Growth and Technical Change : A Smithian Approach

Xavier Ragot
CEPREMAP, 142 rue du Chevaleret
75013 PARIS, FRANCE
e-mail : xavier.ragot@cepremap.cnrs.fr
tel : 33 1 40 77 84 94.

January 31, 2000

Abstract

This paper proposes a model of technical change based on Adam Smith's analysis of the role of division of labor in economic growth. Technical change is the discovery of new capital goods which help workers or even replace workers performing precise tasks. As a consequence, the number of opportunities of innovation is an increasing function of the number of tasks performed in the economy. We first model division of labor within firms and then model the research sector. The probability of innovation is not only a function of the quantity of resources allocated to the research sector, but also of the total number of tasks. We show that this theory yields a growth model without scale effects. The market equilibrium yields an optimal equilibrium growth rate, but a sub-optimal transitory dynamic. This sub-optimality can take two forms : either too many researchers and an internal division of labor too small an internal division of labor, or too few researchers and an too high an internal division of labor. We show that the long run growth rate depends on the assumption made on the evolution of the number of tasks during economic development. If the number of tasks increases in the long run, a positive effect of the diversity of capital goods on productivity is not necessary to obtain a positive per capita growth rate.

Keywords : division of labor, technical change, growth, scale effects.

JEL : O30, O40, L23, D21.

1 Introduction

The analysis of the determinants of growth in models based on the discovery of new ideas lead to the problem of scale effects. The first models, including Romer [1990], Grossman and Helpman [1991], Aghion and Howitt [1992], yield the conclusion that an increase in the size ofthe population induces an increase in the resources allocated to the research sector, which leads to an accelerating growth rate per capita. Jones [1995a] shows that this effect can not be observed and that, on the contrary, the growth rate remained constant whereas the number of researchers and scientists grew rapidly. This finding induced two different lines of research. The first one models an increasing difficulty of finding new ideas (the model of Jones[1995b], Kortum [1997], and Segerstrom [1998]). The second line of research adds a second dimension to growth model with scale effects, . If in this dimension the spillovers are insufficient to sustain growth, then the scale effects disappear. Young [1998] presents a model belonging to this class, where the diversity of capital goods is introduced in a quality ladder model. His argument is based on Gilfillan [1935] :' [A] new configuration of forces call forth a number of independent solutions by different inventors...filling the same need'. In Young's model the increase in the number of needs fulfilled does not raise productivity whereas the increase in quality of intermediate good does. We propose a model of growth without scale effects where the 'needs' of innovation and the difficulty of innovating are made endogenous. Our model is based on Adam Smith analysis of the role of division of labor in the Wealth of Nations. According to Adam Smith, division of labor has three effects, each of which increases productivity : workers get specialized and raises their dexterity, time is saved because workers do not loose time in changing tasks, and it is possible to introduce new machines when tasks are divided and simple enough. The first effects has been modeled and was often considered as a synonym ofdivision of labor. But Young[1928] writes that the most important point lies in the relationship between division of labor and capital goods. In fact, division of labor does not only facilitate the introduction of capital goods, but it is embodied in different capital goods. This entails first that an increase in the diversity of capital goods can be seen as an increase in the division of labor between firm. As a consequence, an increase in the diversity of inputs raises productivity of firms using those inputs, as in Romer[1990]. This also entails that capital goods are introduced in the economy because of the mechanization of the division of labor. The number of capital goods that can be introduced is thus an increasing function of the number of tasks performed in the economy. As a consequence, the number of opportunities of innovation is an increasing function of the total number of tasks. Growth is thus based on a continuous increase in external division of labor, the number of different firms, based on the opportunities of innovation generated by the division of labor at the firms level. We show that this growth mechanism yields a model of growth without scale effect.

First we model division of labor at the firm level. Each firm produces with intermediate goods bought from other firms and chooses its internal division of labor, which raises productivity of labor on the one hand, but generates information costs. These costs are costs of coordinating the work on many tasks. These information costs have been modeled by Bolton and Dewatripont [1994]. Each firm produces with a patent bought from a research sector. The crucial assumption of this model is that the probability of a successful innovation depends not only on the labor devoted to research but also on the total number of tasks performed in the economy. This link endogenize the difficulty of innovation introduced by Kortum [1997] and Segerstrom [1998].

The model yields three main results. First, we propose a theory of technical change based on division of labor which makes the difficulty of innovation endogenous. It yields a growth model without scale effects. Second, we show that the market equilibrium yields an optimal growth rate, but the transitory dynamic is not optimal. Either there are too few researchers and too high an internal division of labor, or there are too many researchers and too small an internal division of labor. The third result concerns the total number of tasks performed in the economy. Is this number roughly constant during economic growth or does it increase ? We show that each assumption yields different result on the long run growth rate.

The article is divided in four parts. First we derive the market equilibrium growth rate. Then, the optimal problem is solved. The third part discusses the evolution of division of labor during economic development. The results are summarized in the conclusion.

2 The market equilibrium

2.1 The households

The size of the population at time t is $L(t) = e^{ut}$. Each household is modeled as a dynastic family which maximizes the discounted utility

$$U = \int_{t=0}^{\infty} e^{ut} e^{-\rho t} v(c_t) dt$$

where $u(t)$ is per person utility at time t, which is given by

$$v(c_t) = \ln c_t$$

At time t there is a continuum of firms of length N_t. The consumption of the households is a function of the quantity of each good consumed. We assume that each good is perfectly substituable for each household

$$c_t = \int_{i=0}^{N_t} c_{i,t} di$$

where $c_{i,t}$ is the quantity of good bought to firm $i \in [0...N_t]$. At the equilibrium each firm charges the same price. This price will be the numeraire. The program of the representative household is thus

$$\max_{c_t} \int_{t=0}^{\infty} e^{-(\rho-u)t} v(c_t) dt$$

subject to the intertemporal budget constraint $\dot{b}(t) = w(t) + r(t)b(t) - c_t - ub(t)$, where $b(t)$ denotes the per capita financial assests, $w(t)$ the wage income of the representative household, and $r(t)$ denotes the instantaneous rate of return. The solution yields

$$\frac{\dot{c}_t}{c_t} = r(t) - \rho \tag{1}$$

2.2 The firms

Each good in the economy can be either consumed by the household or used as an intermediate good by other firms. The various goods are perfect substitute for the household but they are inperfect substitute for the firms which use them as intermediate goods. Each firm has two different ways to produce a good, either it buys a patent which allows it to produce with an non constant return technology or it imitates existing firms. In the case of imitation the firm produces with labor and produces with a constant return technology.

