Quality enforcement as a public good (preliminary and incomplete)

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Abstract

A competitive market for an experience good is considered where high quality is enforced by repeated game trigger strategies, as e.g. in Klein and Leffler (1981). The good is demanded by two types of customers, long run (LR) and short run (SR), the former buying repeatedly, the latter only once. In this setting quality enforcement has public good characteristics: SR buyers can free ride on quality enforcement by LR buyers but, by doing so, they may prevent LR buyers from punishing firms for producing low quality. We characterize equilibria in different market institutions and show that non-exclusivity has a negative impact on quality enforcement when the market institution provides some public information.

1 Introduction

South Tyrol in Northern Italy is the most important apple growing region in Europe. Apple growers have the possibility to sell their products trough three different channels, through a cooperative, in auctions or directly to wholesale traders. Buyers are either long-term whole sale traders or traders that buy only occasionally, e.g. traders from other regions that have to cover a shortfall in the harvest in their own region. An apple grower can increase his quantity on the market by mixing low quality apples together with high quality and trying to sell them off as high quality. The market institution differ in their quality control. Cooperatives inspect and categorize the quality of each shipment by growers. In the auctions, quality is not verified, but the grower is identified by his membership number. The cases are large (approximately 340 kg) and create the cheating possibility for growers. A grower that obtained a reputation for doing so, was only able to sell at low prices in the auction. He tried to sell under a different membership number, but was not allowed by the auction organizers to do so.

This market is characterized by three features that form the basis for our model, the possibility of moral hazard in the provision of high quality, the existence of both long run and short run buyers and incomplete information exchange about past qualities. Also, the market institutions used differ in their treatment of the quality enforcement problem. This paper analyses in a repeated game setting how the outcome and efficiency of different market institutions, especially auctions and decentralized bargaining markets, are affected by these features.

We consider a market for an experience good with a finite number of buyers and sellers where high quality is enforced by repeated game trigger strategies, as e.g. in Klein and Leffler (1981). The good is demanded by two types of customers, long run (LR) and short run (SR), the former buying repeatedly, the latter only once. In the auction setting the identity or type of the seller cannot directly influence the allocation. All bids are treated in the same way and short run buyers cannot be discriminated against in favor of long run buyers. If past trades, but not the produced qualities are publicly observed, quality enforcement obtains public good characteristics. Observing past trading prices and quantities makes it possible that SR buyers buy from firms with high price histories. SR buyers can, therefore, free ride on quality enforcement by LR buyers but, by doing so, they may prevent LR buyers from punishing firms for producing low quality. The presence of short run buyers creates an additional problem for enforcement. When a firm starts to produce low quality, its LR customers migrate to other firms but punishment is weakened by ongoing sales to SR buyers who fill in for the reduction in demand.. With some probability all cheated LR customers get replaced by SR buyers who are not able to signal low quality to the next generation. A partial market breakdown results and only few firms can be supported as high quality producers. As a consequence a partial market breakdown might occur. However, the outcome of auction markets can be improved by coordination among LR buyers. Coordination strategies allows a better use of information generated by a market institution. It uses a signal of the auction institution and makes it to carry information about cheating, which it is not able to without coordinated strategies. Auction markets with a large information flow, as e.g. when all bids are published, produce high punishment probabilities even without coordination. That is, additional information generated through coordination can act as a substitute for information inherent in a market institution.

In decentralized markets, where only match specific information is available, some firms can form a long term relationship with long run buyers. Those firms sell high quality to LR buyers, but SR buyers receive only low quality. Better outcomes can be obtained if firms are allowed to price discriminate between LR and SR buyers. With price discrimination, SR buyers reward the LR buyers for their enforcement activity. Without discrimination or exclusivity, information spill-overs from LR to SR can reduce welfare, while information flows among LR can only increase welfare. Depending on the market institution, more public information can improve the outcome but can also make it worse.

To illustrate the role of exclusionary or preferential treatment in our repeat purchase setting consider the following hypothetical example.

A restaurant has build up a customer base of repeat purchasers and is selling high quality to them. Now, the restaurant receives a very good review in a widely published restaurant guide. Assume, that the restaurant guide will not be updated or only after a long time.

Through the restaurant review many short-term customers, e.g. tourists, learn about the high quality history of the restaurant and will show up at its doors. Given the limited capacity of the restaurant, its owner has to decide whether to reserve seats for its regular customers or to accept short term customers who might be willing to pay more. In the latter case, the restaurant can lower its quality and maybe increase its price and sell to a sequence of short run customers that are attracted by the review. The tourists will see a full restaurant and high prices, but do not learn from the experience of previous tourists. This seems to be, therefore, a profit increasing strategy for the firm. However, with the absence of long term customers, enforcement is lost, and the above scenario cannot be an equilibrium, if tourists have rational beliefs. An equilibrium, where the restaurant attracts both long run and short run customers, might require mixed strategies which result in reduced punishment probabilities and a partial market break down.

If, in contrast, the restaurant gives preferential treatment to its long term customers and restricts sales to tourists, enforcement can be maintained and the long term customers and some short term customers benefit from high quality production. If the restaurant guide is frequently updated, the information asymmetry is strongly reduced and enforcement is done by both long run and short run customers.

Therefore, the interaction of non-discrimination between LR and SR buyers, and the existence of some but limited information spill-overs is the source of the enforcement problem and the potential market breakdown. Past enforcement obtains public good characteristics, whose rivalrous consumption by short run buyers destroys it. Price discrimination, such as loyalty discounts, makes enforcement to a private good and improves welfare.

An evident strand of literature related to our model is the literature on quality enforcement. A seminal contribution is Klein and Leffler (1981). In this model firms face the moral hazard problem of producing either high or low quality. Quality is an experience good, neither observable nor verifiable. hence the only way to enforce high quality is through repeat purchase. In Klein-Leffler's model all the buyers (as well as the firms) are infinitely lived. Furthermore after one buyer has been cheated all other buyers get informed and together collectively punish the firm by never trading with her again. This allows a very effective detection of "deceptive production" (or cheating). This "shouting" of every buyer who got deceived is however not dealed with as a strategic variable. Buyers can not choose whether to use it, nor are there any costs involved in shouting. If shouting were made strategic it is however unclear whether in equilibrium only cheated players would start shouting and whether buyers would invest in a "shouting" technology.. In our model we take only the market institution as exogenous. We also do not allow for a shouting technology, though in our model buyers would not necessarily want to invest in it.[1]

Papers on Folk theorems with a public signal by Fudenberg et al (1991), (1994) and (1994) are in a similar, though much more general spirit. Though also there the public signals remains an exogenous variable.

The next section presents the model and analyzes the repeated game. The following sections analyze the auction and decentralized bargaining markets in turn.

2 The Market

Consider a repeated market for an experience good with a finite number of sellers and buyers. Sellers are identical, infinitely lived and can produce at most q units per period of either high quality $\overline{\theta}$ at unit cost $\overline{c}$, or of low quality $\underline{\theta}$ at unit cost $\underline{c}$, with $\overline{c} > \underline{c}$. The good is not storable. Buyers differ in their valuation and in their time horizon. Consumers want to consume at most one unit per period during their life. A consumer with valuation β receives a utility of $\beta\theta - p$. The distribution of consumer types remains constant in each period and is common knowledge. There are m_k buyers with valuation β_k, with $k = 1..K$ and $\beta_0 < \beta_1 < ... < \beta_K$. A fraction λ of consumers of each valuation type are infinitely lived (long run buyers LR). A fraction $(1 - \lambda)$ live only for one period and are replaced in the next period by a new generation (short run buyers SR). Sellers and long run buyers have the same discount factor δ. (Note: Discount factor of LR buyers is in most settings not relevant, relevant is only their memory.) Consumers cannot observe the quality of the good at the time of purchase, but learn it during consumption. High quality will be enforced by the threat of punishment in the repeated game. With respect to the timing, we assume that firms have to produce at the beginning of the period, so that the firm does not have any information

[1] The point of introducing short run buyers in our model is to introduce buyers who can not convey information that they got cheated, even if a shouting technology were available.

about the consumer that buys the product and the market outcome when deciding which quality to produce. This rules out, that the firm can make its quality choice conditional on the identity of the buyer, if it learns this as a consequence of the market process.

Consumer choice

If consumers know which firms are supposed to produce high quality on the equilibrium path, their demand functions will be the same as under full information.

Short run buyers are unconcerned about the future and will therefore always play their static (one period) best response. Long run buyers could choose an action that does not maximize their period payoff if the repeated game yields a higher present value payoff for another choice. In our market structure it is not necessary for long run buyers to pick an action that does not maximize their period utility on the equilibrium path. Buyers with correct expectations will demand

$$\begin{array}{lccc} \text{high quality if} & \beta\bar{\theta}-\bar{p}\geq\beta\underline{\theta}-\underline{p} & \text{and} & \beta\bar{\theta}-\bar{p}\geq 0 \\ \text{low quality if} & \beta\bar{\theta}-\bar{p}\leq\beta\underline{\theta}-\underline{p} & \text{and} & \beta\underline{\theta}-\underline{p}\geq 0 \end{array}$$

There are two cases to consider. First assume that $\overline{p}/\underline{p} \geq \overline{\theta}/\underline{\theta}$. Then high valuation buyers with $\beta \in \left[\left(\overline{p}-\underline{p}\right)/\left(\overline{\theta}-\underline{\theta}\right), \beta_K\right]$ will buy high quality, medium valuation buyers with $\beta \in \left[\underline{p}/\underline{\theta}, \left(\overline{p}-\underline{p}\right)/\left(\overline{\theta}-\underline{\theta}\right)\right]$ will buy low quality, and low valuation buyers with $\beta \in \left[\beta_0, \underline{p}/\underline{\theta}\right]$ will not buy, with possible empty sets. In the opposite case, where $\overline{p}/\underline{p} < \overline{\theta}/\underline{\theta}$, high valuation buyers with $\beta \in \left[\overline{p}/\overline{\theta}, \beta_K\right]$ will buy high quality, and other buyers (with valuation $\beta \in \left[\beta_0, \overline{p}/\overline{\theta}\right]$) will not buy, again with possible empty sets. We denote the demand for high and low quality units for given prices $\left(\overline{p}, \underline{p}\right)$ by $\overline{D}\left(\overline{p}, \underline{p}\right)$ and $\overline{D}\left(\overline{p}, \underline{p}\right)$, with indices LR and SR when we refer to demand by long run or short run buyers respectively.

In order to reduce the number of possible cases for demand functions to consider and to rule out trivial equilibria we make the following assumptions on the full information Walrasian equilibrium and the full punishment equilibrium. These assumptions are not necessary for the qualitative results.

Assumption 1: $\overline{\theta}/\underline{\theta} = \overline{c}/\underline{c}$.

This assumption has two important implications. First, that total production is independent of the composition of total production between high and low quality since $\underline{\theta}\beta - \underline{c} \geq 0 \Leftrightarrow \overline{\theta}\beta - \overline{c} \geq 0$; and secondly, that producing low quality is never optimal in the first best (FB), (i.e., with verifiable quality) since $\overline{\theta}\beta - \overline{c} \geq \underline{\theta}\beta - \underline{c}$ for all $\beta \geq \overline{c}/\overline{\theta} = \underline{c}/\underline{\theta}$.

Assumption 2: *Free Entry of firms* and $\beta_0 < \underline{c}/\underline{\theta}$, $\beta_1 = \underline{c}/\underline{\theta} + \epsilon$ (for some small ε)

Together with Assumption 1, Assumption 2 implies that the number of active firms in the FB is determined by $n = \sum_1^K \beta_k m_k/q$. Note that the same number of firms will also be active in our second best world (where quality is not verifiable) since free entry ensures that $\underline{p} = \underline{c}$. Notice, that n is not necessarily unique:

Next notice that in our second best world high quality can only be enforced by future "punishment" that is harsh enough to induce the firm to forego a current gain of $\overline{c} - \underline{c}$. In our model the "harshest" punishment occurs if a deviating firm (that is, a firm that sells low quality at the high price $\overline{p}$) is immediately detected and if this firm is unable to participate in the high quality market in any subsequent period (ex post observable quality + grimm strategies by buyers). The harshest punishment will induce a firm to produce high quality if $\overline{c} - \underline{c} \leq (\overline{p} - \underline{p})/\delta$. In the sequel we call the price $\overline{p} = \underline{c} + (\overline{c} - \underline{c})/\delta$, the minimum enforcement price under full punishment and denote it by $\overline{p}^{FP}$. The maximal number of firms that can sell high quality at this price is denoted by $\overline{n}^{FP}$. This number is determined by $\overline{n}^{FP} = \overline{D}(\overline{p}^{FP}, \underline{c}) = \sum_{\overline{k}}^{K} \beta_k m_k / q$, where $\overline{k}$ is the lowest k for which $\overline{\theta}\beta_k - \overline{p}^{FP} \geq \underline{\theta}\beta_k - \underline{c}$. Notice, that $\overline{p}^{FP}$ is strictly higher than the Walrasian equilibrium price for any $\delta < 1$. This implies that asymmetric information is welfare reducing since the number of high quality firms in equilibrium $\overline{n}^* \leq \overline{n}^{FP} < n$ for any $\delta < 1$.

Assumption 3: $\overline{n}^{FP} > 0$. Assumption 3 is a condition on the βs and on δ. It requires that at the minimum enforcement price under full punishment there is positive demand. This rules out cases where high quality can never be enforced.

Under Full information, in a Walrasian equilibrium only high quality units are produced and prices are equal to unit cost, i.e. $\bar{p} = \bar{c}$, $\underline{p} = \underline{c}$, $\bar{n} = n = \overline{D}(\infty, \bar{c}) = \underline{D}(\infty, \underline{c})$ and $\underline{n} = 0$. In any asymmetric information equilibrium $\overline{p}^* > \overline{c}$, $\underline{p}^* = \underline{c}$ and $\overline{n}^* + \underline{n}^* = n$. The welfare loss under these assumption comes from the composition of production, fewer high quality units and more low quality units are produced than in the first best. The aggregate welfare of different market institutions can simply be ranked by the number of high quality units produced.

