Increasing Returns, Long-Run Growth and Financial Intermediation

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Abstract

This paper identifies a novel connection between the banking sector and economic growth. I consider strategic competition among banks in an economic growth model with externalities. The allocation delivered by the banking sector is proven to be different from that in the Walrasian equilibrium, and Pareto superior to it in most cases. This result challenges prevailing views in three literatures. The banking literature has usually assumed some frictions in the economy so that banks can emerge endogenously. Here I show that such assumptions are not necessary for the existence of banks. The literature on strategic intermediaries argues that the equilibrium achieves the Walrasian equilibrium at best. Here I show that the equilibrium delivered by strategic competition among banks often achieves an allocation that is Pareto superior to the Walrasian equilibrium. Finally, the literature of new growth theories with externalities has been concerned with lack of incentives for nonrival goods and inefficiency of the decentralized equilibrium. In some cases, authors have had to assume monopolistic competition in order to sustain economic growth. This paper shows that a decentralized, competitive economy can pay rewards for nonrival goods. In particular, it achieves the Pareto optimal allocation in the widely used case where the production function exhibits the constant returns to the accumulated capital.

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1 Introduction

This paper identifies a novel connection between the financial sector and economic growth. I consider strategic competition by banks in the capital market using the economic growth model with externalities. The resulting allocation might appear to be the same as the Walrasian equilibrium that the literature often uses. However, the allocation delivered by a strategically competitive banking sector is proven to be different from, or even Pareto superior to, that of the Walrasian equilibrium. This result challenges prevailing views in three literatures: banking, strategic intermediation, and economic growth.

The banking literature is concerned with the role of banks and the types of economies in which banks are active. Banks emerge to alleviate frictions such as transaction costs and private information[1]. This is because in an economy with complete markets and no source of friction banks seem to play no role. However, this will be shown not to be true.

In order to compare banks with a market, a clear specification is required for a model in which banks and markets function. I follow a traditional concept of the market, following Arrow and Debreu (1954). Consumers and producers together with an auctioneer achieve a Walrasian equilibrium as a consequence of strategic interactions.

Several papers have attempted to replace the Walrasian auctioneer by strategic firms or middlemen. In particular, Townsend (1983), Stahl (1988) and Yanelle (1998) study strategic competition of middlemen in a frictionless economy[2]. Their common concern is whether strategic intermediaries achieve the Walrasian equilibrium. Results are mixed. Townsend (1983) shows positive results in an exchange economy. In a partial equilibrium framework, Stahl (1988) shows mixed results that depend on the specification of the game, and Yanelle (1998) reports negative ones: the allocation is inefficient. To my knowledge, the literature has not yet addressed the issue in a general equilibrium with production[3], not to mention, with growth and externalities.

The literature on strategic intermediation discusses middlemen in various contexts. The focus of this paper is on financial activity, specifically, the conversion of savings to capital. There are two ways to conduct this activity: direct finance and indirect finance. Banks engage in indirect finance by collecting deposits from consumers and lending them to firms. Firms can raise their capital directly by issuing bonds. This is called direct finance[4].

[1] Among others, banks emerges as a result of costly state verification in Townsend (1979), as delegation of monitoring in Diamond (1984), as a coalition for project selection in Boyd and Prescott (1986), and as liquidity provision in Diamond and Rajan (1998). Also see extensive discussion in Freixas and Rochet (1997).

[2] Townsend (1978) addresses the same issue but in a economy with transaction costs. Yanelle (1997) studies an economy with informational problems.

[3] Townsend (1978) studies a general equilibrium model with production but also with transaction costs.

[4] In the real world, manufacturing firms sometimes engage in lending to other manufacturing firms. This paper regards inner teams of these firms dealing with such indirect financial services as financial intermediaries. A firm in this paper is regarded as a production unit and

Indirect finance is one of the fundamental roles, or even the very definition, of a banking sector. It is different from direct finance in two aspects. First, indirect finance encompasses both raising and lending funds, while direct finance involves only raising them. Second, the amount of deposits that a bank collects from consumers need not be the same as the amount of lending from the bank to firms. The bank can adjust its balance sheet in the interbank market. On the other hand, a firm has to use the funds it collects. In other words, indirect finance breaks the link between the sources and uses of funds.

When a firm directly raises capital, it takes the use of capital into account. Demand for capital, then, depends on the marginal productivity of capital. In this case, the Walrasian equilibrium prevails.

On the contrary, the possibility of adjustment of funds between sources and uses may give incentives to banks to collect as much deposits as they can. Banks can and will compete more aggressively for savings than Walrasian firms would do. This leads to an allocation of output that is favorable to investors. As a result, equilibrium is characterized by a higher interest rate, higher savings, and a higher growth rate as compared to the Walrasian equilibrium.

In much current growth literature, starting with Romer (1986), investment by one firm increases other firms' productivity. This Marshallian externality is the key characteristic inherited by many growth models, including models with differentiated goods. It is well known that the Walrasian equilibrium in these models cannot achieve the first-best Pareto optimal allocation. This is because, as Romer (1990a,b) argues, nonrival goods such as ideas cannot be rewarded in an economy with marginal pricing. Papers that admit externalities[5] have, then, been focusing on monopolistic competition. However, these models are criticized for the lack of empirical evidence of monopolistic competition[6], and for philosophical questions about the necessity of monopoly for economic growth[7].

In the proposed allocation delivered by a competitive banking sector in this paper, banks force firms to invest more than is suggested by the private marginal product of capital. The banking sector acts as a disciplining device for each firm to prevent free-riding on externalities created by other firms. As a result, rewards are paid to nonrival goods. Hence the allocation is often Pareto superior to the Walrasian allocation[8].

This paper should be distinguished from a closely related literature that studies the consequences of an exogenous structure of the financial markets, such as transaction costs and agency problems[9], on economic growth. As stressed

is not allowed to lend capital to other production units by definition.

[5]Recent empirical studies using cross-country data suggest positive spill-overs of technology among countries while admitting persistent productivity gaps. See Coe and Helpman (1995a,b), Klenow and Rodriguez-Clare (1997a,b) and Prescott (1997).

[6]For example, Basu (1995) finds no evidence for large positive profits of firms in U.S. data.

[7]Boldrin and Levine (1997) establish a Schumpeterian growth model with a perfect competitive market and linear technology.

[8]There are empirical studies supporting a positive role of the financial sector on economic growth. See, for example, King and Levine (1993) and Rajan and Zingales (1998a).

[9]See a seminal paper on financial development and growth written by Greenwood and Jovanovic (1990), and its generalization in a detailed study by Townsend and Ueda (1999).

at the outset, the models in this paper do not assume any transaction costs or informational problems. Therefore, the consequences of any such frictions in the capital market could be studied in future work within the framework proposed in this paper.

Section 2 presents a simple two-period example. A generalized economy is presented in section 3. It is an infinite period one sector growth model with general utility function. The technology exhibits Marshallian externality with constant returns to accumulated capital. Section 4 discusses the policy implications of opening a country to the international capital flows. Section 5 examines the case of other institutional settings. Section 6 studies the case of decreasing returns to capital accumulation, and the case with elastic labor supply. Finally, section 7 concludes.

2 A Simple Example

2.1 Financial Activity

Let me clarify the financial activity first, and then present a simple two-period example.

Investment must be financed from savings. This conversion from savings to capital is the fundamental role, or the very definition, of finance. In this paper, on the contrary, if an individual owns a firm exclusively, and only he invests in it, then no financial activity is involved.

I introduce two institutions under which the financial activity is carried out. One is a bond market, where a Walrasian auctioneer is assumed to clear the capital market as shown below.

$$Savings \longrightarrow auctioneer \longrightarrow Capital$$

The other is a banking sector, in which many banks labeled $\{1, 2, \cdots, H\}$ strategically clear the capital market as shown below.

$$Savings \longrightarrow \boxed{\begin{array}{c} 1 \\ \vdots \\ H \end{array}} \longrightarrow Capital$$

In the real world, we observe moneylenders, wealthy financiers, and large finance departments in manufacturing firms. However, according to my model, the financial activity must be distinguished clearly from manufacturing activity and consumers' behavior.

Firms are engaged in manufacturing activity, although they may possibly take part in direct finance, that is, they may raise capital by issuing bonds or equity. However, if some entities borrow and lend funds, I define them as banks.

Also see Aghion and Howitt (1998) for extensive discussion of agency concerns in Schumpeterian growth models. From the corporate finance literature, see Gertler (1988) and Rajan and Zingales (1998b) for an extensive discussion on how the financial system affects the economy.

Similarly, households decide on how much to save using the financial products that financial intermediaries and firms offer. If some entities design financial contracts and offer them to firms, I will call such entities banks.

2.2 Simple Economy

One consumer lives two periods. Let c_1 and c_2 denote consumption at period 1 and 2, respectively. His life time utility is

$$\log(c_1) + \beta \log(c_2). \tag{1}$$

His initial wealth is m_1. He consumes c_1 from m_1, and the remaining s will be saved:

$$c_1 + s = m_1. \tag{2}$$

Savings will be invested in the production process by firms, which return an income of m_2 to consumers at the beginning of period 2. The consumer's budget constraint in the period 2 is thus

$$c_2 \leq m_2. \tag{3}$$

In period 1, two firms, $j = 1, 2$, invest capital into production process and at the beginning of period 2, output of firm j, y_j, is realized. Production is characterized by a simple Romer (1986) type technology, which exhibits a Marshallian externality. Productivity of each firm depends not only on its own investment k_j but also on the other's k_{-j},

$$y_1 = Ak_2^{1-\alpha}k_1^{\alpha}, \tag{4}$$

and

$$y_2 = Ak_1^{1-\alpha}k_2^{\alpha}, \tag{5}$$

Let r be an interest rate. Profits of these firms are

$$w_1(k_1, k_2, r) = Ak_2^{1-\alpha}k_1^{\alpha} - rk_1, \tag{6}$$

and

$$w_2(k_2, k_1, r) = Ak_1^{1-\alpha}k_2^{\alpha} - rk_2. \tag{7}$$

We assume that the ownership of firms is allocated to the consumer. Profit income of the consumer is then

$$w(r, k_1, k_2) \equiv w_1(r, k_1, k_2) + w_2(r, k_1, k_2). \tag{8}$$

These constitute income in the period 2, hence the budget constraint (3) in the period 2 becomes

$$c_2 \leq m_2 = rs + w(r, k_1, k_2). \tag{9}$$

By substituting (2) and (9) into (1), the life time utility of a representative consumer becomes:

$$U(s, k_1, k_2, r) = \log(m - s) + \log(rs + w(r, k_1, k_2)). \tag{10}$$

In this section, then we assumed that firms allocate their output directly to consumers as return to capital and as profit income.

The economy-wide resource constraints are: (i) capital must be converted from savings,

$$k_1 + k_2 \leq s, \tag{11}$$

and (ii) consumption at 2 is bounded by total output,

$$c_2 \leq y_1 + y_2. \tag{12}$$

There are well-known results regarding the first best Pareto optimal allocation and the Walrasian equilibrium.

2.3 First Best Allocation

The first best allocation is given by solving for society-wide efficiency of production and then the corresponding optimal consumption amounts. Given the resource constraint (11) with any savings amount s, the efficient production for the whole economy is obtained by solving the following Lagrangian with the Lagrange multiplier λ_p,

$$\max_{k_1, k_2} Ak_2^{1-\alpha} k_1^{\alpha} + Ak_1^{1-\alpha} k_2^{\alpha} + \lambda_p(s - k_1 - k_2). \tag{13}$$

The FOC for firm 1 is

$$\alpha A k_2^{1-\alpha} k_1^{\alpha-1} = \lambda_p, \tag{14}$$

and that for firm 2 is

$$\alpha A k_1^{1-\alpha} k_2^{\alpha-1} = \lambda_p, \tag{15}$$

These give $k_1 = k_2 \equiv k = s/2$ for any s. The production technology for the economy then becomes

$$y_1 + y_2 = 2Ak = As. \tag{16}$$

Hence the return from s is A. At the rate A, firms have no positive profits.

The optimal consumer's choice with this specification $r = A$ and $w = 0$ is to maximize

$$\log(m - s) + \beta \log(As). \tag{17}$$

The FOC is

$$\frac{1}{m - s} = \frac{\beta}{s}. \tag{18}$$

Hence Pareto optimal savings amount is

$$s_p = \frac{\beta}{1 + \beta} m. \tag{19}$$

2.4 Walrasian Equilibrium

Next, we consider a Walrasian equilibrium. Here, each firms and the consumer are assumed to act as price takers and to take other firm's investment as given.

Firm 1 decides on its investment given the one from firm 2. The FOC is:

$$\alpha A k_2^{1-\alpha} k_1^{\alpha-1} = r. \tag{20}$$

This can be rewritten as

$$k_1 = \left(\frac{\alpha A}{r}\right)^{\frac{1}{1-\alpha}} k_2. \tag{21}$$

Similarly, for firm 2,

$$k_2 = \left(\frac{\alpha A}{r}\right)^{\frac{1}{1-\alpha}} k_1. \tag{22}$$

Apparently, $r = \alpha A$ is the only positive interest rate that satisfies both FOCs of firm 1 and 2. Hence the interest rate is αA.

Given symmetric investment $k_1 = k_2 = k$, the equilibrium profit will be

$$w_j = (1-\alpha)Ak, \tag{23}$$

for $j = 1, 2$. Given the interest rate αA and the equilibrium investment by firms k, the consumer's optimal choice is to maximize

$$\log(m-s) + \beta \log(\alpha A s + 2(1-\alpha)Ak). \tag{24}$$

The FOC is

$$\frac{1}{m-s} = \beta \frac{\alpha A}{\alpha A s + 2(1-\alpha)Ak}. \tag{25}$$

Savings becomes a function of income and the firms' investments:

$$s = \frac{\beta}{1+\beta} m - \frac{2(1-\alpha)}{(1+\beta)\alpha} k. \tag{26}$$

But the capital market is cleared in an equilibrium, i.e., savings must be equal to investments,

$$s = 2k. \tag{27}$$

Hence the equilibrium savings amount is

$$s_w = \frac{\beta\alpha}{1+\beta\alpha} m. \tag{28}$$

Comparing (19) and (28), note that $s_p > s_w$. Under-investment and under-savings are the results of the externality.

