Specific Human Capital, Trade and the Wealth of Nations[*]

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Abstract

We develop a general equilibrium model of trade with endogenous human capital acquisition in job specific skills and imperfectly observable skills. A country with a relatively skilled labor force will specialize in production of goods that are intensive in skilled labor. Incentives to invest in human capital depend on aggregate investments *within the country* and the relative factor prices. Demand for skilled workers at home decreases when the world has more human capital and we show that incentives to invest are strictly decreasing in aggregate investments in the other country. Hence, even if there are no fundamental differences between countries there may be equilibria under international trade with specialization. In particular, this may happen even if there is a unique equilibrium under autharchy and protectionism may in this case be a welfare enhancing policy for the poor country.

1 Introduction

There is room for debate about the magnitudes, but few would argue against the claim that there are enormous differences in living standards across countries. Quite naturally these differences have

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inspired vast amounts of both theoretical and empirical research and there are many competing explanations for the large cross country differences.

Despite of the abundance of theories we will in this paper propose yet another model that can explain cross country differences in standards of living. Our model has in common with many other approaches that externalities are central[1]. Furthermore, as in most macro-oriented research on the topic, differences in standard of living are generated by differences in human capital accumulation. However, contrary to most previous work we provide explicit microfoundations for the externalities. The external effects are *derived* from what we view as a rather natural informational problem and will as a consequence of barriers to labor mobility be local to the country or region that labor can move freely within.

Our model is a rather stylized model of human capital accumulation and trade between two ex ante identical economies. There are two jobs, two final goods, two countries and a continuum of consumers/workers in each country. We refer to the jobs as the *skilled job* and the *unskilled job* respectively. We also make the assumption that a worker must have made a costly human capital investment in order to be productive in the skilled job, while all workers are equally productive in the unskilled job. Hence, human capital is specific to the skilled job

Firms produce two final consumption goods, one which is more intensive in skilled labor than the other, and we assume that firms act competitively and all have access to the same constant returns technology.

What distinguishes our model from most of the previous literature is that we assume that human capital investments are imperfectly observable. Competitive firms pay a worker the expected value of her marginal product. This expectation depends observable characteristics and, though standard Bayesian updating, also on the aggregate level of human capital investments in the economy. But rational workers make a standard cost benefit calculation when deciding how much to invest, so the distribution of wages in turn affects the incentives for human capital investments and therefore investments. The assumption that human capital investments are imperfectly observable therefore leads to externalities in human capital investments.

[1] Well known examples from the growth literature where externalities are important are Lucas [6] and Romer [12]. In trade theory it is also common to analyze models where cross country differences are generated from external effects and Eithier [?] is an early contribution to that literature.

These externalities may create multiple equilibria also if we would assume that there is a single consumption good and consequently no role for international trade. At low levels of human capital investments incentives to invest are increasing in aggregate human capital since skilled workers are so scarce. However, at sufficiently high levels of investment, skilled labor is not so scarce and the prior (aggregate investments) starts to be more important than the noisy signals in determining the wage resulting in a less high powered incentive scheme. Hence, the relationship between aggregate investments and returns to human capital investments is non-monotonic. Depending on details of the distribution of investment costs and the signaling technology there may or may not be multiple equilibria.

Cross country differences in income levels may therefore be generated also in a simplified version of the model with a single good due to coordination on different equilibria. We view this feature more as an unavoidable nuisance due to the informational problem than an interesting theory of income differentials. It is well understood that models with multiple equilibria can explain cross country differences as a result of coordination on different equilibria and there are many other models that are capable of generating multiple equilibria.

What we view as interesting is something that can only happen in the version with two goods. Then there is an interesting interplay between the local informational externality and standard general equilibrium price effects that makes incentives to invest in human capital dependent on aggregate investment behavior also in the other country. In particular, the higher are aggregate investments in the other country, the less valuable workers with skill are in the home country and we show that this will tend to reduce the compensation for workers in the skilled sector and increase wages for unskilled workers. This results in a less high powered incentive scheme at home, so the returns to human capital investments in any country is decreasing in human capital investments abroad.

What is described above is a standard scarcity reasoning that would be true even if there is no informational asymmetry. However, the informational asymmetry means that incentives are affected *differently* in the country where the change occurs and in the country where no change occurs. While it is true that an increase in the number of investors abroad makes investors less a scarce resource which tends to decrease the compensation for workers that are assigned to the skilled job there is also a direct effect on incentives which comes from the fact that when more agents abroad invest the posterior probability that a worker has human capital increases given any observable characteristics. This effect tends to improve incentives.

We show that the interplay between the informational externality and the price effects described above implies that when countries trade, then there is a possibility that identical countries specialize and therefore maintain a different level of wealth and human capital, *even if parameters are such that there is a unique autharchic equilibrium*. This type of trade equilibrium is not based on a country-wide coordination failure: what stops citizens of the poor country to invest more in human capital is the fact that the technologically intensive good can be imported from the other country. While this type of specialization typically is to the disadvantage for the country that specializes in the good with low skill-content it actually helps in alleviating the informational problem and production possibilities are increased when moving in the direction of increased specialization in human capital investments when holding the total quantity of investors constant. These points are illustrated in Section 4 by analyzing a simple example that allows for closed form solutions and later sections show that the effects on prices and factor prices that drive the example hold also under much more general circumstances.

A useful way of thinking about our model is in terms of the Hecksher-Ohlin-Samuelsson (HOS) model. Given human capital investments, the model works essentially as the HOS model. However, the production possibility set is not exogenously given, but determined by individual human capital investments. Hence, trade policy changes the production possibility set of each country and it is possible that this has the effect that a country specializing in the good with low content of skilled labor may be better off in autharchy[2].

Another interesting feature of our model is that it has implications for cross country comparisons of wage *distributions.* In particular, higher levels of human capital corresponds to higher incentives for human capital accumulation, which empirically corresponds to higher variability in wages that can not be accounted for by observable characteristics.

We view our model as too stylized for quantitative implementations, but we think that the model may be successfully embedded in a richer dynamic framework and that our more explicit microfoundations may be a useful way to "see" the externalities. For example, it is crucial for

[2]As is well-known, trade models with increasing returns can produce the same result for more or less the same reasons.

the quantitative analysis in Lucas [6],[7] that external benefits of human capital across national borders are assumed to be zero. Lucas himself views this as a troubling aspect and argues that some spillovers should be local while others should be world wide, but that it seems difficult to get evidence on the relative importance of local versus global spillovers. In our model on the other hand, the externality is derived from fundamental assumptions and there are no cross country externalities, although the price effects create complementarities in investment behavior that look "as if" there is a negative externality.

The paper is structured as follows...

2 The Model

Our model borrows from our earlier work on economics of discrimination in Moro and Norman [9], which in turn is inspired by Arrow [1], Coate and Loury [2], Phelps [11] and others. The main departure from the models of discrimination is that we now need more than one good, so relative prices between consumption goods must be determined in equilibrium. This part of the model is completely standard and given endowments of human capital in each country, much of the equilibrium characterization parallels that in the Hecksher-Ohlin model very closely.

2.1 Human Capital Investments

In each country there is a continuum of workers with heterogenous costs of investment in their human capital. Agents are distributed on $[\underline{k}, \overline{k}]$ according to a distribution function G which is assumed to be continuous and strictly increasing in the more general version of the model. Prior to entering the market each agent k has to choose whether to invest in human capital or not. An agent who invests incurs (his personal) investment cost k, while agents who don't invest incur no cost. We assume that $\underline{k} \leq 0$ and $\overline{k} > 0$

2.2 Information Technology

After the investments, nature assigns each worker a signal $\theta \in \Theta$. The example in Section 4 uses a discrete set of signals, but in the general model it is more convenient to assume that $\Theta = [0, 1]$, distributed according to density f_q if the worker invested in Stage 1 and f_u otherwise. It is assumed that f_q and f_u are continuously differentiable, bounded away from zero and satisfy the strict monotone likelihood ratio property $f_q(\theta)/f_u(\theta) < f_q(\theta')/f_u(\theta')$ for all θ, θ' such that $\theta < \theta'$. This implies that qualified workers are more likely to get higher values of θ than unqualified workers. We let F_q and F_u denote the associated cumulative distributions.

2.3 Production Technology

There are two consumption goods, x_1 and x_2, both produced solely from labor. However, labor input is needed in two separate jobs in order to produce output. We refer to these jobs as *the complex task* and *the simple task.* Call workers who invested in the human capital *qualified* workers and workers who did not *unqualified.* We assume that unqualified workers who are employed in the complex task do not contribute at all to output, so we let the effective input of complex labor in industry i, c_i, be the quantity qualified workers who are employed in the complex task. In the simple task on the other hand the investment decision is immaterial for productivity, so the effective input of simple labor in industry i, s_i, is simply the number of workers (qualified and unqualified) employed in this task. It is crucial that human capital investments affect productivity differently in the different jobs. However, the extreme assumptions that non-investors are completely useless in the skilled job and that the investment does not improve productivity at all in the unskilled job are only for expositional simplicity.

Given inputs c_i and s_i the output in industry i is $y^i(c_i, s_i)$ where $y^i : R_+^2 \rightarrow R_+$ is a standard continuously differentiable neoclassical production function, satisfying constant returns to scale. As in most trade models, it is convenient to rule out "factor intensity reversals" and for this reason we assume that,

A1 $\frac{\frac{\partial y^1(c,s)}{\partial c}}{\frac{\partial y^1(c,s)}{\partial s}} > \frac{\frac{\partial y^2(c,s)}{\partial c}}{\frac{\partial y^2(c,s)}{\partial s}}$ for all $c, s > 0$.

The condition says that, given any (common) factor ratio, the increase needed in complex labor to keep output constant after a decrease in the input of simple labor is smaller in sector one. Following the language in traditional trade theory we refer to sector 1 as more intensive in complex labor than sector 2. Again, the example in Section 4 will be slightly different: there we assume that $y^1(c_1, s_1) = c_1$ and $y^2(c_2, s_2) = s_2$, which can be viewed as a limiting case of a technology that satisfies assumption A1.

2.4 Preferences

The agents in the model care about consumption and whether they undertake the investment or not. Preferences over consumption bundles (given investment behavior) are the same for all agents in the economy, so the only heterogeneity is in terms of investment costs. We also assume additive separability between the consumption part of preferences and costs of investment. The utility of an agent k consuming x_1, x_2 is taken to be $u(x_1, x_2) - k$ if the agent invests and $u(x_1, x_2)$ otherwise, where u is homothetic, strictly quasi-concave and differentiable.

3 Equilibrium

All agents are rational decision makers that take prices as given and equilibrium is defined in direct analogy with competitive equilibrium in a symmetric information environment. However, the informational problem makes it necessary to use somewhat nonstandard notions of wages and labor demands and for clarity we provide a rather detailed definition of equilibrium.

3.1 The Problem of Consumer/Workers

When wages are realized the only thing left to do for a consumer/worker is to allocate her earnings between the two goods. We assume that the utility function over consumption goods is strictly quasi-concave, so the problem

$$\begin{aligned} & \max_{x_1,x_2} u(x_1, x_2) \\ \text{s.t } p_1 x_1 + p_2 x_2 & \leq w \end{aligned} \tag{1}$$

has a unique optimal solution. We use common abuse of notation and let $x_1(w,p), x_2(w,p)$ denote the demand functions, which are identical for all agents in the economy. For notational convenience we also define

$$\begin{aligned} v(w,p) &= u(x_1(w,p), x_2(w,p)) = \\ &= \max_{x_1,x_2} u(x_1, x_2) \\ \text{s.t. } w &\leq p_1 x_1 + p_2 x_2 \end{aligned} \tag{2}$$

3.2 The Problem for the Firms

Firms observe the signal θ for each worker, but don't know whether the worker invested or not. The choice for a firm is to decide "how many" workers of each θ to employ in each task, so formally the firm chooses a pair $\langle l_i^c, l_i^s \rangle$, where $l_i^t : \Theta \to R_+$ is restricted to be integrable for $t = c, s$[3]. The input of labor in the simple task is simply the mass of workers employed in the task, that is $\int l_i^s(\theta)\, d\theta$. In order to specify the input of labor in the complex task we assume that the law of large numbers applies, so that if a fraction π of the workers are equipped with human capital and the firm employs workers in the complex task according to l_i^c, then $\int l_i^c(\theta)\, P(\theta, \pi)\, d\theta$ is the quantity of agents who are equipped with human capital, where

$$P(\theta, \pi) \equiv \frac{\pi f_q(\theta)}{\pi f_q(\theta) + (1 - \pi) f_u(\theta)}, \tag{3}$$

is the posterior probability that a worker with signal θ has invested given prior π. Since $P(\theta, \pi)$ is strictly increasing in θ it is clear that we have to allow wages to depend on the signal as well in order to be able to satisfy market clearing. Thus, we let the firm take wages $w : \Theta \to R_+$ and output price p_i as given and write the profit maximization problem as

$$\max_{\{l_C^i(\cdot), l_S^i(\cdot)\}} p_i y^i \left(\int l_i^c(\theta)\, P(\theta, \pi)\, d\theta, \int l_i^s(\theta)\, d\theta \right) - \int w(\theta) \sum_{t=c,s} l_i^t(\theta)\, d\theta \tag{4}$$

for the representative firm in sector $i = 1, 2$.

3.3 Equilibrium Human Capital Investments

All agents have rational expectations and can thus predict the wage scheme w and the prices. However, there is uncertainty about the realization of the noisy signal when agents invest. The expected utility for an agent with investment cost c is $\int_\theta v(w(\theta), p) dF_q(\theta) - k$ if agent k invests and $\int_\theta v(w(\theta), p) dF_u(\theta)$ otherwise. Rational investment behavior is thus to invest if and only if $\int_\theta v(w(\theta), p) dF_q(\theta) \geq \int_\theta v(w(\theta), p) dF_u(\theta)$ and the corresponding fraction of investors is given by

$$\pi = G\left(\int_\theta v(w(\theta), p) dF_q(\theta) - \int_\theta v(w(\theta), p) dF_u(\theta) \right) \tag{5}$$

[3] More generally we could let the firm choose a measure over Θ and out formulation clearly rules out for example any measure with a mass-point. The more general formulation does however not add anything to the analysis.

