Non-Parametric Dependent Data Bootstrap for Conditional Moment Models

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Abstract

A new non-parametric bootstrap is introduced for dependent data. The bootstrap is based on a weighted empirical-likelihood estimate of the one-step-ahead conditional distribution, imposing the conditional moment restrictions implied by the model. This is the first dependent-data bootstrap procedure which imposes conditional moment restrictions on a bootstrap distribution. The method can be applied to form confidence intervals and p-values from hypothesis tests in Generalized Method of Moments estimation The bootstrap method is illustrated with an application to autoregressive models with martingale difference errors.

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1 Introduction

The bootstrap of Efron (1979) is a powerful nonparametric method to approximate the sampling distributions of estimators and test statistics. For dependent data, nonparametric versions of the bootstrap have been implemented by dividing the sample into blocks and sampling the blocks independently with replacement. The blocks, whose lengths increase with sample size, may be nonoverlapping (Carlstein (1986)) or overlapping (Kunsch (1989)) and may have random lengths (Politis and Romano (1994)). The method of nonoverlapping blocks was extended by Hall and Horowitz (1996) to the case of Generalized Method of Moments (GMM) estimation (Hansen (1982)). In the absence of a structural model that reduces the data-generation process to a transformation of independent random variables, a blocking scheme is the dominant bootstrap method for dependent data.

There are a number of reasons, however, to expect a blocking bootstrap to be inefficient. First, the block bootstrap does not impose on the bootstrap distribution any of the information contained in the given econometric model. For example, in overidentified GMM estimation, as argued by Brown and Newey (1995) in the case of independent observations, the data generated by the Hall-Horowitz bootstrap distribution does not satisfy the moment conditions implied by the estimating equations. The solution proposed by Hall-Horowitz is to re-center the bootstrap moments at their sample values. As Brown and Newey (1995) show, however, this solution is inefficient, pointing out a generic inefficiency with blocking methods. Second, the block bootstrap creates a bootstrap time-series process with serial dependence patterns which are quite different from the original time-series, and this discrepancy does not disappear as the sample size increases. To see this, consider the one-step-ahead conditional distribution of a bootstrap observation, conditional on its past history, in a blocking scheme with blocks of fixed length b. We see that the one-step-ahead conditional distribution is degenerate for $(b-1)$ out of every b observations, and for the remaining observations the one-step-ahead conditional distribution is independent of the past history. This one-step-ahead conditional distribution is obviously quite distinct from the one-step-ahead conditional distribution of the actual data, and this difference suggests that the bootstrap distribution may be an inefficient estimate of the sampling distribution.

In some GMM contexts, such as autoregressions (ARs) or vector autoregressions (VARs), time-series econometricians routinely use bootstrap methods which exploit the recursive structure of the model, by treating the model errors as an independent series. An inherent problem with this approach is that its usefulness depends on the accuracy of the auxiliary assumption about the independence of the model errors. In most applications, it is probably more accurate to view the AR or VAR errors as martingale differences, leaving higher-order dependencies unspecified. In particular, time-varying conditional heteroskedasticity appears quite prevalent in many economic time-series, and the treatment of such errors as independent necessarily leads to inaccurate distributional approximations.

This paper introduces a new non-parametric dependent-data bootstrap which incorporates conditional moment restrictions. It may be viewed as an extension of the recent proposal of Brown and Newey (1999) for a the case of independent data. Our bootstrap employs the smoothed empirical likelihood estimator of Ahn, Kitamura and Tripathi (1999) which efficiently incorporates information contained in a conditional moment restriction.

Section 2 describes the bootstrap problem. Section 3 describes the method to estimate the conditional distribution. Section 4 describes how to use the conditional distribution to generate a bootstrap sample by recursion. Section 5 presents a numerical simulation study.

2 The Bootstrapping Problem

The data $y_t \in R^p$ is a strictly stationary and weakly dependent time series, observed over $t = -m+1, ..., n$ for some finite integer m. Let $x_t = (y_{t-1}, ..., y_{t-m})$. The time-series is known to satisfy the conditional moment restriction

$$E\left(g\left(y_t, x_t; \theta_0\right) \mid \mathcal{F}_{t-1}\right) = 0 \tag{1}$$

where g is a known R^s-valued function, θ_0 is an unknown R^q-valued parameter, and $\mathcal{F}_t = \sigma(y_t, y_{t-1}, ...)$ is the Borel sigma-field generated by the history of the series through time t.

A simple example is an AR(1) with martingale difference errors:

$$y_t = \mu + \alpha y_{t-1} + e_t, \qquad E\left(e_t \mid \mathcal{F}_{t-1}\right) = 0. \tag{2}$$

Here, $\theta = (\mu, \alpha)$ and $g\left(y_t, x_t; \theta\right) = y_t - \mu - \alpha y_{t-1}$.