2.5 Strategically Intermediated Economy

This section provides some intuition for the main results of the paper. Consider the case in which two banks, $h = 1, 2$, instead of an auctioneer, clear the capital market strategically. Banks compete in both the deposit and loan markets. Depending on the result of competition in the deposit market, however, the market structure of the loan market will be either monopolistic or competitive. The economy is described as a two-stage game[10]. The first stage is for the deposit market and the second stage for the loan market.

In the deposit market, bank h just offers the deposit interest rate r_h, and competes with the other bank à lá Bertrand[11]. In the loan market, bank h offers a loan contract, which consists of loan interest rate R_{hj} and a loan amount k_{hj} to firm j, though the loan amount may not always be specified.

First, we consider the loan market with a monopolist. This is the case when a bank captures all the savings in the deposit market. The monopolist solves

$$\max_{R_1, k_1, R_2, k_2} R_1 k_1 + R_2 k_2 \tag{29}$$

subject to a participation constraint for firm 1,

$$A k_2^{1-\alpha} k_1^{\alpha} - R_1 k_1 \geq 0, \tag{30}$$

that for firm 2

$$A k_1^{1-\alpha} k_2^{\alpha} - R_2 k_2 \geq 0, \tag{31}$$

and the balance sheet condition of the bank,

$$k_1 + k_2 \leq S, \tag{32}$$

where S stands for total savings.

Since the monopoly bank can internalize the externality, his revenue becomes economy-wide output as in the first best regime (16). He can charge the highest technologically feasible return A. In other words, the monopolist actually maximizes economy-wide output

$$\max_{k_1, k_2} A k_2^{1-\alpha} k_1^{\alpha} + A k_1^{1-\alpha} k_2^{\alpha} \tag{33}$$

subject to the balance sheet condition (32). The resulting loan contract is to charge the highest technologically feasible return A and specifying the loan amount $S/2$ to each firm lending out all the deposits, i.e., $(R_j, k_j) = (A, S/2)$ for $j = 1, 2$.

Next, we consider the loan market with two banks. This is the case when both banks survive in the deposit market. Competition in the loan market

[10] It will be clear, however, that the case of simultaneous competition is also analyzed similarly.

[11] This is not assumed in the general model, where the price competition will be shown to be the result of optimal choice.

brings the Walrasian result: the private marginal product of capital is equal to the loan rate. This leads to $R = \alpha A$. This is because firms maximization problem is the same as in the Walrasian economy and their FOC is given by (21) and (22), if firms chooses quantities of loans and hence capital. Moreover, loan contracts by banks specifying the loan amount will be beaten by contracts without specification of amounts.

In the deposit market, however, the only candidate for an equilibrium deposit rate is $r = A$. To see this, note that $r = \underline{A} < A$ cannot be an equilibrium, because there always exists an opportunity to become the monopoly bank by offering a deposit rate slightly higher than $\underline{A}$, say $\underline{A}+\epsilon$. Note that $r > A$ cannot be an equilibrium, because the highest loan rate is A.

In brief, in order to try to capture the monopoly profit, both banks bid up until $r = A$ in the deposit market. But if both banks survive, their competition leads the loan rate in lending to be $R = \alpha A < A$ as mentioned above, and hence banks realize negative profits. Thus banks do not want to bid up the deposit rate until $r = A$, but as shown, $r = A$ is the only candidate for the equilibrium deposit rate. Therefore, there is no Nash equilibrium in this game.

Here then I introduce an interbank market. Bank h can adjust its fund size via the interbank market, but it has to balance its balance sheet. The asset side of the balance sheet of bank h can be decomposed into total lending to firms, $k_h \equiv \sum_{j=1}^{2} k_{hj}$, and an interbank lending I_{Ah}. The liability side consists of total deposits from the consumer, s_h and interbank borrowing I_{Bh}. Let us define net borrowing in the interbank market as $B_h = I_{Bh} - I_{Ah}$, and the interbank market rate submitted by bank h as ρ_h.

I assume that banks exert some efforts to clear the interbank market. To this end, I allow banks a second chance to offer contracts[12]. They can offer tentative contract in the morning and then offer decisive contracts in the afternoon. More specifically:
(i) In the morning, banks $h = 1, 2$ offer tentative loan contracts (R^1_{hj}, k^1_{hj}) to firm $j = 1, 2$.
(ii) Firms submit their tentative decision based on the offered contracts to banks on the offers they face[13].
(iii) Banks submit tentative interbank rates and net borrowing amounts (ρ_h, B_h) to the interbank market, and a tentative match of demand and supply is undertaken. It is not guaranteed that both banks can always balance their balance sheet. Note that if all banks do balance their balance sheet, the interbank market is cleared.
(iv) If a bank can balance its balance sheet, it sends a confirmation letter to firms and the interbank market to finalize its tentative contracts.
(v) If all banks send a confirmation letter, the firms accepts them and all contracts are finalized. Otherwise, firms expect better contracts in the afternoon

[12]Without the second chance, the result is the same as the economy without the interbank market.

[13]This assumption implies that the firm's decision is myopic. In other words, firms do not consider the possibility of better deal in the afternoon. This assumption simplifies the analysis here, but it is not assumed in the general model below.

and reject them.
(vi) If the contracts are not finalized in the morning, banks offers the decisive contract (R^2_{hj}, k^2_{hj}) to firms in the afternoon. This contract is allowed to differ from the original contract[14]. Note that in the afternoon a bank's fund is restricted to the deposits that the bank has taken from consumers, i.e., $k^2_h = s_h$.

I denote this procedure *strategic tâtonnement*. It maintains the assumption in the Walrasian tâtonnement that a contract is not finalized until the demand and supply match, but differs in that banks strategically choose price and quantity (in the interbank market) simultaneously.

In the loan market and the interbank market, banks maximize their own profits given their previously determined deposit amounts and deposit interest rate. Knowing that Ak_j is the technologically feasible revenue from firms, both banks want to offer the loan contract $(R^1_h = A, k^1_{hj} = S/2)$, that is, the monopoly solution and Pareto optimal. This is true even if their share in deposit market is different, say $s_2 < S/2 < s_1$. Note that net transaction in the interbank market is $S/2 - s_2 = s_1 - S/2$. However, when bank 1 offers this contract, bank 2 with less fund size may earn more revenue by offering another contract to firm 2 $(R^1_2 = A + \epsilon, k^1_{22} = s_2)$ without using the interbank market.

This is actually profitable for bank 2, if this contract is finalized. But bank 1 can detect this "opportunistic" behavior of bank 2. As long as bank 1 offers the Pareto optimal contract $(R^1_1 = A, k_{1j} = S/2)$, and offers funds $s_1 - S/2 > 0$ in the interbank market, bank 2's deviant strategy always prevents the interbank market from being cleared, because the deviant always prefers under-investment in order to free-ride on the other firm's investment.

Here, an equilibrium strategy resembles a *Tit-for-Tat* strategy: in the morning banks offer Pareto optimal contracts as the target, and the Walrasian contracts as the punishment in the afternoon:

$$\{(R^1_h = A, k^1_{hj} = S/2), (\rho_h = A, B_h = S/2 - s_h), (R^2_h = \alpha A, k^2_{hj} = N.S.)\} \quad (34)$$

where $N.S.$ stands for "not specified".

If both banks offer this contract, firms will accept it and the interbank market is cleared at once[15]. If either bank deviates, this strategy brings less profits for both banks than the target contract because the solution reverts to the Walrasian one. In other words, there is no profitable deviation. Therefore, this simple *Tit-for-Tat* like strategy is a Nash equilibrium[16].

This supports the main result: there exists a unique Nash equilibrium out-

[14]Here the recontracting is allowed only once. In the general model many sessions are allowed.

[15]If firms are not myopic, they reject the offer and wait next period to get a lower loan rate αA. Because of this, I assumed here that firms are myopic. In the general model, firms are not myopic and thus more careful argument is required.

[16]Other punishment strategies like $R^2_h = 0$ can support a Nash equilibrium, too.

come with the interbank market[17]:

$$r_h = R_h = A, \quad S(A) = 2k_j = \frac{\beta}{1+\beta}m. \tag{35}$$

Note that this outcome is Pareto optimal.

3 The Model

This section presents a more rigorous treatment of the game in an environment which is common to the economic growth theory, that is, an infinite period model with a general utility function and a Cobb-Douglas production function. Other differences from the simple example above are that firms' strategic behavior is more carefully taken care of, and that the strategic tâtonnement is repeated many times.

Again, I first describe the economy without any intermediaries at all and then display the Walrasian equilibrium and the symmetric first best allocation. These are benchmarks for the strategically intermediated economy, which I describe later in this section.

3.1 Demography, Preferences and Technology

The economy is populated by consumers, indexed by $i \in \{1, \cdots, I\}$, and firms, indexed by $j \in \{1, \cdots J\}$.

All consumers are identical in preferences. A consumer i who has wealth m_{it} at the beginning of period t, decides on consumption c_{it} and savings s_{it} in period t. Consumption and savings must satisfy the budget constraint:

$$c_{it} + s_{it} \leq m_{it}. \tag{36}$$

Let $\beta \in (0, 1)$ be a discount rate, and $u(\cdot)$ be the period-utility function with the property that $u : \mathbb{R}_+ \to \mathbb{R}$, $u \in C^2$, $u' > 0$ and $u'' < 0$. Given an initial wealth m_{i0}, consumer i maximizes his utility[18]:

$$\sum_{t=0}^{\infty} \beta^t u(c_{it}). \tag{37}$$

I assume that initial wealth m_{i0} is equal for all consumers and that the ownership of firms is allocated to all consumers equally at the initial date. For simplicity, I assume that the ownership structure remains unchanged over time[19]. Let w_{jt} denote profit income of the j-th firm, and ψ_i^f be the ownership

[17] Equilibrium strategies can be many, although the equilibrium realizations of interest rates, savings and investment amounts are unique.

[18] In order to define different stages within the same period, I formulate the problem in discrete time.

[19] Homogeneous consumers and no technological shocks provide no reason for consumers to trade these ownership shares.

of j-th firm by i-th consumer in period t. The feasibility conditions are, for all $j \in J$,

$$\sum_{i=1}^{I} \psi_{ij}^{f} = 1. \tag{38}$$

Let w_{cit} denote the total profit income of consumer i at date t. It is defined as

$$w_{cit} \equiv \sum_{j=1}^{J} \psi_{ij}^{f} w_{jt}. \tag{39}$$

Wealth at $t+1$, m_{it+1}, consists of profit income and the gross return on savings at t. Wealth of consumer i at t can then be written as

$$m_{it+1} = r_{it} s_{it} + w_{it}, \tag{40}$$

where r_{it} denote the gross interest rate on savings.

A firm j raises capital at the beginning of each period and returns it as output with gross borrowing rate R_{jt}. This makes firms' decisions period-by-period decisions. The production technology is a simple version of Romer (1986). Firms have an identical technology, which exhibits a Marshallian externality: productivity of each firm depends on the average capital level. Let y_{jt} denote output of firm j. A firm produces its output from capital k_{jt}, given the population average capital[20] K_t, as

$$y_{jt} = f(k_{jt}, K_t) \equiv A K_t^{\eta} k_{jt}^{\alpha}. \tag{41}$$

In the base model below, I consider the case of $\eta = 1 - \alpha$. Other cases are also discussed later.

Let R_{jt} be the borrowing rate of capital that firm j pays at t. Profits of firm j are then

$$w_{jt}(k_{jt}, k_{-jt}, R_{jt}) = A K_t^{1-\alpha} k_{jt}^{\alpha} - R_{jt} k_{jt}. \tag{42}$$

Profit income of consumer i at t then becomes a function of $\{k_{jt}, R_{jt}\}_{j=1}^{J}$:

$$w_{cit}(\{k_{jt}, R_{jt}\}_{j=1}^{J}) \equiv \sum_{j=1}^{J} \psi_i^f w_{jt}(\{k_{jt}, R_{jt}\}_{j=1}^{J}). \tag{43}$$

Wealth at period $t+1$ becomes a function of $(r_{it}, s_{it}, \{k_{jt}, R_{jt}\}_{j=1}^{J})$.

$$m_{it+1}(r_{it}, s_{it}, \{k_{jt}, R_{jt}\}_{j=1}^{J}) = r_{it} s_{it} + w_{cit}(\{k_{jt}, R_{jt}\}_{j=1}^{J}). \tag{44}$$

[20] To avoid circularity associated with finite number of the firm, it is assumed to take average of capital from the remainder of the firms, $-j$.

The economy-wide resource constraints are:
(i) capital must be converted from savings for each t,

$$\sum_{j=1}^{J} k_{jt} \leq \sum_{i=1}^{I} s_{it}, \tag{45}$$

(ii) consumption and savings at t are bounded by total output at $t-1$,

$$\sum_{i=1}^{I} (c_{it} + s_{it}) \leq \sum_{j=1}^{J} y_{jt-1}, \tag{46}$$

(iii) and a fixed point condition:

$$K_t = \frac{1}{J-1} \sum_{l \in -j} k_{lt}. \tag{47}$$

3.2 Walrasian Equilibrium

I consider a Walrasian equilibrium as a (generalized) game in the style of Arrow-Debreu (1954), and as defined recursively in the style of Prescott and Mehra (1980). In addition to I consumers and J firms, I add one more agent: an auctioneer.