2.1 The repeated Game

In our model only the consumer of a firms product learns directly about the produced quality. Other buyers and sellers learn only from observed consequences in the market stage game. The situation, therefore constitutes a repeated game with imperfect monitoring where players observe public and also private signals about the actions of other players. The equilibrium in this case is in general difficult to characterize. The case of public signals, i.e. all players receive the same (imperfect) signals, has been thoroughly analyzed. Fudenberg, Levine, Maskin (1994) showed that under some assumption on the signal a Folk theorem obtains. Fudenberg Levine (1994) derived sufficient conditions when short run players are also present. If players are symmetrically informed about actions of others they agree on the need to punish. If players receive conflicting signals, only those buyers that were cheated will avoid the cheating firm, while other buyers will still buy from it.

Consequently, punishment by some buyers might not have any immediate payoff consequences for the cheating firm. Fudenberg and Levine (1991) showed that a partial Folk Theorem is possible if players are either perfectly patient or are only required to play ε-best strategies. Recently Kandori and Matsushima (1998) and Compte (1998) show that a Folk theorem can be obtained when players are allowed to send messages.

In our case, we are interested in the informational properties of market institutions, therefore we do not allow players to directly communicate and we restrict ourselves to strictly positive discount rates. We derive approximate bounds on punishment probabilities and size of the high quality market depending on the market institution and the amount of coordination among buyers. Using the special structure of our market game, we obtain strategies that are functions of a simple state space. The evolution of the game has as a consequence a Markov property and the standard tools of dynamic programming can be used.

The market is divided into two submarkets, one designed for high quality and one for low quality. Enforcement of high quality production requires that the access of firms to the high market is rationed. As in Klein Leffler (1981), firms need a positive expected excess profit stream that they will lose if they start to produce low quality. The required price premium reduces the demand and only a restricted number of firms can participate in the high quality market. Initially, firms are arbitrarily selected for participation in the high quality market. Firms that have been detected cheating will be replaced by new firms from the pool of non producing and low quality producing firms. This, however, does not occur on the equilibrium path. Production for the low quality market does not have any incentive problems. Therefore, in the following analysis we can restrict attention exclusively on the high quality market.

High quality production will be enforced with trigger strategies. Buyers that have been cheated or learned from others that a firm is cheating will belief that this firm will produce low quality for the rest of the game and never again buy from this firm. This means that a firm that cheats some LR buyers will lose their demand and will lose all high quality demand once the deviation has been publicly signalled.

If a short run buyer has been cheated, this information disappears from the game. SR buy only once and have no possibility to signal to other players which quality they received.

The history of the game with respect to each firm is in one of four different phases, the no-cheating phase on the equilibrium path, the cheating phase, when the firm produces low quality but it is not yet publicly known to do so, the detection phase, when the market signals indicate that a firm is cheating, and a punishment phase, when the firm has left forever the high quality market. Players have the same information and beliefs in all phases except for the cheating phase. In the cheating phase, only the firm and the already cheated buyers know this. Other players cannot distinguish cheating and no-cheating phase. A long run buyer chooses his actions conditional on being cheated and in some settings also on the number of periods that passed since he was cheated.

A cheating firm and cheated long run buyers have to form beliefs about the number of LRs that have been cheated. Therefore, the state during a cheating phase includes the number of cheated long run players, the time that passed since they were cheated and the beliefs about this. We are mostly looking for strategies in which only the support of beliefs matters. This support is encoded in the definition of the state. Actual beliefs need not be part of the state, which greatly simplifies the analysis. We are therefore able to rank market institutions and equilibria in their efficiency using approximate bounds, without having to derive a complete solution. Our main objective is to shed light on the qualitative differences and problems that various market micro structures have in order to enforce good performance with imperfect monitoring and imperfect information aggregation.

2.2 The general repeated game

The general model of the repeated game follows Fudenberg, Levine, however, in contrast to their model we have not only imperfect information about players action but also incomplete information about players types. The valuation of individual buyers for the good are private information. We assume that the distribution of valuations is common knowledge. Many markets institutions operate under anonymity of traders. Firms in our model always have names and can be identified. Whether names or characteristics of buyers plays a role in the equilibrium depends on two types of assumptions, anonymity and discrimination. By anonymity we mean that the name (and the type) of the buyer is not revealed, which implies that only aggregate (statistical) information for the action of all buyers is observed. Discrimination denotes the property of the market institution and thereby of the game structure that does not allow players to condition their behavior on the identity of other players. For example in the auction setting firms are not able to accept only offers by long run buyers. Exclusion by name or identity is assumed to be illegal in this case. We assume that the identity of buyers cannot be observed in the auction markets, however in the decentralized bargaining markets firms can recognize a long run buyer that they have met in the past.

With these informational assumptions, buyers and sellers have to form beliefs about the aggregate distribution of actions and histories.

For the following let i denote players, player $i \in I$ is of type $v_i \in \mathcal{V}$, with I and $\mathcal{V}$ finite. Firms are all of the same type. The type of a buyer is given by his valuation β and his time horizon, either LR or SR. The distribution of types in the economy is given by a counting measure $m(v)$. The market game is played in each period. Each player chooses a stage game strategy or action a_i. Each action profile $a = (a_1, .., a_n)$ induces a probability distribution over outcomes $z = (z_1, .., z_n)$. The outcome reflects a stochastic element ξ in the market institution and possibly mixed strategies. At the end of a period t, each player i observes an outcome z_{it} which contains a private signal as well as a public component, and the players action. A players realized payoff depends only on his observed outcome. The history for player i at the beginning of period t is $h_{it} = (h_{it-1}, z_{it-1})$. Let H denote the set of all possible histories. A strategy σ_i of a player is a map from his set of private histories H_i to stage game strategies. Players form beliefs $\mu_i(h_{it}) \in M(I \times \mathcal{V} \times H)$ about the distribution of histories of other players and their types conditional on his own history. Beliefs of players have to be consistent with the distribution of types and with the strategy profile σ. Given a system of beliefs μ and a history h_t, the strategies and the random element induce a distribution over action and histories of all players at all times. The expected payoff in a period for player i for given history is

$$u_i(\sigma, h_{it}) = \int u_i(a, \xi) dF(\xi) d\mu_i(h_{it})$$

where $a = (\sigma_i(h_{it}))_{i \in I}$ is the action profile given strategies and histories at period t, and the discounted average continuation payoff of long run players

$$V_{it} = (1-\delta) E_t \sum_{\tau=1}^{\infty} \delta^{\tau} u_i(\sigma, h_{i\tau})$$

where the expectation is taken over possible continuation histories conditional on the history of player i at time t. A Perfect Equilibrium requires that players form consistent beliefs and maximize their utility for every continuation game. Short run players will always play a best response in the stage game given the public history. In the equilibria that we look at, it is not necessary for long run buyers to play an action on the equilibrium path that is not a stage game best response. Off the equilibrium path after a firm has started to cheat, the behavior of long run buyers determines the probability of public detection. This probability can possibly be increased if long run buyers are punished for not signalling. However, with imperfect information about individual purchase histories, this is a difficult problem. For most cases we concentrate therefore on the enforcement of behavior on the seller side.

2.3 A simplified repeated game

The following provides a simplified version of the repeated game that makes it easier to understand the structure and dynamics of the game (off-the equilibrium path). We concentrate on the high quality market, simplify the payoff structure and use detection probabilities that are exogenous.

In each period a simultaneous move stage game is played. Each firm chooses to produce either high (H) or low (L) quality, each buyer announces a set of firms from which he is willing to buy.

The payoff to a firm is $p - c$, $c \in \{\underline{c}, \bar{c}\}$, if at least one buyer announces a demand for the firms products, and zero otherwise. The expected payoff to a buyer is an increasing function of the average quality of all firms which are in his announced set. In contrast to the actual market game this abstracts from buyer interaction and market clearing off the equilibrium path. Each buyer is randomly assigned to a firm in the demanded set and learns the quality that this firm has produced. If a long run buyer learns that a firm has produced low quality, he will expect the firm to always produce low quality and stop buying from this firm. If low quality was sold to a short run buyer, then no other buyers will learn about this. The probability that a firm is punished depends on the joint information of all buyers. Cheating a short run buyer, therefore, does not affect the joint information of buyers in future periods. The information on the buyer side can be summarized by the number of long run buyers x that have been cheated. For now, we assume that a deviating firm can observe how many long run buyers it has cheated. Long run buyers that have been cheated will avoid the cheating firm. In the market game this results in actions that lead to publicly observable outcomes that signal that the firm is producing low quality. Here, we assume that there is an exogenous probability of public detection $\pi(x, D)$ which is increasing in the number of cheated long run buyers x. With trigger strategies, public detection of a firm's cheating leads to punishment by all buyers and to zero payoff for the firm.

Under these assumptions, the information of buyers with respect to a firm can be summarized with the following state variables:[2]

State $\mathbf{0}$: no long run buyer has been cheated,

state $x \in X$: number of LR cheated but cheating not publicly signalled,

state detection D : cheating is publicly signalled, and

state P : every buyer knows that firm has cheated and is being punished.

Histories that map into the same state induce the same distribution of beliefs across buyers, i.e. are indistinguishable for buyers subject to permutations of buyers' names. Each firm i has a state space Y_i with generic element $y_i \in X \cup \{0, D, P\}$, the joint state is given by $y = (y_1, ..., y_n)$ $\in Y = \underset{i}{\mathbf{X}}\, Y_i$.

The strategy of a buyer has to be measurable with respect to his information. SR and LR that have not been cheated can distinguish the 3 types of histories and states $\{D\}$, $\{P\}$ and $X \cup \{0\}$. Cheated LR buyers know that the firm has produced low quality, i.e. $y_i \in X$, but not the total number of buyers x that know this. However, given the current simplifications, this later information is not directly payoff relevant to the cheated buyers and they do not need to condition their behavior on beliefs about x.

We consider only strategies that are independent of calender time which implies that the continuation of the game at a time t depends only on the state y. In the equilibrium that we are interested in, traders follow the following strategies

$$\begin{aligned} \sigma_i(y_i) = & \quad \begin{cases} H & \text{if } y_i = \mathbf{0} \\ L & \text{if } y_i \neq \mathbf{0} \end{cases} \qquad \text{for each firm } i \\ \sigma_j((y_i)_i) = & \quad \{i : \text{supp}\mu_j(y_i) = \{\mathbf{0}\}\} \quad \text{for each buyer } j \end{aligned} \tag{1}$$

Beliefs of buyers $\mu_j(y_i)$ for all firms i and buyers j are given by the above observability assumptions, i.e.

[2] This is not the minimal state space possible, but used to make exposition easier and more compatible with later version of game.

$$\text{supp}\mu_j(y_i) = \begin{cases} \{\mathbf{0}\} & \text{if } y_i \in X \cup \{\mathbf{0}\} \text{ and } j \text{ has not been cheated by firm } i \\ X & \text{if } y_i \in X \text{ and } j \text{ has been cheated by firm } i \\ \{D\} & \text{if } y_i \in D \\ \{P\} & \text{if } y_i \in P \end{cases}$$

The strategy profile of all players induces a probability distribution over the evolution of the state. The evolution of the state conditional on strategies has a Markov property and is therefore fully summarized by the transition matrix and the initial state. Initially, all firms that are on the high quality market are in state 0. The transition matrix Π at a period t is a function of the firms action in this period, i.e. Π_a where $a \in \{L, H\}$, conditional on the strategies of buyers as given above. The probability $\pi(x, D)$ of being detected given state x is independent of whether the chosen action was H or L.

The transition matrix when the firm produces low quality has the form

Π_L	0	...	x	...	x'	...	$\lambda\overline{n}$	D	P
0	$\pi_L(0,0)$		$\pi_L(0,x)$		$\pi_L(0,x')$		$\pi_L(0,\lambda\overline{n})$	0	0
$\vdots$					$\vdots$				
x	0		$\pi_L(x,x)$		$\pi_L(x,x')$		$\pi_L(x,\lambda\overline{n})$	$\pi(x,D)$	0
$\vdots$					$\vdots$				
x'	0	...	0	...	$\pi_L(x',x')$	...	$\pi_L(x',\lambda\overline{n})$	$\pi(x',D)$	0
$\vdots$					$\vdots$				
$\lambda\overline{n}$	0		0		0		$1-\pi(\lambda\overline{n},D)$	$\pi(\lambda\overline{n},D)$	0
D	0		0		0		0	0	1
P	0		0		0		0	0	1

and when the firm produces high quality it has the following form

Π_H	0	...	x	...	x'	...	$\lambda\overline{n}$	D	P
0	1		0		0		0	0	0
$\vdots$					$\vdots$				
x	0		$1-\pi(x',D)$		0		0	$\pi(x,D)$	0
$\vdots$					$\vdots$				
x'	0	...	0	...	$1-\pi(x',D)$	...	0	$\pi(x',D)$	0
$\vdots$					$\vdots$				
$\lambda\overline{n}$	0		0		0		$1-\pi(\lambda\overline{n},D)$	$\pi(\lambda\overline{n},D)$	0
D	0		0		0		0	0	1
P	0		0		0		0	0	1

The probability of being detected after exactly τ periods of cheating when the initial state is y_t is

$$\phi_{L,\tau}(y_t) = \pi_L^\tau(y_t, D)$$

where $\pi_a^\tau(y,y') = \Pr(y_{t+\tau} = y'|y_t = y, a)$ is the τ-period transition probability from state y to y' if the firm always chooses action a. The expected discount factor until detection when starting from state y_t is $\widehat{\delta}_L(y_t) = \sum_{\tau=1}^{\infty} \delta^\tau \phi_{L,\tau}(y_t)$.

