Matching Between Heterogeneous Workers and Firms[*]

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Abstract

A model of the labor market under search frictions is developed, where participants are heterogeneous with respect to their productivity types and the individual decision of which type of agents to match with is endogenized. Wages are negotiated, so that all gains from trade are exploited. This has important implications for the equilibrium outcomes. In particular, two applications are studied. It is observed that countries with high (low) unemployment tend to exhibit low (high) wage dispersion. And there is evidence showing that individual and firm characteristics have more explanatory power for the French than for the American data. The model is able to replicate these two observations, underscoring the relevance of considering matching patterns between heterogeneous agents in the different economies. Since the model does not feature a minimum wage, I thus provide a theory of endogenous wage compression.

JEL Classification Codes: E24, J31, J41, J64

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1 Motivation

The paper looks at how agents who di¤er with respect to their productivities, decide to match in an economy characterized by search frictions. When this is the case, a central question is: what matching patterns are sustainable in equilibrium? In other terms, who matches with whom? The question is asked in the context of a labor market, where workers are characterized by di¤erent skill levels and …rms by di¤erent technologies used. Interestingly, using this framework, it is possible to replicate two empirical regularities observed in the U.S. and European labor markets. First, it is well documented in Bertola and Ichino (1995), Abraham and Houseman (1995), Katz, Loveman and Blanch‡ower (1995), that the former is characterized by low unemployment but high wage dispersion, and the latter by high unemployment but low wage dispersion. Bertola and Ichino even point out the fact that within-type wage dispersion (i.e. after controlling for education and experience) is also higher in the U.S. than in Europe. This can be explained in this model by only relying on di¤erent matching behaviors among heterogeneous agents. It is shown that the characteristics of the European and American labor markets, as shaped by their respective labor market policies, as well as the actually observed matching patterns, are consistent with equilibria generating the corresponding observations on wage dispersion and unemployment. Second, again contrasting U.S. and European labor markets, Abowd, Kramarz, Margolis and Troske (2000) …nd that, accounting for observable and unobservable heterogeneity, individual characteristics plus establishment e¤ects explain about 20% more of the annual variation in annual wages for the French sample, as opposed to the American one. This is an observation, that can also be explained in the context of this model. Hence, the analysis of matching patterns across countries, even though relatively ignored, may be very relevant to the study of labor markets.

Matching models can be divided into two categories, depending on how the match payo¤s are determined. The …rst one is comprised of models where non-transferable utility is assumed. In these, individuals take the characteristics of the counterpart they consider as a potential partner as given, which determines the utility they derive from the match. There is no possibility for any one partner to induce the other one to accept the match by transferring some of the utility they get from the match to the other partner. Hence, a match will take place if and only if both individuals derive su¢cient utility from it, given these …xed payo¤s. This typically results in the creation of "classes", where individuals only match with partners of similar characteristics, thereby prohibiting individuals with very di¤erent characteristics from forming a pair[1]. The standard references are Burdett and Coles (1997, 1999) who, without loss of generality, apply the non-transferability assumption to the marriage market. The second category is comprised of models where transferable utility is assumed: a meeting between two agents creates a local surplus, whose division between the two partners is bargained over. Therefore, as long as there are gains from trade, the possibility to negotiate the division of the surplus ensures that a partner can always induce the other one to accept the match, while also retaining a positive surplus for herself. Hence, a match will take place if and only if the combined match surplus is positive. The labor market is the prototypical application of transferable utility, since …rms and workers can negotiate wages to split output. The paper focuses on the transferable utility case, with heterogeneous agents. We will see that this results in matching patterns that may be very di¤erent from the ones observed with non-transferable utility.

The few related papers addressing the issue of matching between heterogeneous agents with transferable utility are Burdett and Coles (1999), Sattinger (1995), and Shimer and Smith (2000). Burdett and Coles look at all the basic ingredients required for a general theory of partnership formation under several di¤erent settings: transferable utility, non-transferable utility and match speci…c heterogeneity. Sattinger looks more speci…cally at the case of transferable utility and focuses on how ex-ante di¤erences in worker quality may generate sorting externalities, as the workers' matching patterns a¤ect the composition of the pool of unemployed workers in equilibrium and therefore other workers' decisions. Shimer and Smith de…ne a search equilibrium in the case where there is a continuum of types and …nd su¢ cient conditions for existence of equilibrium and for the agents' matching sets to be convex. The present model uses a framework similar to Shimer and Smith. A matching equilibrium is de…ned and its characteristics presented. By outlining the consequences of assuming transferable utility, the model is able to account for the two observations mentioned at the beginning. Thus, this paper is also an attempt to show that taking matching patterns between heterogeneous agents into account, is important in explaining certain labor market outcomes.

The paper is organized as follows. A matching equilibrium is de…ned in section 2. The general characteristics of such an equilibrium are presented in section 3, emphasizing the consequences of assuming transferable

[1] At least when the individual payo¤s are only a function of, and linear in the partner's charm.

utility. As an illustration, the case of two productivity types is developed as well. Two applications are studied in section 4. First, comparing U.S. and European labor markets, countries with high (low) wage dispersion experience low (high) unemployment. The model has the property that equilibria where agents match with a larger set of productivity types tend to result in a higher wage dispersion and lower unemployment than equilibria where agents match with a smaller set of types. Hence, high wage dispersion can be associated with low unemployment and low wage dispersion with high unemployment. A simulation and some empirical evidence, consistent with the labor market policies in place, are provided to support the notion that Europe may be exhibiting the kind of matching patterns that would result in higher unemployment and lower wage dispersion than in the U.S. I thus provide a theory of endogenous wage compression. Second, I give evidence on French and U.S. wage data, indicating that the combination of individual characteristics and establishment e¤ects have more explanatory power for the French than for the American data. Again, the model can explain this observation, when taking into account the matching patterns between heterogeneous agents. These two applications emphasize the importance of incorporating the matching behavior of participants in the labor market, when studying these markets across countries. Finally, section 5 concludes and presents possible future extensions.

2 Matching equilibrium

2.1 Assumptions

The economy is composed of (i) a pool of searching agents looking for a partner to match with, and (ii) a pool of matched agents who are producing and splitting the output of the match. Exogenous breakdowns in the matched pool are the source of new entrants into the searching pool. It implies that both the ‡ows in and out of the searching pool are endogenous. In addition, it is assumed that the utility is fully transferable between agents in a match and that the wage is determined through bargaining. This simple set up is designed to closely replicate the workings of a labor market. It is also expendable to situations very di¤erent from one. The main characteristics are that heterogeneous agents are looking for partners to form a long-term relationship, where some output is to be produced and shared (no output can be produced by a single agent). In addition, the matches may be stochastically broken, in which case the search process has to resume.

Because of search frictions, finding a partner to engage in production with, is a time consuming process and agents get to meet each other only randomly, according to a Poisson process. Consider that there are n productivity types, p_i, $i \in \{1,...,n\}$ and that there is a constant total number of agents of each type in the entire economy. There is no uncertainty about the type of the agents met. These agents are referred to as partners (no firm or worker). This is purely for simplicity, since the nature of the relationship between two partners is the same as between a worker and firm. There is no further search once the match is formed. Total symmetry is needed between partners (workers or firms). As a result, it is assumed that both unemployment benefits and vacancy posting costs are equal to zero[2]. Each type has a bargaining power $\mu = \frac{1}{2}$ (in the Nash bargaining solution). The output from a match is determined by a strictly positive, increasing and symmetric production function. The production function will be assumed to be additive, so that all firms can be considered as just a worker-job pair.

