

LOCAL PUBLIC INVESTMENT AND COMPETITION FOR A FIRM

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Abstract

Most of the work in the field of competition between jurisdictions for the attraction of a large plant focuses on financial offers, bids or tax holidays. In this paper we add to the competition game an initial stage in which jurisdictions can invest in an infrastructure capital to enhance their attractiveness and modify the outcome of the competition stage. We characterize the Nash equilibrium of this game. In an example we show how the parameters of the model change the outcome of the game. In particular, the size of a jurisdiction is a powerful attraction force for the firm but it can be bypassed by a well specialized infrastructure capital, even if the competing jurisdiction is big.

INTRODUCTION

The attraction of a large plant by competing jurisdictions can be viewed as a part of the fiscal competition literature where the private capital to be invested in a jurisdiction is indivisible. These models focus on the relationships between jurisdictions and a firm and especially when the firm is large enough to have significant economic effects at a local level. In this framework, the large firm creates a competition between jurisdictions to get the better financial conditions for its future localization. The relationships between the jurisdictions and the firm have been modeled through a bidding game between jurisdictions where the firm has all the bargaining power (Black and Hoyt, 1989 ; King, McAfee and Welling, 1993 ; King and Welling, 1992)¹. The salient point of this literature is its efforts to explain the magnitude and the duration of the tax exemption offered to the large firm. Historically, the duration of the tax exemption has been the major concern. The tax holiday models (Bond and Samuelson, 1986 ; King and Welling, 1992 ; King, McAfee and Welling, 1993) consider that the competition between jurisdictions and the ability to offer financial bids depends mainly on the dynamics of the mobility of the firm attracted. The fixed cost of delocalization of the firm after its localization enable the jurisdiction to finance its first stage tax exemption by a second stage taxation under the constraint that the firm, at the second stage, is indifferent between staying in the jurisdiction and leaving the jurisdiction. In these models, there is no explicit link between the large firm and the economy of the jurisdiction. Here, the inducement policy of a jurisdiction is financed by the attracted firm. By explaining the magnitude of tax exemption, Black and Hoyt (1989), develop a model of competition where the jurisdictions maximize local welfare and finance their attraction of a large firm by a local taxation based on the local economic surplus induced by the localization of the firm². Our work is mainly inspired with the second approach because it allows us to link inducement policies with local economic development.

As the emphasis put on the tax instrument is motivated with the widespread use of financial offers in inducement policies, we think that it covers only one part of the problem. Public investments in infrastructure capital can be used by

¹ Only a pioneering work by Doyle and van Wijnbergen (1994) has explicitly described a more complex bargaining procedure.

² In this model, local economies of scale in the production of a public good are the source of the economic surplus induced by the localization of the firm and are created by the increase in the labor force and in the number of taxpayers.

jurisdictions to compete for attracting a firm (Taylor, 1992). As King, McAfee and Welling (1993) mentioned in their work, investment in infrastructure may predetermine the condition of attraction in the competition stage and then the financial offers. This idea is consistent with the fact that empirical works on the effect of fiscal incentives on the location of industries do not present strong evidence that the tax structure can be a good indicator of the attractiveness of jurisdictions. Moreover, Helms (1985) consider that focusing only on tax structure does not allow to describe all the parameters that can affect the decision of localization of the firms. Public goods can have productive effects on firms and then have to be introduced in the model. These matters echoes to the huge empirical literature about the impact of infrastructure capital on private productivity at national level (Aschauer, 1989; Berndt and Hansson, 1991; Munnell, 1990b; Tatom, 1991b; Lynde and Richmond, 1993; Conrad and Seitz, 1994; Nadirii and Numuneas, 1994) but also at regional (Morrisson and Schwartz, 1996; Holtz-Eakin, 1994; Costa and alii, 1987; Munnell, 1990b; Evans and Karras, 1994; Seitz and Licht, 1995; Garcia-mila and McGuire, 1992) or at the firm level (Shah, 1992; Seitz, 1994). This economic impact of the infrastructure capital can explain the competition between regions and the regional relocation patterns of firms (Seitz and Licht, 1995; Hulten and Schwab, 1991). Tax exemption and public factors affect the location decision of the firms and have to be introduced in a model of competition between jurisdiction for the attraction of a firm. As financial offers are powerful incentives that can be dedicated to a special location project, infrastructure capital, by its public nature and its ability to be accumulated is a complementary instrument improves the conditions of realization of economic activities and increases the profitability of local firms, but also the profitability of the attracted firm. For that purpose, we built a two stage game where we add to the competition game between jurisdictions in financial offers a preliminary stage where jurisdictions can make costly investments in a public infrastructure capital. Introducing an accumulation process in an infrastructure capital alters the overall competition between the jurisdictions because we have to take into account both the opportunity to invest that depends on the outcome of the competition and the opportunity of localization of the firm that depends on the investments and financial offers of the jurisdiction.

The first part of the paper is devoted to the characterization of the Nash equilibrium of this game. We reduce the determination of the investment level and the financial offer to an investment problem. We show that the outcome of the game, i.e. the localization of the large firm depends both on the profit maximization of the firm and the welfare objective of the jurisdictions. For each outcome we exhibits the conditions under which the jurisdictions invest. Given

these investments, we model the competition game as a two players game where the reaction function of the jurisdiction takes into account the behavior of the large firm. We finish this part by considering the conditions under which different equilibrium configurations may occur. In a second part of the paper, we illustrate the equilibrium concept by two examples with asymmetric jurisdictions. One example where the infrastructure is specific to the attracted firm and one when it is public. In each example we show that the size of a jurisdiction predetermines its ability to invest and represents a strong attraction force. Under some conditions, we are able to prove that differences in size can be bypassed by a good network of local firm or a well designed infrastructure capital that may enable the smallest jurisdiction to attract the large firm.

1. THE MODEL

We consider a two stage game with two players, a large firm and two jurisdictions. In a first stage, each jurisdiction $i = \{A, B\}$ can invest in its public capital Z_0^i . The unit cost of investment r is determined in a common market. Initial endowments of each jurisdiction are given by the profits of the firms that are located in it $n^i \pi^i(Z_0^i)$. If we consider that each firm sells one unit of a good on a national market at a unit price of 1, the profit for a given number of firms is $\pi^i(Z^i) = 1 - c^i(Z^i)$. Following the literature on the public factors (Kaizuka, 1965 ; Boadway, 1973 ; Henderson, 1974 ; Hillman, 1978 ; Feehan, 1989 ; McMillan, 1979a) we consider that the stock of the public capital is an unpaid factor that reduces the cost of the firms located in each jurisdiction and creates a rent $[c^i(0) - c^i(Z^i)]^3$. The cost of a firm is a decreasing function of the public capital stock $c^{i'}(Z^i) < 0$ and $c^{i''}(Z^i) > 0$. The unit cost of production is fixed, $c^i(0) = \bar{c}$. All the economic gains earned in a jurisdiction are fully distributed in it, and finance overall consumption of the inhabitants $n^i d_t^i$ and the actions of the local government. A firm is an entrepreneur, a consumer and a taxpayer. In the first stage, the balanced budget condition for a jurisdiction is given by :

$$n^i \pi^i(Z_0^i) = r(Z_1^i - Z_0^i) + n^i d_0^i \quad (1)$$

In the second stage the large firm wants to locate in one of the two jurisdictions. To attract the large firm, each jurisdiction can make a financial offer to the

³ Public capital and especially social overhead capital (education ...) increases the productivity of private factors of production. In this paper we focus only on economic capital like public infrastructure, see Diewert (1986) for a complete definition.

firm Ω^i . The net profit of the large firm is given by $\Pi^i(Z_1^i) + \Omega^i$. Two balanced budget conditions have to be considered at the local level :

- If the large firm locates in the jurisdiction :

$$n^i \tilde{\pi}^i(Z_1^i) = n^i d_1^i + \Omega^i \quad (2)$$

- Else :

$$n^i \pi^i(Z_1^i) = n^i d_1^i \quad (3)$$

One can notice that the location of the large firm modifies the profits of the local firms $\tilde{\pi}^i(Z^i) = 1 - \tilde{c}^i(Z^i)$. We consider that $\tilde{c}^{i'}(Z^i) < 0$ and $\tilde{c}^{i''}(Z^i) > 0$. This variation of the profits can be explained by the interactions between local firms and the large firm through technological spillovers, induced economic effects or agglomeration effects⁴. This local surplus created by the location of the large firm draw a link between the attraction policy and the local economic development⁵.

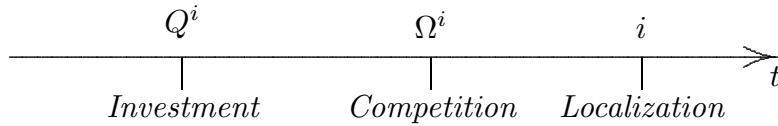
For the large firm, the balanced budget condition is given by :

$$\Pi^i(Z_1^i) + \Omega^i = D_1^i \quad (4)$$

The profit function of the large firm is defined by $\Pi^i(Z^i) = 1 - C^i(Z^i)$ with $C^{i'}(Z^i) < 0$ and $C^{i''}(Z^i) > 0$.