Given (1), there are standard GMM methods to estimate θ and conduct asymptotic inference. In this paper we are not concerned with the choice of estimators and test statistics, we simply suppose that there is some estimator $\hat{\theta}$ for θ and some test statistic $t_n = t_n(y_1, ..., y_n; \theta_0)$. The test statistic t_n can take the form $t_n = \hat{\theta}$, or alternatively $t_n = \hat{\theta} - \theta_0$, but most likely takes a studentized form, e.g. $t_n = \left(\hat{\theta} - \theta_0\right)/\hat{s}$, where $\hat{s}$ is an asymptotic standard error for $\hat{\theta}$. The purpose of t_n is that if the distribution of t_n were known, then the distribution can be inverted to yield confidence statements about θ.

For concreteness, we now give three examples of how the choice of statistic t_n yields different conventional confidence intervals for θ. For any choice of t_n, let q_α denote the α quantile of the distribution of t_n (e.g., $P(t_n \leq q_\alpha) = \alpha$). If $t_n = \hat{\theta}$, then a 90% percentile-type confidence interval for θ is formed as $[q_{.05}, q_{.95}]$. If $t_n = \hat{\theta} - \theta_0$, then Hall's percentile-type interval takes the form $[\hat{\theta} - q_{.95}, \hat{\theta} - q_{.05}]$. If $t_n = \left(\hat{\theta} - \theta_0\right)/\hat{s}$, then the percentile-t-type interval takes the form $[\hat{\theta} - \hat{s}q_{.95}, \hat{\theta} - \hat{s}q_{.05}]$.

In principle, the distribution of t_n can be calculated from $F(y_1, ..., y_n)$, the joint distribution of the data, but F is generally unknown. The bootstrap method approximates the distribution of t_n by that of $t_n^* = t_n\left(y_1^*, ..., y_n^*; \hat{\theta}\right)$ where the random sample $(y_1^*, ..., y_n^*)$ has some joint distribution F^* which is designed to mimic the data distribution F. The so-called bootstrap distribution F^* is thus an approximation (or estimate) of F. One difference between alternative bootstrap procedures is the choice of estimate F^*.

Brown and Newey (1995, 1999) argue that bootstrap inference will be efficient when the distribution estimate F^* is efficient. Intuitively, bootstrap inference can be made exact if we set $F^* = F$, and inferential error (the deviation of Type I error from the nominal level) only arises through the deviation of F^* from F. The larger this deviation, the larger the potential inferential error.

3 Non-Parametric Estimation of Conditional Distribution

We saw in the previous section that the problem of bootstrap inference reduces to the problem of selecting an estimate F^* of F, the unknown data distribution, and that it is desirable for this estimate to be efficient. Since all that is known about F^* is the conditional moment restriction (1), the problem appears to reduce to the problem of efficient non-parametric estimation subject to a conditional moment restriction. Brown and Newey (1999) consider the case of independent observations; we propose here an analog for the case of dependent observations.

It is helpful to decompose F using the conditional factorization

$$F(y_1, ..., y_n) = \prod_{t=1}^{n} G(y_t \mid \mathcal{F}_{t-1}) F_0(x_1)$$

where $G(y \mid \mathcal{F}_{t-1}) = P(y_t \leq y \mid \mathcal{F}_{t-1})$ denotes the one-step-ahead conditional distribution function and F_0 denotes the distribution of the initial condition x_1. Thus if we have non-parametric estimates G^* and F_0^* of G and F_0, respectively, we can form the natural non-parametric estimate of F :

$$F^*(y_1, ..., y_n) = \prod_{t=1}^{n} G^*(y_t \mid \mathcal{F}_{t-1}) F_0^*(x_1).$$

We now turn to the problem of non-parametric estimation of the one-step-ahead conditional distribution G. Supposing that G depends only on the most recent m lags[1] of the series, we can write as $G(y \mid \mathcal{F}_{t-1}) = G(y \mid x_t)$. Then (1) can be written as

$$\int g(y, x_t; \theta_0)\, dG(y \mid x_t) = 0. \tag{3}$$

Observe that (3) is a restriction on the one-step-ahead conditional distribution function G.

In the context of independent observations, Brown and Newey (1999) argue that efficient estimation of G requires that the estimate G^* satisfy the empirical analog of (3):

$$0 = \int g\left(y, x_t; \hat{\theta}\right) dG^*(y \mid x_t). \tag{4}$$

Independently, Brown and Newey (1999) and Ahn, Kitamura and Tripathi (1999) have proposed similar asymptotically efficient estimators[2] which satisfy (4).