Denote by f_{ij} the output produced in a match between type p_i and type p_j. Hence:

$$\forall i; j;\ f_{ij} = f_{ji}$$

$$\forall (i; j; k) \in \{1,...,n\};\ j > k \Rightarrow f_{ij} > f_{ik}$$

2.2 Matching between heterogeneous agents

Denote by U_i the discounted lifetime expected value of search for an unmatched partner of type p_i, and by M_{ij} the discounted lifetime expected value of a match to a type p_i partner, when matched with a type

[2] This is not a totally innocuous assumption, though. If partners were receiving income during search, that might preclude some equilibria, since search income would affect partners' search values. It follows that the output of certain matches might not be large enough to compensate both parties in these matches.

p_j partner. When considering whether to match, the searching partner's decision is a combination of several factors. It depends on how frequent the matching opportunities are, and how long the matches are expected to last. These are represented by λ, the meeting rate and by δ, the rate at which productive matches break down. The searching partner also needs to take into account the distribution of types of the other partners looking for a match. Denote by α_i the proportion of type p_i's in the searching pool, and by N_i the number of that same type in the pool. Hence:

$$\alpha_i = N_i / \sum_{j=1} N_j$$

Similarly, call γ_i the ratio of type p_i's in the entire economy and by L_i their respective number in the economy or type p_i labor force. Hence:

$$\gamma_i = L_i / \sum_{j=1} L_j$$

The partner searching for an opportunity to produce also has to take into consideration the wages offered in the market. These are given by c_{ij}, the compensation to type p_i when matched with type p_j. Finally, the partner must have expectations regarding the matching behavior of others. Denote by Π_{ij} the probability that a representative agent of type p_i is willing to match with type p_j. Anticipating rational expectations in the model, this corresponds to type p_j's beliefs about type p_i's willingness to match with her. With all these considerations in mind, a partner of type p_i has to choose a probability π_{ij} of accepting to match, upon meeting type p_j. The probability π_{ij} is the only decision variable for the partner. Notice that Π_{ij} defines how a representative agent in the market behaves, while π_{ij} is the corresponding individual value.

Maximizing behavior by the partners implies that the value of search, in flow terms, is given by (in steady state):

$$\forall i \in \{1 \ldots n\};\ rU_i = \lambda \sum_{k=1} \alpha_k \Pi_{ki} \underset{\pi_{ik} \in [0;1]}{Max} \{\pi_{ik} (M_{ik} - U_i)\} \tag{A}$$

When calculating her discounted expected value of search, type p_i considers the probability of a meeting (at rate λ per period of time). In case of an encounter, there is a probability α_k that the partner met is of type p_k. Type p_i believes there is probability Π_{ki} that type p_k is willing to match with her, in which case, she then has to decide whether to accept the match or continue search. She accepts to match if her surplus from the match is positive, randomizes if indifferent and rejects it otherwise (if the partner met is not willing to match, type p_i continues to search). Equation (A) accounts for the fact that type p_i may encounter any one of the n types.

When matched with type p_j, type p_i receives instantaneous compensation c_{ij}. Matches break down at a rate δ per unit of time. Hence:

$$\forall (i;j) \in \{1...n\};\ rM_{ij} = c_{ij} + \delta(U_i - M_{ij}) \tag{B}$$

Output is divided between partners, so that:

$$\forall (i;j) \in \{1...n\};\ c_{ij} + c_{ji} = f_{ij} \tag{C}$$

The wage negotiated is derived from the Nash bargaining solution (Nash 1950), with disagreement points equal to the value of search for the respective partners. Hence, partners split the surplus from matching, where the surplus is defined as the value of a match less the value of search. This results in an equal split of the surplus, since partners have equal bargaining powers. Therefore:

$$\forall (i;j) \in \{1...n\};\ M_{ij} - U_i = M_{ji} - U_j \tag{D}$$

The value functions depend on the proportion of the different types of partners in the searching pool. In steady state, equality of the flows in and out of the searching pool implies:

$$\forall i \in \{1...n\};\ \delta[L_i - N_i] = \lambda\Big(\sum_{k=1} \alpha_k \pi_{ik}\Big)N_i \tag{E}$$

The left-hand side of (E) represents the number of type p_i partners going back into the searching pool (per period of time). The right-hand side represents the number of type i's leaving that same pool. There are N_i of them in the searching pool and they meet at a rate λ. There is a probability α_k that a meeting is with type k and these encounters lead to matches with a probability π_{ik}.

2.3 Definition of a matching equilibrium

Definition A matching equilibrium is comprised of value functions (U_i, M_{ij}), compensations (c_{ij}), skill distribution of searching partners (N_i), individual decision rules (π_{ij}), beliefs (Π_{ij}), such that $\forall (i; j) \in \{1 \ldots n\}$:

i) The value functions correspond to maximizing behaviors by the partners, i.e. they solve the Bellman equations (A) and (B), with $\pi_{ij} = 1\ (\in [0; 1]; = 0)$ if $M_{ij} - U_i > 0\ (= 0; < 0)$,

ii) Upon matching, the bargaining outcome satisfies (C) and (D),

iii) The numbers of each type in the searching pool satisfy (E),

iv) The beliefs are rational and there is consistency of individual and aggregate behavior, i.e. $\pi_{ij} = \Pi_{ij}$.

Remark 1: The focus of this paper is on steady state pure strategy equilibria. Chen (1999) proved existence of equilibrium, possibly in mixed strategy.

Remark 2: Because of the assumptions on the bargaining outcome, it is always the case that $\Pi_{ij} = \Pi_{ji}$, $\forall (i; j) \in \{1 \ldots n\}$.

3 Characteristics of a matching equilibrium

3.1 General characteristics

Now that a matching equilibrium has been defined, it is possible to look at the general properties that all possible equilibria exhibit. The proofs are given in Appendix A.

Proposition 1 The value of search is strictly increasing in the partners' type, i.e. $i > j \Rightarrow U_i > U_j$.

Interpretation: A higher type can always follow the strategy of a lower type and get a higher value from it, because of higher output when matched. Therefore, the strategy actually chosen by the higher type has to result in a higher value of search.

Proposition 2 Upon matching, partners do not split output equally, but rather they equally split the output plus the differential in their value of search. In other words, $\forall (i, j) \in \{1, \ldots, n\}$, $c_{ij} = \frac{1}{2}[f_{ij} + r(U_i - U_j)]$.

Proposition 3 When matching, the higher productivity partner always retains a strictly bigger share of output, i.e.: $\forall (i, j) \in \{1, \ldots, n\}$, if $i > j$, then $c_{ij} > c_{ji}$.

Interpretation: When matching, heterogeneous partners have to split some output. Even though both partners split the match surplus equally, since the higher productivity partner has a higher value of search, he retains the larger share of output.

Proposition 4 The compensation that a higher productivity partner receives from matching with a particular type is strictly larger than the compensation that a lower productivity type receives from matching with that same type: $\forall (i, j, k) \in \{1, \ldots, n\}$, if $i > j$, then $c_{ik} > c_{jk}$.

Interpretation: When a higher productivity partner matches with a given type k, not only does the match produce more output than when a lower productivity partner were to match with that same type k, but also the higher type has a higher value of search. Hence, the more productive partner receives a bigger compensation. However, notice that it is not necessarily always the case that $c_{ij} > c_{ik}$, when $j > k$. This is because, even though more output is produced in the (i, j) match than in the (i, k) match, type j also has a higher value of search. So, an agent does not always want to match with the most productive types of partners.

Propositions 2-4 are characteristic of models where utility is assumed transferable, such as in the labor market, where wages are negotiated between workers and firms. We know that as long as there is a positive surplus to a match, the two partners will stop search and accept to match and produce. Because match payoffs are not fixed, but are determined through negotiations, a partner can always induce the other one to match: as long as the total surplus is positive, both partners can be better off matched than searching. Proposition 2 is a direct consequence of the transferable utility assumption. Because more productive types have higher search values, the less productive ones have to compensate them to accept the match, and hence the high types must receive more than half the output. Formally, from Proposition 2, the compensation c_{ij} that type i receives from matching with type j is equal to $\frac{1}{2}[f_{ij} + r(U_i - U_j)]$[3]. It is greater than half the output if $i > j$. Propositions 3 and 4 are direct applications from Propositions 1 and 2.

3.2 Conditions for perfect sorting

As established in Burdett and Coles (1997), the non-transferability assumption leads to the creation of "classes". As we will see, this result does not necessarily hold with transferable utility. Of all possible matching patterns, the equilibrium where partners only match with their own types stands out. The following proposition sets necessary (and sometimes sufficient) conditions for within type matching only to be an equilibrium.

