, h_t)

1. $\sigma_{i,t}^f(h_t)$ solves the problem of firm i,

   and, given $p(i)_t = \sigma_{i,t}^{f,1}(h_t)$,

2. $\upsilon_t^i(h_t, p(i)_t) = \lambda_{i,t}(p(i)_t)$, and

3. $(n_t, y(i)_t) = \sigma_t(h_t, p(i)_t)$ solves the problem of the household, given beliefs $\upsilon_t^i(h_t, p(i)_t)$, and satisfies the market clearing condition $\int_0^1 y(i)_t q(i)_t di = n_t$.

"Sequential Monopolistic Competition Equilibrium" (SMCE) provides a natural framework to study the interactions between competition and reputation. On the one hand, as long as μ is strictly larger than one, the economy exhibits monopolistic power, and as μ gets close to one, the competition between firms is increased. On the other hand, in making quality decisions, firms care about their reputation since quality provision has strategic implications.

Notice that the (3) requirement is simply that consumers's allocations satisfy their demands. In particular, given $(h_t, p(i)_t)$, with $p(i)_t < \infty$, and $\eta = \upsilon_t^i(h_t, p(i)_t)$ consumers' demands are given by

$$y(i)_t = y_t \left[\frac{U'(y_t)/\alpha}{p(i)_t/\eta}\right]^{\frac{\mu}{\mu-1}}$$

In what follows, we restrict attention to *symmetric equilibria* in the sense that all firms behave the same way, so expectations about quality are the same for every good.

In order to stress the pervasive effects of assuming that the quality is only observed after purchasing the good, let us consider an equilibrium where strategies do not depend on histories. If current actions of the firms do not affect the consumers' expectations about future quality, then, no matter what the price is, it is a dominant strategy for the firms to choose to provide the low quality, $q(i)_t = 0$, to save on production costs. If the firm produces low

Bayesian Equilibrium we impose consistency conditions. However, in our imperfect information environment, a simple form of consistency on relative beliefs suffices (see, for example, Fudenberg and Tirole, 1991, Ch. 8 and Battigalli, 1996).

quality, then for any price $p(i)_t$ if $\upsilon^i_t(h_t, p(i)_t) = 0$, consumer's expectations are fulfilled. The resulting payoffs are zero it follows that this is *the worst SMCE*. More formally,

Proposition 1 *There exist* low quality SMCE, *supported by strategies* $\sigma^f_{i,t}(h_t) = (p(i)_t, 0)$*, and beliefs* $\upsilon^i_t(h_t, p(i)_t) = 0$*, with their corresponding allocations* $(n_t, y(i)_t) = (0, 0)$*, for all* i *and* (t, h_t)*. Furthermore, there is no SMCE with lower payoffs for consumers and firms.*

Incidentally, note that this is the unique SMCE (payoff) in which strategies do not depend on histories. In this case no reputation considerations arise. Note also that this would be the unique outcome if firms where anonymous players not accountable for their past quality decisions.

The next step is to determine under what conditions the equilibrium outcome with perfect observability, described in the previous section, is the outcome of a SMCE. In order to check this, we consider the standard trigger strategies of reversion to the worst SMCE strategies.

Consider the stationary path

$$p(i)_t = \overline{p}, \lambda_{i,t}(\overline{p}) = 1, \; y(i)_t = y_{\overline{p}}, n_{\overline{p}} = y_{\overline{p}}$$

where $U'(y_{\overline{p}}) = \alpha\overline{p}$. We want to find conditions under which this outcome is supported by the following "revert to low quality" strategies and beliefs and allocation rules:

$$\begin{aligned}
\sigma^f_{i,0} &= (\overline{p}, 1), \\
\sigma^f_t(h_t) &= (\overline{p}, 1), \text{ if } p^q(i)_n = p(i)_n = \overline{p} \text{ for } 0 \leq n < t, \\
&= (\overline{p}, 0), \text{ otherwise.}
\end{aligned}$$

$$\begin{aligned}
\upsilon^i_0(h_0, \; p(i)_0) &= 1 \text{ if } p(i)_0 = \overline{p} \text{ and } \upsilon^i_0(h_0, \; p(i)_0) = 0, \text{ otherwise} \\
\upsilon^i_t(h_t, p(i)_t) &= 1, \text{ if } p^q(i)_n = p(i)_n = \overline{p}, \; 0 \leq n < t \text{ and } p(i)_t = \overline{p} \\
\upsilon^i_t(h^c_t, p(i)_t) &= 0 \text{ otherwise.}
\end{aligned}$$

$$\begin{aligned}
\sigma_0(h_0, p(i)_0) &= (n_{\overline{p}}, y_{\overline{p}}) \text{ if } p(i)_0 = \overline{p}, \\
\sigma_0(h_0, p(i)_0) &= (0, 0) \text{ if } p(i)_0 \neq \overline{p} \\
\sigma_t(h_t, p(i)_t) &= (n_{\overline{p}}, y_{\overline{p}}) \text{ if } p^q(i)_n = p(i)_n = \overline{p}, \; 0 \leq n < t \text{ and } p(i)_t = \overline{p} \\
\sigma_t(h_t, p(i)_t) &= (0, 0) \text{ otherwise.}
\end{aligned}$$

Consider first the monopolistic competition outcome with perfect observability; that is, $\overline{p} = \mu$ and $y_{\overline{p}} = \overline{y}$. Notice that, if it is sequentially rational for firms to produce high quality with probability one, then the above strategies correspond to sequentially rational choices, since they where optimal choices with perfect observability.