This estimator G^* is a hybrid mixture of non-parametric density and empirical likelihood estimation. The estimator can be described briefly as follows. For any given value of x_t, find a multinomial distribution on the support points $(y_1, y_2, ..., y_n)$ described by the probabilities $p = (p_1, p_2, ..., p_n)$ such that the distribution satisfies the conditional moment restriction (4), yet is close, in the sense of locally weighted empirical likelihood distance, to a non-parametric kernel estimator of the distribution.

[1]If G depends on k lags of the series, and the function g depends on l lags of the series, then without loss of generality we can set $m = \max(k, l)$.

[2]We follow the treatment of Ahn, Kitamura and Tripathi (1999) since their estimator is guaranteed to produce non-negative probability weights.

More precisely (again x_t is held fixed), for some non-negative kernel $K(\cdot)$ and bandwidth h, define the kernel weights

$$w_i = \frac{K\left(\frac{x_i - x_t}{h}\right)}{\sum_{j=1}^{n} K\left(\frac{x_j - x_t}{h}\right)}, \qquad i = 1, ..., n, \tag{5}$$

and local empirical likelihood estimator

$$\hat{p} = (\hat{p}_1, ..., \hat{p}_n) = \underset{p_1,\dots,p_n}{\operatorname{argmax}} \sum_{i=1}^{n} w_i \log(p_i)$$

where the p_i are constrained to satisfy

$$p_i \geq 0, \qquad \sum_{i=1}^{n} p_i = 1, \qquad \sum_{i=1}^{n} g\left(y_i, x_t; \hat{\theta}\right) p_i = 0. \tag{6}$$

The estimator $G^*(y \mid x_t)$ is the multinomial conditional probability distribution such that $P(y_t^* = y_i \mid x_t) = \hat{p}_i$, $i = 1, ..., n$. The constraint (6) ensures that this is a valid probability distribution and the conditional moment restriction (4) is satisfied.

Numerically, a convenient method to obtain the estimator is via a Lagrange multiplier technique. Ahn, Kitamura and Tripathi (1999) show that the solution satisfies

$$\hat{p}_i = \frac{w_i}{n + \lambda' g\left(y_i, x_t; \hat{\theta}\right)} \tag{7}$$

where λ solves

$$\sum_{i=1}^{n} \frac{g\left(y_i, x_t; \hat{\theta}\right) w_i}{n + \lambda' g\left(y_i, x_t; \hat{\theta}\right)} = 0. \tag{8}$$

The multiplier λ can be found by numerically solving the s nonlinear equations in (8), yielding the probabilities (7).

Relevant issues for empirical implementation include the choice of kernel and bandwidth. For the kernel, a convenient choice is the multivariate normal

$$K(u) = \exp\left(-\left(u'\Sigma^{-1}u\right)/2\right) \tag{9}$$

(normalizing constants are irrelevant because of the definition (5)), where

$$\Sigma = n^{-1} \sum_{t=1}^{n} (x_t - \overline{x})(x_t - \overline{x})',$$

the sample covariance matrix of the conditioning variables x_t. This scaling by Σ results in an estimator which is invariant to linear transformations of x_t. The choice of normal kernel is convenient because it technically precludes the possibility that for some x, $K\left(\frac{x_i - x}{h}\right) = 0$ for all i, which can happen for kernels with bounded support if x is unusual and the bandwidth h is small.

Neither Ahn, Kitamura and Tripathi (1999) nor Brown and Newey (1999) give a rule to select the bandwidth h. Since the estimation problem is very similar to that of m-variate density estimation (see (5) above), we use the plug-in rule suggested by Silverman (1986, p. 45, 86-87) for multivariate density estimation:

$$h = c_m n^{-1/(4+m)} \tag{10}$$

where c_m is determined by the choice of kernel. For the normal kernel,

$$\begin{aligned} c_1 &= 1.06 \\ c_2 &= 0.96 \\ c_m &= \left(\frac{4}{1+2m}\right)^{1/(4+m)}, \qquad m > 2. \end{aligned}$$

It would be useful in the future to investigate alternative rules for selection of h, including cross-validation.

4 Bootstrap Recursion

For a given initial condition $x_1^* = \left(y_0^*, ..., y_{-m+1}^*\right)$, the conditional probability distribution G^* described in the previous section defines a non-parametric bootstrap distribution F^*. Namely, given x_1^*, the probability weights $\hat{p}_i$, $i = 1, ..., n$ are calculated from (5), (7) and (8), then a random draw is made from $(y_1, y_2, ..., y_n)$ with each receiving probability $(\hat{p}_1, \hat{p}_2, ..., \hat{p}_n)$, yielding y_1^*. Then set $x_2^* = \left(y_1^*, ..., y_{-m+2}^*\right)$, recalculate the probability weights $\hat{p}_i$, $i = 1, ..., n$, and draw from $(y_1, ..., y_n)$ with these weights to yield y_2^*. This defines a Markov chain on the points $(y_1, y_2, ..., y_n)$ and creates a bootstrap time series $(y_1^*, y_2^*, ..., y_n^*)$. As discussed above, given this bootstrap time series, the statistic of interest, namely $t_n^* = t_n\left(y_1^*, ..., y_n^*; \hat{\theta}\right)$ can be calculated. Since the distribution of $(y_1^*, y_2^*, ..., y_n^*)$ can be described by this Markov recursion, the distribution of t_n^* can be calculated by simulation.