Proposition 5 A necessary condition for an equilibrium, where partners only match with their own type is that $\forall (i;j) \in \{1:::n\}; i \neq j; f_{ii} + f_{jj} > 2f_{ij}$[4]. If $\lambda \gg r + \delta$, this condition becomes necessary and sufficient[5].

[3] Notice the connection with the wage expression in models such as Pissarides (1990) and Mortensen and Pissarides (1994). Assuming equal bargaining power, wage is given by $\frac{1}{2}[f + rU]$, where f is match output and U is the worker's value of search. In these models too, wage is a function of output and the differential in search value between the two parties. In these models, the firm's search value is always zero, because of the free entry condition. Even if workers were to not receive utility from non-market production, U would still be strictly greater than zero, because as long as there are firms posting vacancies, workers can expect to enter in a productive match at some point in the future

[4] This is the case of strictly supermodular production functions.

[5] Notice that the case, where the different frictions in the economy disappear, satisfies $\lambda \gg r + \delta$.

One can see that matching with partners of similar characteristics does not hold generally. This results, however, extends the findings of Becker (1973, 1974). Indeed, when there is no friction in the economy and the types are complementary inputs in the production function, equilibrium implies assortative matching. However, when frictions arise, this condition on the types is only necessary for perfect sorting.

For the sake of illustration and to carry some intuition, the next section looks at the simple case, where there are only two types of agents: high and low productivity.

3.3 An illustration: the two-type-case

In this section, I illustrate the model, by considering the case of two productivity types. The reader will see that, even in this simple set-up, several matching patterns may arise, including the possibility of multiple equilibria. However, this will allows us to illustrate some consequences of assuming transferable utility. The different equilibria are denoted in the following manner: the first letter represents who the low productivity type is willing to match with, and the second letter who the high productivity type is willing to match with. Partners may match with low types only (L), high types only (H), or both types (B). Given that partners cannot refuse to match with both types (since there is no income during search) and since, if type p_i is willing to match with type p_j, then the reverse is true (from the Nash bargaining assumption), these are the only possible equilibria. Hence:

$$\begin{aligned}
\text{LH equilibrium if } ¦_{ll} &= ¦_{hh} = 1 \text{ and } ¦_{hl} = ¦_{lh} = 0 \\
\text{BB equilibrium if } ¦_{ll} &= ¦_{lh} = ¦_{hl} = ¦_{hh} = 1 \\
\text{HB equilibrium if } ¦_{lh} &= ¦_{hl} = ¦_{hh} = 1 \text{ and } ¦_{ll} = 0 \\
\text{BL equilibrium if } ¦_{ll} &= ¦_{lh} = ¦_{hl} = 1 \text{ and } ¦_{hh} = 0 \\
\text{HL equilibrium if } ¦_{lh} &= ¦_{hl} = 1 \text{ and } ¦_{ll} = ¦_{hh} = 0
\end{aligned}$$

It is possible to look graphically at which equilibrium matching patterns emerge for a given production function. Before, existence conditions have to be derived and an intuitive methodology is provided in Appendix

B[6]. An example is given in figure 1. For that, the skill distribution $\{\gamma_l; \gamma_h\}$, the meeting rate (λ), the breakdown rate (δ), and the discount rate (r) are fixed. The regions where a particular equilibrium is sustainable, are determined in $(f_{hh}; f_{ll})$ space (the value of f_{lh} being fixed, this restricts the possible values for f_{hh} and f_{ll}). The dashed line represents the boundary between supermodular and submodular production functions. For clarity of exposition, the horizontal and vertical axes do not have the same scale. This example demonstrates the possibility of multiple matching equilibria[7][8]. This particular result was also established in Burdett and Coles (1999) and Sattinger (1995). It is due to the fact that an agent's matching decision is a function of the distribution of types in the searching pool, which is the result of the matching decisions of all other agents. This sorting externality is the source of multiple equilibria. One can observe that, when the production function is supermodular, only three pure strategy equilibria can arise: $LH; BB$ and HB (this property can actually be established for all parametrizations). Matches where high types never match with other high types are precluded in this case. Also, when the partners' types exhibit a high degree of complementarity in the production function[9], only LH behavior is sustainable. The counter-intuitive equilibria where high types refuse to match with each other $(BL; HL)$ correspond to production functions where f_{hh} is not much higher than f_{lh} and f_{ll} is low. This can be explained very easily in the context of transferable utility. Imagine a task, which requires two agents for completion. Assume that two low types cannot complete the task successfully (i.e. $f_{ll} \approx 0$), but that two high types together are not much more productive than a low type and a high type (i.e. $f_{hh} \simeq f_{lh}$). For the sake of illustration, one can imagine that the task is surgery, which requires a surgeon and an assistant to wipe the surgeon's forehead. Of course, two medical assistants cannot perform the surgery, but having two surgeons performing the operation does not produce better results than a surgeon with her assistant. Since assistants are totally unproductive working together, they have no other option than matching with a surgeon. Under these conditions, a surgeon is well compensated for accepting a match with an assistant. It may even be preferable for her not to work with another surgeon, since she would then have to evenly split roughly the same match product, rather than retain most of it. One can see that this result depends crucially on the transferable utility assumption, whereby a

[6] Existence conditions are available upon request.

[7] One can actually prove that, if $\gamma_l = \gamma_h = \frac{1}{2}$, then a multiple LH/BB equilibrium cannot exist. This is because these two matching behaviors would lead to the same skill composition in the search pool, and thus could not both be sustained as equilibria.

[8] It also shows that for some production functions, no pure strategy equilibria are sustainable.

[9] i.e. in the right-hand portion of the graph.

low type can induce a high to match with him, by compensating the high type with a higher wage.

The next section looks at two applications for the model, underscoring the importance of considering matching patterns between heterogeneous agents in the labor market.

4 Are matching patterns between heterogeneous agents relevant to the study of labor markets ?

4.1 Unemployment and Wage Dispersion

There has been great interest recently among economists in explaining the di¤erent labor market outcomes in the U.S. and in Europe, as witnessed by the abundant literature (Bentolila and Bertola (1990), Bertola and Ichino (1995), Bertola and Rogerson (1997), Lazear (1990), Millard and Mortensen (1997), Mortensen and Pissarides (1997) just to name a few). It is observed that European countries experience higher unemployment than the U.S. and exhibit lower wage dispersion. The explanations put forward all rely on having di¤erent labor market policies in Europe and the U.S. To the exception of Bertola and Ichino (1995), however, the focus of these papers is only on one aspect of the divergence between American and European labor markets, namely unemployment di¤erences. While these types of explanations have de…nite merits, it is possible, using the model, to take a di¤erent approach and investigate whether matching patterns between heterogeneous agents can simultaneously explain both the unemployment and wage dispersion di¤erences across markets. In particular, it is often taken as given, without much rationalization, that wage setting institutions in Europe result in more compressed wages. I want to propose an explanation that does not posit wage compression to induce high unemployment, but rather one where the two phenomena naturally arise together, i.e. I want to propose a rationale for endogenous wage compression[10].

[10] Notice that a legally imposed minimum wage may also generate higher unemployment and lower wage dispersion in the context of this model, by precluding the formation of some matches (either by preventing matches between two low productivity

