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Empirical Bayes Forecasts of One Time Series Using Many Predictors

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ABSTRACT

We consider the problem of forecasting a single time series, y_{t+1}, using a linear regression model with k predictor variables, X_t, when each predictor makes a small but nonzero marginal contribution to the forecast. It is well known that OLS is inadmissable when $k \geq 3$. Although Bayes estimators are admissable, the associated forecasts are unappealing because they can have large (frequentist) risk for some parameter values. We therefore consider Empirical Bayes estimators of the regression coefficients and their associated forecasts, when both the prior and regression error distributions are unknown. To focus attention on large k, we adopt a nesting where k is proportional to the sample size (T), and focus on the asymptotic properties of the true Bayes, Empirical Bayes, and OLS forecasts. We consider Bayes estimators that are functions of the OLS estimates, and propose a nonparametric Empirical Bayes estimator that is asymptotically optimal, in the sense that it achieves the Bayes risk of the best infeasible Bayes estimator when the true error distribution is normal. This result suggests that the Empirical Bayes estimator will have desirable frequentist risk as well. Both nonparametric and parametric Empirical Bayes estimators are examined in a Monte Carlo experiment, with results that are encouraging from both a Bayes and frequentist risk perspective. The new estimators are then applied to the problem of forecasting a few monthly postwar aggregate U.S. economic time series using the first 146 principal components from a large panel of predictor variables.

Key Words: Large model forecasting, random coefficients model

JEL Codes: C32, E37

1. Introduction

Modern advances in data availability make it possible to contemplate the real time forecasting of a single time series using very many predictors. The example of immediate interest in this paper arises in macroeconomic forecasting. Suppose one is interested in forecasting the rate of price inflation in the United States using monthly data, using data since 1959. Economic theory and empirical experience suggests that inflation might be predicted by many different variables, including interest rate spreads, monetary aggregates, output gap measures, capacity utilization measures, and unemployment rates; Stock and Watson (1999) examine 189 such potentially useful predictors of inflation, and surely other series could be added to the list. One approach to handling these data is to suppose a factor structure exists, to extract estimates of the latent factors, and to use the first few of these for forecasting. But suppose that a small-dimensional factor structure does not exist and that each variable makes a small independent contribution towards forecasting inflation. How then can one usefully exploit the information in, say, 200 predictors to forecast inflation with approximately 400 time series observations?

This paper considers this problem in the context of the linear regression model,

$$y_{t+1} = \beta' X_t + \epsilon_{t+1},\ t=1,\ldots,T \tag{1.1}$$

where X_t is a vector of k predictor time series and ϵ_{t+1} is an unforecastable i.i.d. error, which is assumed throughout to have a distribution that does not depend on β, has mean zero and variance σ_ϵ^2, and that $\{\epsilon_t\}$ and $\{X_t\}$ are independent. In addition, throughout it is assumed that $X'X/T = I_k$, the $k\times k$ identity matrix; in the empirical application in section 4, the X's are the first k principal components computed from a set of predictor variables and thus are orthonormal by construction.

We are interested in out-of-sample forecasting, specifically forecasting y_{T+2} using X_{T+1} under quadratic loss. Let $\tilde{y}_{T+2} = \tilde{\beta}'X_{T+1}$ be a candidate forecast, where $\tilde{\beta}$ is an estimate of β based on $(X_t, y_{t+1}), t=1,\ldots,T$; then the loss function is,

$$L(\tilde{\beta},\beta) = (y_{T+2}-\tilde{y}_{T+2})^2 = [\epsilon_{T+2} + (\tilde{\beta}-\beta)'X_{T+1}]^2, \tag{1.2}$$

and the associated risk is the expected loss,

$$EL(\tilde{\beta},\beta) = \sigma_\epsilon^2 + \mathrm{tr}[E(\tilde{\beta}-\beta)'(X_{T+1}X'_{T+1})(\tilde{\beta}-\beta)]. \tag{1.3}$$

Because the Xs are orthonormal, we assume that $E[X_T X'_T | \{X_t, y_{t+1}\}, t=1,\ldots,T] = I_k$. Because σ_ϵ^2 does not depend on the estimator, the risk (1.3) is thus equivalent to the risk,

$$R(\tilde{\beta},\beta) = E(\tilde{\beta}-\beta)'(\tilde{\beta}-\beta). \tag{1.4}$$

It follows from Stein (1955) that, for $k \geq 3$, ordinary least squares (OLS) is inadmissable under the mean quadratic risk function (1.4), and indeed for large k/T the risk of OLS can be quite large. James and Stein (1960) constructed an estimator that risk-dominates OLS using shrinkage (although the original James-Stein estimator is not admissable). Because Bayes estimators are admissable, one approach to this problem is to use a Bayes estimator in which a prior distribution over β is posited. However this estimator depends on the prior and the (frequentist) risk (1.4) can be quite poor for values of β that are unlikely under the prior. This difficulty can be overcome by allowing the prior to adapt to the data, that is, by adopting empirical Bayes methods (Robbins [1955, 1964]). Efron and Morris (1972) showed that the James-Stein estimator can be derived as an empirical Bayes estimator. This all suggests that empirical Bayes estimators provide a potentially fruitful approach to minimizing the frequentist risk (1.4).

We therefore consider forecasting y_{T+2} using an empirical Bayes estimator of β. The elements of β, β_i, $i=1,\ldots,k$, are thus modeled as i.i.d. draws from a prior distribution, $g(\beta_i)$ (remarks about the non-i.i.d. case are made in section 2). Were the prior and the distribution of ϵ known, the (admissable) Bayes estimator of β would be the posterior mean. When the error distribution is Gaussian with a known variance, the OLS estimators are sufficient for β so the Bayes estimator is a function only of the OLS estimators. Because of the simplifications this provides, we consider the Bayes estimator that arises under normal errors. This Bayes estimator is infeasible because the error variance and prior are unknown; the empirical Bayes estimator is its feasible counterpart based on estimates of the prior and of σ_ϵ^2.

The formal results examine the asymptotic properties of this estimator when the true error distribution is potentially nonnormal. If k is held fixed as $T\rightarrow\infty$, the risk (1.4) of all mean square consistent estimators converges to zero, and such a nesting does not do justice to the empirical problem with $k=200$ and $T=400$. We therefore adopt a nesting that treats the number of regressors as proportional to the number of observations (an assumption used, in a different context, by Bekker [1994]). It is shown that the Bayes risk of the proposed empirical Bayes estimator converges to the Bayes risk of the infeasible normal Bayes estimator. Thus the empirical Bayes estimator is asymptotically admissable in this leading case.

Although this paper adopts the terminology of empirical Bayes estimation, our results can alternatively be thought of as applying directly to the random coefficients model where β_i are i.i.d., but the coefficient distribution is unknown. This interpretation is elaborated in section 2.

The remainder of the paper is organized as follows. Theoretical results are presented in section 2. A Monte Carlo study of the empirical Bayes estimators in this model is given in section 3. Empirical results, in which these methods are used to forecast several U.S. macroeconomic time series, are given in section 4. Section 5 concludes.

2. Assumptions and Theoretical Results

2.1 Asymptotic nesting

All estimators and results are stated in a transformed version of the parameter space that is consistent with the asymptotic nesting of $k/T \to \rho$ as $T \to \infty$. It will be notationally convenient to ignore integer constraints and to set $k=\rho T$. Because k and T are linked, various objects are doubly indexed arrays, and β and its estimates are sequences indexed by k; to simplify notation however this indexation is usually suppressed. It should be emphasized that all limits in this paper are taken along this sequence unless explicitly stated otherwise.

Under this nesting, if β is O(1), the population R^2 tends to one, which is unrepresentative of the empirical problems of interest. We therefore adopt a nesting in which each predictor is imagined as making a small but generally nonzero contribution to the forecast, specifically, we adopt the local parameterization,

(2.1) $$\beta = b/\sqrt{T},$$

where $b = (b_1,\ldots,b_k)$ and $\{b_i\}$ are i.i.d. draws from the proper prior distribution g. Accordingly, let $\hat{b}$ be the OLS estimator of b,

(2.2) $$\hat{b} = T^{-1/2}\sum_{t=1}^{T} X_t y_{t+1}$$

so that $\hat{b}_i - b_i = T^{-1/2}\sum_{t=1}^{T} X_{it}\epsilon_{t+1}$. Under the maintained assumptions that $\{\epsilon_t\}$ is serially uncorrelated and indepedent of $\{X_t\}$, then $E\hat{b}_i = b_i$, and $E(\hat{b}-b)(\hat{b}-b)' = \sigma_\epsilon^2 I_k$. Let $f_k(\hat{b}|b)$ denote the conditional pdf of $\hat{b}$ given b, which under the stated assumptions has the location form, $f_k(\hat{b}|b) = f_k(\hat{b}-b)$.

Under this nesting, the frequentist risk (1.4) of the estimator $\tilde{\beta}$ is,

$$R(\tilde{\beta},\beta) = \rho k^{-1}\sum_{i=1}^{k}(\tilde{b}_i - b_i)^2, \tag{2.3}$$

where $\tilde{b} = \sqrt{T}\tilde{\beta}$. When $\hat{\beta}$ is a sufficient statistic, or equivalently when the Bayes estimator is constructed as a function of $\hat{\beta}$, then the Bayes, or average, risk of $\tilde{\beta}$ given the prior g is,

$$r_g(\tilde{b},f_k) = \rho \int\int k^{-1}\sum_{i=1}^{k}(\tilde{b}_i - b_i)^2 f_k(\hat{b}-b)g(b)d\hat{b}db. \tag{2.4}$$

If k were fixed as $T\rightarrow\infty$, standard central limit theory for $\Re^k$ implies that, under suitable moment conditions, the OLS estimator would converge in distribution to $N(0,\sigma_\epsilon^2 I_k)$. It is useful to denote the pdf of this limiting distribution by $\phi_k(\hat{b}-b) = \Pi_{i=1}^{k}\phi(\hat{b}_i - b)$, where $\phi(\bullet)$ is the normal density with mean zero and variance σ_ϵ^2. If the distribution of ϵ is normal, then $f_k = \phi_k$.