The optimal strategy of the firm is the solution to the following Bellman equation
$V(y) = \max\{V_H(y), V_L(y)\}$
where

$$V_H(y) = \begin{cases} (\bar{p} - \bar{c})(1-\delta) + \delta V(\mathbf{0}) & \text{if } y = \mathbf{0} \\ (\bar{p} - \bar{c})(1-\delta) + \delta\left[\pi_H(x,x)V(x) + \pi(x,D)V(D)\right] & \text{if } y = x \in X \\ (\underline{p} - \bar{c} + v_D)(1-\delta) + \delta V(P) & \text{if } y = D \\ (\underline{p} - \bar{c})(1-\delta) + \delta V(P) & \text{if } y = P \end{cases}$$

$$V_L(y) = \begin{cases} (\bar{p} - \underline{c})(1-\delta) + \delta \sum_{\tilde{x}} \pi_L(\mathbf{0},\tilde{x})V(\tilde{x}) & \text{if } y = \mathbf{0} \\ (\bar{p} - \underline{c})(1-\delta) + \delta\left[\sum_{\tilde{x}} \pi_L(x,\tilde{x})V(\tilde{x}) + \pi(x,D)V(D)\right] & \text{if } y = x \in X \\ (\underline{p} - \underline{c} + v_D)(1-\delta) + \delta V(P) & \text{if } y = D \\ (\underline{p} - \underline{c})(1-\delta) + \delta V(P) & \text{if } y = P \end{cases}$$

Assumptions:

(A1) $\pi(x, D)$ is increasing in x

(A2) $\pi(x, D)$ is concave in x

(A3) $\sum_{k \geq 1} \pi_L(x, x+1)) \geq \sum_{k \geq 2} (\pi_L(x+1, x+k) - \pi_L(x, x+k))$, for all x

(A4) $\pi_L(x+1, x+k) \geq \pi_L(x, x+k)$, for all $k \geq 2$ and for all x

Proposition 1 *Assume A1 to A4 hold, then prices* $\underline{p}^* = \underline{c}$, $\bar{p}^* = \underline{c} + (\bar{c} - \underline{c})/\hat{\delta}$, *where* $\hat{\delta} = \sum_{\tau=1}^{\infty} \delta^\tau \pi_L^\tau(\mathbf{0}, D)$ *and quantities* $\overline{n}^* = \max \overline{D}(\overline{p}^*, \underline{c})$, $\underline{n}^* = n - \overline{n}^* = \underline{D}(\overline{p}^*, \underline{c})$ *are the outcome of the subgame perfect equilibrium with strategies given by (1). For given transition probabilities* π *this is the equilibrium outcome with the lowest welfare loss.*

Proof: The following lemmas show that if the gains to cheating $V_L(x) - V_H(x)$ is positive for some state x then it is also positive for all higher states $x + k$. A firm for which it was optimal to cheat a buyer will therefore always produce low quality. No buyer that has learned that a firm has produced low quality in the past will therefore demand from this firm. We show later that if the price is at least $\bar{p}^*$, then the firm does not have an incentive to start producing low quality after a good history. $\bar{p}^*$ is therefore the minimum enforcment price and $\overline{n}^*$ the largest number of firms that can be supported to produce high quality in any SPE.

Lemma 2 *Assume $V(D)$ is independend of x and $V(D) < (\bar{p} - \bar{c})$.*
If (A1) $\pi(x, D)$ is increasing in x then $V_H(x)$ is decreasing in x.
If (A2) $\pi(x, D)$ is concave in x,then $V_H(x)$ is convex in x.

The payoff for the firm for producing high quality at state x is given by
$V_H(x) = (\bar{p} - \bar{c})\frac{1-\delta}{1-\delta(1-\pi(x,D))} + \frac{\delta\pi(x,D)}{1-\delta(1-\pi(x,D))}V(D)$
The lemma can be easily verified.

Lemma 3 *If $V(x)$ is decreasing and convex and (A3) holds then the incentives to cheat are increasing in the number of cheated long run buyers x.*

Proof:
Recall that
$V_L(x) = (\bar{p} - \underline{c})(1 - \delta) + \delta\left[\sum_{\tilde{x}} \pi_L(x, \tilde{x})V(\tilde{x}) + \pi(x, D)V(D)\right]$
$V_H(x) = (\bar{p} - \bar{c})(1 - \delta) + \delta\left[\pi_H(x, x)V(x) + \pi(x, D)V(D)\right]$
then the payoff difference is

$$\Delta(x) = V_L(x) - V_H(x) = (\bar{c} - \underline{c})(1 - \delta) + \delta\left[\sum_{\tilde{x}} \pi_L(x, \tilde{x})V(\tilde{x}) - \pi_H(x, x)V(x)\right]$$

This difference is increasing in x if the term in brackets ist increasing in x. i.e.
$\sum_{\tilde{x}\in X} \pi_L(x, \tilde{x})V(\tilde{x}) - \pi_H(x, x)V(x) \leq \sum_{\tilde{x}\in X} \pi_L(x + 1, \tilde{x})V(\tilde{x}) - \pi_H(x + 1, x + 1)V(x + 1)$
collecting terms, we have

$$\begin{aligned} & [\pi_H(x, x) - \pi_L(x, x)]\, V(x) + \sum_{\tilde{x}\neq x, \tilde{x}\neq x+1} [\pi_L(x + 1, \tilde{x}) - \pi_L(x, \tilde{x})]\, V(\tilde{x}) \\ \geq \; & [\pi_H(x+1, x+1) - \pi_L(x+1, x+1) + \pi_L(x, x+1)]\, V(x+1) \end{aligned}$$

note $\pi_H(x, x) = \sum_{\tilde{x}\in X} \pi_L(x, \tilde{x})$

The sum of the coefficients on the left and right hand sides are equal. Dividing by the expression in brackets on the left hand side, implies that the right hand side is a convex combination as long as $\pi_L(x + 1, \tilde{x}) \geq \pi_L(x, \tilde{x})$.for all $\tilde{x} \geq x + 2$ and all x

$$\frac{\sum_{\tilde{x}\neq x} \pi_L(x, \tilde{x}))}{\pi_L(x, x + 1) + \sum_{\tilde{x}\neq x+1} \pi_L(x + 1, \tilde{x})}V(x) + \sum_{\tilde{x}\neq x, \tilde{x}\neq x+1} \frac{(\pi_L(x + 1, \tilde{x}) - \pi_L(x, \tilde{x}))}{\pi_L(x, x + 1) + \sum_{\tilde{x}\neq x+1} \pi_L(x + 1, \tilde{x})}V(\tilde{x}) \geq V(x+1) \tag{2}$$

If $V(x)$ is a decreasing, convex function, then the above inequality (2) will hold as long as $E(x) \geq x+1$, where the expectation is evaluated at the probability distribution defined by the left hand side([3]). A sufficient condition for this to hold is that the first term is larger than a half, or equivalently

$$\sum_{k\geq 1} \pi_L(x,x+1)) \geq \sum_{k\geq 2} (\pi_L(x+1,x+k) - \pi_L(x,x+k))$$

In the case $q=1$ this simplifies to $\pi_L(x,x+1)) \geq (\pi_L(x+1,x+2)$, i.e. the probability of cheating an additional long run buyer and not being dedected is a decreasing function of the number of already cheated long run buyers x.

Remark: Second Order Stochastic Dominance cannot be used, because it is not satisfied in most relevant examples, e.g. never if $q=1$.

Lemma 4 *Assume (A1,A2,A3) hold. If* $\Delta(\mathbf{x}) \geq 0$, *then* $\Delta(x') \geq 0$ *for all* $x' > x$.
If $\Delta(\mathbf{0}) \geq 0$, *then* $\Delta(x) \geq 0$ *for all* $x \in X$.

If a firm is willing to cheat after a good history, i.e. in state $y = \mathbf{0}$, then it is willing to always cheat. This implies that the relevant incentive constraint for the minimum enforcement price compares never cheating and always cheating. It also implies that the strategy of the firm in the case when the firm cannot observe the number of cheated long run buyers depends only on the support of the beliefs and not on the exact distribution.

Proof:

We have proven that under the above assumptions $V_H(x)$ is decreasing and convex and the gains to cheating and switching back to high quality production are increasing in x. We did not proof it for the optimal value function $V(x)$ and optimal deviation payoffs $\Delta(x)$. However, the former is sufficient for the proof of the lemma as the following shows for the case $q=1$.
Define

$$\Delta_{aa'}(x) = V_{La}(x) - V_{Ha}(x) = (\bar{c} - \underline{c})(1-\delta) + \delta\left[\pi_{a'}(x,x+1)V_{a'}(x+1) - \pi_a(x,x)V_a(x)\right] \qquad (3)$$

where a and a' stand for the continuation strategy $\sigma(x) = a$ and $\sigma(x+1) = a'$. Note $\Delta_{aa'}(x)$ is increasing in $V_a(x+1)$ and decreasing in $V_a(x)$. Let $a = *$ and $a' = *$ represent the optimal continuation strategies.
The previous lemmas imply $\Delta_{HH}(x)$ is increasing in x.
Note the following properties which follow from optimal choice and equation (3).

$\Delta_{H*}(x) = 0 \Leftrightarrow \Delta_{L*}(x) = 0$

$\Delta_{H*}(x) > 0 \Leftrightarrow \Delta_{L*}(x) > 0$

$\Delta_{aa'}(x) \geq 0 \Rightarrow \Delta_{a*}(x) \geq 0$ for all a, a'

[3] $EV(x) \geq V(E(x)) \geq V(x+1)$; where the first inequality follows from convexity for an extension of the function V to real numbers; the second inequality follows from monotonicity, V (weakly) decreasing, and from $E(x) \geq x+1$.

Assume $\Delta_{**}(x) \geq 0$. Then either (a) $\Delta_{**}(x) = \Delta_{aH}(x) \geq 0$ or (b) $\Delta_{**}(x) = \Delta_{aL}(x) \geq 0$ for any a.

Case (a): $\Delta_{HH}(x) \geq 0$ implies $\Delta_{HH}(x+1) \geq 0$ and by monotonicity and optimality also all $\Delta_{**}(x) \geq 0, x \geq x+1$

Case (b): $\Delta_{LL}(x) \geq 0$ implies $\Delta_{**}(x+1) = \Delta_{L*}(x+1) \geq 0$

In both cases, if producing low quality is preferred at state x then it is also preferred at state $x+1$ and by induction at all higher states.The same is true for $x = 0$, i.e. $y = \mathbf{0}$. For general q the same argument applies when appropriate changes are made (consider a' as a vector and consider all higher states $x+k,\ k = 1, 2, 3, ...$).

At the minimum enforcement price, i.e the price such that $\Delta(\mathbf{0}) = 0$ the incentive constraint of cheating and then switching back to producing high quality is not binding since $\Delta_{*H}(x) \leq \Delta_{*L}(x)$ under the assumptions of the lemma.

Minimum Enforcement Price

In the following we derive the minimum enforcment price in a representation comparable to the full punishment case. The incentive constraint is conditional on the information of the firm when it starts to produce low quality for the first time. The ex ante detection probabilities $\phi_\tau = \pi_L^\tau(\mathbf{0}, D)$ and the profit v_D in the detection period summarize the essential features of the market game for the firms incentive constraint. As in standard repeated game analysis it suffices to look only at single deviations from an equilibrium. The following incentive constraint is derived under the assumption that a firm that finds it optimal to cheat once will produce low quality products in all continuation games, which, as shown, is satisfied if assumptions A1 through A4 hold. $\Delta v_D = v_D - (\underline{p} - \underline{c})$ is used to correct for the difference between the payoffs in the detection and punishment periods.

The expected profit from cheating is then given by

$$V_{\mathbf{L}} = \sum_{\tau=1}^{\infty} \left[(\bar{p} - \underline{c})(1 - \delta^\tau) + (\underline{p} - \underline{c})\delta^\tau + (1 - \delta)\delta^\tau \mathbf{E}_{|\tau} \Delta v_D \right] \phi_\tau + (\bar{p} - \underline{c})\phi_\infty$$

where $\phi_\infty = (1 - \sum\limits_{t=1}^{\infty} \phi_\tau)$ is the probability of never being detected. The term in brackets is the average profit if detection occurs τ periods after the firm starts to produce low quality. The average discounted profit from always producing high quality is $\bar{V} = \bar{p} - \bar{c}$. The incentive constraint $\bar{V} \geq V_L$ implies

$$\bar{p} \geq \underline{p} + \frac{\bar{c} - \underline{c}}{\hat{\delta}} + \frac{(1 - \delta)}{\hat{\delta}} \mathbf{E} \left[\delta^\tau \Delta v_D \right]$$

where $\hat{\delta} = \sum\limits_{\tau=1}^{\infty} \delta^\tau \phi_\tau$ is the expected discount factor until detection. This is the same characterization as in the standard case with full punishment in the period following cheating when $\hat{\delta}$ is replaced by δ. If the hazard rate $\alpha = \phi_\tau / (1 - \sum\limits_{s=1}^{\tau-1} \phi_s)$ is constant, then $\hat{\delta} = \frac{\alpha\delta}{1-(1-\alpha)\delta}$.

The smallest price $\bar{p}$ that satisfies this inequality is the minimum enforcement price. The minimum enforcement price is lower with earlier expected detection. Therefore, market institutions can be welfare-ranked by their implied $\hat{\delta}$.

2.4 Market equilibrium

(incomplete)

The outcome of the repeated game is a constant market equilibrium in each period characterized by prices $(\bar{p}^*, \underline{p}^*)$ and number of firms $\bar{n}^*$, $\underline{n}^*$ producing high or low quality respectively. Total supply and demand function imply equilibrium price for low quality along the equilibrium path $\underline{p}^* = \underline{c}$ The price and number of firms on the high quality market are determined by the minimum enforcement price and the demand curve. The detection probabilities are the outcome of the interaction between market micro structure and stage game strategies of buyers and are a (weakly) decreasing function of the size of the high quality market $\phi(\bar{n})$. Together with the incentive constraint on the firm, this implies a minimum enforcement price $\bar{p}_E(\bar{n})$ that is increasing in $\bar{n}$, which is the incentive compatible "supply" curve. Any price-quantity pair above it can be enforced on the firm side. The intersection with the demand function defines the maximum size of the high quality market $\bar{n}^*$ that can be supported by an equilibrium of a specific market game. The number of units that can be produced is $q\bar{n}^*$.

The market equilibrium can be calculated in the following steps:

- Market micro structure implies $(\phi_\tau)_{\tau=1}^{\infty}$, and v_D for given n^*
- ϕ_τ and v_D and incentive constraint for firms imply minimum enforcement price $\bar{p}^*$
- Total supply and demand function imply equilibrium price for low quality along the equilibrium path $\underline{p}^*$
- minimum enforcement price $\bar{p}^*$, low quality price $\underline{p}^*$ and demand function imply the demand for high quality units
- which in turn implies the number of firms n^* that can be supported as high quality producers, other firms can only produce low quality (rationing effect because of profit premium for high quality production)
- find fixed point in n^*

In the next section, we derive the detection and punishment probabilities if the market is organized in the form of sequential auctions

3 Auctions

We now consider an environment where the units of high quality are sold at an auction, whereas there remains a separate low quality market for which we do not specify a particular market mechanism.