At the end, the large firm locates in the jurisdiction where it earns the highest net profit. The unfolding of the game can be illustrated by the following figure :

GRAPHIQUE 1 : UNFOLDING OF THE GAME



To explain the competition between jurisdictions and the impact of public capital and the financial offers, we have to define their objective functions. Following Black and Hoyt (1989) we suppose that each jurisdiction try to maximize the welfare of its inhabitants. We measure the local welfare by the total consumption in the jurisdiction $n^i d_t^i$. The welfare function is given by the discounted sum of individual consumption levels over the two periods :

$$V^i = n^i (d_0^i + \rho d_1^i)$$

⁴ In this paper we do not model these interactions to concentrate our attention on the attraction game and we deal only with positive effects.

⁵ Black and Hoyt (1989) consider that this surplus come from economies of scale in the production of the public good induced by the increase in the number of workers, taxpayers in the jurisdiction.

The levels of consumption in the second stage depend on the outcome of the competition between the jurisdictions, i.e the location decision of the large firm. If we use equations (1), (2) and (3) to replace the consumption levels by their equivalents, the local welfare is a function of the public capital Z_1^i , the financial offer Ω^i and of the location decision of the large firm :

$$\begin{aligned} V^i(Z_1^i, b^i) = & n^i \pi^i(Z_0^i) - r(Z_1^i - Z_0^i) \\ & + \rho [n^i \pi^i(Z_1^i) + e^i \{n^i [\tilde{\pi}^i(Z_1^i) - \pi^i(Z_1^i)] - \Omega^i\}] \end{aligned} \quad (5)$$

Where $e^i = 1$, if the large firm locates in i , $e^i = 0$ else.

The competition for the attraction of the firm in the second stage leads to the strategic determination of the level of the financial offer Ω^i given the stock of infrastructure in each jurisdiction at the beginning of the stage. These financial offers are functions of the current stock of infrastructure. To set the level of its bid, a jurisdiction compares the total surplus created by the location of the firm in each territory. This total surplus is given by :

$$T^i(Z_1^i) = \Pi^i(Z_1^i) + B^i(Z_1^i)$$

The welfare of a jurisdiction i is directly increased by the wealth of the new located firm $\Pi^i(Z_1^i)$, but also indirectly by the induced economic surplus over all inhabitants $B^i(Z_1^i) = n^i [\tilde{\pi}^i(Z_1^i) - \pi^i(Z_1^i)]$. The function $B^i(Z_1^i)$ also gives the value of the maximal bid of a jurisdiction. At this maximum, a jurisdiction can offer all the induced economic surplus. Alternatively, the function $T^i(Z_1^i)$ can be viewed as the maximal gain of the large firm when it locates in the jurisdiction i .

A jurisdiction has no incentives to give all its induced economic surplus to the firm if it sets its bid so as to made the firm indifferent between the two locations⁶. The surplus remaining in a jurisdiction is the difference between the total surpluses, $T^i(Z_1^i) - T^j(Z_1^j)$ ⁷. Let us note Q^i ($i = A, B$) the level of investment in the stock of infrastructure. As $Z_1^i = Z_0^i + Q^i$, we can rewrite the objective function of a jurisdiction i as a function of the decision of investment in the first stage :

$$\begin{aligned} V^i(Q^i, Q^j) = & n^i \pi^i(Z_0^i) - rQ^i + \rho [n^i \pi^i(Z_0^i + Q^i) \\ & + e^i \{T^i(Z_0^i + Q^i) - T^j(Z_0^j + Q^j)\}] \end{aligned}$$

⁶ This procedure of determination of the bids is identical to a second price sealed bid auction (Black and Hoyt, 1989 ; King, Welling and McAfee, 1992 and 1993).

⁷ The financial offer $\Omega^i = T^j(Z_1^j) - \Pi^i(Z_1^i)$ is a decreasing function of the investment in the stock of infrastructure. The determination of the bid can be described in the following figure. The bid is positive if $T^B > F^A$ (Figure a) and negative if $T^B < F^A$ (Figure b).

2. EQUILIBRIUM

The optimization problem can be defined as the setting of the optimal level of the investment in the stock of infrastructure under the budget constraint, given the investment of the competing jurisdiction. The optimization program depends on the hypothetical outcome of the competition between the two jurisdictions. This hypothetical outcome is determined by the preferences of the large firm and by the willingness to invest of the jurisdictions. The attraction of a firm have an implicit price, in terms of investment required, that can be too high in some case.

A jurisdiction is attractive for a large firm if the maximal gain that it can earn in the jurisdiction is greater than in the other :

$$T^i(Z_1^i) - T^j(Z_1^j) \geq 0 \quad (6)$$

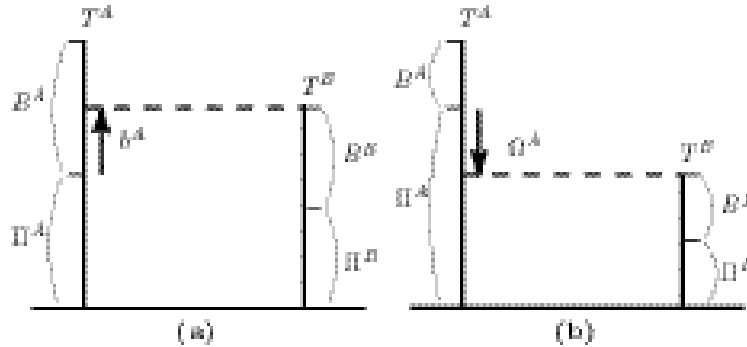
Depending on the value of the attractiveness constraint (6) a jurisdiction has to consider two cases.

- If the jurisdiction i ($i = A, B$) wants to attract the large firm, it sets its optimal investment Q_W^{i*} as the solution of the following maximization program :

$$(W^{i*}) \left\{ \begin{array}{ll} \text{Max}_{Q^i} & V_W^i(Q^i, Q^j) = n^i \pi^i(Z_0^i) - rQ^i + \rho[n^i \pi^i(Z_0^i + Q^i) \\ & \qquad \qquad \qquad \qquad \qquad + T^i(Z_0^i + Q^i) - T^j(Z_0^j + Q^j)] \\ \text{sc} & \begin{array}{ll} 0 \leq Q^i \leq \bar{Q}^i & d(i) \\ T^i(Z_0^i + Q^i) - T^j(Z_0^j + Q^j) \geq 0 & g(i) \end{array} \end{array} \right.$$

- If the jurisdiction i does not want to attract the firm, it sets its optimal investment Q_L^{i*} as the solution of the following maximization program :

$$(L^{i*}) \left\{ \begin{array}{ll} \text{Max}_{Q^i} & V_L^i(Q^i) = n^i \pi^i(Z_0^i) - rQ^i + \rho[n^i \pi^i(Z_0^i + Q^i)] \\ \text{sc} & \begin{array}{ll} 0 \leq Q^i \leq \bar{Q}^i & d(i) \\ T^j(Z_0^j + Q^j) - T^i(Z_0^i + Q^i) > 0 & p(i) \end{array} \end{array} \right.$$



The constraint $d(i)$ implies that a jurisdiction cannot invest more than it can collect from its taxpayers. Rewriting (1) leads :

$$\bar{Q}^i = \frac{1}{r} [n^i \pi^i (Z_0^i)] \quad (7)$$

Inequalities $g(i)$ and $p(i)$ are attractiveness constraints that describe the location decision of the large firm.

In this description we have omitted the second condition of attraction. A jurisdiction i will want to attract a large firm if the welfare obtained after the location of the firm (program W^i) is greater than the welfare obtained if the jurisdiction does not attract the firm (program L^i). Depending on the value of the investment of the competing jurisdiction j , the local authorities have to change their investment decisions but can also choose different optimization program. The two optimization programs are linked. The determination of the level of investment in the infrastructure stock, best reply to the investment of the competing jurisdiction, is complexified by the strategic interactions that passes through the attractiveness constraints.

DEFINITION 1 : An equilibrium of this attraction game is a subgame perfect Nash equilibrium in the levels of infrastructure investments. At the Nash equilibrium the winning jurisdiction wants to attract the large firm and the large firm wants to locate in this jurisdiction.

In first period the jurisdictions have to determine non cooperatively their investment levels anticipating the outcome of the second period subgame. The second period is a bidding game between the two jurisdictions that try to attract the large firm with financial offers. The equilibrium of this subgame requires that a jurisdiction wants to attract the large firm and the firm wants to locate in the jurisdiction. We will show that this coincidence of interests can be reduced to the satisfaction of the jurisdiction objective : if a jurisdiction wants to attract the large firm, then the firm wants to locate in this jurisdiction. The consequence is that the bidding subgame can be reduced to a subgame between the two jurisdictions. As the bids depends on the initial investments, the equilibrium of the game is determined by the optimal investment strategy of the jurisdictions, in terms of restricted reaction functions.

To construct the restricted best reply functions we have to describe how jurisdictions set their investment levels depending on the possible outcome of the competition. We first consider light optimization programs where attractiveness constraint have not been introduced. We obtain the hypothesis under which the levels of investment can be determined. Second, we describe the selection procedure

of the level of investment that depends on the value of the attractiveness constraint and of the value of the objective function in the two light optimization programs. This selection will give the restricted best reply functions that enable us to describe all the equilibrium configurations.