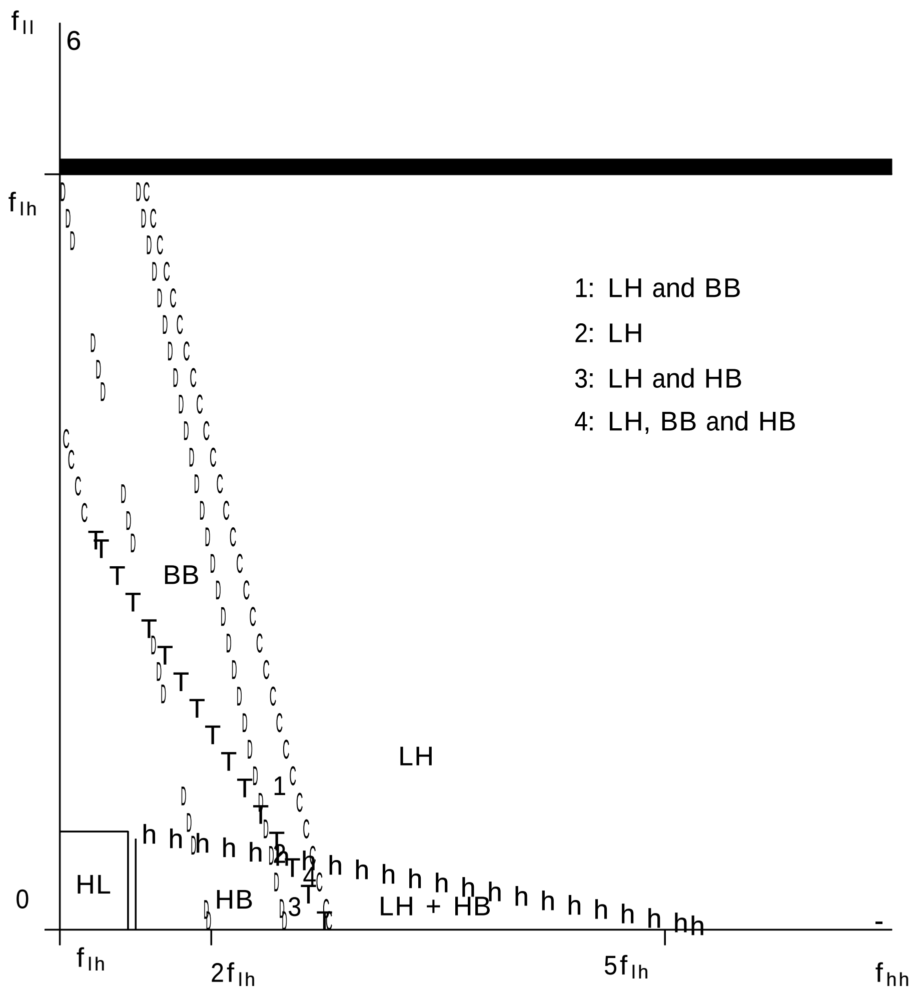

Figure 1: Equilibrium regions: $°_h$ = 1=3; ¸ = 1; ± = :1; r = :02; f_{lh} = 1

This model also contributes to the literature on wage inequality. Kremer and Maskin (1996) and Krusell, Ohanian, Rios-Rull and Violante (2000) look at the e¤ects of a skill-biased technological change on wage dispersion. However, because they look at competitive economies, their models do not have implications on unemployment. Acemoglu (1999) and Albrecht and Vroman (2000) build models where the labor market is characterized by search frictions. They have ex-ante heterogeneity in workers (high- or low-skill), but their set-ups di¤er from mine, since they assume that …rms endogenously post vacancies. Although their models di¤er along several dimensions, they both …nd that equilibria with endogenous segmentation along worker skill lines result in both higher wage dispersion and unemployment than equilibria where high- and low-skill workers may accept the same type of jobs[11]. While that literature focuses primarily on accounting for the recent trend in wage inequality in the U.S., the present model is interested in explaining the di¤erences in wage inequality and unemployment in the U.S. and Europe.

The model is able to determine equilibrium values for unemployment and wages. Of course, di¤erent types of equilibria result in distinct steady state values. Therefore, matching patterns in‡uence the proportion of workers looking for a job, as well as the wage distribution. The measure of wage inequality retained is the ratio of highest wage observed to lowest wage observed. This is similar to the ratio of i^{th} percentile to j^{th} percentile often used in the literature. I retain this particular measure among others, since it …ts the theoretical framework quite well. Hence, it is possible, using the model, to consider the possibility that Europe and the U.S. are in di¤erent equilibria, with the US. being in a low employment/high wage dispersion equilibrium and Europe being in a high unemployment/low wage dispersion equilibrium.

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partners or by restricting a low productivity type from transferring enough utility to a high productivity partner to induce her to match). However, there is evidence showing that there is smaller wage dispersion within type in Europe, even after controlling for the usual skill proxies (education, experience). A minimum wage can not explain smaller wage dispersion at high skill levels in Europe. Therefore, I do not pursue an explanation in this direction, but rather attempt to provide an explanation for endogenous wage compression.

[11] Because these authors have di¤erent assumptions, they also have di¤erent matching patterns in their respective other possible non-segmented equilibrium. Acemoglu's assumptions imply that all jobs are of the same type and attract high- and low-skill workers. Albrecht and Vroman's assumptions imply that there are two types of jobs and that high-skill workers accept to work both for high and low productivity …rms, while low-skill workers only match with low productivity …rms.

As we have seen in section 3.3, distinct matching patterns may be either due to di¤erent fundamental parameters or to di¤erent beliefs. Since it is known that breakdown rates, for example, have di¤erent values in Europe and in the U.S., I will emphasize the …rst approach. Because the labor market policies in place a¤ect some of these parameters (in particular meeting or breakdown rates), this methodology will generate a link between policies and matching patterns. Hence, the exercise is to see how parameter values in‡uence equilibrium matching between heterogeneous agents. I will …rst present simulations outlining which sets of parameter values give rise to particular equilibria. This will be done by varying for the meeting and breakdown rates, which may be in‡uenced by labor market policies, and leaving the other parameters constant (skill distribution, technology and rate of time preference). Then, I will present some evidence that, in line with the respective policies in place, actual match breakdown rates are consistent with individual European workers matching in equilibrium with a limited set of productivity types, and American workers with a larger set of types. In other words, the characteristics of the American and European labor markets lead to more homogeneity within European matches than within American ones. Using the literature on under- and over-education, I will also provide direct additional evidence showing that, indeed, European labor market matches tend to be more homogeneous. After establishing that the two economies are distinguished by di¤erent matching patterns, I will provide some intuition, based on the transferable utility assumption, why these patterns lead to the unemployments/wage dispersions actually observed. Finally, I will also present some theoretical support that more heterogeneity in matches lead to lower unemployment together with higher wage dispersion.

4.1.1 Are the U.S. and Europe characterized by di¤erent matching patterns?

The model is simulated to determine the parameter regions compatible with particular equilibria. To that e¤ect, the technology parameters (f), the skill distribution (°$_i$) and the rate of time preference (r) are …xed, while the meeting and breakdown rates (¸, ±) are allowed to vary. For clarity of exposition, I again assume that partners are of two types (low and high productivity). From Proposition 5, we know that the production function has to be supermodular to possibly generate LH equilibria. Since the simulation results do not qualitatively depend on the production function assumed[12], f_{ll}, f_{lh} and f_{hh} are set at 0:3, 0:5 and 0:6, respectively. The discount rate r is set at 0:2, which corresponds to a real interest rate of 2% per quarter[13]. Finally, I chose $°_h$ = 1=3 (implying that there are twice as many low-skilled than high skilled workers). In the picture, ¸ 2 [0; 6], which implies that the average time before a meeting takes place is no greater than two weeks and ± 2 [0; 1], which is equivalent to assuming that the average employment duration is at least one quarter. The equilibrium region graph is shown in …gure 2.

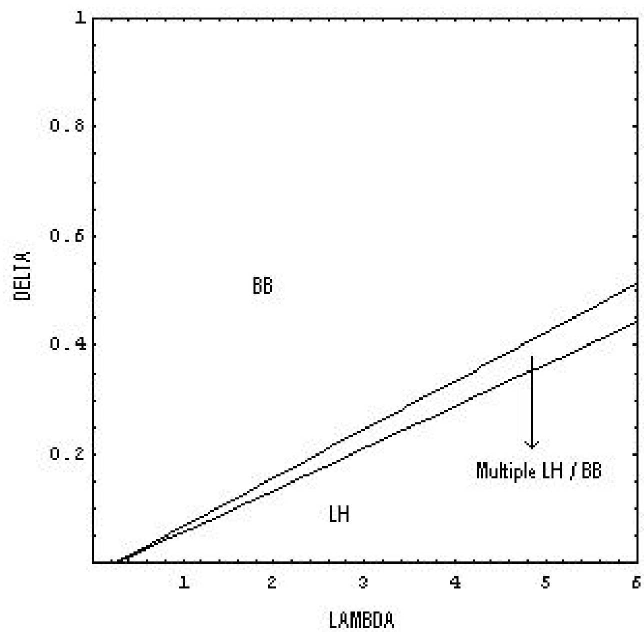

Figure 2: Equilibrium regions: $°_h$ = 1=3; r = 0:2; (f_{ll}; f_{lh}; f_{hh}) = (:3; :5; :6)

LH equilibria are consistent with higher values of ¸=±, while BB equilibria are associated with lower such ratios. These results are intuitive, since a high meeting rate or a low breakdown rate justify high types being patient and waiting to meet other high types. Under such parameters, a low type cannot compensate a high

[12] All the production functions simulated returned an equilibrium region graph similar to the one presented below, with LH, BB and multiple equilibria (LH/BB) regions. The only way to have HB as an equilibrium is to choose f such that f_{hh} + f_{ll} **&** 2f_{lh} (but not too close) and $°_h$ ¸ $\frac{1}{2}$. In that case, …xing ¸ and increasing ±, one would move from an LH to an HB to a BB region.