Some interesting results for the OLS estimator and forecast can now be obtained by straightforward calculation. The asymptotic frequentist risk of the OLS estimator is $R(\hat{\beta},\beta) \rightarrow \rho\sigma_\epsilon^2$. The asymptotic relative efficiency of the forecast based on $\tilde{b}$, relative to infeasible forecast, $\beta' X_t$, is $[\sigma_\epsilon^2 + R(\tilde{\beta},\beta)]/\sigma_\epsilon^2$. Thus, the asymptotic relative efficiency of the OLS forecast is $1+\rho$. Because the frequentist risk of OLS does not depend on b, the Bayes risk of OLS is $r_g(\hat{b},f_T) = \rho\sigma_\epsilon^2$ for all proper priors g.

2.2 Estimators

The estimators are motivated by assuming that the errors are normally distributed, so that $\{\hat{b}_i\}$ are independent, and that σ_ϵ^2 is known. Then $\hat{b}$ is sufficient for b. Thus, when g is known, the corresponding Bayes estimator of b under normal errors is,

$$\hat{b}_i^{NB} = \int b_i\phi(\hat{b}_i-b_i)g(b_i)db_i / \int \phi(\hat{b}_i-b_i)g(b_i)db_i. \tag{2.5}$$

In practice g and σ_ϵ^2 are unknown, so this estimator is infeasible. The empirical Bayes (EB) estimator is a feasible version of (2.5), with g and σ_ϵ^2 estimated. Adopt the notation that $\hat{\phi}$ is ϕ, with $\hat{\sigma}_\epsilon^2$ replacing σ_ϵ^2, where $\hat{\sigma}_\epsilon^2$ is the usual estimator of σ_ϵ^2, $\hat{\sigma}_\epsilon^2 = (T\text{-}k)^{-1}\sum_{t=1}^{T}(y_{t+1}-\hat{\beta}'X_t)^2$. The EB estimator is,

$$\hat{b}_i^{EB} = \int b_i\hat{\phi}(\hat{b}_i-b_i)\hat{g}(b_i)db_i / \int \hat{\phi}(\hat{b}_i-b_i)\hat{g}(b_i)db_i. \tag{2.6}$$

As expressed in (2.6), computation of $\hat{b}^{EB}$ requires estimation of g. The specific estimator of g considered here is a deconvolution estimator of g, constructed in the manner of Fan (1991) and Diggle and Hall (1993). Let m denote the marginal distribution of $\hat{b}_i$. Under the normality assumption,

$$m(\hat{b}_i) = \int \phi(\hat{b}_i-b_i)g(b_i)db_i. \tag{2.7}$$

Let $\chi_m(t) = \int m(x)e^{-itx}dx$, etc. Then (2.7) implies, $\chi_m(t) = \chi_\phi(t)\chi_g(t)$, so $\chi_g(t) = \chi_m(t)/\chi_\phi(t)$. Let $\hat{m}$ be a kernel density estimator of m. This suggests the nonparametric estimator of the characteristic function of g, $\hat{\chi}_g(t) = \chi_{\hat{m}}(t)/\chi_{\hat{\phi}}(t)$. Following Diggle and Hall (1993), we therefore consider the nonparametric deconvolution estimator of g, (2.8) $$\hat{g}(x) = \int \omega(t)\hat{\chi}_g(t)e^{itx}dx + \int (1-\omega(t))\chi_{g*}(t)e^{itx}dx$$

where $\omega(t)$ is a weight function and g* is a fixed density. In particular, for the theoretical results we follow Diggle and Hall (1991) and assume $\chi_{g*}(t)=0$ and $\omega(t)=\mathbf{1}(|t|\leq p_T)$, where $p_T>0$ and $p_T\to\infty$ as $T\to\infty$. Other choices for $\omega(t)$ and g* are examined in the numerical work.

2.3 Results

In addition to the previous normalizations and assumptions, we make the following assumptions. The first assumption places restrictions on moments, the second pertains to the prior, and the third pertains to the estimator of the marginal.

Assumption 1. $\sup_{it} EX_{it}^4 < \infty$ and $E\epsilon_t^4 < \infty$.

Assumption 2. The prior pdf g is bounded and has compact support.

Assumption 3. $\hat{m}(\hat{b}_i)$ is such that $E[\hat{m}(x)-m(x)]^2 \to 0$ pointwise for all x.

The next assumption restricts the asymptotic dependence between $\hat{b}_i$. Define $f_k^{\perp}(\hat{b}_i-b_i) = \Pi_{i=1}^k f_{0k}(\hat{b}_i-b_i)$, where f_{0k} denotes the marginal distribution of $\hat{b}_1-b_i$, conditional on b_i, for a given T and $k=\rho T$; thus $f_k^{\perp}$ is the product of the marginals and represents the measure under which $\hat{b}_i$ are independent but have the same marginals as does $\hat{b}_1-b_1$ under f_k. For assumption 4, we extend the measures on $\Re^k$ to measures on $\Re^\infty$ by completing them with independent normals ϕ, denoting this completion by ϕ_{-k}. Accordingly let the completed measures be $\tilde{f}_k = f_k\phi_{-k}$, etc. We adopt the metric $d_\infty(x,y) = \Sigma_{i=1}^\infty 2^{-i}|x_i-y_i|/(1+|x_i-y_i|)$ on $\Re^\infty$ (cf. Billingsley [1968]).

Assumption 4. $\{\tilde{f}_k(\hat{b}-b)/\tilde{f}_k^{\perp}(\hat{b}-b)\}_{k=1}^{\infty}$ is a family of functions (on $\Re^{\infty}$) which is equicontinuous under the metric $d_{\infty}(x,y)$, and is pointwise bounded by a function (on $\Re^{\infty}$) $M(\hat{b}-b)$ such that $\int\int M(\hat{b}-b)\tilde{\phi}(\hat{b}-b)\tilde{g}(b)d\hat{b}db < \infty$, and likewise for every $f_k^{\perp}\phi_{-k}$ measure.

Our first result shows the consistency of $\hat{\sigma}_{\epsilon}^2$.

Proposition 1. Under assumption 1, $\hat{\sigma}_{\epsilon}^2 \rightarrow \sigma_{\epsilon}^2$.

The following theorem shows that the average risk of the EB estimator with an unknown error distribution asymptotically attains the average risk of the infeasible normal Bayes estimator under a normal error distribution.

Theorem 1. Assume that assumptions 1-4 hold. Then $r_g(\hat{b}^{EB},f_k) - r_g(\hat{b}^{NB},\phi_k) \rightarrow 0$.

Several remarks are in order. First suppose that the true error distribution is Gaussian, so $f_k=\phi_k$. Then theorem 1 states that the Bayes risk of the EB estimator asymptotically equals the Bayes risk of the infeasible true Bayes estimator, $\hat{b}^{NB}$. It follows that $\hat{b}^{EB}$ is admissible and is asymptotically optimal in the sense of Robbins (1965). Because the Bayes risk function was derived from the forecasting problem, this and related statements about the properties of the EB estimator carry over directly to the forecast based on the EB estimator.

Second, theorem 1 goes further and provides conditions under which this risk is achieved by the EB estimator, even if the finite sample distribuiton of $\hat{b}$ is nonnormal. The key observation here is the weak convergence of $\tilde{f}_k$ to $\tilde{\phi}_k$ on $\Re^{\infty}$, which follows from the stated assumptions (which imply convergence of finite dimensional distributions) and from tightness under the metric d_{∞} (cf. Billinsley [1968]).

Third, assumption 3 is stated as a high level assumption on the consistency of the nonparametric estimator of the marginal distribution of $\hat{b}_i$. Such an estimator is discussed and used in the section 3.

Fourth, assumption 4 is somewhat unusual and merits some discussion. Clearly assumption 4 holds with equality if the $\{\hat{b}_i\}$ are independent. This will be so for general error distributions if the X's are Gaussian, or alternatively for general distributions of the (orthonormal) X's if ϵ has a Gaussian distribution. More generally, assumption 4 holds for general error distributions if the X's are m-dependent. This is assumption is sufficient but does not appear to be necessary, and work is ongoing to relax this assumption.

Fifth, it is possible to provide a result similar to theorem 1 for other EB estimators. A particularly convenient EB estimator exploits the asymptotic independence and normality of $\hat{b}$, so that $\hat{b}_i^{NB}$ can be computed as a so-called simple empirical Bayes (SEB) estimator (cf. Maritz and Lwin [1989]). The SEB estimator has the form,

$$\hat{b}_i^{EB} = \hat{b}_i + \hat{\sigma}_\epsilon^2 \hat{\ell}(\hat{b}_i), \tag{2.9}$$

where $\hat{\ell}$ is an estimate of the score ℓ of the marginal distribution of $\hat{b}_i$. A result like theorem 1 for this estimator can be shown when $\hat{\ell}$ is mean-square consistent for ℓ under the sequence of measures ϕ_k. Such an estimator is provided by Bickel et. al. (1993). In the numerical work, we consider both deconvolution EB estimators and SEB estimators based on (2.9) and estimated scores.