3.4 Conditions for Equilibrium

The fraction of investors π summarizes all relevant information about investment behavior in the population: if a fraction π invests and all agents behave optimally, then it must be that all workers with costs less than or equal to $G^{-1}(\pi)$ invest and all workers with higher costs do not invest. To avoid excessive notation we therefore leave out the trivial individual investment rules in our definition of equilibrium. For brevity we also use $f_\pi(\theta)$ as shorthand notation for $\pi f_q(\theta) + (1-\pi) f_u(\theta)$ and later on in the paper we will use $F_\pi(\theta)$ instead of $\pi F_q(\theta) + (1-\pi) F_u(\theta)$.

Definition 1 *Output prices $p^* = (p_1^*, p_2^*)$, wages $w^* : \Theta \to R_+$ together the fraction of investors π^* demand functions $x_1(w,p), x_2(w,p)$, outputs (x_1^*, x_2^*) and factor input distributions represented by $\{l_i^{c*}, l_i^{s*}\}_{i=1,2}$ constitutes a competitive equilibrium under autharky if:*

1. *l_i^{c*}, l_i^{s*} solves (4) taking p^* and π^* as given and $x_i^* = y^i\left(\int l_i^{c*}(\theta) P(\theta, \pi^*) d\theta, \int l_i^{s*}(\theta) d\theta\right)$ is the associated output for sector $i = 1, 2$.*
2. *$(x_1(w,p), x_2(w,p))$ solves (1)*
3. *$x_i^* = \int_\theta x_i(w^*(\theta), p^*) f_\pi(\theta) d\theta$ for $i = 1, 2$*
4. *$\sum_{i=1,2}(l_i^{c*}(\theta), l_i^{s*}(\theta)) = f_\pi(\theta)$ for all $\theta \in [0,1]$*
5. *$\pi^* = G\left(\int_\theta v(w^*(\theta), p^*) dF_q(\theta) - \int_\theta v(w^*(\theta), p^*) dF_u(\theta)\right)$, where $v(w,p)$ is defined in (2).*

The first condition says that factor demands and outputs must be optimal for the firm given output and factor prices and individual investment behavior, the second that each individual agent must choose her utility maximizing consumption bundle given the income received in equilibrium and prices, the third condition states that the goods market clears given these profit and utility maximizing supply and demand decisions and the fourth condition says that the factor market must clear. Finally, the last condition says that human capital investments must be individually optimal given the equilibrium wage scheme and relative prices.

We defer the introduction of the notation for the model with trade until later, but equilibrium is defined in the same way except that all variables except for goods prices then must be indexed by country and that the good market clearing conditions must be adjusted in the obvious way.

4 A Special Case

In order to develop some intuition we will first consider a special case of the model, where the production technology, the information technology and preferences are chosen in such a way that there are closed form solutions. We let the production functions be given by $y^1(c) = c$ and $y^2(s) = s$. Hence, we assume that *only* skilled labor is used in the production of the skilled good and that *only* unskilled labor is used in the production of the unskilled good and choose units so that input and output quantities are the same in each sector. Furthermore, we assume that there are only two signals and label these as $\Theta = \{b, g\}$ and assume that the conditional probability distributions are given by

$$\begin{array}{lcc} & b & g \\ \text{invest} & 1-\eta & \eta \\ \text{don't invest} & \eta & 1-\eta \end{array} . \tag{6}$$

In order for the labels to make sense we let $\eta > 1/2$, meaning that an investor is more likely to get the signal g (the good signal) than an agent who did not invest, while the opposite holds for signal b (the bad signal). We also assume that preferences are of Cobb-Douglas form, $u(x_1, x_2) = \sqrt{x_1 x_2}$[4]. Finally we will work with a discrete cost distribution, which will be described in detail below.

4.1 Autarchy Equilibrium

We normalize prices by using good 2 as a numeraire, which with our functional form assumption means that the individual demands from the consumer optimization problem (1) are given by

$$\begin{aligned} x_1(p_1, 1, w) &= \frac{w}{2p_1} \\ x_2(p_1, 1, w) &= \frac{w}{2}. \end{aligned} \tag{7}$$

The maximized utility in (2) is thus given by

$$v(w, p) = \frac{w}{2\sqrt{p_1}}. \tag{8}$$

In the investment stage workers can rationally predict p_1 as well as the wage for each signal. Workers who invest get signal g with probability η and b with probability $1-\eta$, while workers who don't

[4] Observe that because of the investment stage, the "cardinal" properties of u matters. We have chosen the form $\sqrt{x_1 x_2}$ since homogeneity of degree one of the utility function implies risk neutrality in money income.

invest get signal g with probability $1-\eta$ and b with probability η. Computing the expectation of $v(w,p)$ *conditional on investment* and subtracting from this the expectation of $v(w,p)$ *conditional on not investing* we get what we refer to as the *gross benefits of investment,* which may be written as

$$\frac{(2\eta-1)(w_g-w_b)}{2\sqrt{p_1}}. \tag{9}$$

Here, w_g is the wage earned by a worker with the good signal and w_b is the wage of a worker with a bad signal. In a full equilibrium a worker invest if and only if the gross benefits of investment given by (9) is larger than the worker specific cost of investment.

Wages and prices must also be consistent with profit maximization by firms, utility maximization by consumers and market clearing conditions. Indeed, it turns out that *given any investment behavior* there exists a unique *continuation equilibrium*[5]. Depending on the fraction of investors this unique equilibrium can take on different forms and for a better understanding of the mechanics of the model we will describe the three possible types of continuation equilibria and give exact conditions on exogenous parameters and the endogenous fraction of investors for each of these three types of equilibria to exist. For brevity we have excluded most derivations, which are available from the authors on request.

4.1.1 Possibility 1: Equilibria with workers divided across sectors in accordance to their signals

We first check whether it can be an equilibrium to employ all workers with the good signal in sector 1 and all workers with the bad signal in sector 2. Given that a fraction π invest and that a fraction η of these workers get the high signal there will then be $\pi\eta$ investors in sector 1. Of the investors, a fraction $1-\eta$ get the bad signal and of the $1-\pi$ non-investors a fraction η get the bad signal, so the associated outputs are

$$\begin{aligned} x_1 &= c = \pi\eta \qquad (10) \\ x_2 &= s = \pi(1-\eta) + (1-\pi)\,\eta. \end{aligned}$$

[5]We refer to something as a continuation equilibrium when if for some given investment behavior, all other equilibrium conditions except possible the condition that investments are optimal given the implied gross benefits of investment hold.

Now, with optimizing consumers the aggregate demand satisfies $x_1/x_2 = 1/p_1$. Hence, in order for markets to clear it must be that the price is of good one expressed in units of good 2 is

$$p_1 = \frac{\pi(1-\eta) + (1-\pi)\,\eta}{\pi\eta}. \tag{11}$$

Finally $w_b = 1$ in order to guarantee zero profits in sector 2 and

$$p_1 \pi\eta = w_g(\pi\eta + (1-\pi)\,(1-\eta)) \tag{12}$$

in order to guarantee zero profits in sector 1[6]. Moreover, $w_g \geq 1$ in order for the firms in sector 2 not to be able to make a profit by deviating and hiring workers with good signals. Also, w_g has to be low enough so that firms in sector 1 does not want to hire workers with bad signal. Combining these two considerations with the prices and wages given by (11) and (12) we find that this is indeed an equilibrium given that $\frac{1-\eta}{\eta} \leq 1$, which is always true, and if

$$w_g = \frac{\pi(1-\eta) + (1-\pi)\,\eta}{\eta\pi + (1-\eta)\,(1-\pi)} \geq 1. \tag{13}$$

Rearranging (13) it is easy to see that the inequality holds whenever $\pi \leq 1/2$.

At first glance, it may seem surprising that equilibria where workers are assigned to task *exactly* in accordance to the signals is not a knife-edge case. However, the expected productivity for the marginal worker in the high job "jumps down" when the first worker with the low signal is employed in the skilled job and this discontinuity makes this a robust possibility. The way this discontinuity can be seen in the example is that there is some flexibility on how to set w_g without violating other equilibrium conditions than zero profits within the sector.

4.1.2 Possibility 2: Equilibria where workers with good signals are in the unskilled sector

For this to be an equilibrium it must be that firms in sector 2 are indifferent between which type of worker to hire, so $w_g = w_b = 1$. Zero profits in sector 1 then immediately nails down the candidate equilibrium price as

$$p_1 = \frac{\eta\pi + (1-\eta)\,(1-\pi)}{\eta\pi}. \tag{14}$$

[6] To understand (12), observe that $p_1\pi\eta$ is the revenue of the sector and that w_g is the wage given to each worker. The quantity of workers employed is $\pi\eta + (1-\pi)(1-\eta)$, where of course the term $(1-\pi)(1-\eta)$ consists of the workers that don't contribute to output (but can not be distinguished from the others).

The only thing left to verify is that we can find good quantities consistent with market clearing given this price such that some workers with good signals are working in sector 2. Imposing optimization by consumers we find that this is possible whenever

$$\underbrace{\frac{(1-\eta)\pi + \eta(1-\pi)}{\eta\pi + (1-\eta)(1-\pi)}}_{\text{what would be } w_g \text{ if all good in skilled sector}} \quad < \quad 1$$

Hence, this type of equilibrium exists precisely when the first type of equilibrium fails to exists.

4.1.3 Possibility 3: Equilibria where workers with bad signals are in the skilled sector

The final possibility is that some workers with bad signals are in the skilled sector. However, due to the symmetry of the demand functions this can never be an equilibrium for this particular example. There is no particular intuition for this other than that the price of good 1 must be high enough so that firms in the sector make zero profits from hiring agents with bad signals to the skilled sector. When actually checking what high enough is it turns out that given this price the relative demand for good 1 is too small to rationalize that workers with the bad signal are in the skilled sector[7], so there can be no such equilibrium.

Equilibria of other forms than the three possibilities described are easy to rule out. This means that there is a unique "continuation equilibrium" for each π.

4.1.4 Equilibrium investments

By straightforward substitution from the equilibrium characterization above we get that the unique continuation equilibrium benefits of investment are

$$B(\pi) = \max\left\{\sqrt{\frac{\pi\eta}{\pi(1-\eta) + (1-\pi)\eta}}\left(\frac{(2\eta-1)^2(1-2\pi)}{2(\eta\pi + (1-\eta)(1-\pi))}\right), 0\right\}. \tag{15}$$

By thinking of π as an independent variable (15) gives the relation between actual investments and incentives to invest, which we can plot as in Figure 1 below.Any π such that $\pi = G(B(\pi))$ constitutes an equilibrium under autarchy and it is instructive to go back to the general definition

[7] However, with a slightly more general form of the utility function, $u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$, this type of equilibrium exists if and only if

$$\frac{\alpha}{1-\alpha}\frac{1-\eta}{\eta} > 1.$$

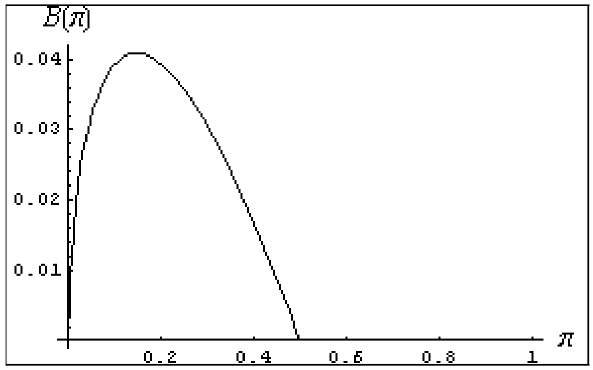

Figure 1:

of equilibrium 9 and note what we have done. In essence, (15) is condition 5 in the definition of equilibrium with all the other conditions reduced to an expression which is purely in terms of parameters of the model and the fraction of investors.

We now specialize the example further and assume that $\eta = 2/3$, which means that (15) simplifies to

$$B(\pi) = \max\left\{\frac{1}{6}\sqrt{\frac{2\pi}{2-\pi}}\left(\frac{1-2\pi}{1+\pi}\right), 0\right\}. \tag{16}$$

To make the fixed point problem as simple as possible we consider a discrete distribution of investment costs. In order for the example to be rich enough to generate the possibilities we want it turns out that at least 4 distinct cost levels are needed. We let $k_1 < k_2 < k_3 < k_4$ where $k_1 < 0$ and $k_2 > 0$ and assume that the cumulative cost distribution $G(k)$ is

$$G(k) = \begin{cases} \frac{1}{100} & \text{for } k_1 \leq k < k_2 \\ \frac{1}{4} & \text{for } k_2 \leq k < k_3 \\ \frac{1}{2} & \text{for } k_3 \leq k < k_4 \\ 1 & \text{for } k_4 \leq k \end{cases}. \tag{17}$$

Since $B(\pi) = 0$ for $\pi \geq 1/2$ it is immediate that $\pi < 1/2$ in any equilibrium, so abstracting from indifferences this leaves 1/100 and 1/4 as the only remaining potential equilibria. Observe also that if $k_2 < B(1/100)$, then $\pi = 1/100$ can not be an equilibrium since all agents with costs equal to k_2 would want to invest as a best response to the relevant equilibrium wages. I.e., $G(B(1/100)) \geq 1/4 > 1/100$, so this would not be an equilibrium. To guarantee that 1/4 is an equilibrium is equivalent to checking that $k_2 \leq B(1/4) < k_3$.

Computing numerical values for the gross benefits of investment is just a matter of plugging the relevant values for π into (16) and these are $B(1/100) = 0.016212198$ and $B(1/4) = 0.035634832$. Thus, sufficient conditions for $\pi = \frac{1}{4}$ being the *unique equilibrium under autharky* is that $k_2 < 0.016212198$ and $k_3 > 0.035634832$. Observe that mixed equilibria can be ruled out in this case as well since it is easy to show that $B(\pi) \geq B(1/100) > k_2$ for all $\pi \in [1/100.1/4]$, which makes it impossible for workers with cost k_2 to be indifferent.

4.2 Trade Equilibria

We now assume that there are two equal sized countries, h (home) and f (foreign) that can costlessly trade goods with each other, while labor is assumed completely immobile. The most straightforward way to characterize equilibria would be to approach it as in the autharky model: derive the (unique) continuation equilibrium for each (π^h, π^f) and then solve the relevant fixed point problem. This is clearly possible, but the number of potential forms of equilibria expands considerably with trade and there are seven different cases that can not be ruled out by simple arguments, so we proceed slightly differently.

