Job Search, Savings and Wealth Effects*

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Abstract

In this paper we consider a risk averse worker who at any point in time is either employed or unemployed; layoffs are random and beyond the worker's influence while the re-employment chance is directly affected by her search effort. Wealth is used to smooth consumption; run down during spells of unemployment and accumulated during spells of employment. The current savings decision depends on the current level of wealth and in general the consumption path is not going to be completely smooth over time. We characterize when search effort increases (decreases) as wealth decreases (increases) implying that the probability of leaving unemployment increases (decreases) with the duration of the unemployment spell. We simulate the model for specific utility functions.

Keywords: Search, Consumption Smoothing, Duration Dependence.

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1 Introduction

Job search is, like the savings decision, an important intertemporal allocation problem facing a worker. When an unemployed worker decides how much effort to put into job search, she essentially weighs the shortening of the jobless period and thereby increasing future expected income against loss of leisure today. A worker who is making an intertemporal allocation decision of this kind is also expected to carefully allocate consumption over time. In the face of fluctuating income, for instance due to the worker alternating between employment and unemployment, this amounts to another intertemporal allocation problem - the savings decision. Hence, search and savings are intimately related; yet they are rarely analyzed as interrelated problems but usually studied in isolation. In some cases, the savings decision is mute, say because of risk neutrality of the worker or if wealth cannot be stored and cannot be transferred between individuals. Risk neutrality seems to be somewhat unsatisfactory when one is considering an ordinary worker's search for a job. Also insurance and capital markets are rarely completely imperfect. And even so, there are always other ways of going about smoothing consumption over time, say via the timing of purchases of durable goods (see Browning and Crossley (1998)). That is, the assumption that wealth cannot be stored is also unsatisfactory.

In this paper, we consider a risk averse worker who at any point of time is either employed or unemployed. Layoffs are random and beyond the worker's influence while the re-employment chance can be affected by the worker's search effort. We derive the worker's optimal savings and search behavior and characterize the resulting consumption paths and wealth formation. For a broad range of model specifications it is shown that wealth increases durings spells of employments while it decreases during spells of unemployment. Also, it is shown that search effort increases as wealth decreases and therefore, the probability of leaving unemployment increases with the duration of the unemployment spell.

Thus, we obtain positive duration dependence by allowing the job searcher to smooth consumption and to choose search intensity optimally. It is a common empirical observation that the unemployment hazard rate (i.e. the rate with which an unemployed worker leaves the unemployment pool) is a function of the length of the unemployment spell. The controversy is over the sign of the effect, that is, whether the hazard rate is negative or positive duration dependent. There are quite a few explanations in the literature as to why we should expect the hazard rate to exhibit positive duration dependence so that the longer the experienced unemployment spell is to lower is the probability of getting a job. Berkovitch (1990), for instance, suggests there is a stigma associated with long unemployment spells so that the hazard rate would show negative duration dependence. In Mortensen (1986) a simple liquidity constraint is build into a basic search model which generates a decreasing reservation wage as the unemployed worker moves closer and closer to the constraint. This would point to positive duration dependence. Danforth (1979) states a similar result in a somewhat more general setting. Workers are assumed risk averse and it is established that the reservation wage is lower, the lower the level of the worker's wealth. However, the analysis of these paper rests on the assumption that once a job is found, the worker never leaves this job, that is, employment is an absorbing state. Thus there is no savings motivated by the wish of smoothing consumption between periods of employment and periods of unemployments.

Another common explanation of the duration dependence of the hazard rate is that there is unobserved heterogeneity in the work force. If some workers tend to leave the unemployment pool faster than others, then the pool of long time unemployed will have a relatively high rate of workers with low transition probabilities. This will lead to the observation that the hazard rate exhibits negative duration dependence. While in fact, the hazard rate is constant for each worker. Van den Berg and Van Ours (1996) attempt to estimate duration dependence while controlling for unobserved heterogeneity on U.S. data. They find significant effects of unobserved heterogeneity in the data. Also, they find that the hazard rate for white males exhibits negative duration dependence, not much duration dependence for white females and positive duration dependence for black males and females. They conclude that this may be due to larger stigmatization effects for whites than for blacks. Stigmatization is hard to measure. And this article will show, that one could also possibly try to explain these differences via wealth effects.

All the above arguments are augmentations of the basic search model. The basic search model with the assumption of expected income maximizing agents does not in itself display duration dependence. This paper shows that if one assumes that agents are risk averse, that they maximize expected utility and that they have access to capital markets, then duration dependence can occur with no other assumptions. Essentially, the hazard rate is a function of wealth as in Danforth (1979). These results suggest that if possible, researchers should include wealth measures in their estimations of hazard rate duration dependence.

The literature on job search theory is of course not unaware of the shortcomings of assuming risk neutral job searchers. First of all, focusing on the job search problem in itself has facilitated analysis of other aspects of search models. To the extend that the savings decision has little or no effect on the issues at hand, simplifying assumptions such as risk neutrality make sense in the way that agents can then be assumed to maximize expected income rather than expected utility. However, while many aspects of the search model are probably not affected much by the savings decision, this paper will show that for certain strands of the literature, the simplification can be misleading. Second, there are some notable exceptions to the expected income maximization assumption in the literature; Burdett and Mortensen (1978) and Danforth (1979) look at risk averse job searchers who smooth income over time. Danforth (1979) considers how a risk averse unemployed worker forms her reservation wage when consumption smoothing is explicitly modelled. While Burdett and Mortensen (1978) combine search and an intertemporal income-leisure allocation problem with risk averse workers in an analysis of the labor supply decision.

The paper proceeds by presenting the model in section 2. Then comparative statics and duration dependence are discussed in sections 3 and 4. Section 5 presents simulations of the model and section 6 concludes.