If the firm does indeed deliver the high quality good, then the profits, each period, will be given by $\Pi(i) = (\mu - 1)\overline{y}$ and, therefore, the present value profits, after high quality is observed all previous periods and the current price is μ, are given by $V(\mu, 1) = (\mu - 1)\overline{y}/(1 - \beta)$. On the other hand, if the firm deviates -say, in period t- and delivers the low quality good, while setting the price $p(i)_t = \mu$, the current profits will be $\mu\overline{y}$ and the present value profits, after $p^q(i)_t = \infty$ is observed the last period (or any previous period), are $V(\mu, 0) = 0$. Thus, the firm chooses not to deviate and produce high quality (i.e.,$\lambda_{i,t}(\mu) = 1$ is sequentially rational) if

$$\begin{aligned} & (\mu - 1)\overline{y} + \beta V(\mu, 1) \\ \geq \ & \mu\overline{y} + \beta V(\mu, 0) \end{aligned}$$

that is, if

$$\beta V(\mu, 1) = \frac{\beta}{(1 - \beta)}(\mu - 1)\overline{y} \geq \overline{y} \tag{8}$$

Let $\beta = 1/(1 + \rho)$, then the firm will choose not to deviate when

$$\mu \geq 1 + \rho.$$

Thus, we have shown the following proposition

Proposition 2 *If the market power is high enough, so that the mark-up is higher than the discount rate, then the perfect information equilibrium is a SMCE.*

The intuition of the last proposition is clear. Given that the firm has the option of making a short run profit by selling low quality goods, the equilibrium mark-up must be high enough for the firm not to choose to do it. As the equilibrium profits are accrued over time, the discount rate matters.

This is the intuition of the Industrial Organization literature on unobservable quality, and the first quotations go back to Adam Smith. If by reducing the quality the firm can make short run profits, a reputation argument can explain why firms decide not to do so. As we have just seen, reputation is valuable when firms make positive profits in equilibrium. But, as competition gets tighter, i.e., in our model μ gets arbitrarily close to 1, monopolistic rents disappear and the equilibrium with perfect observability may not be sustainable through reputation if the discount rate is high enough. Notice that the time period can be seen as the time that it takes for consumers to observe the quality of the goods. The shorter is the information lag, the smaller is the discount rate, and the easier to sustain the equilibrium with perfect observability. Nevertheless, as long as $\rho > 0$, the Pareto efficient solution is never attained.

So far, we have only shown under which conditions is the perfect information equilibrium a SCE. However, it should be clear that the above argument applies to any outcome defined by $\overline{p} = \mu'$ and $y_{\overline{p}} = y'$, as long as $\mu' \geq 1 + \rho$. In this case, the choices are sequentially rational since consumers satisfy their demands at $p(i)_t = p^q(i)_t = \mu'$, firms make non-negative profits and any deviation is punished. In particular, a price deviation is instantaneously punished by triggering the beliefs that the firm is producing low quality. It follows that a price deviation is dominated by choosing to announce $p(i)_t = \mu'$ and delivering high quality. In summary,

Proposition 3 *For any $\mu' \geq 1 + \rho$ there exists a stationary SMCE where the price per unit of service is μ' and firms always produce high quality.*

This analysis shows how high quality can be maintained through reputation, but there is no role for competition. In fact, if consumer expectations depend on price and quality history, there may be no dimensions along which the firms can actually compete. There is a sense in which consumers expectations alone determine the whole set of restrictions that the firms face, and the actions of a single firm end up by being irrelevant for the others. The problem, however, is more with the definition of SMCE than with competition. We next discuss this problem, showing that a more "sensible" definition of equilibrium results in competition playing a role.

2.2.1 Regular beliefs

As we have just seen, in a SMCE consumers' beliefs may be fairly unreasonable. For example, conditional on past history, beliefs can be highly discontinuous in current prices, a consumer may have had his expectations of high quality always fulfilled in the past and, yet, a price announcement trigger a complete distrust. In fact, nothing precludes that beliefs could be such that observing high quality were to lead to down grading consumers' expectations of high quality. It seems more natural (more consistent with any reasonable learning process) to assume that beliefs satisfy some minimal continuity and monotonicity properties. We impose a regularity condition on beliefs that takes into account these intuitive ideas.

Definition 4 *A consumer has* $\varepsilon-$ positive beliefs *if there exist a* $\varepsilon > 0$ *such that, for all* $i, t, h_t, p(i)_t \geq 1$, $\upsilon_t^i(h_t, p(i)_t) \geq \varepsilon$ *if there is no period* $n \geq 0$, $n < t$, *with* $p^q(i)_n \neq p(i)_n$

Definition 5 *A consumer has* weakly monotone beliefs *if, for all* i, t, h_t, $p(i)_t$, $\upsilon_{t+1}^i(h_t, p(i)_t, p^q(i)_t, p(i)_{t+1}) \geq \upsilon_t^i(h_t, p(i)_t)$, *whenever* $p^q(i)_t = p(i)_t$ *and* $p(i)_{t+1} \geq p(i)_t$

Definition 6 *An* $\varepsilon-$ Regular SMCE *is a SMCE where agents' beliefs are* ε*-positive and weakly monotone.*

Definition 7 *A* Regular Monopolistic Competition Equilibrium *(RMCE) is a SMCE with beliefs* $\{\upsilon_t^i\}$, *such that there is a sequence of* ε_n*-Regular SMCE, with beliefs* $\{\upsilon_t^i\}_n$ *satisfying* $\{\upsilon_t^i\}_n \to \{\upsilon_t^i\}$ *as* $\varepsilon_n \searrow 0$.