We conjecture that $\pi^h = 1/100$ and $\pi^f = 1/2$ is now an equilibrium, i.e., both countries are investing at rates different from the unique autharky equilibrium. To check that this is indeed an equilibrium we derive the gross benefits of investment in each country as a function of investments within the country *conditional on the other country following the assumed equilibrium behavior.* This generates conditions on the cost function G that are necessary and sufficient for the assumed behavior to be consistent with equilibrium. Again, most derivations are omitted.

4.2.1 An Equilibrium with Inequality

We conjecture that the equilibrium will be of the form where country h produces only the unskilled good and it turns out that in this case country f must (given large enough π^f) use all workers with the good signal in the skilled sector *as well as some workers with the bad signal.* If the equilibrium takes on this form prices can be determined immediately. First of all we have that $w_b^h = w_g^h = w_b^f = 1$ for zero profits in sector 2. Moreover, firms in f must make zero profits on workers with the bad signal that work in the skilled sector, implying that

$$p_1 = \frac{\pi^f (1-\eta) + \left(1-\pi^f\right)\eta}{\pi^f (1-\eta)} = \frac{\left(2-\pi^f\right)}{\pi^f}. \tag{18}$$

Morover, for zero profits on workers with good signal in sector 1 we have that

$$w_g = \frac{p_1 \pi^f \eta}{\pi^f \eta + (1-\pi^f)(1-\eta)} = \frac{2\left(2-\pi^f\right)}{1+\pi^f}. \tag{19}$$

Hence, prices and wages are immediately nailed down. What is left to determine is the fraction of workers with bad signals that are employed in the skilled sector in order for the prices and quantities to be consistent with market clearing and optimization by consumers. Let this fraction of workers with bad signal that are in sector 1 be given by κ. The outputs are than

$$\begin{aligned} x_1 &= \pi^f \eta + \kappa \pi^f (1-\eta) && (20) \\ x_2 &= 1 + (1-\kappa)\left(\pi^f (1-\eta) + \left(1-\pi^f\right)\eta\right). \end{aligned}$$

We need to check that here are no incentives for country h to enter sector 1. This is checked by computing the expected revenue from a worker with the good signal in country h (who is paid a wage $w_g^h = 1$) and making sure that the profit is weakly negative. For $\pi^h = \frac{1}{100}$ this turns out to be satisfied for each $\pi^f \geq 0.038834951$. Moreover, it must be that there exists some $\kappa \in [0,1]$ such that the price determined in (18) is also consistent with market clearing and individual utility maximization. This condition however turns out to be implied by the other conditions.

The conclusion is that this is consistent with a continuation equilibrium in the model given that $\pi^h = \frac{1}{100}$ and $0.038834951 \leq \pi^f \leq 1$ and we can derive the gross benefits of investment simply by substituting wages and prices into (9). This gives

$$B^f\left(\pi^f; \pi^h = \frac{1}{100}\right) = \frac{1}{6}\sqrt{\frac{\pi^f}{(2-\pi^f)}}\left[\frac{3\left(1-\pi^f\right)}{1+\pi^f}\right] \tag{21}$$

in this range. Now note that for $\pi^f = \pi^h = \frac{1}{100}$, the benefits will be the same as in the autharky model. For all other values in the table below the benefits for country f are approximations of numbers computed from the expression above[8]. For h it turns out that given that $\pi^f = 1/2$, the

[8] The table hides that fixing $\pi^h = 1/100$, there are ranges of values for π^f where the equilibrium is of a different form from either the equilibrium described above or the form of the autharky equilibrium. I.e., for $\pi^f < 0.00336$ (rougly) and for $0.02922 < \pi^f < 0.03884$ the continuation equilibrium takes on different forms. However, in the first case gross benefits of investment are zero and in the second case it is strictly increasing in the interval and always below the benefits for the type of equilibrium described in this section.

benefits of investment will be zero for all $\pi^h \in [0,1]$[9]

#INVESTORS	1/100	1/20	1/10	1/5	1/4	1/3	1/2	2/3	3/4	1
$B(\pi)$ AUTHARKY	0.016	0.032	0.039	0.039	0.036	0.026	0	0	0	0
$B^f(\pi^f; 1/100)$ TRADE	0.016	0.072	0.094	0.111	0.113	0.112	0.097	0.071	0.055	0
$B^h(\pi^h; 1/2)$ TRADE	0	0	0	0	0	0	0	0.027	0.039	0

(22)

Observe that if we go back the cost distribution with four mass-points that we considered in the autharky case we see that if $k_3 < 0.096225045$, then investments consistent with $\pi^f = 1/2$ is a best response to $(\pi^h, \pi^f) = (1/100, 1/2)$. Indeed, given the previous assumption that $k_2 < 0.016212198$ the fraction $\pi^f = 1/2$ is the only possible fraction consistent with best responses by workers in f when investments are at the lowest level in country h. Thus, if

$$\begin{aligned} 0.035634832 &< k_3 < 0.096225045 \\ k_2 &< 0.016212198, \end{aligned}$$

then the example is complete since the assumed behavior in f is consistent with equilibrium and it is trivial to check that incentives in h are zero (since $w_b^h = w_g^h = 1$), so the best response to a fraction $\frac{1}{2}$ in f is for only the agents with negative costs of investment to invest. Hence $(1/100, 1/2)$ is an equilibrium.

Intuition for the rather dramatic differences in incentives that are due to the behavior in the other country can be gained from considering how the type of continuation equilibrium changes with π^f and π^h. Working out the details one can show that there are 5 different ranges where equilibria are qualitatively different as illustrated in the picture below.Region corresponds to the region where the equilibrium is qualitatively similar to autharky equilibria for $\pi \leq 1/2$. In this region good signals are in sector 1 and bad signals are in sector 1. In regions b and d the country with the lower investment rate sends some of their workers with good signals to sector 2, which implies that workers with good signals must have the same wage as workers with bad signals. The country with the higher rate of investment on the other hand uses all workers with good signals in the skilled sector, so here there are some wage differences and correspondingly some incentives for investment. Finally, in regions a and e the country with the lower investment rate is fully specialized in sector 2 and the country with the higher investment rate sends some workers with

[9] For $\pi^h \leq 0.2$ all workers in h will be in sector 2 and for for $\pi^h > 0.2$ workers with the good signal at h will be mixed between sectors. In each case $w_g^h = w_b^h = 1$.

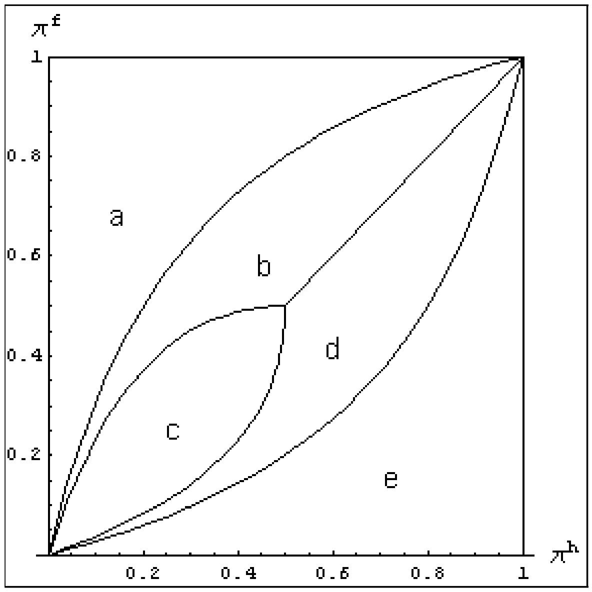

Figure 2:

bad signals to the skilled sector. Again, this means no incentives in the country with the lower rate of investments and positive incentives in the country with the higher rate.

4.2.2 Interpretation of the Example-Economics of Specialization

The crucial point of the example is that trade allows countries to specialize and that this may be consistent with equilibrium even though there are no intrinsic differences between the countries. For any fixed investment behavior in country f, the higher are investments in the other country the lower will the price of the skilled good be relative to the price of the unskilled good, which in a competitive environment implies wage in that sector will decrease in the fraction of investors in the other country. Symmetrically, the lower are the investment in h, the higher is the price and the higher is the wage in the skilled sector in country f.

While this logic is just standard textbook price theory it is important to observe that the informational asymmetry is crucial in order to generate specialization in equilibrium. If there was no informational asymmetries in the model, the standard scarcity reasoning above would still hold, *but the wage for an investor in country f would always be equal to the wage of an investor in country h since otherwise the industry in one country would be more profitable than in the other* *country.* With the informational asymmetry, wages are equalized in the sense that workers in each sector are paid the same *per expected unit of output.* However, the expected output of two otherwise identical workers from different countries will differ unless investment behavior is the same in the two countries since rational firms will use the domestic equilibrium investment behavior as their priors when evaluating their workers.

In a sense, the specialization may be viewed as an imperfect "solution" to the informational problem in the model[10]. Under much more general circumstances than in the example it can be shown that the production possibilities set expands if the differences in investment behavior is increased, but the total quantity of investors is held constant. In our example, it is trivial to use (10) to see that the equilibrium outputs in the unique autharky equilibrium are $(x_1, x_2) = (2/12, 7/12)$. In the asymmetric trade equilibrium[11] it is a little bit more tricky since we need to determine the fraction of bad signal workers in f that are working in the skilled sector to derive world outputs. For $\pi^f = 1/2$ this fraction can be computed to be $1/2$ and plugging this value into (20) we get world outputs $(x_1, x_2) = (5/12, 18/12)$. Clearly world outputs under trade are more than twice the autharky outputs, so specialization means increased output of both goods with a constant number of total investors.

A simple intuition for the increase in world production is that specialization reduces the number of "mistakes" in how workers are matched to jobs: in autharky, a fair number of workers are working in sector 1 although they are completely useless in that sector. This inefficiency is reduced in the equilibrium with specialization.

We will not dwell on welfare implications of the model, but it should be noted that increased output in the world does not necessarily translate into higher welfare even in a utilitarian sense. For the right welfare calculations we would have to take also the investment costs into consideration and here it is kind of obvious that increased specialization implies increased average investment costs. Some low cost agents in the poor country will not invest and some high cost agents in the rich country will invest. Thus, the net effect is inconclusive. However, it is a *possibility* that the symmetric equilibrium is worse in a utilitarian sense and in this case it may be that somehow

[10] For a detailed elaboration on this point in the context of discrimination, see Norman [10].

[11] The reader may object that there are now more investors on the aggregate, but no outputs or prices would change if $\pi^h = 0$, which would mean that the aggregate investments are constant.

forcing the world economy to the symmetric equilibrium is Pareto dominated by a situation where countries specialize and the rich country compensates the poor country with transfers.

5 The General Model

In many ways the equilibrium characterization for the general case follows the example. For any fixed investment behavior we show that there is a unique continuation equilibrium that can be characterized in terms of a tangency between the production possibilities set (for that fixed investment behavior) and a fictitious representative consumer. Corresponding to this tangency we get a threshold in terms of the noisy signal that determines who will be assigned to the skilled job as well as relative prices of the consumption goods. This information is enough to determine the gross benefits of investment and in a full equilibrium an agent invests if and only if the benefits exceeds the cost. Hence, from any benefit of investment we can derive the fraction of investment which is consistent with equilibrium and vice versa. Hence, in the second step, full equilibria are characterized in terms of a fixed point equation in the fraction of investors.

5.1 Notation and Preliminary Results

For sectors $i = 1, 2$ we let c_i and s_i denote generic inputs of labor in the complex and simple task respectively. In order for $\{c_i, s_i\}_{i=1,2}$ to be feasible when the fraction of investors is π there must be some "labor demand" $l : \Theta \to R^4_+$ such that

$$c_i = \int_\theta l_i^c(\theta) P(\theta, \pi) \, d\theta \text{ and } s_i = \int_\theta l_i^s(\theta) \, d\theta, \tag{23}$$

for $i = 1, 2$ and

$$\sum_i (l_i^c(\theta) + l_i^s(\theta)) \leq f_\pi(\theta) \tag{24}$$

Given any π, we let $Z(\pi)$ denote the set of feasible factor inputs, that is

$$Z(\pi) = \left\{ c_1, s_1, c_2, s_2 \middle| \, \exists l : \Theta \to R^4_+ \text{ such that (23) and (24) hold} \right\}. \tag{25}$$

From this set of feasible factor inputs we can define the production possibilities set for any given investment behavior in the obvious way as

$$X(\pi) = \left\{ (x_1, x_2) \in R^2_+ \left| x_i = y^i(c_i, s_i) \right. \text{ for some } (c_1, s_1, c_2, s_2) \in Z(\pi) \right\}. \tag{26}$$

Convexity properties of $X(\pi)$ and $Z(\pi)$ will be important for the analysis and for future reference we list the relevant results that are used in later sections.

Lemma 1 *The set of feasible factor inputs in the economy is given by*

$$Z(\pi) = \left\{(c_1, s_1, c_2, s_2) \in R^4_+ \,|g(c_1 + c_2, s_1 + s_2; \pi) \geq 0 \right\}, \tag{27}$$

where

$$g(c, s; \pi) \equiv \pi - c - s + (1 - \pi) F_u \left(F_q^{-1} \left(\frac{\pi - c}{\pi} \right) \right), \tag{28}$$

Lemma 2 $g(c, s; \pi)$ *is strictly quasi-concave in* (c, s) *for any given* $\pi > 0$

Lemma 3 $Z(\pi)$ *is convex for every* $\pi \in [0, 1]$.

Lemma 4 *Suppose that* y^i *is concave for* $i = 1, 2$. *Then,* $X(\pi)$ *is a convex set for every* $\pi \in [0, 1]$.