2 The Model

Consider a simple search model in which a worker moves back and forth between unemployment and employment according to a simple two-state Markov process. It is assumed that the worker has a strictly concave utility function which implies that she will want to smooth consumption over states. This smoothing is accomplished by use of capital markets where the worker can place her wealth.

If employed, the worker has no control over the transition probabilities. However, if unemployed, she can manipulate the probability of moving back into employment via her choice of search intensity. To simplify matters, it is assumed that utility is intertemporally separable. This rules out issues such as habit persistence. Furthermore, the instantaneous utility function is separable in consumption and search intensity. Hence, in any given period, the workers utility from a consumption level c and a level of search intensity s, is $v(c,s) = u(c) - e(s)$. As mentioned $u(\cdot)$ is assumed strictly increasing and strictly concave. $e(\cdot)$ is assumed strictly increasing and convex with $e(0) = 0$. Also, for convenience the functions are assumed to be differentiable to the point where it is needed. Wage offers are assumed to come from a degenerate distribution such that all jobs offer the same wage w. k_t will denote the workers level of wealth. $r > 0$ is the constant rate of interest and $\rho > 0$ is the consumers discount rate. The worker's compensation outside employment is captured by b. This will be referred to as unemployment benefits but this could also include income in a second labor market, utility from more leisure time, etc.

The worker's wealth is assumed to be bounded both above and below. The lower bound can be justified as a borrowing limit imposed by the capital market. Aiyagari (1994) points out that a lower bound on wealth can also be motivated by requiring asymptotic present value budget balance (i.e. $\lim_{t\to\infty} k_t/(1+r)^t \geq 0$) combined with non-negative consumption. The upper bound is imposed in order to bound the problem and ensure existence of a solution. A general equilibrium argument could justify that in equilibrium, the interest rate must be such that people do not have infinite wealth (and thereby an infinite supply of capital). So, in general equilibrium and if set high enough, the upper limit will not be binding.

The workers problem can be written as:

$$
\begin{aligned}
\max_{\{c_t, s_t\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} (1+\rho)^{-t} v(c,s) \\
s.t. \quad : \quad & k_{t+1} = (1+r)k_t + wn_t + b(1-n_t) - c_t \\
& c_t \geq 0 \ \forall t \\
& k_{t+1} \in \left[\underline{k}, \bar{k}\right] \\
& s_t \in [0,1] \ \forall t \\
& n_t \in \{0,1\} \text{ follows a Markov process with transition function } Q(n, n', s),
\end{aligned}
$$

where the transition function is given by:

$Q(n,n')$	$n'=0$	$n'=1$
$n=0$	$1-s$	s
$n=1$	η	$1-\eta$

.

By use of the Bellman equation, the problem can be re-written. Let $V(k)$ be the value of employment for a given level of wealth k. Similarly, let $U(k)$ be the value of unemployment for a given k. Denote by k' next periods wealth. Then the problem can be re-stated in functional form as:

$$
\begin{aligned}
V(k) \quad = \quad & \max_{c} \left[u(c) + (1+\rho)^{-1}\left((1-\eta)V(k') + \eta U(k')\right)\right] \\
& s.t. : k' = (1+r)k + w - c \\
& c \geq 0 \\
& k' \in \left[\underline{k}, \bar{k}\right],
\end{aligned}
$$

where the optimal search intensity when working is obviously zero. The value function for unemployment is given by:

$$\begin{aligned} U(k) &= \max_{c,s}\left[u(c)-e(s)+(1+\rho)^{-1}\left(sV(k')+(1-s)U(k')\right)\right] \\ & \quad s.t.: k'=(1+r)k+b-c \\ & \quad c\geq 0 \\ & \quad k'\in\left[\underline{k},\bar{k}\right]. \\ & \quad s\in[0,1]. \end{aligned}$$

Define the feasibility constraint $\Gamma(k,y)=\left\{x\in\mathbb{R}|\underline{k}\leq x\leq\min\left[(1+r)k+y,\bar{k}\right]\right\}$. The problem can be re-stated in the following form:

$$V(k) = \max_{k'\in\Gamma(k,w)}\left[u((1+r)k+w-k')+\frac{(1-\eta)V(k')+\eta U(k')}{1+\rho}\right] \tag{1}$$

$$U(k) = \max_{(k',s)\in\Gamma(k,b)\times[0,1]}\left[u((1+r)k+b-k')-e(s)+\frac{sV(k')+(1-s)U(k')}{1+\rho}\right], \tag{2}$$

which in turn can be written in the form of the two mappings ϕ and ψ:

$$V(k) = \phi(V,U)(k) \tag{3}$$

$$U(k) = \psi(V,U)(k). \tag{4}$$

The question of existence of a solution can be resolved by establishing that ϕ and ψ are contraction mappings on complete metric spaces. First of all, requiring that wealth must lie in the interval $\left[\underline{k},\bar{k}\right]$ ensures boundedness of the problem. That ϕ and ψ are indeed contraction mappings can be verified by appealing to Blackwell's sufficient conditions.[1] Suppose $V_2(k)\geq V_1(k)$ and $U_2(k)\geq U_1(k)$ for all $k\in\left[\underline{k},\bar{k}\right]$. Then it is seen that $\phi(V_2,U_2)\geq\phi(V_1,U_1)$ and $\psi(V_2,U_2)\geq(V_1,U_1)$. Hence it is shown that monotonicity is satisfied. As for the discounting condition it is seen that:

$$\begin{aligned} \phi(V+\alpha,U+\alpha)(k) &= \max_{k'\in\Gamma(k,w)}\left[u((1+r)k+w-k')+\frac{(1-\eta)\left(V(k')+\alpha\right)+\eta\left(U(k')+\alpha\right)}{1+\rho}\right] \\ &= \phi(V,U)(k)+\frac{\alpha}{1+\rho}, \end{aligned}$$

and

$$\begin{aligned} \psi(V+\alpha,U+\alpha)(k) &= \max_{(k',s)\in\Gamma(k,b)\times[0,1]}\left[u((1+r)k+b-k')-e(s)\right. \\ & \qquad \left.+\frac{s\left(V(k')+\alpha\right)+(1-s)\left(U(k')+\alpha\right)}{1+\rho}\right] \\ &= \psi(V,U)(k)+\frac{\alpha}{1+\rho}. \end{aligned}$$

Thus, the discounting condition is also satisfied.