That is, ε*-positive beliefs* incorporate elements of trust and induction. Consumers' beliefs must assign at least ε probability of delivering high quality, as long as firms have not deceived consumer's expectations. Although we are not aware that such restriction on beliefs has previously been used, it can also be seen as containing two restrictions that have been used elsewhere. One is a weak form of the *induction hypothesis*[6] requiring, for example, that $\upsilon_t^i(h_t, p(i)_t) \geq \varepsilon$ as long as, for $n < t$, $p^q(i)_n = p(i)_n \geq 1$. The second is a non-degeneracy condition on intial beliefs requiring, for example, that $\upsilon_0^i(h_0, p(i)_0) \geq \varepsilon_n$ if $p(i)_0 \geq 1$, and then letting $\varepsilon_n \searrow 0$, as in perfection requirements[7]. *Weakly monotone beliefs* also incorporate an element of in-

[6]Forms of the induction hypothesis have been used, for example, by Cho and Matsui (1995) and Sargent (1999).

[7]It should be noticed that most of our results hold if we do not restrict initial beliefs, except that more equilibria exist with such a weaker restriction on beliefs. In particular, the low quality equilibrium and a multiplicity of non-stationary high quality equilibria with arbitrary prices in period zero. It should also be noticed that imposing *perfection* (i.e., $\upsilon_t^i(h_t, p(i)_t) \geq \varepsilon_n$, unconditionally, and then $\varepsilon_n \searrow 0$) would be too strong of a restriction since deviations could not be followed by reversals to low quality equilibrium paths.

duction, in that, given that beliefs were fulfilled, beliefs cannot be lower that will not be fulfilled the following period when a firm has no greater incentive to deviate (i.e., the posted price is not lower)[8]. As it can be seen, our *regularity* conditions are fairly weak and reasonable. They are typically satisfied when beliefs evolve (i.e., are updated) according to some learning procedure. For example, *Bayesian updating* will satisfy *weak monotonicity* if consumers consider that they are observing a stationary path; as in fact they are in a stationary RMCE. Similarly, ε*-positiveness* is satisfied when the learning process starts with not degenerated initial beliefs and, again, satisfies minimal monotonicity conditions guaranteeing that a forecast of low quality with probability one will not be made after only having observed high quality.

To see the role that such regularity conditions can play, consider that firms' stationary strategies and allocations are the ones described above supporting the price $\overline{p} > 1 + \rho$, $\overline{p} \neq \mu$, and $\upsilon_t^i(h_t, \overline{p}) = \lambda_{i,t}(\overline{p}) = 1$, $\overline{p} = U'(y_{\overline{p}})/\alpha$ as a SMCE, and firm i considers to deviate in period t and set another price $p(i) \geq 1$. Given that, along the path, consumers expectations are not deceived, $\upsilon_t^i(h_t, p(i)) \geq \varepsilon_n$ in a ε_n-RSCE. In this case, firm i will deliver high quality in period t if the following incentive condition, equivalent to (8), is satisfied

$$\beta \sum_{n=0}^{\infty} \beta^n (p(i) - 1) y_{\overline{p}} \left(\frac{\overline{p}}{p(i)} \right)^{\frac{\mu}{\mu-1}} \left(\upsilon_{t+n+1}^i \left(h_t, p(i), (p(i)/1, p(i))_{t+1}^{t+n+1} \right) \right) \right)^{\frac{\mu}{\mu-1}}$$

$$\geq \; y_{\overline{p}} \left(\frac{\overline{p}}{p(i)} \right)^{\frac{\mu}{\mu-1}} \left(\upsilon_t^i(h_t, p(i)) \right)^{\frac{\mu}{\mu-1}}$$

where $(p(i)/1, p(i))_{t+1}^{t+n+1}$ denotes the sequence of prices $(p^q(i)_m, p(i)_{m+1})$,with $p^q(i)_m = p_m(i)/1 = p(i)_{m+1} = p(i)$, $m = t, ..., t+n$. That is,

$$\beta(p(i) - 1) \sum_{n=0}^{\infty} \beta^n \left(\upsilon_{t+n+1}^i (h_t, p(i), (p(i)/1, p(i))_{t+1}^{t+n+1}) \right)^{\frac{\mu}{\mu-1}} \geq \left(\upsilon_t^i(h_t, p(i)) \right)^{\frac{\mu}{\mu-1}} \tag{9}$$

However, if beliefs satisfy the *weak monotonicity* condition, (9) reduces to

$$\rho^{-1}(p(i) - 1) \geq 1$$

or

$$p(i) \geq 1 + \rho$$

[8] Analyzing reputation games, Kreps and Wilson (1982) considered a similar restriction on beliefs.

In other words, firm i will maintain high quality as long as the price satisfies the mark up reputational condition. Then, $v_t^i(h_t, p(i)) = 1$, for all t. It follows that a profit maximizing firm will deviate to

$$p(i) = \max\{\mu, 1+\rho\} \tag{10}$$

Therefore, since for any ε_n-RSCE (10) is satisfied, it must also be satisfied in a RSCE. Furthermore, the previous argument also shows that the worst SCE is not a RSCE since, given that beliefs are not degenerate in period zero, firms always prefer to start offering high quality. Nevertheless, in the zero probability event that low quality is observed, the worst SCE path is part of the RSCE since after low quality has been observed our regularity conditions do not place any restriction on beliefs. The above argument shows the uniqueness of stationary RMCE. In the Appendix we show that such uniqueness result is satisfied even when we consider non stationary paths. We can now state the main proposition that relates competition and reputation.

