# Inflation, Equilibrium and Government Commitment

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## 1 Introduction

This paper stems from a recent heated debate on the relationship between the price level and fiscal policy. While the relationship between inflation, government deficits and debt has a long tradition in macroeconomics, a recent string of papers has put a new twist on it by proposing a "fiscal theory of the price level", which shows that the government can target directly the price level by using fiscal variables, such as the present value of future surpluses and the current level of nominal debt.[1]

At stake are some basic ideas about how to control inflation. The traditional view has highlighted the importance of an independent central bank and has held that high inflation can only be ultimately fueled by high rates of money growth.[2] In this view, the fiscal policy is important, but mainly so because excessive deficits may eventually force the central bank to monetize. According to the fiscal theory, the price level is primarily determined by deficits and the debt alone, and a central bank that successfully keeps a low money growth rate *even for ever* may actually force the economy into a hyperinflation spiral. In this case, price stability calls much more for a responsible fiscal policy accompanied by nominal interest-rate targeting.

The key difference between the fiscal theory and the traditional view lies in the interpretation of the government budget constraint, which links the real value of debt to the present value of primary surpluses the government will run in the future. The advocates of the theory view this link as an equilibrium condition: an imbalance between the real value of debt and the surpluses would trigger changes in the price level that would lead back towards an equilibrium, either by reducing or by increasing the value of the nominal debt. The traditional view interprets the link

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[1]See e.g. Leeper [12], Sims [22] and Woodford [24, 25, 26, 28, 27]. Cochrane [3] has extended the analysis to long-term debt, and Dupor [5] to the exchange-rate determination in an open-economy framework. Loyo [14] has applied the theory to study inflation episodes in Brazil.

[2]The traditional view reflects heavily the ideas of Milton Friedman; these ideas are best represented in Friedman and Schwartz [6]. Sargent and Wallace [19] and Sargent [17] highlight the importance of the relationship between fiscal and monetary policy, and are the precursors to the literature on the independence of the central bank.

as a constraint on policy, which forces government action, either through a fiscal adjustment or through a default on debt, whenever the real value of debt and the present value of primary surpluses tend not to be equal.

These two views stem from important differences in the equilibrium concept different researchers have in mind. A first step in this paper is to study in depth which definitions of equilibrium are useful and appropriate in a general environment in which a government interacts with a multitude of private decision-makers that have a negligible impact on the aggregate. In particular, this research focuses on the meaning and consequences of commitment, whereby a government "policy" is set ex ante, either endogenously or exogenously. Some of the commonly used definitions of equilibrium in this environment turn out to be remarkably ambiguous on features that are key to understand the consequences and the consistency of government actions. The goal of this first part is not only to provide tools to address the controversy on the fiscal theory of the price level, but to reach a better understanding of the choices and constraints policy-makers face in designing their policies. I address these issues in section 2.

In the second part of this paper I use the tools developed in the first part to address the specific issue of the fiscal theory of the price level. As it is shown below, a more-complete specification of the economic environment is important in establishing when and how it is possible to control the price level by relying on the government surplus and debt. Section 3 develops a simple model to this end, and discusses how it can be extended to address important aspects that are left out of the basic version.

The goal of this paper is to reach a clearer and hopefully less-controversial understanding of the constraints imposed on monetary and fiscal policy by their interdependence. The tools that are developed in this project are also useful to analyze the design of a broader class of government policies and to explore the limits of the power of commitment.

# 2 Equilibrium and Government Commitment

In this paper I focus on environments in which the government can commit to future actions. I thus abstract from time-consistency issues. The motive for studying the consequences of commitment is twofold.

(i) To this date, most macroeconomic models treat government policy as exogenous, and aim at understanding the consequences of different choices by the government without simultaneously explaining how policies are chosen. The conclusions to be learned from these models are more limited but more robust than those of models with endogenous policy, as they do not rely on specific assumptions on the political system that determines government actions.

As it will be shown below, the standard definition of a competitive equilibrium with exogenous policy is very similar to that of an equilibrium with endogenous policy and commitment, and is instead very different from the one that is used to study environments in which the government cannot commit.

All of the papers I will discuss in section 3 take the government policy as exogenous, so it is natural to focus on the same setup.

(ii) The nature of equilibria without commitment may be quite complicated and very dependent on specific details of the model. As examples, governmental actions may be constrained by "constitutions", that limit the power each policymaker is able to wield; when many policies are chosen at the same time, the possibility of "reputational spillovers" can arise. A complete description of the government choices should take all of these factors into account. In some circumstances, the assumption of full commitment might be a better approximation of the equilibrium outcome than the opposite assumption of absolute lack of commitment. It also provides an upper bound to what the government could achieve just by designing institutions that lessen the impact of time inconsistency.

By "government", I refer in this paper both to the fiscal and the monetary authorities. I do not model here a strategic interaction between different policy makers, although overall consistency is a key focus of the analysis.

The goal of this section is to show that a complete description of an equilibrium with commitment (or of a competitive equilibrium) requires a much more extensive characterization than the one embedded in the standard definitions. It also shows that, if the government has sufficient "flexibility", the standard definition is satisfactory to describe what equilibrium *outcomes* the government will be able to attain. This flexibility is however in contrast with the rigid rules that are contained in the standard version of the fiscal theory of the price level. Furthermore, the process of looking for a proper definition of an equilibrium suggests a natural restriction on the government strategies. This restriction becomes particularly important if the assumption of a continuum of households (which much of the macroeconomic literature uses) is viewed as the approximation of an environment with a large but finite number of them.

In what follows, I adopt the notation used by Stokey [23].

The economy is modeled as a game. The players are a continuum of private households and a government. I will only look at symmetric equilibria, in which all households take the same actions and play in pure strategies. Consequently, I will only describe the game along branches in which the same action is taken by all households, except at most one. The analysis could be extended to describe the whole game, although sometimes this may be quite complicated.