So, it has been established that ϕ and ψ are contraction mappings which ensures existence of a solution. In the following section, the solution will be characterized.

[1] See for example Stokey and Lucas (1989), theorem 3.3.

3 Wealth and Search

The main focus of this paper is to determine how the choice of search intensity varies with wealth. This will play a crucial role in designing for instance optimal unemployment insurance benefit profiles as such wealth effects will imply duration dependence in important variables; first of all, the probability of leaving unemployment will depent on the period already spend in unemployment.

Let $c^w(k)$ and $c^u(k)$ be the optimal choices of consumption given wealth k under employment and unemployment, respectively. Furthermore, let $s(k)$ be the optimal choice of search when unemployed. Finally, let the optimal choices of next period's wealth be defined by $k^w(k) \equiv (1+r)k + w - c^w(k)$ and $k^u(k) \equiv (1+r)k + b - c^u(k)$. It is assumed that the solution is interior. The first order conditions associated with equations (1) and (2) can then be written as:

$$V_k(k) = \frac{1+r}{1+\rho}\left[(1-\eta)V_k(k^w(k)) + \eta U_k\left(k^w(k)\right)\right] \tag{5}$$

$$U_k(k) = \frac{1+r}{1+\rho}\left[s(k)V_k(k^u(k)) + (1-s(k))U_k\left(k^u(k)\right)\right] \tag{6}$$

$$e_s(s(k)) = \frac{V(k^u(k)) - U(k^u(k))}{1+\rho}. \tag{7}$$

And by the envelope theorem it follows that:

$$\begin{aligned} V_k(k) &= u_c(c^w(k))(1+r) \\ U_k(k) &= u_c\left(c^u(k)\right)(1+r). \end{aligned}$$

Unless otherwise stated, subscripts denote derivatives with respect to the subscript variable.

The objective is to characterize how the choice of search intensity varies with wealth. Towards this, (7) is differentiated with respect to k. This yields:

$$\frac{\partial s(k)}{\partial k} = \frac{V_k(k^u(k)) - U_k(k^u(k))}{(1+\rho)e_{ss}(s(k))}\frac{\partial k^u(k)}{\partial k}. \tag{8}$$

Therefore, if $k^u_k(k) > 0$ and $c^w(k) > c^u(k)$ for all k, then $s_k(k) < 0$ globally. Which is to say that search intensity increases when k falls. Furthermore, if k is decreasing over unemployment spells this will yield positive duration dependence for search intensity. In other words, the worker will search harder the longer she has been unemployed. So, now it remains to show these three things and duration dependence will have been established. It is worth noting, that had one followed the traditional approach and assumed the workers to be risk neutral, then search intensity would be constant over the duration of an unemployment spell.

4 Positive Duration Dependence

In the following, duration dependence in the transition probability of leaving unemployment is characterized in terms of the workers marginal valuation of wealth. The key expression being the relative valuation of more wealth ini the state of unemployment relative to employment, $V_k(k) - U_k(k)$. The motivation for characterizing the wealth effect in terms of this difference is that it is our conjecture that assumptions on standard utility functions such as the attitude towards risk implies that $V_k(k)-U_k(k)$ can be signed. Extensive experimentation suggests that $V_k(k)-U_k(k)$ is negative for all k in our model for utility functions in the CARA and DARA classes. This is true, for instance, for the CARA utility function $u(c)=\frac{1-\exp(-\theta c_t)}{\theta}$, where θ is the coefficient of absolute risk aversion.

We establich duration dependence in two steps; first we look at the optimal search intencity as a functon of wealth and second we look at how wealth is formed over the unemployment spell.

Lemma 1 *Suppose V and U are strictly concave, that $r \leq \rho$, and that $e(\cdot)$ is sufficiently convex, then $k_k^u(k)>0$ if and only if $V_k(k)-U_k(k)<0$ for all k.*

Proof. Suppose counter to the claim that for some k^*, $\frac{\partial k^u(k^*)}{\partial k} \leq 0$. By assumption it is given that $V_k(k)-U_k(k)<0$ for all k. Hence, by (8) it must be that $s_k(k^*) \geq 0$. Equation (6) states that:

$$U_k(k^*) = \frac{1+r}{1+\rho}\left[s(k^*)V_k(k^u(k^*)) + (1-s(k^*))U_k\left(k^u(k^*)\right)\right]. \tag{9}$$

By differentiation of (9) with respect to k^*, it is given that:

$$U_{kk}(k^*) = \frac{\frac{[V_k(k^u(k^*))-U_k(k^u(k^*))]^2}{(1+\rho)e_{ss}(s(k^*))} + s(k^*)V_{kk}(k^u(k^*)) + (1-s(k^*))U_{kk}\left(k^u(k^*)\right)}{1+\rho}\frac{\partial k^u(k^*)}{\partial k}.$$

By assumption of concavity of V and U and assuming that $e(\cdot)$ is sufficiently convex, the right hand side will be positive. But this yields contradiction with concavity of U which implies that the left hand side is negative. ■