Lemma 5 *If in addition to the hypotheses in Lemma 4 the factor intensity assumption A1 is satisfied, then for each* $x', x'' \in X(\pi)$ *where* $x', x'' >> 0$ *and each* $\lambda \in (0, 1)$ *there is a neighborhood* B *of* $\lambda x' + (1 - \lambda) x''$ *such that* $x \in X(\pi)$ *for all* $x \in B$ *(that is, the frontier of* $X(\pi)$ *can be described by a strictly concave downward sloping function as in Figure 3).*

All proofs are in the appendix. For Lemma 1 the crucial step is to verify that the efficiency frontier of $Z(\pi)$ will be achieved by "threshold rules" meaning that all workers with signals below a certain threshold will work in the simple task and all above in the complex task. Given such a threshold θ^* and full employment the aggregate input of complex labor is $c = \pi(1 - F_q(\theta^*))$ and the aggregate input of simple labor is $s = \pi F_q(\theta^*) + (1 - \pi) F_u(\theta^*)$ and by eliminating the threshold we get the condition $g(c, s; \pi) = 0$, while points where $g(c, s; \pi) < 0$ can't be achieved by any threshold and points where $g(c, s; \pi) > 0$ can be achieved by (for example) free disposal of resources. Since the two kinds of labor can be divided arbitrarily between the sectors (27) describes the feasible set. The intuition for Lemma 2 is that as one moves along a given level curve to g and shifts workers from the simple task to the complex task, the average quality of workers in the complex task decreases. Hence, the quantity of c that can be obtained by giving up a unit of s is strictly decreasing in c. Lemma 3 follows almost trivially from Lemma 2. Lemma 4 is more or less immediate since taking a convex combination of the factor inputs corresponding to any two points in $X(\pi)$ results in at least a convex combination of the different output levels by concavity of the production function. Some grinding is needed to demonstrate strict convexity in Lemma 5, but the reasoning is rather straightforward. It is already established that $X(\pi)$ is convex, so the only remaining possibility is that there is a linear segment of the production possibilities set. This implies that the marginal rate of transformation between the different labor inputs is constant on this segment. Since resources are transferred away from the sector intensive in skilled labor this in turn implies that the aggregate input of complex (simple) labor must increase (decrease). But the set of feasible aggregate inputs is strictly convex, so *all* factor inputs can be increased relative a convex combination of the initial inputs.

5.2 Equilibrium Characterization

While there is a distortion in the model which means that the production possibilities set describes feasible output taking the informational problem into account we expect that the competitive assumptions will mean that all gains from trade will be exhausted given that production must be a point in $X(\pi)$. All agents have identical homothetic utility functions over consumption bundles, so the economy is like a representative agent economy when investment costs are sunk. While the distribution across agents is indeterminate we can characterize which combinations of aggregate consumption of each good that are consistent with (restricted) Pareto optimality by solving

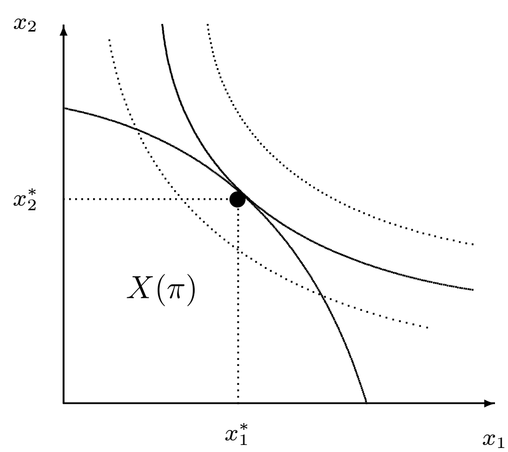

Figure 3: Efficient Production and Consumption Given Investment Behavior

$$\max_{x_1,x_2} u(x_1,x_2) \tag{29}$$
$$\text{subj to. } (x_1,x_2) \in X(\pi)$$

$X(\pi)$ is convex and compact and u is strictly quasi-concave, so there is a unique solution to (29), fully characterized as a tangency between the production possibilities set and the highest achievable level curve to u, as depicted in Figure 3. Our first proposition confirms the intuition that equilibrium allocations will be fully characterized by the restricted Pareto problem (29). We define $\theta(c,\pi)$ as the threshold signal needed in order to generate a labor input c in the complex task when a fraction π invests, that is

$$\theta(c,\pi) \equiv F_q^{-1}\left(\frac{\pi-c}{\pi}\right). \tag{30}$$

Proposition 1 *Aggregate outputs* (x_1^*,x_2^*) *and prices* (p_1^*,p_2^*) *are consistent with equilibrium conditions 1-4 of the model if and only if* (x_1^*,x_2^*) *solves (29) and* (p_1^*,p_2^*) *is a normal to a hyperplane that separates* $X(\pi)$ *and the set of bundles such that* $u(x_1,x_2) \geq u(x_1^*,x_2^*)$. *Moreover, the equilibrium wages must satisfy*

$$w^*(\theta) = \begin{cases} p_i^* \frac{\partial y^i(c_i^*,s_i^*)}{\partial s_i} & \text{for } \theta \leq \theta(c_1^*+c_2^*,\pi) \\ p_i^* P(\theta,\pi) \frac{\partial y^i(c_i^*,s_i^*)}{\partial c_i} & \text{or } \theta > \theta(c_1^*+c_2^*,\pi) \end{cases}, \tag{31}$$

where $(c_1^*,c_2^*,s_1^*,s_2^*)$ *are effective factor inputs consistent with outputs* (x_1^*,x_2^*) *and the threshold signal* $\theta(c_1^*+c_2^*,\pi)$ *must satisfy*

$$P(\theta(c_1^*+c_2^*,\pi),\pi)\frac{\partial y^i(c_i^*,s_i^*)}{\partial c_i} = \frac{\partial y^i(c_i^*,s_i^*)}{\partial s_i} \tag{32}$$

A rigorous proof is in the appendix, but since we use Proposition 1 extensively in the rest of the paper we provide a rather detailed heuristic argument. First of all we observe that since the utility function is homothetic aggregate consumption must be the solution to

$$\max_{x_1,x_2} u(x_1,x_2) \tag{33}$$
$$\text{s.t } p_1^*x_1 + p_2^*x_2 \leq \int w^*(\theta) f_\pi(\theta)\, d\theta.$$

Moreover, the budget constraint must bind with equality and the equilibrium outputs must be affordable, so the right hand side of the budget constraint must equal $p_1^*x_1^* + p_2^*x_2^*$.

Next, note that we can think of $P(\theta,\pi)$, the posterior probability that the worker invested, as the "efficiency units" of labor provided by a worker with signal θ if employed in the skilled task.

Clearly, there would be arbitrage possibilities unless $w(\theta) = w_c P(\theta, \pi)$ for all θ employed in the skilled task and $w(\theta) = w_s$ for all θ employed in the unskilled task. Moreover, $w(\theta) = w_c P(\theta, \pi) \geq w_s$ for all θ employed in the skilled task, since otherwise the workers in the unskilled task could be replaced by cheaper workers. Similarly, $w_s \geq w_c P(\theta, \pi)$ for θ in the simple task since otherwise the workers in the skilled task could be replaced by cheaper workers. Now, $P(\theta, \pi)$ is monotonically increasing in θ, so we conclude that also in a competitive equilibrium there must be a threshold θ^* such that workers above the threshold are assigned to the skilled task and those below to the unskilled task and $w_s = w_c P(\theta^*, \pi)$ since otherwise it would be profitable to replace some workers in one task with workers currently in the other. Given these arbitrage conditions on wages, the problem for the representative firm in each sector reduces to $\max_{c_i, s_i} p_i y^i(c_i, s_i) - w_c c_i - w_s s_i$, so equilibrium requires that

$$p_i \frac{\partial y^i(c_i^*, s_i^*)}{\partial c_i} = w_c \text{ and } p_i \frac{\partial y^i(c_i^*, s_i^*)}{\partial s_i} = w_s, \tag{34}$$

and combining with $w_s = w_c P(\theta^*, \pi)$ this gives condition (32) which also has the interpretation of saying that the relative price of the factors must equal the rate at which one factor can be transformed into the other. From marginal it also follows that if conditions on u are imposed that guarantees that (33) has an interior solution, then

$$\frac{\frac{\partial u(x_1^*, x_2^*)}{\partial x_1}}{\frac{\partial u(x_1^*, x_2^*)}{\partial x_2}} = \frac{p_1^*}{p_2^*} = \frac{\frac{\partial y(c_1^*, s_1^*)}{\partial c_1}}{\frac{\partial y(c_2^*, s_2^*)}{\partial c_2}} = \frac{\frac{\partial y(c_1^*, s_1^*)}{\partial s_1}}{\frac{\partial y(c_2^*, s_2^*)}{\partial s_2}} = \frac{dx_1(x_2)}{dx_2}. \tag{35}$$

Thus, in equilibrium the relative prices must separate $X(\pi)$ and the set of better bundles for the fictitious representative consumer.

Proposition 1 immediately implies that equilibria are unique in all relevant respects.

Corollary 1 *Given any $\pi > 0$ there is a unique aggregate bundle (x_1^*, x_2^*) that is consistent with equilibrium and equilibrium prices and wages are unique up to a multiplicative constant.*

Uniqueness of the aggregate bundle follows directly from Proposition 1 since (29) has a unique solution by the strict convexity of preferences and the production possibilities set. This pins down a unique relative price between the goods. Normalizing by setting $p_2^* = 1$ and noting that (A9) has a unique solution for every x_2 since the constraint set is convex and the objective is strictly quasi-concave, this in turn implies that the equilibrium wages given by (31) are unique as well up to the choice of numeraire. Thus, equilibria are fully characterized in terms of the solution to (29).

5.3 Equilibrium Investments

We choose good 2 as our unit of account and let $p(\pi)$ be the equilibrium price of good one. Furthermore, we let $x_1(\pi), x_2(\pi)$ be the equilibrium outputs, $c_i(\pi), s_i(\pi)$ the (unique) equilibrium factor inputs, $\widetilde{\theta}(\pi) \equiv F_q^{-1}\left(\frac{\pi - c_1(\pi) - c_2(\pi)}{\pi}\right)$ the associated (unique) threshold signal and $r_i(\pi) = c_i(\pi)/s_i(\pi)$ the corresponding factor ratio given a fraction of investors π. We may then write the unique equilibrium wage scheme $w(\theta;\pi)$ as

$$w(\theta;\pi) = \begin{cases} \frac{\partial y^2(r_2(\pi),1)}{\partial s_2} & \text{for } \theta \leq \widetilde{\theta}(\pi) \\ P(\theta,\pi)\frac{\partial y^2(r_2(\pi),1)}{\partial c_2} & \text{or } \theta > \widetilde{\theta}(\pi) \end{cases}. \tag{36}$$

If the final equilibrium condition (5) is satisfied for $w(\theta;\pi)$ and $p(\pi)$ generated above then all equilibrium conditions are satisfied, while if this is not the case, then the economy can not be in equilibrium for that particular fraction of investors. The equilibria of the model is thus fully characterized as fixed points to

$$\pi = G\left(\int_\theta v\left(w(\theta;\pi), p(\pi)\right) dF_q(\theta) - \int_\theta v\left(w(\theta;\pi), p(\pi)\right) dF_u(\theta)\right), \tag{37}$$

where $v(w,p)$ is defined in (2). For ease of notation we define the equilibrium benefits of investment,

$$B(\pi) \equiv \int_\theta v\left(w(\theta;\pi), p(\pi)\right) dF_q(\theta) - \int_\theta v\left(w(\theta;\pi), p(\pi)\right) dF_u(\theta). \tag{38}$$

Intuitively it is rather clear that since higher signals are better rewarded and since investing increases the probability of a higher signal we have that $B(\pi) > 0$. However, there are two exceptions. For $\pi = 0$ and for $\pi = 1$ the posterior is constant in the signal, so the wage is constant as well, so $B(0) = B(1) = 0$. We summarize the important properties of the function B as a proposition:

Proposition 2 *The function B defined in (50) satisfies the following properties: 1) B is continuous in π, 2) $B(0) = 0$, 3) $B(1) = 0$, and 4) $B(\pi) > 0$ for all $\pi \in (0,1)$.*

Hence, the intermediate value theorem can be used to establish that equilibria always exists and if $G(0) > 0$ any equilibrium much be non-trivial.

6 Trade

We now assume that two countries, a, b trade in goods on a free market, but that workers are unable to cross national borders. We let λ_a and $\lambda_b = 1 - \lambda_a$ denote the fractions of workers in each country. We write $w_j : \Theta \to R_+$ for the wages in country j and π_j for the fraction of investors and abuse previous notation by letting $\pi = (\pi_a, \pi_b)$ be the vector of fractions of investors rather than a scalar. Outputs are denoted x_{ij} where the first index refers to the good $(i = 1, 2)$ and the second to the country and $x = (x_{1a}, x_{2a}, x_{1b}, x_{2b})$ denotes the vector of outputs. When factor input distributions are needed explicitly we add a country index and write l_{ij}^t for the labor demand in sector i, country j and task t.

6.1 Trade Equilibrium

An equilibrium is defined exactly as in Definition 1 except that all variables except for goods prices now are indexed by country. The only equilibrium condition that needs any other modification than an addition of a country index is the goods market clearing condition. Since goods are allowed to be freely traded on the world market this condition now becomes

$$\sum_{j=a,b} \lambda_j x_{ij}^* = \sum_{j=a,b} \int_\theta x_i \left(w_j^* (\theta), p^*\right) \lambda_j f_{\pi^j} (\theta) \, d\theta \tag{39}$$

for each good i.