I will look at one-shot games; the generalization of the arguments to dynamic games is straightforward. Also, I will mainly focus on pure strategies, although the analysis can be easily generalized.

I first describe the game whose equilibria I will call the *no-commitment outcomes*, following Stokey [23]. In this game, households move first; each household chooses an action $x$ from a set $X$, without observing the actions being taken by other households. The action taken by individual households is not observable by other households and/or the government. What everybody can observe is an "average" of the actions taken by the individual households; along the branches in which all households (except one at most) take action $x$, $x$ is just such average.[3]

[3] I do not specify what "average" means outside of those branches. Levine and Pesendorfer [13] address this issue in more detail.

After the households have moved, the government chooses an action $y$ from a set $Y(x)$, where $x$ is the observed average choice of the households.

The payoff to the households is described by a function

$$u : X \times D \to \mathbb{R}, \tag{1}$$

where

$$D \equiv \{(x, y) \in X \times Y : y \in Y(x)\}, \tag{2}$$

The first argument of $u$ represents the action taken by the individual household, the second action is the observed average action in the population and the third argument is the action taken by the government.

If the average action taken by the households is $x$ and the government takes $y$, the pay-off to the government is $v(x, y)$. A common assumption about $v$ is $v(x, y) \equiv u(x, x, y) \quad \forall (x, y) \in D$. In this case, the government is benevolent, i.e., it maximizes the utility of the representative household. Many studies of macroeconomics do not specify the function $v$ and take the government "policy" as exogenous. The analysis I carry out here will offer useful insights even for these models, as it will allow me to look at the constraints the government faces in choosing a policy and at the potential for multiple equilibria.

A strategy for a household is just a choice of $x$.[4] A strategy for the government is a function $\sigma$ that maps an average action by the private households into a government action that is feasible:

$$\sigma : X \to Y, \quad \sigma(x) \in Y(x) \, \forall x \in X. \tag{3}$$

I define a *competitive equilibrium* as an outcome in which each individual household is taking an optimal choice given the action taken by other households and given the government action. A competitive equilibrium is thus a couple $(x^*, y^*)$ such that

$$u(x^*, x^*, y^*) = \max_{x \in X} u(x, x^*, y^*) \tag{4}$$

Let $E$ denote the set of competitive equilibria.

A symmetric sequential equilibrium[5] of this game is a strategy pair $(\phi, \sigma)$ that satisfies the following:

$$(\phi, \sigma(\phi)) \in E \tag{5}$$

[4]With some abuse of notation, I will refer to $x$ as both the households' strategy and their action. A more-proper definition would introduce additional notation and display the strategy as a function of the history of the game after which households move. However, households move after the empty history, so this function would just be a constant.

[5]I use sequential equilibrium as my concept because the actual choice of each household is private information to that household, and only the average $x$ is observed. Notice that the government only cares about the average action taken by the households, and each household only cares about what other households do through their average choice. For this reason, the out-of-equilibrium beliefs become irrelevant. The only consistent belief on a symmetric equilibrium path requires of course the government to attribute probability 1 to all the households having chosen the same action. I thus do not include beliefs in the definition, although they are part of it in general.

and

$$v(x, \sigma(x)) = \max_{y \in Y(x)} v(x, y) \quad \forall x \in X \tag{6}$$

I define a *no-commitment outcome* as any outcome of a symmetric sequential equilibrium. It is easy to see that this definition is equivalent to the one provided by Stokey [23].

I next turn to what Stokey [23] calls a Ramsey equilibrium and Chari and Kehoe [2] call a commitment equilibrium. This is the equilibrium of a different game, which I will call Ramsey game to distinguish from the commitment game proposed below. In this game, the government moves first, choosing an action $y$ from a set $Y$. Following the government move, households choose simultaneously and independently an action $x$ from a set $X(y)$. While I still use $u$ and $v$ to describe the household and government preferences, their domains are now different: $u$ is defined on $\{(\hat{x}, x, y) : y \in Y, \hat{x}, x \in X(y)\}$ and $v$ is defined on $\{(x, y) : x \in X(y)\}$. In the no-commitment game, the set of feasible actions for the government may depend on what the household chose. For instance, the combinations of spending and tax rates that are feasible may depend on the taxable income, which depends on the household labor and/or saving decisions. In the game that corresponds to the Ramsey outcome, the government moves first and its feasible actions cannot depend on what the households will choose later. It is instead possible that the household's choice is constrained by what the government did. Not only the burden of consistency between private and government choices in the two games lies on different players; in the first game, consistency constrains only the *aggregate* choice of the households and the government action, while in the latter it forces the action *each* household takes to be consistent with what was chosen by the government. It would thus be incorrect to say that the only change between the no-commitment game and the Ramsey game stems from the timing of moves, except for the special case in which the actions the government and the players can take are independent of each other, so that $Y(x) \equiv \bar{Y}$ in the no-commitment game and $X(y) \equiv \bar{X}$ in the Ramsey game.[6] Chari and Kehoe [2] overcome this problem by changing the government preferences: they expand the sets $Y(x)$ so that the government can choose from a set $Y$ independently of the household preferences, but include a large negative penalty in the case feasibility (e.g., spending no more than the total tax revenues) is violated. Given the large penalty, a violation will never occur on the equilibrium path; however, this gives the government the power to violate feasibility on branches out of equilibrium!

As before, I can define the set of competitive equilibria ($E^R$) as the set of pairs of $(x, y)$ such that $x$ is the optimal choice for the household when the government chooses $y$:

$$E^R \equiv \{(x, y) : y \in Y, x \in X(y), x \in \arg \max_{\hat{x} \in X(y)} u(\hat{x}, x, y)\} \tag{7}$$

A Ramsey outcome is the best outcome for the government among the outcomes of subgame perfect equilibria of the game I just described. Formally, a pair $(x^R, y^R)$ is a Ramsey outcome when the following holds:

$$(x^R, y^R) = \max_{(x,y)} v(x, y) \text{ s.t. } (x, y) \in E^R \tag{8}$$

[6] Levine and Pesendorfer [13] and Fudenberg, Levine and Pesendorfer [7] fall in this special case.