At each period t, there is a continuum of firm of length N_t. When a firm has bought a patent, it has access to a technology that allow the firm to choose its internal division of labor d_t which is the number of tasks performed in the firm, and the time spent on each task x_l $(0 \leq l \leq d_t)$. Tasks are not perfect substitute for each other. They enter the production function through an aggregate, which combines the time spent on each task $\left(\int_{l=0}^{d_{i,t}} x_l^{\frac{s-1}{s}} dl\right)^{\frac{s}{s-1}}$. The elasticity of substitution between tasks is thus $1/s$. The firm also chooses the quantity of capital k_j, $(\ 0 \leq j \leq N_t)$ bought from other firms. The production function is

$$Q_{i,t} = N_t^{\gamma-\frac{\eta}{\varepsilon}+\eta} \left(\int_{j=0}^{N_t} k_j^{\varepsilon} dj\right)^{\frac{\eta}{\varepsilon}} \left(\int_{l=0}^{d_{i,t}} x_l^{\frac{s-1}{s}} dl\right)^{\frac{s}{s-1}\sigma} \tag{2}$$

with $\gamma, \eta, \varepsilon, \sigma > 0$ and $s > 1$. To understand the parameters in this production function, assume that the firm uses a total amount of labor equal to L, and a total amount of capital equal to K. Assume moreover, that the firm uses its labor and capital uniformly among all tasks and intermediate goods ($x_l = L/d_{i,t}$ and $k_j = K/N_t$), the output is then $Q_{i,t} = N_t^{\gamma} d_{i,t}^{\frac{\sigma}{s-1}} K^{\eta} L^{\sigma}$. First, σ and η are the elasticity of production with respect to the total amount of labor and capital. When the amount of total capital is fixed, the effect of an increase in the number of types of capital goods available to the firm raises output by an amount proportional to N_t^{γ}. As noted by Benassy[1996], γ is the correct mesure of the effect of diversity of capital goods on productivity. The effect on output of an increase in the number of tasks performed in the firm depends on the term $d_{i,t}^{\frac{\sigma}{s-1}}$. This effect is greater the smaller $s > 1$, and the greater σ. The parameter s represents the effect of the internal division of labor on productivity of labor.

Contrary to the types of capital goods available to the firm, the internal division of labor is chosen by the firm. The relation between division of labor and productivity are stressed by Adam Smith in the first chapter of the *Wealth of Nations*. But division of labor entails additional costs. Coase [1937] noted that the entrepreneurship function exhibits decreasing returns to scale, which provides a rationale for the division of the production in different firms. Bolton and Dewatripont [1994] stress the costs of information acquisition and Becker and Murphy [1992] claims that the degree of division of labor is limited by various costs of coordinating workers. We assume that these organisation costs require $hd_{i,t}^{\theta}$ units of labor.

The existence of imitators implies that each patented firm faces a limit price to avoid the production of imitators which would drive the profit of the patented firm to 0. Each patented firm faces the same limit price because of the constant return technology of imitators . Without loss of generality, we assume that the limit price is equal to 1. The firm i maximizes its profits

$$\max_{d_{i,t},x_j,k_j} \pi_{i,t} = \max_{d_{i,t},x_j,k_j} Q_{i,t} - \int_{j=0}^{N_t} p_j k_j dj - \left(\int_{l=0}^{d_{i,t}} x_l dl + hd_{i,t}^{\theta} \right) w_t$$

First order conditions are

$$\eta k_j^{\varepsilon-1} N_t^{\gamma-\frac{\eta}{\varepsilon}+\eta} \left(\int_{j=0}^{N_t} k_j^{\varepsilon} dj \right)^{\frac{\eta}{\varepsilon}-1} \left(\int_{l=0}^{d_{i,t}} x_l^{\frac{s-1}{s}} dl \right)^{\frac{s}{s-1}\sigma} = 1 \quad (3)$$

$$\sigma x_j^{-\frac{1}{s}} N_t^{\gamma-\frac{\eta}{\varepsilon}+\eta} \left(\int_{j=0}^{N_t} k_j^{\varepsilon} dj \right)^{\frac{\eta}{\varepsilon}} \left(\int_{l=0}^{d_{i,t}} x_l^{\frac{s-1}{s}} dl \right)^{\frac{s}{s-1}\sigma-1} = w_t$$

$$\frac{s}{s-1} \sigma x_{d_{i,t}}^{\frac{s-1}{s}} N_t^{\gamma-\frac{\eta}{\varepsilon}+\eta} \left(\int_{j=0}^{N_t} k_j^{\varepsilon} dj \right)^{\frac{\eta}{\varepsilon}} \left(\int_{l=0}^{d_{i,t}} x_l^{\frac{s-1}{s}} dl \right)^{\frac{s}{s-1}\sigma-1} = \left(x_{d_{i,t}} + \theta h d_{i,t}^{\theta-1} \right) w_t \quad (4)$$

The second order conditions require that $\sigma < 1$, $\eta < 1$, and $\frac{s}{s-1}\sigma < \theta$. The first equation shows each firm chooses the same amount of its intermediate goods. Because of the equality of limit prices, each firm demands the same amount $\overline{k}$ of the good produced by other firms.The second condition implies that each firm allocates the same time $\overline{x}$ on each task. Each firms chooses the same internal division of labor and produces the same amount of good, thus we can write d_t the internal division of labor of each firm instead of $d_{i,t}$, and the production of each firm Q_t instead of $Q_{i,t}$.

Rearranging the first order conditions leads to

$$\begin{aligned} \eta Q_t &= N_t \overline{k} & (5)\\ \sigma Q_t &= w_t d_t \overline{x} & (6)\\ \frac{s}{s-1}\sigma Q_t &= \left(d_t\overline{x} + \theta h d_t^{\theta}\right) w_t & (7) \end{aligned}$$

The two previous equations yield

$$h d_t^{\theta} = \frac{1}{\theta}\frac{\sigma}{s-1}\frac{Q_t}{w_t} \tag{8}$$

The profits π_t of each firm is

$$\pi_t = m Q_t \tag{9}$$

with $m = \left(1 - \eta - \sigma - \frac{1}{\theta}\frac{\sigma}{s-1}\right)$, which is the proportion of profits in the firm output. We assume that the firm produces with a positive profit, such that $m > 0$. The positive profits do not come a monopolistic competion under constant return asumption. It comes from the other choice variable, the division of labor, which induces that the overall returns are non-constant. As a consequence, price taker firms make a positive profit.