In a first stage, after sellers produced their desired quality, they decide whether to participate in the auction - i.e. enter the "high" quality market or remain in the low quality market.

We will try to characterize the most efficient equilibrium of particular auctions, i.e. the highest amount of high quality that can be sustained by the specific auction mechanism.

An important determinant of the punishment probability in an auction market is the amount of information available to the buyers

(i) instantaneously, i.e., during the bidding process; and

(ii) ex post (to all buyers, or only to LR?).

The following summarizes the main possiblities of signals in the auction process that convey the information that a firm has been cheating.

Ad (i) the amount of instantaneous information is mainly predetermined by the chosen auction format: In a sealed bid auction, for instance, there is no instantaneous information, while mimicking (uninformed buyers try to imitate the bidding behavior of informed ones) and tricking (informed LRs try to induce uninformed buyers to buy low quality at the high price to avoid an increase in $\overline{p}$) possibilities are present in an English auction. To keep things simple, we concentrate on two sealed bid formats (first and second price).

Ad (ii) The amount of ex post information is a matter of assumption. LRs know, of course, the past prices and quantities of the different firms, and their "ranks". In the sequel we assume that this information is also available to SRs. If it is not, the punishment probability is lower, of course. In some real auctions additional information on the "order state" is made available. A raw indicator of the order state is the price-supplement "EDI" (existence of extra demand at market price, "repartiert" in German) for "the demand for the unit(s) produced by a given firm was higher than the supply (=1, since we concentrate on the special case $q = 1$ here)". We will discuss the impact of this information on the punishment probability below. For the sealed bid first price auction we will also consider the case where the number of bids at the selling-price is made public ex post. We call this the "demand overhang observed" version.

When will a firm be punished in an auction? Punishment (the firm is unable to sell its unit at the high price) will only take place if some publicly available information indicates that the firm is or has been (either with some probability or for sure) cheating. The relevant signal may be the quantity or the price. A *negative quantity signal* (a firm was unable to sell its unit) identifies the cheater exactly. In the "EDI" and in the "demand overhang observed" version, buyers might also be able to observe *positive quantity signals* (the demand for the unit sold on an earlier rank was "too high"). High demand can, however, only have signalling value if the buyers coordinate their bids (without coordination all buyers that haven't got a unit yet will submit a bid for the unit sold by a given non-cheating firm). A positive quantity signal usually (except for rank $\overline{n}-1$) means "set detection": It indicates that some firm on a later rank is (or was) cheating. *Negative price signals* (the selling-price is lower than $\overline{p}$) are never observed in the auctions considered below since firms are assumed to announce reservation prices. We allow firms to announce reservation prices to avoid the bilateral monopoly problem mentioned earlier (ex post, once high quality has been produced, buyers have an incentive to bid less than $\overline{p}$; if firms take this into account, they will never produce high quality). A *positive price signal* (the selling-price exceeds $\overline{p}$) on an earlier rank is similar to a positive quantity signal: It indicates that some firm on a later rank has cheated in the past. In the sealed bid first price format a single buyer that offers more than $\overline{p}$ for a unit on an earlier rank suffices to generate a positive price signal. In the sealed bid second price auction at least two buyers that offer more than $\overline{p}$ for the same unit are needed to change the selling-price.

3.1 Sealed Bid Second Price Auction

Format: The units designated for the high quality market are put up for sale in random sequence. The auction begins with the auctioneer announcing the sequence of units (= the "ranks" of the firms) and the firms' reservation prices[4]. Then sealed bids for the first unit (= for the unit sold by the firm on rank 1) are solicited. The highest bid is accepted at a price equal to the 2nd highest bid. Then sealed bids for the unit produced by the firm on rank 2 are solicited. The process continues until the unit of the firm on rank $\overline{n}$ is sold.

We consider two different strategy-specifications. In the first, LR-buyers bid $\overline{p}$ along the equilibrium path. In the second, they bid $\overline{p}+\Delta$.

3.1.1 Second Price with $\overline{p}$ Bids

Strategies: *On the equilibrium path* all buyers whose type β_k is greater or equal than $(\overline{p}-\underline{c})\ /\ \left(\overline{\theta}-\underline{\theta}\right)$ bid $\overline{p}$ on each rank until they get their unit. *Off the equilibrium path* the behavior of the cheated LRs depends on their beliefs on how many LRs have already been cheated. These beliefs imply a critical rank. For a given period t we denote the critical rank for a LR buyer first cheated τ periods ago by $\widehat{r}^t_\tau$. If the rank of the cheating firm, denoted by r^t_f, is strictly lower than $\widehat{r}^t_\tau$ then the LR buyer cheated τ periods ago bids $\overline{p}$ on all $r\neq r^t_f$. Otherwise he bids $\overline{p}$ on all ranks strictly lower than $\widehat{r}^t_\tau-\tau$ (provided $\widehat{r}^t_\tau-\tau\geq 1$; otherwise...). If he doesn't get a unit on one of these ranks he bids $\overline{p}+\Delta$ on rank $\widehat{r}^t_\tau-\tau$.

Pessimistic Beliefs: Cheated LR buyers assume that with strictly positive probability all other LRs have already been cheated. Pessimistic beliefs are the worst beliefs regarding the support.

[4]Here and throughout the rest of the auction-analysis we assume that all high price firms announce $\overline{p}$ as their reservation price.

Thus, they give an upper bound for the punishment probability. Pessimistic beliefs ignore initial periods and are not fully consistent if the LR buyer under consideration has bought high quality from the cheating firm a short time before (provided "hit and run" cheating is not optimal). With pessimistic beliefs $\widehat{r}_{\tau}^{t} = (1-\lambda)\,\overline{n}+1$ for all cheated LRs independently of τ (and t). (If $(1-\lambda)\,\overline{n}$ is not an integer then take the next lower integer.)

Optimistic Beliefs: The LR buyer cheated τ periods ago believes that cheating firm began cheating τ periods ago. Optimistic beliefs are best beliefs regarding the support. For a LR buyer with optimistic beliefs that has been cheated τ periods ago $\widehat{r}_{\tau}^{t} = \overline{n}+1-\tau$ if $\tau \leq \lambda\overline{n}$ and $\widehat{r}_{\tau}^{t} = (1-\lambda)\,\overline{n}+1$ otherwise. That is, $\widehat{r}_{\tau}^{t} = \min\{\overline{n}+1-\tau, (1-\lambda)\,\overline{n}+1\}$.

Results:

Claim 3: With pessimistic beliefs the probability of (set) detection is zero for at least $(1-\lambda)\,\overline{n}$ cheating periods. That is, with pessimistic beliefs $\phi_{L,s} = 0$ for $s \leq (1-\lambda)\,\overline{n}$. Similarly, with optimistic beliefs the probability of (set) detection is zero for at least $\overline{n}-1$ cheating periods.

Proof: Cheating is detected (set detection) if more than one LR buyer bids $\overline{p}+\Delta$ for the same unit. With the described strategies this cannot happen in the first ... cheating periods.

3.1.2 Second Price with $\overline{p}+\Delta$ Bids

Strategies: *Along the equilibrium path* the SRs bid $\overline{p}$ from rank 1 on until they get their unit. LRs do the same in the first round. From then on they bid $\overline{p}+\Delta$ for the firm from which they bought previously. *Off the equilibrium path* the behavior of the cheated LRs depends again on their beliefs which determine their critical rank $\widehat{r}_{\tau}^{t}$. If the rank of the cheating firm is strictly lower than $\widehat{r}_{\tau}^{t}$ then the LR buyer cheated τ periods ago bids $\overline{p}$ on all ranks $r \neq r_{f}^{t}$. Otherwise he bids $\overline{p}$ on all ranks lower than $\widehat{r}_{\tau}^{t}-2$. If he doesn't get a unit on one of these ranks he bids $\overline{p}+\Delta$ on rank $\widehat{r}_{\tau}^{t}-1$. From then on he bids $\overline{p}+\Delta$ for the firm he gets until he is cheated again.

Results: Results depend upon whether "EDI" is observed or not. The following claim holds for the not-observed version.

Claim 4: With pessimistic beliefs $\phi_{L,1} < \lambda^{3}$, and $\phi_{L,\tau} = 0$ for all $\tau > 1$.

Proof: Set detection occurs if more than one LR bids $\overline{p}+\Delta$, that is, if (i) the cheating firm had a LR buyer (probability λ), (ii) the cheating firm gets one of the rear ranks (probability λ), (iii) the cheated LR doesn't get a unit before rank $(1-\lambda)\,\overline{n}$, and (iv) the firm on rank $(1-\lambda)\,\overline{n}$ has a LR given that the cheating firm has a LR and the cheated LR hasn't got a unit before rank $(1-\lambda)\,\overline{n}$ (probability strictly lower than λ).

3.2 Sealed Bid First Price Auction

Format: The $\overline{n}$ units designated for the high quality market are put up for sale in random sequence. We assume that the firms can set reservation prices.[5] The auction begins with the auctioneer announcing the sequence of units (= the "ranks" of the firms). Then sealed bids for the first unit (= for the unit sold by the firm on rank 1) are solicited. The highest bid is accepted at the stated price. Then sealed bids for the unit produced by the firm on rank 2 are solicited. The process continues until the unit of the firm on rank $\overline{n}$ is sold.

We will consider several cases, depending on the amount of information bidders get, apart from the transactions.

3.2.1 No History and reservation prices observed

Consider the case where all units, high and low quality, are sold in a single auction and buyers cannot observe the prices of units sold at previous auctions nor at the current auction of units other than their own. In this case the first price auction resembles in its information transmission the decentralized bargaining market. However the difference is that in the auction the firms lose any "exclusivity" right, i.e. they can not deter buyers who are making the highest bid from winning the auction. This is mostly a hypothetical case that we consider in order to distinguish the role of information and the role of exclusion, i.e. making the offers conditional on the type or identity of the buyers.

Consider first the case where

$$\beta_K < \frac{(1+\delta)(\overline{c}-\underline{c})}{\delta(\overline{\theta}-\underline{\theta})\frac{\overline{n}^{ER}}{n}} = \overline{\beta}^{ER}_{SR}$$

The following strategies constitute an equilibrium.

Strategies **The Firms:** Firms that had a LR in the first period continue to produce high quality and set a reservation price $\overline{p}^{ER}$. When they can not sell their good, they produce next period low quality and set a reservation price of $\underline{p}$. Whenever they sell their low quality unit at a price $p > \underline{p}$ they believe that a LR bought their good and next period produce high quality and set a reservation price of $\overline{p}^{ER}$. If a firm sells her good only at a price $\underline{p}$, she produces in the next period a low quality unit and sets her reservation price to $\underline{p}$.

The Long Run Buyers: Each LR bids $\overline{p}^{ER}$ for the firm from which he bought from last period. *Beliefs out of equilibrium.* *LRs* without a regular firm bid only $\underline{p}+\varepsilon$, and then next period bid $\overline{p}^{ER}$ for the unit of that same firm, since they believe that the firm produced a high quality unit and set a reservation price of $\overline{p}^{ER}$.

[5]Here and throughout the rest of the auction-analysis we assume that all high price firms announce $\overline{p}$ as their reservation price.

The Short Run Buyers: Each SR only bids $\underline{p}$. SRs believe that only firms that produced low quality set a reservation price below $\overline{p}^{ER}$.

Proposition 1 (Exclusive Relationship Equilibrium) *In the case of* $\beta_K < \frac{\bar{p}^{ER}-\underline{p}}{(\overline{\theta}-\underline{\theta})\frac{\bar{n}^{ER}}{n}}$ *there is an equilibrium where in each period after the first one the number of high quality units equals the number of long run players with valuation* $\beta_k \geqslant \overline{\beta}^{ER} = (\bar{c}-\underline{c})/\delta(\overline{\theta}-\underline{\theta})$*. Each Long Run buyer with valuation* $\beta_k \geqslant \overline{\beta}^{ER}$ *gets a high quality unit at a price* $\overline{p}^{ER} = \underline{c} + (\bar{c}-\underline{c})/\delta$ *from his "regular" firm. LRs with valuation* $\beta_k < \overline{\beta}^{ER}$ *and Short Run buyers get low quality units from other firms at a price* $\underline{p} = \underline{c}$*. Hence,* $\bar{n}^{ER} = \lambda \sum_{\beta_k \geqslant \overline{\beta}^{ER}} m_k$.

Proof: No firm in a relation with a LR Buyer has an incentive to deviate: If the firm sets a reservation price $p > \overline{p}^{ER}$ she will not be able to sell to her LR,who only bids $\overline{p}^{ER}$.

A firm that starts to produce low quality can expect earnings of

$$\overline{p}^{ER} - \underline{c} + \frac{\delta}{1-\delta}(\underline{p}-\underline{c})$$

since if she produces low quality once, she will lose her LR forever. If she continues to produce high quality her expected earnings are

$$\overline{p}^{ER} - \overline{c} + \frac{\delta}{1-\delta}(\overline{p}^{ER}-\overline{c}).$$

Hence, we have the condition

$$\overline{p}^{ER} - \overline{c} + \frac{\delta}{1-\delta}(\overline{p}^{ER}-\overline{c}) \geqslant \overline{p}^{ER} - \underline{c} + \frac{\delta}{1-\delta}(\underline{p}-\underline{c}) \quad \Leftrightarrow \quad \bar{p}^{ER} \geqslant \frac{(\bar{c}-\underline{c})}{\delta} + \underline{p}.$$

For $\underline{p} = \underline{c}$ we have

$$\bar{p}^{ER} \geqslant \underline{c} + \frac{\bar{c}-\underline{c}}{\delta}.$$

No firm producing low quality has an incentive to deviate: Will the firm try to set a reservation price $p > \underline{p}$? Along the equilibrium path no LR will bid $p > \underline{p}$ neither a SR, hence no incentive to do so. Furthermore, for the same reason, the firm will not start to produce high quality since she won't be able to get a price higher than $\underline{p}$.