[13] The discount rate turns out to be the least sensitive of all parameters in determining the equilibrium regions.

type enough to accept to match with him. In other words, matching with the …rst partner occurs when it is not justi…ed to wait for a better match (in the same spirit, and using unreported simulations, LH equilibria are associated with large proportions of high-skilled workers and low discount rates).

The empirical evidence supports the notion that match breakdowns are more frequent in the U.S. than in Europe. For example, Mortensen and Pissarides (1997) report that European unemployment is characterized by longer, but less frequent spells than in the U.S. These authors …nd that only 10% of the unemployed have been in that state for above a year in the U.S., while the same number is between 40% and 50% in Europe. At the same time, the in‡ow rates into unemployment are two to eight times higher in the U.S. than in Europe[14]. Further evidence from the OECD Job Study (1994) shows that the average job tenure is greater in Europe, while the percentage of tenure of less than one year is greater in the U.S. All of this indicates that matches break down at a lower rate in Europe than in the U.S., which according to the model, promotes within type matching[15]. Finally, notice that these results are consistent with all the theoretical literature (Bentolila and Bertola (1990), Bertola (1990), Delacroix (1998), Hopenhayn and Rogerson (1993), Millard and Mortensen (1997) and Mortensen and Pissarides (1997)), which …nds that higher …ring costs, such as those observed in Europe, lead to lower rates of job separation.

In addition to …nding that employment stability in Europe promotes within-type matching, one can …nd direct evidence about matching patterns in the respective economies. In order to do that, one can look at the extent of under- and over-education in various labor markets. One can do this by comparing the level of education required for a particular job with the education actually completed by the worker holding that position. Following Hartog (2000), required schooling is typically measured in three di¤erent ways. The …rst method uses Job Analysis (JA) data. This involves the evaluation of the required level and type of education

[14] As measured by the number of unemployed for less than a month as a percent of the population aged 15-64 less the unemployed (target population).

[15] Using microeconomic evidence from the OECD Employment Outlook (1994), Bertola and Rogerson (1997) …nd that job destruction rates (job destruction as a percentage of total employment) are of the same order of magnitude in Europe and the U.S. While this data cannot be easily reconciled with the evidence presented above, note that Bertola and Rogerson mention that there may be some measurement issues associated with their evidence (di¤erences in data collection, compositional e¤ects). And the evidence on worker turnover, as opposed to job turnover, is in line with what is reported in the text.

for the job titles in an occupational classification, by professional job analysts[16]. The second method, uses Worker Self-Assessment (WA) data, which consists of the worker specifying the education required for the job. This can be done using PSID data, as in Sicherman (1991). Finally, the information can be obtained from realized matches (RM), where the required education is derived from what workers usually have attained, i.e. the mean or the mode of that distribution. Using one of these methodologies, it is possible to measure the incidence of (i) matches where workers are over-educated, (ii) matches where workers are under-educated and (iii) proper matches, i.e. where workers have the correct education level for the job.

The data presented comes from Hartog (2000) for the Netherlands, Portugal and the U.K., from Daly, Büchel and Duncan (2000) for Germany, and from Acemoglu (1999) for the United States. Hartog (2000) also provide results identical to Acemoglu (1999) for the U.S. The method used is WA for Germany, the Netherlands, the U.K. and the U.S., and JA for Portugal. This choice is due to the availability of the data. However, when both methodologies were available, WA and JA provided similar results. Additional information in Hartog and Oosterbeek (1988) and Sicherman (1991) confirms the patterns observed in the U.S. and the Netherlands, for slightly different periods. In the U.S., Sicherman (1991) reports that in a sample of about 5,000 male households aged 18-60, when asked "how much formal education is required to get a job like yours?", and when compared to the respondents' actual completed education level, 57% of the sample reported either under-education or over-education. This study was conducted using PSID data from 1976 and 1978 (Acemoglu (1999) conducted the same study for 1985 and still found that 54% of the individuals sampled reported that their own education level was different than the one required). For purpose of comparison, Hartog and Oosterbeek (1988) find that incidence of over- or under-education is more common in the U.S. than in Netherlands.

Figure 3 reports the proportion of proper matches, unemployment rate, and wage dispersion (measured as the log-difference between the 90th and the 10th percentile of the wage distribution) for the five countries, in particular years. The limited availability of data on proper matches dictated the choice of country and years reported. However, all the data falls in a relatively short time period (1981-86). Portugal was the only country for which data was available for several years between 1981 and 1986, and it exhibited roughly

[16] One such example is the United States Dictionary of Occupational Titles (USDOT).

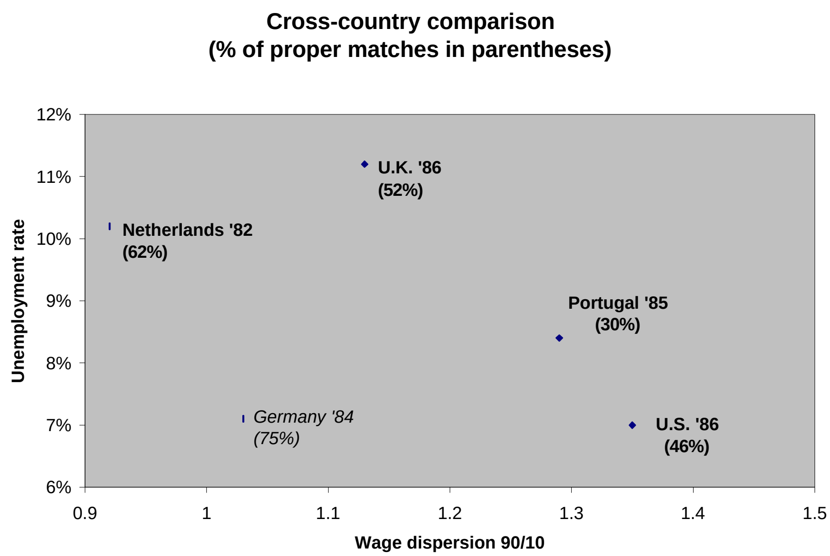

Figure 3:

constant incidence of proper matches. The data on wage dispersion comes from the OECD Employment Outlook (1996) and the unemployment rate data from the Bureau of Labor Statistics database. Figure 4 reports average unemployment and wage dispersion between 1981 and 1986 for the same countries (not enough data was available to compute the average incidence of proper matches over the same period).

This evidence is compatible with the notion that partners match with a larger set of productivity types, or a greater percentage of the population, in the U.S. than in Europe. The latter has a higher incidence of proper matches, i.e. a higher proportion of matches with the required productivity type. In the U.S., however, matches tend to show more heterogeneity. This claim can also be supported by other considerations. Since European countries are characterized by more generous and longer unemployment bene…ts, this enables workers looking for a job to be more picky when choosing whether to agree to match with a given type or to wait for a better match[17]. Assuming di¤erent matching patterns across the ocean also …ts the fact that higher unemployment is observed in Europe for all skill categories (and therefore is not only due to labor market

[17] Also, the prevalence of advance notice for high skill workers in Europe may increase the proportion of matches between high types at the expense of matches with low types.