The auctioneer's strategy is to set the interest rate $r_t \in \mathbb{R}_+$ so as to maximize her payoff, which is the value of excess demand in capital market,

$$\pi_a(r_t, s_{it}, k_{jt}) = r_t \left(\sum_{j=1}^{J} k_{jt} - \sum_{i=1}^{I} s_{it} \right). \tag{48}$$

The strategy for firm j is investment $k_j \in \mathbb{R}_+$, and its objective is to maximize its profit. The strategy for consumer i is to set savings $s_{it} \in [0, m_{it}]$ so as to maximize his utility. I denote $S(r_t)$ for aggregate savings function.

Since the auctioneer's problem is the same for every period, and the production function is constant returns to accumulated capital k, consumers' expectation on an equilibrium interest rates are the same in all periods, $E_t[r_{t+l}] = r$ for all $l \geq 0$. Then consumer's problem can be written recursively as[21]

$$V_w(m_{it}) = \max_{s_{it} \in [0, m_{it}]} u(m_{it} - s_{it}) + \beta V_w(m_{it+1}) \tag{49}$$

This recursive formulation of consumer's problem enables us to describe the dynamic economy as if it is a static game. Time subscripts are often dropped hereafter.

[21] It is not until recently that existence and uniqueness of value functions arising from unbounded return functions in perpetual growth model have been given rigorous treatment. See Alvarez and Stokey (1998), Nakajima (1999) and Townsend and Ueda (1999).

Definition 1. A *Walrasian economy* is the game Γ_W, which consists of $I+J+1$ agents (I consumer, J firms, and one auctioneer), their strategy sets (savings, investments, and interest rate), and their utilities (V_w, w_j and π_a):

$$\Gamma_W = (I + J + 1, ([0, m], \mathbb{R}_+, \mathbb{R}_+), (V_w, w_j, \pi_a)). \tag{50}$$

Consumer i's best response is defined as

$$BR_{ci}(\{k_j\}_{j=1}^J, r) = \arg \max_{s_i \in [0, m_i]} u(m_i - s_i) + \beta V_w(rs_i + w_{ci}). \tag{51}$$

Firm j's best response is

$$BR_{fj}(\{k_j\}_{l \in -j}, r) = \arg \max_{k_j} AK^{1-\alpha} k_j^\alpha - rk_j, \tag{52}$$

which can be simplified to

$$BR_{fj}(K, r) = \left(\frac{\alpha A}{r}\right)^{\frac{1}{1-\alpha}} K. \tag{53}$$

Auctioneer's best response is defined as

$$BR_a(\{s_i\}_{i=1}^I, \{k_j\}_{j=1}^J) = \arg \max_r r \left(\sum_{j=1}^J k_j - \sum_{i=1}^I s_i\right), \tag{54}$$

which can be written as

$$\begin{aligned} BR_a(\{s_i\}_{i=1}^I, \{k_j\}_{j=1}^J) &\equiv 0 \quad \text{if} \sum_{j=1}^J k_j - \sum_{i=1}^I s_i < 0, \\ &\equiv \overline{R}_+ \quad \text{if} \sum_{j=1}^J k_j - \sum_{i=1}^I s_i = 0, \\ &\equiv \infty \quad \text{if} \sum_{j=1}^J k_j - \sum_{i=1}^I s_i > 0. \end{aligned} \tag{55}$$

Let the best response correspondence $BR(\{s_i\}_{i=1}^I, \{k_j\}_{j=1}^J, r)$ for the game Γ_W be defined as a Cartesian product of each best response:

$$\begin{aligned} &BR(\{s_i\}_{i=1}^I, \{k_j\}_{j=1}^J, r) \\ &= \Pi_{i=1}^I BR_{ci}(\{k_j\}_{j=1}^J, r) \times \Pi_{j=1}^J BR_{fj}(K, r) \times BR_a(\{s_i\}_{i=1}^I, \{k_j\}_{j=1}^J). \end{aligned} \tag{56}$$

Definition 2. $(\{s_i^w\}_{i=1}^I, \{k_j^w\}_{j=1}^J, r^w)$ is a *Nash equilibrium* for the game Γ_W if it is a fixed point of the best response correspondence for the game:

$$(\{s_i^w\}_{i=1}^I, \{k_j^w\}_{j=1}^J, r^w) \in BR(\{s_i^w\}_{i=1}^I, \{k_j^w\}_{j=1}^J, r^w). \tag{57}$$

Definition 3. An *equilibrium* of the Walrasian economy is a Nash equilibrium of the game Γ_W.

Proposition 1. *The Walrasian economy Γ_W has a unique equilibrium except trivial one $(s = k_1 = k_2 = 0)$ such that*

$$r^w = \alpha A, \tag{58}$$

and

$$k_j^w = \frac{S(\alpha A)}{J}. \tag{59}$$

Proof. First I show that the proposed solution is a fixed point of BR. Given $k_j^w = \frac{S(\alpha A)}{J}$, the auctioneer's best response is any number in $\overline{R}_+$. Hence $r^w = \alpha A \in BR_a(s^w, k^w, r^w)$. Given $r^w = \alpha A$, firms' best response is $k_j = K \in \mathbb{R}_+$. Hence $k_j^w = K^w = S/J \in BR_{fj}(s^w, k^w, r^w)$. Given $r^w = \alpha A$, consumer's best response is $S(\alpha A)$ by construction.

Next, I show it is unique except for $(s = k_1 = k_2 = 0, r = 0)$. $r < \alpha A$ cannot be an equilibrium. Since $k_j > K$ by BR_{fj} and thus no fixed point in $\mathbb{R}_{++}$. $r > \alpha A$ cannot be an equilibrium either except for $(s = k = 0)$. Because $k_1 < k_2$ by BR_{f1} and $k_2 < k_1$ by BR_{f2} and thus the fixed point is $k_j = K = 0$. Then $r = 0$ if $s > 0$. But if $r = 0$, $s = 0$. Hence if $r < \alpha A$, $s = k_1 = k_2 = 0$ is the only fixed point. □

For CRRA utilities, $u(c) = c^{1-\sigma}/(1-\sigma)$, I can further specialize the result.

Corollary 1. *The growth rate for the case of CRRA utility is*

$$g_w = (\beta \alpha A)^{1/\sigma}. \tag{60}$$

Proof. It is immediate from the Euler equation derived from (49) with $r = \alpha A$.

$$u'(c_{it}) = \beta \alpha A u'(c_{it+1}). \tag{61}$$

□

3.3 First Best Allocation

The first best solution is a natural benchmark for a welfare comparison of several institutional settings. Let us denote $s = \{s_{it}\}_{i=1}^I$ and $k = \{k_{jt}\}_{j=1}^J$ as the vectors of savings and investment, respectively.

Definition 4. The *symmetric first best Pareto optimal allocation* is (s^p, k^p) that maximizes the equally weighted sum of the consumer's utility

$$\sum_{i=1}^{I} \sum_{t=1}^{\infty} \beta^t u(c_{it}), \tag{62}$$

subject to technological constraints (41), and economy-wide resource constraints, (45), (46) and (47).

Proposition 2. *The symmetric first best Pareto optimal allocation is*

$$s_{it}^p = \frac{S}{I}, \tag{63}$$

and

$$k_{jt}^p = \frac{S}{J}. \tag{64}$$

Proof. Given the resource constraint (45) with any total savings amount S, the efficient production for the whole economy is obtained by solving the Lagrangian at each t with Lagrange multiplier λ_p:

$$\max_{\{k_j\}_{j=1}^J} \sum_{j=1}^J AK^{1-\alpha}k_j^\alpha + \lambda_p(S - \sum_{j=1}^J k_j). \tag{65}$$

For each $j \in J$, the FOC is

$$\alpha AK^{1-\alpha}k_j^{\alpha-1} = \lambda_p. \tag{66}$$

This gives $k_j = K = \frac{S}{J}$ for any S. The economy-wide production technology then becomes

$$\sum_{j=1}^J Ak = JAk = AS. \tag{67}$$

Hence the return on savings is A. By definition, the total savings amount is $S(A)$. □

Using the socially efficient return A, representative consumer's problem corresponding to the social planner's problem can be written recursively as

$$V_p(m) = \max_{s_t} u(m-s) + \beta V_p(m'), \tag{68}$$

where $m' = As$. Using this, for CRRA utilities $u(c) = c^{1-\sigma}/(1-\sigma)$, I can further specialize the result.

Corollary 2. *For the case of CRRA utility functions, the growth rate is*

$$g_p = (\beta A)^{1/\sigma}. \tag{69}$$

Proof. This follows immediately from the Euler equation associated with (68):

$$u'(c_t) = \beta Au'(c_{t+1}). \tag{70}$$

□

Note that the growth rate of Walrasian equilibrium (61) is lower than that of Pareto optimal level (70), i.e., $g_w < g_p$.

3.4 Strategically Intermediated Economy

3.4.1 Basic Structure

Now I consider the case in which banks replace the auctioneer, and clear the capital market strategically period by period. The assumption that banks live one period only is in line with the growth literature in which firms live one period only [22].

Banks compete in both the deposit and loan markets. Although traditionally strategic competition has been thought of in terms of Bertrand and Cournot competition, this paper considers a more general competitive notion, that is, competition in both price and quantity. It will be shown, however, that the competition of banks in the deposit market endogenously becomes Bertrand competition.

There exists H banks. A deposit contract of bank $h \in \{1, \cdots, H\}$ to consumer i consists of a deposit interest rate r_{hi} and a recommended savings amount s_{hi}. A loan contract of bank h to firm j consists of a loan interest rate R_{hj}, and a recommendation of loan amount k_{hj}.

Bank h has to balance its balance sheet. Bank h's asset can be decomposed into total loan to firms, $k_h \equiv \sum_{j=1}^{J} k_{hj}$, and an interbank loan I_{Ah}. The liability side consists of total deposits from consumers, $s_h \equiv \sum_{i=1}^{I} s_{hi}$ and interbank borrowing I_{Bh}. Define net borrowing on the interbank market as $B_h = I_{Bh} - I_{Ah}$, and the interbank market rate submitted by the bank h as ρ_h. Naturally, there is a balance sheet constraint for each bank:

$$B_h = k_h - s_h. \tag{71}$$

The ownership of banks is assumed to be allocated equally to consumers and is left unchanged over time as in the case of firms. The profit of bank is denoted as π_h,

$$\pi_h = \sum_{j=1}^{J} R_{hj}\tilde{k}_{hj} - \rho_h B_h - \sum_{i=1}^{I} r_{hi}\tilde{s}_{hi}, \tag{72}$$

where $\tilde{k}_{hj}$ and $\tilde{s}_{hi}$ denotes realized transactions, rather than the contracts offered by the bank.

Let ψ_{ih}^{b} be the ownership of h-th bank by i-th consumer with $\sum_{i=1}^{I} \psi_{ih}^{b} = 1$ for all $h \in H$. Then consumer i's profit income changes from (43) to (73)

$$w_{ci} \equiv \sum_{j=1}^{J} \psi_{ij}^{f} w_j + \sum_{h=1}^{H} \psi_{ih}^{b} \pi_h. \tag{73}$$

[22] See Prescott and Mehra (1980). In game theory, this is a Markov perfect equilibrium depending only on the level of physical capital. Underlying story is that CEOs of banks do not serve infinite times, nor are selected from the same dynasty forever. This assumption implies that once an agent becomes a banker, he is very unlikely to become a banker again. Therefore, there is little incentive for banks to consider multi-period tie-ins.

I assume a two stage game in each period[23]. In the first stage, there is competition in the deposit market whereas in the second stage there is the competition in the loan market (and the interbank market). The competition in the deposit market creates two distinct market structure in the second stage: monopolistic or competitive.

Assumption 1. *Only banks with positive deposits can be active in the loan market and the interbank market.*

3.5 Deposit Market

First I describe the first stage, the deposit market. The strategy of bank h consists of the deposit rate r_{hi} and the deposit amount (recommendation) s_{hi} to consumer i, for $i = 1, \cdots I$, and denoted as $z_{h0} \equiv (r_{hi}, s_{hi})_{i=1}^{I}$. A bank need not specify each value of its strategy. In other words, "not specified" can be taken as a strategy, and it is abbreviated as $N.S.$. Hence the strategy set is defined as $Z_0 \equiv ((\mathbb{R}_+ \cup \{N.S.\}) \times (\mathbb{R}_+ \cup \{N.S.\}))^I$. Let $z_0 = \{z_{h0}\}_{h=1}^{H}$ denote the vector of the strategy over all banks.

Similarly, a consumer i's strategy is denoted as $z_c = (\{r_{hi}\}_{h=1}^{H}, \{s_{hi}\}_{h=1}^{H})$, which is chosen from the strategy set $Z_c \equiv (\mathbb{R}_+ \times \mathbb{R}_+)^H$. However, this strategy set is constrained by strategies of banks z_0. The constrained choice set of consumer i is written as $G_c(z_0)$. Let $G_{ch}(z_{h0})$ be an element of $G_c(z_0)$ corresponding to the constrained choice set of consumer i constrained by the contract offered by bank h. I assume that $s_{hi} = 0$ is always in a choice set.

$$\begin{aligned} G_{ch}(z_{bh\tau}) &\equiv r_{hi} \times \mathbb{R}_+ \quad \text{if bank } h \text{ specifies } r_{hi} \text{ only,} \\ &\equiv \mathbb{R}_+ \times (s_{hi} \cup \{0\}) \quad \text{if bank } h \text{ specifies } s_{hi} \text{ only,} \\ &\equiv r_{hi} \times (s_{hi} \cup \{0\}) \quad \text{if bank } h \text{ specifies both } R_{hj} \text{ and } s_{hi}. \end{aligned} \tag{74}$$

Note that the choice set of the last case is either (r_{hi}, s_{hi}) or $(r_{hi}, 0)$, and I label these choices "*accept*" and "*reject*", respectively. The whole constrained choice set is now defined as the Cartesian product of G_{ch} over $h \in H$:

$$G_c(z_{h0}) \equiv G_{c1}(z_{10}) \times G_{c2}(z_{20}) \times \cdots \times G_{cH}(z_{H0}). \tag{75}$$

A consumer's problem can be formulated recursively,

$$V_b(m) = \max_{z_c \in G_c(z_{h0})} u(m - s) + \beta V_b(m'), \tag{76}$$

where $m' = rs + w_c$.