The production possibilities set in a country with λ^j workers and fraction of investors π^j is simply $\lambda^j X\left(\pi^j\right)$, where X is defined as in (A8). The world production possibilities are thus given by

$$X_w (\pi) = \lambda_a X (\pi_a) + \lambda_b X (\pi_b), \tag{40}$$

which inherits all relevant properties from the production possibilities set of the autharky model. In particular, since a linear combination of convex sets is convex we have that;

Lemma 6 $X_w (\pi)$ *is strictly convex*

Also the characterization of equilibrium follows the model without trade closely and the analogue to Proposition 1 is;

Proposition 3 *World outputs x_w^* and prices p^* and are consistent with equilibrium if and only if x_w^* solves (29) and p^* is a normal to a hyperplane that separates $X_w (\pi)$ and the set of bundles such that $u (x_1, x_2) > u (x_1^{w*}, x_2^{w*})$. Moreover, the equilibrium wages must satisfy*

$$w_j^* (\theta) = \begin{cases} p_i^* \frac{\partial y^i \left(c_{ij}^*, s_{ij}^*\right)}{\partial s_i} & \text{for } \theta \leq \theta \left(c_{1j}^* + c_{2j}^*, \pi_j\right) \\ p_i^* P (\theta, \pi) \frac{\partial y^i \left(c_{ij}^*, s_{ij}^*\right)}{\partial c_i} & \text{or } \theta > \theta \left(c_{1j}^* + c_{2j}^*, \pi_j\right) \end{cases}, \tag{41}$$

for each good i that is produced in country j, where $(c^*_{1j}, c^*_{2j}, s^*_{1j}, s^*_{2j})$ are effective factor inputs consistent with outputs x^*_j and where $x^*_w = \lambda_a x^*_a + \lambda_b x^*_b$. Finally, the threshold signals $\theta\left(c^*_{1j} + c^*_{2j}, \pi_j\right)$ must satisfy

$$P\left(\theta\left(c^*_{1j} + c^*_{2j}, \pi_j\right), \pi_j\right) \frac{\partial y^i\left(c^*_{ij}, s^*_{ij}\right)}{\partial c_i} = \frac{\partial y^i\left(c^*_{ij}, s^*_{ij}\right)}{\partial s_i} \tag{42}$$

Most arguments from the proof of Proposition 1 generalize trivially and the only place where any additional work is needed is to show that x^*_w must be on the frontier of $X_w(\pi)$, which is done using a "revealed profit maximization" argument: If x^*_w is not on the frontier aggregate world revenue can be increased without increasing aggregate world costs for labor, given assumed equilibrium prices and factor prices. But then it must be that profits increases in at least one sector and one country, which contradicts the assumption that x^*_w is part of an equilibrium.

Except for the trivial indeterminacy that arises in the model since there are many different labor demands that give the same effective factor inputs the equilibrium is unique.

Proposition 4 *Given any π the world output x^*_w , the country specific outputs x^*_j and factor inputs $(c^*_{1j}, c^*_{2j}, s^*_{1j}, s^*_{2j})$ are all uniquely determined. Output prices and wages are also unique up to a multiplicative constant.*

6.2 Traditional Trade Results

When the fraction of investors is fixed in each country the model is almost identical to the standard HOS model. The only substantial difference is that while factors are fixed in that model we have a strictly convex set of available effective factor inputs. However, let

$$w^*_{sj} = p^*_i \frac{\partial y^i\left(c^*_{ij}, s^*_{ij}\right)}{\partial s_i} \text{ and } w^*_{cj} = p^*_i \frac{\partial y^i\left(c^*_{ij}, s^*_{ij}\right)}{\partial c_i}. \tag{43}$$

Applying (41) we see that the profit maximization problem for a firm in country j and sector i reduces to $\max_{c_i, s_i} p^*_i y^i(c_{ij}, s_{ij}) - w^*_{cj} c_i - w^*_{sj} s_i$, which is as in any standard trade model, so any conclusion that does not rely on the endowment of factors will carry over to our framework. In particular;

Proposition 5 (factor price equalization) *If both countries produce both goods, then (effective) factor prices are equalized, i.e., $(w^*_{ca}, w^*_{sa}) = (w^*_{cb}, w^*_{sb})$. If country a does not produce good 1 , then $\frac{w^*_{ca}}{w^*_{sa}} \geq \frac{w^*_{cb}}{w^*_{sb}}$, while if country a does not produce good 2 the inequality is reversed.*

There are several ways to show the result and the reader may consult any rigorous textbook in international trade for a formal proof. The most intuitive argument is probably that unit costs must be equalized for both goods if both goods are produced in both countries since otherwise zero profits will fail somewhere. By choice of units we may take this to be equal to unity for both goods, which graphically means that, for each good i, the same level curve to y^i must be tangent to the unit cost line *for both countries.* From the graph (insert one) it is easy to see that this violates the factor-intensity assumption A1.

Proposition 6 (factor abundance hypothesis) *If $\pi_a < \pi_b$, then country a is a net importer of the skill intensive good and country b is a net exporter of the skill intensive good.*

While very intuitive in light of standard trade models, Proposition 6 is not a direct translation of the Hechsher-Ohlin Theorem in the same way as the factor price equalization result. In the proof we have to deal with the fact that effective factors of production are endogenously determined, which is an additional concern compared to the standard framework.

6.3 How Effective Factor Prices Respond when Investments Change

Our main goal is to analyze the effects of changes in the fraction of investors in one country on the benefits to invest for the other country. To acheive this goal we need to analyze the changes in incentives that are due to several effects. Intuitively we expect an increase in the fraction of investors in the other country to increase the production of skilled goods in the other country and decrease the production of skilled goods at home, as well as make the skilled good relatively cheaper. Hence, it seems that the need for skilled labor is reduced in the country where the fraction of investors is unchanged, which should reduce the relative price for skilled labor and therefore reduce incentives.

As before, good 2 is the unit of account and $p(\pi)$ is the equilibrium price of good one. We let $x_{1j}(\pi), x_{2j}(\pi)$ be the equilibrium outputs, $c_{ij}(\pi), s_{ij}(\pi)$ the (unique) equilibrium factor inputs, $\theta_j(\pi) \equiv F_q^{-1}\left(\frac{\pi - c_{1j}(\pi) - c_{2j}(\pi)}{\pi}\right)$ the associated (unique) threshold signal and $r_{ij}(\pi) = c_{ij}(\pi)/s_{ij}(\pi)$ the corresponding factor ratios. We may then write the unique equilibrium wage scheme in country j as

$$w_j(\theta;\pi) = \begin{cases} p(\pi)\frac{\partial y^1(r_{1j}(\pi),1)}{\partial s_1} = \frac{\partial y^2(r_{2j}(\pi),1)}{\partial s_2} & \text{for } \theta \le \theta_j(\pi) \\ p(\pi)P(\theta,\pi)\frac{\partial y^1(r_{1j}(\pi),1)}{\partial c_1} = P(\theta,\pi)\frac{\partial y^2(r_{2j}(\pi),1)}{\partial c_2} & \text{or } \theta > \theta_j(\pi) \end{cases}. \tag{44}$$

For any π and $j = a, b$ we let $c_j(\pi) = c_{1j}(\pi) + c_{2j}(\pi)$ and $s_j(\pi) = s_{1j}(\pi) + s_{2j}(\pi)$ and let

$$\begin{aligned} w_{jc}(\pi) &= p(\pi) \frac{\partial y^1(r_{1j}(\pi), 1)}{\partial c_1} = \frac{\partial y^2(r_{2j}(\pi), 1)}{\partial c_2} \\ w_{js}(\pi) &= p(\pi) \frac{\partial y^1(r_{1j}(\pi), 1)}{\partial s_1} = \frac{\partial y^2(r_{2j}(\pi), 1)}{\partial s_2} \end{aligned} \tag{45}$$

be the effective equilibrium factor prices (expressed in units of good 2). We begin with making a simple "revealed profit maximization" argument that gives us some discipline on how the correlation of factor prices and effective factor uses relates to the correlation between prices and outputs.

Lemma 7 *For any $\pi \neq \pi'$ we have that*

$$\begin{aligned} (p(\pi) - p(\pi'))(x_{1j}(\pi) - x_{1j}(\pi')) \geq &\left(w_{jc}(\pi) - w_{jc}(\pi')\right)\left(c_j(\pi) - c_j(\pi')\right) \\ &+ \left(w_{js}(\pi) - w_{js}(\pi')\right)\left(s_j(\pi) - s_j(\pi')\right) \end{aligned} \tag{46}$$

Moreover, if $r_{ij}(\pi) \neq r_{ij}(\pi')$ for some industry i, the inequality is strict.

Remark 1 *In trade theory it is common to normalize prices in such a way so that the nominal value of output is constant and we could do such a renormalization of our prices in which case the left hand side of the inequality would be zero. In this case this would look exactly like the standard trade theory correlation between factor prices and factor endowments, although the effective factors in the expression above are endogenous. However, the form above will be sufficient for our purposes, so we'll stick to the more straightforward convention to use one good as a numeraire. Also note that the inequalities come only from profit maximization by the firms and therefore hold in the autharky model as well*

The proof of Lemma 7 is a standard revealed profit maximization argument and can be found in the appendix. The main usefulness of the result is that combined with optimal consumer behavior and the characterization of equilibrium wages it guarantees the expected comparative statics results for how the effective factor prices depend on how many agents invest in the economy.

Proposition 7 *Suppose that π, π' are such that $x_{ij}(\pi), x_{ij}(\pi') > 0$ for all i, j (which implies that effective factor prices are equalized across countries) and suppose that $\pi_a < \pi'_a$ and $\pi_b \leq \pi'_b$. Then $w_c(\pi) > w_c(\pi')$ and $w_s(\pi) < w_s(\pi')$.*

The idea of the proof is as follows. If factor prices don't change in accordance with the proposition, then the factor ratio *in each industry* must decrease even though the fraction of workers who invest increases (in each country) if moving from π to π'. In each sector and country, this means that the expected marginal productivity in the skilled task increases and the productivity in the unskilled task decreases since *both* the posterior probability that a worker is productive and the marginal effect of a reallocation of resources between the skilled and unskilled tasks are more favorable in the equilibrium with investments π'. Now, in equilibrium the expected marginal productivity must be equalized across tasks for the threshold agent in each country and it follows that the threshold must be higher with investment behavior π than in the equilibrium with investments π'. From this it is intuitive and easy to show that the *aggregate* input of skilled labor must increase and the *aggregate* input of unskilled labor must decrease when moving from π to π'. This in itself is *not* a contradiction since the aggregate factor ratio and the factor ratios in the two sectors may actually change in the opposite directions if resources are reallocated across sectors. However, it is clear that in order for the aggregate factor ratio to go up and the factor ratio in each industry to go down it must be that the output in the skill-intensive industry must increase and the output in the other industry must decrease, which in turn implies that the relative price of the skilled good must go down when moving from π to π'. At this point, each term in the expression of Lemma 7 has been signed under the assumption that the proposition is false and the right hand side is negative, while the left hand side is positive, which is the desired contradiction.

6.4 Cross Country Effects on Incentives

In analogue with the autharky model, the benefits of investment in country j are

$$B^j(\pi) \equiv \int_\theta v\left(w_j(\theta;\pi), p(\pi)\right) dF_q(\theta) - \int_\theta v\left(w_j(\theta;\pi), p(\pi)\right) dF_u(\theta). \tag{47}$$

While no particular assumptions on risk preferences are needed to guarantee existence of equilibria it turns out that assuming risk neutrality in money income simplifies the analysis considerably when analyzing cross country effects on incentives. Risk neutrality in the investment stage means that $\int v(w,p)h(w)dw = v(\int wh(w)\,dw, p)$ for every p and every money lottery $h(w)$ money. It follows from standard analysis of risk preferences that $v(w,p) = \alpha(p)w + \beta(p)$ for some $\alpha(p), \beta(p)$.

We've already assumed that the utility function u is homothetic, which implies that

$$x_i(w,p) = \frac{x_i(w',p)}{x_2(w',p) + px_1(w',p)} w \tag{48}$$

for every w, w' and goods $i = 1, 2$. Thus, if u is homogenous of degree one, then

$$v(w,p) = u(x_1(w,p), x_2(w,p)) = \frac{u(x_1(w',p), x_2(w',p))}{x_2(w',p) + px_1(w',p)} w, \tag{49}$$

so this specification leads to reduced form preferences over money that exhibits risk-neutrality. The reason why this is a convenient formulation is that we may write $B(\pi)$ as

$$B^j(\pi) = v(1, p(\pi)) \left(w_{js}(\pi) \left(F_q\left(\widetilde{\theta}(\pi)\right) - F_u\left(\widetilde{\theta}(\pi)\right)\right) + w_{jc}(\pi) \int_{\widetilde{\theta}(\pi)}^1 P(\theta,\pi)(f_q(\theta) - f_u(\theta))\, d\theta \right). \tag{50}$$

To be completed. Need to figure out in exactly what way the negative cross effects from the example generalizes in the general model. May be that we can only say definite things about differences in the effects since a change in π^j has an effect on factor prices (expressed in good 2 units) as well as on the price of good 1 (in good 2 units). Seemingly these effects will go in opposite directions (higher investments in other country$\Rightarrow$relative factor prices change in such a way as incentives are decreased in the other country, but price on good 1 goes down, so there is a positive "wealth effect" which affects the incentives in the "wrong" way.

7 Example with Capital That Can Move Freely Across Borders

To be written. Main points. By setting the capital share of income appropriately in each sector essentially anything can be explained. To figure out: if data can be used to tie our hands in a natural way.

8 Concluding Remarks

To be written. Main points:

1. Information problem makes it easier to make sense of human capital differences as a driving force between cross country externalities since it is possible that a country is poor in human

capital and at the same time the benefits to accumulate are lower. Not possible in perfect information environments unless "persistence assumptions" of the type "it takes human capital to build human capital" are invoked (or exogenous differences are assumed).

2. Model generates implications on how GDP and wage distributions relate that we intend to see how they compare with data.

3. Model can explain "skill-biased technological change" in developed countries as a consequence of cheap imports when trade barriers are reduced.