The intuition behind this results is this. Consider (9) and increase k marginally by Δ. By strict concavity of U it must be that $U_k(k^*+\Delta) < U_k(k^*)$. It also follows from concavity of V and U that $V_k(k^u(k^*+\Delta)) \geq V_k(k^u(k^*))$ and $U_k(k^u(k^*+\Delta)) \geq U_k(k^u(k^*))$. In isolation, this tends to yield contradiction in that the left hand side of (9) is decreasing and the right hand side is increasing. However, it is given that $s(k^*+\Delta) \geq s(k^*)$. Since $0 < V_k < U_k$ this means that in the convex combination between V_k and U_k weight is being switched towards the smaller of the two. This effect counteracts the other effect that tends to increase the right hand side of (9). Hence, in order to establish contradiction with (9) one must make assumptions so as to ensure that this last effect is not too large and that the net effect is that the right hand side of (9) is increasing. One such sufficient condition is that $e(\cdot)$ is sufficiently convex such that $s_k(k^*)$ is sufficiently small.

Finally, in order to establish duration dependence, it must be shown that $k^u(k) < k$ for all k. This is shown in the following lemma.

Lemma 2 *Suppose V and U are concave, then $k^u(k) < k$ for all k if and only if $V_k(k)-U_k(k)<0$ for all k.*

Proof. Equation (6) can be re-written as:

$$U_k(k) - \frac{1+r}{1+\rho}U_k\left(k^u(k)\right) = \frac{1+r}{1+\rho}s(k)\left[V_k(k^u(k)) - U_k\left(k^u(k)\right)\right].$$

Now, assume to the contrary that $k^u(k) \geq k$ for some k. By concavity of $U(\cdot)$ and $r < \rho$, this implies that the left hand side is strictly positive. But by assumption that $V_k(k) - U_k(k) < 0$ for all k it must be that the right hand side is negative, yielding contradiction. ■

This establishes positive duration dependence given concavity of V and U. An unemployed worker will monotonically reduce her wealth during an unemployment spell. And since search intensity is a negative function of wealth, this means that search intensity goes up as an unemployment spell carries on.

Setting $\eta = 0$, which is to say that employment is an absorbing state in the Markov process, significantly simplifies the model. This assumption is made in both Acemoglu and Shimer (1999) and Danforth (1979). In Danforth (1977), it is shown that under the assumption of DARA utility, the certainty equivalent associated with a generalized lottery is an increasing function of wealth. The main insight of Danforth (1979) is that if the state of employment is a trap in the Markov process, then there is no lottery associated with employment. However, the state of unemployment offers a lottery where with some probability the worker remains unemployed and with the residual probability she enters into employment. The worker in Danforth (1977) faces a distribution of possible wage offers and the main problem here is to determine the worker's reservation wage which equates the value of going into employment at that wage with the value of remaining unemployed and sample another offer. Thus, as wealth increases the certainty equivalent associated with unemployment increases while the same effect does not apply to the state of employment. Hence, the reservation wage must increase in wealth. Combined with showing that wealth must decrease with the duration of the unemployment spell, duration dependence is obtained, in that the reservation wage must fall over the duration of an unemployment spell. While, as of yet, duration dependence has not been established in the present paper for $\eta = 0$, the above results suggest that for main parts of the proof of duration dependence, the assumption generates what we are looking for. However, the results also emphasize that once employment is no longer a trap, duration dependence is much harder to establish. Essentially, in this case employment is also a lottery, in which case the certainty equivalent associated with employment also increases with wealth. So, now the question becomes; which certainty equivalent rises by the most. And this is less obvious.

Thus, previous literature would suggest that CARA utility functions will be instrumental in generating positive duration dependence. This is essentially done by making the value function in the state of unemployment relatively more concave in wealth than the value function in the state of employment. Experimentation with the model in this paper suggests that this is true. Another effect that this paper brings into the light is the effect of the lower bound of wealth. The computer simulations show that this liquidity constraint forces concavity into the value functions and in particular U which is directly affected by a lower bound since wealth is decreasing in the state of unemployment. Thus, the liquidity constraint also has the potential of driving positive duration dependence.

4.1 Concavity of the Value Functions

The above duration dependence is proven under the assumption that V and U are concave. It is not clear that concavity is always satisfied. The following arguments may provide some insights into the problem.

One way of proving concavity would be to argue via the fact that if the contraction mappings ϕ and ψ map concavity into concavity then one would know that the fix points have to be within the set of concave functions. It is immediately shown that if one assumes that V^n and U^n are concave functions, then it must be that $V^{n+1} = \phi(V^n, U^n)$ is strictly concave. The problem is showing that $U^{n+1} = \psi(V^n, U^n)$ is concave. The argument goes something like this.

Define $F(k,k') \equiv u((1+r)k + b - k')$. By definition of $u(\cdot)$, $F(k,k')$ is strictly concave in (k,k'). Pick some $k_0, k_1 \in \left[\underline{k}, \bar{k}\right]$, $\lambda \in (0,1)$ and define:

$$\begin{aligned}
k'_0 &\equiv k^u(k_0),\ k'_1 \equiv k^u(k_1)\\
\vec{k}_0 &\equiv (k_0, k'_0),\ \vec{k}_1 \equiv (k_1, k'_1)\\
s_0 &\equiv s(k_0),\ s_1 \equiv s(k_1)\\
\vec{k}_\lambda &\equiv (k_\lambda, k'_\lambda) = \lambda \vec{k}_0 + (1-\lambda)\vec{k}_1\\
s_\lambda &= \lambda s_0 + (1-\lambda)s_1.
\end{aligned}$$