2.1. Restricted best reply functions

A jurisdiction wins the competition and makes the required investments if the jurisdiction wants to attract the firm and if the large firm wants to locate in the jurisdiction. In a first stage, we will study the investment decision of the jurisdiction without taking into account these constraints. To carry out this study we restrict the game by considering only the jurisdictions and define light optimization programs with respect to the outcomes of the competition between jurisdictions :

- The jurisdiction i is attractive for the large firm and wants to welcome it. It sets its investment in infrastructure Q_W^i as the solution of the following light optimization program :

$$(W^i) \begin{cases} \text{Max}_{Q^i} & V_W^i(Q^i, Q^j) = n^i \pi^i(Z_0^i) - rQ^i + \rho[n^i \pi^i(Z_0^i + Q^i) \\ & + T^i(Z_0^i + Q^i) - T^j(Z_0^j + Q^j)] \\ \text{sc} & \\ & 0 \leq Q^i \leq \bar{Q}^i \end{cases} \quad d(i)$$

- The jurisdiction i is not attractive for the large firm and sets its investment Q_L^i as the solution of the following light optimization program :

$$(L^i) \begin{cases} \text{Max}_{Q^i} & V_L^i(Q^i) = n^i \pi^i(Z_0^i) - rQ^i + \rho[n^i \pi^i(Z_0^i + Q^i)] \\ \text{sc} & \\ & 0 \leq Q^i \leq \bar{Q}^i \end{cases} \quad d(i)$$

The solutions $\{Q_W^i, Q_L^i\}$ are not functions of the investments made by the jurisdiction j , because the objective function of the jurisdiction i is additively separable with respect to Q^j ⁸. The values of the solutions depends both on the hypothesis about the economic profitability of the infrastructure capital and the magnitude of the induced economic effects created by the location of the firm. Let us precise these hypothesis.

HYPOTHESIS 1 : The function $V_L^i(Q^i)$ is concave with respect to Q^i and admits and interior maximum $0 < Q_L^i < \bar{Q}^i$.

⁸ To be convinced, the reader can refer to the examples of Section 3.

The program (L^i) admits an interior solution Q_L^i , such that $0 < Q_L^i < \bar{Q}^i$. The optimal investment is solution of the first order condition :

$$\frac{\partial V_L^i(Q_L^i)}{\partial Q^i} = \rho \left(n^i \frac{\partial \pi^i}{\partial Q^i}(Z_0^i + Q_L^i) - \frac{r}{\rho} \right) = 0 \quad i = A, B$$

So Q_L^i is :

$$n^i \frac{\partial \pi^i}{\partial Q^i}(Z_0^i + Q_L^i) = \frac{r}{\rho} \quad (\text{H1})$$

Then we must have :

$$0 < Q_L^i \quad \Leftrightarrow \quad n^i \frac{\partial \pi^i}{\partial Q^i}(Z_0^i) > \frac{r}{\rho} \quad (\text{H1.a})$$

$$\bar{Q}^i > Q_L^i \quad \Leftrightarrow \quad n^i \frac{\partial \pi^i}{\partial Q^i}(Z_0^i + \bar{Q}^i) < \frac{r}{\rho} \quad (\text{H1.b})$$

We obtain these relations because $c^{i''}(Z^i) > 0$ and $\pi^{i''}(Z^i) < 0$.

This hypothesis have two implications. First, for a given initial stock of infrastructure Z_0^i , the net profits of a marginal investment in the stock are positive. The condition (H1.a) make a link between the size of the jurisdiction and the profitability of the public investment, given the initial stock.

Second, the condition (H1.b) says that a jurisdiction have no incentives to invest all its financial resources in the infrastructure stock because the marginal benefits of infrastructure are decreasing with the level of investment.

An optimal investment exists and satisfy the budget constraint. This optimal investment is found where the marginal social benefits are equals to the discounted marginal costs. The hypothesis 1 is an existence condition for an investment that satisfy the samuelsonian condition for public factors (Kaizuka, 1965 ; Sandmo, 1972).

HYPOTHESIS 2 : If the large firm locates in the jurisdiction i , the overall economic surplus $T^i(Z_1^i)$ is an increasing function of the stock of infrastructure :

$$n^i \frac{\partial \tilde{\pi}}{\partial Q^i}(Z_0^i + Q^i) + \frac{\partial \Pi^i}{\partial Q^i}(Z_0^i + Q^i) > 0 \quad \forall Q^i \in [0, \bar{Q}^i] \quad i = A, B$$

The investment in infrastructure expand the induced effects produced by the location of the large firm. One can think about infrastructure that enhances transports, communication networks or that reduces negative externalities (noise, pollution...).

HYPOTHESIS 3 : The function $V_W^i(Q^i, Q^j)$ is strictly concave with respect to Q^i and admits an interior maximum $0 < Q_W^i < \bar{Q}^i$.

The program (W^i) admits an interior solution Q_W^i , such that $0 < Q_W^i < \bar{Q}^i$. The optimal investment for the jurisdiction $i = A, B$ is given by the first order condition :

$$\frac{\partial V_W^i(Q_W^i, Q^j)}{\partial Q^i} = \rho \left(n^i \frac{\partial \pi^i}{\partial Q^i}(Z_0^i + Q_W^i) + \frac{\partial T^i}{\partial Q^i}(Z_0^i + Q_W^i) - \frac{r}{\rho} \right) = 0$$

We obtain Q_W^i as the solution of :

$$n^i \frac{\partial \pi^i}{\partial Q^i}(Z_0^i + Q_W^i) + \frac{\partial T^i}{\partial Q^i}(Z_0^i + Q_W^i) = \frac{r}{\rho} \quad (\text{H3})$$

To check the condition about the admissible values of Q^i determined by B(i), it is sufficient to have :

$$0 < Q_L^i \quad \Leftrightarrow \quad n^i \frac{\partial \pi^i}{\partial Q^i}(Z_0^i) + \frac{\partial T^i}{\partial Q^i}(Z_0^i) > \frac{r}{\rho} \quad (\text{H3.a})$$

$$\bar{Q}^i > Q_L^i \quad \Leftrightarrow \quad n^i \frac{\partial \pi^i}{\partial Q^i}(Z_0^i + \bar{Q}^i) + \frac{\partial T^i}{\partial Q^i}(Z_0^i + \bar{Q}^i) < \frac{r}{\rho} \quad (\text{H3.b})$$

To be valid these conditions rely on the assumption that $T''(Z^i) < 0$, which is satisfied because $\tilde{\pi}''(Z^i) < 0$ and $\Pi^{i''}(Z^i) < 0$. We have assumed in the Hypothesis 2 that $\partial T^i / \partial Q^i \geq 0$. Then it is necessary that the marginal profitability of the infrastructure do not increase too much if this condition have to be satisfied.

The condition (H3.b) will be difficult to be satisfied if the attracted firm is a large unit whose produce significative economic induced effects. To be able to take into account this kind of situation we have to define an alternative to the hypothesis 3.

HYPOTHESIS 4 : The program (W^i) admits a corner solution, such that $Q_W^i = \bar{Q}^i$. We take the hypothesis 3 and we change the condition (H3.b) :

$$n^i \frac{\partial \pi^i}{\partial Q^i}(Z_0^i + \bar{Q}^i) + \frac{\partial T^i}{\partial Q^i}(Z_0^i + \bar{Q}^i) > \frac{r}{\rho} \quad (\text{H4.b})$$

The attraction of the large firm is highly profitable for the jurisdiction because the expected social benefits go beyond the costs of investment. In this situation, a jurisdiction invests all its financial resources to attract the firm. It is a stronger assumption than the assumption made in [H3], but we think that it covers some situations experienced by small jurisdictions that compete for a big firm⁹.

⁹ The location of a big firm can generate such induced economic effects that the financial offers to this firm does not satisfy the budget constraint of the jurisdiction. In these cases, the public authorities of higher level often give their financial support. This case of interest is not modeled.

HYPOTHESIS 5 : The program (L^i) admits a corner solution, such that $Q_L^i = 0$. This hypothesis is based on [H1], when we suppose that the jurisdiction made specific investment for the large firm. In this case the function V_L^i does not depend on Q^i . When the large firm does not locate in the jurisdiction the optimal investment is $Q_L^i = 0$. This case will be use as a benchmark in the example.

All these hypothesis define the following properties :

[P1] : Q_L^i interior maximum of the program (L^i) : $0 < Q_L^i < \bar{Q}^i$.

[P2] : Positive effect of infrastructure :
 $\partial T^i(Z_0^i + Q^i)/\partial Q^i > 0$, $Q^i \in [0, \bar{Q}^i]$.

[P3] : Q_W^i interior maximum of the program (W^i) : $0 < Q_W^i < \bar{Q}^i$.

[P4] : Maximal admissible investment for the program (W^i) : $Q_W^i = \bar{Q}^i$.

[P5] : Minimal admissible investment for the program (L^i) : $Q_L^i = 0$.

Before the construction of the best reply functions, we can compare the optimal investment Q_W^i and Q_L^i .