This Markov recursion requires a choice for initial condition x_1^*. One option is to set it to the values in the sample, vis., $x_1^* = (y_0, ..., y_{-m+1})$. This has the natural advantage of conditioning on relevant information, as argued by Sims and Zha (1999). Another option is to draw x_1^* from the unconditional distribution of $(y_t^*, y_{t-1}^*, ..., y_{t-m}^*)$ defined by the Markov recursion. While intuitively attractive, it is unclear how this could be calculated, or even how to easily and generically verify that such a distribution exists. We propose a simplifying shortcut, and draw x_1^* as a random $m-$block from the original data series.

Simulating bootstrap samples via this recursion requires considerably greater computation time than conventional bootstrap methods. Table 1 shows the computation time (in minutes using Gauss32 on a 300Mhz Pentium II processor) required to calculate 90% confidence intervals using 999 bootstrap simulations, on AR(k) models, for a variety of lag orders k and sample size n. For these calculations, we set $m = k$. These computation times should only be taken as rough guidelines, since the actual time will depend on the number of iterations required to solve the equations (8). What is apparent from Table 1 is that the computation time depends quite strongly on the sample size n, and more mildly on the Markov order m. For small samples, computation time is quite modest, for example, for a sample of size $n = 50$, computation time was approximately a half-minute, and for a sample of size 100, computation time ranged from one to two minutes. But for large samples, computation time can be quite demanding, as illustrated by the case $n = 500$, where computation time ranged from 18 to 49 minutes.

Table 1: Computation Time for Non-Parametric Bootstrap Confidence Intervals (Minutes Using Gauss32 on a 300Mhz Pentium II Processor)

$m =$	1	2	4	6	8	12
$n = 50$	0.5	0.5	0.6	0.6	0.7	0.7
$n = 100$	1.1	1.3	1.5	1.6	1.8	2.4
$n = 250$	4.6	5.2	6.5	8.0	9.1	12
$n = 500$	18	20	26	32	37	49

5 Monte Carlo Evidence

We explore the behavior of alternative bootstrap methods in a simple sampling experiment. The model is an AR(1)

$$y_t = \mu + \alpha y_{t-1} + e_t, \qquad E(e_t \mid \mathcal{F}_{t-1}) = 0.$$

or constrained AR(2)

$$y_t = \mu + \alpha y_{t-2} + e_t, \qquad E(e_t \mid \mathcal{F}_{t-1}) = 0$$

where the martingale difference errors e_t are generated by an ARCH(1) process with student-t innovations (with degree of freedom parameter 5):

$$\begin{aligned} e_t &= z_t \sqrt{\omega + \gamma e_{t-1}^2} \\ z_t &\sim \text{iid } t_5 \end{aligned}$$

We normalize $\omega = 1$ and set $\mu = 0$. We vary γ among $\{.1, .3, .5, .7\}$ to assess the effect of conditional heteroskedasticity, and vary α among $\{0, .5, -.5, .8, -.8\}$ to assess the effect of serial correlation. In this experiment, we consider samples of size $n = 30$, 60 and 120, and generate 3000 Monte Carlo replications.

We are primarily interested in the comparison of our non-parametric bootstrap with the non-parametric block bootstrap. To implement the latter, we used overlapping blocks (as in Kunsch (1989)), and tried three choices of block-length $b \in \{4, 6, 8\}$. (for the $n = 120$ case we used $b \in \{6, 8, 10\}$.) Because the qualitative results were not very sensitive to the selection of block-length, we only report the results for $b = 6$ for $n = 30$ and 60, and $b = 8$ for $n = 120$. To implement our non-parametric bootstrap we set $m = 1$ for the AR(1) and $m = 2$ for the AR(2), and used the kernel (9) with bandwidth determined by the Silverman rule (10).

The parameter of interest is taken to be the autoregressive parameter α. We consider tests of $H_0 : \alpha = \alpha_0$, and report results for tests of nominal size 5%. We report results for two-sided tests and one-sided tests (against both alternatives, since the sampling distributions are asymmetric). These results can alternatively be interpreted as coverage probabilities of bootstrap confidence intervals, constructed via either the symmetric two-sided or asymmetric equal-tailed methods.