Hence, by concavity of $F(\cdot)$ and convexity of $e(\cdot)$, we have:

$$\begin{aligned}
F(\vec{k}_\lambda) &> \lambda F(\vec{k}_0) + (1-\lambda)F(\vec{k}_1)\\
e(s_\lambda) &< \lambda e(s_0) + (1-\lambda)e(s_1).
\end{aligned}$$

A proof of concavity would then go something like:

$$\begin{aligned}
\lambda U^{n+1}(k_0) + (1-\lambda)U^{n+1}(k_1) &= \lambda\left[F(\vec{k}_0) - e(s_0) + \frac{s_0 V^n(k'_0) + (1-s_0)U^n(k'_0)}{1+\rho}\right]\\
&\quad +(1-\lambda)\left[F(\vec{k}_1) - e(s_1) + \frac{s_1 V^n(k'_1) + (1-s_1)U^n(k'_1)}{1+\rho}\right]\\
&= \left[\lambda F(\vec{k}_0) + (1-\lambda)F(\vec{k}_1)\right] - \left[\lambda e(s_0) + (1-\lambda)e(s_1)\right]\\
&\quad + \frac{s_0\lambda V^n(k'_0) + s_1(1-\lambda)V^n(k'_1)}{1+\rho}\\
&\quad + \frac{(1-s_0)\lambda U^n(k'_0) + (1-s_1)(1-\lambda)U^n(k'_1)}{1+\rho}.
\end{aligned}$$

By strict concavity of $F(\cdot,\cdot)$ and strict convexity of $e(\cdot)$, it follows that:

$$\begin{aligned}
\lambda U^{n+1}(k_0) + (1-\lambda)U^{n+1}(k_1) &< F(\vec{k}_\lambda) - e(s_\lambda)\\
&\quad + \frac{s_\lambda\left(\lambda V(k'_0) + (1-\lambda)V(k'_1)\right) - (s_\lambda - s_0)\,\lambda V(k'_0)}{1+\rho}\\
&\quad + \frac{(1-s_\lambda)\left(\lambda U(k'_0) + (1-\lambda)U(k'_1)\right) + (s_\lambda - s_0)\lambda U(k'_0)}{1+\rho}\\
&\quad - \frac{(s_\lambda - s_1)(1-\lambda)\left(V(k'_1) - U(k'_1)\right)}{1+\rho}.
\end{aligned}$$

Concavity of V^n and U^n then yields:

$$\begin{aligned}
\lambda U^{n+1}(k_0)+(1-\lambda)U^{n+1}(k_1) &< F(\vec{k}_\lambda)-e(s_\lambda)+\frac{s_\lambda V^n(k'_\lambda)+(1-s_\lambda)U^n(k'_\lambda)}{1+\rho}\\
&\quad -\frac{(1-\lambda)(s_1-s_0)\lambda V^n(k'_0)-\lambda(s_1-s_0)(1-\lambda)V^n(k'_1)}{1+\rho}\\
&\quad +\frac{(1-\lambda)(s_1-s_0)\lambda U^n(k'_0)-\lambda(s_1-s_0)(1-\lambda)U^n(k'_1)}{1+\rho}\\
&= F(\vec{k}_\lambda)-e(s_\lambda)+\frac{s_\lambda V^n(k'_\lambda)+(1-s_\lambda)U^n(k'_\lambda)}{1+\rho}\\
&\quad -\frac{\lambda(1-\lambda)(s_1-s_0)}{1+\rho}\left\{\left[V^n(k'_0)-V^n(k'_1)\right]-\left[U^n(k'_0)-U^n(k'_1)\right]\right\}.
\end{aligned}$$

By the principle of optimality, we finally get:

$$\begin{aligned}
\lambda U^{n+1}(k_0)+(1-\lambda)U^{n+1}(k_1) &< U^{n+1}(k_\lambda)\\
&\quad +\frac{\lambda(1-\lambda)(s_0-s_1)}{1+\rho}\left\{\left[V^n(k'_0)-U^n(k'_0)\right]-\left[V^n(k'_1)-U^n(k'_1)\right]\right\}\\
&= U^{n+1}(k_\lambda)+\lambda(1-\lambda)(s_0-s_1)\left(e_s(s_0)-e_s(s_1)\right). \qquad (10)
\end{aligned}$$

The last term on the right hand side of (10) is always positive, which means that it is not given that $U^{n+1}(k_\lambda)\geq \lambda U^{n+1}(k_0)+(1-\lambda)U^{n+1}(k_1)$, which would prove concavity. But one can make the last term arbitrarily small by assuming that $e(\cdot)$ is sufficiently close to linear in which case $e_s(s_0)-e_s(s_1)$ can be made arbitrarily close to zero and this would yield concavity. However, this approach violates one of the assumptions made in claim 1. So, this is potentially a problem. On the other hand, in general the more convex $e(\cdot)$ is, the closer s_0-s_1 is to zero. But of course, this does not mean that $(s_0-s_1)\left(e_s(s_0)-e_s(s_1)\right)$ is going to zero.

Other ways of trying to prove concavity yield similar oppositely working effects. Consequently, the theoretical analysis is short of a concrete and direct answer as to the direction of the duration dependence of search intensity. In the next section, simulations of the model show that concavity of the value functions are obtained for a broad range of model specifications.

5 Simulation of the Model

The following section presents the results of a few numerical simulations of the model. The model is simulated by use of value function iteration.