LEMMA 1 : Under [P1] and [P2] (or [P5] and [P2]) we have :

$$Q_W^i > Q_L^i \quad i = A, B$$

PROOF : Let us define the function :

$$\begin{aligned} \Delta^i(Q^i) &= n^i \frac{\partial \pi^i}{\partial Q^i}(Z_0^i + Q^i) - \frac{r}{\rho} \\ \tilde{\Delta}^i(Q^i) &= n^i \frac{\partial \pi^i}{\partial Q^i}(Z_0^i + Q^i) + \frac{\partial T^i}{\partial Q^i}(Z_0^i + Q^i) - \frac{r}{\rho} \\ &= \Delta^i(Q^i) + \frac{\partial T^i}{\partial Q^i}(Z_0^i + Q^i) \end{aligned}$$

The property [P1] implies that :

$$\Delta^i(Q_L^i) = 0$$

With this equation and [P2] we have :

$$0 < \tilde{\Delta}^i(Q_L^i) = \frac{\partial T^i}{\partial Q^i}(Z_0^i + Q_L^i)$$

As the profit functions are concave, the function T^i is concave too, so :

$$\frac{\partial \tilde{\Delta}^i}{\partial Q^i} < 0 \quad \Rightarrow \quad Q_W^i > Q_L^i \quad i = A, B$$

□

Given the value of the optimal investment in each situation we can define the best reply function for each jurisdiction by the following proposition :

PROPOSITION 1 : *Given the lemma 1 and under [P1], [P2] and [P3] (or [P5], [P2] and [P3]) the optimal investment for a jurisdiction, given the investment of the other, can take two value depending on the outcome of the competition for the attraction of the large firm.*

$$i \neq j, \quad Q^{i*} = \begin{cases} Q_W^i, & \text{if } 0 < Q^j \leq \tilde{Q}^j \\ Q_L^i, & \text{if } \tilde{Q}^j < Q^j < \bar{Q}^j \end{cases}$$

with $\tilde{Q}^j = T^{j-1} \left[T^i (Z_0^i + Q_W^i) - \frac{1}{\rho} H^i \right] - Z_0^j$ $i \neq j$, and $H^i = V_L^i(Q_L^i) - V_L^i(Q_W^i) > 0$.

PROOF : To prove this proposition, we start with the jurisdiction A . Results for the jurisdiction B are alike.

To construct the retracted best reply function for the jurisdiction A , $Q^{A*} = R^A(Q^B)$ we consider two situations depending on the values taken by Q^B . These situations are determined by a double condition. If we take the best issue.

- i) the firm wants to locate in the jurisdiction i . The net surplus for the local firms after the attraction of the firm is positive as in (6).
- ii) the jurisdiction i wants to attract the firm. The welfare in the jurisdiction where the large firm locates have to be greater than the welfare in this jurisdiction if the large firm does not locate so as to incite the jurisdiction to attract the firm.

Let us focus on the condition ii). The condition ii) is fulfilled, the jurisdiction will invest Q_W^i . Else, the jurisdiction will invest Q_L^i . The jurisdiction A wants to attract the firm if and only if :

$$\phi^A(\cdot, Q^B) = V_W^A(Q_W^A, Q^B) - V_L^A(Q_L^A) \geq 0$$

With $\cdot = (Q_L^A, Q_W^A)$ fixed and only Q^B is variable. This condition is equivalent to :

$$\phi^A(\cdot, Q^B) = V_W^A(Q_W^A, Q^B) - V_L^A(Q_W^A) - H^A \geq 0$$

With $H^A = V_L^A(Q_L^A) - V_L^A(Q_W^A)$, so :

$$\phi^A(\cdot, Q^B) = \rho [T^A (Z_0^A + Q_W^A) - T^B (Z_0^B + Q^B)] - H^A \geq 0$$

The condition $\phi^i \geq 0$ implies that the condition $i)$ is fulfilled. As $H^i > 0$, the condition $\phi^i \geq 0$ implies that $T^i(Z_0^i + Q_W^i) - T^j(Z_0^j + Q^j) \geq 0$

The threshold \tilde{Q}^B is the level of investment of the competing jurisdiction that let the jurisdiction A indifferent between investing Q_W^A to attract the large firm or not attracting the firm. The level \tilde{Q}^B is the solution of the following equation :

$$\phi^A(\cdot, Q^B) = \rho[T^A(Z_0^A + Q_W^A) - T^B(Z_0^B + Q^B)] - H^A = 0$$

Then,

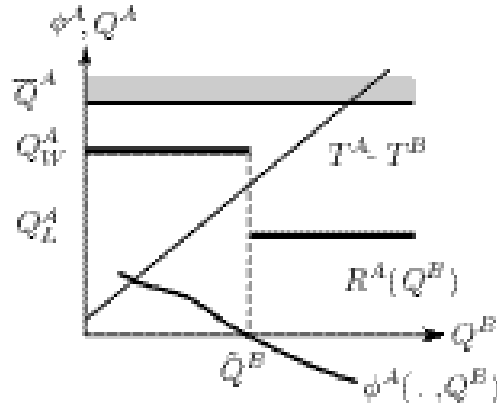
$$\tilde{Q}^B = T^{B-1} \left[T^A(Z_0^A + Q_W^A) - \frac{1}{\rho} H^A \right] - Z_0^B$$

We know that the function $\phi^A(\cdot, Q^B)$ is a decreasing function of Q^B . The two areas over which the best reply function is constructed are determined by the position of Q^B with respect to \tilde{Q}^B :

- If $Q^B \leq \tilde{Q}^B$ then $\phi^A(Q_W^A, Q^B) \geq 0$ and the best reply for the jurisdiction A is given by the solution of the program (W^A) : Q_W^A .
- If $Q^B > \tilde{Q}^B$ then $\phi^A(Q_L^A, Q^B) < 0$ and the best reply for the jurisdiction A is given by the solution of the program (L^A) : Q_L^A .

The restricted best reply function is a piecewise constant function with two values because the objective functions V_W^A and V_L^A are additively separable with respect to the investment of the jurisdiction B ¹⁰. The following figure gives an illustration of the construction of a reaction function¹¹.

GRAPHIQUE 2 : BEST REPLY FUNCTIONS



□

¹⁰ The profit function only varies with the stock of infrastructure located in their jurisdiction. We suppose that there are no spillovers between jurisdictions or that the infrastructure do not affect the competition between firms on the market for products. The later case where location are endogenously determined have been studied by Markusen, Morey et Olewiler (1994) and more specifically by Richter (1994), in the presence of public factors.

¹¹ To simplify we consider a linear attraction constraint.

The proposition says that the investment of a jurisdiction depends on the outcome of the game, ie. the locational choice of the large firm. But this outcome does not depend only on the preference of the firm between localization. The jurisdiction has to be incited to attract the firm and to invest. So there exists cases where a firm could locate in a jurisdiction if it invests Q_W , but the jurisdiction does not want to attract the firm and invests Q_L . Depending on the value taken by Q^B , we can have $V_W^A(Q_W^A, Q^B) < V_L^A(Q_L^A)$. In this situation, the attraction of the large firm is too costly for the jurisdiction A . The jurisdiction A will not want to invest to attract the firm¹². The consequence of the proposition, is that the jurisdiction wants to invest to attract the large firm if the local surplus created by the coming of the large firm is greater than the potential loss of welfare induced by the financing of a higher level of infrastructure capital. This situation is defined by the condition $\phi^A(., Q^B) \geq 0$.

We can also remark that if the condition $\phi^A(., Q^B) \geq 0$, we have necessary that $T^A(Z_0^A + Q^A) - T^B(Z_0^B + Q^B) > 0$. If a jurisdiction wants to attract the large firm, then the large firm wants to locate in the jurisdiction. So, the problem of the localization of the large firm is embodied in the problem of setting the right level of investment in the infrastructure capital by the competing jurisdictions.

COROLLARY 1.1 : *Under [P1], [P2] and [P4] (or [P5], [P2] and [P4]) the optimal investment for a jurisdiction, given the investment of the other, is equal to :*

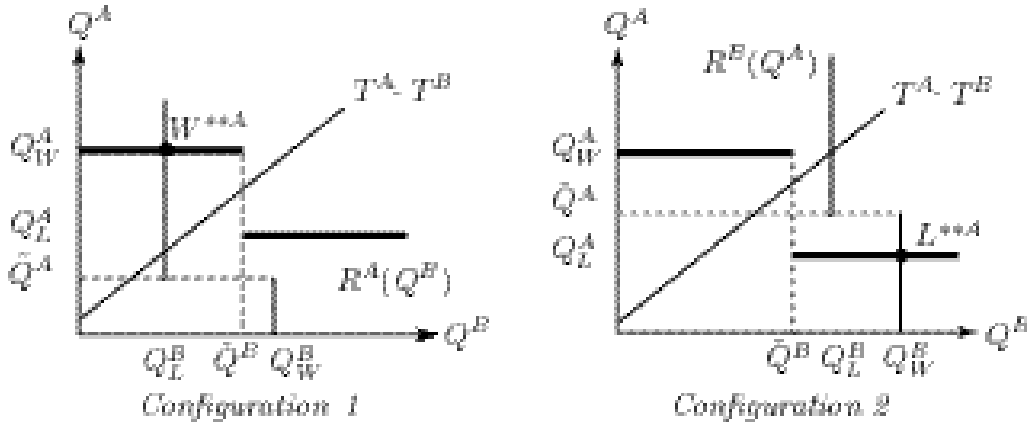
$$i \neq j, \quad Q^{i*} = \begin{cases} \bar{Q}^i, & \text{if } 0 < Q^j \leq \tilde{Q}^j \\ Q_L^i, & \text{if } \tilde{Q}^j < Q^j < \bar{Q}^j \end{cases}$$

2.2. Equilibrium configurations and selection

To describe the outcome of the competition we have to know if a jurisdiction attracts the firm when it plays its optimal strategy, given the optimal strategy of the competing jurisdiction. As there are two possible investment levels for the two jurisdictions (Q_W^i, Q_L^i) , $i = \{A, B\}$, four configurations can be considered. We will say that a configuration is “normal” (configuration 1 and 2) if the outcome of the competition leads to an equilibrium where one jurisdiction invests Q_W^i and the other invests Q_L^j , $\forall i \neq j$.