2.4 Parametric priors

The preceding results treat g nonparametrically. An alternative is to treat g as belonging to a finitely parameterized family, say, $g=g(b_i,\theta)$, where θ is a finite-dimensional parameter vector. If $\hat{\theta}$ is a consistent estimator of θ and if g is continuous in θ, then the nonparametric estimator of $\hat{g}$ in the preceding results can be replaced by the parametric estimator, $g(\bullet,\hat{\theta})$. Inspection of the proof of theorem 1 reveals that theorem 1 also holds when g is accordingly estimated parametrically, assuming the true prior is in the family of parametric priors. In principal this approach also extends to priors that admit dependence among $\{b^i\}$, as long as this dependence is characterized by an identified, finite dimensional parameter vector.

For some parameterizations of g, θ can be estimated by method of moments based on the marginal distribution of b_i. The usual advantages and disadvantages of parametric vs. nonparametric estimation apply in this case, and it ultimately is an empirical question whether the parametric or nonparametric approach is preferred. Both are explored in the Monte Carlo simulations in the next section. For additional discussions of parametric empirical Bayes estimation see Lehmann and Casella (1998) and Maritz and Lwin (1989).

2.5 An interpretation in terms of a random coefficient model

The model has an alternative, frequentist interpretation in terms of a random coefficients model. Suppose (1.1) holds with Gaussian errors and adopt the renormalization in section 2.1. In the random coefficients interpretation, b_i are i.i.d. g and the Bayes risk is the forecasting risk (up to the additive constant σ_ϵ^2), averaged over the random parameters. If g and σ_ϵ^2 are known, the efficient estimator of b under quadratic loss is $E(b|\hat{b},g,\sigma_\epsilon^2)$, and this estimator minimizes the average forecasting risk. When g and σ_ϵ^2 are unknown, this conditional mean is infeasible but the EB estimator provides a feasible counterpart. Under theorem 1, this feasible estimator achieves the same average forecast risk under normal errors as the infeasible conditional mean and moreover sufficient conditions are given for achieving this risk even if the errors are nonnormal.

3. Monte Carlo Results

3.1 Estimators

Parametric Gaussian EB estimator. The parametric Gaussian EB estimator posits that b_i is i.i.d. $N(\mu,\tau^2)$ and that the errors are $N(0,\sigma_\epsilon^2)$. Method of moments estimators are obtained from this parameterization of the posterior distribution, or more simply by manipulation of $\hat{b}_i = b_i + \zeta_i$, where b_i is i.i.d $N(\mu,\tau^2)$ and ζ_i has mean zero and variance σ_ϵ^2. Given μ, τ^2, and σ_ϵ^2, the Bayes estimator is given by the usual conjugate prior expressions. These expressions are evaluated using the method of moments estimators $\hat{\mu} = k^{-1}\sum_{i=1}^{k}\hat{b}_i$ and $\hat{\tau}^2 = (R^2-\rho)/[\rho(1-R^2)]\hat{\sigma}_\epsilon^2 - \hat{\mu}^2$, where R^2 is the sample R^2 from the OLS regression of y on X. When $\hat{\tau}^2 < 0$, $\hat{\tau}^2$ was replaced by abs($\hat{\tau}^2$).

Deconvolution EB estimator. The deconvolution estimator is computed using (2.8), where the integrals are evaluated numerically. The kernel density estimator $\hat{m}$ was computed from $\{\hat{b}_i\}$ using t-distribution kernel with five degrees of freedom, and a bandwidth $c(T/100)^{-2/7}/s_{\hat{b}}$, where $s_{\hat{b}}$ is the sample standard deviation of $\hat{b}$ and c is a constant (referred to as the t-kernel bandwidth parameter). This heavy tailed kernel was found to perform better than truncated kernels because $\hat{m}$ appears in the denominator of the EB estimate of the posterior mean. Although Diggle and Hall (1993) chose χ_{g*} in (2.8) to be zero, so that the deconvolution estimate was shrunk towards a uniform distribution, empirical experimentation indicated that it was better to shrink towards the parametric Gaussian prior, so this is the choice of g* used for the results here. The weight function $\omega(t)$ was chosen to be triangular so $\omega(0)=1$ and $\omega(p_{max})=0$.

Simple EB estimator. We also consider the simple EB estimator (2.9). Following Härdle et. al. (1992), the score function is estimated using the bisquare kernel with bandwidth rate proportional to $(T/100)^{-2/7}$. Preliminary investigation found advantages to shrinking the nonparametric score estimator towards the parametric Gaussian score estimator. Specifically, the modified score estimator was,

$$(3.1) \qquad \tilde{\ell}(x) = \lambda_T(|x|)\hat{\ell}(x) + (1-\lambda_T(x))\hat{\ell}_g(|x|)$$

where $\hat{\ell}$ is the bisquare kernel nonparametric score estimator, $\hat{\ell}_g$ is the score of the marginal distribution of $\hat{b}_i - b_i$, estimated assuming normal errors and an i.i.d. $N(\mu,\tau^2)$ prior, with the estimates discussed above used for σ_ϵ^2, μ and τ^2. The shrinkage weights are $\lambda_T(|x|) = \exp(-\frac{1}{2}\kappa^2 x^2)$, where x represents $(\hat{b}_i - b)/\hat{\sigma}_\epsilon$. Results are presented for various shrinkage parameters κ; small values of κ represent less shrinkage, and with $\kappa=0$, $\tilde{\ell} = \hat{\ell}$.

Both the deconvolution and simple EB nonparametric estimators occassionally produced extremely large Bayes estimates, and some results were sensitive to these outliers. We therefore implemented the upper truncation $|\hat{b}_i^{EB}| \le \max_i |\hat{b}_i|$ on both the deconvolution and simple nonparametric EB estimates.

Other benchmark estimators. Results are also reported for some estimators that serve as benchmarks: the true Bayes estimator, the OLS estimator, and the BIC estimator. The true Bayes estimator is the exact Bayes estimator based on the prior and σ_ϵ^2, which while infeasible in practice can be computed in this Monte Carlo setting. The BIC estimator is the estimator that estimates b_i either by $\hat{b}_i$ or by zero, depending on whether this regressor is included in the regression according to the BIC criterion. Enumeration of all possible models and thus exhaustive BIC selection is straightforward in this design because of the orthonormality of the X′s.

3.2 Experimental Design

The data were generated according to (1.1), with ϵ_t i.i.d. $N(0,\sigma_\epsilon^2)$, where X_t are the k principal components of the $\{Z_t, t=1,\ldots,T\}$, where Z_{it} are i.i.d. N(0,1); X_t was rescaled so that $X'X/T = I_k$. The parameter σ_ϵ^2 was chosen to achieve a desired population R^2. The population R^2 was fixed at 40% for all simulations reported here. The number of regressors was set at $k = \rho T$.

Two sets of calculations were performed. The first examines the finite sample convergence of the Bayes risk of the various estimators to the Gaussian Bayes risk of the true Bayes estimator; that is, this calculation examines the relevance of Theorem 1 to the finite sample behavior of these estimators. For these calculations, the parameters $b_i = \beta_i/\sqrt{T}$ were generated from the mixture of normals prior

$$\text{(3.2)} \qquad b_i \text{ i.i.d. } N(2,1) \text{ w.p. } q,\ N(0,½) \text{ w.p. } (1\text{-}q).$$

Under this prior, a fraction q of the variables are important in the sense that they are likely to have large coefficients, while 1-q of the coefficients are unimportant, having nonzero but typically small coefficients. For these calculations, the free parameters of the design are T, q, and ρ. For each estimator, the Bayes risk r_g was computed by Monte Carlo, with 1000 Monte Carlo repetitions, where each repetition entailed redrawing (b, X, ϵ). Two sets of results are reported, for $\rho = 0.4$ and $\rho = 0.7$, for various values of q.

While the formal results pertain to the Bayes risk, the original motivation for this investigation was to produce estimators with good frequentist properties. The second set of calculations therefore evaluates the performance of the various estimators using frequentist risk (1.4). For repitions b_i was fixed according to,

$$\text{(3.3)} \qquad b_i = 1,\ i=1,\ldots,qk, \text{ and } b_i = 0,\ i=qk+1,\ldots,k$$

where q is a design parameter between 0 and 1. For these results, ρ was set at 0.4.

3.3 Results and Discussion

The Bayes risk results are presented in tables 1 (ρ=0.4) and table 2 (ρ=0.7). First consider the results in table 1 when q=1 (first panel). In this case the prior is a single normal and the parametric Bayes estimator nests the true prior. For T$\geq$200, the Bayes risk of the parametric EB estimator is quite close to the Bayes risk of the true Bayes estimator. In contrast, the Bayes risk of OLS is quite large. It is noteworthy that, for these parameters, model selection using the BIC has a higher Bayes risk than OLS, presumably because BIC is in part selecting variables for which the classical estimation error makes the estimated coefficient inordinately large. When q=1, one would expect the nonparametric EB estimators to do less well than the parametric EB estimator, and this is the case, especially for small T. Still, the nonparametric EB estimators do remarkably well for T=400 (for which k=160), with several of the nonparametric EB estimators having Bayes risks within 15% of the risk of the true Bayes estimator.