5.1 Value Function Iteration

This approach attempts to approximate the value functions $V(k)$ and $U(k)$. Based on these approximations, one can then deduce the policy functions. As stated in section 2, the model can be written as:

$$\begin{aligned} V(k) &= \max_{k' \in \Gamma(k,w)} \left[u((1+r)k + w - k') + \frac{(1-\eta)V(k') + \eta U(k')}{1+\rho} \right] \\ &= \phi(V,U)(k) \\ U(k) &= \max_{(k',s) \in \Gamma(k,b)\times[0,1]} \left[u((1+r)k + b - k') - e(s) + \frac{sV(k') + (1-s)U(k')}{1+\rho} \right] \\ &= \psi(V,U)(k), \end{aligned}$$

where ϕ and ψ were shown to be contraction mappings. The value function iteration method is motivated by the properties of these contractions. One poses some initial guess $V^0(k)$ and $U^0(k)$, and by use of the mappings ϕ and ψ, the iteration method then computes the sequences:

$$\begin{aligned} V^{n+1}(k) &= \phi(V^n, U^n)(k) \\ U^{n+1}(k) &= \psi(V^n, U^n)(k), \text{ for } n = 0, 1, 2, \ldots . \end{aligned}$$

Since ϕ and ψ are contraction mappings, it is given that this sequence will eventually converge to the fix point of this mapping, $V(k)$ and $U(k)$.[2]

Specifically, the simulations use a Chebyshev polynomial specification.[3] One then tries to fit the polynomials to the value functions on a given grid of wealth levels, $\vec{k} = (k_0, k_1, \ldots, k_l)$. This grid is based on the zeros of a Chebyshev polynomial. In this way, the grid places more weight in the tails of the wealth interval $\left[\underline{k}, \bar{k}\right]$. The simulation chosen is a collocation method. Hence, in each step, the method solves for as many parameter values in the polynomials as there are grid points in the grid. In this way, the problem will be exactly identified.

A Chebyshev polynomial of degree l can be written as $\Upsilon^{l-1}(k) = a_0\Upsilon_0(k) + a_1\Upsilon_1(k) + a_2\Upsilon_2(k) + \cdots + a_l\Upsilon_l(k)$, where $\Upsilon_0(k) = 1$, $\Upsilon_1(k) = k$, and $\Upsilon_i = 2k\Upsilon_{i-1}(k) - \Upsilon_{i-2}(k)$ for all $i \geq 2$. Define $\Upsilon(k) \equiv (\Upsilon_0(k), \Upsilon_1(k), \ldots, \Upsilon_l(k))$, $\alpha^n \equiv (\alpha_0^n, \alpha_1^n, \ldots, \alpha_l^n)'$, and $\beta^n \equiv (\beta_0^n, \beta_1^n, \ldots, \beta_l^n)'$. Consequently, the n-step approximation of the value function can be written as,

$$\begin{aligned} V^n(k) &= \Upsilon(k) \cdot \alpha^n \\ U^n(k) &= \Upsilon(k) \cdot \beta^n. \end{aligned}$$

In each step, given V^n and U^n, the mappings ϕ and ψ map each point in the grid $\vec{k}$ into to some point on the real line. Let this mapping be given by the $(l+1 \times 2)$ matrix $Y^n(\vec{k})$,

$$Y^n(\vec{k}) \equiv \left(\phi(V^n, U^n)(\vec{k}), \psi(V^n, U^n)(\vec{k}) \right).$$

Hence, the $n+1$ step approximation is given by the solution to the following,

$$\Upsilon(\vec{k}) \left(\alpha^{n+1}, \beta^{n+1} \right) = Y^n(\vec{k}), \tag{11}$$

[2]This solution method is by no means fast. In particulay, this problem is an infinite horizon type problem and the rate of convergence is only linear at rate $(1+\rho)^{-1}$. Alternatively, one could consider acceleration methods such as policy function iteration. Both Judd (1998) and Christiano and Fisher (1997) consider this in more detail.

[3]A fair amount of simulations were done on ordinary polynomials as well. Both specifications seem to capture the essentials of the model.

where

$$\Upsilon(\vec{k}) \equiv \begin{pmatrix} \Upsilon(k_0) \\ \Upsilon(k_1) \\ \vdots \\ \Upsilon(k_l) \end{pmatrix}.$$

The use of Chebyshev polynomials has the advantage that one does not have to invert a potentially unpleasant matrix. It is given that:[4]

$$\Delta \equiv \left(\Upsilon(\vec{k})'\Upsilon(\vec{k})\right)^{-1} = \frac{1}{l+1}\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2 \end{pmatrix}.$$

Hence, (11) can be re-written as:

$$\begin{aligned} \Delta\Upsilon(\vec{k})'\Upsilon(\vec{k})\left(\alpha^{n+1},\beta^{n+1}\right) &= \Delta\Upsilon(\vec{k})'Y^n(\vec{k}) \\ &\Updownarrow \\ \left(\alpha^{n+1},\beta^{n+1}\right) &= \Delta\Upsilon(\vec{k})'Y^n(\vec{k}). \end{aligned} \tag{12}$$

(12) is then used to update the parameter values. One can choose one's stopping rule in many ways. The stopping rule applied in the present simulations, was to stop once the absolute value of $\left(\alpha^{n+1}-\alpha^n,\beta^{n+1}-\beta^n\right) < \xi$, where ξ is chosen appropriately small and for the implied policy functions not changing much between iterations either.

