May the worst monetary standard be the best?
A re-enactment of “the crimes of 1873”

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ABSTRACT: I establish new existence and welfare results for the Kiyotaki-Wright model [1989]. For a general version of the model, I construct many equilibria in which goods with **poor** storage properties are widely accepted. Furthermore, I show that these equilibria may be socially desirable because more trade occurs that in the alternative equilibria in which **better** goods are widely accepted. By analogy, these results may be helpful in analyzing societies that chose among commodity standards; say, between a gold and a silver standard. For instance, we may have a general-equilibrium-model reconstruction of the so-called crimes of 1873 in US and France.

[100 words]

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1. INTRODUCTION

This paper establishes new existence and welfare results for the Kiyotaki-Wright model [1989] considering mixed strategies that do not restrict agents to play a unique strategy for each opportunity set. For a **general version** of the model, I construct many equilibria in which goods with **poor** storage properties are widely accepted while better goods are less accepted. Furthermore, I show that these equilibria may be socially desirable because more trade occurs that in the alternative equilibria in which **better** goods are those which are widely accepted. The nontechnical intuition is that if intrinsically attractive objects have great acceptance, people would be very reluctant to trade them away. In contrast, if intrinsically unattractive objects are the objects that are widely accepted, people would be less reluctant to trade them away and, consequently, more trade may occur.

By analogy, those results may be helpful in analyzing historical episodes in which a society faced the choice of a commodity standard; say, the election between a gold standard and a silver standard. For instance, my welfare results may help us to evaluate the controversial rush after 1867 to adopt the gold standard, the **appreciating and possibly the convenient standard,** by the great commercial nations of the time. In fact, we may have a reconstruction in a general equilibrium model of the so-called crimes of 1873 in US and France (see Friedman [1990 b] and Flandreau [1996]). That is, my results may suggest that keeping (as in China, India, Mexico, etc.) the silver standard, the **depreciating and possibly the inconvenient standard,** was a sound economic policy decision as discussed for some protagonists. Therefore, we may have some support to the alleged claim of some silver advocates that the "worst standard was the best" (Conant et al. [1903, p. 501]); that is, perhaps with more precision, that the intrinsically worst commodity standard may be the socially best one.

General description of the model

The Kiyotaki-Wright model is a discrete time model in which infinitely-lived agents are randomly matched pairwise in each period. The agents maximize expected discounted utility by choosing trading strategies for indivisible objects that are perfectly durable but costly to store. When there are more than two goods, there must be other than double coincidence trade because of the assumed pattern of specialization in production and consumption. According to Aiyagari-Wallace [1997], the Kiyotaki-Wright model "is the only attractive model with an endogenous transaction pattern" (p. 2-3; see also Wallace [1996, p. 251]). In turn, Wallace [1997] indicate that this model is the first one "in which several objects are potential media of exchange and in which the relationship between the physical properties of those objects and their role as media of exchange can be studied" (p. 3).

General description of the results

I describe next the results exhibited in this paper which are for storage costs of goods nearly equal. For any number of goods larger than two and an open set of parameters and initial conditions, I prove the existence of multiple mixed-strategy equilibria in which the most costly to-sore good is universally accepted in trade. In this equilibria, agents play trading strategies near to always trade. I also prove that there exists a continuum of stable steady states of this kind. Notice that these results are for a general version of the model. Moreover, assuming three goods, I show that these equilibria Pareto dominate to alternative equilibria in which less costly to-store goods are universally accepted. Furthermore, they also Pareto dominate the alternative mixed strategy equilibrium displayed in Renero [1999] and in which the most costly to-store good has the highest acceptance rate and agents are restricted to play a unique strategy for each opportunity set. Notice that these results are for equilibria, not steady states.

The mixed-strategy equilibria are interesting because agents have to be indifferent in trade among goods which are not their consumption good. Hence, acceptability, saleability, or "liquidity" has to compensate for poor storage properties. That is, the acceptance of a good, which is endogenous in the model, varies directly with its storage cost, which is exogenous in the model.

Technical intuition for the results

The technical intuition of these results are closely related to the properties of paths with always-trade strategies. First of all, always-trade strategies maximize the fraction of agents who consume. Hence, always-trade strategies may be socially desirable. However, they are equilibrium strategies if and only if goods are equally costly to store.

If goods are equally costly to store and always-trade strategies are played, then agents are indifferent among holding goods other that their consumption good. Consequently, always-trade strategies are individually optimal. This situation is very suggestive. If objects are not equally costly to store but not too different, then one may suspect the existence of individually optimal mixed strategies where the acceptance of an object varies directly with its storage cost and agents are indifferent among goods. In fact, for small enough differences in storage costs, I show that for any number of goods larger than two, many equilibria which such strategies exist. Moreover, those trading strategies are near to always trade. Therefore, one may conjecture that this type of equilibria may have good welfare properties relative to equilibria in which less costly to-store goods have the highest acceptance. For the case of three goods, I verify this conjecture.

However, the mixed-strategy equilibrium exhibited in Renero [1999], which is for the case of three goods, cannot come very close to always trade because it satisfies the restriction that agents play a unique strategy for each opportunity set –a restriction which, as shown by Kehoe-Kiyotaki-Wright [1993], implies that the number of steady states is generically finite. Even though, this restricted mixed-strategy equilibrium Pareto dominates to the alternative equilibria in which less costly to-store goods are universally accepted. The technical intuition for this welfare result is that mixed strategies are still closer to always trade. Moreover, the implied distribution of goods is more symmetric relative to those alternative equilibria.

Since the distribution of goods of the unrestricted mixed-strategy equilibria exhibited in this paper are practically as symmetric as that of the restricted mixed-strategy equilibrium, the Pareto ranking among these equilibria has to be explained for the closeness of the strategies to always trade.

Early work on the model

Early work on the Kiyotaki-Wright model [1989] seemed to be consistent with the intuitive idea that good intrinsic properties (say, storability, the depreciation rate, or the rate of return[1]) were necessary or desirable for objects to be universally accepted in equilibrium. For instance, for any number of goods and one fiat object, Aiyagari-Wallace [1991] prove the existence of a steady state in which the least costly to-store object is universally accepted in trade. In fact, this result seems to be the conspicuous part of the first analysis of a general version of the model.[2] Also, assuming two goods and an one fiat object, Aiyagary-Wallace [1992] show that the fiat object is accepted in equilibrium only if it is the least costly to-store object. Moreover, they exhibit three multiple equilibria which are Pareto ranked according to the acceptability of the fiat object: the greater the acceptability of the fiat object the better. (Other work apparently with the same spirit on modified versions of the model is Marimon-McGrattan-Sargent [1990] and Williamson-Wright [1994].)

However, two early results for the Kiyotaki-Wright model [1989] are inconsistent with the extreme idea that only objects with the best intrinsic property could be universally accepted in equilibrium. For the case of three goods and no fiat objects, Kiyotaki-Wright [1989] show the existence of a steady state in which the universally accepted good is the second least costly to-store. For the case of three goods and one fiat object, Aiyagari-Wallace [1992] claim a numerical example of a steady state in which the fiat object being the second least costly to-store is universally accepted. However, it is not known if these kinds of steady states generalize to more than three goods.

Finally, there is one early result which may contradict the idea that good intrinsic properties are necessary for an object to be universally accepted in equilibrium. Although they did not call attention to it, Kehoe-Kiyotaki-Wright [1993] display for three goods and no fiat objects a steady state in which the worst object is universally accepted. However, their steady state is different from those I show exist and it is not known also whether their kind of steady state generalizes to more than three goods. Moreover, for the strategy spaces that Kehoe-Kiyotaki-Wright [1993] assume, it is shown in Renero [1998] that for almost any initial condition (or given any initial condition and for almost any parameters), there does not exist an equilibrium approaching to that steady state.

Organization of the paper

This paper is organized as follows. In Section 2, I give the description of the environment, the notation, and the equilibrium definition. In Section 3, I establish properties of paths with always-trade strategies which are used in many of the proofs. In Section 5, I deal with the case of three goods and no fiat objects. In Section 6, I make some concluding remarks. Technical proofs and a graph mentioned in Section 6 are in the Appendix.

[1] The rate of return is considered explicitly as an intrinsic property in Aiyagari-Wallace-Wright [1996, p. 398] and in Wallace [1997, p. 11].

[2] As far as I know, my general results constitute the other analysis so far of the same version of the model and for that matter of any general version of the model.