The similarity between the definition of a Ramsey outcome and the definition of a competitive equilibrium stems from the fact that both refer just to a pair of actions $(x, y)$, rather than to strategies: neither concept describes out-of-equilibrium behavior. In fact, given that $v$ is arbitrary, I can easily reconcile any model in which we write the government policy as exogenous with a Ramsey outcome by giving the government ad-hoc preferences on its choice of $y$.

If we interpret commitment as implying a Ramsey game, a comparison between the no-commitment outcome(s) and the Ramsey outcome(s) is not as interesting, because the sets of possible outcomes are in general different across the two games.

The Ramsey game does not follow the usual definition of commitment in game theory pioneered by Schelling [20].[7] Under the usual definition, the commitment game adds an initial stage to the game without commitment. Before households make their choice the government is allowed to "tie its hands" by deleting some of the actions that it will be able to take ex post. I will take an extreme version of commitment, in which the government can costlessly delete ex ante as many actions as it wants from the sets $Y(x)$, provided at least one action is left in each of these sets. For this case, I can restrict the analysis to the case in which the government deletes all actions but the one it plans to take from each of the sets $Y(x)$.[8] In this form, the government chooses in the first stage a function $\sigma$ as described in (3): a government *action* in the first stage of the commitment game corresponds to a government *strategy* in the no-commitment game. After the government has chosen $\sigma$, the game unfolds as before: the households choose an action $x \in X$, and the government chooses an action $y$. However, I can disregard the final choice by the government, as only one action is left at that stage: $\sigma(x)$.

While in the Ramsey game the government chooses a fixed action $y$, the commitment game as defined here forces the government to respect the same feasibility restrictions as the no-commitment game: whatever mechanism the government uses to tie its hands, it cannot force itself to take an action that is impossible ex post.

A strategy for the private households in the commitment game is a function $\phi^C(\sigma)$ that maps a choice of $\sigma$ by the government into an action $x$. A strategy for the government is a choice of $\sigma$.[9] I define $\Sigma$ as the space of such choices:

$$\Sigma \equiv \{\sigma : X \to Y, \sigma(x) \in Y(x) \forall x \in X\} \tag{9}$$

As a first step, I look for conditions under which the government can implement uniquely the Ramsey outcome. In order to do so, the government must ensure that $(x^R, y^R)$ is the unique (sequential) equilibrium of the subgame following its initial commitment to an appropriate strategy, which I label $\sigma^R$. Formally, this requires

(i) $\sigma^R(x^R) = y^R$. If the households take the action prescribed by the Ramsey outcome, the government must be committed to do so as well. This makes sure that $\phi^C(\sigma^R) = x^R$ is optimal for the households if they expect other households to choose $x^R$ after observing a government commitment to $\sigma^R$.

[7] See also the discussion in Fudenberg and Tirole [8].

[8] By deleting all actions but one from each set,I restrict the number of equilibria of the game, but not the equilibrium outcomes.

[9] As in the no-commitment game, I do not distinguish between an action and a strategy for the player that moves first, although a rigorous definition would introduce a distinct notation.

(ii) $(x, \sigma^R(x))$ is not a competitive equilibrium if $x \neq x^R$. When this happens, a household that expects all other households to choose $x$ after the government commits to $\sigma^R$ will find choosing $x$ strictly dominated by some other action. This implies that any such expectation is self-defeating, given that all households are assumed to be identical.

If we do not add any other requirement on the strategy space $\Sigma$, $\sigma^R$ will exist under very weak conditions.[10] It will be enough that, for any action $x$ that the private sector can take (except possibly $x^R$), there exists at least one action $y \in Y(x)$ that the government can take and does not form a competitive equilibrium with $x$. This observation justifies focusing on the Ramsey outcome in environments in which the government is assumed to have commitment powers: generically, the government will always be able to commit to a strategy that leads the economy to Ramsey as the unique equilibrium outcome of the subgame.

However, in some economic problems we are not interested just in outcomes, but also in the process that leads to them. As an example, we might be interested in knowing how households expect the government to behave if the economy moves out of the equilibrium path. This is a recurring theme in the debate on the fiscal theory of the price level, which I address in section 3, as the following statements show:

> The way that fiscal disturbances affect the price level is through a wealth effect upon private consumption demand. A tax cut not balanced by any expectation of future tax increases would make households perceive themselves to be able to afford more lifetime consumption, if neither prices nor interest rates were to change (...). That would lead them to demand more goods than they choose to supply (...). The resulting imbalance between demand and supply of goods drives up the price of goods, until the resulting reduction in the real value of households' financial assets causes them to curtail demand (or increase supply) to the point at which equilibrium is restored.[11]

> The government budget constraint (...) represents a constraint on the government's choice variables that must be satisfied for all admissible values of those variables appearing in the government's budget constraint that are not government choice variable but that are endogenous in the model as a whole.[12]

In order to assess the validity of these arguments, it is necessary to fully specify $\sigma^R$ and the game government and households are playing. This is always true when all the equilibrium, rather than just its outcome, is deemed of interest.

We may be especially interested in studying the commitment game in more detail in environments in which there are natural restrictions on the set of strategies that the government can commit to. A natural restriction arises when we interpret the assumption of a continuum of households as the approximation to a game with a large, but finite number of households whose actions are observed with noise, as in Levine and Pesendorfer [13] and Fudenberg, Levine and

[10] For this reason, $\sigma^R$ will typically not be unique.
[11] Woodford [28].
[12] Buiter [1].