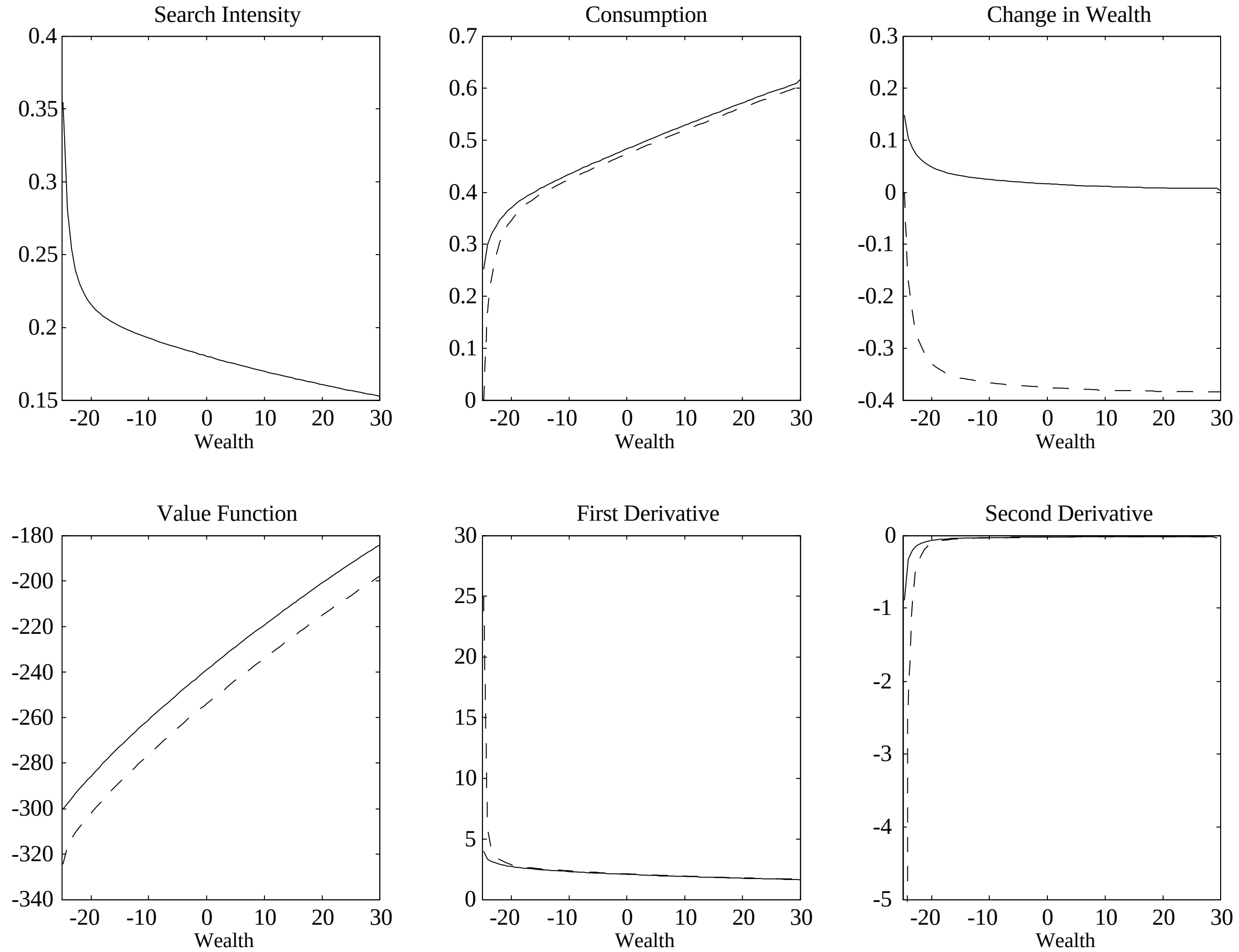
5.2 Simulations

The simulations show that concavity obtains for a broad range of model specifications. Extensive sensitivity analysis was performed with utility functions in the CARA and DARA classes and with well behaved convex search cost functions. For these specifications it was not possible to generate simulation results that did not display concavity.

The following simulation shows a fairly typical picture. The period length is supposed to be one month. The model specifications are: $u(c) = \ln(c)$, $e(s) = \frac{10s}{1-s}$, $r = (1+.0495)^{\frac{1}{12}} - 1$, $\rho = (1+.05)^{\frac{1}{12}} - 1$, $\eta = .02$, $w = .5$, $b = .1$, $\underline{k} = \frac{-b}{r} + .01 = -24.78$, $\bar{k} = 30$. The lower bound is set such that the unemployed worker can exactly cover her interest payments at the lower bound, but consumption is subsequently almost zero. The model does not allow for default on the loans. Hence, it is not clear how one would deal with wealth levels below this lower bound. Figure 1 shows the policy functions associated with this model. The dashed lines depict the function for the unemployed state. First of all, it is seen that search intensity is a decreasing function of wealth.

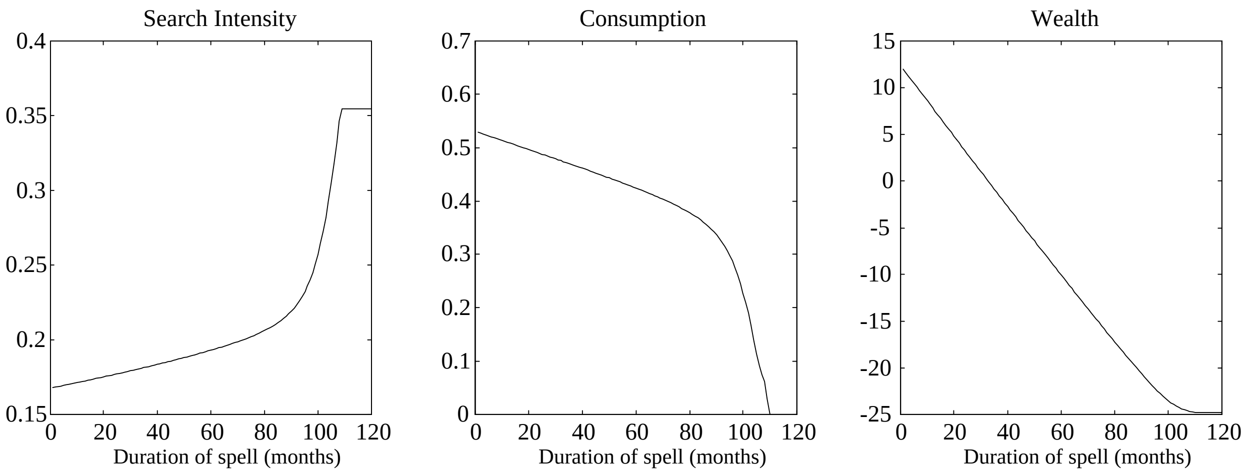
[4]See for example Christiano and Fisher (1997). It is at this particular point, this approach has an advantage over say, ordinary polynomials. If one wants to do a rather large number of grid points, inverting the equivalent to $\Upsilon^n(\vec{k})$ can yield numerical problems in that some numbers will be too small and matlab starts rounding them off. Usually this would not be too much of a problem. However, in some cases, this badly scaled matrix would result in cycles in the iteration procedure and convergence could not be obtained.