2. PHYSICAL ENVIRONMENT AND EQUILIBRIUM DEFINITION

I will use the physical environment, notation, and essentially the equilibrium concept of the Aiyagari-Wallace [1992] exposition of the Kiyotaki-Wright model [1989]. I give next a brief description of these items. For reader's convenience, I abstract of fiat objects.[3]

2.1 The Physical Environment

Time is discrete and represented by the positive integers. There exist N 3 goods which are assumed to be perfectly-durable and indivisible goods. The goods are indexed by the set {1, ..., N}. There are N types of infinitely-lived agents indexed also by the set {1, ..., N}. Type i consumes good and produces good i+1. There is a [0,1/N] continuum of agents for each type.

Agents maximize expected discounted utility with discount factor of (0,1). In any period, a type-i agent's utility of neither consuming nor storing anything is zero, the utility of consuming one unit of good i without storing anything is $u_i > 0$, and the utility of not consuming and storing one unit of object j from the given period until the next period is $-c_{ij}$ 0. After consuming one unit of good i, agent of type i produces one unit of good i+1 which appears at the beginning of next period. At the beginning of the initial period t = 1, each agent is endowed with one unit a good. Finally, each period each agent is paired randomly with one other agent. It is assumed that paired agents know each other's type and current inventory but not trading histories.

2.2 Definition of Equilibrium

Next, I define a class of Nash equilibria with rational expectations. In particular, I assume that the strategies are *symmetric,* i.e., all agents of the same type in the same situation use the same strategy, and that trading strategies are *nondiscriminatory,* i.e., willingness to trade does not depend on the type of agents one meets. Moreover, we assume that agents do not dispose of objects and do not postpone consumption. We require that in equilibrium the actions of each agent are individually optimizing given the actions of the other agents and the path of inventory distribution of stocks. The notation presumes that each agent start each period with one unit of a good. That is, we assume additionally that agents never give a good away to another agent for nothing.[4]

Therefore, the timing of an agent's activities in period t is that he or she starts with one good, meets another agent, ends up with some good after meeting, and then stores or consumes and produces to start at period t+1 with some good.

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[3] For a treatment which include fiat objects see Renero [1994].

[4] In fact, we can ignore gift giving because, as noticed by Aiyagari-Wallace [1991], it is never an equilibrium strategy given the following small and otherwise innocuos change to the model: let agents derive some small enough utility from consuming any good.

I give next notation for the trading strategies of the agents and other items. The probability of choosing to trade good j for good k by those who are type i, hold good j, and meet another agent with good k at period t, is denoted by $s_{ij}^{k}(t)$. The vector of these over (j,k) is denoted by $s_i\ (t)$ and the vector of these over (i,j,k) is denoted by $s(t)$.

The symbol $p_{ij}(t)$ denotes the proportion of agents who are type i and hold good j at the start of period t. The symbol $p(t)$ denotes the vector of these elements or the distribution of inventories; that is, $p(t)$ is the state of system. The law of motion of the sequence $\{p(t)\}$ is denoted by

$$p(t+1) = h[p(t), s(t)],$$

where h is defined by the following two equations

$$\text{(1.1)} \qquad a_{ij}(t) = p_{ij}(t) - p_{ij}(t) \sum_{k} \sum_{l \neq j} p_{kl}(t)\, s_{ij}^{l}(t)\, s_{kl}^{j}(t) \; + \sum_{k} \sum_{l \neq j} p_{il}(t)\ p_{kj}(t)\ s_{ij}^{j}(t)\, s_{kj}^{l}(t)$$

$$\text{(1.2)} \qquad p_{ij}(t+1) = \left(1 - \boldsymbol{d}_{ij}\right) a_{ij}(t) + \boldsymbol{d}_{i+1,\,j}\ a_{ii}(t)$$

where $_{kn}$ = 1 and 0 otherwise.

In (1.1), the symbol $a_{ij}(t)$ represents the proportion of agents who are type i and end up with object j after the random meetings at period t. On the RHS of (1.1), the first term is the proportion of agents who are type i and hold object j at the start of period t and, consequently, before the random meetings at period t; the second term is the proportion of agents who are type i and held object j before the random meetings and traded for a different object; and the third term is the proportion of agents who are type i and who did not hold object j before the random meetings and who traded for object j at period t. In (1.2), if $j = i$, then $p_{ij}(t+1) = 0$ because agents do not postpone consumption; if $j = i+1$, then $p_{ij}(t+1) = a_{ij}(t) + a_{ii}(t)$ because $a_{ii}(t)$ is the fraction of agents who produce good i+1; lastly, if $j \quad i$ and $j \quad i+1$, then $p_{ij}(t+1) = a_{ij}(t)$.

Let $v_{ij}(t)$ denote the expected discounted utility of agent of type i ending with object j after trade at period t, but before consuming, storing, or disposing, and let $v(t) \in \Re^{N(N)}$ denote the vector of expected utilities. Given the sequence $\{p(t+1), s(t+1)\}$, there exists a unique bounded sequence of vectors $\{v(t)\}$ which satisfy the dynamic-programming equations

$$\text{(2.1)} \qquad v_{ij}(t) = -c_{ij} + \boldsymbol{r} \sum_{k} \sum_{l} p_{kl}(t+1) \left[s_{ij}^{l}(t+1)\, s_{kl}^{j}(t+1)\, v_{il}(t+1) + \left(1 - s_{ij}^{l}(t+1)\, s_{kl}^{j}(t+1)\right) v_{ij}(t+1) \right]$$

if $i \quad j$ and

(2.2) $v_{ii}(t) = u_i + \boldsymbol{r} \sum_k \sum_l$

$$p_{kl}(t+1)\left[s^{l}_{i,i+1}(t+1)\, s^{i+1}_{kl}(t+1)\, v_{il}(t+1) + \left(1 - s^{l}_{i,i+1}(t+1)\, s^{i+1}_{kl}(t+1)\right) v_{i,i+1}(t+1)\right]$$

The existence and uniqueness of the bounded sequence $\{v(t)\}$ follow from the fact that the RHS of equations (2.1) and (2.2) are contractions as functions of the components of the vector *v(t)*.

The individual optimizing conditions for trading strategies are

(3.1) $v_{ij}(t) \geq v_{ik}(t) \quad \textit{if} \quad s^{k}_{ij}(t) = 0$

(3.2) $v_{ij}(t) \leq v_{ik}(t) \quad \textit{if} \quad s^{k}_{ij}(t) = 1$

(3.3) $v_{ij}(t) = v_{ik}(t) \quad \textit{if} \quad 0 < s^{k}_{ij}(t) < 1.$

The condition about optimality of consumption after acquiring the consumption good is

(3.4) $u_i + c_{ij} + v_{i,i+1}(t) \geq -c_{ii} + \boldsymbol{r}\left[u_i + c_{ij} + v_{i,i+1}(t)\right]$

The condition about otimality of nondisposal is

(3.5) $v_{ij}(t) \geq 0$

Therefore, we have the next definitions.

DEFINITION 1. A *symmetric equilibrium* from p(1), in which agents (i) do not play discriminatory strategies, (ii) do not dispose of objects, and (iii) do not postpone consumption, is a path $\{p(t+1), s(t)\}$ such that

(1) $p(t+1) = h[p(t), s(t)]$, and

(2) there exists a bounded sequence $\{v(t)\}$ such that equations (2.1)-(2.2) and (3.1)-(3.5) hold.

Notice that the equilibrium condition (2) requires that the sequence $\{s_i(t)\}$ be the best response of any agent of type i taken as given the path $\{p(t+1), s(t)\}$.

DEFINITION 2. A *symmetric steady state,* in which agents (i) do not play discriminatory startegies, (ii) do not dispose of objects, and (iii) do not postpone consumption, is a constant *(p,s)* such that $[p(t+1), s(t)] = (p, s)$ for all *t* *1* satisfies Definition 1 when *p(1) = p.*

3. ALWAYS TRADE

I will prove next some properties of paths with always-trade strategies. All these facts are crucial to prove claims of section 4. Kiyotaki-Wright [1989], considering stationary paths for the case of three goods and no fiat objects, notice that always-trade strategies, though possibly not individual optimal, could give the agents higher consumption rates than a particular steady state. In Lemma 3.1, which is proved in the Appendix, I establish that always-trade strategies played by good holders maximize the fraction of people who get their consumption good, $\sum_i a_{ii}(t)$, if no agents holds his or her consumption good at the start of any period; that is, if agents do not postpone consumption.

LEMMA 3.1. Let $[p(t), s(t)]$ be a pair of vectors of distribution of inventories and of trading strategies. Let also $a_{ii}(t)$ be given accordingly by equation (1.1), i.e.,

$$a_{ii}(t) = \sum_{k} \sum_{l \neq i} p_{il}(t)\, p_{ki}(t)\, s^{i}_{il}(t)\, s^{l}_{ki}(t) \quad .$$

Then, $\sum_i a_{ii}(t) \leq 1/N$ and with equality if all $s^{k}_{ij}(t) = 1$ and $p_{ii}(t) = 0$.