A Appendix: Proofs

Proof of Lemma 1. Given any measurable labor demand $l : \Theta \rightarrow R_+^4$ satisfying (23) and (24) define $t(\theta) \equiv \frac{l_1^c(\theta)+l_2^c(\theta)}{f_\pi(\theta)}$, which intuitively may be interpreted as the fraction of workers with signal θ that is employed in the complex task. Obviously, $0 \leq t(\theta) \leq 1$ for feasibility and it is also clear that efficiency requires that $\sum_i \sum_t l_i^t(\theta) = f_\pi(\theta)$ for almost all θ for each $l : \Theta \rightarrow R_+^4$ that generates factor inputs z on the efficiency frontier of $Z(\pi)$we begin by showing that all remaining workers must be in the other task for efficiency. Thus, for each z on the efficiency frontier of $Z(\pi)$ there exists some $t : [0,1] \rightarrow [0,1]$ such that

$$\begin{aligned} c_1 + c_2 &= \int t(\theta) f_\pi(\theta) P(\theta,\pi) d\theta = \int t(\theta) \pi f_q(\theta) d\theta \qquad \text{(A1)} \\ s_1 + s_2 &= \int (1 - t(\theta)) f_\pi(\theta) d\theta. \end{aligned}$$

Next we note that For each z on the efficiency frontier of $Z(\pi)$ there exists some $\theta^* \in [0,1]$ such that

$$\begin{aligned} c_1 + c_2 &= \pi(1 - F_q(\theta^*) \qquad \text{(A2)} \\ s_1 + s_2 &= \pi F_q(\theta^*) + (1-\pi) F_u(\theta^*) \end{aligned}$$

To see this we note that if this is not the case, then there is a point z' on the efficiency frontier of $Z(\pi)$ and some $\theta' \in \Theta$ such that $\int_0^{\theta'} \pi t(\theta) f_q(\theta) d\theta > 0$ and $\int_{\theta'}^1 (1 - t(\theta)) f_\pi(\theta) d\theta > 0$. By choice of θ' we may without loss assume that $\int_0^{\theta'} t(\theta) f_\pi(\theta) d\theta = \int_{\theta'}^1 (1 - t(\theta)) f_\pi(\theta) d\theta > 0$. Consider some alternative labor demand that generates some $\widehat{t}$, where $\widehat{t}(\theta) = 0$ for $\theta \leq \theta' + \varepsilon$ and $\widehat{t}(\theta) = 1$ for $\theta > \theta' + \varepsilon$ and let $z'' = (c_1'', s_1'', c_2'', s_2'')$ denote some feasible inputs that satisfies $c_1'' + c_2'' = \int \pi \widehat{t}(\theta) f_q(\theta) d\theta$ and

$s_1'' + s_2'' = \int (1 - \widehat{t}(\theta)) f_\pi(\theta) d\theta$ (while we are not explicit about it in the notation, these should be seen as a function of ε). Clearly,

$$
\begin{aligned}
s_1' + s_2' &= \int_0^{\theta'} (1 - t(\theta)) f_\pi(\theta) d\theta + \int_{\theta'}^1 (1 - t(\theta)) f_\pi(\theta) d\theta = \\
&= \int_0^{\theta'} (1 - t(\theta)) f_\pi(\theta) d\theta + \int_0^{\theta'} t(\theta) f_\pi(\theta) d\theta = \int_0^{\theta'} f_\pi(\theta) d\theta = \\
&= \int_{\theta \in [0,1]} (1 - \widehat{t}(\theta)) f_\pi(\theta) d\theta - \int_{\theta'}^{\theta' + \varepsilon} f_\pi(\theta) d\theta = s_1'' + s_2'' - \left(F_\pi\left(\theta' + \varepsilon\right) - F_\pi(\theta)\right)
\end{aligned}
\tag{A3}
$$

The input of labor in the complex task in the original plan is

$$
\begin{aligned}
c_1' + c_2' &= \int_0^{\theta'} t(\theta) p(\theta, \pi) f_\pi(\theta) d\theta + \int_{\theta'}^1 t(\theta) \pi f_q(\theta) d\theta < \\
&< p\left(\theta', \pi\right) \int_0^{\theta'} t(\theta) f_\pi(\theta) d\theta + \int_{\theta'}^1 t(\theta) \pi f_q(\theta) d\theta = \\
&= p\left(\theta', \pi\right) \int_{\theta'}^1 (1 - t(\theta)) f_\pi(\theta) d\theta + \int_{\theta'}^1 t(\theta) \pi f_q(\theta) d\theta = \\
&< \int_{\theta'}^1 \pi f_q(\theta) d\theta = \int_{\theta \in [0,1]} \pi \widehat{t}(\theta) f_q(\theta) d\theta + \int_{\theta'}^{\theta' + \varepsilon} \pi f_q(\theta) d\theta = \\
&= c_1'' + c_2'' + \pi \left(F_q\left(\theta' + \varepsilon\right) - F_q(\theta)\right)
\end{aligned}
\tag{A4}
$$

$c_1'' + c_2'' + \pi \left(F_q(\theta' + \varepsilon) - F_q(\theta)\right) > c_1' + c_2'$ for all $\varepsilon > 0$ and F_q is continuous and so $F_q(\theta' + \varepsilon) \to F_q(\theta)$ as $\varepsilon \to 0$, so there must exists some $\overline{\varepsilon} > 0$ such that $c_1'' + c_2'' > c_1' + c_2'$ given that $\varepsilon < \overline{\varepsilon}$ and since $s_1'' + s_2'' > s_1' + s_2'$ for all $\varepsilon > 0$ it follows that the input in both tasks increase if ε is sufficiently small. Hence, any point on the efficiency frontier must satisfy (A2) for some $\theta^* \in [0,1]$. Eliminating θ^* we get that z if is on the boundary, then

$$
\begin{aligned}
s_1 + s_2 &= \pi F_q(F_q^{-1}\left(\frac{\pi - c_1 - c_2}{\pi}\right)) + (1 - \pi) F_u(F_q^{-1}\left(\frac{\pi - c_1 - c_2}{\pi}\right)) = \\
&= \pi - c_1 - c_2 + (1 - \pi) F_u(F_q^{-1}\left(\frac{\pi - c_1 - c_2}{\pi}\right)),
\end{aligned}
\tag{A5}
$$

i.e., $g(c_1 + c_2, s_1 + s_2; \pi) = 0$ where g is defined in (28). It is easy to verify that $g(c_1 + c_2, s_1 + s_2; \pi) > 0$ for interior points and that $g(c_1 + c_2, s_1 + s_2; \pi) < 0$ for infeasible points, which completes the proof. ■

Proof of Lemma 2. We define $H(c) = F_u\left(F_q^{-1}\left(\frac{\pi - c}{\pi}\right)\right)$ and apply the inverse function theorem to conclude that $H'(x) = -\frac{f_u\left(F_q^{-1}\left(\frac{\pi - c}{\pi}\right)\right)}{f_q\left(F_q^{-1}\left(\frac{\pi - c}{\pi}\right)\right)}$. The ratio $\frac{f_u(\theta)}{f_q(\theta)}$ is the inverse of the likelihood ratio, which is strictly decreasing, F_q^{-1} is strictly increasing $\frac{\pi - c}{\pi}$ is strictly decreasing, so we conclude that $H'(c)$

is strictly decreasing, that is H is strictly concave in c. Now take a pair of inputs $(c', s'), (c'', s'')$ and let (c^λ, s^λ) denote a convex combination given any $\lambda \in (0,1)$. If $c' \neq c''$ we have that,

$$\begin{aligned} g\left(c^\lambda, s^\lambda; \pi\right) &= \pi - c^\lambda - s^\lambda + H\left(c^\lambda\right) = \qquad \text{(A6)} \\ &= \pi - c^\lambda - s^\lambda + H\left(c^\lambda\right) + \lambda H(c') - \lambda H(c') + (1-\lambda) H(c'') - (1-\lambda) H(c'') = \\ &= \lambda g(c', s'; \pi) + (1-\lambda) g(c'', s'', \pi) + H\left(c^\lambda\right) - \lambda H(c') - (1-\lambda) H(c'') > \\ &> \lambda g(c', s'; \pi) + (1-\lambda) g(c'', s'', \pi) > \min\left\{g(c', s'; \pi), g(c'', s'', \pi)\right\} \end{aligned}$$

since H is strictly concave. Assume without loss that $s' < s''$. Then

$$g(c', s'; \pi) - g(c'', s'', \pi) = s'' - s' > 0, \qquad \text{(A7)}$$

which implies that

$$\begin{aligned} g\left(c^\lambda, s^\lambda; \pi\right) &= g(c'', s'', \pi) + \lambda\left(g(c', s'; \pi) - g(c'', s'', \pi)\right) > \\ &> g(c'', s'', \pi) = \min\left\{g(c', s'; \pi), g(c'', s'', \pi)\right\}. \end{aligned}$$

Hence, g is strictly quasi-concave as claimed[12]. ■

Proof of Lemma 3. Let $Z^*(\pi) = \left\{c, s \in R_+^2 | g(c, s; \pi) \geq 0\right\}$ be the set of feasible aggregate factor inputs. It follows immediately from the (strict) quasi-concavity of g that $Z^*(\pi)$ is convex. Convexity of $Z(\pi)$ is then rather clear and can be established in a number of different ways. Easiest is to note that if there exists $z, z' \in Z(\pi)$ and $\lambda \in [0,1]$ such that $\lambda z + (1-\lambda) z' \notin Z(\pi)$, then we may let (c, s) and (c', s') represent the aggregate factor inputs corresponding to z and z'. Clearly, $\lambda z + (1-\lambda) z' \notin Z(\pi)$ means that $g(\lambda c + (1-\lambda) c', \lambda s + (1-\lambda) s'; \pi) < 0$, which violates the convexity of $Z^*(\pi)$. ■

Proof of Lemma 4. Let $x, x' \in X(\pi)$ be two plans with associated factor inputs $z, z' \in Z(\pi)$. Since $Z(\pi)$ is convex, any convex combination $z^\lambda = \lambda z + (1-\lambda) z \in Z(\pi)$ and,

$$y^i\left(c_i^\lambda, s_i^\lambda\right) \geq \lambda y^i(c_i, s_i) + (1-\lambda) y^i(c_i', s_i') = \lambda x_i + (1-\lambda) x_i',$$

for $i = 1, 2$. Hence $x^\lambda = \lambda x + (1-\lambda) x' \in X(\pi)$. ■

The production possibilities set for our model economy is given by

$$X(\pi) = \left\{(x_1, x_2) \in R_+^2 \,|x_i = y^i(c_i, s_i) \text{ for some } (c_1, s_1, c_2, s_2) \in Z(\pi)\right\}. \qquad \text{(A8)}$$

[12] For $c' = c''$ we have that $g(c^\lambda, s^\lambda; \pi) = \lambda g(c', s'; \pi) + (1-\lambda) g(c'', s'', \pi)$, so g is not strictly concave.

The frontier of $X(\pi)$ can be characterized in the usual way in terms of solutions to

$$\begin{aligned} x_1(x_2) &= \max_{c_1,c_2,s_1s_2} y^1(c_1,s_1) \\ \text{s.t } y^2(c_2,s_2) &\geq x^2 \\ g(c_1+c_2,s_1+s_2;\pi) &\geq 0 \end{aligned} \tag{A9}$$

and by a straightforward application of the envelope theorem if follows that the effect of a small increase in x^2 on the maximal choice of x_1 equals the negative of the multiplier on the first constraint. By inspection of the first order conditions to (A9) we find that

$$\frac{dx_1(x_2)}{dx_2} = -\lambda(x_2) = -\frac{\frac{\partial y^1(c_1(x_2),s_1(x_2))}{\partial c_1}}{\frac{\partial y^1(c_2(x_2),s_2(x_2))}{\partial c_2}} = -\frac{\frac{\partial y^2(c_1(x_2),s_1(x_2))}{\partial s_1}}{\frac{\partial y^2(c_2(x_2),s_2(x_2))}{\partial s_2}}, \tag{A10}$$

where the x_2 argument indicates that variables are chosen optimally given the value of x_2. The most important observation from (A10) is the technical rate of substitution between factors must be equalized across industries. Moreover, the ratio of complex to simple labor will be higher in sector the sector more intensive in complex labor than in the other sector. This is the crucial fact underlying the second part of our next preliminary result.

Proof of Lemma 5 Since $X(\pi)$ is weakly convex a failure of strict convexity implies that there is a linear segment on the frontier, i.e., there is some $x_2' < x_2''$ and corresponding $x_1(x_2') > x_1(x_2'')$ such that

$$x_1\left(\lambda x_2' + (1-\lambda)x_2''\right) = \lambda x_1\left(x_2'\right) + (1-\lambda)x_1\left(x_2''\right) \tag{A11}$$

for every $\lambda \in [0,1]$. In terms of derivatives, this means that

$$\begin{aligned} \frac{dx_1(x_2)}{dx_2} &= -\frac{\frac{\partial y^1\left(c_1\left(x_2'\right),s_1\left(x_2'\right)\right)}{\partial c_1}}{\frac{\partial y^1\left(c_2\left(x_2'\right),s_2\left(x_2'\right)\right)}{\partial c_2}} = -\frac{\frac{\partial y^2\left(c_1\left(x_2'\right),s_1\left(x_2'\right)\right)}{\partial s_1}}{\frac{\partial y^2\left(c_2\left(x_2'\right),s_2\left(x_2'\right)\right)}{\partial s_2}} = \\ &= -\frac{\frac{\partial y^1(c_1(x_2),s_1(x_2))}{\partial c_1}}{\frac{\partial y^1(c_2(x_2),s_2(x_2))}{\partial c_2}} = -\frac{\frac{\partial y^2(c_1(x_2),s_1(x_2))}{\partial s_1}}{\frac{\partial y^2(c_2(x_2),s_2(x_2))}{\partial s_2}} \end{aligned} \tag{A12}$$

for all $x_2 \in [x_2', x_2'']$. Hence

$$\begin{aligned} \frac{\frac{\partial y^1(c_1(x_2),s_1(x_2))}{\partial c_1}}{\frac{\partial y^1(c_1(x_2),s_1(x_2))}{\partial s_1}} &= \frac{\frac{\partial y^1\left(c_1\left(x_2'\right),s_1\left(x_2'\right)\right)}{\partial c_1}}{\frac{\partial y^1\left(c_2\left(x_2'\right),s_2\left(x_2'\right)\right)}{\partial s_2}} \\ \frac{\frac{\partial y^2(c_2(x_2),s_2(x_2))}{\partial c_2}}{\frac{\partial y^2(c_2(x_2),s_2(x_2))}{\partial s_2}} &= \frac{\frac{\partial y^2\left(c_2\left(x_2'\right),s_2\left(x_2'\right)\right)}{\partial c_2}}{\frac{\partial y^2\left(c_2\left(x_2'\right),s_2\left(x_2'\right)\right)}{\partial s_2}} \end{aligned} \tag{A13}$$

and similarly for sector 2, which implies that $\frac{c_1(x_2)}{s_1(x_2)} = \frac{c_1(x_2')}{s_1(x_2')}$ and $\frac{c_2(x_2)}{s_2(x_2)} = \frac{c_2(x_2')}{s_2(x_2')}$ for all $x_2 \in [x_2', x_2'']$. For any $\lambda \in [0,1]$ let $x_2^\lambda = \lambda x_2' + (1-\lambda) x_2''$. Also, let $r_i \equiv \frac{c_i(x_2')}{s_i(x_2')}$ for sector $i = 1,2$ and note that since sector 1 is more intensive in c than is sector 2 (A13) implies that $r_1 > r_2$. Using (A11), the fact that the factor ratios are convex and properties of constant returns technologies we than have that[13].