Following the recommendation of Hall (1992), we consider tests based on the percentile-t method. That is, our selected test statistic is $t_n = (\hat{\alpha} - \alpha_0)/\hat{s}$, where $\hat{s}$ (with one exception discussed below) is the Eicker-White heterskedasticity-consistent standard error for $\hat{\alpha}$. In this model, t_n is asymptotically distributed as $N(0, 1)$.

For the one exception to this rule, we report results for the model-based autoregressive bootstrap using conventional standard errors (for the AR(1) model only). We report this case merely because this method is widely used in practice, and we wish to emphasize the distortions from using nonpivotal methods. This method uses the OLS estimates and independent draws from the OLS residuals $\{\hat{e}_1, ..., \hat{e}_n\}$ to generate bootstrap samples, and conducts inference using the percentile-t method using a conventional OLS standard error.

In addition to the above method, we report two baseline model-based bootstrap methods. The first is identical to the model-based bootstrap method described above, except that inference is based on the percentile-t method using the Eicker-White heterskedasticity-consistent standard error. This might be viewed as a naive hybrid bootstrap, as the test statistic is explicitly robust to heteroskedasticity, yet the bootstrap algorithm explicitly generates bootstrap time-series which are conditionally homoskedastic (with iid errors).

For the second baseline model-based bootstrap, we use an estimated AR/ARCH model. An ARCH(1) is fit to the AR residuals $\hat{e}_t$ by Gaussian quasi-likelihood, yielding fitted conditional variances and normalized residuals $\hat{z}_t = \hat{e}_t / \sqrt{\hat{\omega} + \hat{\gamma}\hat{e}_{t-1}^2}$. The bootstrap algorithm proceeds by iid resampling from the empirical normalized residuals $\{\hat{z}_2, ..., \hat{z}_n\}$, and the AR/ARCH process is then generated (using the estimated parameters) as specified by the model. We report this bootstrap as a baseline comparison, not because it is a realistic method for empirical practice, (as it requires correct knowledge of the conditional variance equation) but to illustrate the "best-case" for comparison with the non-parametric methods.

As stated above, our tests are based on a t-statistic t_n. Each bootstrap method generates a bootstrap distribution for the t-statistic which we can denote by t_n^* with conditional probability measure P^*. Then the bootstrap p-value for a symmetric two-sided test of $H_0 : \alpha = \alpha_0$ against $H_1 : \alpha \neq \alpha_0$ is $p_1 = P^*(|t_n^*| > |t_n|)$, the bootstrap p-value for a one-sided test of $H_0 : \alpha = \alpha_0$ against $H_1 : \alpha > \alpha_0$ is $p_2 = P^*(t_n^* > t_n)$, and the bootstrap p-value for a one-sided test of $H_0 : \alpha = \alpha_0$ against $H_1 : \alpha < \alpha_0$ is $p_3 = P^*(t_n^* < t_n)$. For each Monte Carlo sample and bootstrap method, these bootstrap p-values are calculated by simulation using 399 bootstrap replications. The test rejects at the 5% nominal level if the bootstrap p-value is smaller than 0.05.

We report the frequency of rejection (across the 3000 Monte Carlo samples) in Tables 2, 3, and 4 for the AR(1) model for the cases $n = 30$, 60, and 120, respectively, and in Tables 5 and 6 for the AR(2) model for the cases $n = 30$ and $n = 60$.

The results in all tables tell essentially the same story. As expected, the naive model-based bootstrap using conventional standard errors has quite poor performance when the ARCH parameter γ is large. The modified naive model-based bootstrap, using White standard errors, performs surprisingly well, but does show meaningful size distortion when the ARCH parameter γ is large. For $n = 30$, the AR/ARCH bootstrap performs only slightly better, but there is a more noticeable improvement for larger sample sizes. The blocking bootstrap does not do well in any experiment. The results for the two-sided tests are often reasonable, but the one-sided tests have enormous size distortions. More disturbingly, the extent of the size distortion does not seem to diminish as the sample size increases.

Our non-parametric bootstrap, however, does quite well in most cases. The size distortion is typically no worse than the (infeasible) bootstrap based on the correct model, and in some cases has less size distortion. A notable exception, however, is the one-tailed test of $H_0 : \alpha = \alpha_0$ against $H_1 : \alpha < \alpha_0$ for the case of negative AR parameters, $\alpha = -.5$ and $\alpha = -.8$. In these cases, the non-parametric bootstrap does less well than the baseline model-based bootstrap methods. The performance is dramatically better than the block bootstrap,however. Another exception is the one-tailed test of $H_0 : \alpha = \alpha_0$ against $H_1 : \alpha > \alpha_0$ for the AR(2) model with $n = 30$ and $\alpha = .8$, where the non-parametric bootstrap is notably undersized. Overall, the numerical evidence shows that the non-parametric bootstrap does reasonably well even in very small samples, at least in the simple class of models considered.