Definition 5. Denote the deposit market (the first stage) as Γ_1, which consists of $(H + I)$ agents (banks and consumers), their constrained strategy space, and utilities of consumers[24]:

$$\Gamma_1 \equiv (H + I, (Z_0, G_c), V_b). \tag{77}$$

[23] It will be clear, however, simultaneous moves in both deposit and loan market could be analyzed similarly.

[24] This is not a game, just a part of a game.

A consumer's best response in the first stage is as follows. First, consumer i calculates its maximum utility from each contract offered by the banks. There are three cases.

1. For the contract $(r_{hi} = N.S., s_{hi} = s)$, consumer i's optimal choice is to set $r_{hi} = \infty$, and maximum utility is $V_b(m) = \infty$.

2. For the contract $(r_{hi} = r, s_{hi} = s)$ consumer i's maximum utility is

$$\begin{aligned} V_b(m) &= u(m-s) + \beta V_b(rs+w), \quad \text{for } r > 0, \\ &= u(m), \quad \text{for } r = 0. \end{aligned} \tag{78}$$

3. For the contract $(r_{hi} = r, s_{hi} = N.S.)$, consumer i maximizes his utility with respect to s_{hi} as in the Walrasian economy.

$$V_b(m) = \max_{s\in[0,m]} u(m-s) + \beta V_b(rs+w). \tag{79}$$

Consumer i then chooses the best contract.

3.5.1 The Second Stage with a Monopoly Bank

Now I look at the monopoly case in the loan market. This is the case where one bank captures the whole savings in the deposit market.

The monopoly bank h's strategy in the loan market is the loan contract to firm j that consists of the loan rate R_{hj} and the loan amount k_{hj}, and defined as $z_M \equiv (R_{hj}, k_{hj})$. The strategy set is defined as $Z_M \equiv ((\mathbb{R}_+ \cup \{N.S.\}) \times (\mathbb{R}_+ \cup \{N.S.\}))^J$, meaning loan rates and loan amounts all nonnegative but the bank might not specify one or the other or both in which case the firms choose.

Firm j's strategy when it faces offers from the monopoly bank is to choose $z_{Mf} \equiv (R_{hj}, k_{hj})$ from its strategy set $Z_{Mf} \equiv (\mathbb{R}_+ \times \mathbb{R}_+)$. However, this strategy set is constrained by strategies of banks z_M. The constrained correspondence is written as $G_{Mf}(z_M)$. I assume that $k_j = 0$ is always in the choice set.

$$\begin{aligned} G_{Mf}(z_M) &\equiv R_j \times \mathbb{R}_+ \quad \text{if the bank specifies } R_j \text{ only}, \\ &\equiv \mathbb{R}_+ \times (k_j \cup \{0\}) \quad \text{if the bank specifies } k_j \text{ only}, \\ &\equiv R_j \times (k_j \cup \{0\}) \quad \text{if bank } M \text{ specifies both } R_j \text{ and } k_j. \end{aligned} \tag{80}$$

Note that the choice set of the last case is either (R_j, k_j) or $(R_j, 0)$, and I label these choices "*accept*" and "*reject*", respectively.

The monopoly bank maximizes the profit, π_M given deposit amount s_{hi} per consumer and its deposit rate r_{hi},

$$\max_{\{R_{hj}, k_{hj}\}_{j=1}^J} \pi_M \equiv \sum_{j=1}^J R_{hj} k_{hj} - \sum_{i=1}^I r_{hi} s_{hi}, \tag{81}$$

subject to the resource (capacity) constraint[25]

$$\sum_{j=1}^{J} k_{hj} \leq \sum_{i=1}^{I} s_{hi}. \tag{82}$$

Definition 6. The *second stage with a monopoly bank* is the game Γ_M, which consists of one bank and J firms, their strategy sets, and their utilities.

$$\Gamma_M \equiv (1+J, (Z_M, G_{Mf}), (\pi_M, w_j)). \tag{83}$$

Lemma 1. *Nash equilibrium of* Γ_M *is characterized by the optimal decision by the monopoly bank h*

$$(i) \quad \sum_{j=1}^{J} k^*_{hj} = \sum_{i=1}^{I} s^*_{hi}, \tag{84}$$

$$(ii) \quad k^*_{hj} = K = \frac{S}{J}, \tag{85}$$

$$(iii) \quad R^*_{hj} = A, \tag{86}$$

and (iv) the decision by firms, "accept".

Proof. (i) Given $\sum_{i=1}^{I} r_{hi} s_{hi}$, profit π_h is increasing in k_{hj} for any R_{hj}. Hence the monopoly bank lend out all deposits.
(ii) Since the monopoly bank can charge as much as $AK^{1-\alpha}k_j^\alpha$, it maximizes the total output

$$\sum_{j=1}^{J} AK^{1-\alpha}k_j^\alpha. \tag{87}$$

Since k_j^α is a concave function, the monopoly bank lends equal amount of capital to every firm.
(iii) Since all firms invest the same amount, the production function of a representative firm becomes AK. Then highest interest rate that the bank can charge is A, and it is optimal for the monopolist to charge this amount to each

[25] The monopoly bank cannot lend out the capital more than the savings that it has collected in the deposit market, that is, it faces the resource (capacity) constraint (82). If the monopoly bank does not specify the amount of loan and firms demand capital more than the aggregate savings, then the monopoly bank should expect this situation and specifies the amount in the first place. Hence the monopoly bank does not observe excess demand in the equilibrium. Still, the allocation of capital and payoffs in this situation should be described in the game. The strategy space could be constrained or expanded to address the case of excess demand, but for simplicity and without loss of generality, I assume here that the monopoly bank allocate the savings equally to each firms if the aggregate demands for capital is larger than the aggregate savings.

firm.
(iv) Each firm's best response is to accept this offer, because there is no other contract available and profit is nonnegative:

$$w_j = AK^{1-\alpha}k_j^{\alpha} - Ak_j = 0. \tag{88}$$

□

3.6 Institution-Free Results of the Whole Game

Before studying the competitive second stage, I consider some results of the whole game. This is because I can characterize robust results driven by the competition in the deposit market, even without detailed study of the competitive second stage.

Let us define the set of active banks, which is a function of the strategies in the first stage (z_0, z_c): $D(z_0, z_c) \equiv \{h \in H : s_h > 0\}$. In other words, the set H is the pool of potential entrants to the banking sector, and the set D is the set of actual entrants in the banking sector. If D is singleton, the loan market is monopolized. Otherwise, it is competitive.

Let $\Gamma_C(z_0, z_c)$ denote the competitive second stage, depending on the result of the deposit market. In the competitive second stage, banks are assumed to behave non-cooperatively. By non-corporative behavior, I mean that the bank's revenue per firm is symmetric or, if it is asymmetric, the probability to achieve each asymmetric outcome is the same among banks. In other words, no bank is treated more favorably than others. Knowing that symmetric lending to its client firms is most profitable for any bank, we can summarize this assumption as follows.

Assumption 2. *For all (h, j), for all (l, k) such that $k_{hj} > 0$ and $k_{lk} > 0$, followings are true:*

$$Rev \equiv R_{hj}k_{hj} = R_{lk}k_{lk} \tag{89}$$

or

$$\begin{aligned} &\textit{if} \quad Rev_{hj} \equiv R_{hj}k_{hj} \neq Rev_{lk} \equiv R_{lk}k_{lk}, \\ &\textit{then} \quad prob(Rev_{hj})Rev_{hj} = prob(Rev_{lk})Rev_{lk}. \end{aligned} \tag{90}$$

where probabilities (prob) to obtain specific revenue are common among banks.

Definition 7. A *second stage* is the game

$$\begin{aligned} \Gamma_2(z_0, z_c) &\equiv \Gamma_M \quad \text{if } D(z_0, z_c) \text{ is singleton,} \\ &\equiv \Gamma_C(z_0, z_c) \quad \text{otherwise.} \end{aligned} \tag{91}$$

I will show a detailed description of a specific institution as the competitive second stage later.

Definition 8. A *strategically intermediated economy* is the game Γ. This is defined as the extensive game, which consists of

- the first stage Γ_1,
- the set of all possible histories for the second stage, which consist of all possible strategies in the first stage, $Z_0^H \times Z_c^I$,
- the second stage $\Gamma_2(z_0, z_c)$,

In sum

$$\Gamma \equiv (\Gamma_1, Z_0^H \times Z_c^I, \Gamma_2) \tag{92}$$

I must admit that it is somewhat courageous to characterize the equilibrium of the whole game before describing it completely, but the following lemmas are robust to any institutional assumption of the competitive second stage.

Lemma 2. *Under assumption 2, a bank's average revenue per firm is the highest if a bank is monopolist. It is given by A, the technologically highest return.*

Proof. If the outcome of the competitive second stage is symmetric, this lemma is an immediate result from the fact that a monopoly bank is maximizing the economy-wide output. This is also applicable to the asymmetric outcome case of the competitive second stage. In this case some banks free-ride on others to get better return than A, and the other banks get worse return than A. However, because of the non-corporative behavior, the probability of obtaining either return should be the same for every bank. But as shown in the proof of lemma 1, symmetric lending maximizes economy-wide output. Hence expected revenue from asymmetric lending is less than that of symmetric lending. Therefore, a monopoly bank collects higher return from firms than a competitive bank's average return from any asymmetric outcome. □

Lemma 3. *The deposit rate r is equal to A in a Nash equilibrium of the whole game, if it exists.*

Proof. An deposit rate of $r > A$ cannot be an equilibrium outcome of the competitive second stage, because the loan rate R is at most A by lemma 2. Hence $r > A$ yields negative profits.

An deposit rate of $r = \underline{A} < A$ cannot be an equilibrium either, because bank h will deviate to offer $r_h = \underline{A} + \epsilon$ for small $\epsilon > 0$ to become a monopoly bank, and then earn a positive profit $(A - \underline{A} - \epsilon)S(\underline{A})$. There always exists $\epsilon > 0$ to make this profit larger than the original one, $(R - \underline{A})\gamma_h S(\underline{A})$, where $\gamma_h \in (0, 1)$ is the market share of bank h. □

Note that competition in the deposit market becomes endogenously Bertrand, since for $r_{-h} = \underline{A} < A$ the best response of bank h is $(r_h = \underline{A} + \epsilon, s_{hi} = \{N.S.\})$, i.e., it does not specify the amount of savings and competes only in price. However, we still need the characterization of the deposit amount at rate A.

This result is robust. Banks bid up their deposit rate until $r = A$ in an attempt to capture the monopoly profit. But then banks have to charge at least this rate in the loan market $R \geq A$ to meet their own non-negative profit condition.

Lemma 4. *The realized loan contract in a Nash equilibrium of the whole game is equal to the monopoly loan contract* $(R_{hj} = A, k_{hj} = S/J)$, *if an equilibrium exists.*

Proof. By lemma 3, the equilibrium deposit rate is $r = A$. Realized loan rate of $R = \underline{A} < A$ implies that all banks are offering $(r = A, R = \underline{A})$ at the equilibrium. This in turn implies that they earn a strictly negative profit. But then, bank h will deviate to $(r = \underline{A}, R = \underline{A})$ so that bank h become inactive with zero profit. $R > A$ cannot be equilibrium either by lemma 2.

To obtain $R = A$ from firms, symmetric lending is necessary, because the aggregate production function is maximized at symmetric investment given the fixed amount of total lending as shown in the proof of lemma 1. □

Lemma 5. *At least one bank does not specify the savings amount, i.e.,* $s_h = \{N.S.\}$, *in a Nash equilibrium of the whole game, if it exists. No bank specifies the savings amount in a symmetric Nash equilibrium of the whole game, if it exists.*

Proof. Suppose that every bank restricts the savings amount in a Nash equilibrium. There exists then an excess supply of capital. Hence bank h will deviate to offer $r < A$ and earn an extra positive profit by lemma 4 ($R = A$ in a Nash equilibrium).

If all banks specify the savings amount larger than the optimal level, then one bank will deviate to offer a non-specified amount with slightly less interest rate $(r_h = A - \epsilon, s_h = \{N.S.\})$ to capture the whole deposit market and earn a positive monopoly profit. □

This lemma completes the argument that competition in the deposit market endogenously becomes price competition or Bertrand competition. This is true in a strong sense for the case of a symmetric Nash equilibrium in which every bank competes only in price. It is true in a weak sense for any Nash equilibrium in which some banks compete only in price.

By summarizing lemmas 2 to 5, we have following institution free result.

Proposition 3. ***[Main Result 1]*** *Under assumption 2, the Nash equilibrium outcome of the strategically intermediated economy* Γ *is unique for any competitive second stage, if a Nash equilibrium exists. It is characterized by unique interest rates, savings and investment amounts:* $r_{hi} = R_{hj} = A$ *and* $k_j = S(A)/J$. *This is the Pareto optimal allocation and many banks are possibly active.*

Note that the Pareto optimality is specific to constant returns to the accumulated capital case. However, as Romer (1986) shows, it is the only case that allows the economy to grow perpetually. The new growth theory with externalities thus usually focuses on the production functions that display constant returns to the accumulated capital.