With equation 2, the production of each firm is thus

$$\begin{aligned} Q_t &= N_t^{\gamma - \frac{\eta}{\varepsilon} + \eta}\left(N_t \overline{k}^{\varepsilon}\right)^{\frac{\eta}{\varepsilon}} \left(d_t \overline{x}\right)^{\sigma} d_t^{\frac{\sigma}{s-1}}\\ &= N_t^{\gamma}\left(\eta Q_t\right)^{\eta}\left(\frac{\sigma Q_t}{w_t}\right)^{\sigma} d_t^{\frac{\sigma}{s-1}}\\ &= E N_t^{\gamma} Q_t^{\eta} d_t^{\sigma\left(\theta + \frac{1}{s-1}\right)} \end{aligned}$$

Thus

$$Q_t^{1-\eta} = E N_t^{\gamma} d_t^{\sigma\left(\theta + \frac{1}{s-1}\right)} \tag{10}$$

With $E = \eta^{\eta}\sigma^{\sigma}\left(\frac{1}{\theta h}\frac{\sigma}{s-1}\right)^{\frac{1}{\theta}\frac{\sigma}{s-1}}$. This constant relates the level of per firm output to the level of division of labor and to the number of firms. The study of the optimal problem will show that the market dynamic is ineficient only during the transitory dynamic. As a consequence, the study of the transitory dynamic and 'level effects', which do not appear in the growth rate is as important as the determinant of the growth rate. All the change that raises E increases the level of output. Quite intuitively, E depends negatively on the costs of coordination h, and θ. E gets smaller as the effect of division of labor on labor productivity decreases (i.e. s increases). Naturally, E increases with the productivity of labor and capital.

Each firm sells its good to all the other firms and to the representative household. As each firms charges the same price, and as goods enter symetricaly in the utility function, the household consumes an equal amount of the product of each firm. Moreover the demand of all the other firms for the intermediate good produced by a precise firm is the demand of a firm $\overline{k}$ times the number of firms N_t. Thus, the equilibrium of the product market for each firm is

$$Q_t = \frac{c(t)}{N_t} + N_t\overline{k}$$

With (5), this equation leads to

$$Q_t = \frac{1}{1-\eta}\frac{c(t)}{N_t} \tag{11}$$

The production of each firm is the consumption of the household times a constant $\frac{1}{1-\eta} > 1$. This constant is an intersectorial multiplier : if the household increases its consumption of 1 unit, the firms have to use more intermediate goods, so the overall increase in production is greater than 1 unit.

2.3 The research sector

The arbitrage equation in the research sector implies that the value V_t of a patent at date t is the discounted expected profit of a patented firm :

$$V_t = \int_{\tau=t}^{\infty} e^{-R(t)}\pi_\tau d\tau$$

With the interest rate $R(t) = \int_{z=t}^{\tau} r(z)dz$. Differentiating the expression (11) with respect to t yields $r(t)V_t = \dot{V_t} + \pi_t$. As the financial markets are perfect, the net return of hoding a patent is equal to its return on the finanacial market at the riskless interest rate $r(t)$. The value of a patent is equal to the sum of the profits at time t and the increase of the value of the patent at time t. This equation rewrites

$$\frac{\dot{V}_t}{V_t} = r(t) - \frac{\pi_t}{V_t} \tag{12}$$

The probability of discovery of a new patent, which is a new type of intermediate good, is proportional to the quantity of labor n_t used in that sector times the probalbility λ_t per unit of labor. The crucial assumption of this model is that λ_t is a function of the number of opportunities of innovation that are available in the economy. According to the smithian view of technical change, this probability is a function of the number of tasks performed in the economy. The assumption is that each of them can be mechanised, after the introduction of new intermediate goods. The number of tasks performed in the economy depends on the assumption made on the division of labor in the firms. Indeed, if each firm divides labor in a different way, then the number of tasks in the economy is the number of tasks performed in a firm times the number of firms $N_t d_t$. But if all firms divide labor the same way, the tasks performed in each firms are the same. The total number of tasks performed in the economy is $d_{t.}$We assume first that the number of tasks performed in the economy is $N_t d_t$. In the discussion of the results, we assume that the total number of tasks performed is d_t and we derive further result. The probability of a successful innovation of one unit of labor in the research sector is thus

$$\lambda_t = d_t N_t \tag{13}$$

The arbitrage equation implies that the reseach sector makes no profits, that is the cost of one unit of labor is equal to its value, which is the expected value of an innovation times its probability

$$w_t = \lambda_t V_t \tag{14}$$

The increase in the number of patents at each period is the number of new discoveries

$$\dot{N}_t = \lambda_t n_t \tag{15}$$

We assume that the labor market is competitive. The size of the population is L_t. The equilibrium on the labor market implies

$$L_t = N_t \left(\int_{l=0}^{d_t} x_l dl + h d_t^{\theta} \right) + n_t \tag{16}$$

This equation states that all the population is employed in a firm or in the research sector. The fraction of the population hired by firms can be employed either for production, or for coordinating the division of labor.

The model is now fully specified. The resolution needs few calculus because of the study of the stability of the balanced growth path. Before the main proposition of this model, we can simplify the model to show where the absence of scale effects come from. Consider that there are no workers in the research sector, the labor market equilibrium writes

$$L_t = N_t \left(\int_{l=0}^{d_t} x_l dl + h d_t^\theta \right)$$

Using 6 and 8, the previous equation yields

$$L_t = (1 + \theta(s-1)) h d_t^\theta N_t \tag{17}$$

Thus, when the number of firms increases, their internal division of labor decreases. Indeed, when the number of firms increases each of them produces less. Because of the cost of coordinating the workers, division of labor increases with the production of a firm. Thus the greater the number of firms, the smaller the division of labor. One can say that there is a substitution effects between the internal division of labor d_t and the external division of labor between firms N_t. As the probability of discovery of new patents is an increasing function of the internal division of labor, the increase in the number of firms decreases the rate of arrival of new firms. Thus, if the population is not growing the endogenous arrival of new firms is not self-sustaining, it becomes harder and harder to discover new patents. Thus, this theory of technical change makes the difficulty of discovery endogenous, which has been considered exogenous in Segerstrom [1998] and Kortum [1999]. Moreover, equation (17) shows that the greater θ, the smaller the effect of N_t on d_t. Indeed, the elasticity of d_t with respect to N_t is $-1/\theta$. As a consequence, the decrease in the number of the opportunities of innovation is smaller when θ increases. Thus, the growth rate is expected to be an increasing function of θ.