Proposition 8 *There is a unique* Regular Monopolistic Competition Equilibrium *(RMCE), which is characterized by the production of high quality services being sold at a per unit price of* $\mu' = \max\{\mu, 1+\rho\}$ *.*
Proof. *In Appendix.*

Notice that, by making very weak assumptions on beliefs, we have obtained very strong results. With competition and reputation, there is a unique stationary equilibrium where the mark up is the maximum between the mark up of the competitive equilibrium with perfect observability and the discount rate. As we noticed before, it follows that the RMCE can not be efficient (even if $\mu \searrow 1$) as long as $\rho > 0$.

3 A model of currency competition

In this section we modify the model described above to introduce competition between profit maximizing currency issuers. Our aim is to show how competition and reputation interact in the private provision of money and, in particular, if it can be an efficient monetary arrangement.

As in the monopolistic competition model, there is a continuum of goods or services and consumers' preferences are given by $\sum_{t=0}^{\infty} \beta^t [U(y_t) - \alpha n_t]$, where U satisfies the same monotonicity and concavity assumptions and $y_t = \left[\int_0^1 y(i)_t^{1/\mu} di\right]^\mu$ with $\mu \geq 1$. In contrast with the monopolistic competition model, we allow for free entry in the production of each of the goods. That is, product markets are competitive and, since the technology is linear in labor, firms will make zero profits in equilibrium. It follows that all goods or services have a per unit price of *one* and that the real wage will be constant and equal to one.

The central characteristic of our model of currency competition is the existence of currency issuers, each one having the right (monopoly) to issue its own distinct currency, and each specific currency being needed to purchase a corresponding specific good. More precisely, we impose a money-specific cash-in-advance constraint on each good. To simplify, we will denote by currency i the currency needed to purchase good i. It follows that as μ approaches one currency substitution increases. In particular, currencies are perfectly substitutable in the limiting case of $\mu = 1$. We assume that currency issuers take into account the demands for their currencies, taking real interest rates as given. These good-specific cash-in-advance constraints have been used, for example, by Woodford (1990) and resemble monetary arrangements where goods are indexed by their location. Alternatively, one could directly assume that a single good can be bought with the monetary aggregate $m_t = \left[\int_0^1 m(i)_t^{1/\mu} di\right]^\mu$, instead of deriving explicitely the demand for real balances as we do.

Thus, the representative consumer maximizes utility subject to the following budget constraint

$$b_{t+1} + \int_0^1 \left[\frac{M(i)_{t+1}}{P(i)_t} + y(i)_t\right] di \leq n_t + b_t(1 + r_t) + \int_0^1 \frac{M(i)_t}{P(i)_t} di + \Pi_t$$

where $P(i)_t$ is the price of good i in units of money i, $\Pi(i)_t$ are the current profits of the provider of currency $i, \Pi_t = \int_0^1 \Pi(i)_t di$, b_{t+1} are real bonds measured in units of the composite good, and r_t is the real interest rate.

The cash-in-advance constraints are

$$P(i)_t y(i)_t \leq M(i)_t$$

for all i and t.

It follows that the demand for good i in period t is given by

$$U'(y_t) y_t^{\frac{\mu-1}{\mu}} y(i)_t^{\frac{1-\mu}{\mu}} = \alpha R(i)_t \tag{11}$$

where $R(i)_t$ is the gross nominal interest rate

$$R(i)_t = (1 + r_t)(1 + \pi(i)_t)$$

and $\pi(i)_t$ is the -currency i- inflation rate between period $t-1$ and t. Furthermore, given that utility is linear in leisure, equilibrium real interest rates satisfy $r_t = \rho$

The demand for good i, can be written as

$$y(i)_t = y_t \left[\frac{R(i)_t}{R_t} \right]^{\frac{\mu}{1-\mu}} \tag{12}$$

where

$$R_t \equiv \left[\int_0^1 R(i)_t^{1/1-\mu} di \right]^{1-\mu}$$

As long as the cash-in-advance constraints are binding, (12) results in a demand for currency i

$$m(i)_t = m_t \left[\frac{R_t}{R(i)_t} \right]^{\frac{\mu}{\mu-1}} \tag{13}$$

where $m_t = \left[\int_0^1 (m(i)_t^{1/\mu} \right]^{\mu} di$.

The issuer of currency i faces an intertemporal budget constraint given by

$$\frac{M(i)_{t+1}}{P(i)_t} + d(i)_{t+1} = \frac{M(i)_t}{P(i)_t} + d(i)_t(1+\rho) + \Pi(i)_t$$

where $d(i)_t$ is the debt of the i-currency issuer at time t, in units of the consumption good, and $\Pi(i)_t$ are the profits of the money issuer in units of the consumption good. It also faces the corresponding non-Ponzi constraints guaranteeing that the present value budget constraint is well defined. The present value of profits are

$$\sum_{t=0}^{\infty} \beta^t \Pi(i)_t = \sum_{t=1}^{\infty} \beta^t \left((R(i)_t - 1)\, m(i)_t \right) - \frac{M(i)_0}{P(i)_0} - \frac{d(i)_0}{\beta} \tag{14}$$

where $m(i)_t = \frac{M(i)_t}{P(i)_t}$.