A general version of this proposition is that the Nash equilibrium interest rate of the strategically intermediated economy is given by the monopoly loan rate, and the equilibrium savings and investment amount is given by consumer's optimal choice at the monopoly loan rate. The proof would run exactly the same as above, and specific examples are given in section 5.

3.7 Why We Need the Interbank Market?

Now we turn to the question of whether a Nash equilibrium of the whole game exists. It depends on whether the competitive second stage supports an equilibrium loan rate $R = A$. However, this is not an easy task. Because at the rate A, if a firm chooses its own investment amount, i.e.,

$$\max_{k_j} AK^{1-\alpha}k_j^{\alpha} - Ak_j, \tag{93}$$

then by the FOC

$$k_j = \alpha^{\frac{1}{1-\alpha}}K, \tag{94}$$

zero investment is chosen as a fixed point $k_j = K = 0$. In other words, the only fixed point of loan rate to assure positive investment is $R = \alpha A$. In this case, a simple competition in loan market results in no Nash equilibrium[26].

One way to restore the equilibrium is to assume that the government or consumers choose one bank if the banks offer the same deposit rate $r = A$ and introduce usury law to set the highest interest rate at A. Then the loan market is always provided by monopolist and $R = A$ is achieved. This is similar to the study of Yanelle (1998)[27]. The industrial organization literature would call this solution as a monopolist with a contestable market[28].

However, we do not usually see such a monopoly bank in our economy[29]. One of the main questions of growth theory with externalities is whether we need a monopolist to sustain economic growth. Here it changes its form: do we need a monopoly bank to ensure the existence of equilibrium? The answer is no[30].

[26] This case is exactly the same as the case with decentralized direct finance by firms, in which firms issue bonds directly to consumers without any intermediaries. This case is analyzed in detail in section 4.

[27] She studies the partial equilibrium static model with simultaneous move in both side of intermediaries of consumption goods. Her equilibrium is based on the Nash equilibrium existence theorem of Simon and Zame (1992). It requires boundedness of strategy space. However, interest rate of our model can be taken as large as deviant want, and thus theorem of Simon and Zame (1992) cannot be applicable. To see this, without usury law, the deviant bank can offer the rate $A+\epsilon$ with specifying small amount of investment so that even monopoly contract offering A will be upset.

[28] Again, a monopolist with a contestable market does not usually require any regulation on price cap.

[29] Usury laws are omnipresent, though.

[30] We also show that usury laws are not necessary for the existence of equilibrium.

A reasonable institutional setting such as the interbank market with allowing banks to exert some efforts to clear it yields the same outcome as a monopolist with a contestable market. The idea behind this is that the interbank market is restricted to participant banks only, and hence banks can achieve their own Pareto optimal allocation by competition.

The institutional setting below may seem very specific. But actually, as shown above, the equilibrium rate must be $r = R = A$, and thus banks must somehow invent some institution to implement this. Actually it can be shown that the strategic interbank market with a Walrasian auctioneer also achieves the same allocation. I believe there will be many similar decentralized and competitive ways to restore the equilibrium.

The assumption below is the formal statement to avoid the monopoly case when banks offer the same deposit rate.

Assumption 3. *When two or more banks offer the same highest deposit rate without specification of the amount, $(r_{hi} = r, s_{hi} = N.S.)$, then consumers are assumed to deposit the positive amount to each of these banks.*

3.8 Competitive Second Stage with the Interbank Market

The existence of an interbank market is assumed. In order to clear the interbank market, I assume *strategic tâtonnement* to be consistent with strategic behavior of banks who can select both the price and quantity[31]. However, I maintain the Walrasian assumption that the transaction is not finalized until the demand and supply coincide.

I call the following procedure as *strategic tâtonnement*. There are substages or rounds which are possibly repeated infinitely many times. In each round $\tau \in \{1, 2, \cdots, T\}$, there are five phases
(i) bank $h \in H$ offers a tentative loan contract $(R^{\tau}_{hj}, k^{\tau}_{hj})$ to firm $j \in J$,
(ii) firms submit their tentative decision on offered contracts to banks, and
(iii) banks submit tentative interbank rates and net borrowing amounts $(\rho^{\tau}_h, B^{\tau}_h)$ to the interbank market, and a tentative match of demand and supply is undertaken. It is not guaranteed that all banks can always balance their balance sheet. Note that if all banks do balance their balance sheet, the interbank market is cleared.
(iv) If a bank can balance its balance sheet, it sends a confirmation letter to firms and the interbank market to finalize transaction. This decision is denoted as $d^{\tau}_{bh} = 1$ if confirmed and $d^{\tau}_{bh} = 0$.
(v) Firms respond to the confirmation letters, $d^{\tau}_{fj} = 1$ if accept, $d^{\tau}_{fj} = 0$ if reject the confirmation letters. In the case that a firm does not receive the confirmation letters, it trivially decides $d^{\tau}_{fj} = 0$. If all banks send confirmation letters and all firms accepts them, then all contracts are finalized. Otherwise, banks

[31] It will be clear that analysis is almost the same for the case of traditional Walrasian *tâtonnement* in the interbank market. In this case, the interbank rate offer is not a strategy of banks.

and firms that did not reach agreement proceed to the next round.

At a terminal round T, if the contracts are not finalized before, banks offers the decisive contract (R_{hj}^T, k_{hj}^T) to firms and firms submit their decision decisively, and the interbank market contracts are also decisive. In the case $T = \infty$, if some strategies of banks and firms do not finalize contract forever, no investment by any firms, $k_j = 0$ for all j, is assumed.

If there is only one chance to clear the market, i.e., $T = 1$, then $R = \alpha A$ is the only candidate for equilibrium loan rate as discussed above. So, we assume $T \geq 2$. $T = 2$ suggests there exists the morning and the afternoon session of the interbank market. Finite T suggests the similar situation. Infinite T implies continuous talking among banks and firms all over the day. Even if T is infinite it is just in a day, and the next day will come[32]. I only study the case of $T = \infty$, but the case of finite T is analyzed similarly[33].

In each round τ, the strategy of bank h is defined as

$$z_{bh\tau} \equiv \{(\rho_h^\tau, B_h^\tau), (R_{hi}^\tau, k_{hj}^\tau)_{j=1}^J, (d_{bh}^\tau)\}, \tag{95}$$

which is chosen from the strategy set

$$Z_b \equiv (\mathbb{R}_+ \cup \{N.S.\}) \times (\mathbb{R} \cup \{N.S.\}) \times (\mathbb{R}_+ \cup \{N.S.\})^J \times (\mathbb{R}_+ \cup \{N.S.\})^J \times \{0, 1\}. \tag{96}$$

Note that the strategy set is equal for all banks h and rounds τ. We also denote $z_{b\tau} \equiv \{z_{bh\tau}\}_{h=1}^H$.

Firm j's strategy when it faces offers from banks in round τ is to choose $z_{f\tau} \equiv (R_{hj}^\tau, k_{hj}^\tau, d_{fj}^\tau)$ from its strategy set $Z_f \equiv (\mathbb{R}_+ \times \mathbb{R}_+)^H \times \{0, 1\}$. However, this strategy set is constrained by strategies of banks $z_{b\tau}$. The constrained correspondence is written as $G_f(z_{b\tau})$. We assume, however, that $k_j = 0$ is always in the choice set. Let $G_{fh}(z_{bh\tau})$ be an element of G_f corresponding to the constrained choice set of firm j constrained by the contract offered by bank h.

$$\begin{aligned} G_{fh}(z_{bh\tau}) &\equiv R_{hj} \times \mathbb{R}_+ \quad \text{if bank } h \text{ specifies } R_{hj} \text{ only}, \\ &\equiv \mathbb{R}_+ \times (k_{hj} \cup \{0\}) \quad \text{if bank } h \text{ specifies } k_{hj} \text{ only}, \\ &\equiv R_{hj} \times (k_{hj} \cup \{0\}) \quad \text{if bank } h \text{ specifies both } R_{hj} \text{ and } k_{hj}. \end{aligned} \tag{97}$$

Note that the choice set of the last case is either (R_{hj}, k_{hj}) or $(R_{hj}, 0)$, and we label these choices "*not reject*" and "*reject*", respectively. The constrained choice set of each firm is now defined as Cartesian product of G_{fh} over $h \in H$:

$$G_f(z_{bh\tau}) \equiv G_{f1}(z_{b1\tau}) \times G_{f2}(z_{b2\tau}) \times \cdots \times G_{fH}(z_{bH\tau}). \tag{98}$$

[32] When I was drafting this paper at a cafe in Tokyo, two businessmen sat near my table and start a discussion. One man was owner-manager of a small company and the other was an employee of the big bank. For more than two hours, they were negotiating the conditions of a loan amount and a loan rate. Finally, they seemed to reach tentative decision, and the banker said he would call owner-manager to confirmation of the deal after he discussed with his boss in the bank.

[33] Just read a subgame perfect equilibrium for $T = \infty$ as a Nash equilibrium for $2 \leq T < \infty$, because only difference is the final round.

For simplicity and without loss of generality, we assume exclusive contract offer and thus exclusive submission of demand. In other words, a firm have to choose a contract from one bank[34].

Let $z_\tau \equiv (z_{b\tau}, z_{f\tau})$, the strategies of all agents in round τ. The strategy set for z_τ is $Z \equiv \{Z_b, Z_f\}$.

Definition 9. A *history* $z^{\tau-1}$, for $\tau = 1, 2, \cdots, \infty$, denotes sequences of strategies before round τ, i.e., $(z_0, z_1, z_2, \cdots, z_{\tau-1})$.

The space of history is denoted as $\Omega_{\tau-1}$ for $\tau = 1, 2, \cdots, \infty$,

$$\begin{aligned} \Omega_{\tau-1} &\equiv Z_0 \quad \text{for } \tau = 1, \\ &\equiv Z_0 \times Z^{\tau-1} \quad \text{for } \tau \geq 2. \end{aligned} \tag{99}$$

Definition 10. A *round* τ, $\tau = 1, 2, \cdots, \infty$, with a history $z^{\tau-1}$ is defined as the extensive game with perfect information and simultaneous moves, $\Phi_\tau(z^{\tau-1})$. This consists of

- $(H + J)$ agents (banks and firms),
- phases $p = 1, 2, 3, 4, 5$,
- a player function $P(p)$ that assigns agents to each step p, i.e.,
  - $P(1) = \{1, 2, \cdots H\}$ (banks offer loan contracts),
  - $P(2) = \{H + 1, H + 2, \cdots, H + J\}$ (firms submit demands),
  - $P(3) = \{1, 2, \cdots H\}$ (banks offer interbank contracts),
  - $P(4) = \{1, 2, \cdots H\}$ (banks' send confirmation letters), and
  - $P(5) = \{H + 1, H + 2, \cdots, H + J\}$ (firms' accept/reject confirmation)
- constrained strategy spaces $G(p)$ for each player $P(p)$, i.e.,
  - $G(1) = (\mathbb{R}_+ \times \mathbb{R}_+)^J$ (a bank's offer of loan contracts to firms),
  - $G(2) = G_f$ (a firm's choice),
  - $G(3) = \mathbb{R}_+ \times \mathbb{R}_+$ (a bank's offer to the interbank market),
  - $G(4) = \{0, 1\}$ (a bank's confirmation), and
  - $G(5) = \{0, 1\}$ (a firm's confirmation).
- their profit functions (π_h, w_j).

In sum,

$$\Phi_\tau(z^{\tau-1}) \equiv (H + J, p, P, G, (\pi_h, w_j)), \tag{100}$$

for all τ and all $z^{\tau-1} \in \Omega_{\tau-1}$.

[34] Since demand submission from each firm is observable to anyone, banks can require this exclusive response. Moreover, following "main bank" clause can mimic the same situation without exclusive contract: if a firm submit demands for several banks it must choose a "main bank" and the "main bank" specifies a total amount of loan.

Note that strategy z_τ can be conditioned on history $z^{\tau-1}$.

Definition 11. The *strategic tâtonnement* is a settlement procedure represented as infinitely repeated rounds. More specifically, it consists of set of all possible history and corresponding each round as a function of history:

$$\Phi^\infty \equiv \{\Omega_\infty, \{\Phi_\tau\}_{\tau=1}^\infty\}, \tag{101}$$

and the *competitive second stage* is the game equivalent to this strategic tâtonnement,

$$\Gamma_C \equiv \Phi^\infty. \tag{102}$$

Note that the strategy set for Γ_C is Z^∞ and a typical element is $z \equiv \{z_\tau\}_{\tau=1}^\infty$.

The next assumption takes care of the case where there are multiple best contracts. In this case, an assumption on a sharing rule is necessary.

Assumption 4. *When two or more banks offer the same lowest loan rate without specification of the amount, ($R_{hj} = R, k_{hj} = N.S.$), then firms are assumed to submit positive demand to each of these banks.*

Now we see how the strategic tâtonnement achieves the equilibrium. Bank $h \in D$ maximizes its profit, given the other banks strategy x_{-h} for $-h \in D$,

$$\max_{\rho_h, B_h, \{R_{hj}, k_{hj}\}_{j=1}^J} \sum_{j=1}^J e_{hj} - \rho_h B_h - \sum_{i=1}^I r_{hi} s_{hi}, \tag{103}$$

where

$$\begin{aligned} e_{hj} &= \quad \text{if firm } j \text{ chooses } h, \\ &= 0 \quad \text{otherwise,} \end{aligned} \tag{104}$$

subject to the balance sheet constraint

$$\sum_{j=1}^J k_{hj} = \sum_{i=1}^I s_{hi} + B_h, \tag{105}$$

and each firm's participation constraint (111).