Table 2: **Size of 5% Bootstrap Tests, AR(1), $n = 30$**

	$P(p_1 < .05)$				$P(p_2 < .05)$				$P(p_3 < .05)$			
$\gamma =$	.1	.3	.5	.7	.1	.3	.5	.7	.1	.3	.5	.7
$\alpha = 0$												
AR/OLS	.086	.137	.183	.204	.083	.130	.144	.165	.095	.120	.155	.165
AR/White	.075	.073	.088	.091	.078	.099	.101	.107	.082	.077	.088	.083
AR/ARCH	.074	.069	.077	.080	.076	.090	.087	.088	.084	.074	.085	.080
Block	.096	.084	.078	.075	.140	.178	.179	.199	.167	.166	.200	.199
Non-Par	.059	.055	.053	.045	.095	.094	.101	.113	.087	.088	.093	.112
$\alpha = .5$												
AR/OLS	.086	.116	.155	.186	.081	.100	.124	.149	.100	.128	.152	.177
AR/White	.070	.070	.084	.088	.079	.086	.102	.110	.076	.084	.088	.094
AR/ARCH	.066	.066	.072	.070	.077	.076	.089	.092	.070	.082	.082	.085
Block	.081	.074	.077	.078	.210	.224	.234	.247	.103	.109	.119	.140
Non-Par	.065	.064	.065	.065	.072	.092	.081	.116	.085	.089	.097	.104
$\alpha = -.5$												
AR/OLS	.080	.128	.150	.181	.087	.121	.141	.163	.086	.109	.116	.126
AR/White	.072	.078	.075	.076	.081	.090	.099	.098	.075	.086	.078	.073
AR/ARCH	.063	.074	.062	.063	.079	.077	.080	.079	.072	.082	.076	.070
Block	.096	.081	.071	.062	.095	.115	.122	.139	.262	.258	.254	.260
Non-Par	.052	.048	.054	.046	.081	.098	.097	.097	.081	.091	.116	.122
$\alpha = .8$												
AR/OLS	.106	.143	.152	.172	.072	.096	.108	.120	.114	.150	.160	.176
AR/White	.082	.094	.092	.095	.072	.094	.102	.110	.090	.102	.096	.101
AR/ARCH	.078	.086	.081	.081	.067	.085	.088	.087	.090	.096	.089	.088
Block	.091	.090	.081	.067	.237	.251	.236	.253	.101	.103	.100	.099
Non-Par	.078	.077	.077	.073	.060	.068	.078	.085	.094	.092	.092	.099
$\alpha = -.8$												
AR/OLS	.054	.089	.114	.133	.072	.101	.130	.132	.063	.078	.085	.113
AR/White	.045	.064	.073	.082	.062	.083	.091	.092	.065	.065	.069	.079
AR/ARCH	.042	.058	.060	.066	.062	.074	.075	.072	.062	.074	.075	.072
Block	.060	.067	.056	.063	.063	.079	.088	.092	.353	.341	.339	.341
Non-Par	.050	.052	.053	.057	.074	.095	.098	.097	.095	.109	.125	.136

Table 3: Size of 5% Bootstrap Tests, AR(1), $n = 60$

	$P(p_1 < .05)$				$P(p_2 < .05)$				$P(p_3 < .05)$			
$\gamma =$	.1	.3	.5	.7	.1	.3	.5	.7	.1	.3	.5	.7
$\alpha = 0$												
AR/OLS	.118	.181	.235	.301	.093	.150	.176	.201	.105	.132	.168	.210
AR/White	.077	.076	.083	.099	.074	.096	.090	.103	.076	.068	.066	.085
AR/ARCH	.069	.062	.070	.067	.072	.082	.071	.070	.069	.064	.059	.073
Block	.075	.060	.055	.054	.133	.158	.171	.183	.146	.148	.159	.185
Non-Par	.058	.055	.049	.047	.072	.089	.096	.109	.083	.091	.088	.109
$\alpha = .5$												
AR/OLS	.098	.155	.221	.260	.085	.120	.151	.176	.101	.137	.178	.210
AR/White	.069	.079	.078	.080	.073	.080	.093	.108	.074	.075	.074	.078
AR/ARCH	.065	.065	.058	.055	.072	.068	.079	.079	.072	.069	.066	.062
Block	.058	.054	.042	.046	.262	.253	.274	.278	.071	.076	.086	.101
Non-Par	.057	.054	.053	.041	.066	.082	.086	.100	.071	.076	.084	.076
$\alpha = -.5$												
AR/OLS	.098	.157	.216	.280	.088	.128	.172	.205	.086	.120	.150	.166
AR/White	.066	.074	.084	.098	.065	.075	.092	.096	.070	.074	.074	.078
AR/ARCH	.061	.065	.066	.067	.063	.064	.075	.068	.064	.071	.066	.073
Block	.054	.052	.047	.044	.054	.069	.091	.106	.282	.276	.286	.274
Non-Par	.047	.047	.040	.043	.063	.078	.081	.093	.078	.090	.109	.123
$\alpha = .8$												
AR/OLS	.089	.134	.175	.213	.088	.102	.117	.131	.099	.143	.174	.210
AR/White	.067	.071	.075	.084	.082	.089	.091	.104	.074	.078	.085	.090
AR/ARCH	.064	.060	.057	.058	.080	.083	.074	.079	.071	.068	.071	.072
Block	.044	.040	.038	.040	.371	.375	.361	.338	.044	.047	.053	.062
Non-Par	.052	.060	.054	.046	.065	.070	.087	.103	.069	.078	.072	.064
$\alpha = -.8$												
AR/OLS	.064	.120	.153	.201	.072	.115	.142	.178	.069	.087	.109	.126
AR/White	.056	.066	.068	.083	.057	.078	.083	.099	.061	.065	.068	.066
AR/ARCH	.051	.056	.056	.062	.054	.063	.064	.067	.062	.064	.064	.061
Block	.032	.030	.034	.035	.028	.034	.046	.058	.481	.460	.431	.419
Non-Par	.045	.056	.043	.044	.064	.074	.069	.082	.093	.122	.125	.141