$$\begin{aligned} s_i\left(x_2^\lambda\right) y^i(r_i,1) &= y^i\left(c_i\left(x_2^\lambda\right), s_i\left(x_2^\lambda\right)\right) \equiv x_i\left(x_2^\lambda\right) = \lambda x_i(x_2') + (1-\lambda) x_i(x_2'') = \\ &= \lambda y^i\left(c_i(x_2'), s_i(x_2')\right) + (1-\lambda) y^i\left(c_i(x_2''), s_i(x_2'')\right) = \\ &= \left[\lambda s_i(x_2') + (1-\lambda) s_i(x_2'')\right] y^i(r_i,1) \end{aligned}$$

Hence, $s_i\left(x_2^\lambda\right) = \lambda s_i(x_2') + (1-\lambda) s_i(x_2'')$ and since $c_i(x_2) = r_i s_i(x_2)$ for $x_2 \in [x_2', x_2'']$ it also follows that $c_i\left(x_2^\lambda\right) = \lambda c_i(x_2') + (1-\lambda) c_i(x_2'')$. Now let $c(x_2) \equiv c_1(x_2) + c_2(x_2)$ and $s(x_2) \equiv s_1(x_2) + s_2(x_2)$ and observe that

$$\left(c\left(x_2^\lambda\right), s\left(x_2^\lambda\right)\right) = \left(\lambda c(x_2') + (1-\lambda) c(x_2''), \lambda s(x_2') + (1-\lambda) s(x_2'')\right). \tag{A14}$$

Moreover, using the constant factor ratio and constant returns to scale we find that

$$\begin{aligned} c_1(x_2'') - c_1(x_2') &= \frac{r_1}{y^1(r_1,1)}\left(x_1(x_2'') - x_1(x_2')\right) \\ c_2(x_2'') - c_2(x_2') &= \frac{r_2}{y^2(r_2,1)}\left(x_2'' - x_2'\right) \\ s_1(x_2'') - s_1(x_2') &= \frac{1}{y^1(r_1,1)}\left(x_1(x_2'') - x_1(x_2')\right) \\ s_2(x_2'') - s_2(x_2') &= \frac{1}{y^2(r_2,1)}\left(x_2'' - x_2'\right), \end{aligned} \tag{A15}$$

so if $c(x_2'') \geq c(x_2')$, then

$$\begin{aligned} 0 &\leq \frac{r_1}{y^1(r_1,1)}\left(x_1(x_2'') - x_1(x_2')\right) + \frac{r_2}{y^2(r_2,1)}\left(x_2'' - x_2'\right) \\ &\Downarrow \ /\left(x_2'' - x_2'\right) > 0 \text{ and } r_1 > r_2/ \\ 0 &< r_1\left(\frac{1}{y^1(r_1,1)}\left(x_1(x_2'') - x_1(x_2')\right) + \frac{1}{y^2(r_2,1)}\left(x_2'' - x_2'\right)\right) = \\ &= r_1\left(s_1(x_2'') - s_1(x_2') + s_2(x_2'') - s_2(x_2')\right), \end{aligned} \tag{A16}$$

which means that $s(x_2'') > s(x_2')$, a contradiction since this means that $c_1(x_2'), c_2(x_2'), s_1(x_2'), s_2(x_2')$ can not solve (A9) given $x_2 = x_2'$. Hence we conclude that $c(x_2'') < c(x_2')$ and (symmetric argument)

[13] The notation below is somewhat awkward for sector 2. The convention is that $x_2(x_2) = x_2$.

that $s\left(x_2''\right) > s\left(x_2'\right)$. But then, by Lemma 2 it follows that

$$g\left(c\left(x_2^\lambda\right), s\left(x_2^\lambda\right); \pi\right) > \min\left\{g\left(c\left(x_2'\right), s\left(x_2'\right); \pi\right), g\left(c\left(x_2''\right), s\left(x_2''\right); \pi\right)\right\} \geq 0,$$

so $\left(c_1\left(x_2^\lambda\right), s_1\left(x_2^\lambda\right), c_2\left(x_2^\lambda\right), s_2\left(x_2^\lambda\right)\right)$ is not on the frontier of $Z(\pi)$, which means that $\left(x_1\left(x_2^\lambda\right), x_2^\lambda\right)$ is not on the frontier of $X(\pi)$, which contradicts the definition of $\left(x_1\left(x_2^\lambda\right), x_2^\lambda\right)$. ■

Proof of Proposition 1. (only if) Suppose $p^*, w^*, x_1(w,p), x_2(w,p), (x_1^*, x_2^*)$ and $\{l_i^{c*}, l_i^{s*}\}_{i=1,2}$ satisfy conditions 1-4 in the definition of equilibrium given $\pi = \pi^*$ and let $c_i^* = \int l_i^{c*}(\theta) P(\theta, \pi) d\theta$ and $s_i^* = \int l_i^{s*}(\theta) d\theta$ for $i = 1, 2$.

Claim 1 $\Pi_i^* = p_i^* y^i(c_i^*, s_i^*) - \int w^*(\theta) \sum_{t=c,s} l_i^{t*}(\theta) d\theta = 0$ for $i = 1, 2$.

Proof. If $\Pi_i^* < 0$, then $l_i^{t\prime}(\theta) = 0$ is better than $\{l_i^{c*}, l_i^{s*}\}$ since it yields zero profits and if $\Pi_i^* > 0$, then $l_i^{t\prime}(\theta) = 2l_i^{t*}(\theta)$ is better than $\{l_i^{c*}, l_i^{s*}\}$ since the associated profits are $2\Pi_i^*$. ■

Claim 2 Let $\Theta^c = \{\theta | l_1^{c*}(\theta) + l_2^{c*}(\theta) > 0\}$ and $\Theta^s = \{\theta | l_1^{s*}(\theta) + l_2^{s*}(\theta) > 0\}$. Then there must exist w_c, w_s such that $w^*(\theta) = w_c P(\theta, \pi)$ for almost all $\theta \in \Theta^c$ and $w^*(\theta) = w_s$ for almost all $\theta \in \Theta^s$.

Proof. For contradiction assume (without loss) that there is a constant κ and sets A, B where $A = \{\theta | w^*(\theta) > \kappa P(\theta, \pi)\}$ and $B = \{\theta | w^*(\theta) < \kappa P(\theta, \pi)\}$ and suppose that $\int_{\theta \in A} l_i^{c*}(\theta) d\theta > 0$ for some sector i. Consider an alternative labor demand by the firm where $l_i^{s\prime} = l_i^{s*}$ and $l_i^{c\prime}(\theta) = 0$ for all $\theta \in A$, $l_i^{c\prime}(\theta) = l_i^{c*}(\theta) + \beta$ for $\theta \in B$ and the demand for skilled labor is unchanged for all other θ. We set β so that $\beta \int_{\theta \in B} P(\theta, \pi) d\theta = \int_{\theta \in A} P(\theta, \pi) l_i^{c*}(\theta) d\theta$, which means that effective factor inputs are the same as in the assumed equilibrium. The change in profits is thus the same as the change in wages, that is

$$\begin{aligned} \Pi_i' - \Pi_i^* &= -\int_{\theta \in B} \beta w^*(\theta) d\theta + \int_{\theta \in A} w^*(\theta) l_i^{c*}(\theta) d\theta = \\ &> -\kappa \int_{\theta \in B} \beta P(\theta, \pi) d\theta + \kappa \int_{\theta \in A} P(\theta, \pi) l_i^{c*}(\theta) d\theta = 0, \end{aligned} \tag{A17}$$

which contradicts the assumption that the original labor demand maximized profits given prices. Hence there must be some w_c such that $w^*(\theta) = w_c P(\theta, \pi)$ for almost all $\theta \in \Theta^c$. The second half follows by a symmetric argument. ■

Claim 3 $w^*(\theta) = \max(w_s, w_c P(\theta, \pi))$ for almost all $\theta \in [0,1]$ and, ignoring deviations on sets of measure zero, $\Theta^s = [0, \theta^*]$ and $\Theta^c = [\theta^*, 1]$, where θ^* satisfies $\frac{w_s}{w_c} = P(\theta^*, \pi)$.

Proof. If there would be a set A with positive measure such that $w^*(\theta) < w_c P(\theta, \pi)$ for all $\theta \in A \subset [0,1]$ or set $B \subset [0,1]$ with positive measure such that $w^*(\theta) < w_s$ for all $\theta \in B$, then identical arguments as in the previous claim may be used to show that the presumed equilibrium demands can not be optimal given wages. Hence $w^*(\theta) \geq \max(w_s, w_c P(\theta, \pi))$ for almost all $\theta \in [0,1]$. Moreover, if $w^*(\theta) > \max(w_s, w_c P(\theta, \pi))$ for θ on some set C, then $\sum_{i=1,2} \sum_{t=c,s} \int_{\theta \in C} l_i^t(\theta)\, d\theta = 0$ in order for firms to maximize profits, which contradicts condition 4 in the definition of equilibrium (factor market clearing), so $w^*(\theta) = \max(w_s, w_c P(\theta, \pi))$ for almost all $\theta \in [0,1]$. Since $y^i(0, s) = y^i(c, 0) = 0$ and since $P(\theta, \pi)$ is strictly decreasing there must therefore be θ', θ'' such that $w_s > w_c P(\theta', \pi)$ and $w_s < w_c P(\theta'', \pi)$. Hence there is some $\theta^* \in (0,1)$ such that $w_s = w_c P(\theta^*, \pi)$ and $w^*(\theta) = \max(w_s, w_c P(\theta, \pi))$ for almost all $\theta \in [0,1]$. It follows by computations as in the previous claim that $\Theta^s \cap [\theta^*, 1]$ and $\Theta^c \cap [0, \theta^*]$ has measure zero. For example, if there is some set $D \subset [\theta^*, 1]$ with positive measure such that $l_i^{s*}(\theta) > 0$ for all $\theta \in D$, then $\int_{\theta \in D} l_i^{s*}(\theta)\, d\theta > 0$ and $\int_{\theta \in D} w^*(\theta)\, l_i^{s*}(\theta)\, d\theta > w_s \int_{\theta \in D} l_i^{s*}(\theta)\, d\theta$. Obviously, profits would be higher by letting $l_i^s(\theta) = l_i^{s*}(\theta) + \beta$ for $\theta \leq \theta^*$ and $l_i^s(\theta) = 0$ for $\theta > \theta^*$. ■

Claim 4 If x_1^*, x_2^* is part of an equilibrium with $p^* >> 0$, then x_1^*, x_2^* must be on the frontier of $X(\pi)$.

Proof. If x_1^*, x_2^* is not on the frontier of $X(\pi)$ there is some $z' \in Z(\pi)$ such that $y^1(c_1', s_1') > x_1^*$ and $y^2(c_2', s_2') \geq x_2^*$. Total revenue in the economy increases and factor market clearing (condition 4 in Definition 1) implies that total costs are unchanged and equal to $\int w^*(\theta) f_\pi(\theta)\, d\theta$. Hence, at least one sector must make higher profits, which contradicts that the original labor demands were part of an equilibrium. ■

To complete the only if part we note that individual utility maximization implies that $p^* x > p^* x^*$ for all x such that $u(x_1, x_2) > u(x_1^*, x_2^*)$. By Claim 4, x^* is on the frontier of $X(\pi)$, so p^* must separate $X(\pi)$ and $\left\{x \in R_+^2 | u(x_1, x_2) > u(x_1^*, x_2^*)\right\}$ and the marginal conditions in (35) must hold. Using the structure on wages imposed by Claim 3 the profit maximization problem reduces to $\max_{c_i, s_i} p_i^* y^i(c_i, s_i) - w_c c_i - w_s s_i$, so

$$w_c = p_1^* \frac{\partial y^1(c_1^*, s_1^*)}{\partial c_1} = p_2^* \frac{\partial y^2(c_2^*, s_2^*)}{\partial c_2} \text{ and} \tag{A18}$$

$$w_s \quad = \quad p_1^* \frac{\partial y^1 (c_1^*, s_1^*)}{\partial s_1} = p_2^* \frac{\partial y^2 (c_2^*, s_2^*)}{\partial s_2}.$$

Finally, combining Claim 3 and factor market clearing we find that $c_1^* + c_2^* = \pi(1 - F_q(\theta^*))$, or $\theta^* = F_q^{-1}\left(\frac{\pi - c_1^* - c_2^*}{\pi}\right)$, which by again using Claim 3 implies that the wage scheme is of the form in (31).