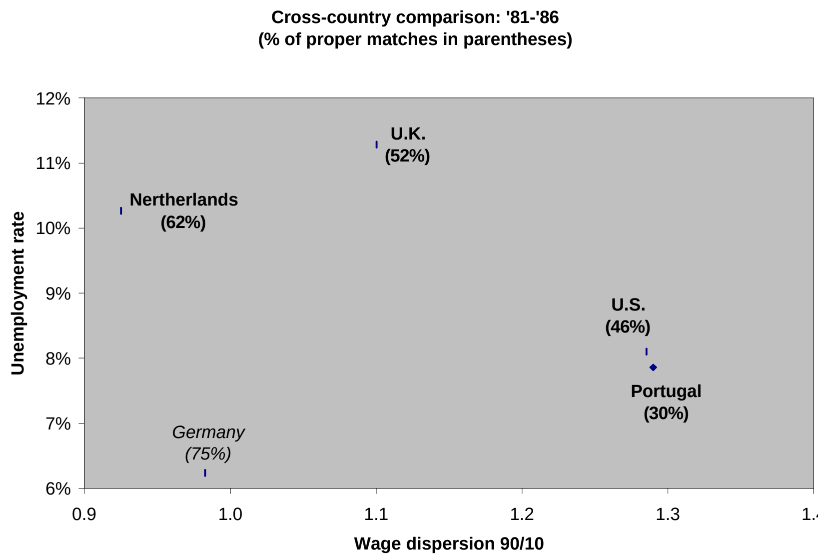

Figure 4:

policies a¤ecting primarily the lesser skilled workers). It also implies that the duration of unemployment is higher in Europe than in the U.S., as observed empirically.

In conclusion, …gures 3 and 4 show that countries with a high incidence of proper matches tend to exhibit both high unemployment and low wage dispersion, as expected when partners' heterogeneity is taken into account. The Netherlands, Portugal, the U.K. and the U.S. …t that description exactly. Germany, which has the highest proportion of proper matches, also has the lowest wage dispersion. However, it exhibits relatively low unemployment. This was pointed out in Nickell and Bell (1996), who underline the speci…city of the German educational system. In Germany, two-thirds of the teenagers participate in an apprenticeship training system. Apprentices receive both classroom and on-the-job training. This can be expected to promote the formation of matches between an apprentice and her …rm, and therefore to reduce unemployment.

In the next section, I provide some intuition why the matching patterns observed in the U.S. and Europe simultaneously generate the correct unemployment and wage dispersion patterns and give a theoretical result showing that heterogeneity in matches is associated with lower unemployment and higher wage dispersion, under transferable utility.

4.1.2 How do matching patterns affect unemployment and wage dispersion?

We know that Europe[18] is in an equilibrium where every category of workers matches with a limited set of other productivity types, while, in the U.S., workers match with a larger set of partners. What does this imply for unemployment and wage dispersion? In an economy, like the U.S, where agents match with productivity types very different from theirs, this results in more matches for all types, and hence lower unemployment. In that economy, a high productivity type may accept to match with a lower type. However, there is an opportunity cost to the high type of matching with a low type. Hence, in the bargaining, the low type needs to compensate the high type to induce him to match. We know that, due to the transferable utility assumption, wages split output plus the differential in search values. When agents match with a larger set of types, this differential can become large in matches between agents that are quite different in productivity. Hence, one observes higher wage dispersion, because low types had to compensate higher types more for accepting to match. This is similar, in spirit, to the "opportunity cost effect" mentioned in Acemoglu (1997). These considerations can even explain higher within-type wage dispersion[19] in the U.S., as reported in Bertola and Ichino (1995).

One can again simulate the model to verify that unemployment is higher and wage dispersion lower in an economy with homogenous matches only (LH equilibrium), than in an economy with heterogeneous matches (BB equilibrium). Keeping the same values for f, γ_h and r and fixing $\lambda = 6$ (i.e. an average of two weeks between meetings), one can allow the breakdown rate to vary. Starting from $\delta = 0$ and increasing its value, the equilibrium changes from LH to BB, as in figure 5. One can verify that, as long as matches do not break down too frequently, unemployment (wage dispersion) is higher (lower) in an LH than in a BB equilibrium (notice that $U\%(BB)$ is greater than $U\%(LH)$ when average job tenures (1 quarter) are significantly less

[18] Among the countries for which we had relevant evidence, European workers match with a smaller set of types than their American counterparts, to the exception of Portugal. However, Portugal still fits very well in our framework, since it had (in the early 80's) relatively low unemployment, and at the same time, high wage dispersion.

[19] i.e. wage dispersion for workers of identical characteristics.

than the ones observed in Europe or the U.S.)[20].

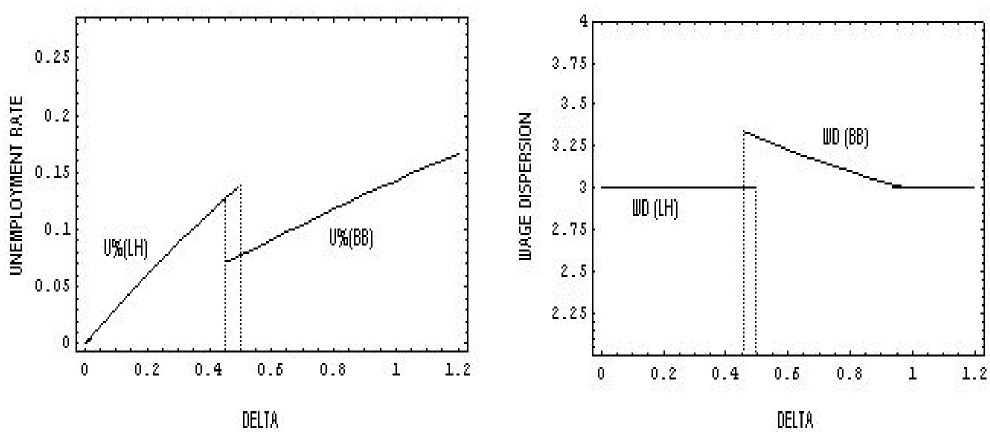

Figure 5: Unemployment and wage dispersion: $°_h = 1{=}3$; $r = 0{:}2$; $(f_{ll}; f_{lh}; f_{hh}) = (:3; :5; :9)$; ¸ = 6

One can also use the model to look for theoretical support for the claim that the unemployment and wage dispersion observed in Europe and the U.S. are consequences of di¤erent matching patterns. Proposition 6 looks at the issue of within-type wage dispersion and establishes that, when a given type's unconditional probability of matching is higher in a particular equilibrium matching pattern, then the wage dispersion for this type of partners also tends to be higher in that equilibrium. If all types tend to match with larger matching sets, then greater within-type wage dispersion leads to greater overall wage dispersion.

De…ne the within-type wage dispersion WD_i for type p_i, as the ratio of the highest to the lowest wage observed for that type in equilibrium. Also de…ne type p_i's unconditional probability of matching $\hat{p}_i$, as the product of the meeting rate ¸ times the proportion of the population type p_i is willing to match with ($\hat{p}_i$ =

[20] When ± is very high, only BB is an equilibrium, since there is no point rejecting a particular match in favor of a better one, when matches do not last long anyway. Hence, when ± is high, every partner matches with the …rst potential partner, and because of large job destruction rates, unemployment is high. Also, when ± tends to $+1$, the values of search U_l and U_h tend to zero, and there is no advantage to being a high productivity type. In that case, the output of an hl match is split equally, i.e. $c_{lh} = c_{hl} = \frac{1}{2} f_{lh}$ and $c_{ll} < c_{lh} = c_{hl} < c_{hh}$ and $WD(BB) = WD(LH)$. Heterogeneity in productivity within a match has an e¤ect on wage dispersion only, when there is an advantage to being of higher productivity.

$\lambda \sum_{k=1} \alpha_k - ik)$.

Proposition 6 For a given type p_i, when the unconditional probability of matching $\hat{p}_i$ increases from one equilibrium to another, then the lower bound on the wage dispersion for type p_i increases. The lower bound is given by: $WD_i \geq \frac{\hat{p}_i}{r+\delta+\hat{p}_i}$.

When types p_i match with a great variety of other types, some of the matches will be with partners very different from them, which implies that they will be either well compensated to accept the match or that they will have to accept low wages themselves. This will tend to increase WD_i. And of course, when $\hat{p}_i$ is high, a higher proportion of meetings lead to matches and thus unemployment is lower.