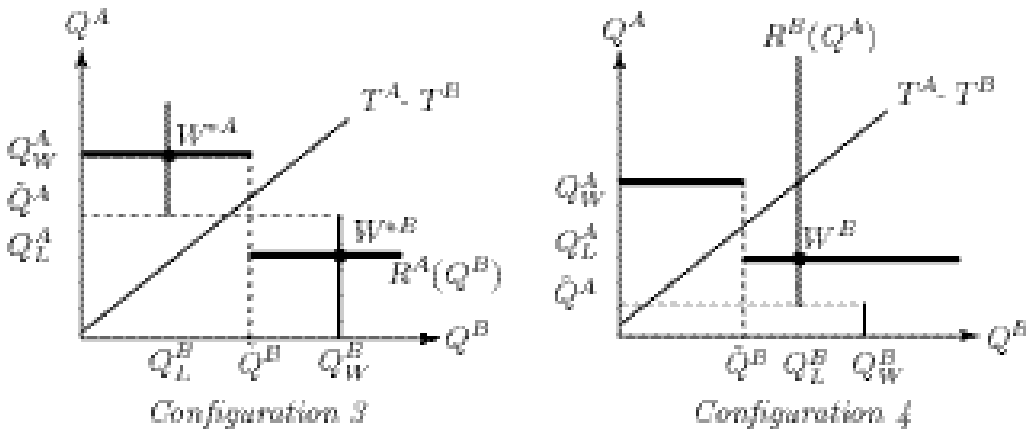
¹² This phenomenon, where a jurisdiction renounces to invest for the attraction of the large firm is not taken into account in the model of King, Welling and McAfee (1993), because the jurisdictions do not have positive and endogenous reservation utility in the competition.

GRAPHIQUE 3 : "NORMAL" EQUILIBRIA



We can exhibit two special configurations : one where the two jurisdictions may invest to attract the firm (configuration 3), and one where no jurisdiction wants to invest to attract the firm (configuration 4).

GRAPHIQUE 4 : "SPECIAL" EQUILIBRIA



A careful study of these configurations leads to the following propositions.

PROPOSITION 2 : *The outcome of the game where the two jurisdictions invest (Q_L^i, Q_L^j) does not exist.*

PROOF : Let us prove that the jurisdiction can play (Q_L^i, Q_L^j) . To be correct, this statement implies that :

$$T^i(Z_0^i + Q_W^i) - T^j(Z_0^j + Q_L^j) - \frac{H^i}{\rho} < 0$$

$$T^j(Z_0^j + Q_W^j) - T^i(Z_0^i + Q_L^i) - \frac{H^j}{\rho} < 0$$

We know that :

$$V_W^i(Q^i, Q^j) = V_L^i(Q^i) + \rho[T^i(Z_0^i + Q^i) - T^j(Z_0^j + Q^j)]$$

Let us assume that, for (Q_L^i, Q_L^j) and for the jurisdiction i ¹³ we have :

$$T^i(Z_{1L}^i) - T^j(Z_{1L}^j) \geq 0$$

This leads to :

$$V_W^i(Q_L^i, Q_L^j) - V_L^i(Q_L^i) = T^i(Z_{1L}^i) - T^j(Z_{1L}^j) \geq 0$$

Under [P3] or [P4], Q_W^i is the investment where V_W^i is maximal, we get :

$$V_W^i(Q_W^i, Q_L^j) - V_L^i(Q_L^i) > V_W^i(Q_L^i, Q_L^j) - V_L^i(Q_L^i) \geq 0$$

The jurisdiction i will play Q_W^i because $\phi^i(Q_W^i, Q_L^j) \geq 0$. Contradiction. \square

We have constructed the best response functions so as to embody the location decision of the large firm. When a jurisdiction want to attract a firm, the firm want to locate in the jurisdiction. To be consistent, this construction, based on the decision of the jurisdictions, implies that there is always a jurisdiction that want to attract the large firm. This proposition shows that it is the case because the large firm will always locate in a jurisdiction¹⁴.

PROPOSITION 3 : *Given the proposition 2, there exists always a jurisdiction i for which the winning outcome $W^i = (Q_W^i, Q_L^j)$ is a Nash equilibrium of the attraction game.*

i) There is one equilibrium where jurisdiction i attracts the large firm if :

$$\phi^i(Q_W^i, Q_L^j) = T^i(Z_0^i + Q_W^i) - T^j(Z_0^j + Q_L^j) - \frac{H^i}{\rho} \geq 0 \quad (\text{C3.a})$$

$$\phi^j(Q_W^j, Q_L^i) = T^j(Z_0^j + Q_W^j) - T^i(Z_0^i + Q_L^i) - \frac{H^j}{\rho} < 0 \quad (\text{C3.b})$$

ii) There are two equilibria W^i and W^j :

$$\phi^i(Q_W^i, Q_L^j) = T^i(Z_0^i + Q_W^i) - T^j(Z_0^j + Q_L^j) - \frac{H^i}{\rho} \geq 0 \quad (\text{C3.c})$$

$$\phi^j(Q_W^j, Q_L^i) = T^j(Z_0^j + Q_W^j) - T^i(Z_0^i + Q_L^i) - \frac{H^j}{\rho} \geq 0 \quad (\text{C3.d})$$

¹³ Same argument apply for j .

¹⁴ In our model, we implicitly assume that the reservation profit of the large firm is zero.

PROOF : Let us assume that¹⁵ :

$$T^i(Z_{1L}^i) - T^j(Z_{1L}^j) \geq 0$$

Proposition 2 implies that the jurisdiction i facing Z_{1L}^j invests Q_W^i because $\phi^i(Q_W^i, Q_L^j) \geq 0$. This condition is equivalent to $Q_L^j < \tilde{Q}^j$.

Two equilibrium configurations can occur depending on the position of Q_L^i with respect to \tilde{Q}^i .

- If $Q_L^i > \tilde{Q}^i$ then $\phi^j(Q_W^j, Q_L^i) < 0$. Facing Q_L^j , the best reply for the jurisdiction j is Q_L^j . A fortiori, the jurisdiction j will play Q_L^j facing Q_W^i . In this configuration, (Q_W^i, Q_L^j) is an equilibrium if (Q_W^j, Q_L^i) is not an equilibrium.
- If $Q_L^i \geq \tilde{Q}^i$ then $\phi^j(Q_W^j, Q_L^i) \geq 0$. Then it is possible that the jurisdiction plays Q_W^j . There are two equilibria. □

In the first case, this proposition describes the conditions under which only one winning Nash equilibrium exists for a jurisdiction (configuration 1 and 2). A jurisdiction attract the large firm if two conditions are satisfied. The large firm wants to locate in the jurisdiction and the jurisdiction wants to invest Q_W^i in the infrastructure capital to welcome the firm. Depending on the value of the investments, the second part of the proposition exhibits conditions under which two winning equilibria may exist (configuration 3).

The location of the large firm is difficult to anticipate because of the relative position of the attraction constraints with the values of the stocks of the jurisdictions. These factors depends directly on the relative size of the jurisdiction, thus on the fiscal resources of each jurisdiction, and on the profitability of each sites for the firm, measured by the gross profit $F^j(Z_1^j)$. The outcome of the competition can be modified indirectly by the magnitude of the induced effects created by the location of the firm in each jurisdiction. These factors are resumed by the magnitude of the function $B^j(Z_1^j)$ which is the maximal value of the bid that a jurisdiction can made to induce the large firm to locate. The outcome of the competition is pre-determined by the setting of the level of the infrastructure capital which depend also on the willing to invest of each jurisdiction. To clarify the magnitude and the significance of each factor we characterize the equilibrium of this game in two examples.

¹⁵ The reverse inequality holds for j .

3. CHARACTERIZATION OF THE EQUILIBRIUM

To keep the analysis as simple as possible we consider that the jurisdiction do not have any infrastructure installed at the initial stage, $Z_0^i = 0$. The investment is also the stock of infrastructure capital : $Z^i \equiv Z_1^i = Q^i$.

If the large firm is not attracted, the firms initially located in the jurisdiction i have the following cost function :

$$c^i(Z^i) = \bar{c} - 2g^i\sqrt{Z^i}$$

Their profit function is :

$$\pi^i(Z^i) = 1 - c^i(Z^i) = 1 - \bar{c} + 2g^i\sqrt{Z^i}$$

The parameter g^i is a measure of the adaptation of the infrastructure capital to the activity of the firm. We assume that the firms initially installed in a jurisdiction share the same parameter. Differences in the profit between jurisdictions come from differences in the level and the adaptation of the infrastructure capital.