We can now state our main proposition.

Proposition 1 *The values $d_t^{\theta+1} N_t$ and $d_t L_t$ are constant along a balanced growth path. The growth rate of the number of firms and of the internal* *division of labor are*

$$\left\{ \begin{array}{c} \dot{N}_t/N_t = (1+\theta)\,u \\ \dot{d}_t/d_t = -u \end{array} \right. \tag{18}$$

Moreover, the balanced growth path is a saddle-path equilibrium.

The proof which need few calculus is in appendix. First, as it was expected, the number of firms increases with θ. Second, the internal division of labor decreases whereas the number of firm increases : there is a substitution of internal division of labor by external division of labor (N_t). Third, the change in division of labor needs an increase in the size of the population. If it is not the case, the division of labor stops at a stationary states.

These growth rate allow us to deduce the growth rate of the economy. Indeed, equation (10) yields

$$\frac{\dot{Q}_t}{Q_t} = \frac{\gamma}{1-\eta}\dot{N}_t/N_t + \frac{1}{1-\eta}\sigma\left(\theta + \frac{1}{s-1}\right)\dot{d}_t/d_t \tag{19}$$

The total production per capita is N_tQ_t/L_t. Its growth rate, g^e, is equal to $\dot{N}_t/N_t + \dot{Q}_t/Q_t - u$. Equation (11) shows that g is also the growth rate of consumption per capita.

Using the equilibrium growth rate of d_t and N_t, the growth rate of the economy is then

$$g^e = \left[\left(\frac{\gamma}{1-\eta}+1\right)(1+\theta) - \frac{1}{1-\eta}\sigma\left(\theta+\frac{1}{s-1}\right) - 1\right]u$$

using the definition of $m = 1-\eta-\sigma\left(1+\frac{1}{\theta}\frac{1}{s-1}\right)$, the previous equation yields

$$g^e = \frac{1}{1-\eta}\left[\gamma\,(1+\theta) + \theta m\right]u$$

An increase in consumption per capita, as in division of labor, necessitates an increase in the size of the population. This result is similar to those of Jones[1995a,b], Segerstrom [1998]and Kortum [1998] among others. The growth rate exhibits an inter-industrial multiplier effect $\frac{1}{1-\eta}$ because each good can be either consumed or used as an intermediate good. The growth rate is a positive function of the effect of product diversity on the productivity of factors, but even if ther are no effect of product diversity ($\gamma = 0$), the growth rate is positive and equal to $\frac{1}{1-\eta}\theta m u$. This result comes from the endogenous increase in the number of firms. As a consequence, even if there are no productivity gains, the increase of the number of firms, and the non-constant returns at the firm level, sustains growth. Furthermore, the growth rate is an increasing function of m which is the proportion of profit in the firm output. This relation is similar to Aghion and Howitt [1992] result. It is the traditional channel of the remunenration of the research effort, the greater m the more profitable an innovation. If the share of profits in output is near 0, growth per capita can be sustained if the effect of diversity on productivity ($\gamma > 0$) is great enough. Finally, the growth rate is an increasing function of θ for the same reason as the number of firms N_t is. As a consequence, θ has a negative level effect and a positive effect on growth.

We can check that the resources allocated to the research sector are growing along a balanced growth path. Indeed, equation (16) yields the size of the quantity of labor n_t^e allocated to the research sector in the market equilibrium

$$n_t^e = L_t - N_t \left(\int_{l=0}^{d_t} x_l dl + h d_t^{\theta} \right) \tag{20}$$

Using (6) and (8), the previous equation yields

$$n_t^e = L_t \left(1 - (1 + \theta(s-1))\, h \frac{d_t^{\theta+1} N_t}{d_t L_t} \right) \tag{21}$$

The previous proposition entails that the bracket is constant along a balanced growth path. As a consequence, the population allocated to the research sector is growing. The rate of patenting r_p is constant along a balanced growth path and it is equal to the rate of growth firms number. Indeed, this rate is

$$r_p = \frac{\dot{(\lambda_t n_t)}}{\lambda_t n_t} = \dot{N}_t / N_t + \dot{d}_t / d_t + \dot{n}_t^e / n_t^e = (1 + \theta)\, u$$

3 Optimal growth

The social planner maximizes the discounted utility of all the future generation. The constraints are first the production function of a firm, the equilibrium of the good market, the equilibrium of the labor market and the law of probability of discovering new products.

$$\max_{c(t),x_{l,t},k_{i,t},d_{i,t},n_t} \int_{t=0}^{\infty} e^{-(\rho-u)t} v\left(c(t)\right) dt$$

$$s.t. \begin{cases} Q_{i,t} = N_t^{\gamma-\frac{\eta}{\varepsilon}+\eta} \left(\int_{j=0}^{N_t} k_{j,i}^{\varepsilon} dj\right)^{\frac{\eta}{\varepsilon}} \left(\int_{l=0}^{d_{i,t}} x_{l,i}^{\frac{s-1}{s}} dl\right)^{\frac{s}{s-1}\sigma} \\ c(t) = \int_{i=0}^{N_t} Q_{i,t} di - \int_{i=0}^{N_t} \left(\int_{j=0}^{N_t} k_{j,i} dj\right) di \\ \int_{i=0}^{N_t} \left(\int_{l=0}^{d_{i,t}} x_{l,i} dl + h d_{i,t}^{\theta}\right) di + n_t = L_t \\ \dot{N}_t = \lambda \left(\int_{i=0}^{N_t} d_{i,t}\, di\right) n_t \end{cases}$$

The last equation of this program shows that the probability of a discovery depends on the sum of all tasks performed in each firm. It is thus assumed that all tasks are different. The resolution is given in appendix. The main results are given in the folowing proposition.