Pesendorfer [7]. In such an environment, the government should be restricted to play strategies that are *continuous* in the average observed $x$, as any discontinuity is smoothed by the noise. [to be completed]

## 2.1 Example

To clarify the theoretical points, we apply them to a simple example. We consider a two-period economy. Households start with some initial endowment (normalized to 1) and must decide how much to consume and how much to save. To keep the problem as simple as possible, the rate of return on savings is also assumed to be 1. Households do not have any endowment in the second period. In the second period, the government provides two public goods, A and B.[13] The government does not have access to lump-sum taxes, and can only tax savings in the second period at a proportional rate.

The household's choice $x$ are thus savings, and $X \equiv [0, 1]$. The government's choice $y$ is a tax rate on savings ($\tau$) and the level of spending on both goods, $g^A$ and $g^B$. The set $Y(x)$ takes thus the following form:

$$\{(\tau, g^A, g^B), \tau \in [0, 1], g^A \geq 0, g^B \geq 0, \tau x \geq g^A + g^B\} \tag{10}$$

The households' preferences are described by:

$$u(\hat{x}, x, y) = \sqrt{1 - \hat{x}} + \sqrt{\hat{x}(1 - \tau)} \tag{11}$$

where $\hat{x}$ is the savings decision taken by the single household and $x$ are average savings by all other households.

The government preferences are given by

$$v(x, y) = -(g^A - 1/9)^2 - (g^B - 1/18)^2 \tag{12}$$

In this example, households do not value the public goods[14] and the government simply uses a target for the public goods.

Since there is a continuum of households, each household does not perceive its choice of $\hat{x}$ to have an impact on the aggregate; households thus choose $\hat{x}$ to maximize (11) taking $(x, y)$ as given. This yields

$$\hat{x} = \frac{1 - \tau}{2 - \tau} \tag{13}$$

The set of competitive equilibria of this economy is therefore

$$\{(x, \tau, g^A, g^B) : \tau \in [0, 1], x = \frac{1 - \tau}{2 - \tau}, g^A + g^B \leq \tau x\} \tag{14}$$

[13] As we argue below, having two public goods prevents mapping this problem into one that is consistent with a Ramsey game.

[14] As usual, the results would be the same if the public goods entered in the household utility in a strongly separable way.

In order to find the equilibria of the no-commitment game, we solve the government's problem for a given $x$:

$$\max_{\tau,g^A,g^B} -(g^A - 1/9)^2 - (g^B - 1/18)^2 \text{ s.t.} g^A + g^B \le \tau x \tag{15}$$

The government optimal strategy $\sigma$ as a function of $x$ is thus

$$\tau = \min\{\frac{1}{6x}, 1\}, g^A = \min\{1/9, x/2 + 1/36, x\}, g^B = \min\Big\{1/18, \max\{x/2 - 1/36, 0\}\Big\} \tag{16}$$

In words, the government sets spending at its target level if possible; if not, it taxes savings at a 100% rate and allocates the revenues according to the sharing rule described in (16).

A (symmetric) equilibrium is obtained by requiring that $\hat{x} = x$ and that (13) and (16) hold at the same time. There are 3 equilibria of the game; in each of them, the government strategy is described by (16). The three possible choices by the households are: $x = 1/3, x = 1/4, x = 0$.[15] In the 3 equilibrium outcomes, the tax rates are respectively $1/2, 2/3, 1$, government spending in good A is $1/9, 1/9, 0$ and in good B is $1/18, 1/18, 0$. Given the households' preferences, the economy exhibits a Laffer curve. In the first two equilibria, the government raises the same amount of revenues: $\tau = 1/2$ is at the left of the peak of the Laffer curve, whereas $\tau = 2/3$ is at the right. With the given timing, there is nothing the government could do to get out of one of the "bad" equilibria; if the households expect the government set high tax rates on savings, they will work few hours and force the government ex post to confirm their expectations in order to raise enough revenues for its spending target.

Suppose now the government is given the power to commit. The example is designed in such a way to make apparent that it is hard to let the government move first, independently of the household actions, as in the Ramsey game.[16] One possible way to design a Ramsey game would be to let the government choose $\tau, g^A, g^B$ first, subject only to $\tau \in [0, 1]$, $g^A \ge 0$, $g^B \ge 1$ and $g^A + g^B \le 1$. Households would the move second by choosing $x \in [0, 1]$. The pay-off would be the same as that of the game with no commitment, except that an arbitrarily large negative penalty would be assessed to the government is the feasibility restriction $\tau x \ge g^A + g^B$ is violated. It is easy to show that the outcome of this game is the Ramsey outcome, i.e., the best competitive equilibrium ($\tau = 1/2, x = 1/3, g^A = 1/9, g^B = 1/18$). On the equilibrium path, the threat of the large penalty makes sure that feasibility is satisfied. However, feasibility is violated on out-of-equilibrium paths: when the government chooses its equilibrium action $\tau = 1/2, g^A = 1/9, g^B = 1/18$, feasibility is violated if average savings do not reach at least 1/3. This is not a problem for the equilibrium: if each household expects the others to save at least 1/3, then it expects feasibility to be satisfied and payoffs are well defined. However, it is far from clear what the households should think if they expected everybody else to save less than 1/3. With the payoffs as stated in the Ramsey game, households do not care about feasibility and they

[15] Since households move at the start of the game, a strategy for them coincides with their choice.

[16] If there were just one public good, then one possible design would have let the government set the tax rate $\tau$, and government spending would have been assumed to be equal to $\tau x$. With two public goods, it becomes obvious that modeling timing properly is important.

should base their decisions only on the tax rate set by the government. This is hardly a plausible characterization of the interaction between the government and the households, though.