The profit of the residents when the large firm locate in their jurisdiction is given by :

$$\tilde{\pi}^i(Z^i) = 1 - \tilde{c}^i(Z^i) = 1 - \bar{c} + \tilde{c}^i + 2g^i\sqrt{Z^i}$$

with the cost :

$$\tilde{c}^i(Z^i) = (\bar{c} - \tilde{c}^i) - 2g^i\sqrt{Z^i}$$

The localization of the large firm generates economies of agglomeration induced by a physical or a technological proximity with the firms initially located in the jurisdiction. These agglomeration effects, like marshallian type of externality, reduce the unit cost of production of the firms located in the selected jurisdiction by \tilde{c}^i .

The profit of the large firm is given by :

$$\Pi^i(Z^i) = 1 - \bar{C}^i + 2G^i\sqrt{Z^i}$$

A location specific parameter \bar{C}^i change the cost of the large firm because of different dotation in resources or amenities. The adaptation parameter of the infrastructure capital for the large firm is given by G^i . The value of this parameter is determined by specific factors in each jurisdiction that allow a better use of the stock of infrastructure.

The surplus remaining in a jurisdiction after the localization of the large firm is :

$$T^i(Z^i) - T^j(Z^j) = 2G^i\sqrt{Z^i} - 2G^j\sqrt{Z^j} + (n^i\tilde{c}^i - n^j\tilde{c}^j) + (\bar{C}^j - \bar{C}^i)$$

Two types of cost differentials modify the magnitude of this surplus. An internal differential $\Delta I = (n^i\tilde{c}^i - n^j\tilde{c}^j)$ that measures the ability of the firms initially located in a jurisdiction to take advantage of the attraction of the large firm. An external differential $\Delta X = (\bar{C}^j - \bar{C}^i)$ that measures the economies of localization that the large firm earns when it locates in the jurisdiction i rather than j . The infrastructure capital and its adaptation for the attracted firm in each jurisdiction modifies also the surplus. This surplus can be rewritten :

$$T^i(Z^i) - T^j(Z^j) = 2G^i\sqrt{Z^i} - 2G^j\sqrt{Z^j} + \Delta I + \Delta X \quad (8)$$

3.1. The specific investment case

We begin with a simple example to study the method of characterization of the equilibrium. Let us assume that the jurisdictions invest in a specific infrastructure that is dedicated to the large firm. In this case the infrastructure capital is not public and we focus on the competition and on the role of the impact of the localization of the large firm on firms initially located in the jurisdiction.

There is no marginal benefit of the infrastructure capital for local firms, $g^i = 0, \forall i$. The profit function of the local firms are :

$$\begin{aligned} \pi^i &= 1 - \bar{c} \\ \tilde{\pi}^i &= 1 - (\bar{c} - \tilde{c}^i) \end{aligned}$$

To simplify, we assume that the unit cost for the large firm does not depend of its location : $\bar{C} = \bar{C}^i = \bar{C}^j$. The total economic surplus in the jurisdiction i is :

$$T^i(Z^i) = \Pi^i(Z^i) + n^i(\tilde{\pi}^i - \pi^i) = (1 - \bar{C}) + 2G^i\sqrt{Z^i} + n^i\tilde{c}^i$$

Using (8), we get the following surplus after the attraction :

$$T^i(Z^i) - T^j(Z^j) = 2(G^i\sqrt{Z^i} - G^j\sqrt{Z^j}) + n^i\tilde{c}^i - n^j\tilde{c}^j$$

3.1.1. Investment levels

Each jurisdiction, sets its investment for the two issues. In the case where the jurisdiction is not attractive, the investment is the solution of following light optimization program :

$$(L^i) \begin{cases} \text{Max}_{Z^i} & V_L^i(Z^i) = (1 + \rho)n^i\pi^i - rZ^i \\ \text{st} & 0 \leq Z^i \leq \bar{Z}^i \end{cases} \quad \text{d}(i)$$

In the specific investment case, there is no gain to invest in the infrastructure capital when the large firm does not come. So hypothesis [H5] apply and we get :

$$Z_L^i = 0$$

The investment will be positive, only in the issue where the large firm locate in the jurisdiction. In the case where the jurisdiction is attractive the investment is the solution of the following light optimization program :

$$(W^i) \begin{cases} \text{Max}_{Z^i} & V_W^i(Z^i, Z^j) = (1 + \rho)n^i\pi^i - rZ^i + \rho[T^i(Z^i) - T^j(Z^j)] \\ \text{st} & 0 \leq Z^i \leq \bar{Z}^i \end{cases} \quad \text{d}(i)$$

From the first order condition we get :

$$Z_W^i = \left(\frac{\rho}{r}G^i\right)^2 \quad (9)$$

This investment does not depend on the initial population of a jurisdiction or its fiscal resources. The level of the investment depends on the profitability of the infrastructure for the large firm and its cost for the jurisdiction. In this case we have to check the domain condition to be sure that the jurisdiction have enough fiscal resources (population) to finance its investment. Using (9) and (7) we deduce that a jurisdiction will invest Z_W^i if and only if :

$$n^i > \frac{(\rho G^i)^2}{r\pi^i} = \bar{n}^i$$

Otherwise, we apply the hypothesis [H4] and following (7) the investment is :

$$Z_W^i = \bar{Z} = n^i \frac{\pi^i}{r}$$

3.1.2. Equilibrium

We have check the conditions of attraction ϕ^i for different values of the parameters and characterize the different equilibrium configurations.

PROPOSITION 4 : *With specific investment, if :*

$$n^i > \frac{1}{r\pi^i}(\rho G^i)^2 = \tilde{n}^i$$

there is always one winning equilibrium :

i) *There is one winning Nash equilibrium for the jurisdiction i if :*

$$n^i > \frac{\tilde{c}^j}{\tilde{c}^i} n^j + \frac{\rho}{r} G^j{}^2$$

ii) *There are two Nash equilibria if :*

$$\frac{\tilde{c}^j}{\tilde{c}^i} n^j + \frac{\rho}{r} G^j{}^2 > n^i > \frac{\tilde{c}^j}{\tilde{c}^i} n^j - \frac{\rho}{r} G^i{}^2$$

PROOF : We know that :

$$\begin{aligned} V_L^i(Z_L^i) &= (1 + \rho)n^i\pi^i \\ V_L^i(Z_W^i) &= (1 + \rho)n^i\pi^i - \frac{1}{r}(\rho G^i)^2 \end{aligned}$$

It follows :

$$H^i = V_L^i(Z_L^i) - V_L^i(Z_W^i) = \frac{1}{r}(\rho G^i)^2$$

To obtain a W^i equilibrium, and using conditions (C3.a) and (C3.b) we obtain :

$$\begin{aligned} \phi^i(Z_W^i, Z_L^j) &= \rho[T^i(Z_W^i) - T^j(Z_L^j)] - H^i \geq 0 \\ 2\frac{\rho}{r}G^i{}^2 + n^i\tilde{c}^i - n^j\tilde{c}^j - \frac{\rho}{r}G^i{}^2 &\geq 0 \end{aligned}$$

And :

$$\begin{aligned} \phi^j(Z_W^j, Z_L^i) &= \rho[T^j(Z_W^j) - T^i(Z_L^i)] - H^j < 0 \\ 2\frac{\rho}{r}G^j{}^2 + n^j\tilde{c}^j - n^i\tilde{c}^i - \frac{\rho}{r}G^j{}^2 &< 0 \end{aligned}$$

We get the following system of inequalities :

$$n^i\tilde{c}^i - n^j\tilde{c}^j + \frac{\rho}{r}G^i{}^2 \geq 0 \tag{a}$$

$$n^i\tilde{c}^i - n^j\tilde{c}^j - \frac{\rho}{r}G^j{}^2 > 0 \tag{b}$$

Only the condition (b) must hold. After some manipulation we get the condition *i*) of the proposition.

Following proposition 3 and the conditions (C3.c) and (C3.d), two equilibria exist if :

$$\begin{aligned}\phi^i(Z_W^i, Z_L^j) &= \rho[T^i(Z_W^i) - T^j(Z_L^j)] - H^i \geq 0 \\ 2\frac{\rho}{r}G^{i2} + n^i\tilde{c}^i - n^j\tilde{c}^j - \frac{\rho}{r}G^{i2} &\geq 0\end{aligned}$$

And :

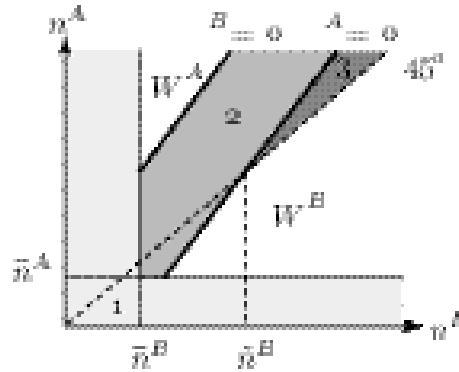
$$\begin{aligned}\phi^j(Z_W^j, Z_L^i) &= \rho[T^j(Z_W^j) - T^i(Z_L^i)] - H^j \geq 0 \\ 2\frac{\rho}{r}G^{j2} + n^j\tilde{c}^j - n^i\tilde{c}^i - \frac{\rho}{r}G^{j2} &\geq 0\end{aligned}$$

Some calculations lead to the conditions *ii*) of the proposition.