A similar picture emerges for the second and third panels, for which q<1. Here the parametric EB estimator uses an approximation that does not nest the true mixture of normals prior, so the parametric EB estimator cannot achieve the Bayes risk of the true Bayes estimator. Nonetheless, it comes within 10% of this risk, especially for large T. It is noteworthy that some of the nonparametric EB estimators have lower Bayes risks than the parametric EB estimator when q=.25; presumably these nonparametric EB estimators are picking up the deviations from normality of the true mixture of normals prior.

The results in table 2, for which ρ=0.7, present a similar picture. The Bayes risks of the OLS and BIC estimators is typically poor and is worse than the parametric or nonparametric EB estimators. The parametric and nonparametric EB estimators have Bayes risk approaching that of the true Bayes estimator.

The frequentist risk results are given in table 3. No prior is specified so the Bayes estimator is not relevant here. The main conclusion from table 3 is that the EB estimators all have lower frequentist risk than OLS or BIC. The parametric EB estimator typically does as well or better than the nonparametric EB estimator. This is surprising because the parametric EB estimator models the distribution of b_i as a single normal, when in fact it is highly nonnormal, with point mass on either zero or one. In several cases, however, especially for q small, the nonparametric EB estimator has lower frequentist risk than the parametric EB estimator.

4. Empirical Results *[Incomplete]*

4.1 Data

Forecasts were computed for eight major monthly macroeconomic variables for the United States. Four of these are the measures of aggregate real economic activity: total industrial production (ip); real personal income less transfers (gmyxpq); real manufacturing and trade sales (msmtq); and the number of employees on nonagricultural payrolls (lpnag). The remaining four series are aggregate price indexes: the consumer price index (punew); the personal consumption expenditure implicit price deflator (gmdc); the CPI less food and energy (puxx); and the producer price index for finished goods (pwfsa). The forecasts were constructed using a set of 146 predictors that cover eight broad categories of available macroeconomic and financial time series. The series are listed in appendix B. The complete data set spans 1959:1-1998:12.

4.2 Construction of the Forecasts

Forecasts were constructed from regressions of the form

$$y_{t+h} = \beta' X_t + \epsilon_{t+h} \qquad (4.1)$$

where X_t is composed of the first k principal of the standardized predictors. Forecasts are constructed for horizons h = 1, 3, 6, and 12 months. The coefficient vector β was estimated by OLS and by three parametric Empirical Bayes estimators. All are versions of the Gaussian Empirical Bayes estimator described in Section 3.1. The first estimator is PGEB1 studied in the monte carlo experiment. The other two estimators robustify the method of moments calculations for deviations from the classical regression assumptions that underly PGEB1.

The second estimator, PGEB2, computes OLS estimators of β over the first and second half of the sample and forms $\hat{\tau}^2 = k^{-1}\sum(\hat{b}_{i,1}-\hat{\mu})(\hat{b}_{i,2}-\hat{\mu})$, where $\hat{b}_{i,1}$ and $\hat{b}_{i,2}$ denote the OLS estimates of b_i from the first and second half of the sample. The scale factor σ_ϵ^2 is then estimated as $\hat{\sigma}_3^2 = k^{-1}\sum(\hat{b}-\hat{\mu})^2-\hat{\tau}^2$. By using the covariation in the OLS estimates across the two sample periods to estimate τ^2, this estimator is robust to the short-run dependence in the ϵ_{t+h} in the multistep forecast errors.

The third estimator, PGEB3, estimates the Gaussian Empirical Bayes combining weights directly by predictive least squares. That is, it constructs estimates of b as $\hat{b}^{EB} = \lambda\hat{b}+(1-\lambda)\hat{\mu}$ where λ is estimated by OLS regressions of y_{t+h} onto the $\hat{\beta}_t'X_t$ and $\bar{\beta}_t 1'X_t$, where $\hat{\beta}_t$ is the OLS estimate of β computed at date t, $\bar{\beta}_t=k^{-1}1'\hat{\beta}_t$ is the sample mean of the OLS coefficients estimated at date t, and 1 denotes a k-vector of 1's..

As benchmarks, forecasts are also computed using the DIAR model from Stock and Waton (1998). These forecasts are computed using a small and fixed number of principal components (k=2, for the results shown below).

All forecasts are computed recursively, that is in simulated real time, beginning in 1970:1. Thus, for example, to compute the forecasts for month T, principal components of the predictors were computed using data from 1960:1 through month T. The first k = min(146,ρT) principal components were used as X_t. To capture serial correlation in the variables being predicted, residuals from univariate autoregressions were used for y_{t+h}. Thus, letting z_t denote the variable to be forecast, then y_{t+h} was formed as the residual from the regression z_{t+h} onto $(1, z_t, z_{t-1}, \ldots z_{t-p})$ with data from t=1960:1 through T-h with the lag length p determined by BIC. The regression coefficients in (4.1) were then estimated using the methods described above with data through from t=1960:1-T-h. These estimated coefficients, together with the coefficients from the autoregression were used to construct forecasts for z_{T+h}. This procedure was carried out for T=1970:1 through 1998:12-h.

4.3 Results

Results are presented in Tables 4 and 5 for $\rho=0.2$ and $\rho=0.4$, respectively. The entries in these tables are the mean square error of the simulated forecast errors relative to the mean square error from the univariate autoregression. Results are shown for forecast horizons h=1,3,6,12. Thus, for example, the first row of table 4 shows the results for the 1-month-ahead predictions of industrial production. The value of 0.98 under the column heading "OLS" implies that the mean square of the forecast constructed using the OLS estimates of β had a mean square error that was 2% less than the forecasts that set $\beta=0$ (the univariate autoregressive forecast). Results are also shown for the three Empirical Bayes estimators described above and for the DIAR estimator.

Several findings stand out in table 4. First, in all cases PGEB1 and PGEB2 improve upon OLS. Second, PGEB3 performs marginally worse than PGEB1 and PGEB2 and in a few cases is dominated by OLS. Third, in most cases the Empirical Bayes estimators improve upon the univariate autoregression; the improvements are more pronounced for the real series than for inflation. Finally, the best performing models for all series and all horizons are the DIAR models. Apparently, it is better to forecast using only the first two principal components of the predictors with no shrinkage, than to use many of the principal components and shrink them toward a common value.

Taken as a whole the table suggests that improvement of the Empirical Bayes estimators relative to the univariate autoregression are modest. This is somewhat surprising given the performance of the Empirical Bayes estimators in the Monte Carlo experiments reported in section 3.3. The explanation seems to be that the predictive power of the regression (measured by the regression R^2) is not as great in as in the Monte Carlo design. In the Monte Carlo experiment, the R^2 was 0.40, and for the eight series considered here it is considerably less than 0.40. For example, suppose for a moment that the DIAR results give a good estimate of the forecastibility of y given all of the predictors. Thus, h=1 the R^2 for the real variables is approximately 10-15% and the R^2 for the price indexes is less than 5%. A calculation shows that if the population R^2 is 10% and $\rho=0.2$, then the asymptotic relative MSE for OLS is 1.08 and for EB it is 0.96. If the R^2 is 5%, then the asymptotic mse for OLS is 1.14 and for EB it is 0.99. These are roughly what is evident in the table.

Table 5 shows the results with $\rho=0.4$, so that more principal components are included. Not surprisingly, OLS performs worse that when $\rho=0.2$. Indeed, each of the Empirical Bayes forecasts also performs worse than the corresponding model with $\rho=0.2$. Again, this is what would be expected if the DIAR results are reasonably accurate estimates of the potential predictive power of the regressions, so that the additional factors added in table 5 make negligible forecasting contributions.

The final question addressed in this section is whether the Empirical Bayes methods can be used to improve upon the DIAR models. To answer this question the forecasting experiment was repeated, but now using the DIAR model as baseline regression rather than the univariate autoregression. Thus, residuals from the DIAR forecasts were used for y_{t+h} in the Empirical Bayes regressions. The results for this experiment are shown in table 6 ($\rho=0.2$) and table 7 ($\rho=0.4$). There is some evidence that the EB estimators can yield modest improvements on the DIAR model particularly for the real activity variables. For example, at the 6 month forecast horizon, PGEB1 yields an average 4% improvement over DIAR for the real series. There is no evidence of improvement for the price variables.

5. Discussion and Conclusion

These initial empirical results suggest that the EB estimators offer only slight improvements upon estimators that use only the first two estimated factors for forecasting these eight major macroeconomic time series. The fact that the Empirical Bayes estimator yield considerable improvement in the Monte Carlo design, and indeed approached the efficiency of the true Bayes estimator, but deliver only small improvements in the empirical application suggests that the empirical finding is not the result of using an inefficient forecast, but rather that there simply is little predictive content in these macroeconomic principal components beyond the first few. If true, this has striking and, we believe, significant implications for empirical macroeconomics and large-model forecasting. Additional work remains, however, before we can be confident of this negative finding. This work includes implementation of the nonparametric EB estimators and development and implementation of alternative parametric EB estimators.

Additional theoretical work and extensions are ongoing. Work is underway on relaxing assumption 4. Also, we conjecture that $\hat{b}^{NB}$ can be shown to be minimax with respect to the Bayes risk across error distributions f_k, in which case $\hat{b}^{EB}$ can be further interpreted as being asymptotically equivalent to this minimax estimator in the sense that it achieves the minimax Bayes risk for all error densities (subject to some regularity conditions); work along these lines is ongoing as well.

Appendix A: Proofs of Theorems

[In preparation.]

Brief sketch of arguments for ES World Congress reviewers:

Proposition 1. By direct calculation using Chebyschev's inequality.

Theorem 1.