Thus, always-trade strategies of good holders maximize the fraction of agents who consume if agents do not postpone consumption and, consequently, they may be socially desirable. Moreover, since the fraction of agents who consume are equal across types if the inventory distribution is constant (Aiyagari-Wallace [19991]), always-trade strategies maximize the fraction of agents who consume by type in constant paths. In the next section, I will show that there exist parameters for which the fraction of agents who consume is close to $1/N$, the upper bound for aggregate consumption.

I establish in Lemma 3.2, which is proved in the Appendix, that the law of motion h of paths with always-trade strategies, s^{AT}, nondisposal, and nonpostponement of consumption, is a z-stage contraction for some positive integer z. It follows that this kind of path converges to p^{AT}, where $p^{AT}_{ij} = N^{-2}$, j i and j $i+1$, and $p^{AT}_{i,i+1} = 2N^{-2}$.

LEMMA 3.2. There exists $z \in \aleph$ such that $h[\cdot, s^{AT}]$ is a z-stage contraction on the space A = $\{p \in \Re^{N(N)}_{+} : \sum_k p_{ik} = 1/N \ \ \textit{and} \ \ p_{ii} = 0\}$ and with the *sup* metric defined by $d(p, p') = \max_{(i,j)} \{|p_{ij} - p'_{ij}|\}$.

4. MANY EQUILIBIA AND MANY STEADY STATES

I will use continuity arguments to prove that for an open set of parameters (which include equal storage costs) and initial conditions, there exist many multiple equilibria (i.e., equilibria from the same initial condition) and even a continuum of steady states with mixed trading strategies. Moreover, for each steady state of this continuum there exists a neighborhood of the steady state such that for any initial condition in that neighborhood there exists an equilibrium converging to the steady state.

The proofs of these claims are long and technical. Therefore, I exhibit next the key and possibly instructive steps as lemmas leaving the proofs to the Appendix.

The next lemma assumes implicitly that agents do not give gifts, do not dispose of objects, and do not postpone consumption, and it gives necessary and sufficient conditions for agents to be indifferent in trade for all goods but their respective consumption.

LEMMA 4.1: Given $\{p(t+1), s(t+1)\}$, let $\{v(t)\}$ satisfy equations (2.1)-(2.2). Then, $v_{ij}(t) = v_{in}(t)$, $j \neq i$, $n \neq i$, for all t, *if and only if*

$$c_{in} - c_{ij} = \boldsymbol{r}\left[u_i + c_{i,i+1}\right]\left(\sum_k p_{ki}(t+1)s_{ki}^n(t+1) - \sum_k p_{ki}(t+1)s_{ki}^j(t+1)\right) \tag{4.1}$$

for all t.

The last lemma says that agents are indifferent all along the path in holding different goods but their respective consumption goods if and only if the difference of storage costs across goods is compensated by the difference in expected discounted utility from consumption next period all along the path. Notice that Lemma 4.1 implies that always-trade strategies are equilibrium strategies if and only if goods are equally costly to store[5].

Part of the next work is proving that the condition of Lemma 4.1 is satisfied. For convenience, I use the next notation: e[i] $\in$ *argmax*$\{c_{ij} : j \neq i\}$; that is e[i] is one of the most costly to-store goods, with exception of the respective consumption good, for agents of type i.

The next lemma establishes that for sequences of inventory distributions and parameters $(\boldsymbol{r}, u_i, c_{ij})$ in an open set, there exist individually-optimal actions for all type of agents: trading strategies, not postponement of consumption, and not freely disposing. Further, the trading strategies can be arbitrarily close to always-trade strategies, s^{AT}, and some strategies can be fixed at values out of a continuum.

LEMMA 4.2: Assume $c_{ij} < \boldsymbol{r}\left[u_i + c_{i,i+1}\right](1/N)$. For every neighborhood Δ of s^{AT}, there exist a neighborhood M of p^{AT}, an open set W of parameters $(\boldsymbol{r}, u_i, c_{ij})$ (a set that includes equal

[5] Aiyagari-Wallace[1991] give a proof for steady states.

storage costs), and a positive number $\boldsymbol{w} < 1$, such that for any sequence $\{p(t+1)\}$ of M, any parameters $(\boldsymbol{r}, u_i, c_{ij})$ in W, and any w in $(\boldsymbol{w}, 1)$,

(1) there exists a sequence $\{s(t)\}$ and a bounded sequence $\{v(t)\}$ such that equations (2.1)-(2.2) and (3.1)-(3.5) hold,

(2) $s(t) \in \Delta$,

(3) $s_{ki}^{e[i]}(t) = w$, k ≠ e[i].

Notice that w in (3) of Lemma 4.2 may be indexed by k and i (see Renero [1994]). That is, the continuum at which some strategies can be fixed may be multidimensional but for simplicity a unidimensional continuum is used.

Notice also that Lemma 4.2 does not provide equilibria because nothing is said about the initial condition p(1) and the law of motion $p(t+1) = h[p(t), s(t)]$. Here the difficulty is that the path $\{p(t+1), s(t)\}$ satisfy this law of motion and $\{p(t+1)\}$ be in the neighborhood M of Lemma 4.2. The next lemma deals with this problem for initial conditions p(1) in an open set.

LEMMA 4.3: For every neighborhood M of p^{AT}, there exist a neighborhood C of p^{AT} and a neighborhood Δ of s^{AT}, such that for any p(1) in C and any sequence $\{s(t)\}$ in Δ, the path $\{p(t+1)\}$ given by $p(t+1) = h[p(t), s(t)]$ is in M.

Therefore, for parameters and initial conditions in an open sets, Lemmas 4.2 and 4.3 imply the existence of multiple equilibria from p(1) in which some strategies are fixed, all along the path, at values out of a continuum.

Proposition 4.4, whose proof is left to the Appendix, establishes the existence of a continuum of steady states which are approachable from nearby initial conditions. I take advantage that the notation is simpler for steady states to present and explain important characteristics of steady states, and equilibria, with mixed trading strategies.

PROPOSITION 4.4: Assume $c_{ij} < \boldsymbol{r}\lfloor u_i + c_{i,i+1} \rfloor (1/N)$. For every neighborhood Δ of s^{AT}, there exist an open set W of parameters $(\boldsymbol{r}, u_i, c_{ij})$ (a set that includes equal storage costs) and a positive number $\boldsymbol{w} < 1$, such that for any parameters $(\boldsymbol{r}, u_i, c_{ij})$ in W and any w in $(\boldsymbol{w}, 1)$, there exists a steady state (p,s) in which

1) $s \in \Delta$,

2) $s_{ki}^{e[i]} = w$, k ≠ e[i],

3) $$\sum_k p_{ki} s_{ki}^j = \sum_{k \neq e[i]} p_{ki} w + p_{e[i],i} - \frac{c_{i,e[i]} - c_{ij}}{r\,[u_i + c_{i,i+1}]}$$

4) $$N\,a_{ii} = (1/N)\left(\sum_i \sum_k p_{ki}\, w\right) - \sum_i \sum_l p_{il} \frac{c_{i,e[i]} - c_{il}}{r\,[u_i + c_{i+1}]}$$

Moreover, at least for steady states (p,s) that satisfy $s_{ki}^j = s_{li}^j$, j ≠ k, j ≠ l, there exist a neighborhood O of the steady-state p such that for any p(1) ∈ O, there exist equilibria from p(1) converging to the steady state (p,s).

Notice that for simplicity w in 2) of Prop. 4.4 is not indexed by k and i (see Renero [1994]).

Notice also that according with 3) of Proposition 4.4, holding object e[i] gives the highest probability to agents of type i of trading for their consumption good. Such probability is given by $\sum_{k \neq e[i]} p_{ki} w + p_{e[i],i}$ while the probability of trading for their consumption good when holding object j is given by $\sum_{k \neq e[i]} p_{ki} w + p_{e[i],i} - \frac{c_{i,e[i]} - c_{ij}}{r\,[u_i + c_{i,i+1}]}$.

Part 4) of Proposition 4.4 gives the fraction of agents who consume $N\,a_{ii}$. If $w = 1$ and $c_{i,e[i]} - c_{ij} = 0$, agents always trade and $N\,a_{ii} = (1/N)$, i.e., the fraction of agents who consume is the biggest possible according to Lemma 3.1.

Notice also that assuming e[i] = e, i ≠ e (i.e., the good e is the most costly to-store good for everybody except possibly for agents of type e), Proposition 4.4 says that there exist steady states in which the most costly to-store good for everybody is accepted with probability near to one (or probability one if $w = 1$) for parameters in an open set. In fact, there exists a continuum of steady states, labeled by w with this feature.