Proposition 2 *The optimal growth rate of the economy g^* is equal to the market equilibrium growth rate :*

$$g^* = \frac{1}{1-\eta}\left[\gamma\left(1+\theta\right) + \theta m\right] u$$

The market equilibrium has either too much labor allocated to the research sector and too small an internal division of labor ($n_t^e > n_t^*$ and $d_t^e < d_t^*$), or not enough labor allocated to the research sector and a to high an internal division of labor ($n_t^e < n_t^*$ and $d_t^e > d_t^*$).

The equality of the optimal growth rate and the decentralized growth rate can be found in other models of growth without scale effects, such as Segerstrom [1998]. In growth models with scale effects the externalities which generate the global increasing returns entail the market growth rate to differ from the optimal growth rate. Here the externalities have only a level effects and do not effect the long run growth rate. The externalities of this model are the dependance of the probability of discovery on the internal division of labor. The calculation shows that the market equilibrium level of internal division of labor and level of labor allocated to research are not optimal. The internal division of labor can be to high are to small. When the effect of the division of labor on productivity is high (small s), then the internal division of labor is too high, and when the effect of the internal division of labor is small, the internal division can be too small.

4 Discussion of the model : What is the number of tasks performed in the economy ?

We assumed that all firms were dividing labor in a different way, such that the number of tasks performed in the economy was the number of tasks performed in a firm times the number of firms, $N_t d_t$. The number of tasks plays a crucial role in this model because it is the aggregate mesure of the number of innovation opportunities, and it thus affects the probability of a successful innovation. We can study quite simply an other assumption, which is to assume that all firms divide labor the same way, such that the total number of tasks performed is now d_t. The results are given below in a proposition. It is difficult to choose between rhe two assumption. On the one hand, the existence of a range of qualifications of workers in all sectors seem to confirm the second assumption, the d_t-assumption. These qualifications make each worker competent on a small number of tasks, and many qualifications are widespread across sectors. This would entail that all firms perform more or less the same tasks, such as accouncy, secretaryship, financial analysis, and so on. One must acknowledge that many tasks are common across sectors. On the other hand, there are some qualifications which are linked to peculiar setors, such as teachers or physicians (see Becker and Murphy [1992]). Moreover, even if the qualification is very general, on the job training make workers more productive on a small number of tasks. These tasks can be linked to the type of capital goods used in the firm, or to the firm organization. A semi-skilled worker with a precise qualification will do quite different tasks if he or she works in an automobile factory in Japan or in the USA. As a consequence the link between qualifications and tasks performed is unclear. These reflections made us put firward the $N_t d_t$-assumption : The number of tasks increases with the number of firms. The results in the d_t-case are sumarized in the following proposition. The proof is left in appendix.

Proposition 3 *If all firms divide labor the same way, the probability of a successful innovation is $\lambda_t = d_t$. The rates of growth of the internal and external division of labor are*

$$\begin{cases} \dot{d}_t/d_t = 0 \\ \dot{N}_t/N_t = u \end{cases}$$

The growth rate of the economy is

$$g = \frac{\gamma}{1-\eta} u$$

The internal division of labor tends toward a constant in the long run and the number of firms increases at a rate which is the population growth rate. The growth rate of the economy depends only on the effect of the diversity of inputs on the productivity of factors in the firm. When there is no effect, the growth rate per capita does not increase. The difference between the growth rate of the number of firms and the growth rate of the population may lead to put this assumption in doubt. Nethertheless, we could find a positive difference along the transitory dynamic even with this assumption. An other difficult aspect is to determine the change in the level of internal division of labor of firms in the long run. It may be difficult to use the level of division of labor in the firm, because at this level many other factors may affect the division of labor in the firm.

5 Conclusion

This model proposes a theory of technical change based on Adam Smith's intuition of the role of division of labor in economic growth. Technical change is the discovery of new capital goods which help workers or even replace workers performing precise tasks. As a consequence, the number of opportunities of innovation is an increasing function of the number of tasks performed in the economy. We first model division of labor within firms and then model the research sector. The probability of innovation is not only a function of the quantity of resources allocated to the research sector, but also of the total number of tasks. We show that this theory yields a growth model without scale effects. The market equilibrium yields an optimal equilibrium growth rate, but a sub-optimal transitory dynamic. This sub-optimality can take two forms : either too many researchers and an internal division of labor too small an internal division of labor, or too few researchers and an too high an internal division of labor. We show that the long run growth rate depends on the assumption made on the evolution of the number of tasks during economic development. If the number of tasks is increasing, a positive effect of the diversity of capital goods on the productivity is not necessary to obtain a positive growth rate per capita.

A Proof of proposition 1

Using the product market equilibrium (11) and the labor market equilibrium (16), we can exhibit two differrentials equation, whose solution gives the balanced growth path an its stability. Equation (14) and (13) yields $\dot{w}_t/w_t = \dot{\lambda}_t/\lambda_t + \dot{V}_t/V_t = \dot{d}_t/d_t + \dot{N}_t/N_t + \dot{V}_t/V_t$. Using this expression in (12) yields

$$r(t) - \frac{\pi_t}{V_t} = \frac{\dot{w}_t}{w_t} - \frac{\dot{d}_t}{d_t} - \frac{\dot{N}_t}{N_t} \tag{22}$$

using (9), the previous equation is equivalent to

$$r(t) - m\frac{Q_t}{V_t} = \frac{\dot{w}_t}{w_t} - \frac{\dot{d}_t}{d_t} - \frac{\dot{N}_t}{N_t}$$

The rate of growth of wages can be deduced from (8) : $\frac{\dot{w}_t}{w_t} = \frac{\dot{Q}_t}{Q_t} - \theta\frac{\dot{d}_t}{d_t}$. With equation (22) the previous equation yields

$$r(t) - \frac{\dot{Q}_t}{Q_t} = m\frac{Q_t}{V_t} - (1+\theta)\frac{\dot{d}_t}{d_t} - \frac{\dot{N}_t}{N_t} \tag{23}$$

We can compute Q_t/V_t with (14), (8) and (13)

$$\frac{Q_t}{V_t} = \frac{\lambda_t Q_t}{w_t} = h\frac{\theta}{\sigma}(s-1)\lambda_t d_t^{\theta} = h\frac{\theta}{\sigma}(s-1)N_t d_t^{\theta+1}$$