4.2 Noise in wages

Another point can be made to further emphasize the importance of studying the matching patterns of heterogeneous agents in the labor market. Abowd, Kramarz, Margolis and Troske (1998) showed, using individual data on wages, matched with firm data, that the combination of observed and unobserved individual characteristics and establishment effects, explain more of the French wage data than the American one. In other terms, there is more noise in the U.S. than in the French wage data.

From Proposition 2, we know that, for given productivity types p_i and p_j, the wage c_{ij} is not only a function of the productive characteristics of the two partners engaged in the match (f_{ij}), but also of their respective values of search, U_i and U_j. In equilibrium, type p_i optimally matches with a certain set of types. These matching opportunities affect her value of search, and hence the wage she can negotiate with type p_j. In conclusion, the wage c_{ij} depends on more than just f_{ij}. Because the values of search U_i and U_j reflect all the matching opportunities that p_i and p_j have in equilibrium, besides just matching with each other, c_{ij} also depends on the characteristics of members of type p_i's and p_j's matching sets (and even on which types the latter are matching with, and so on)[21].

Applying this reasoning to Europe and the U.S., one can expect more noise in wage data in the U.S., after controlling for the partners' characteristics: if partners match with larger sets in the U.S., then the characteristics of the match participants are less relevant in the wage determination. We know from Abowd, Kramarz, Margolis and Troske (1998) that this is indeed the case.

5 Conclusion and future extensions

An equilibrium model was developed where agents of different productivities have to decide which kind of partners to match with, when frictions make finding a partner a difficult and time consuming process. The model was designed to replicate the salient features of a labor market. In equilibrium, several matching patterns may arise, as illustrated in the two-type case. General characteristics of a matching equilibrium were underlined, emphasizing the importance of assuming transferable utility. The model was then applied to the issues of (i) wage dispersion and unemployment in Europe and the U.S. and of (ii) the relationship between wage and firms' and workers' characteristics in France and the U.S. It was shown that matching patterns may explain the differences between these labor markets. In fact, the model emphasized the need for more cross-country empirical research on matching behavior between agents of different productivities.

A natural extension of this framework would be to directly introduce labor market policies, such as unemployment insurance, firing costs or minimum wages, into the model and see how they affect matching patterns. One would then better be able to analyze how a skill-biased technological change might interact with labor policies to induce changes in unemployment and wage dispersion in the U.S. and Europe, as observed

[21] Denote by Ω_i, type p_i's matching set. We know that c_{ij} depends on f_{ij}, U_i and U_j. It is established in Appendix A that $U_i = \frac{\lambda \sum_{k \in \Omega_i} \alpha_k c_{ik}}{r\left(r+\delta+\lambda \sum_{k \in \Omega_i} \alpha_k\right)}$. Hence, U_i depends on all the wages type p_i can expect to receive with partners in his matching set. As c_{ik}, in turn, depends on f_{ik}, U_i and U_k, we can see that c_{ij} is a function of f_{ij}, f_{ik}, U_k ($k \in \Omega_i$), but also f_{jl}, U_l ($l \in \Omega_j$). This is not all c_{ij} depends on, though. One can notice, by iteration, that c_{ij} also depends on f_{km} ($m \in \Omega_k$) and f_{lp} ($p \in \Omega_l$) ...

since the 1980's. In addition to wage dispersion, the di¤erence in earnings mobility between the two labor markets could also be examined with such matching considerations.

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A Proofs

Proof of Proposition 1:

Let us call Ω_i, type i's matching set. By definition, $\Omega_i = \{j; M_{ij} \geq U_i\}$. In equilibrium, $\forall i \in \{1 \ldots n\}$; $rU_i = \lambda \sum_{k \in \Omega_i} \alpha_k (M_{ik} - U_i)$ and $\forall (i;j) \in \{1 \ldots n\}$; $rM_{ij} = c_{ij} + \delta(U_i - M_{ij})$. Hence, (B) implies that: $(r+\delta)(M_{ij} - U_i) = c_{ij} - rU_i$. Then (A) implies that: $rU_i = \frac{\lambda}{r+\delta} \sum_{k \in \Omega_i} \alpha_k (c_{ik} - rU_i)$. Since the surplus is split equally between matching partners, $c_{ij} - rU_i = c_{ji} - rU_j = f_{ij} - c_{ij} - rU_j$. Hence, $c_{ij} = \frac{1}{2}[f_{ij} + rU_i - rU_j]$ and $c_{ij} - rU_i = \frac{1}{2}[f_{ij} - rU_i - rU_j]$. Therefore, $rU_i = \frac{\ell}{2(r+\delta)} \sum_{k \in \Xi_i} \alpha_k (f_{ik} - rU_i - rU_k)$. It is clear that $(M_{ij} - U_i)$, $c_{ij} - rU_i$, and $f_{ij} - rU_i - rU_j$ have the same sign, so that $j \in \Xi_i \Leftrightarrow f_{ij} - rU_i - rU_j \geq 0$.

Let us now show that for any type and any subset S of $\{1,\ldots,n\}$, $rU_i \geq \frac{\ell}{2(r+\delta)} \sum_{k \in S} \alpha_k (f_{ik} - rU_i - rU_k)$.

Suppose that: $\exists i \in \{1,\ldots,n\}; \exists S \subset \{1,\ldots,n\};\ rU_i < \frac{\ell}{2(r+\delta)} \sum_{k \in S} \alpha_k (f_{ik} - rU_i - rU_k)$ (H).

Then, $\sum_{k \in \Xi_i} \alpha_k (f_{ik} - rU_i - rU_k) < \sum_{k \in S} \alpha_k (f_{ik} - rU_i - rU_k)$. If $\Xi_i \backslash S \cap \Xi_i = \emptyset$, then $\Xi_i \subset S$. But this would be in contradiction with (H), since for every $k \in S \backslash \Xi_i$; $f_{ik} - rU_i - rU_k < 0$. If $S \backslash S \cap \Xi_i = \emptyset$, then $S \subset \Xi_i$. But this would also be in contradiction with (H).

Therefore, $\sum_{k \in \Xi_i \backslash S \cap \Xi_i} \alpha_k (f_{ik} - rU_i - rU_k) < \sum_{k \in S \backslash S \cap \Xi_i} \alpha_k (f_{ik} - rU_i - rU_k)$. However, by definition of Ξ_i, the left-hand side term is non-negative while the right-hand side term is negative, leading to a contradiction.

Hence, $\forall i \in \{1,\ldots,n\}; \forall S \subset \{1,\ldots,n\};\ rU_i \geq \frac{\ell}{2(r+\delta)} \sum_{k \in S} \alpha_k (f_{ik} - rU_i - rU_k)$.

Now, take $(i,j) \in \{1,\ldots,n\};\ i > j;\ rU_i \geq \frac{\ell}{2(r+\delta)} \sum_{k \in \Xi_j} \alpha_k (f_{ik} - rU_i - rU_k)$.

Hence, $rU_i - rU_j \geq \frac{\ell}{2(r+\delta)} \sum_{k \in \Xi_j} \alpha_k (f_{ik} - f_{jk} - rU_i + rU_j)$

and $r(U_i - U_j)\left(1 + \frac{\ell}{2(r+\delta)} \sum_{k \in \Xi_j} \alpha_k\right) \geq \frac{\ell}{2(r+\delta)} \sum_{k \in \Xi_j} \alpha_k (f_{ik} - f_{jk}) > 0$. This implies that $U_i > U_j$.

Proof of Proposition 2:

Proposition 2 was established in the proof of proposition 1.

Proof of Proposition 3:

From equilibrium condition (D), $U_i - U_j = M_{ij} - M_{ji}$. From equilibrium condition (B), $rM_{ij} = c_{ij} + \delta(U_i - M_{ij})$ and $rM_{ji} = c_{ji} + \delta(U_j - M_{ji})$. Hence, $c_{ij} - c_{ji} = r(M_{ij} - M_{ji}) = r(U_i - U_j)$. From Proposition 1, if $i > j$, then $U_i > U_j$, and therefore $c_{ij} > c_{ji}$.