Moreover, the last part of Proposition 4.4 establishes that at least for each steady state of the continuum of steady states satisfying $s_{ki}^j = s_{li}^j$, j ≠ k, j ≠ l, there exists a neighborhood of the steady state such that for any initial condition in that neighborhood there exists an equilibrium converging to the steady state.

Proposition 4.4 may seem to contradict a proposition provided by Kehoe et al. [1993] for the case of three goods and no fiat objects. This proposition establishes a finite number of symmetric steady states for almost any parameters; i.e., with exeption possibly of a set of parameters of Lebesque measure zero. Actually, there is no contradiction because they are dealing with a somehow modified version of the model or equilibrium definition. The modification is to require that $s_{ij}^k + s_{ik}^j = 1$. That is, if the probability of choosing to trade good j for good k by those who are of type i, hold good j, and meet another agent with good k, is s_{ij}^k, then the probability of choosing to trade good k for good j by those who are of type i, hold good k, and meet another agent with good j, is trade good j for good k by those who are of type i, hold good j, and meet another agent with good j, is $1 - s_{ij}^{k}$.

5. WELFARE IN THE MODEL WITH THREE GOODS

I deal in this section with the original model specification of three goods and no fiat objects. However, I do not restrict the analysis either to steady states or to pure strategies. I will show that there exist two-period convergent equilibria in which the most costly to-store good is universally accepted and in which all agents play mixed trading strategies. Moreover, I will show that these equilibria Pareto superior to other equilibria with a different transaction pattern; in particular, equilibria with universal acceptance of less costly to-store goods or with highest acceptance rate but not with universal acceptance of the most costly to-store good.

There are more than one way to prove this claim. The one that we will follow here, which seems to be the easiest one, is to make use of the results in Renero [1999, Prop. 3.4]. There, allowing only mixed strategies that restrict agents to play a unique strategy for each opportunity set, I prove that there exists a two-period convergent equilibrium in which agents play mixed strategies and which is Pareto superior to the other equilibria in which less costly to-store goods are universally accepted. Consequently, we need just to show here that (1) there exist two-period convergent equilibria in which all agents play mixed strategies and in which the most costly to-store good is universally accepted, and that, by transitivity, (2) these equilibria Pareto superior that equilibria with mixed strategies which restrict agents to play a unique strategy for each opportunity set. The proof of the welfare ranking relies on the fact that all multiple equilibria converge and consequently the welfare ranking is determined by the ranking of the steady-state expected utilities for a discount factor ρ close enough to 1. For reader's convenience and to establish some notation, I will review next the associated steady states.

For an open set of parameters, Kiyotaki-Wright [1989] exhibit a pure-strategy steady state $\left(p^{f}, s^{f}\right)$ in which the least costly to-store good has universal acceptance if the storage costs of goods, c_{ij}, satisfy either the inequalities $c_{i1} < c_{i2} < c_{i3}$ or the inequalities $c_{i1} < c_{i3} < c_{i2}$. The vector s^f is given by $s^{f} = \left(s_{12}^{3}, s_{13}^{2}, s_{23}^{1}, s_{21}^{3}, s_{31}^{2}, s_{32}^{1}\right) = (1,0,1,0,0,1)$ and p^f is given by

$$p^{f} = \begin{bmatrix} p_{11}^{f} & p_{12}^{f} & p_{13}^{f} \\ p_{21}^{f} & p_{22}^{f} & p_{23}^{f} \\ p_{31}^{f} & p_{32}^{f} & p_{33}^{f} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 0 & 2^{-\frac{1}{2}} & 1 - 2^{-\frac{1}{2}} \\ 2 - 2^{\frac{1}{2}} & 0 & 2^{\frac{1}{2}} - 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

The trading startegies of agents of type 1 are optimal if $\dfrac{c_{13} - c_{12}}{r\,\left[u_1 + c_{12}\right]} < \left(p_{31}^{f} - p_{21}^{f}\right) = \dfrac{1}{3}\left(2^{\frac{1}{2}} - 1\right)$.

Actually, the pair $\left(p^{f}, s^{f}\right)$ is the only steady state with universal acceptance of the east costly to-store good if the inequality above holds.

For an open set of parameters, Kiyotaki-Wright [1989] also exhibit a pure-strategy steady state $\left(p^s, s^s\right)$ in which the second least costly to-store good has universal acceptance if the storage costs of goods, c_{ij}, satisfy either the inequalities $c_{i1} < c_{i3} < c_{i2}$. (Remember that type-i agents produce good i+1 with modulus 3.) The vector of trading strategies s^s is given by $s^f = \left(s_{12}^3, s_{13}^2, s_{23}^1, s_{21}^3, s_{31}^2, s_{32}^1\right) = (1,0,0,1,1,0)$ and p^s is given by

$$p^s = \begin{bmatrix} p_{11}^s & p_{12}^s & p_{13}^s \\ p_{21}^s & p_{22}^s & p_{23}^s \\ p_{31}^s & p_{32}^s & p_{33}^s \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 & 2^{1/2}-1 & 2-2^{1/2} \\ 0 & 0 & 1 \\ 2^{-1/2} & 1-2^{-1/2} & 0 \end{bmatrix}.$$

I prove in Renero [1999] that there exists a steady state $\left(p^c, s^c\right)$ with mixed strategies which restrict agents to play a unique strategy for each opportunity set if all goods are equally to-store. The vector s^c is given by $s^c = \left(s_{12}^3, s_{13}^2, s_{23}^1, s_{21}^3, s_{31}^2, s_{32}^1\right) = \left(2/3, 1/3, 2/3, 1/3, 2/3, 1/3\right)$ and p^c is given by

$$p^c = \begin{bmatrix} p_{11}^c & p_{12}^c & p_{13}^c \\ p_{21}^c & p_{22}^c & p_{23}^c \\ p_{31}^c & p_{32}^c & p_{33}^c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 & 2/9 & 1/9 \\ 1/9 & 0 & 2/9 \\ 2/9 & 1/9 & 0 \end{bmatrix}.$$

Notice that $p^{AT} = p^c$. I also prove in Renero [1999] that for an open set of parameters and initial conditions there exists a two-period converging equilibrium close to $\left(p^c, s^c\right)$.

The next proposition establishes that for an open set of parameters and initial conditions, there exist multiple equilibria which are Pareto ranked. In particular, that there exist many, many two-period convergent equilibria in which agents play mixed trading strategies and in which the most costly to-store good has universal acceptance. Moreover, these equilibria Pareto superior to other equilibria with universal acceptance of less costly to-store goods or with mixed startegies that restrict agents to play a unique strategy for each opportunity set.

PROPOSITION 5: Assume N = 3 and either $c_{i1} < c_{i2} < c_{i3}$ or $c_{i1} < c_{i3} < c_{i2}$. There exists a neighborhood *P* of $p^{AT} = p^c$ and an open set *Y* of parameters $\left(\boldsymbol{r}, u_i, c_{ij}\right)$ (a set that includes equal storage costs), such that for any p(1) $\in$ P and any parameters in *Y*,

(i) the path $\left\{p(t+1), s^f\right\}$ from p(1) converges and it is an equilibrium and the only equilibrium in which $s_{ij}^1(t) = 1$, j $\neq$ i;

(ii) for $c_{i1} < c_{i3} < c_{i2}$, the path $\{p(t+1), s^f\}$ from p(1) converges and it is an equilibrium and the only equilibrium in which $s_{ij}^3(t) = 1$, j ≠ i, if $s_{ij}^k(t) + s_{ik}^j(t) = 1$;

(iii) there exists a two-period convergent equilibrium from p(1) in which agents play mixed trading strategies satisfying $s_{ij}^k(t) + s_{ik}^j(t) = 1$ and which Pareto superior to the equilibria in (i) and (ii);

(iv) there exists a continuum of equilibria from p(1) converging in two periods to the same steady state (out of a continuum) and in which agents play mixed trading strategies and in which the most costly to-store good is universally accepted; moreover, these equilibria Pareto superior to the equilibria in (i), (ii), and (iii).

Proof: Claims (i)-(iii) are Prop. 3.4 in Renero [1999]. The existence claim of (iv) follows by establishing first steady states (p^w, s^w) using a more general version of Prop. 4.4 (which indexes *w* of 2) by k and i) either for $s_{ij}^{w,3} = 1$ and $s_{13}^{w,2} = w_{13}$ if $c_{i1} < c_{i2} < c_{i3}$, or for $s_{ij}^2(t) = 1$ and $s_{12}^{w,3} = w_{12}$ if $c_{i1} < c_{i3} < c_{i2}$. Secondly, the two-period convergence follows by using any strategies s(1) which satisfy the equalities

$$s_{i,i+1}^{i+2}(1) = \frac{p_{i,i+2}^w - p_{i,i+2}(1)\left[1 - p_{i,+2,i}(1) - p_{i+2,i+1}(1)\, s_{i,i+2}^{i+1}(1) - p_{i+1,i}(1)\, s_{i+1,i}^{i+2}(1)\right]}{p_{i,i+1}(1)\; p_{i+1,i+2}(1)}$$

for any p(1) close enough to p^{AT} to have p^w = h[p(1),s(1)].