The formulation of the game with commitment that I recommended in the previous subsection overcomes the shortcomings of the Ramsey game. In this game, the government starts by choosing a conditional response to the average savings by the households, i.e., what corresponds to a strategy in the no-commitment game. Subsequently, households make their savings decisions, and finally the government implements the choice it committed itself to. An example of a government strategy that attains the Ramsey outcome in the game with commitment is the following:

$$\tau = 1/2, g^A = \frac{2}{3}\tau x, g^B = \frac{1}{3}\tau x \tag{17}$$

Under this strategy, the households expectations are defined in a sensible way in and out of equilibrium: suppose e.g. that a household expects everybody else to save 1/4. According to (17), it will expect the government to cut spending below its target, but to keep its tax rate at 1/2, a policy that is feasible, although not optimal ex post. The commitment assumption implies that, for reasons not specified in the model,[17] the government cannot (or is better of not to) revise its plan; the household thus chooses optimally to save 1/3 even if it expects everybody else to save 1/4, which means that 1/4 cannot be an equilibrium savings level. In fact, 1/3 is the unique equilibrium choice by the households if the government commits to (17); by writing the game in the suggested form, we establish the result providing households (and the government) with a proper way of forming beliefs about the outcomes following all possible histories. The strategy (17) is continuous and hence is part of a sequential equilibrium of the game in which we restrict the government to play continuous strategies.

The reasoning above applies also to the case in which government preferences are not specified and the government policy is taken as exogenous. Even in this case, it is important for the households to be able to form beliefs about outcomes for all possible histories of the game, which requires the government to follow a well-specified policy in all contingencies. Accordingly, a full description of the government choices should include a specification of (feasible) out-of-equilibrium play even when the government choices are exogenous.

# 3 A Game-Theoretic Approach to Ricardian and non-Ricardian Policies

One of the key sources of debate on the fiscal theory of the price level is the distinction between "Ricardian" and "non-Ricardian" government policy, following the definition used by Woodford [25]. In a Ricardian policy, taxes, government spending and the monetary policy (whether a money supply or an interest rule) are specified in such a way that the government budget

[17] Any of the reasons we previously mentioned could be at work. It could be for instance connected to a separation of powers between different organs of the state, or to a constitutional requirement that forbids the government to revise retroactively its policy. Taking these features as given is acceptable unless modeling the option of defaulting on the institutional setup is judged to be important in driving the results.

constraint[18] is satisfied for any price vector: for this reason, it is called a "constraint". The opponents of the fiscal theory view specifying a policy as Ricardian as a requirement, and any other "policy" is simply a misspecification. This view is expressed best by Buiter [1].[19] On the contrary, a non-Ricardian policy specifies taxes and transfers that do not have to satisfy the government budget constraint. Proponents of the fiscal theory view the government budget constraint as an equilibrium condition: when it is violated, imbalances in the demand and supply force price changes that lead back to equilibrium, provided the specified path for taxes, spending and money (or nominal interest rates) is at least consistent with some equilibrium. For this reason, Cochrane [4] recommends the term "government valuation equation" to replace "government budget constraint". A non-Ricardian policy, if one is possible, may be a powerful tool for determining the price level; by balancing its budget only for some specific prices, the government can use its choices of taxes and spending to achieve price level determinacy even while running monetary policies that have traditionally be associated with indeterminacy, such as interest rate pegs.[20]

The previous section introduced the natural tools to address this issue. However, the problem at hand contains an extra element compared to the simple example of the previous section: prices. In order to describe the economy as a game, it is necessary to describe in detail how the price system arises from the choices the households and the government take, in and out of equilibrium.

In what follows, I adopt a version of trading posts that is similar to Shubik [21].[21] While I make a number of assumptions on the details of how trading takes place, it is straightforward to show that these details could be changed without affecting the results. What can potentially make a difference is the main assumption that trading takes place simultaneously and through trading posts.[22]

I start by going in detail through a two-period "cashless" economy, in which money is purely a unit of account. In this environment there is no monetary policy, and all of the action comes from the fiscal side.

I then explain how the approach can be generalized and suggest a way of introducing the transaction role of money. As I argue below, the transaction role of money, even when tiny, cannot be neglected in a complete and satisfactory inquiry, although many insights are gained already from the model with no money.

Let us consider an economy with a continuum of identical households that live for two periods (1 and 2) and a government. Households receive a constant exogenous endowment of a single

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[18] The government budget constraint includes the transversality condition, in an economy with an infinite sequence of markets.

[19] Other papers that express similar views are by McCallum [15] and Kocherlakota and Phelan [11].

[20] See Sargent and Wallace [18] for the "traditional" result.

[21] I assume enough symmetry that these trading rules yield the Walrasian outcome. As Shubik [21] points out, this is far from guaranteed in general. A more-complicated version with multilateral trading posts could overcome this problem.

[22] An alternative model of the microstructure of the determination of prices in a competitive equilibrium is provided by the search-theoretic approach developed by Rubinstein and Wolinsky [16] and Gale [9, 10]. However, this approach is considerably more cumbersome to deal with, and introducing a government in their environment would require significant adaptations that are currently beyond the scope of this project.

homogeneous good in each period.[23] Each household starts the first period with $B_1$ units of government bonds. A government bond is a claim to 1 "dollar", a unit of account whose value is determined in a way that I describe below. All debt is assumed to mature in one period; once again, this is not an important assumption, but saves on notation considerably. The government has access to lump-sum taxes in both periods; with the tax revenues $T_1$ and $T_2$, it finances some government spending in either period ($G_1$ and $G_2$), as well as repayment of its original debt.