No LR Player has an incentive to deviate: Consider a LR with $\beta \geqslant \overline{\beta}^{ER}$. Since the firm sets a reservation price of $\bar{p}^{ER}$ the LR will bid $\bar{p}^{ER}$.

No SR has an incentive to deviate: When will a high valuation SR be willing to pay $\hat{p} > \overline{p}^{ER}$? When the SR bids $\hat{p} > \overline{p}^{ER}$ he gets market average quality, hence is willing to do that only if

$$\beta_k(\frac{\bar{n}^{ER}}{n}\overline{\theta} + (1-\frac{\bar{n}^{ER}}{n})\underline{\theta}) - \widehat{p} \geqslant \beta_k\underline{\theta} - \underline{p} \qquad \Leftrightarrow \qquad \widehat{p} \leqslant \underline{p} + \beta_k\frac{\bar{n}^{ER}}{n}(\overline{\theta}-\underline{\theta})$$

However the SR will only bid marginally more than $\bar{p}^{ER}$ hence we would need that

$$\beta_K \geqslant \frac{\bar{p}^{ER} - \underline{p}}{(\overline{\theta} - \underline{\theta})\frac{\overline{n}^{ER}}{n}}$$

hence in order to have the SR not wanting to bid $\hat{p} \geqslant \overline{p}^{ER}$ we need that

$$\beta_K < \frac{\bar{p}^{ER} - \underline{p}}{(\overline{\theta} - \underline{\theta})\frac{\overline{n}^{ER}}{n}}$$

Hence the SRs are only bidding for the low quality units hence will bid $\underline{p}$. ■

Remark Note that

$$\frac{\bar{p}^{ER} - \underline{p}}{(\overline{\theta} - \underline{\theta})\frac{\overline{n}^{ER}}{n}} < \frac{(1+\delta)(\overline{c} - \underline{c})}{\delta(\overline{\theta} - \underline{\theta})\frac{\overline{n}^{ER}}{n}} = \frac{\bar{p}^{ER} - \underline{p} - \delta(\overline{c} - \underline{c})}{(1-\delta)(\overline{\theta} - \underline{\theta})\frac{\overline{n}^{ER}}{n}} = \overline{\beta}_{SR}^{ER}$$

since $\bar{p}^{ER} > \overline{c}$.

Remark In the case where $\beta_K < \frac{\bar{p}^{ER} - \underline{p}}{(\overline{\theta} - \underline{\theta})\frac{\overline{n}^{ER}}{n}}$ the sealed bid first price auction where buyers can neither observe other buyers behavior nor can the firm use the reservation price as a signal, fares as good as the decentralized bargaining market.

In the case where $\frac{\bar{p}^{ER} - \underline{p}}{(\overline{\theta} - \underline{\theta})\frac{\overline{n}^{ER}}{n}} < \beta_K < \overline{\beta}_{SR}^{ER}$ the sealed bid first price auction fails to replicate the Exclusive Relationship equilibrium of the Decentralized bargaining market. The reason is that high valuation SRs by bidding $\bar{p}^{ER} + \varepsilon$ can make sure to obtain market average quality $\frac{\bar{n}^{ER}}{n}\overline{\theta} + (1 - \frac{\bar{n}^{ER}}{n})\underline{\theta}$ and since their valuation is high enough, this is a desirable option for them. The performance of the new equilibrium is worse in terms of efficiency than the decentralized bargaining market.

Hence the exclusivity feature of the decentralized bargaining market - in the way we model the decentralized bargaining market - is a crucial component in increasing efficiency. The main difference is that firms in the decentralized bargaining market do not have to sell their unit to the buyer who is willing to pay the most, the firm can remain exclusive towards her LR buyer, which the auction setting does not allow for.

However the welfare implications are unclear, since the high valuation SRs actually prefer this equilibrium.

Let us now characterize the equilibria when we also eliminate the information constraints.

3.2.2 Trading history without demand overhang observed (DONO)

In this case buyers observe the price history and the number of units sold but no information as to whether there was excess demand for the unit of a firm.

Strategies: *Along the equilibrium path* all consumers with a type $\beta_k \geqslant (\overline{p} - \underline{c}) / (\overline{\theta} - \underline{\theta})$ bid $\overline{p}$ until they get a unit.

Off the equilibrium path the behavior of cheated LRs depends on their beliefs regarding the number of already cheated LRs. These beliefs imply a critical rank. For a given period t we denote the critical rank for a LR buyer first cheated τ periods ago by $\widehat{r}_\tau^t$. If the rank of the cheating firm, denoted by r_f^t, is lower or equal than $\widehat{r}_\tau^t$ then the LR buyer cheated τ periods ago bids $\overline{p}$ on all $r \neq r_f^t$. In the case of $r_f^t > \widehat{r}_\tau^t$ the LR bids $\overline{p}$ on all ranks strictly lower than $\widehat{r}_\tau^t$. If he doesn't get a unit on one of these ranks he starts to bid $\overline{p} + \Delta$ starting from rank $\widehat{r}_\tau^t$ with a certain probability ρ. However when he didn't get a unit on one of the ranks $\widehat{r}_\tau^t \leqslant r \leqslant r_f^t - 1$, he will not bid on rank r_f^t and continue to bid $\overline{p}$ on all ranks $r > r_f^t$.

The evolution of $\widehat{r}_\tau^t$ depends on the beliefs of the LRs about the maximal possible number of LRs that have been cheated before by that same firm. We call these beliefs pessimistic if cheated LR buyers assume that with strictly positive probability all other LRs have already been cheated. Pessimistic beliefs ignore initial periods and are not fully consistent if the LR buyer under consideration has bought high quality from the cheating firm only a short time before. Beliefs are optimistic when the cheated LR assumes that for sure he is the first cheated LR, hence the firm only started cheating (producing low quality) when he bought the good.

Pessimistic Beliefs: With pessimistic beliefs $\widehat{r}_\tau^t = (1-\lambda)\,\overline{n}$ for all cheated LRs independently of τ (and t). (If $(1-\lambda)\,\overline{n}$ is not an integer then take the next lower integer.)

Optimistic Beliefs: The LR buyer cheated τ periods ago believes that cheating firm began cheating τ periods ago. For a LR buyer with optimistic beliefs that has been cheated τ periods ago $\widehat{r}_\tau^t = \overline{n} - \tau$ if $\tau \leq \lambda\overline{n}$ and $\widehat{r}_\tau^t = (1-\lambda)\,\overline{n}$ otherwise. That is, $\widehat{r}_\tau^t = \min\{\overline{n} - \tau, (1-\lambda)\,\overline{n}\}$.

Pessimistic beliefs will deliver an upper bound for the detection probability and optimistic beliefs a lower bound.

Optimistic beliefs are best beliefs regarding the support pessimistic the worst.

The goal is to find $\overline{n}$ such that none of the $\overline{n}$ firms producing high quality have an incentive to deviate. For that we first have to determine how fast a firm that cheats get detected. For the time being let us only consider set detection. That means once a set of firms is detected to contain a cheating firm all those firms will be replaced by other firms.

Hence let us first assume that we found such an $\overline{n}$, and that the strategies for the buyers are optimal.

Proposition 1: For $\overline{n}$ the players have no incentive to deviate from their strategies.

Proof: How will a LR buyer that received a low quality unit at the high price in the past (a "cheated LR") behave in an auction with reservation price $\overline{p}$? The best he can get is a high quality unit at the price $\overline{p}$. Whether he will be able to realize the associated payoff depends on several factors, an important one being the rank of the cheating firm. If the sequential auction starts with the unit produced by the cheating firm (the cheating firm "is on rank 1") then the cheated LR, by not participating in the bidding for this unit, can make sure to get a high quality unit for $\overline{p}$: An uninformed (SR or LR) buyer will get the low quality unit and the price for the units on later ranks remains constant. Things might change if the cheating firm has a higher rank. Suppose, for example, the unit produced by the cheating firm is sold as the last unit (cheater has the rear rank $\overline{n}$). Then a cheated LR that hasn't got a unit before rank $\overline{n}-1$ has to decide: If he bids $\overline{p}$ at rank $\overline{n}-1$ he has a chance of at most 50 percent to get the unit sold at this rank and a chance of at least 50 percent not to get this unit and to be left with two bad alternatives: to buy the unit produced by the cheating firm for $\overline{p}$ or not buy at all. Given this, he has an incentive to bid more than $\overline{p}$ at rank $\overline{n}-1$. How much more will he bid? We assume, that there exists a smallest monetary unit which we denote by Δ. Has the cheated LR an incentive to bid more than $\overline{p}+\Delta$ at rank $\overline{n}-1$? If he is *sure* that there is no other (cheated LR) buyer who prefers the unit sold at rank $\overline{n}-1$ to that sold at rank $\overline{n}$, hence if he is sure that no other LR has yet been cheated then bidding $\overline{p}$ is the right choice: With this strategy he gets a unit from a non-cheating firm at the price $\overline{p}$ for sure while he has to pay more than $\overline{p}$ (with strictly positive probability) for such a unit with any other strategy.

What, however, if he thinks that there could be competition for the unit sold at rank $\overline{n}-1$? Then he has an incentive to bid $\overline{p}+\Delta$ on an earlier rank. Then his best strategy, i.e. on which rank will he start to bid $\overline{p}+\Delta$ will depend on the size of Δ and on his belief on the number of already cheated LRs. The analysis below is based on the assumption that Δ is very small (Δ converges to zero). In this case a cheated LR will incur this cost whenever there is an arbitrarily small chance that he will have to fight for a high quality unit against other informed LRs if he continues to bid $\overline{p}$. In other words, in this case the relevant thing to consider is the upper bound of the support of the cheated LRs belief on the number of already cheated LRs.

However once the cheated LR happens to be bidding in a rank $r > r_f^t$ there is no need to bid more than $\overline{p}$, since the number of firms in the last $\overline{n}-r$ ranks equals the numbers of buyers who are willing to pay at least $\overline{p}$.

Pessimistic Beliefs: Suppose, for instance, that a cheated LR believes that all other LRs could already have been cheated. If the rank of the cheating firm r_f^t is lower than or equal to $(1-\lambda)\,\overline{n}$ then he will bid $\overline{p}$ for all non-cheating firms. With this strategy he gets a high quality unit at price $\overline{p}$ for sure. (If the cheating firm is on a rank lower or equal than $\overline{n}-\lambda\overline{n}$ then she will be able to sell its unit to an ignorant SR even in the worst case where all $\lambda\overline{n}$ LR-buyers have already been cheated and none of them has got a unit on one of the front ranks.) In the case where $r_f^t > (1-\lambda)\,\overline{n}$ the LR will bid $\overline{p}$ on all ranks strictly lower than $(1-\lambda)\,\overline{n}$. If he doesn't get a unit on one of these ranks he bids $\overline{p}+\Delta$ on rank $(1-\lambda)\,\overline{n}$ since he could have to fight against other informed LRs if he continues to bid $\overline{p}$. (Note that at rank $(1-\lambda)\,\overline{n}$ there are still enough high quality units for all

LR, namely $\overline{n}-(1-\lambda)\overline{n}=\lambda\overline{n}$.)[6]

In fact the above strategy is not an equilibrium. Since suppose that the cheating firm has rank $r>(1-\lambda)\overline{n}$. Then even in the case of all $\lambda\overline{n}$ cheated LR still waiting for their good, if all start bidding $\overline{p}+\Delta$ starting with rank $(1-\lambda)\,\overline{n}$ then this particular LR will just wait for the firm at $r=\overline{n}$ and bid $\overline{p}$ and get the unit, since there is no other bidder left. It is however not an equilibrium for all bidders to do so, since then there is a chance that the remaining SR who should get the unit from the cheating firm gets it before. Hence the equilibrium involves a mixing between bidding $\overline{p}+\Delta$ and bidding $\overline{p}$.

Optimistic Beliefs: The LR buyer believes that the firm only started cheating at the period he bought the good. Hence after τ periods there are at most $\tau-1$ other LRs who have been cheated. Hence if $r_f^t \leqslant \widehat{r}_\tau^t=\overline{n}-\tau$ then even in the worst case where all those LRs didn't yet get a unit from one of the firms in ranks $r<r_f^t$, they can still all get their unit in the last τ ranks. However in the case where $r_f^t>\widehat{r}_\tau^t$ there is a chance that all the τ cheated LRs are still waiting to buy. If the LR bids $\overline{p}$ then he might not get the good. If he continues to bid $\overline{p}$ or $\overline{p}+\Delta$ then he might not get the good. However once the other buyers who are still waiting for their good learn of a price $p>\overline{p}$ the price will go up, since the high valuation SR might be happy to bid more for average quality. Hence for our LR it might be better to bid $\overline{p}+2\Delta$ since else there is a chance he has to pay much more or not get the good at all. The chances for him to get the good at $\overline{p}$ are $1/(1+\tau)$ the chances to get it at $\overline{p}+2\Delta$ are $\tau/(1+\tau)$, hence he rather bid $\overline{p}+\tau$ in $\widehat{r}_\tau^t$.

The SR have no incentive to deviate.

What we still have to specify is what happens once a bid of $p>\overline{p}$ is announced.$\blacksquare$

The equilibrium $\overline{n}$ will be determined by the detection probability a firm faces if it announces high quality but only produces low quality. A first upper bound on this detection probability is given by

Claim 1: *Both, in pessimistic and in optimistic beliefs case the hazard rate (= probability of detection in period τ given survival till τ)[7] is strictly lower than λ for all τ and all x_t.*

Proof: (Set) detection is impossible if $r_f^t \leq (1-\lambda)\,\overline{n}$.

Problem: Very lousy upper bound!

Claim 6: *With pessimistic beliefs the (set-) detection probability, $\pi_L(x,D)$, is given by*

[6] Will he get the unit at this rank? Not necessarily. There could be another cheated LR who bids $\overline{p}+\Delta$ here. Then the unit is assigned randomly. Independently of the outcome of the randomization there are enough high quality units left for all other LRs.