The claim of the Pareto ranking follows by transitivity, continuity, and for the discount factor ρ close enough to 1 since (1) all equilibria converge for p(1) in a neighborhood of $p^{AT} = p^c$ and (2) the components of the expected-utility vectors v^c associated to the steady state (p^c, s^c) are smaller than those of the expected-utility vector v^{AT} associated to (p^{AT}, s^{AT}) if for every type of agents all goods are equally costly to store. ∎

Notice that because the strategies s(1) are undetermined if agents play mixed strategies all along the path, we choose strategies s(1) such that the paths with mixed strategies converge in two periods; that is, such that $[p(t+1), s(t+1)] = (p, s)$ for $t \geq 1$. In fact, there exists a continuum of such strategies (as the equality given in the proof above shows) for unrestricted mixed strategies that make the equilibrium paths to converge to the same steady state. Notice also that there exists a continuum of steady states (indexed by $s_{13}^{w,2}$ if $c_{i1} < c_{i2} < c_{i3}$ or $s_{12}^{w,3}$ if $c_{i1} < c_{i3} < c_{i2}$) with this feature.

6. SOME CONCLUDING REMAKS

In order to make an interpretation of my analytical results, I recall next briefly some historical facts. Around the last quarter of the XIX century, gold was the appreciating standard and the silver the depreciating standard; say, gold was the best standard and silver was the worst standard. In fact, the gold-silver price ratio “skyrocketed”. According to Friedman [1990 b, p. 1169], from “15.4 in 1870, it jumped to 16.4 by 1873, 18.4 by 1879, and 30 by 1896”. A stylized explanation may be that many important countries, specially European countries and US, quit successively in and after 1873 the free coinage of silver and embrace sooner than later a gold standard. Consequently, as it was expected, the demand for monetary purposes increased for gold and decreased for silver.

It seems that after The International Monetary Conference held in Paris in 1867 the posture of “gold standard for all civilized nations” had an increasing acceptation around the world (see Casasús [1893, p. 83] and Russell [1898, p. 84]). However, the adoption of the gold standard has been at least controversial in US and, perhaps, in other countries as well. In fact, some countries kept in practice a silver standard for a while –Mexico, for instance, until 1905. Taking into account the possible changes in the demand of silver and gold, for countries like US and Mexico the options were apparently to have (1) a higher growth of the price level, even inflation, with a silver standard; or (2) a lower growth in the price level, even deflation, with a gold standard.

United Kingdom adopted a gold standard in 1819. US quit the free coinage of silver in 1873 and it resumed a specie standard on the basis of gold in 1879. According to Friedman [1990 b, p. 1170] the deflation from 1875 to 1896 for US was about 1.7 percent per year and for the United Kingdom 0.8 percent per year. To compare with the data available for Mexico, we can use the data provided by Friedman [1990 b, Table A1] to obtain the average annual rate of change in the price level for the periods 1877-1886 and 1886-1904. In this way we obtain -1.65% and 0.4%, respectively, for the US; and –1.22% and 0.32%, respectively, for the United Kingdom.

For Mexico, the only price index available for the years of 1877 and 1886 gives an annual average rate of 1.77.[6] The inflation might have been no bigger in average than 3.1% per year for the period 1886-1904.[7] Furthermore, Mexican protagonists seemed to have observed price stability, specially in consumption goods, at least for the 1890’s (see Romero 1898, p.600 including fn. 1) which seems to be consistent with modern computations of price indexes.[8]

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[6] See El Colegio de México, p. 172. (These data are reproduced by INEGI as Cuadro 19.2.)

[7] See Zabludowsky [1992, p.299] who, as far as I know, gives figures with the biggest credible increase of prices for the period by considering apparently prices of Mexican exports. The biggest annual inflation was 13% where periods of inflation alternate with years of deflation of up 11% per year. Taking the geometric average for five years, the change of the price level would be between –1% and 6% for the period 1886-1904. Solís [1975, 1991] gives index figures for the period 1887-1904 with an average growth of prices of 6.50 and 6.02% respectively. However, Solís have been criticized by Zabludowsky [1992, p. 292, fn. 6] for not giving his methodology and his figures have been qualified as doubtful.

[8] See, for instance, El Colegio de México, p. 156-7 (reproduced as Cuadro 19.3 in INEGI), p. 172 (reproduced as Cuadro 19.2 in INEGI), and Cuadro 19.7 of INEGI)

Therefore, it seems that these rates of change of the price level were not too different to zero at least for today's standards. That is, one may say that the choice was not between hyperinflation and deflation, much less hyperinflation against hyperdeflation; say, moderate inflation against moderate deflation. Even more, one may say that the choice was among near rates of deflation or among near rates of inflation for some years.

Hence, those close change rates of price levels may give me the excuse to use the welfare results exhibited in this paper, which are for nearly equal storage costs, to try to evaluate those policy options. Furthermore, it seems that silver depreciated not only relative to gold as the data for Mexico may suggest. In fact, silver depreciated also relative to the US an UK index baskets of goods for long periods of time.[9] That is, at least in average, silver was apparently the intrinsically worst good in terms of value appreciation. In the same sense, although less sharply, gold was apparently the intrinsically best good.

Notice that the silver standard implied the devaluation of the national currency relative to the currencies based on gold. This seems to have caused great concern among Mexicans. Moreover, silver was possibly less convenient than gold. In fact, according to Friedman [1990 a, p.97], Jevons placed "great emphasis on the inconvenience to wealthier countries of silver money because it weighs so much more than a quantity of gold of the same value." Perhaps, one could argue also that silver was possibly riskier or more volatile than gold in terms of value.[10]

What was the best choice? As far as I know, the only formal attempt to answer this question has been Friedman [1990 b] who simulates with an econometric model the consequences of keeping the free coinage of silver in US; that is, keeping a bimetallic standard. He finds that the US would have been in practice under a silver, not gold, standard. Moreover, he estimates that in US the deflation rate would have been half or less of the actual deflation rate and eventually there would have been a rise in the price level. Friedman concludes that the act of 1873 was "a mistake that had highly adverse consequences" (p. 1177). That is, keeping the free coinage of silver, US would have avoided political agitation and deflation together with economic problems: Economic contractions, "widespread bank failures plus a banking panic in 1893, and a run on U.S. gold reserves by foreigners fearful that silver agitation would force the United States off the gold standard." (Friedman b, p. 1176)

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[9] Using the data given by Friedman [1990 b, Table A1] and "changing of numeraire," one can compute the index of prices in terms of silver for the UK and US. Hence, I have elaborated the graph which appears at the end of the Appendix taking the geometric average change of the corresponding index for periods of five years. One can observe there that the change of the UK index of prices in terms of silver was positive up to 10% between 1877 and 1905. Moreover, this change was bigger than the change of the index of prices in terms of gold for the period 1865-1905. As regards US, the change of index of prices in terms of silver was positive up to 8% between 1980 and 1905. Definitely, this change was bigger than the change of the index of prices in terms of gold for the period 1880-1905. Notice that the US index of prices in terms of gold is given directly by the data because US resumed gold convertibility in 1879.

[10] Using the data given by Friedman [1990 b, Table A1], one can compute the standard deviation of the geometric average of the index change from 1 to five years. In this way, we can observe that for the UK, the standard deviations was bigger for the prices in terms of silver than for the prices in terms of gold in the period 1866-1905, and even for the subperiod 1866-1889. For the US, that standard deviations were also bigger for the period 1880-1905.

To try to determine the best choice, I make next an interpretation of my results. The equilibrium in which the most costly to-store good is universally accepted is analog to an economy with the worst commodity standard; say the silver standard. Similarly, the **alternative** equilibrium in which the least costly to-store good is universally accepted may be analog to **the same** economy with the best standard; say the gold standard. That is, the **alternative** equilibria may be thought as the options for an economy of feasible commodity standards. Therefore, the Pareto ranking displayed in this paper of those equilibria may suggest that the best choice was the silver standard; that is, that the US Act of 1873 was a mistake, or a "crime", as asserted by Friedman.