Finally, we can calculate the difference between the interest rate and the rate of growth of the production of a firm $r(t) - \frac{\dot{Q}_t}{Q_t}$ with the equations (11), which yields $\frac{\dot{Q}_t}{Q_t} = \frac{\dot{c}(t)}{c(t)} - \frac{\dot{N}_t}{N_t}$ and The previous equation and (1) yield $r(t) - \frac{\dot{Q}_t}{Q_t} = \frac{\dot{N}_t}{N_t} + \rho$. (23) yields finally

$$\rho - mh\frac{\theta}{\sigma}(s-1)N_t d_t^{\theta+1} + (1+\theta)\frac{\dot{d}_t}{d_t} + 2\frac{\dot{N}_t}{N_t} = 0 \tag{24}$$

The previous equation is a first non-linear differential equation, which relates the internal division of labor d_t and the number of firms N_t. The second differential equation uses the labor market equilibrium (16). Equation (6) and (8) yield

$$\begin{aligned} L_t &= hd_t^{\theta}N_t + \sigma\frac{N_t Q_t}{w_t} + n_t && (25) \\ &= (1+\theta(s-1))\,hN_t d_t^{\theta} + n_t && (26) \end{aligned}$$

With equation (15), we have $n_t = \frac{\dot{N}_t}{\lambda_t}$. With (26), it yields

$$L_t = (1 + \theta(s-1))\, hd_t^\theta N_t + \frac{\dot{N}_t}{\lambda_t}$$

Substituting λ_t by its value, $N_t d_t$, the previous equation is equivalent to

$$\frac{\dot{N}_t}{N_t} = d_t L_t - (1 + \theta(s-1))\, hN_t d_t^{\theta+1} \tag{27}$$

The previous equation is the second differential equation in d_t and N_t. Equation (24) and (27) can be used to write the differrential system more simply. Substituting in (24), it yields

$$(1+\theta)\frac{\dot{d}_t}{d_t} = \left(mh\frac{\theta}{\sigma}(s-1) + 2\,(1+\theta(s-1))\right) hN_t d_t^{\theta+1} - \rho - 2d_t L_t$$

Finally, we obtain

$$\begin{cases} \frac{\dot{N}_t}{N_t} = d_t L_t - (1+\theta(s-1))\, hN_t d_t^{\theta+1} \\ (1+\theta)\frac{\dot{d}_t}{d_t} = \left(m\frac{\theta}{\sigma}(s-1) + 2\,(1+\theta(s-1))\right) hN_t d_t^{\theta+1} - \rho - 2d_t L_t \end{cases}$$

The balanced growth equilibria is defined by the growth rate of the internal division of labor $\frac{\dot{d}_t}{d_t}$ and the growth rate of the external division of labor $\frac{\dot{N}_t}{N_t}$ being constant. Equation (24) implies that $N_t d_t^{\theta+1}$ is constant along a balanced growth path. The first equation of the system yields $d_t L_t$ being constant. Thus we have

$$\begin{cases} (1+\theta)\frac{\dot{d}_t}{d_t} + \frac{\dot{N}_t}{N_t} = 0 \\ \frac{\dot{d}_t}{d_t} + \frac{\dot{L}_t}{L_t} = 0 \end{cases}$$

The growth rate of the population is constant thus $\dot{L}_t/L_t = u$. The solution of this system is

$$\begin{cases} \frac{\dot{N}_t}{N_t} = (1+\theta)\, u \\ \frac{\dot{d}_t}{d_t} = -u \end{cases} \tag{28}$$

The study of the stability of the balanced growth path is easier with the change of variables $x_t = d_t L_t$, $y_t = hN_t d_t^{\theta+1}$. It yields

$$\begin{cases} \frac{\dot{x_t}}{x_t} = (1+\theta)\, u + \left(m\frac{\theta}{\sigma}(s-1) + 2\,(1+\theta(s-1))\right) y_t - \rho - 2x_t \\ \frac{\dot{y_t}}{y_t} = \left(m\frac{\theta}{\sigma}(s-1) + (1+\theta(s-1))\right) y_t - \rho - x_t \end{cases}$$

The locus of $\dot{x_t} = 0$ and $\dot{y_t} = 0$ are

$$\begin{cases} x_t = \frac{1}{2}\left(u\,(1+\theta) - \rho\right) + \left(\frac{m}{2}\frac{\theta}{\sigma}(s-1) + (1+\theta(s-1))\right) y_t \\ x_t = \left[m\frac{\theta}{\sigma}(s-1) + (1+\theta(s-1))\right] y_t - \rho \end{cases}$$

The equilibrium values of x_t and y_t, x^e and y^e respectively, are

$$\begin{cases} y^e = \frac{\sigma(u(1+\theta)+\rho)}{(1-\eta-\sigma)\theta(s-1)-\sigma} \\ x^e = u\,(1+\theta) + \frac{\sigma(1+\theta(s-1))(u(1+\theta)+\rho)}{(1-\eta-\sigma)\theta(s-1)-\sigma} \end{cases}$$

The study of the dynamic system shows that the system exhibits a saddle-path equilibria.

B Proof of proposition 2

The program of the social planner is

$$\max_{c_t, x_{l,t}, k_{i,t}, d_{i,t}, n_t} \int_{t=0}^{\infty} e^{-(\rho-u)t} v\,(c_t)\, dt$$

$$s.t. \begin{cases} c_t = \int_{i=0}^{N_t} Q_{i,t} di - \int_{i=0}^{N_t} \left(\int_{j=0}^{N_t} k_{j,i} dj\right) di \\ Q_{i,t} = N_t^{\gamma - \frac{\eta}{\varepsilon} + \eta} \left(\int_{j=0}^{N_t} k_{j,i}^{\varepsilon} dj\right)^{\frac{\eta}{\varepsilon}} \left(\int_{l=0}^{d_{i,t}} x_{l,i}^{\frac{s-1}{s}} dl\right)^{\frac{s}{s-1}\sigma} \\ \int_{i=0}^{N_t} \left(\int_{l=0}^{d_{i,t}} x_{l,i} dl + h d_{i,t}^{\theta}\right) di + n_t = L_t \\ \dot{N_t} = \lambda \left(\int_{i=0}^{N_t} d_{i,t}\, di\right) n_t \end{cases}$$

We first reduce the dimensionality of the problem. For a given quantity of labor $\overline{X}_i$ devoted to production, the social planner seeks to maximize in each firm, $\left(\int_{l=0}^{d_{i,t}} x_{l,i}^{\frac{s-1}{s}} dl\right)^{\frac{s}{s-1}\sigma}$ subject to $\int_{l=0}^{d_{i,t}} x_{l,i}\ dl = \overline{X}_i$. The solution is clearly symetrical. As a consequence $x_{l,i} = x_i$.