Exchange happens through trading posts where objects are traded pairwise. In period 1, there will be 3 trading posts: in the first, goods are exchanged for maturing bonds; in the second, goods are exchanged for newly issued bonds that mature in period 2; in the third, maturing bonds can be exchanged for newly issued bonds that mature in period 2. In period 2, the only trading post is one where goods are exchanged for maturing bonds. I often refer to trading posts as markets. As in Shubik [21], each household that wants to trade must submit an unconditional bid for the amount it wishes to sell on this market. The bid must represent a quantity of the good (or bond) *sold*, rather than bought, because only in this way households can meet their binding obligation at any price. In equilibrium, households have perfect foresight about the relative price in each market, and a single household cannot alter any price through its actions. For this reason, households would be strictly indifferent between using unconditional bids or more-sophisticated bid schemes. Being a large player, the government could potentially have an interest in submitting more-complex bids. However, even for the government a bid is still interpreted as a binding commitment, in and out of equilibrium; whatever bidding mechanism the government uses, I thus require it to be able to meet its obligations even when *many* households make an unexpected bid. That would still leave room for potentially complicated government bids, in which perhaps rationing is sometimes involved; I show here that the government can attain many of its goals of a given level of taxes and spending (and, indirectly, a given level of prices) even when it is restricted to making unconditional sell bids,[24] The trading post clears simply by setting the price equal to the ratio of the supply of the two objects to be exchanged; at that price, market clearing is achieved as an identity, independently of the bids, and exchange takes place.[25]

I now describe the game formally. Lower-case letters are used for decisions by single households, upper-case for aggregate variables. In equilibrium, the values of lower- and upper-cased variables coincide, but each household is free to deviate from what everybody else is doing and set its lower-case variable to a different value, whereas aggregate values are given from the perspective of the household.

(i) Households start with 1 unit of the period-1 good and $B_1$ units of government debt maturing

[23] Production does not play any role in what I am interested in, but could be included by adding an appropriate amount of notation and markets.

[24] An easy variant lets the government use unconditional sell bids for goods and set a fixed price in the markets in which it sells bonds (the government can do this because there is no limit to the quantity of bonds it can print).

[25] If the government sets a price in some markets, those markets clear by exchanging any quantity that the households offer to sell for an appropriate quantity of the good they wish to buy, at the relative price set by the government.

in period 1. The government levies a lump-sum tax $T_1 \in [0, 1]$.[26]

(ii) Trading opens. There are bilateral trading posts for each possible exchange; in our case 3 exchanges are possible: goods for maturing government bonds, goods for new bonds issued by the government and maturing bonds for new bonds. Each household may submit a sale bid for $b_1^{C_1}$ units of bonds in the market for goods, and another sale bid for $b_1^{B_2}$ units of bonds in the market for new bonds maturing next period, subject to the constraint that $b_1^{C_1} + b_1^{B_2} \leq b_1 \equiv B_1$, i.e., the sale bids cannot exceed the total amount of bonds the household starts with. I use superscripts to indicate the object the player wishes to buy in each market: e.g., $C_1$ represents period-1 goods, $B_2$ represents bonds maturing in period 2. There is no point in distinguishing between lower- and upper-case on the superscript, as it only refers to the type of good, not the quantity; for this reason, I always use upper-case letters. Each household may also submit a sale bid of $c_1^{B_2}$ units of goods in exchange for new bonds, subject to the constraint that $c_1^{B_2} \leq 1 - T_1$. The government submits a sale bid for $C_1^{B_1}$ units of goods in the market for maturing bonds, subject to $C_1^{B_1} \leq T$. It also submits a sale bid for $B_2^{B_1}$ units of new bonds in exchange for maturing bonds, and $B_2^{C_1}$ units of new bonds in exchange for goods.

(iii) The price at the 3 trading posts is determined as the ratio of the quantities of the unconditional bids, and exchange takes place. Prices are indexed by the two objects that are being exchanged at each trading post; accordingly,

$$\begin{aligned} P_{C_1B_1} &= \frac{B_1^{C_1}}{C_1^{B_1}} \\ P_{C_1B_2} &= \frac{B_2^{C_1}}{C_1^{B_2}} \\ P_{B_1B_2} &= \frac{B_2^{B_1}}{B_1^{B_2}} \end{aligned} \tag{18}$$

The relative price of goods and maturing bonds $P_{C_1B_1}$ determines the value of the unit of account (the "dollar") for the cashless economy. For this reason, I interpret $P_{C_1B_1}$ as the general level of prices. I explain below that this may be quite different in a model in which there is money and I explain how the analysis will be generalized. $P_{B_1B_2}$ is the relative price of the unit of account in the two period, i.e., it is the nominal interest rate in the economy.

(iv) Consumption and government spending take place. Each household consumes

$$c_1 = 1 - T_1 - C_1^{B_2} + \frac{b_1^{C_1}}{P_{C_1B_1}} \tag{19}$$

[26] The tax cannot exceed 1 because that is the total endowment.

and starts period 2 with $b_2 = b_1^{B_2} P_{B_1 B_2} + c_1^{B_2} P_{C_1 B_2}$ units of nominal bonds. The government spends

$$G_1 = T_1 + C_1^{B_2} - C_1^{B_1} \tag{20}$$

units in the first period. Notice that (18) was directly used to substitute prices out of the expression for government spending: while each household could in principle deviate and purchase a different amount of any objects than what other households are doing, the government fully realizes that any deviation by its part is reflected in prices through (18).

(v) Households start with 1 unit of the period-2 good. The government levies a lump-sum tax $T_2 \in [0, 1]$.

(vi) The only market is the one that trades maturing bonds for goods. Each household submits a bid $b_2^{C_2} \leq b_2$. The government submits a bid $C_2^{B_2} \leq T_2$.

(vii) The price is determined as before by the ratio of bids, i.e.

$$P_{C_2 B_2} = \frac{B_2^{C_2}}{C_2^{B_2}} \tag{21}$$

(viii) Each household consumes

$$c_2 = 1 - T_2 + \frac{b_2^{C_2}}{P_{C_2 B_2}} \tag{22}$$

The government spends

$$G_2 = T_2 - C_2^{B_2} \tag{23}$$

The household's preferences over the outcomes are described by

$$u(c_1) + u(c_2) \tag{24}$$

where $u$ is a strictly increasing and concave function satisfying Inada conditions.