(a) Show that, under assumption 4, $f_k/f_k^{\perp} \to 1$ almost surely with respect to $\tilde{\phi}\tilde{g}$ on $\Re^{\infty}$. Argument proceeds using Billingsley (1968, Theorem 5.5) and the Arzela-Ascoli theorem.

(b) Decompose

$$\begin{aligned} r_g(\hat{b}^{EGB},f_k) - r_g(\hat{b}^{NB},\phi_k) &= [r_g(\hat{b}^{EB},f_k) - r_g(\hat{b}^{EB},f_k^{\perp})] \\ &+ [r_g(\hat{b}^{EB},f_k^{\perp}) - r_g(\hat{b}^{EB},\hat{\phi}_k)] \\ &+ [r_g(\hat{b}^{EB},\hat{\phi}_k) - r_g(\hat{b}^{NB},\phi_k)]. \end{aligned}$$

Refer to the terms in brackets as I, II and III respectively. Term I converges to zero using the result (a) (by calculation). Term II converges to zero from Proposition 1, the weak convergence of $f_1^{\perp}$ to ϕ_1, the continuous mapping theorem, and calculations. Term III converges to zero by proposition 1, an extension of the proof in Diggle and Hall (1993), and tedious calculations.

Appendix B: Data Description

This appendix lists the time series used to construct the forecasts discussed in section 4. The format is: series number; series mnemonic; data span used; transformation code; and brief series description. The transformation codes are: 1 = no transformation; 2 = first difference; 4 = logarithm; 5 = first difference of logarithms; 6 = second difference of logarithms. An asterisk after the date denotes a series that was included in the unbalanced panel but not the balanced panel, either because of missing data or because of gross outliers which were treated as missing data. The series were either taken directly from the DRI-McGraw Hill Basic Economics database, in which case the original mnemonics are used, or they were produced by authors′ calculations based on data from that database, in which case the authors calculations and original DRI/McGraw series mnemonics are summarized in the data description field. The following abbreviations appear in the data definitions: SA = seasonally adjusted; NSA = not seasonally adjusted; SAAR = seasonally adjusted at an annual rate; FRB = Federal Reserve Board; AC = Authors calculations

Real output and income

1.	ip	1959:01-1998:12	5	industrial production: total index (1992=100,sa)
2.	ipp	1959:01-1998:12	5	industrial production: products, total (1992=100,sa)
3.	ipf	1959:01-1998:12	5	industrial production: final products (1992=100,sa)
4.	ipc	1959:01-1998:12	5	industrial production: consumer goods (1992=100,sa)
5.	ipcd	1959:01-1998:12	5	industrial production: durable consumer goods (1992=100,sa)
6.	ipcn	1959:01-1998:12	5	industrial production: nondurable condsumer goods (1992=100,sa)
7.	ipe	1959:01-1998:12	5	industrial production: business equipment (1992=100,sa)
8.	ipi	1959:01-1998:12	5	industrial production: intermediate products (1992=100,sa)
9.	ipm	1959:01-1998:12	5	industrial production: materials (1992=100,sa)
10.	ipmnd	1959:01-1998:12	5	industrial production: nondurable goods materials (1992=100,sa)
11.	ipmfg	1959:01-1998:12	5	industrial production: manufacturing (1992=100,sa)
12.	ipd	1959:01-1998:12	5	industrial production: durable manufacturing (1992=100,sa)
13.	ipn	1959:01-1998:12	5	industrial production: nondurable manufacturing (1992=100,sa)
14.	ipmin	1959:01-1998:12	5	industrial production: mining (1992=100,sa)
15.	iput	1959:01-1998:12	5	industrial production: utilities (1992-=100,sa)
16.	ipxmca	1959:01-1998:12	1	capacity util rate: manufacturing,total(% of capacity,sa)(frb)
17.	pmi	1959:01-1998:12	1	purchasing managers′ index (sa)
18.	pmp	1959:01-1998:12	1	NAPM production index (percent)
19.	gmyxpq	1959:01-1998:12	5	personal income less transfer payments (chained) (#51) (bil 92$,saar)

Employment and hours

20.	lhel	1959:01-1998:12	5	index of help-wanted advertising in newspapers (1967=100;sa)
21.	lhelx	1959:01-1998:12	4	employment: ratio; help-wanted ads:no. unemployed clf
22.	lhem	1959:01-1998:12	5	civilian labor force: employed, total (thous.,sa)
23.	lhnag	1959:01-1998:12	5	civilian labor force: employed, nonagric.industries (thous.,sa)
24.	lhur	1959:01-1998:12	1	unemployment rate: all workers, 16 years & over (%,sa)
25.	lhu680	1959:01-1998:12	1	unemploy.by duration: average(mean)duration in weeks (sa)
26.	lhu5	1959:01-1998:12	1	unemploy.by duration: persons unempl.less than 5 wks (thous.,sa)
27.	lhu14	1959:01-1998:12	1	unemploy.by duration: persons unempl.5 to 14 wks (thous.,sa)
28.	lhu15	1959:01-1998:12	1	unemploy.by duration: persons unempl.15 wks + (thous.,sa)
29.	lhu26	1959:01-1998:12	1	unemploy.by duration: persons unempl.15 to 26 wks (thous.,sa)
30.	lpnag	1959:01-1998:12	5	employees on nonag. payrolls: total (thous.,sa)
31.	lp	1959:01-1998:12	5	employees on nonag payrolls: total, private (thous,sa)
32.	lpgd	1959:01-1998:12	5	employees on nonag. payrolls: goods-producing (thous.,sa)
33.	lpcc	1959:01-1998:12	5	employees on nonag. payrolls: contract construction (thous.,sa)
34.	lpem	1959:01-1998:12	5	employees on nonag. payrolls: manufacturing (thous.,sa)
35.	lped	1959:01-1998:12	5	employees on nonag. payrolls: durable goods (thous.,sa)
36.	lpen	1959:01-1998:12	5	employees on nonag. payrolls: nondurable goods (thous.,sa)
37.	lpsp	1959:01-1998:12	5	employees on nonag. payrolls: service-producing (thous.,sa)
38.	lpt	1959:01-1998:12	5	employees on nonag. payrolls: wholesale & retail trade (thous.,sa)
39.	lpfr	1959:01-1998:12	5	employees on nonag. payrolls: finance,insur.&real estate (thous.,sa
40.	lps	1959:01-1998:12	5	employees on nonag. payrolls: services (thous.,sa)
41.	lpgov	1959:01-1998:12	5	employees on nonag. payrolls: government (thous.,sa)
42.	lphrm	1959:01-1998:12	1	avg. weekly hrs. of production wkrs.: manufacturing (sa)
43.	lpmosa	1959:01-1998:12	1	avg. weekly hrs. of prod. wkrs.: mfg.,overtime hrs. (sa)
44.	pmemp	1959:01-1998:12	1	NAPM employment index (percent)
Real retail, manufacturing and trade sales				
45.	msmtq	1959:01-1998:12	5	manufacturing & trade: total (mil of chained 1992 dollars)(sa)
46.	msmq	1959:01-1998:12	5	manufacturing & trade:manufacturing;total(mil of chained 1992 dollars)(sa)
47.	msdq	1959:01-1998:12	5	manufacturing & trade:mfg; durable goods (mil of chained 1992 dollars)(sa)
48.	msnq	1959:01-1998:12	5	manufact. & trade:mfg;nondurable goods (mil of chained 1992 dollars)(sa)
49.	wtq	1959:01-1998:12	5	merchant wholesalers: total (mil of chained 1992 dollars)(sa)
50.	wtdq	1959:01-1998:12	5	merchant wholesalers:durable goods total (mil of chained 1992 dollars)(sa)
51.	wtnq	1959:01-1998:12	5	merchant wholesalers:nondurable goods (mil of chained 1992 dollars)(sa)
52.	rtq	1959:01-1998:12	5	retail trade: total (mil of chained 1992 dollars)(sa)
53.	rtnq	1959:01-1998:12	5	retail trade:nondurable goods (mil of 1992 dollars)(sa)
Consumption				
54.	gmcq	1959:01-1998:12	5	personal consumption expend (chained)-total (bil 92$,saar)
55.	gmcdq	1959:01-1998:12	5	personal consumption expend (chained)-total durables (bil 92$,saar)
56.	gmcnq	1959:01-1998:12	5	personal consumption expend (chained)-nondurables (bil 92$,saar)
57.	gmcsq	1959:01-1998:12	5	personal consumption expend (chained)-services (bil 92$,saar)
58.	gmcanq	1959:01-1998:12	5	personal cons expend (chained)-new cars (bil 92$,saar)
Housing starts and sales				
59.	hsfr	1959:01-1998:12	4	housing starts:nonfarm(1947-58);total farm&nonfarm(1959-)(thous.,sa
60.	hsne	1959:01-1998:12	4	housing starts:northeast (thous.u.)s.a.
61.	hsmw	1959:01-1998:12	4	housing starts:midwest(thous.u.)s.a.
62.	hssou	1959:01-1998:12	4	housing starts:south (thous.u.)s.a.
63.	hswst	1959:01-1998:12	4	housing starts:west (thous.u.)s.a.
64.	hsbr	1959:01-1998:12	4	housing authorized: total new priv housing units (thous.,saar)
65.	hmob	1959:01-1998:12	4	mobile homes: manufacturers' shipments (thous.of units,saar)
Real inventories and inventory-sales ratios				
66.	ivmtq	1959:01-1998:12	5	manufacturing & trade inventories:total (mil of chained 1992)(sa)
67.	ivmfgq	1959:01-1998:12	5	inventories, business, mfg (mil of chained 1992 dollars, sa)
68.	ivmfdq	1959:01-1998:12	5	inventories, business durables (mil of chained 1992 dollars, sa)
69.	ivmfnq	1959:01-1998:12	5	inventories, business, nondurables (mil of chained 1992 dollars, sa)
70.	ivwrq	1959:01-1998:12	5	manufacturing & trade inv:merchant wholesalers (mil of chained 1992 dollars)(s