Moreover, that Pareto ranking may give some original support to the next statements by Matías Romero who was alternatively at least three times Mexican Secretary of the Treasury and Ambassador to US in the last decades of the XIX century:

> …"*Everybody in Mexico, that is, from the educated to the ignorant, from the rich to the poor, from the natives to the foreigners, and even the bankers who in other countries are decidedly favorable to the gold standard, are all in favor of silver. The Government holds the same opinion. As Mexico is now prosperous a large portion of the people attribute its prosperity to the silver standard and are therefore decidedly favorable to the continuance of that standard*". Romero [1898, p. 576-7]

However, the main argument of Romero and other protagonists about such prosperity was that the depreciation of silver made the Mexican products more competitive relative to foreign goods. Needless to say that the Kiyotaki-Wright model is a "closed economy." The other arguments were that Mexico attracted more foreign investment and that money supply became higher because the specie, silver, was exported less. In contrast, according to Limantour [1904a, p. 427], Mexican Secretary of the Treasury in 1892-1911, the occurrence of silver depreciation and prosperity was mostly a coincidence. (For a detailed survey of the protagonists' points of view about keeping in Mexico the free coinage of silver see Hernández-Renero [1998].)

APPENDIX

Proof of Lemma 3.1: If always-trade strategies are played by agents, the proportion of agents of type i who consume, $a_{ii}^{AT}(t)$, is given by

$$a_{ii}^{AT}(t) = \sum_k p_{ki}(t) \sum_{l \neq i} p_{il}(t) = \frac{1}{N} \sum_k p_{ki}(t) \ .$$

Consequently, adding over i,

$$\sum_i a_{ii}^{AT}(t) = \frac{1}{N} \sum_i \sum_k p_{ki}(t) = \frac{1}{N}$$

To see that $1/N$ is an upper bound for aggregate consumption implied for *any* set of trading strategies, notice that

$$\sum_i a_{ii}(t) = \sum_i \sum_k \sum_{l\neq i} p_{il}(t)\; p_{ki}(t)\, s^i_{il}(t)\; s^l_{ki}(t) \leq \sum_i \sum_k \sum_{l\neq i} p_{il}(t)\; p_{ki}(t)$$
$$= \sum_i \sum_k p_{ki}(t) \sum_{l\neq i} p_{il}(t) = \frac{1}{N}$$

for all pairs $[p(t), s(t)]$ of vectors of distribution of inventories and of trading strategies. ∎

Proof of Lemma 3.2: Notice first that eqs. (1.1) which define $h(\cdot\,, s^{AT})$ can be written as

(*) $$a_{ij}(t) = p_{ij}(t)\left[1 - \sum_k \sum_{l\neq j} p_{kl}(t)\right] + \sum_k p_{kj}(t) \sum_{l\neq j} p_{il}(t) = \frac{1}{N}\sum_k p_{kj}(t)\,.$$

Consequently, according to eqs. (1.2),

(**) $$\sum_i p_{ij}(t+1) = \left(1 - \frac{1}{N}\right)\sum_i p_{ij}(t) + \left(\frac{1}{N}\right)\sum_i p_{i,j-1}(t)\ .$$

We deal next with the aggregated proportions of agents who hold a particular good, $q_j(t) \equiv \sum_i p_{ij}(t)$, and their law of motion given by equations (**). Let

$$B \equiv \left\{ q \in \Re^N_+ : \sum\nolimits_j q_j = 1 \quad and \quad q_j \leq \left(1 - \frac{1}{N}\right)\right\}$$

I will prove next that the function $Q : B \rightarrow B$ defined by

$$Q_j(q) = \left(1 - \frac{1}{N}\right) q_j + \left(\frac{1}{N}\right) q_{j-1}$$

is a (N-1)-stage contraction with the sup metric of modulus $1 - \left(\frac{1}{N}\right)^{N-1}$. Let $Q^1 \equiv Q$ and $Q^{n+1} \equiv Q \circ Q^n$. Going backwards, notice that

(***) $$Q_j^{N-1}(q) \equiv \sum\nolimits_{k=0}^{N-1} \begin{bmatrix} N-1 \\ k \end{bmatrix} \left(1 - \frac{1}{N}\right)^{(N-1-k)} \left(\frac{1}{N}\right)^k q_{j-k}$$

where $\begin{pmatrix} n \\ k \end{pmatrix} = \frac{n!}{k!(n-k)!}$. Since $\sum_{k=0}^{N-1} q_{j-k} = 1$ and

$$\sum_{k=0}^{N-1} \begin{bmatrix} N-1 \\ k \end{bmatrix} \left(1 - \frac{1}{N}\right)^{(N-1-k)} \left(\frac{1}{N}\right)^{k} = \left(1 - \frac{1}{N} + \frac{1}{N}\right)^{N-1} = 1,$$

the equations (***) can be simplified to the form

$$Q_j^{N-1}(q) \equiv \left(\frac{1}{N}\right)^{N-1} + \sum_{k=0}^{N-2} b_k \; q_{j-k}$$

where the coefficients b_k are positive and $\sum_{k=0}^{N-2} b_k = 1 - \left(\frac{1}{N}\right)^{N-1}$. Therefore, Q^{N-1} is a contraction of modulus $1 - \left(\frac{1}{N}\right)^{N-1}$. Given this result, we can prove the lemma.

Let $H \equiv h\left(\cdot, s^{AT}\right)$, $H^1 \equiv H$ and $H^{n+1} \equiv H \circ H^n$. Take any pair (p, p') of distributions of inventories and let the pair (q, q') be given by $q_j \equiv \sum_i p_{ij}$ and $q'_j \equiv \sum_i p'_{ij}$. Then, according to eqs. (1.1), (1.2), (*) and using the function Q^n,

$$H_{ij}^{n+1}(p) = \frac{1}{N} Q_j^n(q) \text{ if } j \neq i+1,$$

$$H_{i,i+1}^{n+1}(p) = \frac{1}{N} Q_{i+1}^n(q) + \frac{1}{N} Q_i^n(q)$$

and similarly for p' and q'. Notice that

$$\max_i \left\{ \left| q_i - q'_i \right| \right\} = \max_i \left\{ \left| \sum_k p_{ki} - p'_{ki} \right| \right\} \leq (N-1)\, d(p, p') \; .$$

Then, for $t \in \aleph$, if $j \neq i+1$,

$$\left|H_{ij}^{t(N-1)+1}(p)-H_{ij}^{t(N-1)+1}(p')\right|=\frac{1}{N}\left|Q_j^{t(N-1)}(q)-Q_j^{t(N-1)}(q')\right|\leq\frac{1}{N}\max_i\left\{\left|Q_i^{t(N-1)}(q)-Q_i^{t(N-1)}(q')\right|\right\}$$

$$\leq\frac{1}{N}\max_i\left\{\left|Q_i^{t(N-1)}(q)-Q_i^{t(N-1)}(q')\right|\right\}$$

$$\leq\frac{1}{N}\left(1-\left[\frac{1}{N}\right]^{N-1}\right)^t\max_i\left\{\left|q_i-q_i'\right|\right\}$$

$$\leq\frac{N-1}{N}\left(1-\left[\frac{1}{N}\right]^{N-1}\right)^t d\left(p,p'\right)$$

and similarly

$$\left|H_{i,i+1}^{t(N-1)+1}(p)-H_{i,i+1}^{t(N-1)+1}(p')\right|=\frac{1}{N}\left|Q_{i+1}^{t(N-1)}(q)-Q_{i+1}^{t(N-1)}(q')+Q_i^{t(N-1)}(q)-Q_i^{t(N-1)}(q')\right|$$

$$\leq 2\,\frac{N-1}{N}\left(1-\left[\frac{1}{N}\right]^{N-1}\right)^t d\left(p,p'\right).$$

Thus, for   large enough, $H^{t(N-1)+1}$ is a contraction. ∎

Proof of Lemma 4.1: If the sequence $\{v(t)\}$ satisfy eqs. (2.1)-(2.2) and $v_{ij}(t)=v_{in}(t)$, j ≠ i, n ≠ i, for all t, then

(*) $$v_{ij}(t)=-c_{ij}+\boldsymbol{r}\left[u_i+c_{i,i+1}\right]\left(\sum_k p_{ki}(t+1)s_{ki}^{j}(t+1)\right)+\boldsymbol{r}\,v_{ij}(t+1)$$

(**) $$v_{ii}(t)=u_i+c_{i,i+1}+v_{i,i+1}(t)$$

Subtracting eqs. (*) wich each other we get eqs. (4.1). This proves the necessity.