The interbank market must be cleared in equilibrium. Let define the set of active banks that submit the interbank market rate ρ as $\tilde{D}_\rho \equiv \{h \in D | \rho_h = \rho\}$. Using this notation, aggregate net borrowing at each interbank market rate $\rho \in R_+$ can be defined as

$$\tilde{B}(\rho) \equiv \sum_{h \in \tilde{D}_\rho} B_h. \tag{106}$$

The interbank market clear condition is now written as follows. For all $\rho \in R_+$,

$$\tilde{B}(\rho) = 0. \tag{107}$$

Since banks want to exploit as much revenue from firms as possible, they prefer to offer take-it-or-leave-it contract that specifies both loan rate and amount. This combination of price and quantity may be different from that suggested by the private marginal product of capital. If some banks take advantage of the externality by offering small amount of investment with slightly higher loan rate, any contracts different from the Walrasian one would be beaten. Banks want to detect this deviation. But, with some deviation, the interbank market will not be cleared. Hence they can and will detect such deviation by changing their offer after the interbank market is not cleared.

Banks strategies over the strategic tâtonnement, then, consists of a target contract, detection mechanism and a punishment contract. When they adopt the same target, the target will be realized. Hence it becomes a kind of coordination game.

Firm j's best response to the banks' tentative offer z_τ is as follows.

1. Calculation of maximum profit from each contract $z_{bh\tau}$ offered by bank h. There are three cases.

   (a) For the contract $(R_{hj} = N.S., k_{hj} = k)$, firm j's optimal choice is to set $R_{hj} = 0$, which yields a maximum profit of $AK^{1-\alpha}k^\alpha$.

   (b) For the contract $(R_{hj} = R, k_{hj} = k)$, choice of firm j is either to accept or to reject the offer, and thus firm j's maximum profit is

   $$\max_{accept, reject} \{AK^{1-\alpha}k^\alpha - Rk, 0\} \tag{108}$$

   (c) For the contract $(R_{hj} = R, k_{hj} = N.S.)$, firm j maximizes its profit with respect to k_j as in the Walrasian economy:

   $$k_j = \left(\frac{\alpha A}{R}\right)^{\frac{1}{1-\alpha}} K. \tag{109}$$

   The maximum profit is

   $$A\left(\frac{\alpha A}{R}\right)^{\frac{\alpha}{1-\alpha}} K. \tag{110}$$

2. Firm j then chooses the best contracts among all offers.

3. Firm j decides whether it meets nonnegative profit condition:

   $$w_j = AK^{1-\alpha}k_j^\alpha - R_j k_j \geq 0, \tag{111}$$

   because $k_j = 0$ is always an element of G_f.

4. Firm j submits its tentative demand on offered contract to all banks at each round τ.

5. After observing the confirmation letters sent by all banks, firm j considers possible future change of contracts. If a better contract is expected to firm j in the future rounds firm j rejects the confirmation letter. Otherwise, firm j accept it.

Lemma 6. *If at least one bank does not send confirmation letter, all firms rejects all the confirmation letter.*

Proof. The bank that does not send confirmation letter will offer new contract in the next round. Hence firms can expect better or equal offer in the next round. □

Let define the *Walrasian contract* as[35] $z_w \equiv \{(\rho_h = \alpha A, B_h = k_h - s_h), (R_{hj} = \alpha A, k_{hj} = N.S.)_{j=1}^{J}\}$. Also let define the *Pareto optimal contract* as $z_p \equiv \{(\rho_h = A, B_h = k_h - s_h), (R_{hj} = A, k_{hj} = S/J)_{j=1}^{J}\}$.

Lemma 7. *Repetition of the Walrasian contract $\{z_w\}_{\tau=l}^{\infty}$ with confirmation $d_{bh} = 1$ is Nash equilibrium after any history at any rounds l.*

Proof. First, there is no profitable deviation with $R_h > \alpha A$, because deviant banks will get no demand of capital when other bank offers $R = \alpha A$.

Second, there is no profitable deviation with $R_h < \alpha A$ from the Walrasian contract, either. A firm's optimal choice of k_j given the loan rate R and the average investment level is

$$k_j = \left(\frac{\alpha A}{R}\right)^{\frac{1}{1-\alpha}} K. \tag{112}$$

Suppose bank 1 deviates to offer the loan contract $(R_1 = \alpha A - \epsilon, k_{1j} = N.S.)$ for some $\epsilon > 0$. As long as others stick to the Walrasian contract, the interbank market is not cleared and revenue becomes zero, unless it offers the same interbank contract $(\rho_1 = \alpha A, B_1 = k_1 - s_1)$. Hence even the deviant bank will offer the Walrasian interbank contract. But then, the revenue of bank 1 is $(\alpha A - \epsilon)s_1$, which is lower than the revenue from the Walrasian loan contract. □

Lemma 8. *When two or more banks are active, their revenues from firms are secured at $\alpha A s_h$ as a Nash equilibrium.*

Proof. I will show that the Walrasian contract is a profitable deviation from the contract $\{(\rho_h = \alpha A - \epsilon, B_h = k_h - s_h), (R_h = \alpha A - \epsilon, k_{hj} = N.S.)\}$. If $(R_h = \alpha A - \epsilon, k_{hj} = N.S.)$ is only contract available to firms, there exists an excess demand of capital. (Note that contract restricting k_{hj} with $R_h = \alpha A - \epsilon$ cannot be equilibrium, because they face more demand and thus $k_{hj} = N.S.$ yields higher revenue.) Hence, if bank 1 deviate to $(\rho_1 = \alpha A, B_1 = k_1 - s_1), (R_1 = \alpha A, k_{1j} = N.S.)$, the bank 1 still face the residual demand from firms enough to lend out all of its savings s_1. This yields more revenue than the original contract. □

[35] Note that firm j, $j = 1, \cdots, J$, chooses $k_j = S/J$ as the best response in this contract so that there will be neither excess demand nor excess supply of the capital.

Although the Walrasian equilibrium is $R_{hj} = \alpha A$, banks does not have to offer $r = \alpha A$. Banks would like to get higher return from firms if possible. Consider the Pareto optimal contract ($R_{hj} = A, k_{hj} = S/J$). This contract is feasible, and a monopoly bank will charge this contract. Competitive banks also want to offer this contract.

Let define a target contract

$$z_\phi \equiv ((\rho_h = \phi A, B_h = k_h - s_h), (R_{hj} = \phi A, k_{hj} = S(A)/J)_{j=1}^{J}). \tag{113}$$

Lemma 9. *The following strategies constitute a subgame perfect equilibrium in the competitive second stage with $\phi \in [\alpha, 1]$.*

$$\begin{aligned} z^*_{bh\tau} &= (z_\phi, d_{bh} = 1) \quad \textit{as long as } z^*_{bl\tau-1} = z_\phi \textit{ for all } l \in -h, \\ &= (z_w, d_{bh} = 1) \quad \textit{otherwise} \end{aligned} \tag{114}$$

$$\begin{aligned} z^*_{fj\tau} &= (\textit{reject except one offer of } z_\phi, z_{fj} = 1), \textit{if only } z_\phi \textit{ is available}, \\ &= ((\textit{reject except one offer of } z_w, \textit{ submit demand } k_{hj\tau} = S(A)/J), z_{fj} = 1), \\ &\textit{if } z_w \textit{ is available.} \end{aligned} \tag{115}$$

Proof. Since all banks adopt the same target strategy z_ϕ, the loan contract in z_ϕ is the only contract offered to firms for all rounds τ. Knowing this, there is no gain from waiting. Hence firms accept immediately as in the strategy (115).

The target loan amount $S(\phi A)/J$ and punishment strategy z_w is necessary to sustain above strategy as an equilibrium. With this strategy, no bank wants to deviate from the target strategy z_ϕ. For, if deviates, revenue from firms dwindles.

Consider one-bank and one-firm deviation for the case $\phi = 1$. Bank 1 could think about offering a firm 1 smaller amount of capital, $\underline{\mathrm{k}}$ to share the potential profits of free-riding on externality with the firm. Note that $AK^{1-\alpha}\underline{\mathrm{k}}^\alpha - A\underline{\mathrm{k}} > 0$ for $\underline{\mathrm{k}} < K$.

However, this scheme does not work under the other banks' strategy $z^*_{bl\tau}$, $l \in -j$. Other banks offer $(J-1)$ firms ($R = A, k_{hj} = S/J$). Their net borrowing is $\sum_{h=2}^{H} B_h = (J-1)k_{hj} - (h-1)s_h$, which is equal to $\frac{J-1}{J}S - (S - s_1) = s_1 - \frac{1}{J}S$. If the deposits in bank 1 is equal to average deposits, then this is equal to zero. It implies that there is no borrowing or loan opportunity for deviant bank 1. Then it is optimal for bank 1 to lend out the whole fund s_1.

In the case of residual demand/supply is not equal to zero, residual demand/supply is equal to the amount to make the bank 1's fund size to be average savings $\frac{S}{J}$. Here, if bank 1 decides to clear the interbank market, it is optimal for bank 1 to lend all funds.

If the bank 1 prevent the interbank market from being cleared, the punishment is triggered: in the second round and after, other banks offers ($\rho_h = \alpha A, B_h = k_h - s_h$), ($R_h = \alpha A, k_{hj} = N.S.$). Deviant banks cannot offer $R_1 > A$ anymore, and thus there is no profitable deviation.

Another deviation cases such as one-bank and two-firms are also analyzed similarly. Because deviation is profitable only with less investment than others, if some banks deviate, the interbank market will not be cleared when other banks stick to the target contract z_ϕ.

Since z_w is a Nash equilibrium by lemma 8, strategy $(z^*_{bh\tau}, z^*_{fj\tau})_{\tau=1}^{\infty}$ is subgame perfect equilibrium. □

Note that $k_{hj} = S/J$ is necessary to sustain any target strategy for $\phi \in (\alpha, 1]$. Also note that if the terminal round is finite $T < \infty$, only Nash equilibrium interest rate at the round T is αA as is the case with one shot interbank market. In this case, firms always reject the offer and wait until T round comes true. The banks' strategy here still constitutes a Nash equilibrium (from the viewpoint of the beginning of the strategic tâtonnement), but it is not a subgame perfect equilibrium. Specifically the part "$z^*_{bhT} = z_\phi$ as long as $z^*_{blT-1} = z_\phi$" is not credible promise[36] and thus not a subgame perfect, but is still consistent with definition of Nash equilibrium. Firms may always reject the offer if $R = A$ is the equilibrium offer, because $R = A$ delivers zero profits for firms. This constitutes a Nash equilibrium in the second stage with infinite rounds, too[37].

Lemma 10. *When two or more banks are active, there exist many subgame perfect equilibrium in the second stage. They are characterized by the no-arbitrage of the loan rate and the interbank rate, and upper and lower bounds of the interbank interest rate:*

$$\alpha A \leq R^*_h \leq A. \tag{116}$$

Proof. By lemma 8, $R_h = \alpha A$ is secured. Hence $R_h \geq \alpha A$. By lemma 9, any $R_h = \phi A$ for $\phi \in [\alpha, 1]$ becomes an element of subgame perfect equilibrium. □

There can be many other punishment strategy to support the same target contract as an equilibrium, and any mixed combination of punishment strategies are also the equilibrium strategy. For example, a strategy that is almost the same except that the switch to z_w from z_ϕ requires two rounds of uncleared interbank market.

Whatever amount of deposits each bank collects, the interbank market enables each bank to adjust the fund position of lending. This possibility of fund adjustment distinguishes indirect finance from direct finance.

As long as the target contract is the same for every bank, subgame perfect equilibrium yields targeted loan rate. However, if some banks' target loan rate is less than bank h's, then the equilibrium rate is lower than bank h's target rate. This is a coordination game[38].

Now it is easy to see the following result for the whole game.

[36] Unlike in repeated games, the threat that banks revert to the Walrasian contract if some banks deviate is always credible.

[37] But this does not survive as an equilibrium of whole game. Because if it survives, banks will get zero revenue in the second stage and would not want to bid up deposit rate more than zero. But by lemma 3, this cannot be an equilibrium.

[38] Nash equilibrium with Pareto optimal contract Pareto dominates the other strategies. If strategy space consists of only the Pareto optimal contract and the Walrasian contract,

Lemma 11. *Following strategy z^* is a subgame perfect equilibrium. In the first stage,*

$$z^*_{bh0} = (r_{hi} = A, s_{hi} = \{N.S.\}), \tag{117}$$

$$z^*_{ci} = (r_{hi} = A, s_{hi} = s(A)). \tag{118}$$

In the competitive second stage,

$$\begin{aligned} z^*_{bh\tau} &= (z_\phi, z_{bh} = 1) \quad \textit{as long as } z^*_{bl\tau-1} = z_\phi \textit{ for all } l \in -h, \\ &= (z_w, z_{bh} = 1) \quad \textit{otherwise} \end{aligned} \tag{119}$$

$$\begin{aligned} z^*_{fj\tau} &= (\textit{reject except one offer of } z_\phi, z_{fj} = 1), \textit{if only } z_\phi \textit{ is available}, \\ &= ((\textit{reject except one offer of } z_w, \textit{ submit demand } k_{hj\tau} = S(A)/J), z_{fj} = 1), \\ &\textit{if } z_w \textit{ is available}.. \end{aligned} \tag{120}$$

In the second stage with monopoly bank,

$$z^*_M = (R_h = A, k_{hj} = S(A)/J), \tag{121}$$

$$z^*_{fj} = (\textit{accept}). \tag{122}$$

Proof. Given z^*_{bh0}, z^*_{ci} is the consumer's optimal strategy in the first stage. A proof for the second stage is already shown before. □

Proposition 4. ***[Main Result 2]*** *There exists a unique subgame perfect equilibrium outcome for the strategically intermediated economy Γ, which is the Pareto optimal allocation and where many banks are active. This is characterized by unique interest rates, savings and investment amounts: $r_{hi} = \rho_h = R_{hj} = A$ and $k_j = S(A)/J$.*

Proof. Proposition 3 characterizes equilibrium outcome. Lemma 11 assures the existence of subgame perfect equilibrium. Lemmas 3 and 4 assures $r = R = A$. Hence $\rho = A$, too, because otherwise there is an arbitrage opportunity. Equilibrium loan amount is equal to S/J by lemma 4. Lemma 5 assures $s_i = s(A)$. □

it is pairwisely risk dominates the other strategies, too. Hence this is likely selected as the equilibrium contracts(See Kandori and Rob (1995)). This argument, however, cannot apply for general strategy space.