A household strategy is thus the following:

1. bids $b_1^{C_1}, b_1^{B_2}, c_1^{B_2}$ as functions of the tax $T_1$;
2. a bid $b_2^{C_2}$ as a function of the taxes $T_1$ and $T_2$, of the aggregate bids at the various trading posts in period 1 ($B_1^{C_1}$, $B_1^{B_2}$, $C_1^{B_2}$, $C_1^{B_1}$, $B_2^{B_1}$, $B_2^{C_1}$) and of its individual period-1 bids $b_1^{C_1}, b_1^{B_2}, c_1^{B_2}$.

Consumption was not included, as it can be deducted mechanically from (19) and (22).

A government strategy is:

1. a tax $T_1$ and bids $C_1^{B_1}$, $B_2^{B_1}$, $B_2^{C_1}$;

2. a tax $T_2$ and a bid $C_2^{B_2}$ as functions of $T_1$ and of the aggregate bids at the various trading posts in period 1: $B_1^{C_1}$, $B_1^{B_2}$, $C_1^{B_2}$, $C_1^{B_1}$, $B_2^{B_1}$, $B_2^{C_1}$.

As for the households I dropped consumption, spending was dropped from the government strategy, being determined as a residual by (20) and (23).

I assume that the government can commit to a strategy before the game begins; time inconsistency is not an issue I am interested in, since government preferences are not explicitly modeled. I look for strategies that allow the government to adhere as strictly as possible to "target" levels of spending and taxes that are exogenous. I fix a target level of taxes at some constant value $\bar{T}$. For government spending, it is interesting to consider two cases: in the first one, target spending is identically zero. In the second case, target spending is 0 in period 2, but it is $\bar{G} > \bar{T}$ in the first period. While in the first example the government is only paying off previously-accumulated debt, in the second example it has to issue fresh debt also to finance some primary deficit in period 1. I am interested in knowing when and whether the government can adhere to this target plan both in and out of the equilibrium, and what are the "minimal" deviations that are needed if it is impossible to keep faith to the plan.

When there is no government spending, the government can reach its target both in and out of equilibrium, independently of the price level. The strategy to do so calls for the following actions to be taken independently of the households' response. First, the government raises $T_1 = \bar{T}$ in period 1. The government bids the entire amount $C_1^{B_1} = \bar{T}$ in exchange for maturing bonds; it also bids a strictly positive amount $B_2^{B_1} = \bar{B}$ of new bonds in exchange for maturing bonds, while it does not submit any bid on the market between goods and new bonds. In period 2, the government levies a tax $T_2 = \bar{T}$ and uses the revenues to bid $C_2^{B_2} = \bar{T}$ in exchange for bonds maturing in period 2. It can be immediately verified from the description of the game that these actions can be taken independently of the choices by the households and that they deliver the exogenous targets for taxes and spending. I now study the households' reaction to see the implications for the price level.

I solve the household's problem backwards.

When submitting its bid in period 2, each household inherits as a given its previous consumption $c_1$ and its level of nominal bonds $b_2$. At this stage, the household can only choose how much of $b_2$ to bid in exchange for additional period-2 goods; the price it expects on that market is given by (21), which is a strictly positive number and is independent of its bid (assuming $B_2 > 0$). The household will thus bid all of its $b_2$ bonds and consume $c_2 = 1 - T_2 + \frac{b_2^{C_2}}{P_{C_2 B_2}}$.

In period 1, the household has to submit 3 bids. Given that the government does not offer new bonds in exchange for goods, there is no point for the household to bid on that market, as the bid would just be wasted. The household is thus left with the problem to allocate the initial amount of bonds $b_1$ between the bid for new bonds and that for goods. From the perspective of an individual household, each unit bid for goods yields $1/P_{C_1 B_1}$ units of the consumption good, and each unit bid for new bonds yields $P_{B_1 B_2}$ units of new bonds. While these prices are not known to the household ex ante, in equilibrium the household has perfect foresight about them.[27]

[27] There is no uncertainty because the government is not playing mixed strategies, and the households' choices are uncorrelated (even if we assumed they were playing mixed strategies, which I do not).

The household also knows that each unit of new bonds will fetch $1/P_{C_2B_2}$ units of period-2 goods. Its problem becomes thus:

$$\max_{c_1,c_2,b_1^{C_1},b_1^{B_2}} u(c_1)+u(c_2) \text{ s.t.}$$
$$\begin{aligned} c_1 &= 1-T_1+\frac{b_1^{C_1}}{P_{C_1B_1}} \\ c_2 &= 1-T_2+\frac{b_1^{B_2}P_{B_1B_2}}{P_{C_2B_2}} \\ b_1^{C_1}&+b_1^{B_2}\leq b_1 \end{aligned} \tag{25}$$

This problem shows that the mechanism I designed corresponds to a Walrasian economy from the perspective of each household: each household is simply taking prices as given and maximizing by allocating its resources.[28] While mathematically the problem is identical, conceptually a household faces a more-complex problem in the economy I consider: it has to form beliefs not only about future prices, as in a dynamic Walrasian equilibrium, but also about current prices, which are determined only after the bid has been submitted.

The first-order condition for household bids at an interior yields:[29]

$$u'(c_1)=\frac{P_{B_1B_2}P_{C_1B_1}u'(c_2)}{P_{C_2B_2}} \tag{26}$$

which is the standard Euler equation, together with $B_1^{C_1}+B_1^{B_2}=B_1$.