71.	ivrrq	1959:01-1998:12	5	manufacturing & trade inv:retail trade (mil of chained 1992 dollars)(sa)
72.	ivsrq	1959:01-1998:12	2	ratio for mfg & trade: inventory/sales (chained 1992 dollars, sa)
73.	ivsrmq	1959:01-1998:12	2	ratio for mfg & trade:mfg;inventory/sales (87$)(s.a.)
74.	ivsrwq	1959:01-1998:12	2	ratio for mfg & trade:wholesaler;inventory/sales(87$)(s.a.)
75.	ivsrrq	1959:01-1998:12	2	ratio for mfg & trade:retail trade;inventory/sales(87$)(s.a.)
76.	pmnv	1959:01-1998:12	1	napm inventories index (percent)

Orders and unfilled orders

77.	pmno	1959:01-1998:12	1	napm new orders index (percent)
78.	pmdel	1959:01-1998:12	1	napm vendor deliveries index (percent)
79.	mocmq	1959:01-1998:12	5	new orders (net)-consumer goods & materials, 1992 dollars (bci)
80.	mdoq	1959:01-1998:12	5	new orders, durable goods industries, 1992 dollars (bci)
81.	msondq	1959:01-1998:12	5	new orders, nondefense capital goods, in 1992 dollars (bci)
82.	mo	1959:01-1998:12	5	mfg new orders: all manufacturing industries, total (mil$,sa)
83.	mowu	1959:01-1998:12	5	mfg new orders: mfg industries with unfilled orders(mil$,sa)
84.	mdo	1959:01-1998:12	5	mfg new orders: durable goods industries, total (mil$,sa)
85.	mduwu	1959:01-1998:12	5	mfg new orders:durable goods indust with unfilled orders(mil$,sa)
86.	mno	1959:01-1998:12	5	mfg new orders: nondurable goods industries, total (mil$,sa)
87.	mnou	1959:01-1998:12	5	mfg new orders: nondurable gds ind.with unfilled orders(mil$,sa)
88.	mu	1959:01-1998:12	5	mfg unfilled orders: all manufacturing industries, total (mil$,sa)
89.	mdu	1959:01-1998:12	5	mfg unfilled orders: durable goods industries, total (mil$,sa)
90.	mnu	1959:01-1998:12	5	mfg unfilled orders: nondurable goods industries, total (mil$,sa)
91.	mpcon	1959:01-1998:12	5	contracts & orders for plant & equipment (bil$,sa)
92.	mpconq	1959:01-1998:12	5	contracts & orders for plant & equipment in 1992 dollars (bci)

Stock prices

93.	fsncom	1959:01-1998:12	5	NYSE common stock price index: composite (12/31/65=50)
94.	fspcom	1959:01-1998:12	5	S&P's common stock price index: composite (1941-43=10)
95.	fspin	1959:01-1998:12	5	S&P's common stock price index: industrials (1941-43=10)
96.	fspcap	1959:01-1998:12	5	S&P's common stock price index: capital goods (1941-43=10)
97.	fsput	1959:01-1998:12	5	S&P's common stock price index: utilities (1941-43=10)
98.	fsdxp	1959:01-1998:12	1	S&P's composite common stock: dividend yield (% per annum)
99.	fspxe	1959:01-1998:12	1	S&P's composite common stock: price-earnings ratio (%,nsa)

Exchange rates

100.	exrus	1959:01-1998:12	5	United States effective exchange rate (merm)(index no.)
101.	exrger	1959:01-1998:12	5	foreign exchange rate: Germany (deutsche mark per U.S.$)
102.	exrsw	1959:01-1998:12	5	foreign exchange rate: Switzerland (swiss franc per U.S.$)
103.	exrjan	1959:01-1998:12	5	foreign exchange rate: Japan (yen per U.S.$)
104.	exrcan	1959:01-1998:12	5	foreign exchange rate: Canada (canadian $ per U.S.$)

Interest rates

105.	fygt5	1959:01-1998:12	2	interest rate: U.S.treasury const maturities,5-yr.(% per ann,nsa)
106.	fygt10	1959:01-1998:12	2	interest rate: U.S.treasury const maturities,10-yr.(% per ann,nsa)
107.	fyaaac	1959:01-1998:12	2	bond yield: moody's aaa corporate (% per annum)
108.	fybaac	1959:01-1998:12	2	bond yield: moody's baa corporate (% per annum)
109.	fyfha	1959:01-1998:12	2	secondary market yields on fha mortgages (% per annum)
110.	sfycp	1959:01-1998:12	1	spread fycp - fyff
111.	sfygm3	1959:01-1998:12	1	spread fygm3 - fyff
112.	sfygm6	1959:01-1998:12	1	spread fygm6 - fyff
113.	sfygt1	1959:01-1998:12	1	spread fygt1 - fyff
114.	sfygt5	1959:01-1998:12	1	spread fygt5 - fyff
115.	sfygt10	1959:01-1998:12	1	spread fygt10 - fyff
116.	sfyaaac	1959:01-1998:12	1	spread fyaaac - fyff
117.	sfybaac	1959:01-1998:12	1	spread fybaac - fyff
118.	sfyfha	1959:01-1998:12	1	spread fyfha - fyff

Money and credit quantity aggregates

119.	fm1	1959:01-1998:12	6	money stock: m1(curr,trav.cks,dem dep,other ck'able dep)(bil$,sa)

120.	fm2	1959:01-1998:12	6	money stock:m2(m1+o'nite rps,euro$,g/p&b/d mmmfs&sav&sm time dep(bil$,
121.	fm3	1959:01-1998:12	6	money stock: m3(m2+lg time dep,term rp's&inst only mmmfs)(bil$,sa)
122.	fm2dq	1959:01-1998:12	5	money supply-m2 in 1992 dollars (bci)
123.	fmfba	1959:01-1998:12	6	monetary base, adj for reserve requirement changes(mil$,sa)
124.	fmrra	1959:01-1998:12	6	depository inst reserves:total,adj for reserve req chgs(mil$,sa)
125.	fmrnbc	1959:01-1998:12	6	depository inst reserves:nonborrow+ext cr,adj res req cgs(mil$,sa)

Price indexes

126.	pmcp	1959:01-1998:12	1	napm commodity prices index (percent)
127.	pwfsa	1959:01-1998:12	6	producer price index: finished goods (82=100,sa)
128.	pwfcsa	1959:01-1998:12	6	producer price index:finished consumer goods (82=100,sa)
129.	psm99q	1959:01-1998:12	6	index of sensitive materials prices (1990=100)(bci-99a)
130.	punew	1959:01-1998:12	6	cpi-u: all items (82-84=100,sa)
131.	pu83	1959:01-1998:12	6	cpi-u: apparel & upkeep (82-84=100,sa)
132.	pu84	1959:01-1998:12	6	cpi-u: transportation (82-84=100,sa)
133.	pu85	1959:01-1998:12	6	cpi-u: medical care (82-84=100,sa)
134.	puc	1959:01-1998:12	6	cpi-u: commodities (82-84=100,sa)
135.	pucd	1959:01-1998:12	6	cpi-u: durables (82-84=100,sa)
136.	pus	1959:01-1998:12	6	cpi-u: services (82-84=100,sa)
137.	puxf	1959:01-1998:12	6	cpi-u: all items less food (82-84=100,sa)
138.	puxhs	1959:01-1998:12	6	cpi-u: all items less shelter (82-84=100,sa)
139.	puxm	1959:01-1998:12	6	cpi-u: all items less midical care (82-84=100,sa)
140.	gmdc	1959:01-1998:12	6	pce,impl pr defl:pce (1987=100)
141.	gmdcd	1959:01-1998:12	6	pce,impl pr defl:pce; durables (1987=100)
142.	gmdcn	1959:01-1998:12	6	pce,impl pr defl:pce; nondurables (1987=100)
143.	gmdcs	1959:01-1998:12	6	pce,impl pr defl:pce; services (1987=100)

Average hourly earnings

144.	lehcc	1959:01-1998:12	6	avg hr earnings of constr wkrs: construction ($,sa)
145.	lehm	1959:01-1998:12	6	avg hr earnings of prod wkrs: manufacturing ($,sa)

Miscellaneous

146.	hhsntn	1959:01-1998:12	1	u. of mich. index of consumer expectations(bcd-83)

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Table 1
Bayes Risks of Various Estimators

DGP: $y_t = \beta' X_t + \epsilon_t$, ϵ_t i.i.d. $N(0,\sigma_\epsilon^2)$, X_t i.i.d. $N(0,1)$; σ_ϵ^2 chosen so that population $R^2 = 0.4$

Prior: $\beta_i = T^{-½} b_i$, b_i i.i.d. N(2,1) w.p. q, N(0,½) w.p. (1-q)