Figure 1: Policy and Value Functions.



Furthermore, wealth is decreasing monotonically when the worker is unemployed. Hence, this specification yields positive duration dependence. Consumption is always greater when employed than when unemployed and we also see that the employed worker is always saving some of her income to be able to smooth consumption in the event that she looses her job. The simulation also shows how the liquidity constraint affects the problem in that the closer the worker is to the liquidity constraint, the stronger the precautionary savings motive gets and the stronger the urge to move out of unemployment becomes. This is reflected by the sharply increasing search intensity. Note that decreases in the interest rate can make the even the employed worker reduce her wealth if she is far enough away from the lower bound on wealth. There is still a precautionary savings motive and savings are still working to smooth consumption, but at the higher wealth levels, it is dominated by the fact that the interest rate is so much smaller than the subjective discount rate. An obvious extension to the model is to do a general equilibrium argument where the interest rate

Figure 2: Unemployment Duration Dependence.



is determined by supply and demand of capital.

The second row of figures in figure 1 shows the value functions (again the dashed lines are for the unemployed state) and the derivatives of the value functions. The main point here is that concavity is obtained. In the figure for the second derivative, the lower bound has been truncated. In fact, the second derivative for $U(k)$ close to $\underline{k}$ is much lower than what can be seen from the figure.

Figure 2 explicitly shows the unemployment duration dependence. The unemployment hazard rate is equivalent to the search intensity here and this then shows directly the kind of shape of the hazard rate, the model would predict. It is clear, that the closer the unemployed worker is to the lower bound, the stronger the positive duration dependence is. This would suggest that if the unemployment hazard rate of one group of unemployed workers shows stronger postive duration dependence relative to another group of workers, this could be due to the wealth effect and that the group in question is closer to their borrowing constraint than the other group.

The main conclusions of the simulations seem very rubust to changes in the specifications. Simulations were done with other types of utility functions, such as $u(c) = \frac{1+c^{1-\gamma}}{1-\gamma}$ or $u(c) = \frac{1-e^{-\gamma c}}{\gamma}$. Both yield the same results as above. If one increases the measure of risk aversion, the level of precautionary savings increase. If one imposes too much curvature in the utility function, convergence becomes illusive and is very sensitive to the initial guess. This is most likely due to the approximation procedure that has a harder time capturing the curvature in the value functions. Other functional forms of the cost of search were also tried, i.e. $e(s) = a_1 s^{a_2}$. Again, this did not affect results in any qualitative manner. The more convex the function is, the less variation one sees in the search intensity. Increases in η results in more savings when the worker is employed. Increasing the interest rate generally increases savings. However, once one increases the interest rate too much, convergence is no longer given. The point where convergence fails seem to be around where the interest rate is larger than the sum of ρ and the transition probabilities.[5] This is the only parameter that the model seems to be sensitive to. And again, it is hard to say whether convergence fails because of problems with the approximation or whether convexities actually show up.

Finally, one should make the point that the theoretical analysis differs from the simulations in the way, that the borrowing constraint is not taken into account in the theoretical analysis. And one might suspect that some of the concavity of the value functions stems from the borrowing constraint. Simulations were done for higher values for the lower bound on wealth, where the unemployed worker's consumption at the lower bound would be less devastating than in the above. Clearly, this results in less steep increases in the search intensity as wealth moves towards the lower bound. However, the basic characteristics of the model remain. The value functions are still concave and hence, search intensity still exhibits duration dependence.

6 Conclusion

This paper makes the point that once one introduces risk aversion and an actual utility maximization problem into a basic search model, conclusions about say duration dependence of certain parameters in the model may be qualitatively affected. It was shown via simulations of a basic model with endogenous search intensity that for a broad range of model specifications, search intensity exhibits positive duration dependence via its dependence on the worker's wealth level. This result can most likely be carried directly over to conclusions about reservation wages (negative duration dependence). These thoughts are not new to the literature. In fact, Danforth (1979) analytically establishes negative duration dependence of reservation wages. However, as this paper has shown, Danforth's (1979) assumption that the worker can never be separated from a job is crucial for establishing these results analytically. It still remains to establish duration dependence analytically for the more general case where the worker moves back and forth between employment and unemployment. But one can definitely say, that the wealth effect will make measures such as reservation wages and search intensities move over the durations of employment and unemployment spells. This conclusion is important for micro studies of duration dependence in the labor market literature as it emphasizes that if it is possible to include indicators of each worker's wealth, one should definitely do so. Furthermore, the results also suggest that it is important to include wealth effects into such problems as optimal design of unemployment benefits.

[5] If one analyses the continuous time version of the model, it is in fact an analytical condition for positive duration dependence that $r < \rho + \eta + s(k)$ for all k.

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