An equilibrium in the subgame in which the government strategy is specified, as above, by $T_1=\bar{T}$, $C_1^{B_1}=\bar{T}$, $B_2^{C_1}=0$, $B_{B_1}^2=\bar{B}$, $T_2\equiv\bar{T}$, $C_2^{B_2}\equiv\bar{T}$ is characterized as follows. From the government strategy, $B_2=\bar{B}$ after any history; hence from (21) $P_{C_2B_2}=\bar{B}/\bar{T}$ if the households bid all of their maturing bonds. From the government strategy, (21) and (22) we obtain $C_2=1$ independently of the household bids. Notice that this is a result on $C_2$, which is average consumption; in principle, each household could consume more or less than 1. Similarly, the government strategy, (18) and (19) imply $C_1=1$ independently of the history. (18) describes how prices form, independently of the household bids. In an equilibrium, a household must find it optimal to choose $C_1=1$ and $C_2=1$, so we can solve for the bids using (18) in combination with (26) and $B_1^{C_1}+B_1^{B_2}=B_1$, from which we obtain $B_1^{C_1}=B_1^{B_2}=1/2$. The equilibrium prices are $P_{C_1B_1}=\frac{B_1}{2\bar{T}}$ and $P_{B_1B_2}=\frac{2\bar{B}}{B_1}$.

With the given government strategy, there is a unique equilibrium, in which the unit of account (the "dollar") has a well-defined value. As in Cochrane [4], government debt in this example is essentially a share to a future payoff and a "dollar" simply represents a fraction of the

[28] There is no market for private debt, which makes households borrowing constrained; this is irrelevant in my setup with identical households, but it could be remedied easily by introducing the appropriate trading posts, if there was scope for bilateral trade.

[29] In equilibrium, households must be choosing an interior point when allocating maturing bonds to the 2 markets. If this were not the case, there would be one market where either goods or valuable new bonds are being offered and no bid is made in response to the offer: it would then be enough to bid an arbitrarily small amount to obtain the goods or the bonds essentially for free.

debt. The government policy in this example is non-Ricardian; the government budget constraint would require

$$B_1 = \bar{T} P_{C_1 B_1} + \frac{\bar{T} P_{C_2 B_2}}{P_{B_1 B_2}} \tag{27}$$

which is only satisfied at the equilibrium price level. According to this game, the only way prices can deviate from the equilibrium values happens when households fail to make their equilibrium bids. The simplest example is one in which households fail to redeem part of their bonds: they could e.g. bid less than $B_2$ in the second period, in which case $P_{C_2 B_2}$ decreases and the present value of taxes seems to exceed the value of debt. This excess is only apparent, for it is the result of many households failing to claim their parts of repayments: if we only count debt that is presented for redemption, the government budget constraint holds. If instead households misallocate $B_1$ across the two markets, perhaps redeeming too many bonds and rolling over too few, the consequence is a failure of the Euler equation, but the government is still repaying all of its debt. Substituting (18), it can be easily verified that (27) always holds if households do not ever waste bonds, even when they choose to bid their initial bonds in a suboptimal way across the two trading posts. The policy can be called "Ricardian" for price deviations that occur from misallocation, but no waste of bonds.

It is interesting to study what happens if the government has a target $\bar{G} > \bar{T}$ for spending in the first period, while target spending is 0 in the second period. In this case, there is no government strategy that allows it to achieve the target levels of spending and taxes both in and out of equilibrium. Because target spending exceeds target taxes in the first period, the government is forced to raise resources in the first period by issuing bonds. This attempt can be successful (provided $\bar{G}$ is not too high) in equilibrium, but the government is forced to curtail its spending if for some reason households do not bid enough resources in exchange for government bonds. In this case, the government budget constraint is a real constraint on the government's actions, and cannot be viewed just as a "government valuation equation", at least off the equilibrium, which supports the traditional view. At the same time, there exists a strategy that allows the government to achieve price determinacy through the fiscal side of the economy. The strategy is quite similar to the one that was proposed in the previous case. Intuitively, it calls for the government to raise taxes at the target pace and bid the appropriate mix of bonds in the different markets. By choosing the right ratio of bonds to be offered in exchange for maturing bonds and in exchange for goods, the government chooses the relative fraction of future tax revenues that go to holders of current debt that roll it over vs. households that lend current resources to the government. A higher ratio implies a lower value of debt, which will be reflected in a higher price level, as holders of maturing debt will bid more of the debt against goods today. At the same time, a higher ratio encourages households to lend more goods to the government, as it implies that new buyers of government debt will be entitled to a larger fraction of the future resources. By choosing the appropriate value for this ratio, the government can thus target the revenues it raises by issuing new debt, and can thus achieve its spending target, albeit only at equilibrium prices. [to be completed]

## 3.1 Introducing Money [the setup]

The example I have been presented here looks only at two periods and does not introduce money. The extension to many periods of the environment described above is straightforward. Introducing money is crucial to address the validity of the fiscal theory of the price level in more detail. Money can be introduced in the game described above through a "cash-in-advance" technology that prevents some barter trading posts from opening. I can achieve this by dividing households into $n$ symmetric groups, with $n$ even, that lie on a circle. Each group $i$ produces a good that cannot be bartered with the good at the "opposite extreme", i.e. $i \pm n/2$, so these trades will require money. Because each group is still formed by a continuum of households, each household will behave as price taker. I now assume that each household likes to consume all $n$ types of goods. As $n \to \infty$, the role of money becomes infinitesimal. Trading posts are open for all pairwise combinations of goods (except for the opposite extreme goods), for all goods vs. money, for goods vs. bonds and money vs. bonds. In this case, a "dollar" is the price of money, not national debt. While this is work in progress, I conjecture that the price of a dollar of money and a dollar of debt will coincide identically only if the government explicitly pursues a policy that pegs the relative price; such a policy implies a commitment to monetization of the debt should households wish to get rid of it by selling it on the market rather than rolling it over. This argument shows that the fiscal theory of the price level is unlikely to survive if accompanied by a money-supply rule, which is inconsistent with any monetization. However, it remains to be seen whether the fiscal theory can succeed in an environment in which the government pegs the value of money and maturing bonds on one side, and the nominal interest rate (the relative value of money and future bonds) on the other. [to be completed]

# 4 Conclusion

While this research is unlikely to lay to rest the dispute on the validity of the fiscal theory of the price level, it shows how the question can at least be cast in a more-complete model in which the definition of an equilibrium is not controversial. [to be completed]

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