T	ρ	q	Bayes	OLS	BIC	PGEB1	PGSEB	NPSEB (a)	NPSEB (b)	NPSEB (c)	NPSEB (d)	NPGE2 (a)	NPGE2 (b)	NPGE2 (c)	NPGE2 (d)
50	0.40	1.00	0.300	1.198	1.384	0.415	0.516	0.560	0.553	0.524	0.523	0.454	0.490	0.578	0.747
100	0.40	1.00	0.300	1.193	1.420	0.363	0.412	0.466	0.456	0.431	0.427	0.395	0.417	0.410	0.437
200	0.40	1.00	0.300	1.200	1.464	0.336	0.348	0.409	0.397	0.377	0.371	0.365	0.377	0.367	0.390
400	0.40	1.00	0.300	1.198	1.505	0.322	0.324	0.380	0.369	0.350	0.345	0.344	0.352	0.344	0.358
50	0.40	0.50	0.307	0.629	0.657	0.384	0.398	0.407	0.406	0.394	0.395	0.419	0.439	0.385	0.397
100	0.40	0.50	0.307	0.640	0.677	0.359	0.359	0.374	0.371	0.363	0.362	0.392	0.398	0.365	0.373
200	0.40	0.50	0.307	0.631	0.693	0.339	0.339	0.354	0.351	0.343	0.342	0.366	0.367	0.346	0.351
400	0.40	0.50	0.307	0.630	0.698	0.330	0.330	0.343	0.340	0.333	0.332	0.354	0.354	0.338	0.341
50	0.40	0.25	0.175	0.346	0.307	0.226	0.230	0.230	0.230	0.227	0.227	0.300	0.308	0.256	0.259
100	0.40	0.25	0.174	0.346	0.317	0.214	0.213	0.214	0.213	0.210	0.211	0.284	0.276	0.241	0.235
200	0.40	0.25	0.173	0.345	0.326	0.208	0.207	0.206	0.206	0.203	0.203	0.276	0.259	0.234	0.224
400	0.40	0.25	0.175	0.344	0.336	0.204	0.203	0.200	0.200	0.196	0.197	0.270	0.250	0.228	0.219

<u>Estimators</u>

Bayes	true Bayes estimator based on true prior and error distribution
OLS	OLS
BIC	OLS with included variables chosen by BIC
PGEB1	EB estimator, linear combining formula, based on parametric $N(\mu,\tau^2)$ prior
PGSEB	"Simple" EB estimator based on parametric $N(\mu,\tau^2)$ prior
NPSEB (a)	Nonparametric simple EB, (bw0, κ) = (0.80, 1.00)
NPSEB (b)	Nonparametric simple EB, (bw0, κ) = (0.80, 1.25)
NPSEB (c)	Nonparametric simple EB, (bw0, κ) = (1.00, 1.00)
NPSEB (d)	Nonparametric simple EB, (bw0, κ) = (1.00, 1.25)
NPGE2 (a)	Nonparametric deconvolution EB, (p_{max}, bwt) = (0.40, 0.33)
NPGE2 (b)	Nonparametric deconvolution EB, (p_{max}, bwt) = (0.40, 0.60)
NPGE2 (c)	Nonparametric deconvolution EB, (p_{max}, bwt) = (0.60, 0.33)
NPGE2 (d)	Nonparametric deconvolution EB, (p_{max}, bwt) = (0.60, 0.60)

Based on 1000 Monte Carlo repetitions

Table 2
Bayes Risks of Various Estimators

DGP: $y_t = \beta' X_t + \epsilon_t$, ϵ_t i.i.d. $N(0,\sigma_\epsilon^2)$, X_t i.i.d. $N(0,1)$; σ_ϵ^2 chosen so that population $R^2 = 0.4$

Prior: $\beta_i = T^{-1/2} b_i$, b_i i.i.d. $N(2,1)$ w.p. q, $N(0,1/2)$ w.p. (1-q)

T	ρ	q	Bayes	OLS	BIC	PGEB1	PGSEB	NPSEB (a)	NPSEB (b)	NPSEB (c)	NPSEB (d)	NPGE2 (a)	NPGE2 (b)	NPGE2 (c)	NPGE2 (d)
50	0.70	1.00	0.588	3.675	3.785	0.880	1.320	1.465	1.445	1.315	1.321	1.211	1.117	>10	>10
100	0.70	1.00	0.588	3.675	3.842	0.760	0.953	1.146	1.111	1.025	1.012	0.860	0.925	>10	>10
200	0.70	1.00	0.588	3.675	3.841	0.687	0.757	0.962	0.922	0.852	0.833	0.753	0.804	>10	4.434
400	0.70	1.00	0.588	3.675	3.872	0.647	0.668	0.858	0.821	0.763	0.744	0.693	0.731	>10	2.651
50	0.70	0.50	0.700	1.929	1.864	0.884	1.011	1.063	1.055	1.009	1.012	0.888	0.906	1.138	0.977
100	0.70	0.50	0.707	1.929	1.861	0.826	0.859	0.921	0.909	0.877	0.873	0.830	0.842	0.905	0.865
200	0.70	0.50	0.703	1.929	1.860	0.780	0.784	0.857	0.842	0.816	0.809	0.793	0.802	0.795	0.812
400	0.70	0.50	0.699	1.929	1.837	0.748	0.748	0.812	0.799	0.775	0.769	0.765	0.773	0.756	0.773
50	0.70	0.25	0.420	1.057	0.873	0.561	0.598	0.622	0.620	0.598	0.600	0.609	0.606	0.560	0.566
100	0.70	0.25	0.422	1.057	0.867	0.528	0.533	0.556	0.552	0.536	0.535	0.573	0.560	0.526	0.530
200	0.70	0.25	0.423	1.057	0.845	0.496	0.496	0.517	0.513	0.499	0.499	0.543	0.529	0.499	0.502
400	0.70	0.25	0.421	1.057	0.838	0.482	0.482	0.499	0.496	0.482	0.482	0.530	0.518	0.488	0.489

Estimators are defined in notes to table 1.

Based on 1000 Monte Carlo repetitions.

Table 3
Classical Risks of Various Estimators

DGP: $y_t = \beta' X_t + \epsilon_t$, ϵ_t i.i.d. $N(0,\sigma_\epsilon^2)$, X_t i.i.d. $N(0,1)$; σ_ϵ^2 chosen so that population $R^2 = 0.4$

$\beta_i = T^{-1/2} b_i$, $b_i = 0$ or 1; fraction of $b_i=1$ is q

T	ρ	q	OLS	BIC	PGEB1	PGSEB	NPSEB (a)	NPSEB (b)	NPSEB (c)	NPSEB (d)	NPGE2 (a)	NPGE2 (b)	NPGE2 (c)	NPGE2 (d)	
200	0.40	0.10	0.024	0.040	0.015	0.015	0.010	0.011	0.011	0.012	0.067	0.062	0.068	0.063	
200	0.40	0.20	0.048	0.080	0.029	0.029	0.024	0.025	0.024	0.025	0.059	0.056	0.060	0.056	
200	0.40	0.30	0.072	0.121	0.041	0.041	0.038	0.038	0.038	0.038	0.056	0.057	0.057	0.056	
200	0.40	0.40	0.096	0.161	0.051	0.051	0.051	0.051	0.050	0.050	0.060	0.061	0.059	0.060	
200	0.40	0.50	0.120	0.202	0.058	0.058	0.061	0.060	0.058	0.058	0.063	0.065	0.063	0.064	
200	0.40	0.60	0.144	0.245	0.062	0.062	0.066	0.066	0.063	0.063	0.066	0.067	0.066	0.066	
200	0.40	0.70	0.168	0.289	0.062	0.062	0.068	0.067	0.064	0.063	0.066	0.067	0.066	0.066	
200	0.40	0.80	0.192	0.335	0.055	0.056	0.064	0.063	0.059	0.059	0.060	0.060	0.060	0.059	
200	0.40	0.90	0.216	0.383	0.037	0.041	0.052	0.050	0.046	0.045	0.044	0.046	0.045	0.045	
200	0.40	1.00	0.000	0.240	0.431	0.009	0.013	0.030	0.027	0.021	0.019	0.023	0.024	0.023	0.024

Estimators are defined in notes to table 1.

Based on 1000 Monte Carlo repetitions.

Table 4
Simulated Out-of-Sample Forecasting Results
Mean Square Errors Relative to Univariate Autogression

A. $\rho=0.2$

Series	Forecast Method OLS	PGEB1	PGEB2	PGEB3	DIAR
1 Month Ahead Forecasts					
Industrial Production	0.98	0.92	0.93	0.93	0.89
Personal Income	1.03	0.97	0.98	0.98	0.91
Mfg & Trade Sales	1.00	0.94	0.94	0.95	0.88
Nonag. Employment	1.05	0.97	0.93	0.97	0.82
Consumer Price Index	1.08	1.00	0.98	1.00	0.97
Pers. Cons. Deflator	1.13	1.02	1.02	1.03	0.99
CPI exc. food&energy	1.10	1.00	1.01	1.00	0.97
Producer Price Index	1.17	1.02	1.02	1.02	1.00
3 Month Ahead Forecasts					
Industrial Production	0.83	0.82	0.81	0.85	0.76
Personal Income	1.03	0.97	0.98	0.98	0.82
Mfg & Trade Sales	0.92	0.90	0.89	0.92	0.75
Nonag. Employment	0.86	0.86	0.82	0.92	0.78
Consumer Price Index	1.04	0.99	1.01	1.00	0.91
Pers. Cons. Deflator	1.09	1.01	1.01	1.02	0.96
CPI exc. food&energy	1.06	0.99	1.01	1.01	0.93
Producer Price Index	1.17	1.03	1.02	1.02	0.99
6 Month Ahead Forecasts					
Industrial Production	0.75	0.75	0.72	0.86	0.67
Personal Income	0.94	0.92	0.98	0.97	0.81
Mfg & Trade Sales	0.85	0.84	0.84	0.96	0.68
Nonag. Employment	0.87	0.86	0.82	0.99	0.79
Consumer Price Index	0.94	0.92	0.97	0.98	0.81
Pers. Cons. Deflator	1.04	0.99	1.03	1.01	0.92
CPI exc. food&energy	0.97	0.95	1.00	0.97	0.88
Producer Price Index	1.11	1.03	1.04	1.03	0.93
12 Month Ahead Forecasts					
Industrial Production	0.81	0.81	0.81	0.91	0.67
Personal Income	0.94	0.93	1.00	1.06	0.86
Mfg & Trade Sales	0.87	0.86	0.88	0.97	0.59
Nonag. Employment	0.93	0.92	0.89	0.99	0.77
Consumer Price Index	0.93	0.92	0.93	0.98	0.74
Pers. Cons. Deflator	1.00	0.97	0.97	0.99	0.86
CPI exc. food&energy	0.96	0.95	1.04	0.95	0.81
Producer Price Index	1.07	1.02	1.00	1.00	0.84

Note: The table entries show the simulated out-of-sample forecast mean square error relative the mean square forecast error for a univariate autoregression. All forecasts were computed using recursive methods described in the text with a sample period beginning in 1960:1. The simulated out-of-sample forecast period is 1970:1-1998:12.