Table 4: Size of 5% Bootstrap Tests, AR(1), $n = 120$

	$P(p_1 < .05)$				$P(p_2 < .05)$				$P(p_3 < .05)$			
$\gamma =$	.1	.3	.5	.7	.1	.3	.5	.7	.1	.3	.5	.7
$\alpha = 0$												
AR/OLS	.127	.229	.328	.412	.106	.173	.208	.235	.100	.155	.214	.264
AR/White	.067	.076	.092	.109	.073	.080	.092	.098	.062	.069	.074	.080
AR/ARCH	.062	.060	.061	.066	.069	.067	.063	.066	.063	.060	.063	.065
Block	.066	.054	.047	.047	.128	.141	.162	.174	.123	.133	.159	.195
Non-Par	.044	.046	.039	.033	.056	.072	.082	.099	.065	.075	.083	.097
$\alpha = .5$												
AR/OLS	.108	.197	.304	.376	.093	.148	.175	.209	.096	.152	.224	.262
AR/White	.063	.063	.088	.101	.068	.083	.090	.094	.064	.059	.077	.089
AR/ARCH	.061	.054	.058	.063	.061	.067	.070	.070	.058	.052	.061	.071
Block	.043	.033	.041	.038	.262	.266	.269	.273	.046	.057	.094	.103
Non-Par	.046	.041	.044	.039	.073	.077	.097	.104	.057	.056	.071	.075
$\alpha = -.5$												
AR/OLS	.105	.194	.282	.367	.095	.157	.202	.241	.081	.137	.165	.209
AR/White	.060	.078	.078	.097	.067	.074	.081	.087	.055	.070	.071	.083
AR/ARCH	.058	.063	.059	.061	.058	.060	.047	.051	.058	.065	.066	.077
Block	.041	.040	.037	.040	.041	.066	.078	.103	.253	.265	.263	.276
Non-Par	.048	.048	.029	.029	.060	.066	.065	.076	.066	.091	.101	.126
$\alpha = .8$												
AR/OLS	.092	.148	.211	.278	.076	.126	.141	.166	.095	.138	.182	.233
AR/White	.070	.067	.057	.073	.069	.093	.095	.102	.070	.069	.064	.080
AR/ARCH	.064	.057	.044	.046	.068	.082	.074	.071	.069	.062	.053	.060
Block	.031	.027	.019	.020	.447	.432	.408	.376	.030	.031	.036	.046
Non-Par	.058	.047	.039	.036	.067	.089	.099	.122	.068	.062	.051	.062
$\alpha = -.8$												
AR/OLS	.077	.158	.214	.284	.079	.137	.179	.226	.071	.104	.134	.153
AR/White	.053	.061	.070	.080	.059	.076	.075	.090	.060	.063	.065	.064
AR/ARCH	.049	.055	.054	.049	.057	.063	.049	.048	.058	.061	.061	.067
Block	.017	.020	.021	.022	.016	.024	.033	.054	.524	.476	.446	.412
Non-Par	.039	.044	.038	.030	.055	.071	.063	.069	.092	.116	.131	.163