[7] The hazard rate $\widetilde{\phi}_{L\tau}$ is given by $\widetilde{\phi}_{L\tau}=\phi_{L\tau}/\left(1-\sum_{\tau'=1}^{\tau-1}\phi_{L\tau'}\right)$.

$$\pi_L(x,D) = \lambda\left[1 - f\left(0 \mid \overline{n}, \lambda\overline{n}+1, x\right)\right] = \lambda\left[1 - f\left(x \mid \overline{n}, \overline{n}-\lambda\overline{n}-1, x\right)\right],$$

where $f(\cdot \mid P, S, A)$ stands for the p.f. of the hypergeometric distribution with population size P, sample size S, and number of items of type 1 in the population equal to A, and where $f(l \mid \cdot)$ is the probability that exactly l items of type 1 are in the sample. Similarly, the transition probabilities $\pi_L(x, x+l)$ are given by

$$\pi_L(x, x+l) = f\left(l \mid \overline{n}-x, 1, \overline{n}\lambda - x\right) \cdot \left[1 - \pi_L(x,D)\right].$$

Proof: In the setting under consideration (set-) detection occurs if (i) the cheating firm ends up on one of the $\lambda\overline{n}$ rear ranks, and (ii) at least one of the cheated LRs doesn't get a unit from a firm on one of the $(1-\lambda)\overline{n}-1$ front ranks. The probability of event (i) is λ, and event (ii) doesn't occur if none of the cheated LRs is allocated to one of the $\lambda\overline{n}+1$ "rear units" in a process that randomly matches buyers to units (without replacement). Since the latter event occurs with the indicated hypergeometric probability $f(0 \mid ..)$, $\pi_L(x,D)$ is as stated in Claim 5. Next consider the transition probabilities $\pi_L(x,x+l)$. Transition from state x to state $x+l$ occurs if (i) the firm is not detected (prob. $1-\pi_L(x,D)$), and (ii) exactly $l\,(=0,1)$ new LRs are cheated in the period under consideration. Since the firm is detected if one of the already cheated LRs is allocated to the cheating firm the conditional transition probability is calculated with the reduced population size $\overline{n}-x$.

Implication: With pessimistic beliefs the long run detection probability is given by

$$\pi_L(\overline{n}\lambda, D) = \begin{cases} \lambda\left[1 - \dfrac{\left[(1-\lambda)\overline{n}\right]!}{\overline{n}!} \cdot \dfrac{\left[(1-\lambda)\overline{n}-1\right]!}{\left[\overline{n}-2\lambda\overline{n}-1\right]!}\right] & \textit{if } \lambda \leq (\overline{n}-1)/2\overline{n} \\ \lambda & \textit{otherwise.} \end{cases}$$

Proof: From the definition of the hypergeometric distribution (from the definition of binominal coefficients)

$$f\left(0 \mid \overline{n}, \lambda\overline{n}+1, x\right) = \begin{cases} \dfrac{\left[\overline{n}-x\right]!}{\overline{n}!} \cdot \dfrac{\left[(1-\lambda)\overline{n}-1\right]!}{\left[(1-\lambda)\overline{n}-1-x\right]!} & \text{if } x \leq (1-\lambda)\overline{n}-1 \\ 0 & \text{otherwise.} \end{cases}$$

Implication: With pessimistic beliefs $\phi_{L1} = \lambda^2\left(\lambda + \frac{1}{\overline{n}}\right)$. Furthermore, an upper bound for the hazard rate for general τ is

$$\widetilde{\phi}_{L\tau} < \pi_L(\tau, D) = \lambda\left[1 - \left(\frac{\overline{n}(1-\lambda)-\tau}{\overline{n}-\tau+1}\right)^{\tau}\right] \quad \textit{for } \tau \leq \min\left\{(1-\lambda)\overline{n}-1, \lambda\overline{n}\right\}.$$

Proof: In the first period after initial cheating (set) detection occurs if (i) the cheating firm had a LR player in the previous period (prob. λ), (ii) the cheating firm is allocated to one of the $\lambda\overline{n}$ rear ranks (prob. λ), and (iii) the cheated LR doesn't get a unit in one of $(1-\lambda)\overline{n}-1$ front positions (prob. $(\lambda\overline{n}+1)/\overline{n})$). The upper bound for general τ is calculated under the worst case assumption that the cheating firm cheats a new LR buyer each period. Under this assumption

$$f(0 \mid \overline{n}, \lambda\overline{n}+1, x=\tau) = \frac{[\overline{n}-\tau]!}{n!} \cdot \frac{[(1-\lambda)\overline{n}-1]!}{[(1-\lambda)\overline{n}-1-\tau]!} \quad \text{for } \tau \leq (1-\lambda)\overline{n}-1.$$

A lower bound for this expression is

$$\left[\frac{(1-\lambda)n-\tau}{n-\tau+1}\right]^{\tau}$$

Claim 7: With optimistic beliefs the hazard rate $\widetilde{\phi}_{L,\tau}$ is strictly smaller than $\tau/\overline{n}$ for $\tau \leq \lambda\overline{n}$. It converges to the pessimistic beliefs benchmark in the long run.

Proof: With the optimistic beliefs strategies specified before (set-) detection is impossible if $r_f^t \leq \overline{n}-\tau$. ($LR$ cheated previous period bids $\overline{p}+\Delta$ only if $r_f^t = \overline{n}$, LR cheated two periods ago bids $\overline{p}+\Delta$ only if $r_f^t \epsilon \{\overline{n}-1, \overline{n}\}$, LR cheated...)

Claim 8: With optimistic beliefs the (set-) detection probability $\pi_L(\mathbf{x}, D)$ is given by

$$\pi_L(\mathbf{x}, D) = \frac{1}{\overline{n}} \sum_{l=0}^{T-1} \left[1 - \prod_{k=l+1}^{T} f(0 \mid \overline{n} - \sum_{s=k+1}^{T} x_s, k+1, x_k)\right].$$

Similarly, the transition probability from state $\mathbf{x} = (x_1, x_2, ...x_{T-1}, x_T)$ to state $\mathbf{x}' = \left(x_1', x_2', ...x_{T-1}', x_T'\right)$ is given by

$$\pi_L(\mathbf{x}, \mathbf{x}') = f(l \mid \overline{n} - \sum_{\tau=1}^{T} x_\tau, 1, \overline{n}\lambda - \sum_{\tau=1}^{T} x_\tau)\left[1 - \pi_L(\mathbf{x}, D)\right]$$

for $\mathbf{x}' = (l, x_1, ...x_{T-2}, x_{T-1}+x_T)$ and by $\pi_L(\mathbf{x}, \mathbf{x}') = 0$ otherwise.

Proof: First notice, that all LR buyers that have been cheated $\lambda\overline{n}$ or more periods ago have the same belief support, namely that all LRs could already have been cheated. Thus, $T = \lambda\overline{n}$. With optimistic beliefs (set) detection occurs if (i) one of the LRs that were cheated s periods ago (or more than T periods ago, respectively) doesn't get a unit in one of the $\overline{n}-s-1$ front positions (in the $\overline{n}-\lambda\overline{n}-1$ front positions, respective), and (ii) the cheating firm is allocated to one of the s rear position. Take any admissible $\mathbf{x} = (x_1, x_2, ...x_T)$. First assume that $r_f^t < (1-\lambda)\overline{n}+1$. Then no cheated LR will bid $\overline{p}+\Delta$. Thus, (set) detection cannot occur if $r_f^t < (1-\lambda)\overline{n}+1$. Now assume that $r_f^t \geq (1-\lambda)\overline{n}+1$. On such a rank the firm will *not* be detected if (1.) all LRs that have been cheated T or more periods ago are not matched to one of the $T+1$ rear positions (prob. $f(0 \mid \overline{n}, \lambda\overline{n}+1, x_T)$, (2.) all LRs cheated exactly $T-1$ periods ago are not matched to one of the T rear positions given that the LRs that have been cheated T or more periods ago are not matched to one of these positions (prob. $f(0 \mid \overline{n}-x_T, \lambda\overline{n}, x_{T-1})$, ..., and $(T-l.)$ all LRs

cheated exactly $l+1$ periods ago are not matched to one of the $l+2$ rear positions (where l is defined as $l = \overline{n} - r_f^t$) given that all LRs cheated earlier are not matched to one of these positions (prob. $f\left(0 \mid \overline{n} - \sum_{s=l+2}^{T} x_s, l+2, x_{l+1}\right)$). The formula in Claim 8 takes the product over all these probabilities, summarizes over all relevant ranks (taking into consideration that we are interested in the probability of detection and not in that of non-detection) and divides the sum by the total number of ranks ($\overline{n}$). Next consider the transition probabilities $\pi_L(\mathbf{x}, \mathbf{x}')$. First notice that transition can only occur from a state $\mathbf{x} = (x_1, ..., x_{T-1}, x_T)$ with $x = \sum\limits_{s=1}^{T} x_s < \lambda\overline{n}$ to a state $\mathbf{x}'$ with $\mathbf{x}' = (l, x_1, ...x_{T-2}, x_{T-1} + x_T)$ and $l \leq \lambda\overline{n} - x$. Transition between 2 such states occurs if (i) the cheating firm is not detected (prob. $1 - \pi_L(\mathbf{x}, D)$), and (ii) exactly $l (= 0, 1)$ new LRs are cheated in the period under consideration. Since detection occurs if one of the already cheated LRs is allocated to the cheating firm the conditional transition probability is calculated with the reduced population size $\overline{n} - x$, where $x = \sum\limits_{s=1}^{T} x_s$.

Claim 9: With optimistic beliefs $\widetilde{\phi}_{L1} = \phi_{L1} = 2\lambda/\overline{n}^2$. *Furthermore, an upper bound for the hazard rate* $\widetilde{\phi}_{L\tau}$ *for* $\tau \leq \lambda\overline{n}$ *is*

$$\widetilde{\phi}_{L\tau} < \pi_L(\mathbf{x}(\tau), D) = \frac{1}{\overline{n}} \sum_{l=0}^{\tau-1} \left[1 - \frac{(\overline{n} - \tau - 1)^{\tau - l} (\overline{n} - \tau + l)!}{\overline{n}!}\right],$$

where $\mathbf{x}(\tau) = (x_1(\tau), x_2(\tau), ...x_T(\tau))$ *and where* $x_k(\tau) = 1$ *for* $k \leq \tau$ *and* $x_{kl}(\tau) = 0$ *otherwise.*

Proof: In the *first period* after initial cheating, detection occurs if (i) the cheating firm had a LR player in the previous period (prob. λ), (ii) the cheating firm is allocated to the rear rank (prob. $1/\overline{n}$), and (iii) the cheated LR doesn't get a unit in one of the $\overline{n} - 3$ front positions (prob. $2/\overline{n}$). The *upper bound* for $\tau \leq \lambda n^*$ is again calculated under the worst case assumption that the cheating firm cheats a new LR buyer each period. Under this assumption (set) detection in period τ occurs if (i) the LR buyer that was cheated s periods ago doesn't get a unit one of in the $\overline{n} - s - 1$ front positions (i.e., he would have to buy from one of the $s+1$ rear positions) and (ii) the cheating firm is allocated to one of the s rear positions. Suppose a firm began cheating $\tau \leq \lambda\overline{n}$ periods ago. Also suppose that in the current period this firm is allocated to a rank r_f^t with $r_f^t \geq \overline{n} - \tau + 1$. On this rank it will not be detected if (1.) the LR that has been cheated τ periods ago is not matched to one of the $\tau + 1$ rear positions (prob $f(0 \mid \overline{n}, \tau + 1, 1)$), (2.) the LR cheated $\tau - 1$ periods ago is not matched to one of the τ rear positions given that the LR cheated τ periods ago is not matched to one of these positions (prob $f(0 \mid \overline{n} - 1, \tau, 1)$), ..., and $\left(\tau - \overline{n} + r_f^t.\right)$ the LR cheated $\overline{n} - r_f^t + 1$ periods ago is not matched to one of the $\overline{n} - r_f^t + 2$ rear positions given that all LRs cheated earlier are not matched to one of these positions (prob $f\left(0 \mid 2\overline{n} - r_f^t - \tau + 1, \overline{n} - r_f^t + 2, 1\right)$). The product of all these probabilities is given by

$$\frac{\binom{\overline{n}-1}{\tau+1}}{\binom{\overline{n}}{\tau+1}} \cdot \frac{\binom{\overline{n}-2}{\tau}}{\binom{\overline{n}-1}{\tau}} \cdot \frac{\binom{n-\tau+l}{2+l}}{\binom{\overline{n}-\tau+1+l}{2+l}} = \frac{(\overline{n}-\tau-1)^{\tau-l}\,(\overline{n}-\tau+l)!}{\overline{n}},$$

where l is defined by $l = \overline{n} - r_f^t$. The formula in Claim 9 is obtained by taking the expectation over all relevant ranks taking into consideration that we are interested in the probability of detection (and not in that of non-detection).

Remark to Claim 9: We have also most of the calculations for the upper bound mentioned in Claim 9 for $\tau \epsilon [\lambda\overline{n}, 2\lambda\overline{n}]$. For $\tau \geq 2\lambda\overline{n}$ the upper bound considers with that of the pessimistic beliefs framework; that is, it is given by

$$\widetilde{\phi}_{L\tau} < \pi_L(\overline{\mathbf{x}}, D) = \lambda \left[1 - \left(\frac{\overline{n}(1-\lambda) - \lambda\overline{n}}{\overline{n}(1-\lambda)+1} \right)^{\lambda\overline{n}} \right],$$

where $\overline{\mathbf{x}} = (0, 0, ..., \lambda\overline{n})$.

A Note on Optimistic / Pessimistic Beliefs

Optimistic Beliefs: With optimistic beliefs it can happen that some LR cheated earlier bids $\overline{p} + \Delta$ on an earlier position and that other cheated LRs who haven't got a unit before have to compete for a high quality unit against non-cheated LRs that bought from a non-cheating firm in previous period, a firm that has got a rear position in the current period. The price might move up and cheated LRs may regret ex post that they didn't bid $\overline{p} + \Delta$ on an earlier rank. However, if the cheated LRs have really optimistic beliefs they simply don't believe that this can happen.

Pessimistic Beliefs: By construction no cheated LR bids in the rear ranks. Competition for a high quality unit on rear ranks is therefore impossible with pessimistic beliefs. However, if competition is impossible, then each cheated LR has an incentive to wait (i.e., not to bid $\overline{p} + \Delta$) and buy his unit in one of the $(\lambda\overline{n} - 1)$ non-cheating firms on the rear ranks. If more cheated LRs do this then there is a positive prob. of competition. Therefore, no symmetric equilibrium in pure strategies exists. Solution: Argue that there exists a symmetric mixed strategy equilibrium where cheated LRs wait with small prob. depending on Δ. As $\Delta \to 0$, the mixed strategy equilibrium converges to our pure strategy equilibrium where cheated LRs bid only on the $(1 - \lambda)$ front ranks.