3.9 Discussion of the Model

3.9.1 Role of the Interbank Market

Readers might think that even without the interbank market, banks can accomplish the Pareto optimal contract by adopting the following Tit-for-Tat strategy:
(i) the target contract is $\{(r = A, s_{hi} = N.S.), (R = A, k_{hj} = \frac{S}{J})\}$,
(ii) if some bank restrict the savings amount in the deposit market, then offer the Walrasian contract in the loan market as a punishment, otherwise stick to the target contract.
This strategy works if symmetric deposit amount for the same deposit rate is assured in the deposit market. It does not work, however, when deposit amount are different among banks in case of the same deposit rate. Because a "lucky" bank with less deposits can offer firms less capital level with slightly higher loan rate to free-ride on externalities.

For the same reason, a one-shot interbank market without any possible adjusting process for uncleared situation does not work well. As Walras considered, market should try to match demand and supply of goods. The interbank market with the *strategic tâtonnement* is necessary and sufficient for my result.

3.9.2 Consumers and Firms

A natural question that arises is whether the same result would obtain even if firms had the possibility of creating their own inter-firm market. Of course if a finance department of a manufacturing firm starts to collect funds from the public, adjusting funds in an interbank or an inter-firm market, and lend to many other manufacturing firms, then the resulting allocation coincides with the one we analyzed above. As I stated clearly, however, firms are defined to conduct manufacturing activity and may possibly engage in direct finance. If they start to borrow and lend funds, they would be banks according to the definition in this paper.

Similarly, household's decision on financial activity is to save some capital given offered financial products. If some households start to design and offer financial contract to firms, they would be banks, too.

This argument may not be so confusing if we think producers in the standard microeconomic theory: if the households start to produce everything in production economy, the model does not need any producers even in production economy, or we just call such productive activity by the "households" as producers. In this sense, banks in this paper may contain finance departments of large manufacturing firms and conglomerates in the real economy.

4 Policy Implications for Further Study

This section is intended to point out some policy implications, which will be studied further in future research.

First, an immediate policy implication is that neither monopoly nor support by a government on investments is not necessary to achieve Pareto optimal allocation even if the production function exhibits externalities. This is in stark contrast to the prevailing view that the support by the government or the protection of monopoly rights by patents is necessary to sustain the optimal growth path if spill-over of new inventions is inevitable.

Second, it should be noted that any policy geared towards lowering the competitiveness of banking sector does harm to the economic growth and welfare according to the model in this paper. Reserve requirement and the activity of a central bank in the interbank market could be analyzed along this line, too.

Third, the model predicts that "large countries" will be better off allowing international capital flows. Consider two-country case where banks can compete for savings and loans for only half of the world population, or $I/2$ consumers and $J/2$ firms. Since the banks in each country achieve the monopoly solution with a contestable market, the solution is the same as the two banks case in one country. This is almost the same model as in the simple example, except that the world average of investment includes investments of both countries $K = (k_1 + k_2)/2$ for the symmetric case. The equilibrium interest rate for country 1 is given by solving

$$\max_{k_1} A\left(\frac{k_1 + k_2}{2}\right)^{1-\alpha} k_1^{\alpha} - rk_1. \tag{123}$$

The FOC is

$$\left(\alpha A + \frac{(1-\alpha)Ak_1}{2K}\right)\left(\frac{k_1}{K}\right)^{\alpha-1} = r. \tag{124}$$

The second term of the LHS has an extra term compared to the simple example where k_2 replaces K. It is easy to show that the equilibrium interest rate is set between αA and A by monopolists in each country, $\alpha A < r < A$. This is a Cournot solution[39]. This interest rate is lower than the one country case. Therefore, combining two country together or deregulating any restrictions on the global capital market makes consumers in both countries better off.

Fourth, if we introduce outside money, at rate $\overline{R} < A$, replicating the "small country" case in international economics literature, then the domestic interest rate becomes $\overline{R} < A$. I assume that a country can utilize spill-over of the world average capital investment only when it participates the global market. Because the potential profitability of the small country by free-riding on the externality, positive net capital inflow will be expected. I explore this case below.

Assume the equal number of firms and consumers and drop the subscript of i and j. Note that welfare depends on disposal income of consumers at the end of the period (or the beginning of the next period) m_{t+1}, because this is the argument of the value function $V(m)$. In a closed economy, the income (wealth) of the beginning of the next period is just the output of this period, whereas in

[39] If regulations are different, this is not the case. This needs further study.

open economy it is the output of this period netting out the interest payment to foreign countries.

Per capita output and hence the wealth at the end of the period in a closed economy is

$$m_c = Ak. \tag{125}$$

The end of period wealth in an open economy is, given the world average investment level $\overline{K}$,

$$m_o = A\overline{K}^{1-\alpha}\hat{k}^{\alpha} - \overline{R}(\hat{k} - s(\overline{R})). \tag{126}$$

The country will prefer to refuse the international capital flows when $m_c > m_o$. But

$$\hat{k} = \left(\frac{\alpha A}{\overline{R}}\right)^{1/(1-\alpha)} \overline{K}. \tag{127}$$

Using this, the condition can be rewritten as

$$Ak > \left(\frac{\alpha A}{\overline{R}}\right)^{\frac{1}{1-\alpha}} (A - \overline{R})\overline{K} + \overline{R}s(\overline{R}). \tag{128}$$

Subtracting $As(\overline{R})$ from both sides, and substituting $k = s(A)$, savings in closed economy, and $\overline{K} = s_w(\overline{R})$, the world average per capita savings,

$$A(s(A) - s(\overline{R})) > (A - \overline{R})\left(\left(\frac{\alpha A}{\overline{R}}\right)^{\frac{1}{1-\alpha}} s_w(\overline{R}) - s(\overline{R})\right), \tag{129}$$

or

$$\frac{A(s(A) - s(\overline{R}))}{A - \overline{R}} > \left(\frac{\alpha A}{\overline{R}}\right)^{\frac{1}{1-\alpha}} s_w(\overline{R}) - s(\overline{R}). \tag{130}$$

The LHS is the efficiency loss by lower deposit rate than the first best divided by the relative change of interest rate, and the RHS is the gain from free-riding, which consists of the adjusted world per capita savings minus domestic per capita savings at the world interest rate. Given the world interest rate $\overline{R} < A$, if the domestic per capita savings $s(\overline{R})$ is relatively small compared to the world per capita savings $s_w(\overline{R})$, then this country should open the capital market. However, if the country have accumulated such wealth level that is near the world average level, then, the inequality (130) holds.

This implies that the savings gap between the world and domestic must be larger than some threshold level. In other words, a "middle country", which has the relatively high savings level, but cannot influence the world price and world average savings level, may be better off by shutting down its capital market to the international capital flows. Very "small countries", on the other hand, always prefer to open its capital market.

Note that $\overline{R} < A$ implies that the foreign countries have some inefficiency if I assume that every country has the same production function. If it is not the case, the world interest rate must be equal to the Pareto optimal level, $\overline{R} = A$. In this case the RHS and the LHS of (129) equal to zero, and thus a country become indifferent in opening the capital market to the international capital flows. In other words, only when the foreign country has an inefficient institution ($\overline{R} < A$), then there may be some incentive for a small country to be isolated from the rest of the world.

5 Other Institutional Setting

In this section, I consider the cases (i) where firms offer bonds directly to consumers without an auctioneer nor banks, and (ii) where both direct finance and indirect finance is possible. In the case (i), there is no pure and mixed equilibrium. Preliminary results of the case of (ii) are reported.

5.1 Strategic Direct Finance

One might think that the difference of the Walrasian outcome and the allocation by strategic intermediation comes from rationality of middlemen, and wonder what happens if market is actually a decentralized strategic competition of firms instead of centralized market with an auctioneer.

I consider this interpretation of market in this section. Without an auctioneer and banks, the game has no Nash equilibrium, with arbitrary sharing rule when they firms offer the same interest rate. There exists unique sharing rule and associated Nash equilibrium. This equilibrium, however, characterized by monopolist.

Now firms $j \in J$ offer bonds, $(r_j, k_{ji}) \in \mathbb{R}_+ \times \mathbb{R}_+$, to consumers $i \in I$. Consumer i's decision is now only $\{accept, reject\}^J$. His problem is now

$$V_D(m) = \max_{\{accept, reject\}^J} u(m-s) + \beta V_{Di}(rs+w). \tag{131}$$

Definition 12. A *economy with strategic direct finance* is a game consists of $I+J$ agents, their strategy set, and their utilities:

$$\Gamma_D = (I+J, (\{accept, reject\}^J, (\mathbb{R}_+ \times \mathbb{R}_+)^I), (V_D, w_j)). \tag{132}$$

Proposition 5. *There is no pure-strategy Nash equilibrium in the economy with strategic direct finance.*

Proof. Any $r < A$ are not an equilibrium since a firm offers slightly better coupon rate and become monopolist. This competition for coupon rate would continue until Pareto Optimal interest rate, $r = A$. But Pareto Optimal interest rate is not an equilibrium either, since there exists free riding incentive for firms to invest less than Pareto optimal amount. A deviant firm's optimal bond offering, given the interest rate A and other firm's investment amount K, is

$$k_j = \alpha^{\frac{1}{1-\alpha}} K. \tag{133}$$

This is less than K.

Actually only interest rate that satisfies the fixed point condition $k_j = K$ is $r = \alpha A$, but this cannot be equilibrium as shown above. Hence there is no equilibrium[40]. □

There is no theorem assures the existence of mixed strategy Nash equilibrium, either. Reny (1999) is the latest work that specifies the sufficient condition for existence of Nash equilibrium for discontinuous games. The condition is called *better-reply secure: if*[41] *for every non equilibrium strategy x^* and every payoff vector w^* for which (x^*, w^*) is in the closure of the graph of the game's vector payoff function, some player i has a strategy yielding a payoff strictly above w_i^* even if the others deviate slightly from x^*.*

Consider[42] a non-equilibrium strategy that every firm offers the Pareto optimal bond $x_j^* = (r_j = A, k_j = N.S.)$. Profits are zero, $w_j^* = 0$. Suppose firm 1 slightly deviates to $(r_1 > A, k_1 = N.S.)$. Firm 1 becomes monopolist, but its profit is negative because r_1 is bigger than the technologically highest return A. Other firms $j = 2 \cdots J$ remains at zero profits. Hence, no player has a strategy yielding a payoff strictly above w_j^* when bank 1 deviates slightly from x_1^*. Therefore, the game is not better reply secure[43].

The reason why direct finance by firms supports no equilibrium becomes clear why banks achieves equilibrium in previous section. Interbank market provides the opportunity to secure the non-equilibrium payoff at αA (lemma 8). Banks can adjust fund size after taking deposit, but firms cannot. Even if banks try to restrict savings amount as firms do here at $r = A$, banks can detect deviation because of the interbank market. The interbank market makes free-riding impossible. On the contrary, a firm's fund is restricted to what it raises, there are no opportunity to punish. Moreover, firms care about how much is their private marginal return, but banks only care how much they can exploit firms and thus free from marginal pricing.

Let λ_j be probability measure on (r_j, k_j).

Proposition 6. *There exists no mixed-strategy Nash equilibrium in an economy with strategically competitive firms.*

Proof. Let $\underline{r}_j$ $((\overline{r}_j)$ be the lowest (highest) interest rate of support of the equilibrium mixed strategy of firm j. They must be the same for every firm at equilibrium.

[40] It is similar to Rothchild-Stiglitz model of insurance. Also see Yanelle (1998).

[41] This is from Reny (1999) except that I use w^* to denote payoff instead of u^* in original.

[42] This example is suggested by Philip J. Reny.

[43] Moreover, Reny (1999) also requires boundedness of strategy space, but here $(r, k) \in \mathbb{R}_+ \times \mathbb{R}_+$ is unbounded.

Let $\underline{r}_{-j} \equiv \min\{\underline{r}_l\}_{l\in -j}$. Suppose $\underline{r}_j < \underline{r}_{-j}$. Then firm j cannot get capital from consumers in the region $[\underline{r}_j, \underline{r}_{-j}]$. Hence $\underline{r}_j < \underline{r}_{-j}$ cannot be an equilibrium. Therefore $\underline{r}_j = \underline{r}$ for all $j \in J$.

Let $\overline{r}_{-j} \equiv \max\{\overline{r}_l\}_{l\in -j}$. Suppose $\overline{r}_j > \overline{r}_{-j}$. Then firm j become a monopolist in the region $[\overline{r}_{-j}, \overline{r}_j]$. If $\overline{r}_{-j} \geq A$, then the firm j have less expected profit than other firms, because expected profit from the region $[\overline{r}_{-j}, \overline{r}_j]$ is negative. If $\overline{r}_{-j} < A$, then the firm j have higher expected profit than other firms, because expected profit from the region $[\overline{r}_{-j}, \overline{r}_j]$ is positive. Then other firms should offer until $\overline{r} \geq A$. Therefore, $\overline{r}_j = \overline{r} \geq A$ for all $j \in J$.