The case where there is no equilibrium cannot occur because the conditions $\phi^i(Z_W^i, Z_L^j) < 0$ and $\phi^j(Z_W^j, Z_L^i) < 0$ are not compatible. \square

As population plays an important role in the model by determining the fiscal resources that can be devoted to the investment in the initial stage but also the magnitude of the induced welfare effect created by the location of the firm that is used to finance the attraction policy of a jurisdiction, it is interesting to know how the size of the populations may affect the issue of the competition. The following figure illustrates the proposition in this perspective.

GRAPHIQUE 5 : SPECIFIC INVESTMENT EQUILIBRIA



The areas W^A and W^B that lies over $\phi^B = 0$ and under $\phi^A = 0$ depicts all the situations where, for a given set of parameters, the size of the jurisdictions allows one to attract the large firm. The area 1 is the area where the populations do not allow the jurisdictions to finance their investments. In the area 2, the size of the jurisdictions lead to multiple equilibria (condition *ii*) of the proposition 4. The area 3 is the set of unique winning equilibria for the jurisdiction B such that the size of its population is lower than i .

The size of the areas W^i depends on the levels of the parameters G^i and \tilde{c}^i . As the productivity of the infrastructure capital of the competing jurisdiction G^B increases, it shifts upward the attraction constraint ϕ^B and shrink the area W^A . In this case, the jurisdiction A wins only if its size increases, given the size of the competing jurisdiction. More productive infrastructure capital is a way to affect competition. A jurisdiction can increase the productivity by adapting the infrastructure to the activity of the attracted firm. In this context, the specificity of the infrastructure capital is an attraction force. As it is possible to dedicate some infrastructure to a large firm, it is more difficult to adapt an exiting stock and to exploit strategically this flexibility¹⁶. This part of the proposition focus on the impact of the targeting of the investment in infrastructure on the issue of competition.

As the size of \tilde{c}^i increases, the slope of the attraction constraint ϕ^i increases and the area W^i shrinks. When the induced welfare effect created by the attraction of the firm increases, the competitiveness of the jurisdiction j increases, because it allows the jurisdiction j to collect more financial resources in the second stage and have stronger attraction policy. The relative size of these induced effects, can be an opportunity for a jurisdiction to compete successfully with a larger jurisdiction. This idea is formalized in the following corollary.

COROLLARY 4.1 : *If $\tilde{c}^j > \tilde{c}^i$ there exists a threshold :*

$$\tilde{n}^i = \tilde{n}^j = \frac{\rho}{r} \frac{G^i{}^2}{\tilde{c}^j - \tilde{c}^i}$$

over which the smaller jurisdiction may attract the large firm.

PROOF : The result follow directly from the condition (a) of the proposition 4, when $n^i = n^j$.

This corollary put the emphasis on the internal composition of the industry in the jurisdiction. As induced effects are more likely to be supported by Marshallian type of external economies, specialized territories can expect to develop more larger induced effects, if the internal specialization is closely related to the activity of the attracted firm. The corollary says that this kind of jurisdiction may attract a large firm despite a smaller industry size. The corollary exhibit also that a minimum agglomeration of firms is needed at the local level for this case to occur. Successful attraction policy based on investments in specific infrastructure needs a minimal

¹⁶ One can think about divisible premises...

size in the population. This minimal size decrease as the magnitude of the induced effect in the smaller jurisdiction increases with respect to the other jurisdiction.

3.2. The public investment case

In this section we assume that the infrastructure capital have positive effects both on the large firm and on the firms already located in the jurisdiction ($g^i \neq 0$). In this case, even if the large firm is not attracted, there exist incentives to invest.

3.2.1. Investment levels

We calculate the investment levels solution of the programs (L) and (W) so as to built the reaction functions. We begin by the case where the jurisdiction is not attractive.

LEMMA 2 : *When :*

$$n^i < \frac{r(1 - \bar{c})}{(\rho g^i)^2} \quad , \quad (8)$$

the investment of the jurisdiction i when it is not attractive is given by :

$$Z_L^i = \left[\frac{\rho}{r} (n^i g^i) \right]^2 \quad (9)$$

PROOF : The investment of a jurisdiction when it losses is given by the first order condition of the light optimization program (P) :

$$n^i \frac{\partial \pi^i}{\partial Z} - \frac{r}{\rho} = n^i g^i \frac{1}{\sqrt{Z_L}} - \frac{r}{\rho} = 0$$

We get the investment :

$$Z_L = \left[\frac{\rho}{r} n^i g^i \right]^2$$

To get the property [P1] we have to check the conditions (H1.a) et (H1.b). The condition (H1.a) is fulfilled because :

$$\lim_{Z \rightarrow 0^+} n^i g^i \frac{1}{\sqrt{Z^i}} > \frac{r}{\rho}$$

The condition (H1.b) is fulfilled if :

$$n^i g^i \frac{1}{\sqrt{Z^i}} < \frac{r}{\rho}$$

We know that :

$$\bar{Z}^i = \frac{1}{r} n^i \pi^i(0) = \frac{1 - \bar{c}}{r} n^i$$

If we replace \bar{Z}^i by its value in the preceding condition we get the condition (8). The interior maximum that fulfills the condition (H1) is given by the equation (9). \square

The existence of an interior maximum for the program (L) depends on a restriction on the size of the population initially located in each jurisdiction. Let us calculate the investment of a jurisdiction i when it is attractive.

LEMMA 3 : *When the jurisdiction i is attractive,*

if $\underline{n}^i < n^i < \bar{n}^i$ then :

$$Z_W^i = \left[\frac{\rho}{r} (n^i g^i + G^i) \right]^2 \quad (10)$$

if $n^i \leq \underline{n}^i$ or $n^i \geq \bar{n}^i$ then :

$$Z_W^i = \bar{Z}^i = \left(\frac{1 - \bar{c}}{r} \right) n^i \quad (11)$$

with

$$\underline{n}^i = \frac{-K - \sqrt{\delta}}{2g^{i2}} \quad \bar{n}^i = \frac{-K + \sqrt{\delta}}{2g^{i2}}$$

and $K = 2g^i G^i - \frac{\rho^2}{r} (1 - \bar{c})$ and $\delta = \frac{r}{\rho} (1 - \bar{c}) \left[\frac{r}{\rho} - 4g^i G^i \right]$.

PROOF : The investment of a jurisdiction when it wins is given by the first order condition of the light optimization program (W) :

$$n^i \frac{\partial \pi^i}{\partial Z^i} + \frac{\partial T^i}{\partial Z^i} - \frac{r}{\rho} = n^i g^i \frac{1}{\sqrt{Z_W^i}} + G^i \frac{1}{\sqrt{Z_W^i}} - \frac{r}{\rho} = 0$$

We get the investment :

$$Z_W = \left[\frac{\rho}{r} (n^i g^i + G^i) \right]^2$$

The hypothesis (H3.a) is fulfilled because :

$$\lim_{Z \rightarrow 0^+} (n^i g^i + G^i) \frac{1}{\sqrt{Z^i}} > \frac{r}{\rho}$$

We get an interior maximum if the condition (H3.b) is fulfilled or a corner solution if the condition (H4.b) holds. To determine which condition holds we have to study the sign of the expression :

$$n^i \frac{\partial \pi^i}{\partial Z^i}(\bar{Z}^i) + \frac{\partial T^i}{\partial Z^i}(\bar{Z}^i) - \frac{r}{\rho}$$

If this expression is negative, (H3.b) holds, else it is (H4.b). Let us write this expression for our example. After some calculation we get the following polynomial :

$$(g^i n^i)^2 + [2g^i G^i - \frac{\rho}{r}(1 - \bar{c})]n^i + G^{i2}$$

if $\frac{r}{\rho} > 4g^i G^i$ then the discriminant is positive and there are two positive and distinct roots :

$$\underline{n}^i = \frac{-K - \sqrt{\delta}}{2g^{i2}} \quad \bar{n}^i = \frac{-K + \sqrt{\delta}}{2g^{i2}}$$

with $K = 2g^i G^i - \frac{\rho^2}{r}(1 - \bar{c})$ and $\delta = \frac{r}{\rho}(1 - \bar{c})[\frac{r}{\rho} - 4g^i G^i]$.

Between the roots the polynomial is negative, (H3.b) holds and we have an interior maximum given by (H3). Outside the roots the expression is positive, the condition (H4.b) is fulfilled and we get an a corner solution \bar{Q}^i . \square

When the jurisdiction i is attractive two cases can occur : the investment is interior or the jurisdiction invest all this financial resources. The corner solution can occur when the relative size of the attracted firm is important or the size of the jurisdiction and the economic effects induced by the location of the firm are important.

At the contrary of the specific investment case, the infrastructure levels depends on the size of the jurisdiction because the opportunity to invest is given by the tradeoff between the social profitability of the project and its social cost. The social profitability of the project is measured by the marginal benefit produced by one unit of infrastructure, $(n^i g^i + G^i)$.