To prove the sufficiency, let equations (4.1) hold. We find next a sequence $\{\tilde{v}(t)\}$ as a candidate to satisfy eqs. (2.1)-(2.2). By forward substitution in eqs. (*) and using eqs. (**), we get the numbers $v_{ij}(t)$ as infinite sums of expressions involving only parameters and elements of the sequence $\{p(t+1),s(t+1)\}$. So, the sequence of numbers $v_{ij}(t)$ given by these infinite sums is our candidate $\{\tilde{v}(t)\}$. Accordingly, we denote those numbers by $\tilde{v}_{ij}(t)$. Using eqs. (4.1), notice that the numbers $\tilde{v}_{ij}(t)$ satisfy the equalities $\tilde{v}_{ij}(t)=\tilde{v}_{in}(t)$, j ¹ i, n ¹ i, for all t. Hence, the sequence $\{\tilde{v}(t)\}$ satisfy eqs. (2.1)-(2.2). Therefore, the sufficiency follows since there is one and only one bounded sequence $\{v(t)\}$ which satisfy eqs. (2.1)-(2.2). ∎

The following proofs use the following notation:

$$l_i(j) \equiv \frac{c_{i,e[i]} - c_{ij}}{r\left[u_i + c_{i,i+1}\right]},$$

$$l \equiv \left[l_i(j)\right],$$

$$\Lambda \equiv \left\{ l_i(j) \in \Re^{N(N)} : l_i(j) \geq 0, i \neq j; \textit{ and for each } i, \textit{there exists an } e[i] \textit{ such that } l_i(e[i]) = 0 \right\},$$

$$A \equiv \left\{ p_{ij} \in \Re_+^{N(N)} : \sum_k p_{ik} = \frac{1}{N} \textit{ and } p_{ii} = 0 \right\},$$

the correspondence $\Gamma : A \times [0,1] \times \Lambda \to \prod_{k=1}^{N(N)(N)} [0,1]$ is defined by

$$\Gamma(p; w, l) \equiv \Big\{ s \in \prod_{k=1}^{N(N)(N)} [0,1] : s_{ij}^{i} = 1, s_{ki}^{e[i]} = w \textit{ if } k \neq e[i],$$
$$\sum_{k \neq j} p_{ki} s_{ki}^{j} + p_{ji} = \sum_{k \neq e[i]} p_{ki} w + p_{e[i],i} - l_i(j) \textit{ if } \sum_{k \neq e[i]} p_{ki} w + p_{e[i],i} - l_i(j) > 0,$$
$$s_{ki}^{j} = 0 \textit{ if } j \neq k, j \neq e[i], \textit{and } \sum_{k \neq e[i]} p_{ki} w + p_{e[i],i} - l_i(j) \leq 0 \Big\}$$

Proof of Lemma 4.2: Notice that the correspondence   is upper hemicontinuous since it is closed and its counterdomain is compact. Hence, part (2) of the Lemma follows for $s(t) = \Gamma(p(t); w, l)$ since $s^{AT} = \Gamma(p^{AT};1,0)$. Notice that part (3) follows by construction in the definition of . Now, suppose $s(t+1) = \Gamma(p(t+1); w, l) \in \prod_{k=1}^{N(N)(N)} (0,1]$; that is, strategies *s*(*t*+1) given by   strictly positive. Let $\{v(t)\}$ be the sequence that satisfies eqs. (2.1)-(2.2) given $\{s(t+1), p(t+1)\}$. Hence, by the definition of   and Lemma 4.1, $v_{ij}(t) = v_{in}(t)$, *j*   *i, n*   *i,* for all t. Consequently, eqs. (3.1)-(3.3) hold.

To guarantee that eqs. (3.4)-(3.5) hold, we use the fact that the sequence $V \equiv \{v(t)\}$ that satisfies eqs. (2.1)-(2.2) given $\{s(t+1), p(t+1)\}$ is a “continuous” function of $P \equiv \{p(t+1)\}$, $S \equiv \{s(t+1)\}$, and the parameters $C \equiv (r, u_i, c_{ij})$. Since this may not be obvious, I give next an outline of the proof. For our purposes, the claim has to be about the continuity of that function, say , relative to the box topology in the domain and counterdomain. Since the RHS of eqs. (2.1)-(2.2) are continuous functions of p(t+1), s(t+1), v(t+1), and the parameters (r, u_i, c_{ij}), it follows that the graph of , $\{(V, P, S, C) : V \in f(P, S, C)\}$ is closed relative to the metrizable product topology: the limit of a sequence on the graph of   is in the graph of . Hence,   is continuous relative to the product topology in the domain and counterdomain because its image is contained in a compact set in the product topology. Then, considering this topology, for any open set U there exists an open set W such that v(1) is in U if $\left[\{s(t+1), p(t+1)\}, \left(\boldsymbol{r}, u_i, c_{ij}\right)\right]$ is in W. Hence, v( ) is also in U if $\left[\{s(t+\boldsymbol{t}), p(t+\boldsymbol{t})\}, \left(\boldsymbol{r}, u_i, c_{ij}\right)\right]$ is in W and the claim of box-topology continuity follows.

We are now in conditions of finishing the proof of Lemma 4.2. Let $\{v^{AT}\}$ be the sequence which satisfies eqs. (2.1)-(2.2) given the constant sequence $\{p^{AT}, s^{AT}\} = \{p^{AT}, \Gamma(p^{AT};1,0)\}$. Notice that the sequence $\{v^{AT}\}$ satisfy eqs. (3.4)-(3.5) with strict inequalities if $c_{ij} < \boldsymbol{r}\left[u_i + c_{i,i+1}\right]\ (1/N)$. Therefore, the claim follows by the continuity of the sequence $\{v(t)\}$ on $\{s(t+1), p(t+1)\}$ and the parameters $\left(\boldsymbol{r}, u_i, c_{ij}\right)$, the hemicontinuity of , and the continuity of on the parameters $\left(\boldsymbol{r}, u_i, c_{ij}\right)$. ▮

Proof of Lemma 4.3: Take $\boldsymbol{e} > 0$ and let $M \equiv \{p_{ij} \in A : d(p^{AT}, p) < \boldsymbol{e}\}$. Take $y \in (0,2)$ close enough to 2 such that the compact set

$$C \equiv \left\{ p_{ij} \in \Re_{+}^{N(N)} : \sum_k p_{ik} = \frac{1}{N} \ and\ p_{ii} = 0,\ p_{ij} \geq \left(\frac{y}{2}\right)\left(\frac{1}{N^2}\right) if\ \ j \neq i\ and\ j \neq i+1,\ p_{i,i+1} \geq \left(\frac{y}{N^2}\right) \right\}$$

is contained strictly in *M*.

For, $0 \leq \boldsymbol{d} \leq 1$, let $\Lambda(\boldsymbol{d}) \equiv \left\{ s \in \prod_{k=1}^{N(N)(N)} [0,1] : s_{ij}^{i} = 1, \boldsymbol{d} \leq s_{ij}^{k}\ if\ \ k \neq i \right\}$ and

$$\Psi(\boldsymbol{d}) \equiv \{p' \in A : p' = h[p.s]\ such\ that\ \ p \in C\ and\ s \in \Delta(\boldsymbol{d})\}\ .$$

Let $\Psi^1 \equiv \Psi$ and $\Psi^{n+1} \equiv \Psi \circ \Psi^n$. Notice that Ψ^n is an upper hemicontinuous correspondence. Also notice that $\Psi^n(1) \subset C$ since $\Delta(1) = \{s^{AT}\}$ and equations (1.1) become $a_{ij}(t) = \frac{1}{N}\sum_k p_{kj}(t)$ for $s(t) = s^{AT}$. According to Lemma 3.2, there exists such that $\Psi^t(1)$ is contained in the interior of C C . Hence, there exist a sequence $\{\boldsymbol{d}_n\}_{n=1}^{t}$ such that $\boldsymbol{d}_n < 1$, $\Psi^n(\boldsymbol{d}_n) \subset M$, and $\Psi^t(\boldsymbol{d}_t) \subset C^{\circ}$. Thus, if $\boldsymbol{d} = \max\{\boldsymbol{d}_n\}_{n=1}^{t}$, $s(t) \in \Lambda(\boldsymbol{d})$, and $p(1) \in C$, the path $\{p(t+1)\}$ given by $p(t+1) = h(p(t), s(t))$ is in *M* and, consequently, the claim follows. ▮

Proof of Proposition 4.4: I will prove next the existence of the steady state. Fix (*w*,). Let $T \equiv \prod_{k=1}^{N(N)(N)} [0,1]$. Define the correspondence $g : A \rightarrow T$ by $g(p) = \Gamma(p; w, \boldsymbol{l})$. Notice that g is nonempty, upper hemicontinuous, and convex valued. Define also the correspondence $f : A \times T \rightarrow A \times T$ by $f(p,s) = [h(p,s), g(p)]$. Notice that $A \times T$ is a non-empty, compact, and convex space and f is a non-empty, upper hemicontinuous, and convex valued correspondence. Hence, according to Kakutani's fixed point theorem, the correspondence f has a fixed point; that is, there exists $[\mathbf{p}, \mathbf{s}](w, \mathbf{l}) \in A \times T$ such that $[\mathbf{p}, \mathbf{s}](w, \mathbf{l}) \in f[[\mathbf{p}, \mathbf{s}](w, \mathbf{l})]$.