In order to maximize the present value of profits, firms must choose $\pi(i)_t$ to maximize

$$(R(i)_t - 1)\, m(i)_t$$

taking $r_t = \rho$ and (13) as given. They must also minimize $\frac{M(i)_0}{P(i)_0}$. Notice that, as in standard (single currency) monetary models, a monetary policy for the i-currency issuer consists of a current price level and a sequence of future nominal interest rates: $\left(P(i)_0, \{R(i)_t\}_{t=1}^{\infty}\right)$.

3.1 Currency competition with full commitment

We now assume that currency issuers can fully commit to a monetary policy and characterize the corresponding monetary equilibrium

Regarding monetary policies, optimality requires the initial price level to be arbitrarily high such that the real value of initial outstanding money holdings (liabilities) $\frac{M(i)_0}{p(i)_0}$ become zero. This is achieved through a big open market operation in which the currency is sold back to the consumers. Each currency issuer takes a negative position in bond holdings, in an amount equal to the real quantity of money. In subsequent periods, the currency issuer collects the real rate of interest on these bond holdings, as well as the inflation rate on real money holdings, corresponding to future money issuing..

To characterize the problem of maximizing time t profits, notice that to maximize

$$(R(i)_t - 1)\, m(i)_t$$

subject to (13), results in the choice

$$R(i)_t = \mu$$

This is not surprising, since this maximization problem is the same, in the monopolistic competition model, as that of maximizing (4) subject to (3). We only need to identify the gross nominal interest rate, in the currency competition model, $R(i)_t$ with the price $p(i)_t$ in the monopolistic competition model, and $m(i)_t$ with $y(i)_t$. As in the previous model, with these prices, the consumption of the goods, $\overline{y} = \overline{m}$, is constant and satisfies $U'(\overline{y}) = \alpha\mu$.

It follows that, as currency substitution increases, i.e., $\mu \searrow 1$, nominal interest rates tend to zero, i.e., $(R(i)_t - 1) \searrow 0$, which is supported by a deflationary monetary policy, i.e., $\pi(i)_t \searrow (\beta - 1)$. In other words, with perfect substitution of private currencies the monetary equilibrium is efficient and the Friedman rule is implemented. In summary, *with full commitment, Hayek's conjecture, that efficient monetary equilibria can be achieved through currency competition, is satisfied.*

Nevertheless, as in standard (single currency) monetary models, the full commitment monetary policy is time inconsistent. This can easily be seen by considering how the budget constraints of a currency issuer evolves over time. At time t, the budget constraint is

$$\sum_{j=t}^{\infty} \beta^{j-t} \Pi(i)_j = \sum_{j=t+1}^{\infty} \beta^{j-t} \left((R(i)_j - 1) m(i)_j \right) - \frac{M(i)_t}{P(i)_t} - \frac{d(i)_t}{\beta} \tag{15}$$

Thus, if given the option to change plans at time t,which we rule out when assuming full commitment, the currency issuer will find it optimal to expand the money supply and let $P(i)_t$ increase without bound. The reason is that the real money demand is decreasing in the nominal interest rates, i.e., in expected future prices. However, once consumers have made their currency decisions, they are stuck with the outstanding money holdings and the nominal money demand is rigid with respect to the realized price. We turn now to analyze the case without commitment.

3.2 Currency competition without commitment

As there is a parallel between monopolistic competition with perfect observability and currency competition with full commitment, there is a parallel between monopolistic competition with unobservable quality and currency competition without commitment. More specifically, in both models firms compete in prices that are not observable or that they cannot commit to: in the monopolistic competition model, this is the price of the good per unit of quality; in the currency competition model it is the nominal interest rate, or the inflation rate. With perfect observability in the first model and with full commitment in the second, there is no distinction between set and realized prices. With unobservable quality in the first model and lack of commitment in the second, the *ex-post* realized prices may differ from the *ex-ante* prices. In fact, in such a case, firms maximize short run profits by setting an arbitrarily large realized price, which in the quality model corresponds to choosing low quality and in the currency model to inflate away current money holdings (i.e., in making "the quality of outstanding money" arbitrarily low). In both models, the timing is very important[9]: consumers purchase

[9] In a paper that also addresses the issue of competition in a time inconsistency setting, Kehoe(1989), used a different timing, and obtained the result that competition could solve the time consistency problem.

services before they observe the quality they yield, in one, and they purchase monies before they observe the real return they yield, in the other; in both models, consumers must form their expectations on realized prices, based on past information and current prices, and, in both models, reputation is what may prevent firms from "flying-by-night."

More formally, while monopolistic firms, in the monopolistic competition model, sequentially choose $(p(i)_t, p^q(i)_t)$, currency issuers sequentially choose $\left(R\,(i)_{t+1}\,, R^q(i)_t\right)$, where, in period t, $R\,(i)_{t+1}$is the gross nominal interest rate, between period t and $t+1$, and $R^q(i)_t = (1+\rho)(1+\frac{P(i)_t}{P(i)_{t-1}})$ the realized, or *ex-post*, rate. The quality of money is the inverse of the price level, $\frac{1}{P(i)_t}$, the units of the good that are bought with money. The difference of timing between the two models corresponds to the fact that in the first model, competition is on the unknown current -period t- prices per unit of quality, while in the second on the, also unknown, realized -between period t and $t+1$- interest rates.