3.2. Equilibrium Characterization

To simplify the characterization of the equilibrium we assume that the size of the attracted firm is large enough to impose the property [P4]. So the effects of the localization of the large firm are so important that the winning jurisdiction invests all its financial resources in the infrastructure capital $Z_W^i = \bar{Z}^i$. Let us apply the condition of the lemma 7 and fulfill the condition (H4.B). The investments are given by the equations (9) and (11) :

$$Z_L^i = \left(\frac{\rho}{r} n^i g^i\right)^2 \quad i = A, B$$

and

$$Z_W^i = \left(\frac{1 - \bar{c}}{r}\right) n^i \quad i = A, B$$

The localization of the large firm, and the Nash equilibrium, are determined by the balance between many parameters : The location specific amenities, the economic

effects induced by the localization of the large firm on local firms and the impact of the infrastructure capital on the large firm profit. The following proposition characterize the Nash equilibrium of this attraction game with respect to these parameters.

PROPOSITION 5 : *The jurisdiction i attract the large firm if :*

. *The cost differentials are in favor of i :*

$$\Delta = \Delta I + \Delta X \leq 0 \quad ,$$

. *the marginal productivity of the infrastructure capital for the attracted firm is sufficiently high in the jurisdiction :*

$$\frac{n^j}{n^i} < \left(\frac{G^i}{G^j} \right)^2 \quad ,$$

$$\frac{n^j}{n^i} < \frac{1}{2} \left(\frac{g^i}{g^j} \right) \left(\frac{G^i}{G^j} \right) \quad ,$$

. *and the unit cost of investment is sufficiently low.*

PROOF : We take a normal equilibrium configuration and we express the conditions that insures i to win. We have to fulfill the conditions (C3.a) and (C3.b) of the proposition 3.

The first condition implies that the jurisdiction j have no interest to attract the firm, that is equivalent to the condition $Z_L^i > \hat{Z}_i$. This condition always holds if $T^j(Z_W^j) - T^i(Z_W^j) < 0$. We can rewrite the attraction condition in the (Z^j, Z^i) space as an implicit function $Z^j = \varphi^j(Z^i)$ defined by the equation $T^j(Z^j) - T^i(Z^i) = 0$ where the expression is given by (7) :

$$Z^j = \left(\frac{1}{P^j} \right)^2 [P^i \sqrt{Z^i} - \Delta]^2 \quad \text{avec } \Delta = \Delta I + \Delta X$$

The jurisdiction j do not want to attract the firm if $Z_W^j < \varphi^j(Z_W^i)$:

$$Z_W^j < \left(\frac{1}{P^j} \right)^2 [P^i \sqrt{Z_W^i} - \Delta]^2$$

If we develop we have :

$$0 < Z_W^j < \left(\frac{1}{P^j} \right)^2 [P^{i2} Z_W^i + \Delta^2 - 2P^i \sqrt{Z_W^i} \Delta]$$

Let us assume :

$$\Delta \leq 0$$

This assumption implies the following condition on the populations $n^j/n^i \leq \tilde{c}^i/\tilde{c}^j$.
To have $\bar{Z}^j < \varphi(\bar{Z}^i)$ it is sufficient to satisfy :

$$Z_W^j \leq \left(\frac{P^j}{P^i}\right)^2 Z_W^i$$

If we replace the investments by their expressions found in (8) we get the following condition :

$$\frac{n^j}{n^i} < \left(\frac{G^i}{G^j}\right)^2$$

The second condition that have to be fulfilled is that the jurisdiction i wants to attract the firm, $Z_L^j < \tilde{Z}^j$, or $\phi^i(Z_W^i, Z_L^j) > 0$:

$$\rho[P^i \sqrt{Z_W^i} - P^j \sqrt{Z_L^j}] - \Delta - [V_L^i(Z_L^i) - V_L^i(Z_W^i)] > 0$$

This condition can be rewritten :

$$[\rho P^i + 2\rho n^i g^i] \sqrt{Z_W^i} - \rho P^j \sqrt{Z_L^j} - \Delta - r(Z_L^i + Z_W^i) > 0$$

As we know that $Z_W^i > Z_L^i$ and $\Delta \leq 0$, we can constraint this inequality to satisfy :

$$[\rho P^i + 2\rho n^i g^i] \sqrt{Z_W^i} - \rho P^j \sqrt{Z_L^j} - \Delta > 2r Z_W^i$$

After some manipulations :

$$4r^2 Z_W^i{}^2 + [4r\rho P^j \sqrt{Z_L^j} - (\rho P^i + 2\rho n^i g^i)^2] Z_W^i + (\rho P^j)^2 Z_L^j < 0$$

The discriminant of the polynomial is :

$$\delta = 4(\rho n^i g^i)^2 + 8\rho^2 n^i g^i G^i + 4(\rho G^i)^2 - 16\rho^2 n^j g^j G^j > 0$$

A sufficient condition for this discriminant to be positive is :

$$8\rho^2 n^i g^i G^i - 16\rho^2 n^j g^j G^j > 0$$

Or :

$$\frac{n^j}{n^i} < \frac{1}{2} \frac{g^i}{g^j} \frac{G^i}{G^j}$$

The product of the roots is positive, so the roots are of the same sign and are positives. Then we can deduce that the attraction condition for the jurisdiction i is fulfilled if :

$$\frac{-K - \sqrt{\delta}}{8r^2} < \frac{1 - \bar{c}}{r} n^i < \frac{-K + \sqrt{\delta}}{8r^2}$$

with $K = 2\rho[n^i g^i + G^i - 4\rho n^j g^j G^i]$.

We can bound r :

$$\frac{-K - \sqrt{\delta}}{8n^i(1 - \bar{c})} < r < \frac{-K + \sqrt{\delta}}{8n^i(1 - \bar{c})}$$

□

This proposition says that the jurisdiction with the greater size may not attract the large firm if the competing jurisdiction have a well defined infrastructure capital and set of local firms. If the size of a jurisdiction is a key factor that determine the amount of the investment (see corollary 3.1), the setting-up of the local firms determines also the economic surplus that can be devoted to the attraction of the large firm. Thus, the costs differentials and the difference between the adaptation of the infrastructure capital to the activity of the large firm may overcome a relative small size. The cost of investment plays also an important role because it determines the incentives of a jurisdiction to invest to attract the large firm. If the cost of investment is too high then the small jurisdiction will not want to invest as much as it is necessary to overcome the investment of the competing jurisdiction. Then the competition is too high with the larger jurisdiction.

If we fix the investment cost, a prior disadvantage in the size can be bypassed by two ways. The first depends on the localization attributes. The firm, when it locates in a jurisdiction can benefit from amenities that reduce its cost. The location of the large firm creates spillovers in the jurisdiction that increase the financial resources devoted to the attraction policy. The second depends on the adaptation of the stock of infrastructure to the activity of the firm. Let us consider that the two jurisdictions are identical with respect to their amenities ($\Delta = 0$). In this case the greater is the adaptation of the stock of infrastructure to the large firm, the better the small jurisdiction can overcome its disadvantage in size. The direct impact of the stock of infrastructure on the large firm is determinant.

This point shed light on two problems. Small jurisdictions can attract a large firm only if their infrastructure capital is well specialized and if it correspond to the needs of the large firm. Large jurisdictions with a diversified stock of infrastructure are highly attractive because the relative performance of the competitors is minored. This last problem is connected to the work of Holtz-Eakin et Lovely

(1996) that study the impact of the infrastructure capital over economies of scale and the variety of activities.

CONCLUSION

Introducing prior investment in an infrastructure capital in a problem of competition between jurisdictions for the attraction of a large plant complexify the resolution of the game because the investment choice of the jurisdiction are interdependent, and for a given pair of investments, the objective function of a jurisdiction changes depending on the outcome of the game. We exhibit the condition under which we can characterize the Nash equilibrium of the game. The outcome of the game depends on the relative size of the jurisdictions, the magnitude of the economic effect induced by the location of the firm but also the adaptation of the infrastructure to the activity of the large firm. The size of a jurisdiction may predetermine the outcome of the game, but there exist situations where the smallest jurisdiction can attract the large firm if it invest in a well specialized stock of infrastructure. Specialization of the infrastructure capital can be a way for small jurisdictions to overcome their disadvantage with larger jurisdictions. Two kinds of extensions can be considered.

The first extension of the model deals with the mobility of the local firm. In our model we have suppose that the local firms are immobile. We can think that the local firms could react to the choice of the jurisdiction. They can react a priori by selecting a jurisdiction anticipating the attraction policy of the jurisdiction. In this setting the population of each jurisdiction is endogenously determined. The local firm cannot be mobile initially but can choose to leave the jurisdiction in the view of the outcome of the game. This a posteriori mobility can be interesting because the attraction policy can lead the jurisdictions to select an inducement policy that can affect the profit of the local firms and change their localization decision afterward.

The second extension deals with the nature of our model. We can think that the attraction of a single firm with a predetermined adaptation of the infrastructure capital is extreme. If we consider that the jurisdictions face a continuum of firms and try to attract them with inducement policies based on infrastructure capital, the strategic choice of the adaptation of the stock of infrastructure capital to the activity of the attracted firms can be viewed as a differentiation policy that tries to limit the competition on the financial offers. Marketing policies based on thematic developments observed in practice, can found a theoretical representation. For that purpose a model like that of Salop (1979) can be useful to deal with this issues and to gives some answers to the regulation of this market for territories.

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