Define now the correpondence y with argument $(w, \mathbf{l})$ by the set of fixed points $[\mathbf{p}, \mathbf{s}](w, \mathbf{l})$ of f. Notice that y is upper hemicontinuous since it is closed and its counterdomain compact. Notice also that, according to Lemma 3.2, $[\mathbf{p}, \mathbf{s}](1,0)$ is the unique fixed point of f since $\mathbf{s}(1,0)$ is the vector of always-trade strategies. Then, the steady-state claim follows by continuity and Lemma 4.2 taking appropriate intersections.

Now, I will prove the last part of the proposition. The idea of the proof is to use continuity because any path with always-trade converges according to Lemma 3.2 and the law of motion $h(p, s^{AT})$ is linear in p (see eqs. (*) of the proof of Lemma 3.2). This means that, eliminating appropriately variables, as we will see, the eigenvalues of the matrix associated are less than one in absolute value. Hence, I can prove asymptotic stability for some steady states.

The role of the restriction for the steady state (p,s) to satisfy $s_{ki}^{j} = s_{li}^{j}$, j ≠ k, j ≠ l, is that for a neighborhood M of p^{AT} and a neighborhood U of $(w, \mathbf{l}) = (1,0)$, the "restricted ," say $g : M \times U \rightarrow \prod_{k=1}^{N(N)(N)} [0,1]$, is a continuously differentiable function since it is defined by

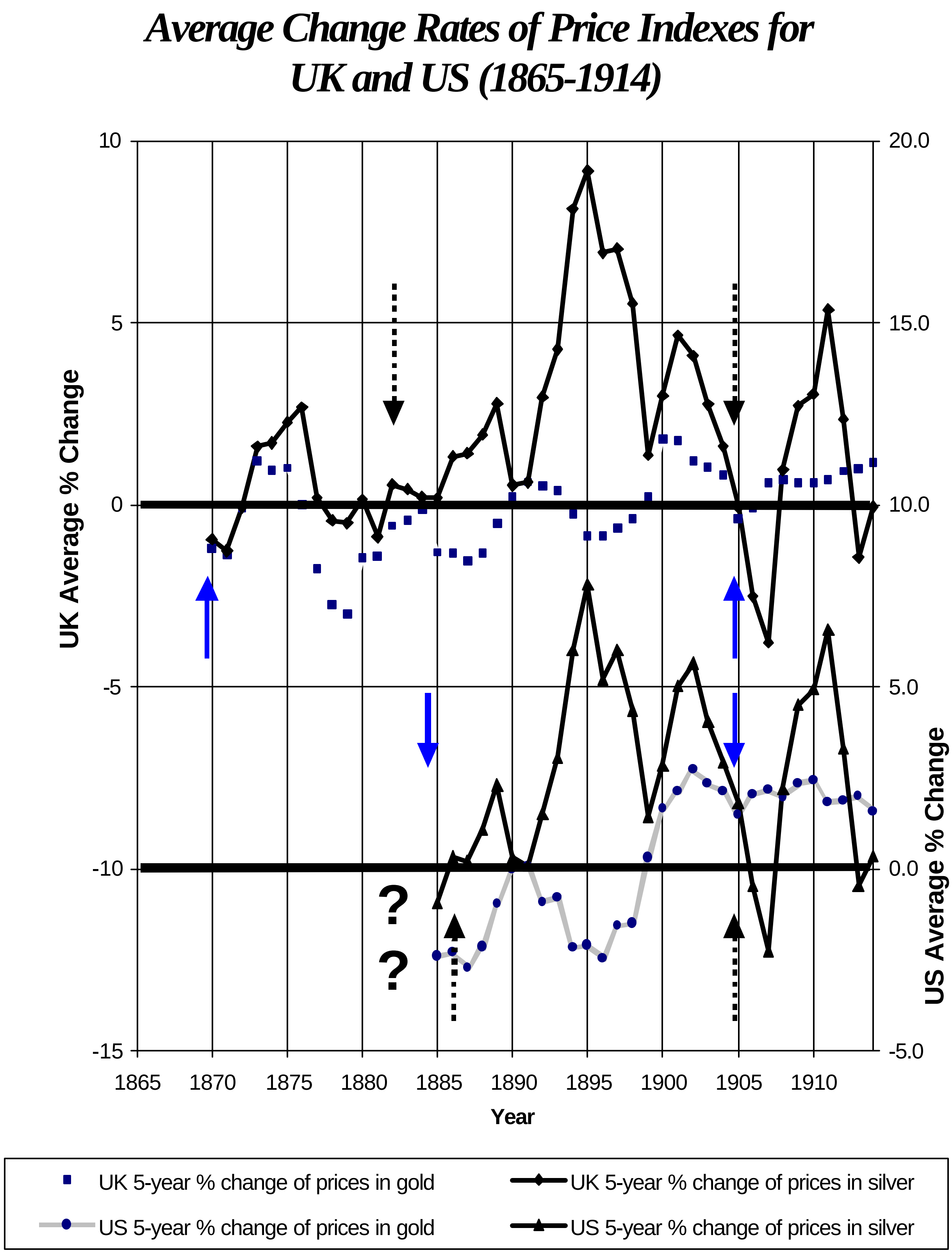
$$g_{ij}^{i}(p; w, \mathbf{l}) = 1,$$
$$g_{ki}^{e[i]}(p; w, \mathbf{l}) = w \quad if \;\; k \neq e[i],$$
$$g_{ki}^{j}(p; w, \mathbf{l}) = \frac{\sum_{k \neq e[i]} p_{ki} w + p_{e[i],i} - \mathbf{l}_i(j) - p_{ji}}{\sum_{k \neq j} p_{ki}} \quad if \;\; k \neq j \, .$$

I will prove asymptotic stability for this kind of restricted steady states by showing that there exists a neighborhood X of $(w, \mathbf{l}) = (1,0)$, such that for any $(w, \mathbf{l}) \in X$, a function which describe the law of motion of inventory distributions, $h(\cdot, g[\cdot, w, \mathbf{l}])$ is continuously differentiable in a neighborhood of the steady state $\mathbf{p}(w, \mathbf{l})$ and its Jacobian evaluated at $\mathbf{p}(w, \mathbf{l})$ has eigenvalues less than one in absolute value.

The difficulty here is that we cannot use directly $h(\cdot, g[\cdot, w, \mathbf{l}])$. To see the reason let G be the matrix associated to the linear function $h(\cdot, g[\cdot, 1, 0]) = h(\cdot, s^{AT})$. Because this function has a unique fixed point different to the null vector, the matrix (I - G) is singular. Another way to see the difficulty is noticing that G has an eigenvalue equal to one because the components of any column of G add to 1.

Using the identities $p_{i,i+1} = \frac{1}{N} - \sum_{j \neq i+1} p_{ij}$ to eliminate $p_{i,i+1}$ from the law of motion $h\left(\cdot , g[\cdot , w, \, l] \right)$, we get the continuously differentiable function f with argument p given $(w, \, l)$; say with argument $(p; w, \, l)$. Since $f\left(p;1,0\right)$ is linear in p, let F the matrix associated. Notice that $f\left(p;1,0\right) = F \cdot p + b$ for some non-null vector b of constants. Since $f\left(p;1,0\right)$ has a unique fixed point, (I - F) is not singular. Moreover, since any path with always-trade strategies converges, all the eigenvalues of F are less than one in absolute value.

Since the correspondence defined by the set of fixed points $\mathbf{p}(w, \, l)$ is upper hemicontinuous, the Jacobian of f as a function of $(w, \, l)$ is upper continuous. Finally, notice that the eigenvalues of a matrix are continuous on the elements of that matrix since the “zeros” of a polynomial are continuous, taking into account multiplicities, on the coefficients of the polynomial. Thus, there exists a neighborhood X of $(w, \, l) = (1,0)$, such that there exists a steady state $\mathbf{p}(w, \, l)$, $f\left(\cdot ; w, \, l \right)$ is continuously differentiable in a neighborhood of $\mathbf{p}(w, \, l)$, and the eigenvalues of the Jacobian of $f\left(\cdot ; w, \, l \right)$ evaluated at $\mathbf{p}(w, \, l)$ are less than one in absolute value if $(w, \, l) \in X$. ∎



Elaborated with data of Friedman [1990 b, Table A1]

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