Table 5
Simulated Out-of-Sample Forecasting Results
Mean Square Errors Relative to Univariate Autogression

B. ρ=0.4

	Forecast Method				
Series	OLS	PGEB1	PGEB2	PGEB3	DIAR
1 Month Ahead Forecasts					
Industrial Production	1.01	0.94	0.96	0.95	0.89
Personal Income	1.07	0.98	0.99	1.00	0.91
Mfg & Trade Sales	1.03	0.95	0.99	0.95	0.88
Nonag. Employment	1.04	0.96	0.98	0.96	0.82
Consumer Price Index	1.16	1.01	1.01	1.01	0.97
Pers. Cons. Deflator	1.18	1.01	1.01	1.02	0.99
CPI exc. food&energy	1.17	1.01	1.02	1.01	0.97
Producer Price Index	1.18	1.01	1.02	1.01	1.00
3 Month Ahead Forecasts					
Industrial Production	0.94	0.92	0.95	0.93	0.76
Personal Income	1.10	1.01	1.02	0.99	0.82
Mfg & Trade Sales	1.04	0.98	0.98	0.98	0.75
Nonag. Employment	0.92	0.90	0.89	0.96	0.78
Consumer Price Index	1.14	1.03	1.05	1.02	0.91
Pers. Cons. Deflator	1.15	1.02	1.05	1.02	0.96
CPI exc. food&energy	1.16	1.04	1.03	1.03	0.93
Producer Price Index	1.21	1.04	1.11	1.01	0.99
6 Month Ahead Forecasts					
Industrial Production	0.88	0.88	0.89	0.97	0.67
Personal Income	1.04	1.00	1.03	1.00	0.81
Mfg & Trade Sales	1.00	0.97	0.97	0.96	0.68
Nonag. Employment	0.97	0.95	0.90	0.97	0.79
Consumer Price Index	1.04	1.00	0.99	0.98	0.81
Pers. Cons. Deflator	1.05	1.00	1.03	1.00	0.92
CPI exc. food&energy	1.02	0.98	1.01	0.97	0.88
Producer Price Index	1.14	1.03	1.04	1.02	0.93
12 Month Ahead Forecasts					
Industrial Production	0.94	0.93	0.92	0.97	0.67
Personal Income	1.03	1.01	1.07	1.01	0.86
Mfg & Trade Sales	0.99	0.97	0.97	0.98	0.59
Nonag. Employment	1.05	1.03	0.99	1.00	0.77
Consumer Price Index	1.06	1.02	1.00	1.02	0.74
Pers. Cons. Deflator	1.07	1.02	1.03	1.03	0.86
CPI exc. food&energy	1.01	0.99	1.05	0.99	0.81
Producer Price Index	1.14	1.06	1.05	1.01	0.84

Note: The table entries show the simulated out-of-sample forecast mean square error relative the mean square forecast error for a univariate autoregression. All forecasts were computed using recursive methods described in the text with a sample period beginning in 1960:1. The simulated out-of-sample forecast period is 1970:1-1998:12.

Table 6
Simulated Out-of-Sample Forecasting Results
Mean Square Errors Relative to DIAR Model

A. ρ=0.2

Series	OLS	PGEB1	PGEB2	PGEB3
		Forecast Method		
1 Month Ahead Forecasts				
Industrial Production	1.05	0.96	0.96	0.97
Personal Income	1.10	1.00	1.02	1.01
Mfg & Trade Sales	1.05	0.97	0.97	0.97
Nonag. Employment	1.14	1.00	0.97	0.99
Consumer Price Index	1.11	1.01	1.02	1.02
Pers. Cons. Deflator	1.14	1.02	1.02	1.03
CPI exc. food&energy	1.15	1.02	1.02	1.01
Producer Price Index	1.17	1.02	1.01	1.02
3 Month Ahead Forecasts				
Industrial Production	0.97	0.93	0.93	0.94
Personal Income	1.14	1.02	1.01	1.01
Mfg & Trade Sales	1.04	0.97	0.99	0.97
Nonag. Employment	1.01	0.96	0.94	0.97
Consumer Price Index	1.12	1.02	1.03	1.02
Pers. Cons. Deflator	1.13	1.02	1.02	1.02
CPI exc. food&energy	1.13	1.03	1.01	1.04
Producer Price Index	1.18	1.01	1.01	1.01
6 Month Ahead Forecasts				
Industrial Production	0.91	0.88	0.90	0.89
Personal Income	1.07	1.00	1.00	0.99
Mfg & Trade Sales	1.00	0.95	0.95	0.93
Nonag. Employment	1.03	0.99	0.96	0.99
Consumer Price Index	1.07	1.00	1.03	1.01
Pers. Cons. Deflator	1.12	1.02	1.06	1.02
CPI exc. food&energy	1.08	1.01	1.01	1.01
Producer Price Index	1.16	1.01	1.02	1.01
12 Month Ahead Forecasts				
Industrial Production	1.03	0.98	0.95	0.98
Personal Income	1.07	1.03	1.00	1.00
Mfg & Trade Sales	1.08	1.00	0.99	0.99
Nonag. Employment	1.11	1.05	0.98	1.04
Consumer Price Index	1.05	1.00	0.98	1.02
Pers. Cons. Deflator	1.06	1.00	1.00	1.01
CPI exc. food&energy	1.10	1.04	1.04	1.03
Producer Price Index	1.13	1.00	1.00	1.00

Note: The table entries show the simulated out-of-sample forecast mean square error relative the mean square forecast error for the DIAR model with 2 factors. All forecasts were computed using recursive methods described in the text with a sample period beginning in 1960:1. The simulated out-of-sample forecast period is 1970:1-1998:12.

Table 7
Simulated Out-of-Sample Forecasting Results
Mean Square Errors Relative to DIAR Model

B. $\rho=0.4$

Series	OLS	Forecast Method PGEB1	PGEB2	PGEB3
1 Month Ahead Forecasts				
Industrial Production	1.04	0.95	0.97	0.96
Personal Income	1.11	0.99	1.01	1.00
Mfg & Trade Sales	1.03	0.96	0.97	0.96
Nonag. Employment	1.11	0.98	1.01	0.97
Consumer Price Index	1.19	1.01	1.01	1.01
Pers. Cons. Deflator	1.19	1.01	1.02	1.02
CPI exc. food&energy	1.22	1.02	1.04	1.02
Producer Price Index	1.18	1.01	1.01	1.02
3 Month Ahead Forecasts				
Industrial Production	1.01	0.95	0.94	0.96
Personal Income	1.18	1.02	0.99	1.00
Mfg & Trade Sales	1.10	0.99	1.00	0.99
Nonag. Employment	0.98	0.94	0.97	0.96
Consumer Price Index	1.20	1.03	1.05	1.01
Pers. Cons. Deflator	1.19	1.02	1.03	1.01
CPI exc. food&energy	1.22	1.05	1.06	1.01
Producer Price Index	1.23	1.03	1.09	1.01
6 Month Ahead Forecasts				
Industrial Production	0.94	0.92	0.95	0.94
Personal Income	1.13	1.03	1.00	1.00
Mfg & Trade Sales	1.09	1.00	0.98	0.97
Nonag. Employment	1.04	0.99	0.98	0.98
Consumer Price Index	1.13	1.02	1.02	1.02
Pers. Cons. Deflator	1.11	1.01	1.02	1.01
CPI exc. food&energy	1.09	1.01	1.03	1.00
Producer Price Index	1.18	1.02	1.05	1.01
12 Month Ahead Forecasts				
Industrial Production	1.07	1.01	0.97	0.97
Personal Income	1.14	1.07	1.00	0.98
Mfg & Trade Sales	1.10	1.01	0.98	0.98
Nonag. Employment	1.14	1.08	0.98	0.99
Consumer Price Index	1.10	1.02	1.01	1.03
Pers. Cons. Deflator	1.07	1.01	1.01	1.01
CPI exc. food&energy	1.05	1.00	1.01	0.99
Producer Price Index	1.15	1.03	1.05	1.01

Note: The table entries show the simulated out-of-sample forecast mean square error relative the mean square forecast error for the DIAR model with 2 factors. All forecasts were computed using recursive methods described in the text with a sample period beginning in 1960:1. The simulated out-of-sample forecast period is 1970:1-1998:12.