We can now define a "*Sequential Currency Competition Equilibrium*" (SCCE) in a similar fashion as we have defined SMCE in the monopolistic competition model. Histories are given by $h_0 = \{\emptyset\}$, $h_1 = \left\{R(i)_1, \frac{1}{P(i)_0}, \text{ all } i\right\}$, and, for $t \geq 2$, $h_t = \left\{h_{t-1}, R\,(i)_{t+1}\,, R^q(i)_t, \text{ all } i\right\}$. The i-currency issuer strategy is given by

$$\sigma^b_{i,t}(h_t) = \left(R(i)_{t+1}, \eta_{i,t}\right),$$

where $\eta_{i,t}$ is a density distribution over R_+ and $\eta_{i,t}(\frac{1}{P(i)_t})$ is the probability that the realized price of money i in period t is $\frac{1}{P(i)_t}$. For $t > 0$, let $\lambda_{i,t}(R(i)_t) = \eta_{i,t}\left(p(i)_{t-1}[R(i)_t(1+\rho)^{-1} - 1]^{-1}\right)$,that is the probability that the ex-post interest rate equals the ex-ante one,

$$\lambda_{i,t}(R(i)_t) = \Pr\{R^q\,(i)_t = R(i)_t\}$$

Consumers behave competitively, deciding, for $t \geq 0$, $\sigma_t(h_t, R(i)_{t+1}) = \{n_t, y(i)_t, M(i)_{t+1}, \text{all } i,\, b_{t+1}\}$ based on their beliefs about future decisions of the currency issuers and corresponding equilibrium prices. Given beliefs about current prices (e.g., let be $\gamma^i_t(h_t, R(i)_{t+1})(\frac{1}{P(i)_t})$ denote the assesed probability of the price of money is $\frac{1}{P(i)_t}$), let $\upsilon^i_t(h_t, R(i)_{t+1})$ denote the assessed probability that, given the history, the ex-post nominal interest rates are equal to the ex-ante ones, i.e., $R(i)_{t+1} = R^q(i)_{t+1}$. Rational expectations requires that their beliefs regarding realized pices (γ^i_t) are consistent with currency issuers strategies $\eta_{i,t}$. In particular,

$$\upsilon^i_t(h_t, R(i)_{t+1}) = \lambda_{i,t+1}(R(i)_{t+1})$$

A SCCE consists of $\left((\sigma^c, \upsilon^c), (\sigma^f_i)\right)$, such that, for every $(t, h_t, R(i)_{t+1})$, $\sigma^f_{i,t}(h_t)$ solves the maximization problem of the i-currency issuer; $\sigma_t(h_t, R(i)_{t+1})$ solves the consumer's problem given consistent beliefs $\upsilon^i_t(h_t, R(i)_{t+1})$, and all markets clear, including the perfectly competitive goods markets.

As with SMCE, there is a worst SCCE in which currencies are not held, since agents expect realized nominal interest rates to be arbitrarily large.

To see more explicitly how currency competition and reputation interact, consider the problem of whether a stationary gross nominal interest rate, $R(i)$, is sustainable as a SCCE. Suppose that the i-currency issuer considers a deviation in period t, setting $R^q(i)_t \neq R(i)_t$, i.e., $\lambda_{i,t}(R(i)_t) = 0$, and printing arbitrarily large amounts of money, i.e., sets $\frac{1}{P(i)_t} = 0$. Suppose that agents' expectations are such that after observing that *ex-post* rate does not correspond with *ex-ante* rate become $\upsilon^{c,i}_{t+s}(h_{t+s}, R(i)_{t+1+s}) = 0$, $s \geq 0$, implying that, given such believes, $\frac{1}{P(i)_t} = 0$ dominates any other deviation. Thus, the demand for currency i becomes zero from time t on, i.e., $m(i)_{t+s} = 0$, $s \geq 1$, which means that the newly issued pieces of paper are worthless (i.e., the big market operation is a big failure).

The value of the outcome after the deviation is zero, except for the value of the outstanding real debt. The reason is that the deviation triggers a currency collapse for that currency, starting tomorrow. But, contrary to the monopolistic competition model with unobserved quality, the demand for money, being an asset, depends on future prices. Thus, the expectations of the currency collapse make the newly injected money be worthless today. Therefore, the present value of the benefits following a deviation is obtained by replacing the real value of money stocks from time t on by zeroes in the expression for profits (15)

$$V^D(i)_t = -\frac{d(i)_t}{\beta}$$

On the other hand, if the issuer does not deviate, the present value of the profits are

$$V^C(i)_t \quad = \quad \beta\frac{(R(i) - 1)m(i)}{1 - \beta} - \frac{M(i)_t}{p(i)_t} - \frac{d(i)_t}{\beta}$$

$$= \rho^{-1}(R(i)-1)m(i) - m(i) - d(i)_t \beta^{-1}$$

The last equality follows from the fact that, in equilibrium, $m(i) = \frac{M(i)_t}{p(i)_t}$. It follows that, the i-currency issuer will choose not to deviate from the stated policy when

$$\left[\rho^{-1}(R(i)-1) - 1\right] \geq 0,$$

$$\text{i.e., } R(i) \geq 1 + \rho$$

or, equivalently, whenever $\pi(i) \geq 0$.

As in the previous model, the set of stationary SCCE is fairly large, although, given a stationary monetary policy $\left(\frac{1}{P(i)_0}, \pi(i)\right)$, i.e., $R(i)_t = (1 + \rho)(1 + \pi(i))$, $t > 0$, the SCCE allocation is uniquely defined. More formally, the following proposition parallels Proposition 3.