Table 5: Size of 5% Bootstrap Tests, AR(2), $n = 30$

	$P(p_1 < .05)$				$P(p_2 < .05)$				$P(p_3 < .05)$			
$\gamma =$	.1	.3	.5	.7	.1	.3	.5	.7	.1	.3	.5	.7
$\alpha = 0$												
AR/White	.054	.057	.057	.067	.068	.076	.075	.084	.071	.064	.063	.073
AR/ARCH	.056	.056	.051	.064	.065	.075	.069	.080	.073	.064	.065	.076
Block	.055	.046	.041	.042	.133	.154	.160	.176	.166	.164	.179	.201
Non-Par	.041	.038	.040	.038	.069	.068	.079	.078	.069	.076	.083	.083
$\alpha = .5$												
AR/White	.060	.064	.059	.069	.069	.067	.082	.075	.068	.069	.067	.080
AR/ARCH	.062	.057	.059	.061	.067	.064	.076	.069	.067	.067	.066	.081
Block	.054	.051	.046	.048	.254	.251	.281	.287	.065	.070	.076	.091
Non-Par	.046	.052	.046	.054	.040	.041	.043	.046	.060	.063	.064	.066
$\alpha = -.5$												
AR/White	.052	.059	.059	.065	.068	.071	.077	.076	.060	.071	.061	.066
AR/ARCH	.052	.052	.054	.062	.064	.066	.075	.067	.060	.068	.059	.069
Block	.040	.045	.040	.037	.050	.064	.073	.078	.348	.345	.342	.365
Non-Par	.041	.037	.046	.049	.062	.065	.070	.072	.069	.079	.094	.113
$\alpha = .8$												
AR/White	.079	.064	.075	.071	.054	.052	.061	.066	.083	.071	.084	.081
AR/ARCH	.076	.065	.075	.066	.052	.052	.055	.058	.084	.072	.083	.077
Block	.047	.043	.052	.040	.352	.368	.349	.341	.049	.045	.060	.055
Non-Par	.059	.058	.058	.063	.010	.012	.016	.018	.082	.077	.073	.081
$\alpha = -.8$												
AR/White	.052	.056	.052	.056	.059	.063	.069	.073	.061	.068	.063	.052
AR/ARCH	.053	.052	.051	.053	.058	.060	.063	.065	.063	.066	.061	.053
Block	.037	.032	.033	.033	.031	.029	.040	.042	.537	.542	.531	.521
Non-Par	.042	.035	.043	.041	.079	.076	.079	.073	.166	.168	.191	.187

Table 6: Size of 5% Bootstrap Tests, AR(2), $n = 60$

	$P(p_1 < .05)$				$P(p_2 < .05)$				$P(p_3 < .05)$			
$\gamma =$	.1	.3	.5	.7	.1	.3	.5	.7	.1	.3	.5	.7
$\alpha = 0$												
AR/White	.066	.056	.066	.072	.064	.068	.072	.086	.064	.058	.068	.065
AR/ARCH	.062	.053	.061	.056	.065	.062	.062	.071	.060	.060	.070	.063
Block	.040	.028	.018	.018	.126	.143	.180	.198	.152	.162	.177	.196
Non-Par	.050	.040	.037	.029	.072	.077	.086	.106	.068	.074	.083	.084
$\alpha = .5$												
AR/White	.053	.052	.059	.065	.057	.073	.071	.081	.053	.058	.065	.070
AR/ARCH	.052	.049	.056	.053	.057	.067	.064	.068	.056	.054	.066	.068
Block	.025	.023	.025	.024	.387	.392	.380	.390	.029	.031	.045	.050
Non-Par	.041	.041	.043	.041	.039	.053	.047	.056	.040	.043	.044	.046
$\alpha = -.5$												
AR/White	.060	.059	.062	.070	.058	.063	.067	.075	.063	.055	.064	.072
AR/ARCH	.061	.056	.056	.062	.052	.061	.061	.061	.066	.057	.065	.070
Block	.025	.024	.018	.015	.024	.031	.037	.043	.452	.457	.456	.452
Non-Par	.048	.045	.042	.048	.047	.053	.046	.049	.079	.082	.105	.124
$\alpha = .8$												
AR/White	.062	.053	.064	.067	.067	.064	.072	.079	.067	.062	.072	.072
AR/ARCH	.060	.051	.053	.058	.064	.059	.064	.066	.067	.060	.066	.069
Block	.016	.010	.010	.011	.614	.619	.601	.581	.015	.009	.011	.016
Non-Par	.053	.046	.045	.043	.023	.021	.028	.026	.079	.069	.068	.062
$\alpha = .8$												
AR/White	.051	.057	.055	.063	.054	.063	.067	.069	.058	.063	.062	.072
AR/ARCH	.049	.055	.048	.057	.054	.059	.059	.057	.061	.065	.061	.064
Block	.007	.006	.010	.008	.005	.007	.009	.015	.739	.723	.705	.690
Non-Par	.030	.030	.031	.033	.065	.069	.069	.068	.181	.209	.229	.260

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