3.2.3 Trading history with demand overhang observed (DOO)

Format: Same as in the sealed bid first price auction with demand overhang not observed (DONO). The only difference concerns the available information. While the prices and the quantities sold were the only signalling variables in the DONO version, now the number of bids at the equilibrium price is also publicly observed.

Strategies: Same as in the DONO version. Note that non-participation in the auction of unit produced by the cheating firm if that firm ends up on some front rank remains optimal because the demand overhang is only observed ex post: The cheating firm on a front rank will still sell its unit at the high price to some ignorant during the detection period. The price at which the (good) units of firms on later ranks are sold therefore remains unaffected. This is different to the English auction where the presence of instantaneous information generates mimicking and tricking incentives.

Results:

Claim 10: Both, in the pessimistic and in the optimistic beliefs case the hazard rate is strictly higher in the DOO- than in the DONO version provided at least one LR buyer has already been cheated.

Proof: If $r_f^t < \overline{n}(1-\lambda)+1$ then the detection probability is zero both in the pessimistic and in the optimistic beliefs case of the DONO version. In the DOO version, on the other hand, this probability is strictly positive on all ranks since non-participation of cheated LRs is publicly observed. On rank 1, for example, the punishment probability is one since the demand at the equilibrium price, denoted by $D(1)$, will necessarily be smaller than $\overline{n}q$ if at least one LR has already been cheated. Denote the demand at the equilibrium price on rank r by $D(r)$. Then $D(r) = \overline{n} - r + 1$ if either (i) the firm on rank r is a non-cheating firm, or (ii) a cheating firm is on rank r but all cheated LRs have already got their unit; and $D(r) < \overline{n} - r + 1$ otherwise. *If* $r_f^t \geq \overline{n}(1-\lambda)+1$ then the detection probability is never lower in the DOO- than in the DONO version and it will sometimes be strictly higher (see the arguments below).

Claim 11: With pessimistic beliefs the (set-) detection probability $\pi_L(x,D)$ *is given by*

$$\begin{aligned}\pi_L(x,D) &= \lambda\left[1-f(0\mid\overline{n},\lambda\overline{n}+1,x)\right]+\tfrac{1}{\overline{n}}\sum_{l=\lambda\overline{n}+1}^{\overline{n}}\left[1-f(0\mid\overline{n},l,x)\right]\\ &= \lambda\left[1-f(x\mid\overline{n},\overline{n}-\lambda\overline{n}-1,x)\right]+\tfrac{1}{\overline{n}}\sum_{l=0}^{(1-\lambda)\overline{n}-1}\left[1-f(x\mid\overline{n},l,x)\right],\end{aligned}$$

where $f(\cdot\mid P,S,A)$ *stands again for the p.f. of the hypergeometric distribution. Similarly, the transition probabilities* $\pi_L(x,x+l)$ *are given by*

$$\pi_L(x,x+l) = f(l\mid\overline{n}-x,1,\overline{n}q\lambda-x)\cdot\left[1-\pi_L(x,D)\right].$$

Proof: First consider the (set-) *detection probability.* On the $(1-\lambda)\overline{n}$ front ranks the cheating firm is not detected if all cheated LRs have got their unit from an earlier firm. Suppose the cheating firm is on a rank r_f^t with $r_f^t < (1-\lambda)\overline{n}+1$. Then the probability that all cheated LRs have already bought on an earlier rank is given by the hypergeometric probability $f\left(x\mid\overline{n},r_f^t-1,x\right)$. The (second part of the) formula in Claim 11 summarizes these probabilities over all relevant ranks (taking into consideration that we are interest in the probability of detection and not in that of non-detection) and divides the sum by the total number of ranks. In the $\lambda\overline{n}$ rear ranks we have the same detection probability as in the DONO version. Thus, the same term as there appears here in the first part of the formula. The conditional *transition probabilities* are the same as in the DONO version.

Implication: With pessimistic beliefs the detection probability / hazard rate for $\tau = 1$ is given by

$$\phi_{L1} = \widetilde{\phi}_{L1} = \lambda \left[(1-\lambda) \frac{\overline{n}(1+\lambda)+1}{2\overline{n}} + \lambda \frac{\lambda\overline{n}+1}{\overline{n}} \right] > \lambda \left(\lambda + \frac{1}{\overline{n}} \right)$$

(while $\phi_{L1} = \lambda^2 \left(\lambda + \frac{1}{\overline{n}}\right)$ in the DONO version). Furthermore, the hazard rate for general values of τ is given by ...

Proof: In the first period after initial cheating detection can only occur if the cheating firm had a LR buyer in the precious period (prob. λ). Suppose this is the case. Then the detection probability is $\lambda + 1/\overline{n}$ if the cheating firm is allocated to one of the $\lambda\overline{n}$ rear ranks (prob. λ) exactly as it was in the DONO version. What is different is the detection probability on the $(1-\lambda)\,\overline{n}$ front ranks. On rank $r_f^t \leq (1-\lambda)\,\overline{n}$ the cheating firm will be detected if the cheated LR hasn't got his unit on an earlier rank. This occurs with probability $1 - \left(r_f^t - 1\right)/\overline{n}$. Summarizing from $r_f^t = 1$ to $r_f^t = (1-\lambda)\,\overline{n}$ and dividing by $\overline{n}$ yields $(1-\lambda)\left[\overline{n}(1+\lambda)+1\right]/2\overline{n}$ which is exactly the factor that distinguishes the formula for the DOO-version from that for the DONO-version.

Claim 12: With optimistic beliefs the (set-) detection probability $\pi_L(\mathbf{x}, D)$ is given by

$$\pi_L(\mathbf{x}, D) = \tfrac{1}{\overline{n}} \sum_{l=\lambda\overline{n}+1}^{\overline{n}} \left[1 - f(0 \mid \overline{n}, l, x)\right] +$$

$$+\tfrac{1}{\overline{n}} \sum_{l=0}^{T-1} \left[1 - f(0 \mid \overline{n} - \sum_{s=l+1}^{T} x_s, l+1, \sum_{s=0}^{l} x_s) \prod_{k=l+1}^{T} f(0 \mid \overline{n} - \sum_{s=k+1}^{T} x_s, k+1, x_k) \right].$$

Similarly, the transition probability from state $\mathbf{x} = (x_1, x_2, ...x_{T-1}, x_T)$ to state $\mathbf{x}' = \left(x_1', x_2', ...x_{T-1}', x_T'\right)$ is given by

$$\pi_L(\mathbf{x}, \mathbf{x}') = f(l \mid \overline{n} - \sum_{\tau=1}^{T} x_\tau, 1, \overline{n}\lambda - \sum_{\tau=1}^{T} x_\tau)\left[1 - \pi_L(\mathbf{x}, D)\right]$$

for $\mathbf{x}' = (l, x_1, ...x_{T-2}, x_{T-1} + x_T)$ and by $\pi_L(\mathbf{x}, \mathbf{x}') = 0$ otherwise. That is, the transition probability is the same as in the DONO version.

Proof: For the $(1-\lambda)\,\overline{n}$ front ranks the argument (and the formula) for the detection probability is exactly the same as in the pessimistic beliefs framework (compare the first term of the formula in Claim 12 with the second term of the formula in Claim 11). The difference concerns the $\lambda\overline{n}$ rear ranks. On such a rank the firm will not be detected if (1.) none of those LRs that have been cheated at least $\overline{n} - r_f^t + 1$ periods ago is matched to a rank higher than its respective critical rank given that none of those LRs that have been cheated earlier is matched to one of these positions either (this is exactly the same story as in the optimistic beliefs framework of the DONO version; thus, the same product of conditional probabilities as there appears here in the last term of the formula), and (2.) none of those LRs that have been cheated $\overline{n} - r_f^t$ or less periods ago is matched to one of $\overline{n} - r_f^t + 1$

rear positions given that all none of those LRs cheated strictly earlier than $\overline{n} - r_f^t$ is matched to one of these positions either. This last event has probability $f\left(0 \mid \overline{n} - \sum_{s=l+1}^{T} x_s, l+1, \sum_{s=0}^{l} x_s\right)$ with $l = \overline{n} - r_f^t$ and explains the remaining term in the formula.

Implication: With optimistic beliefs the detection probability / hazard rate for $\tau = 1$ is given by

$$\phi_{L1} = \widetilde{\phi}_{L1} = \frac{\lambda}{\overline{n}^2} + \frac{(\overline{n}+1)\lambda}{2\overline{n}} = \frac{\lambda}{2} + \frac{\lambda}{2\overline{n}} + \frac{\lambda}{\overline{n}^2}$$

(while $\phi_{L1} = 2\lambda/\overline{n}^2$ in the DONO version).

Proof: In the first period after initial cheating, detection occurs if (i) the cheating firm had a LR player in the previous period (prob. λ), and (ii) the cheated LR hasn't got his unit on a rank strictly lower than that of the cheating firm if $r_f^t < \overline{n}$ and on a rank strictly lower than $\overline{n} - 1$ if $r_f^t = \overline{n}$ (prob. $\left(\overline{n} - r_f^t + 1\right)/\overline{n}$ for $r_f^t < \overline{n}$ and prob. $\left(\overline{n} - r_f^t + 2\right)/\overline{n}$ for $r_f^t = \overline{n}$). Summarizing over all ranks and dividing by $\overline{n}$ yields the formula in the text.

A Note On Set Detection

What happens to the rest of the market after a $\overline{p} + \Delta$ bid has been observed? *In the detection period* all informed buyers will make bids ($\overline{p}$, or $\overline{p} + \Delta$, or even more if next to last non-cheating firm) for non-cheating firms. LRs that in the previous period got a good unit from one of the firms in the set have an incentive to bid at least $\overline{p} + \Delta$ for that firm. Low valuation LRs that bought from firms outside the set in the previous period and low valuation SRs will drop out. Sales in the detection period signal good history. Non-monotonicity in "EDI" (extra supply followed by extra demand) signals bad history. *In the next period* all firms that were on front ranks during the detection period and firms on rear ranks with good sales will attract customers. There will emerge a new $\overline{p}$‘ for this set of firms. Other firms have dubious histories. A different price will be paid to these firms. Problem: The price for these firms is lower and they will not trade with probability one. Thus, their incentive compatibility constraint changes. There are at least 2 possibilities: If the set of firms with dubious history is relatively small than there will be no price sufficiently lower than $\overline{p}$‘ for which the IC constraint holds. Thus, these firms will be replaced by new ones. If the set of firms with dubious history is large, then $\overline{p}$‘ will be large, too. Then there might exist a price $p < \overline{p}$‘ s.t. firms with dubious history supply high quality at a high enough price and with high enough probability of sale such that their IC constraint holds. (Notice that LRs that bought on front ranks during the detection period but from a firm in the detected set a period before will have an incentive to bid an additional Δ for units in that firm. The same is true for uninformed LRs that got a good unit from a firm in the detection set during the detection period.)

3.3 Auction with Coordination

To study the impact of coordination on the punishment probability we return to the sealed bid first price auction in the DONO version.

Information: Our analysis is based on the assumption that "EDI" is observable. The same results should hold if EDI is unobservable. Detection may, however, be delayed by at least one period in the latter case. Assume, for example, (i) that cheating firm has already cheated at least one LR and (ii) that $r_f^t \geq (1-\lambda)\,\overline{n}+1$. Then cheated LRs will bid $\overline{p}$ from rank 1 on. If at least one cheated LR gets a unit in one of the $(1-\lambda)\,\overline{n}$ front ranks then not all SRs will get a unit in one of these ranks. Since the bounced SRs know that they should get a unit on one of the $(1-\lambda)\,\overline{n}$ front ranks along the equilibrium path, they know that some cheating firms is on a rear rank. If their valuation is low, they will drop out. However, not jet cheated LRs will fall in the hole and the firm on rank $\overline{n}$ will be unable to sell its unit. In the next period all players know that (at least) one cheating firm is in the market. They also know that the cheating firm was on one of the $\lambda\overline{n}$ rear ranks in the previous period. LRs know even more: Cheated LRs know exactly which firm has cheated. Not yet cheated LRs know that the firms from which they bought in the previous two periods are good firms. Competition will drive the price up. Cheating firm might have no customers ...

Strategies: *On the equilibrium path* all SR buyers with type β_k greater or equal than $(\overline{p}-\underline{c})\,/\,\left(\overline{\theta}-\underline{\theta}\right)$ bid $\overline{p}$ on each rank (in the "EDI not observed" version: on each rank $r \leq (1-\lambda)\,\overline{n}$) until they get their unit. All LR buyers with type greater or equal than $(\overline{p}-\underline{c})\,/\,\left(\overline{\theta}-\underline{\theta}\right)$ bid $\overline{p}$ from rank $(1-\lambda)\,\overline{n}+1$ on until they get a unit. *Off the equilibrium path* the behavior of the cheated LRs depends on the rank of the cheating firm, denoted by r_f^t. If $r_f^t \leq (1-\lambda)\,\overline{n}$ then the cheated LRs behave as before. If $r_f^t > (1-\lambda)\,\overline{n}$ they bid $\overline{p}$ from first rank on. If they don't get a unit strictly before rank $(1-\lambda)\,\overline{n}$ they

$$
\begin{array}{lll}
\text{bid } \overline{p}+\Delta & \text{on rank } (1-\lambda)\,\overline{n} \Rightarrow & \text{set detection} \\
\text{bid } \overline{p} & \text{on rank } (1-\lambda)\,\overline{n} \Rightarrow & \text{with "EDI. observed", set detection} \\
\text{don't bid} & \text{on rank } (1-\lambda)\,\overline{n} \Rightarrow & \left\{\begin{array}{l} \text{set detection if at least one (out of } q \text{) cheated } LRs \\ \text{has got a unit on an earlier rank or has made a} \\ \text{bid at rank } (1-\lambda)\,\overline{n} \end{array}\right.
\end{array}
$$

The probability of detection conditional on a long run player has been cheated is approximately λ, i.e. the probability that the firm is assigned to one of the last $\lambda\overline{n}$ ranks. Punishment is more likely than in the same auction without coordination strategies. Coordination creates information in that it uses a signal of the auction institution and makes it to carry information about cheating, which it was not able without coordinated strategies.