The choice of $k_{j,i}$ is a static choice

$$\max_{k_{j,i}} c_t = \int_{i=0}^{N_t} Q_{i,t} di - \int_{i=0}^{N_t} \left(\int_{j=0}^{N_t} k_{j,i} dj \right) di$$
$$s.t. \left\{ Q_{i,t} = N_t^{\gamma - \frac{\eta}{\varepsilon} + \eta} \left(\int_{j=0}^{N_t} k_{j,i}^{\varepsilon} dj \right)^{\frac{\eta}{\varepsilon}} \left(\int_{l=0}^{d_{i,t}} x_{l,i}^{\frac{s-1}{s}} dl \right)^{\frac{s}{s-1}\sigma} \right.$$

First, this program shows that each firm use the same amount of input from each other firm : $k_{j,i} = k_i$. As a consequence, $\left(\int_{j=0}^{N_t} k_{j,i} dj \right) = N_t k_i$. Second, the first order conditions are

$$\eta Q_{i,t} = \left(\int_{j=0}^{N_t} k_{j,i} dj \right) = N_t k_i$$

The program rewrites

$$\max_{c_t, x_{l,t}, k_{i,t}, d_{i,t}, n_t} \int_{t=0}^{\infty} e^{-(\rho - n)t} v(c_t) dt$$
$$s.t. \left\{ \begin{array}{c} c_t = (1-\eta) \int_{i=0}^{N_t} Q_{i,t} di \\ Q_{i,t}^{1-\eta} = \eta^{\eta} N_t^{\gamma} d_{i,t}^{\frac{\sigma}{s-1}} (d_{i,t} x_i)^{\sigma} \\ \int_{i=0}^{N_t} \left(d_{i,t} x_i + h d_{i,t}^{\theta} \right) di + n_t = L_t \\ \dot{N}_t = \lambda \left(\int_{i=0}^{N_t} d_{i,t} \, di \right) n_t \end{array} \right.$$

The internal division of labor $d_{i,t}$ of each firm enters symetrically in each constraint. The solution is clearly symetric $d_{i,t} = d_t$. For the same reasons, $x_i = x_t$. With the specification of λ, thhe program rewrites as

$$\max_{d_t, n_t} \int_{t=0}^{\infty} e^{-(\rho - n)t} \ln \left[(1-\eta) N_t \left(\eta^{\eta} N_t^{\gamma} d_t^{\frac{\sigma}{s-1}} \left(\frac{L_t - n_t}{N_t} - h d_t^{\theta} \right)^{\sigma} \right)^{\frac{1}{1-\eta}} \right] dt$$
$$s.t. \left\{ \dot{N}_t = N_t d_t n_t \right. \quad (29)$$

We finally have a system with one state variable N_t and two controls d_t and n_t. We can leave aside the constant value $\int_{t=0}^{\infty} e^{-(\rho - u)t} \ln(1-\eta) \eta^{\frac{\eta}{1-\eta}} dt$ which plays no role in the maximisation. The current-value hamiltonionan is

$$H = \frac{\gamma + 1 - \eta - \sigma}{1 - \eta} \ln N_t + \frac{\sigma}{(s-1)(1-\eta)} \ln d_t + \frac{\sigma}{1-\eta} \ln \left(L_t - n_t - hN_t d_t^{\theta} \right) + \mu_t N_t d_t n_t$$

Where μ_t is the current-value shadow price. The three conditions are

$$\begin{cases} \frac{\partial H}{\partial n} = 0 \\ \frac{\partial H}{\partial d} = 0 \\ \dot{\mu}_t = -\frac{\partial H}{\partial N} + (\rho - u)\mu_t \end{cases}$$

This yields

$$\mu_t d_t N_t - \frac{\sigma}{1-\eta} \frac{1}{L_t - n_t - N_t h d_t^{\theta}} = 0 \tag{30}$$

$$\frac{\sigma}{(s-1)(1-\eta)} - \frac{\sigma}{1-\eta} \frac{\theta h N_t d_t^{\theta}}{L_t - n_t - N_t h d_t^{\theta}} + \mu_t d_t N_t n_t = 0 \tag{31}$$

$$\dot{\mu}_t N_t = -\frac{\gamma + 1 - \eta - \sigma}{1-\eta} + \frac{\sigma}{1-\eta} \frac{h N_t d_t^{\theta}}{L_t - n_t - h N_t d_t^{\theta}} - \mu_t N_t d_t n_t + (\rho - u) N_t \mu_t \tag{32}$$

The study of this system is easier after two changes of variable. The first one is $\zeta_t = \mu_t N_t$. It implies $\dot{\zeta}_t = \dot{\mu}_t N_t + \mu_t \dot{N}_t$. The second change of variable is $\nu_t = \zeta_t^{-1}$, $y_t = N_t h d_t^{\theta+1}$, $x_t = L_t d_t$. The three previous equations yield

$$\begin{cases} \frac{\sigma}{1-\eta}\left(1 - \frac{1}{s-1}\right)\nu_t + (1+\theta)\, y_t - x_t = 0 \\ -\frac{\dot{\nu}_t}{\nu_t} = -\frac{1}{1-\eta}\left(\gamma + 1 - \eta - \frac{\sigma}{s-1}\right)\nu_t - \theta y_t + (\rho - u) + x_t \\ \frac{\dot{N}_t}{N_t} = x_t - \frac{\sigma}{1-\eta}\nu_t - y_t \end{cases}$$

We can study the allocations along a balanced growth equilibrium. First note that along such a path, $\dot{d}_t/d_t$ and $\dot{N}_t/N_t$ are constant. As a consequence, $\dot{x}_t/x_t$ and $\dot{y}_t/y_t$ are constant. Some linear algebra, show that this is possible if $\frac{\dot{\nu}_t}{\nu_t}$ is constant. Then, it implies that x_t, y_t and ν_t are constant. It yields

$$\begin{cases} \frac{\dot{d}_t}{d_t} = -u \\ \frac{\dot{N}_t}{N_t} = (1+\theta)u \end{cases}$$