Proposition 9 *For any $\pi(i) \geq 0$, the policy $(0, \pi(i))$ can be supported as strategy of a stationary symmetric SCCE.*

An equilibrium path with symmetric stationary policies is sustainable if the corresponding inflation rates are positive. The reason why inflation must be positive is because of the timing of collection of revenues for the issuers. Remember that along the commitment solution, the issuers make initial money holdings be valueless and, by an open market operation they sell back the new money balances to the consumers. Thus, at the first period the issuers hold positive assets in an amount equal to the real balances. From those assets they collect the real rate of interest, ρ. Thereafter, they also collect the inflation rate times the real money balances every period. If they deviate, they will keep the real asset holdings only[10]. Thus, as long as the returns they make with the inflation tax are non-negative, they have no incentives to deviate. In other words, future profits must be sufficiently high, and, in this monetary environment, future profits are the gains from future issuance of money. The gains corresponding to the initial issuance of money, the real rate on the real money stock, are sunk.

[10]Note that if the issuer were forced to hold their own currency denominated assets, then the efficient outcome could be supported as a SCCE.

3.2.1 Regular beliefs

As in Section 2, we can restrict the set of SCCE, and allow for competition to play its role, by assuming that beliefs satisfy regularity conditions. *Weak monotonicity* and $\varepsilon-$ *positive beliefs* can be similarly defined by replacing $p(i)$ by $R(i)$ and $p^q(i)$ by $R^q(i)$. Then, the parallel of Proposition 8 is

Proposition 10 *There exist stationary* Regular Currency Competition Equilibrium *(RCCE) and are characterized by the inflation rate* $\pi(i) = \max\left\{\frac{\mu}{1+\rho} - 1, 0\right\}$.

Notice that the only reason that RCCE is not unique is that the initial price $p(i)_0$ is not determined even when beliefs are regular. i.e., $v_1^i(h_1, R(i)_2)$ depends on $1/p(i)_0$, and not on an *ex-post* interest rate, and in response to these -and $v_0^i(h_0, R(i)_0)$- beliefs it may be better to set $1/p(i)_0 > 0$, in contrast with the full commitment case. This inital period indeterminacy is a special feature of the monetary model not present in the monopolistic competition model.

It follows the last propostion that, without full commitment, when privately issued currencies are very close substitutes, inflation is zero and the nominal interest rate ρ. That is, the monetary equilibrium is not efficient and the Friedman rule is not implemented. In summary, *without full commitment, Hayek's conjecture, that efficient monetary equilibria can be achieved through currency competition, is not satisfied.*

4 Conclusions

In this paper we have tried to clarify three related -but distinct- issues. First, how competition and reputation interact when, on the one hand, firms are subject to competition, but, on the other hand, such competitive pressure does apply to all their decisions. These decisions, nevertheless, may be subject to reputational pressures. Second, we have seen how in this general context two -apparently, very different- economic problems share the same basic features: monopolistic competition with experienced goods and currency competition. Third, we have seen that sequential (or sustainable) equilibria may not have much predictable power, and misrepresent the role of competition, in reputational models, but that imposing a weak regularity condition on beliefs results in equilibria, where competition plays a crucial role in enhancing efficiency. Nevertheless, it is within the nature of the reputational mechanism that competitive pressures can not achieve full efficiency. A particular corollary of these results is that Hayek conjecture, that efficient monetary equilibria can be achieved through currency competition, is not satisfied if currency suppliers make sequential decisions. Any of these three issues explored here suggest further work. In particular, alternative forms to model currency competition may be more suitable to relate our results with historical experiences. We leave this for future research.

5 Appendix: Proof of Proposition 8

Before proving the proposition, we prove a series of useful results

Result 1: In any RMCE $p_t \geq \mu$,for all t.

Pf: Assume not. Let t' be such that $p_{t'} < \mu$. If a single firm deviates by increasing the price by a small amount, profits at t' will be higher, so sustainability is satisfied for all $t < t'$. On the other hand, as the price is higher, the quantity sold at t' is lower, so sustainability is also satisfied at t'. As the profits are higher, firms will indeed deviate.

Result 2: In any symmetric RMCE $p_t \geq 1 + \rho$, for all t.

Pf: Assume not. By Result 1, $p_t \geq \mu$, thus the price sequence is bounded below. Let p' be the minimum price of the sequence - if the minimum does not belong to the sequence, see below. Obviously, $p' < \mu$. Let t' be the time period in which the minimum is attained. As we showed above, it is the case that

$$y(p') > \sum_{n=1}^{\infty} \beta^n y(p')(p' - 1)$$

However, as $p_t > p' > \mu$, then $y(p')(p' - 1) > y(p_t)(p_t - 1)$ for all t. Thus

$$y(p') > \sum_{n=1}^{\infty} \beta^n y(p_{t+n})(p_{t+n} - 1)$$

so the sustainability condition is not satisfied.

If the minimum is not attained, the same argument applies with the infimum, since demand functions and profits are continuous on prices.

Result 3: If in any RMCE $p_t > \mu$, then

$$y(p_t) = \sum_{n=1}^{\infty} \beta^n y(p_{t+n})(p_{t+n} - 1)$$

Pf: Assume not. Then, there exists a period t' such that

$$y(p_{t'}) < \sum_{n=1}^{\infty} \beta^n y(p_{t+n})(p_{t+n} - 1)$$

If a single firm lowers the price at t' by a small amount, profits will be higher and the sustainabilty condition will be satisfied for all $t < t'$. On the other hand, as the demand functions are continuous, the sustainability condition will also be satisfied at t'.

Result 4: If in a symmetric RMCE $p_t > \mu$, then $p_{t+n} > \mu$ for all n.

Pf: First, note that by Result 2, $p_t \geq 1 + \rho$, so if $1 + \rho \geq \mu$,the result is obvious. Thus, the only interesting case is when $\mu \geq 1 + \rho$ or $\beta\mu \geq 1$.