(incomplete)

4 Decentralized Bargaining Market (q=1)

The decentralized bargaining market is a market where participants only obtain information about their own trades, but no information about other peoples trade. In each period firms and buyers choose with whom to interact, meaning that firms as well as buyers can reject trading partners.

The most obvious example for a decentralized bargaining institution is the Bazaar market (Geertz 1978) where quality uncertainty is an enormous issue. However also in developed countries in some markets repeat purchase is frequently observed. Most Fish markets are organized as dezentralized bargaining markets with the occurence of repeat purchase (Graddy 1995) and Weisbuch, Kirman and Herreiner (1996). The phenomenon of repeat purchase is also observable in transactions between firms. For instance Macaulay (1963) provides some evidence of repeat purchase. Baker (1990) found that strong relationships (long-term, exclusive ties) between firms and banks coexist next to short-lived, episodic ties.

Assumption 1: *The firms know the identities of the long run players.*

Assumption 2: *Firms can commit to a bargaining protocol and have to make the first announcement.*

In our model both parties in a long run relationship have switching costs *ex post*: the LR buyer, since he is unlikely to find another firm that sells him a high quality unit in the current period; and the firm, since she is unlikely to be able to sell her unit for the high price to another buyer in the current period. A crucial aspect of switching costs is that even though the firm and the LR buyer can select each other *ex ante* in a pool of many firms and many LR buyers, they end up forming an *ex post* bilateral monopoly since there a gains from trading among them rather than with outside parties. It is important that these gains from trade are exploited correctly, i.e., in a way that (i) guarantees an efficient amount of trade *ex post* (if the firm has all the bargaining power and if she does not know her LRs valuation β she may run the risk of forgoing trade in order to get a larger share of the pie in case of trade), and (ii) induces the firm to produce high quality *ex ante* (if the firm knows that the buyer will appropriate a large part of the common surplus *ex post* she may have no incentive to incur the high cost $\overline{c}$ *ex ante*). Assumption 2 helps to solve the *ex ante* quality choice problem. The *ex post* trade problem is solved by appropriately specifying the buyers' out of equilibrium beliefs (see below). Since the focus of the present paper is the quality enforcement problem in a decentralized market and not the hold up problem, our rather crude solution of the latter seems to be justified.

The Matching Process: Firms Search for Buyers In each period, firms approach buyers with a bargaining protocol. If a buyer gets approached by more than one firm, he has to choose with which, if any, firm to bargain. Each firm that gets rejected can approach a new buyer with a new bargaining protocol (but not the same buyer as before, if the bargaining protocol was a take-it-or-leave-it-offer). Rematching and making offers within a period is costless. Bargaining in each match takes place according to the bargaining protocol proposed by the firm. Those pairs that reach an agreement execute their trade and leave the market. The others start anew. When there are no more possible pairs, the period is over.

This matching mechanism corresponds to firms submitting a list of buyers with protocols and buyers submitting a list of firms with protocols and firms making the offers to buyers.

The equilibrium will depend on the beliefs of the LRs that did not get approached by their regular firm in a period. Will they punish forever, i.e. never again buy from this firm, or will they behave as toward a firm that always sold to them? This will determine what quality sample a very high valuation SR player will get.

4.1 The Exclusive Relationship Equilibrium

There exists an exclusive relationship equilibrium where high valuation LRs have stable relationships with firms that produce high quality. In each partnership, the LR buyer exclusively trades with his firm and the firm exclusively sells her unit to her LR buyer. The equilibrium price for the high quality unit is $\overline{p}^{ER} = \underline{p} + (\bar{c} - \underline{c})/\delta$ and each LR with a valuation $\beta_k \geqslant \overline{\beta}^{ER} = (\bar{c} - \underline{c})/\delta(\overline{\theta} - \underline{\theta})$ has a stable partnership. Thus, the number of firms producing high quality is $\bar{n}^{ER} = \lambda \sum_{\beta_k \geqslant \overline{\beta}^{ER}} m_k = \lambda\overline{n}^{SB} < \lambda\overline{n}^{FB} = \lambda n$.

We look at equilibria where the firm and the LR player agree on a bargaining price that is the minimum enforcement price.

Strategies

Firms: Firms that sold to a LR in the first period continue to produce high quality and approach the same LR with a take-it-or-leave-it offer at the equilibrium price $\overline{p}^{ER}$. When their regular consumer does not accept, they approach all other LRs first and only later the SRs, and sell at $\underline{p}$. High quality firms that traded with a LR buyer continue with their strategy as before (i.e. produce high quality units and make a take-it-or-leave-it offer at $\overline{p}^{ER}$). Firms that traded with a SR buyer in the previous period produce low quality and sell at $\underline{p}$, though always trying to reach LRs first (just in case one is free).

Long Run Buyers: Each LR waits to be approached by the firm he bought from last period and accepts any take-it-or-leave-it-offer at a price $\overline{p}^{ER}$.

Beliefs out of equilibrium: If a LR hears a price different from $\overline{p}^{ER}$, or a bargaining protocol different from a take-it-or-leave-it-offer, he suspects the firm of cheating and only accepts offers at $p \leqslant \underline{p}$, preferably from another firm. LRs without a regular firm accept only offers at $\underline{p}$, unless they get approached by the same firm as in the previous period. In this case they believe that the firm produced a high quality unit and accept an offer at $\overline{p}^{ER}$, if their valuation $\beta_k \geqslant \overline{\beta}^{ER}$. Hence, LRs that get approached by a firm they bought from at some time in the past but not in the previous period believe that this firm has produced a low quality unit.

Short Run Buyers: Each SR, when approached by a firm, accepts any take-it-or-leave-it offer of $\underline{p}$ and rejects any other offer. SRs believe that only firms that produced low quality make them an offer.

Proposition 1 (Exclusive Relationship Equilibrium) *There exists an equilibrium where in each period after the first one the number of high quality units equals the number of long run buyers with valuation* $\beta_k \geqslant \overline{\beta}^{ER} = (\bar{c} - \underline{c})/\delta(\overline{\theta} - \underline{\theta})$. *Each Long Run buyer with valuation* $\beta_k \geqslant \overline{\beta}^{ER}$ *gets a high quality unit at a price* $\overline{p}^{ER} = \underline{c} + (\bar{c} - \underline{c})/\delta$ *from his "regular" firm. LRs with valuation* $\beta_k < \overline{\beta}^{ER}$ *and Short Run buyers get low quality units from other firms at a price* $\underline{p} = \underline{c}$. *Hence,* $\bar{n}^{ER} = \lambda \sum_{\beta_k \geqslant \overline{\beta}^{ER}} m_k$.

Proof: No firm in a long run relationship with a LR Buyer has an incentive to deviate: If the firm starts announcing a price $p \neq \overline{p}^{ER}$, or a bargaining protocol different from a take-it-or-leave-it offer at $\overline{p}^{ER}$, then the LR buyer believes that the firm is offering a low quality unit and waits for another firm to buy at $\underline{p}$. However, the firm will not find another LR or a SR who is willing to believe that she has produced a high quality unit, hence she will not produce high quality and then start deviating from making a take-it-or-leave-it offer at $\bar{p}^{ER}$.

The incentive constraint for the firm to produce high quality

$$(1-\delta)\left(\overline{p}^{ER} - \overline{c}\right) + \delta(\overline{p}^{ER} - \overline{c}) \geqslant (1-\delta)\left(\overline{p}^{ER} - \underline{c}\right) + \delta(\underline{p} - \underline{c})$$

implies for $\underline{p} = \underline{c}$ that

$$\bar{p}^{ER} = \bar{p}^{FP} \geqslant \underline{c} + \frac{\bar{c} - \underline{c}}{\delta}.$$

Can a firm try to sell its high quality unit for a price $\hat{p} > \overline{p}^{ER}$ to a SR who might have a high valuation? No, because the beliefs of the SRs are such that any offer is believed to come from a firm that has produced a low quality unit. Therefore the short run is willing to pay only $\underline{p}$. Here it is important that the initiative to sell to a SR is in the hand of the firm. When the firm intends to sell to a SR buyer, it knows that the produced quality has no influence on the continuation play, and will therefore produce low quality. SR buyers will therefore refuse to buy at the high price. If a high valuation SR can announce its intention of buying at a high price, e.g. through cheap talk, the quality decision has been already made and deviations are possible. We come back to this later on.

No firm producing low quality has an incentive to deviate: Will the firm try to make a take-it-or-leave-it offer at some price $p > \underline{p}$? No LR will accept, neither a SR, hence no incentive to do so. Furthermore, for the same reason, the firm will not start to produce high quality since she will not be able to get a price higher than $\underline{p}$.

No LR Player has an incentive to deviate: Consider a LR with $\beta \geqslant \overline{\beta}^{ER}$. Since the firm has all the bargaining power the LR will accept an offer of $\bar{p}^{ER}$ by "his" firm. If he receives any other offer, he thinks that the firm is producing low quality. He will reject the offer and go to different firm. Hence, a firm that asks for $p > \overline{p}^{ER}$ will anticipate that she will lose her LR player and hence start to produce low quality immediately. High valuation LRs (with $\beta > \overline{\beta}^{ER}$) that receive an offer at $p < \overline{p}^{ER}$ do not believe the firm has produced high quality because the price is too low to satisfy the firms incentive constraint. They will also not accept any low quality offer from another firm since we have $\beta_k \bar{\theta} - \overline{p}^{ER} > \beta_k \underline{\theta} - \underline{p}$ for $\beta > \overline{\beta}^{ER}$. The LRs with $\beta < \overline{\beta}^{ER}$ are not willing to pay the price that ensures quality enforcement, hence they will only get approached by firms that produce low quality.

No SR has an incentive to deviate: Since the SRs only get offers from firms that produced a low quality unit, the SR will never accept any offer at a price $p > \underline{p}$. ■

In this equilibrium a LR player that does not receive the high quality unit from his firm - either because the firm produced low quality or the firm sold the unit to somebody else - punishes this firm by never accepting any future offer from her again. This might seem an extreme assumption. However, in the current period the LR has to buy from another firm anyway. If he can convince the new firm that he will also come next period, then the LR player incurs no losses from this punishment strategy.

Refinements:

The above equilibrium always exists. However, the equilibrium is supported by out-of-equilibrium beliefs that may not be very plausible in some cases. Consider the case when the buyers are allowed to talk first, before the firm makes an offer. Then, very high valuation SR buyers can announce that they are willing to pay a very high price that induces firms in a long run relationship that have produced high quality to sell to them. The following condition rules out this case:

$$\beta_K < \frac{(1+\delta)(\overline{c}-\underline{c})}{\delta(\overline{\theta}-\underline{\theta})\frac{\overline{n}^{ER}}{n}} = \overline{\beta}_{SR}^{ER}$$

For the beliefs of SRs to be "plausible", high quality firms should have no incentive to try to sell their units to SRs. If a high quality firm sells its unit for the price $\hat{p}$ to a SR, her payoff in the current period is $\hat{p} - \bar{c}$. However, she loses her LR forever and hence makes in the future only profits of $\delta(\underline{p} - \underline{c})/(1-\delta)$. This is profitable if and only if

$$(1-\delta)(\hat{p}-\bar{c}) + \delta(\underline{p}-\underline{c}) \geqslant (1-\delta)\left(\bar{p}^{ER}-\bar{c}\right) + \delta(\bar{p}^{ER}-\bar{c}) \quad \Leftrightarrow \quad \hat{p} \geqslant \frac{1}{1-\delta}(\bar{p}^{ER} - \delta(\bar{c} + \underline{p} - \underline{c})).$$

For SRs the expected quality off the equilibrium path, i.e. the average market quality, is $\frac{\bar{n}^{ER}}{n}\overline{\theta} +$

$(1-\frac{\bar{n}^{ER}}{n})\underline{\theta}$. Hence, SRs are willing to pay $\widehat{p}$ for average quality if their valuation β_k satisfies

$$\beta_k(\frac{\bar{n}^{ER}}{n}\overline{\theta}+(1-\frac{\bar{n}^{ER}}{n})\underline{\theta})-\widehat{p}\geqslant\beta_k\underline{\theta}-\underline{p}\qquad\Leftrightarrow\qquad\widehat{p}\leqslant\underline{p}+\beta_k\frac{\bar{n}^{ER}}{n}(\overline{\theta}-\underline{\theta})$$

Hence, for SRs' beliefs to be plausible we need that

$$\frac{\delta}{1-\delta}(\bar{p}^{ER}-\bar{c}-(\underline{p}-\underline{c}))+\bar{p}^{ER}>\underline{p}+\beta_K\frac{\bar{n}^{ER}}{n}(\overline{\theta}-\underline{\theta})$$

or

$$\beta_K<\frac{\bar{p}^{ER}-\underline{p}-\delta(\bar{c}-\underline{c})}{(1-\delta)(\overline{\theta}-\underline{\theta})\frac{\overline{n}^{ER}}{n}}.$$

5 Conclusions

(missing)

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Fract.LR (λ)	Mean Time	$\hat{\delta}$	Markup-Factor	Mean Time	$\hat{\delta}$	Markup-Factor
		N=10			N=50	
0.1	57.944	0.190	4.266	46.957	0.190	4.269
0.2	16.035	0.499	1.005	16.308	0.484	1.065
0.3	8.394	0.673	0.486	8.896	0.657	0.523
0.4	5.374	0.770	0.299	5.725	0.757	0.321
0.5	3.769	0.830	0.205	4.007	0.820	0.219
0.6	2.776	0.870	0.149	2.944	0.863	0.158
0.7	2.100	0.900	0.112	2.224	0.894	0.118
0.8	1.607	0.922	0.085	1.703	0.917	0.090
0.9	1.222	0.940	0.064	1.304	0.936	0.069
1.0	1.000	0.950	0.053	1.000	0.950	0.053

$$\frac{\bar{p} - \bar{c}}{\bar{c}} = \frac{\bar{c} - \underline{c}}{\bar{c}} * \frac{1 - \hat{\delta}}{\hat{\delta}}$$