These growth rates determine the optimal growth rate of production per capita, g^*, which is equal to $\dot{N}_t/N_t + \dot{Q}_t/Q_t - u$. the growth rate of the production is given by (19). Thus g^* is equal to the market growth rate

$$g^* = \frac{1}{1-\eta}\left[\gamma\left(1+\theta\right)+\theta m\right]u$$

The equilibrium values of x_t, y_t and ν_t are given by the system

$$\left\{\begin{array}{c} \frac{\sigma}{1-\eta}\left(1-\frac{1}{s-1}\right)\nu_t + \left(1+\theta\right)y_t - x_t = 0 \\ -\frac{1}{1-\eta}\left(\gamma+1-\eta-\frac{\sigma}{s-1}\right)\nu_t - \theta y_t + \left(\rho-u\right) + x_t = 0 \\ x_t - \frac{\sigma}{1-\eta}\nu_t - y_t - (1+\theta)u = 0 \end{array}\right.$$

It yields

$$\left\{\begin{array}{c} y^* = \frac{\sigma(\rho-u)+(1+\theta)u(\gamma+1-\eta-\sigma)(s-1)}{(1+\gamma-\sigma-\eta)\theta(s-1)-\sigma} \\ x^* = \left(\theta\left(s-1\right)+1\right)\frac{\sigma(\rho-u)+(1+\theta)u(\gamma+1-\eta-\sigma)(s-1)}{(1+\gamma-\sigma-\eta)\theta(s-1)-\sigma} - \left(s-2\right)\left(1+\theta\right)u \end{array}\right.$$

The comparaison of the optimal value of x^* and y^* to their market value is not simple because of the relation, which is an externality in the decentralized economy, and wich affects two variables in the optimal problem. First note that equation (29) yields

$$\frac{n_t}{L_t} = \frac{\dot{N}_t}{N_t}\frac{1}{d_t L_t} = \frac{\dot{N}_t}{N_t}\frac{1}{x_t} = (1+\theta)u\frac{1}{x_t}$$

This equation is true in the optimal case and in the decentralized case. As a consequence, if $d_t^* > d_t^e$ then $x_t^* > x_t^e$, the previous equality yields $n_t^e > n_t^*$. Thus if the internal division of labor is to low then the quantity of labor allocated too the research sector is too high. The reverse is also true. The analysis of y^*, x^*, x_t^e and y_t^e with respect to s yields to the conclusion presented in the discussion of the proposition 2.

C Proof of the propostion 3

Quite similar calculations to those in proof of proposition 1 yield the differential system

$$\left\{\begin{array}{l} \frac{\dot{N}_t}{N_t} = d_t N_t^{-1} L_t - \left(\theta(s-1)+1\right)hd_t^{\theta+1} \\ \left(1+\theta\right)\frac{\dot{d}_t}{d_t} = \left(m\frac{\theta}{\sigma}(s-1) + \left(\theta(s-1)+1\right)\right)hd_t^{\theta+1} - \rho - 1)d_t N_t^{-1} L_t \end{array}\right.$$

Along a balanced growth path $d_t^{\theta+1}$. and $d_t N_t^{-1} L_t$ are constant. It yields

$$\begin{cases} \dot{d}_t/d_t\,(1+\theta)=0 \\ \dot{d}_t/d_t - \dot{N}_t/N_t + \dot{L}_t/L_t = 0 \end{cases}$$

The growth rate of the population is constant thus $\dot{L}_t/L_t = u$. The solution of this system is

$$\begin{cases} \dot{N}_t/N_t = u \\ \dot{d}_t/d_t = 0 \end{cases}$$

The total production per capita is $N_t Q_t / L_t$. Its growth rate g is equal to $\dot{N}_t/N_t + \dot{Q}_t/Q_t - n$. Using equation 10, we can calculate the growth rate of the production per firm $\frac{\dot{Q}_t}{Q_t} = \frac{\gamma}{1-\eta}\dot{N}_t/N_t + \frac{1}{1-\eta}\sigma\left(\theta + \frac{1}{s-1}\right)\frac{\dot{d}_t}{d_t}$. It yields

$$g = \frac{\gamma}{1-\eta}u$$

D References

Aghion P. et Howitt [1992] A Model of growth through Creative Destruction. *Econometrica.* 60(2), 323-351.

Amable B. [1995] Endogenous Growth and Cycles through radical and incremental innovation, *Document de travail du CEPREMAP n*°9504.

Basu S. and Fernald J.G. [1987], Returns to Scale in U.S. Production: Estimates and Implications, *Journal of Political Economy,* 105(2).

Becker G. and Murphy K.[1992], The Division of Labor, Coordination Costs, and Knowledge, *Quarterly Journal of Economics, 107*(4).

Bolton P. and Dewatripont M.[1994], The firm as a Communication Network, *Quarterly Journal of Economics, 109*(4).

Coase R. [1937] The Nature of the Firm*, Economica, 1937, p. 386-405.*

Eicher T.S. and Turnovsky S.J [1999], Non-scale Models of Economic Growth, *The Economic Journal,* vol. 109 no. 457

Gilfillan S. [1935], *The Sociology if invention,* Chicago: Follett.

Grossman G.M. and Helpman E. [1991] *Innovation and Growth in the Global Economy,* Cambridge: The MIT Press.

Jones Charles I.[1995], R&D-based Models of Economic Growth, *Journal of Political Economy,* CIII, 759-784.

Jones Charles I.[1999], Growth: with or without scale effects, *American Economic Review (Papers & Proceedings) 89, 139-44.*

Romer [1990] Endogenous Technical Change. *Journal of Political Economy.* 98(5) pt. 2, S71-S102.

Segerstrom P.[1998] Endogenous Growth Without Scale Effects, *American Economic Review,* vol 88 no. 5

Smith A.[1776], *An inquiry into the nature and causes of the wealth of nations,* NLondon: W. Stahan & T. Cadell.

Solow R. [1956] A contribution to the Theory of Economic Growth, *Quaterly Journal of Economics ,* 70, 65-94

Stigler [1951] The Division of Labor is Limited by the Extent of the Market, *Journal of Political Economy*, LIX (June), 185-193.

Yang X. and Borland J. [1991] A Microeconomic Mechanism for Economic Growth. *Journal of Political Economy.* 34, 199-222.

Young A. [1928] Increasing returns and economic progress, *Economic Journal 38,* 527-542.