In equilibrium it has to be the case that

$$y(p_t) \leq \sum_{n=1}^{\infty} \beta^n y(p_{t+n})(p_{t+n} - 1)$$

for all t. If this equation holds with equality for all t, then

$$y(p_t) = \beta y(p_{t+1})p_{t+1} \tag{16}$$

for all t. Using the definition of demand curves

$$\acute{U}^{-1}(\alpha p_t) = \beta \acute{U}^{-1}(\alpha p_{t+1})p_{t+1}$$

Since U is concave, it is straightforward to verify that the equation above defines an ever increasing difference equation except when $p_t = 1 + \rho$ for all t. This means that if some price is higher than μ, then all future prices are higher than μ. If, on the other hand, the condition does not hold with equality in some periods, then by Result 3, it has to be the case that in those periods the price has to be equal to μ. Assume then, contradicting the hypothesis, that there exists a RMCE such that

$$p_t > \mu, p_{t+1} = \mu$$

so

$$\begin{aligned} y(p_t) &= \sum_{n=1}^{\infty} \beta^n y(p_{t+n})(p_{t+n} - 1) \\ &= \beta y(\mu)(\mu - 1) + \beta \sum_{n=1}^{\infty} \beta^n y(p_{t+n+1})(p_{t+n+1} - 1) \end{aligned}$$

and

$$y(\mu) < \sum_{n=1}^{\infty} \beta^n y(p_{t+n+1})(p_{t+n+1} - 1)$$

so

$$\begin{aligned} y(p_t) &> \beta y(\mu)(\mu - 1) + \beta y(\mu) \\ y(p_t) &> \beta y(\mu)\mu \geq y(\mu) \end{aligned}$$

since $\beta\mu \geq 1$. But as the demand functions are decreasing on prices, this means that

$$p_t < \mu$$

contradicting the hypothesis.

Note that the candidate sequences for equilibria are restricted to satisfy equation (16), when he prices are different from μ.

Corollary:

- If $\mu > 1+\rho$, a RMCE is characterized by either $p_t = \mu$,or by a sequence of prices satisfying $p_T > \mu$ for some T, and $p_{t+1} > p_t$ for all $t > T$.
- If $\mu \leq 1 + \rho$, a RMCE is characterized by either $p_t = 1 + \rho$,or by a sequence of prices satisfying $p_T > 1 + \rho$ for some T, and $p_{t+1} > p_t$ for all $t > T$.

Proposition: There is a unique *Regular Monopolistic Competition Equilibrium* (RMCE), which is characterized by the production of high quality
services being sold at a per unit price of $p = \max\{\mu, 1+\rho\}$.

Pf: Assume that $\mu \geq 1+\rho$. It is trivial to show that $p_t = \mu$ for all t is an equilibrium since at that price firm's FOC are satisfied, so profits are lower if they change the price. In the text we showed that firms have no incentives to deviate in this stationary case.

The other candidate for a symmetric RMCE is given by a price sequence such that after some time T period exhibits increasing prices higher than μ. If this is the case, then the price sequence satisfies the following difference equation

$$y(\widetilde{p}_t) = \beta y(\widetilde{p}_{t+1})\widetilde{p}_{t+1}$$

for all $t \geq T$. Consider now a firm i that deviates at time $T+1$ by posting a price above μ but lower than p_{T+1}. For the deviation to be sustainable, all future prices must also be changed such that the following difference equation holds,

$$y(\widetilde{p}_t)\left[\frac{p(i)_t}{\widetilde{p}_t}\right]^{\frac{\mu}{1-\mu}} = \beta y(\widetilde{p}_{t+1})\left[\frac{p(i)_{t+1}}{\widetilde{p}_{t+1}}\right]^{\frac{\mu}{1-\mu}} p(i)_{t+1}$$

but as we assumed that the price sequence is an equilibrium, it must satisfy

$$\frac{y_t(\widetilde{p})}{\beta y_{t+1}(\widetilde{p})} = \widetilde{p}_{t+1}$$

which combined with the previous expression delivers

$$\left[\frac{p(i)_t}{\widetilde{p}_t}\right]^{\mu} = \left[\frac{p(i)_{t+1}}{\widetilde{p}_{t+1}}\right]$$

This implies that if $p(i)_t < \widetilde{p}_t$, then $p(i)_{t+1} > \widetilde{p}_{t+1}$. Thus, the firm can deviate to a sequence of lower prices for ever and still satisfy sustainability. As the prices were higher than the optimum, the firm is making higher profits, so the deviation is optimal. It follows that this cannot be an equilibrium.

Assume now that $\mu < 1+\rho$. If the price sequence satisfies $p_t = 1+\rho$ for all t, then

$$y(1+\rho) = \sum_{n=1}^{\infty} \beta^n y(1+\rho)\rho$$

While it is true that by lowering the current price the firm makes higher profits today, it is also the case that quantities sold will be higher today. Thus, the left hand side of the previous equation will be higher and the firm will therefore have incentives to deliver bad quality. Thus, consistency implies that beliefs of high quality will approach zero in the sequence of epsilon positive beliefs as epsilon goes to zero. Thus, the firm will have no incentives to deviate by reducing the price. As profits are lower for higher prices, it has not incentives to increase the price either. It follows that $p_t = 1 + \rho$ is a price sequence of a RMCE.

The other candidate equilibria also exhibit increasing sequences of prices higher than μ, as before. The same reasoning rules them out.

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