Now let $[\underline{r}^*, \overline{r}^*]$ be the support of the equilibrium mixed strategy. If $\overline{r} > A$, consider the mixed strategy that has the same distribution in $[\underline{r}^*, A]$ but has a mass at A. This strategy is better than λ^*, because in the region $[A, \overline{r}]$ firm has negative profit. But this strategy do not. Hence $\overline{r}^* = A$.

If $\underline{r}^* < A$, consider the mixed strategy that has the same distribution in $(\underline{r}^* + \epsilon, A]$ but has a mass at $\underline{r}^* + \epsilon$. This strategy is better than λ^*. Approximately, firm j looses the opportunity to raise capital at cheap rate, $\xi(A - \underline{r})K/J$, but wins to become monopoly by increasing the interest rate, $\xi(A - (\underline{r} + \epsilon))K$. Here always exists ϵ such that gain is larger than loss.

$$(A - (\underline{r} + \epsilon))K > (A - \underline{r})K/J \tag{134}$$

$$\epsilon < \frac{J-1}{J}(A - \underline{r}). \tag{135}$$

As long as $\underline{r} < A$ firm j can always take ϵ to satisfy inequality (135). Therefore, $\underline{r}$ cannot be less than A. But since $\overline{r}^* = A$, $\underline{r}$ must be equal to A.

In summary, support of mixed strategy is a point A. But proposition 5 says that this pure strategy is not an equilibrium. Therefore, there is no mixed strategy equilibrium. □

Simon and Zame (1992) suggests that there always exists a equilibrium with endogenous sharing rule or public lottery under a general condition. If the sharing rule when firms offer the same price is chosen before competition, but not restricted to the equal share, there exists a sharing rule and associated allocation that consists equilibrium. Although they call this sharing rule as endogenous sharing rule, it has to be exogenously given before the competition of firms.

Yanelle (1998) uses their result and points out further in a slightly different setting that if there exists public lotteries to chose specific share of consumers when firms set the same price, then there will be an equilibrium.

However, this is not applicable here either. There exists no pure strategy equilibrium with positive share for every bank when the offered rate is the same. Now consider the case that only one firm will get all the supply of fund when banks and firms offer the same rate. Even in this case, equilibrium candidate loan rate is also $R = A$. Firms do not restrict the savings, $K = S(A)$, in order to become monopolist. But then, a deviant firm will offer $(A + \epsilon, \xi)$ to free ride and get positive profit. This upsets the equilibrium with monopolist.

5.2 Mixed Institutions

This section discusses preliminary results for the case where both direct and indirect finance are available in the economy.

5.2.1 An Auctioneer and Banks

Consider the case where both an auctioneer and banks try to intermediate the capital market. This case is analyzed by replacing bank 1 in the base model in section 3 to an auctioneer, or restricting the strategy space of bank 1 to price only. It is easy to see that banks 2 to H act the same way in the base model and hence the Pareto optimal contract will be realized. Bank 1 or the auctioneer, then, has to offer the interest rate $r = A$, too. However, no firm issues bond in the capital market at that level, because the auctioneer cannot specify the amount of investment, and all firms are already "over-investing" in terms of private marginal capital.

Therefore, the allocation is the same as before but a bond market cannot exists at the same time with banks[44].

5.2.2 Decentralized Direct Finance and Indirect Finance

I consider here the case where both firms and banks strategically compete for savings. Following the base model, a two stage game at each date is still appropriate. In the first stage, firms and banks compete for consumer savings. In the second stage, if banks collect positive amount of deposits, banks competitively lend to firms while adjusting their fund base in the interbank market.

In this case, the firms now can raise capital and free-ride on others if others offer the Pareto optimal contract by offering consumers the coupon rate slightly higher A with limited amount of bond issue, when banks offer the loan rate A with specification of over-investment S/J. This cannot be prevented in the strategic tâtonnement among banks and firms.

However, this will not happen, if defaults of banks or recontracting with consumers are allowed. Since banks cannot control firms in this regime, they would like to control consumer's behavior of buying privately offered bonds.

A Tit-for-Tat strategy can be applicable to consumers. If no consumer buys the privately offered bonds, then the banks offer Pareto optimal contract to firms, otherwise revert to the Walrasian contract. This time the banks do not ask their shareholders to share losses but pay less for deposit contract than promised. Knowing this, consumers deposit all their savings in banks rather than buying the privately offered bonds from firms[45].

[44]Note that if we consider "main bank" clause that a bank specifies total capital level of a firm but allow firms to borrow fund from others, then bond issues coexists with banks.

[45]Note that again, "main bank" clause works. If a bank specifies total capital level of a firm but allow firms to borrow fund from others, then bond issues coexists with banks.

6 Variant Models

The results needs robustness to alteration of assumptions, including the cases (i) where the production function is not constant returns to accumulated capital but decreasing returns to it[46], (ii) where the production function requires labor as well as capital.

I report here summary of results. In case (i), as long as degree of externality is large enough, allocation delivered by banking sector is Pareto superior to Walrasian outcome. As for case (ii), firms have to pay some amount of wages to produce positive output. In the equilibrium, wages are lower, but the interest rate is higher than in the Walrasian market. Again, more rewards are paid to nonrival goods than in the Walrasian equilibrium, but the amount is less than the first best one. Hence the consumption allocation becomes Pareto superior to the Walrasian outcome, but is not Pareto optimal.

6.1 The Case of Decreasing Returns

Romer (1986) classifies production function with Marshallian externality into three categories: constant returns, decreasing returns, and increasing returns to accumulated factor. In a simple production function,

$$y_h = AK^{\eta} k_j^{\alpha}, \tag{136}$$

these corresponds to the cases of $\eta = 1 - \alpha$, $\eta < 1 - \alpha$ and $\eta > 1 - \alpha$. Since the case of constant returns is already discussed in previous sections, the remaining question is whether the result varies depending on η.

Jones and Manuelli (1990) show that only constant returns to accumulated factor is consistent with both perpetual growth and finite life-time utility, i.e., $\sum_{t=1}^{\infty} \beta^t u(c_t) < \infty$. Hence I report only the case of decreasing returns.

Proposition 7. *When technology exhibits decreasing returns to accumulated capital, $\eta < 1 - \alpha$, the equilibrium of strategically intermediated economy is characterized by higher interest rate and growth rate than the Pareto Optimal allocation:*

$$r_{hi}^* = R_{hj}^* = \rho_h = AK^{\alpha+\eta-1}, \tag{137}$$

and (S, K) is determined by the unique fixed point of

$$K = S(AK^{\alpha+\eta-1}). \tag{138}$$

Proof. The proof is almost the same as before, and I sketch it here.

If more than one bank exist in the second stage, no-arbitrage of loan rate and symmetry of loan amount are proven the same way as before. Because of participation constraint of a firm, maximum loan rate that banks can charge is

[46] If technology exhibit increasing returns to accumulated capital forever, the utility will explodes so that I do not study this case.

less than or equal to average capital of product. Together with symmetric loan, this implies that $R_h \leq AK^{\alpha+\eta-1}$. Also, this maximum rate will be charged, if a bank becomes a monopolist.

Given this Nash equilibrium in the second stage, banks engage in Bertrand competition in the deposit market at the first stage. They will offer depositors the maximum rate.

Since the savings function $S(AK^{\alpha+\eta-1})$ is decreasing function of $K \in \mathbb{R}_{++}$ and approaches $+\infty$ as $K \to 0$ from right, there exists the unique fixed point of K in equation (138). $\square$

If externalities were not present, this equilibrium is worse than the Walrasian equilibrium. However, as long as the externality parameter η is near $1-\alpha$, this equilibrium is superior to the Walrasian equilibrium.

6.2 A Model with Labor

Consider a Cobb-Douglas production function $F : \mathbb{R}^3_+ \to \mathbb{R}_+$. Let Y_{jt} denote the output by firm j, and K_j and L_j denote capital and labor employed by firm j, respectively.

$$Y_j = F(K_{jt}, L_{jt}, K_t) \equiv AK_t^{\eta} K_{jt}^{\alpha} L_{1-\alpha}. \tag{139}$$

Divided by L both side, production function per capita is obtained.

$$y_j = f(k_{jt}, K_t) \equiv AK_t^{\eta} k_{jt}^{\alpha}. \tag{140}$$

This is the same production function as in the base model.

A consumer earns income either as wage or asset return. I assume each consumer is endowed with one unit of time. Instead of profit income in the base model, wage is now equal to the difference between output and interest rate payment to capital. In this case, ϕ_{ijt} is interpreted as the portion of hours-worked in firm j. It must satisfy the feasibility condition, as parallel as equation (38):

$$\sum_{j=1}^{J} \phi_{ijt} = 1. \tag{141}$$

Now w_{cit} denotes the total wage income of consumer i at date t. It is defined as

$$w_{cit} \equiv \sum_{j=1}^{J} \phi_{ijt} w_{jt}. \tag{142}$$

This is the same expression as equation (43).

As long as utility function of a consumer does not exhibits disutility of labor, each consumer spends one unit of time to labor, and labor supply become inelastic. In this case, there is no change as in the base model.

A firm's participation constraint changes from equation (111) to

$$AK^{1-\alpha}k_j^{\alpha} - R_j k_j \geq w_j, \tag{143}$$

where w_j is equilibrium wage rate. But participation constraint of worker is,

$$w_j \geq 0. \tag{144}$$

Proposition 8. *Equilibrium of the intermediated economy with labor in production, but without disutility of labor, is the same allocation as the equilibrium of the base model.*

Proof. Proof is exactly the same as before. A bank raises deposit interest rate up until A. This is because workers inelastically supply labor even at zero wage. □

When the utility function exhibits disutility of labor, then firms have to pay positive amount of wage to hire workers. Let the period-utility function include disutility of labor or utility of leisure:

$$u(c_i) + v(1 - L_i). \tag{145}$$

Let define elasticity of labor supply to wage as ϵ_w^L, and re-weighted labor share

$$\hat{\alpha} = \frac{\alpha + 1/\epsilon_w^L}{1 + 1/\epsilon_w^L}. \tag{146}$$

Proposition 9. *The equilibrium of strategically intermediated economy with labor is* $r = \rho = R = \hat{\alpha}A$ *and* $k_j = K = S(\hat{\alpha}A)/J$. *This implicitly implies labor share is equal to* $(1 - \hat{\alpha}A)$.

Proof. Let $L(w)$ be a labor supply function. A monopoly bank with $s_{hi} = N.S.$ in deposit market maximizes capital share:

$$\max_w AKL(w)^{1-\alpha} - wL. \tag{147}$$

First order condition gives

$$(1 - \alpha)AKL^{-\alpha}L'(w) = L + wL'(w). \tag{148}$$

By dividing both sides by $L'(w)$ and multiplies both side by L, labor share becomes

$$wL = (1 - \alpha)Y - \frac{L}{L'(w)}L. \tag{149}$$

Using the elasticity of labor supply to wage, ϵ_w^L,

$$wL = (1 - \alpha)Y - \frac{wL}{\epsilon_w^L}. \tag{150}$$

Hence

$$wL = \frac{1-\alpha}{1+\frac{1}{\epsilon_w^L}}Y = (1-\hat{\alpha})Y. \tag{151}$$

Then the capital share becomes

$$rK = \frac{\alpha + \frac{1}{\epsilon_w^L}}{1+\frac{1}{\epsilon_w^L}}Y = \hat{\alpha}Y. \tag{152}$$

The remaining argument is the same as before. Competitive banking achieve this monopoly solution. □

If people are poor, elasticity of labor supply should be small. Then the equilibrium wage become small. In the case of elasticity zero is the case of inelastic labor supply, which is analyzed above. If people are rich, elasticity of labor supply should be larger than poor people. Then the equilibrium wage become large.

Social optimal is the same as before. Equilibrium here is not Pareto optimal but Pareto superior to the Walrasian equilibrium. The more wages firms pay, the less inefficient the investment is. But the labor economics literature suggests that the elasticity of labor supply is considered as small and thus the difference from Pareto optimal allocation seems not so large[47].

7 Conclusion

This paper has identified a novel connection between the financial sector and economic growth, and challenged prevailing views in three literatures.

If complete markets exist in an economy without frictions, banks have been considered to play no role. However, this has been shown not to be true in growth models with Marshallian externalities. The function of a banking sector known as indirect finance alone enables banks to exist in a frictionless economy. Since indirect finance with an interbank market breaks the link between sources and uses of capital, banks can and will compete more aggressively than firms[48]. As a result, this strategic competition among banks in the capital market yields an allocation that is more favorable to financiers than the Walrasian allocation involving direct finance.

In new growth theories based on Marshallian externalities or nonrival goods, agents always face less incentive to invest in the Walrasian equilibrium than in

[47] In the homogeneous economy, lower wage brings better consumption allocation in the equilibrium. It should be noted that in the heterogeneous economy, especially where wealth inequality is large, lowering wage implies lowering the consumption sequence of poor people, although the aggregate consumption will be higher than the Walrasian outcome. In this case, we cannot compare those allocation by Pareto criterion.

[48] In this paper, competition among banks in deposit market endogenously becomes Bertrand competition, in which banks compete on interest rate without restricting the amount of deposit that they take. On the contrary, firms always specifies the amount of capital that they raise by issuing bonds. This is consistent with daily observation.

the first-best Pareto optimal allocation. In some cases, no investment is made in the Walrasian equilibrium so that many studies have focused on monopolistic competition. These models, however, are criticized due to lack of empirical support for monopolistic competition, and based on philosophical questions about the necessity of monopolists for economic growth.

In the proposed allocation delivered by a competitive banking sector in this paper, banks force firms to invest more than is suggested by the private marginal product of capital. The banking sector acts as a disciplining device for each firm to prevent free-riding on externalities created by other firms. As a result, rewards are paid to nonrival goods. Hence the allocation is often Pareto superior[49] to the Walrasian allocation, contrary to what the literature of strategic intermediaries has argued.

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