Proof of Proposition 4:

Using Proposition 2, $c_{ik} = \frac{1}{2}[f_{ik} + rU_i - rU_k]$. Hence, $c_{ik} - c_{jk} = \frac{1}{2}[f_{ik} - f_{jk} + r(U_i - U_j)]$. Since, when $i > j$, $f_{ik} > f_{jk}$ and $U_i > U_j$, then $c_{ik} > c_{jk}$.

Proof of Proposition 5:

From (A) and (B), $rU_i = \lambda \sum_{k\in\Gamma_i} \alpha_k (M_{ik} - U_i)$ and $rM_{ij} = c_{ij} + \delta(U_i - M_{ij})$, where α_k is the proportion of type k partners in the searching pool and Γ_i is type i's matching set. After calculations:

$$\forall(i;j); U_i = \frac{\lambda \sum_{k\in\Gamma_i}\alpha_k c_{ik}}{r\left(r+\delta+\lambda\sum_{k\in\Gamma_i}\alpha_k\right)} \text{ and } M_{ij} = \frac{r\left(r+\delta+\lambda\sum_{k\in\Gamma_i}\alpha_k\right)c_{ij} + \lambda\delta\sum_{k\in\Gamma_i}\alpha_k c_{ik}}{r(r+\delta)\left(r+\delta+\lambda\sum_{k\in\Gamma_i}\alpha_k\right)}.$$

Looking for an equilibrium where $\Gamma_i = \{i\}$,

$$\forall i; U_i = \frac{\lambda\alpha_i \frac{1}{2} f_{ii}}{r(r+\delta+\lambda\alpha_i)}$$

$$\forall i; M_{ii} - U_i = \frac{\frac{1}{2} f_{ii}}{(r+\delta+\lambda\alpha_i)} > 0$$

$$\forall j \neq i; M_{ij} - U_i = \frac{(r+\delta+\lambda\alpha_i)c_{ij} - \lambda\alpha_i\frac{1}{2}f_{ii}}{(r+\delta)(r+\delta+\lambda\alpha_i)}$$

For "perfect sorting" under frictions, it is necessary that: $\forall(i;j); i\neq j; (r+\delta+\lambda\alpha_i)c_{ij} < \lambda\alpha_i\frac{1}{2}f_{ii}$. From Proposition 2, $c_{ij} = \frac{1}{2}[f_{ij} + rU_i - rU_j] = \frac{1}{2}\left[f_{ij} + \frac{\lambda\alpha_i\frac{1}{2}f_{ii}}{(r+\delta+\lambda\alpha_i)} - \frac{\lambda\alpha_j\frac{1}{2}f_{jj}}{(r+\delta+\lambda\alpha_j)}\right]$. Hence, we need: $\forall(i;j); i\neq j$;

$$(r+\delta+\lambda\alpha_i)\frac{1}{2}\left[f_{ij} + \frac{\lambda\alpha_i\frac{1}{2}f_{ii}}{(r+\delta+\lambda\alpha_i)} - \frac{\lambda\alpha_j\frac{1}{2}f_{jj}}{(r+\delta+\lambda\alpha_j)}\right] < \lambda\alpha_i\frac{1}{2}f_{ii}$$

$$\Rightarrow \frac{1}{2}f_{ij}(r+\delta+\lambda\alpha_i) - \frac{1}{4}\lambda\alpha_i f_{ii} - \frac{1}{4}\lambda\alpha_j f_{jj}\frac{(r+\delta+\lambda\alpha_i)}{(r+\delta+\lambda\alpha_j)} < 0$$

After a little algebra

$$\Rightarrow (2f_{ij} - f_{ii} - f_{jj})(r+\delta+\lambda\alpha_i)(r+\delta+\lambda\alpha_j) + f_{ii}(r+\delta)(r+\delta+\lambda\alpha_j) + f_{jj}(r+\delta+\lambda\alpha_i)(r+\delta) < 0.$$

So, if $\exists(i;j); i\neq j$; such that $2f_{ij} \geq f_{ii}+f_{jj}$, then the equilibrium would not be supported. Hence, $2f_{ij} < f_{ii}+f_{jj}; \forall(i;j); i\neq j$, is a necessary condition for such an equilibrium to exist. If $\lambda \gg r+\delta$, then the above inequality becomes: $(2f_{ij} - f_{ii} - f_{jj})\lambda^2\alpha_i\alpha_j < 0$. Then, strict supermodularity of f is a necessary and sufficient condition for such an equilibrium to exist.

Proof of Proposition 6:

From the expressions for M_{ij} and U_i established in the proof of Proposition 5, we know that: $j \in \Gamma_i \Leftrightarrow (r+\delta)c_{ij} \geq \lambda\sum_{k\in\Gamma_i}\alpha_k(c_{ik} - c_{ij}) \Leftrightarrow \frac{(r+\delta)}{\lambda} \geq \sum_{k\in\Gamma_i}\alpha_k\left(\frac{c_{ik}}{c_{ij}} - 1\right)$. In particular, calling $c_{i;min}$ and $c_{i;max}$, the smallest and biggest of the compensations in type p_i's matching set, respectively (i.e. smallest and biggest of compensations observed for type p_i), we have that: $\frac{(r+\delta)}{\lambda} \geq \sum_{k\in\Gamma_i}\alpha_k\left(\frac{c_{ik}}{c_{i;max}} - 1\right) \Rightarrow \frac{(r+\delta)}{\lambda} + \sum_{k\in\Gamma_i}\alpha_k \geq \sum_{k\in\Gamma_i}\alpha_k\frac{c_{ik}}{c_{i;max}} \geq \frac{c_{i;min}}{c_{i;max}}\sum_{k\in\Gamma_i}\alpha_k$. Now, if I define wage dispersion for type p_i, WD_i, as $WD_i = \frac{c_{i;max}}{c_{i;min}}$, then, we have that: $WD_i \geq \frac{1}{1+\frac{r+\delta}{\lambda\sum_{k\in\Gamma_i}\alpha_k}}$. The right-hand side term is a lower bound for type p_i's wage dispersion. As an equilibrium is characterized by matching sets $\{\Gamma_i\}_{i=1...n}$ and distribution of types in the searching pool $\{\alpha_j\}_{j=1...n}$, we can compare lower bounds for wage dispersion in different equilibria. Hence, if, for a given type p_i, the probability of matching ($\lambda\sum_{k\in\Gamma_i}\alpha_k$) increases from one equilibrium to the other, then the lower bound also increases.

B Determination of the existence conditions

There are two ways to obtain the existence conditions. One can take each possible equilibrium in turn, and solve the system of equations (A)-(E), given the corresponding values for Π_{ij}. This involves long calculations and little intuition. Alternatively, one can proceeds as follows. First consider the decision problem of an agent facing meeting opportunities at a rate λ, of which a proportion α pay a wage w_1 and a proportion $(1-\alpha)$ pay a wage w_2 (the resulting matches break down at a rate δ). At this point α, w_1 and w_2 are exogenous. It is easy to compute the value of search U, as well as the values of a match at wage w_1, M_1 and at wage w_2, M_2. Knowing these values, the decision problem is trivial. Whether the agent accepts matches at w_1 and at w_2 is determined by inequalities between r, δ, λ, α, w_1 and w_2. Consider that this is the problem faced by a high productivity type. Now consider the same problem faced by another agent, the low productivity type, except that a proportion α of the wages offered pay w_1' and a proportion $(1-\alpha)$ pay a wage w_2'. Again, the values of search and of a match U', M_1' and M_2' can be calculated. The low types' decision problem is determined by inequalities between r, δ, λ, α, w_1' and w_2'. Let α be the proportion of high types in the searching pool. Thus, as per the model notations, $w_1 = c_{hh}$, $w_2 = c_{hl}$, $w_1' = c_{lh}$, $w_2' = c_{ll}$, $U = U_h$, $M_1 = M_{hh}$ and so on. The model can be closed by noticing that (i) due to the nature of the bargaining solution retained, the surplus is equally split between agents, (ii) the sum of the compensations within a given match is equal to the output produced, and (iii) α can be calculated separately for each possible equilibrium considered. Calculations are available upon request. Once the value functions are determined, one can obtain the existence conditions by checking when the match surpluses are consistent with the partners' matching decisions.