(if) Let (x_1^o, x_2^o) be a solution to (29). Since $\left\{x \in R_+^2 | u(x_1, x_2) > u(x_1^o, x_2^o)\right\}$ and $X(\pi)$ are disjoint by the assumption that (x_1^o, x_2^o) solves (29) and convex by assumption (preferences) and Lemma **??** (production possibilities) respectively, it follows that there is some hyperplane separating the sets. Let p^* be a normal to such an hyperplane and let $z^o = (c_1^o, s_1^o, c_2^o, s_2^o)$ be some factor inputs such that $x_i^o = y^i(c_i^o, s_i^o)$ for $i = 1, 2$ and let

$$w^*(\theta) = \begin{cases} p_1^* \frac{\partial y^1 (c_1^o, s_1^o)}{\partial s_1} & \text{for } \theta \leq F_q^{-1}\left(\frac{\pi - c_1^o - c_2^o}{\pi}\right) \\ p_1^* P(\theta, \pi) \frac{\partial y^1 (c_1^o, s_1^o)}{\partial c_1} & \text{or } \theta > F_q^{-1}\left(\frac{\pi - c_1^o - c_2^o}{\pi}\right) \end{cases}. \tag{A19}$$

We note that since$\left\{x \in R_+^2 | px = p^* x\right\}$ is tangent to $X(\pi)$ at (x_1^o, x_2^o) it follows from (A10) that

$$\frac{p_1^*}{p_2^*} = \frac{\frac{\partial y(c_1^o, s_1^o)}{\partial c_2}}{\frac{\partial y(c_2^o, s_2^o)}{\partial c_1}} = \frac{\frac{\partial y(c_1^o, s_1^o)}{\partial s_2}}{\frac{\partial y(c_2^o, s_2^o)}{\partial c_1}}. \tag{A20}$$

Finally, let $l^o : \Theta \to R_+^4$ be any labor demand such that $c_i^o = \int_\theta P(\theta, \pi) l_i^c(\theta) d\theta$ and $s_i^o = \int_\theta l_i^s(\theta) d\theta$ and

$$\begin{aligned} l_1^c(\theta) + l_2^c(\theta) &= \begin{cases} 0 & \text{for } \theta < F_q^{-1}\left(\frac{\pi - c_1^o - c_2^o}{\pi}\right) \\ f_\pi(\theta) & \text{for } \theta \geq F_q^{-1}\left(\frac{\pi - c_1^o - c_2^o}{\pi}\right) \end{cases} \\ l_1^s(\theta) + l_2^s(\theta) &= \begin{cases} f_\pi(\theta) & \text{for } \theta < F_q^{-1}\left(\frac{\pi - c_1^o - c_2^o}{\pi}\right) \\ 0 & \text{for } \theta \geq F_q^{-1}\left(\frac{\pi - c_1^o - c_2^o}{\pi}\right) \end{cases}, \end{aligned} \tag{A21}$$

Existence of labor demands satisfying (A21) follows from the construction of the set of feasible factor inputs, $Z(\pi)$ and equilibrium condition 4 is satisfied by construction of (A21). Now, let $x(w, p)$ solve (1), so that the second equilibrium condition is satisfied, then aggregate demands can be found by integrating $x(w^*(\theta), p^*)$using the distribution $f_\pi(\theta)$. Since preferences are homothetic this implies that for the third condition (market clearing on goods) to be satisfied x_1^*, x_2^* must solve (33) given $p = p^*$ and $w = w^*$, which is the case since

$$\int w^*(\theta) f_\pi(\theta) d\theta \quad = \quad \int_0^{\theta^*} w^*(\theta) (l_1^s(\theta) + l_2^s(\theta)) d\theta + \int_{\theta^*}^1 w^*(\theta) (l_1^c(\theta) + l_2^c(\theta)) d\theta = \tag{A22}$$

$$
\begin{aligned}
&= \int_0^{\theta^*} \sum_{i=1,2} p_i^* \frac{\partial y^i\left(c_i^o, s_i^o\right)}{\partial s_i} l_i^s(\theta) + \int_{\theta^*}^1 \sum_{i=1,2} p_1^* P(\theta,\pi) \frac{\partial y^i\left(c_i^o, s_i^o\right)}{\partial c_i} l_i^c(\theta)\, d\theta = \\
&= \sum_{i=1,2} p_i^* y^i\left(c_i^o, s_i^o\right) = p_1^* x_1^* + p_2^* x_2^*,
\end{aligned}
$$

after use of constant returns to scale. Thus, since p^* is tangent to $X(\pi)$, the aggregate bundle (x_1^*, x_2^*) must also solve $\max_{x_1,x_2} u(x_1,x_2)$ subj to. $p^* x \leq p^* x^*$ and by the calculation above this will be the case. Left to verify is that $l^o : \Theta \rightarrow R_+^4$ is consistent with the first equilibrium condition, profit maximization for the firms. To see this, consider any alternative labor demand for firm i, which generates profits $\Pi_i' = p_i^* y^i\left(c_i', s_i'\right) - \int_\theta w^*(\theta)\left(l_i^{s\prime}(\theta) + l_i^{c\prime}(\theta)\right) d\theta$. From (A19) and (A20) it follows that $w^*(\theta) = \max\left\{p_i^* \frac{\partial y^i\left(c_i^o, s_i^o\right)}{\partial s_i}, p_1^* P(\theta,\pi) \frac{\partial y^i\left(c_i^o, s_i^o\right)}{\partial c_i}\right\}$, so

$$
\begin{aligned}
\int_\theta w^*(\theta) l_i^{s\prime}(\theta)\, d\theta &\geq \int_\theta p_i^* \frac{\partial y^i\left(c_i^o, s_i^o\right)}{\partial s_i} l_i^{s\prime}(\theta)\, d\theta = p_i^* \frac{\partial y^i\left(c_i^o, s_i^o\right)}{\partial s_i} s_i' \\
\int_\theta w^*(\theta) l_i^{c\prime}(\theta)\, d\theta &\geq \int_\theta p_i^* \frac{\partial y^i\left(c_i^o, s_i^o\right)}{\partial c_i} P(\theta,\pi) l_i^{s\prime}(\theta)\, d\theta = p_i^* \frac{\partial y^i\left(c_i^o, s_i^o\right)}{\partial c_i} c_i'
\end{aligned}
\tag{A23}
$$

and we have that

$$
\begin{aligned}
\Pi_i' &\leq p_i^* \left(y^i\left(c_i', s_i'\right) - \frac{\partial y^i\left(c_i^o, s_i^o\right)}{\partial s_i} s_i' - \frac{\partial y^i\left(c_i^o, s_i^o\right)}{\partial c_i} c_i' \right) \leq \\
&\leq p_i^* \left(y^i\left(c_i^o, s_i^o\right) + \frac{\partial y^i\left(c_i^o, s_i^o\right)}{\partial s_i}\left(c_i' - c_i^o\right) + \frac{\partial y^i\left(c_i^o, s_i^o\right)}{\partial c_i}\left(c_i' - c_i^o\right) \right) \\
&\quad - p_i^* \left(\frac{\partial y^i\left(c_i^o, s_i^o\right)}{\partial s_i} s_i' - \frac{\partial y^i\left(c_i^o, s_i^o\right)}{\partial c_i} c_i' \right) = 0.
\end{aligned}
\tag{A24}
$$

A routine calculation verifies that profits are zero if firms hire in accordance to l^o, so this means that all equilibrium conditions 1-4 are satisfied. ■

Proof of Proposition 2. (Continuity) Since the arguments are standard we will only give a sketch of the proof. By Proposition 1 we know that

$$
x(\pi) = \left(x_1(\pi), x_2(\pi)\right) = \arg\max_{x \in X(\pi)} u(x_1, x_2), \tag{A25}
$$

Observe that the constraint may be viewed as a correspondence $X : [0,1] \rightrightarrows R_+^2$ and it is trivial to verify that the constraint correspondence is compact-valued and continuous (i.e. uhc and lhc). It then follows directly from the theorem of the maximum that the set of maximizers is upper-hemi continuous. Since there is a unique maximizer for each π, this implies continuity of $x(\pi)$.. Next one inspects (A9) and observe that the associated factor inputs $z(\pi)$ solves

$$
z(\pi) = \left(c_1(\pi), s_1(\pi), c_2(\pi), s_2(\pi)\right) = \arg\max_z y^1(c_1, s_1) \tag{A26}
$$

$$\begin{aligned} \text{s.t } y^2(c_2, s_2) &\geq x_2(\pi) \\ g(c_1 + c_2, s_1 + s_2; \pi) &\geq 0. \end{aligned}$$

Let the constraint correspondence for this problem be denoted as $\Gamma : [0,1] \rightrightarrows R^4_+$ and note that

$$\Gamma(\pi) = \left\{ z \in Z(\pi) \,| y^2(c_2, s_2) \geq x_2(\pi) \right\}, \tag{A27}$$

which obviously is closed and bounded and continuity of Γ is also immediate. Hence, the theorem of the maximum applies again and since y^i is strictly quasi concave and the constraint set is convex, $z(\pi)$ is uniquely defined. Thus, also the factor inputs are continuous functions of π and direct inspection of the expression for equilibrium wages reveals that $w(\theta, \pi)$ is continuous in π for each $\theta \in [0,1]$. Indeed, it is not hard to verify that for each $\pi \in (0,1)$ and $\delta > 0$ there exists $\varepsilon > 0$ such that $\|w(\theta, \pi) - w(\theta, \pi')\| < \delta$ for all $\theta \in [0,1]$ and since compositions of continuous functions are continuous and $p(\pi)$ is continuous (otherwise $x(\pi)$ can't be continuous) continuity of B follows.

($B(1) = B(0) = 0$) Trivial from inspection of (36).

($B(\pi) > 0$ for $\pi \in (0,1)$) Since the likelihood ratio is strictly increasing there must be some $0 < \theta' < 1$ such that $f_q(\theta) > f_u(\theta)$ for all $\theta > \theta'$. Suppose first that $\theta' > \widetilde{\theta}(\pi)$. Also note that $w(\theta, \pi) = w\left(\widetilde{\theta}(\pi), \pi\right)$ for all $\theta \leq \widetilde{\theta}(\pi)$ so that

$$\begin{aligned} B(\pi) &= \left(F_q(\widetilde{\theta}(\pi)) - F_u(\widetilde{\theta}(\pi))\right) v\left(w\left(\widetilde{\theta}(\pi), \pi\right), p(\pi)\right) + \\ &\quad + \int_{\widetilde{\theta}(\pi)}^1 v(w(\theta, \pi), p(\pi))(f_q(\theta) - f_u(\theta)) \\ &\geq \left(F_q(\widetilde{\theta}(\pi)) - F_u(\widetilde{\theta}(\pi))\right) v\left(w\left(\widetilde{\theta}(\pi), \pi\right), p(\pi)\right) + \\ &\quad + \left(F_q(\theta') - F_q(\widetilde{\theta}(\pi) - \left(F_u(\theta') - F_u(\widetilde{\theta}(\pi))\right)\right) v(w(\theta', \pi), p(\pi)) + \\ &\quad + (1 - F_q(\theta') - (1 - F_u(\theta'))) v(w(\theta', \pi), p(\pi)) \\ &= \left(F_q(\widetilde{\theta}(\pi)) - F_u(\widetilde{\theta}(\pi))\right) v\left(w\left(\widetilde{\theta}(\pi), \pi\right), p(\pi)\right) \\ &\quad + \left(F_u(\widetilde{\theta}(\pi)) - F_q(\widetilde{\theta}(\pi))\right) v(w(\theta', \pi), p(\pi)) \; > 0 \end{aligned} \tag{A28}$$

since v is strictly increasing in w and w is strictly increasing in θ on $[\widetilde{\theta}(\pi), \theta']$ and $F_u(\widetilde{\theta}(\pi)) > F_q(\widetilde{\theta}(\pi))$. If $\theta' \leq \widetilde{\theta}(\pi)$ we may let θ'' be some arbitrary except that $\widetilde{\theta}(\pi) < \theta'' < 1$ and use similar reasoning to show

$$B(\pi) \geq \left(F_q(\widetilde{\theta}(\pi)) - F_u(\widetilde{\theta}(\pi))\right) v\left(w\left(\widetilde{\theta}(\pi), \pi\right), p(\pi)\right) \tag{A29}$$

$$
\begin{aligned}
&+\left(F_q\left(\theta''\right)-F_q(\widetilde{\theta}(\pi)-\left(F_u\left(\theta''\right)-F_u(\widetilde{\theta}(\pi))\right)v\left(w\left(\widetilde{\theta}\left(\pi\right),\pi\right),p\left(\pi\right)\right)+\\
&+\left(1-F_q\left(\theta'\right)-\left(1-F_u\left(\theta'\right)\right)\right)v\left(w\left(\theta'',\pi\right),p\left(\pi\right)\right)\\
=\ &\left(F_q\left(\theta''\right)-F_u\left(\theta''\right)\right)v\left(w\left(\widetilde{\theta}\left(\pi\right),\pi\right),p\left(\pi\right)\right)\\
&+\left(F_u\left(\theta''\right)-F_q\left(\theta'\right)\right)v\left(w\left(\theta'',\pi\right),p\left(\pi\right)\right)\ >\ 0,
\end{aligned}
$$

which completes the proof. ■

Proof of Proposition 3 Opening up a country for trade has no effect on the profit maximization problem for an individual firm or the conditions for factor market clearing. Hence, all arguments in the proof of Proposition 1 that were used in the characterization of equilibrium wages apply also with trade, so wages must be consistent with (41). To see that x_w^* must be on the frontier of $X_w\left(\pi\right)$, suppose that this is not the case. Then there exists alternative factor inputs so that $x_{iw}^* \leq \lambda_a y^i\left(c_{ia}', s_{ia}'\right)+\lambda_b y^i\left(c_{ib}', s_{ib}'\right)$for both goods and with strict inequality for one good. Hence

$$
\begin{aligned}
p_1^* x_{1w}^*+p_2^* x_{2w}^* \ &<\ \lambda_a p_1^* y^1\left(c_{1a}', s_{1a}'\right)+\lambda_b p_1^* y^1\left(c_{1b}', s_{1b}'\right)\\
&+\lambda_a p_2^* y^2\left(c_{2a}', s_{2a}'\right)+\lambda_b p_1^* y^2\left(c_{2b}', s_{2b}'\right)
\end{aligned}
\tag{A30}
$$

Moreover, for the alternative inputs to be feasible it must be that $\sum_i\sum_t l_{ij}^{t\prime}\left(\theta\right) \leq f_\pi\left(\theta\right) = \sum_i\sum_t l_{ij}^{t*}\left(\theta\right)$ for each country and every θ. Let

$$
\begin{aligned}
W_{ij}^* &= \int_\theta w^*\left(\theta\right)\left(l_i^{c*}\left(\theta\right)+l_i^{c*}\left(\theta\right)\right)d\theta \text{ and}\\
W_{ij}' &= \int_\theta w'\left(\theta\right)\left(l_i^{c\prime}\left(\theta\right)+l_i^{c\prime}\left(\theta\right)\right)d\theta
\end{aligned}
\tag{A31}
$$

be the wage costs in the assumed equilibrium and for the alternative factor inputs respectively. Clearly

$$
W_{1j}'+W_{2j}' \leq W_{1j}^*+W_{2j}^*
\tag{A32}
$$

for both j and combining (A30) and (A32) we have that

$$
\sum_{j=a,b}\lambda_j\left(p_1^* x_{1j}^*+p_2^* x_{2j}^*-W_{1j}^*+W_{2j}^*\right) < \sum_{j=a,b}\lambda_j\left(p_1^* y^1\left(c_{1j}', s_{1j}'\right)+p_2^* y^2\left(c_{2j}', s_{2j}'\right)-W_{1j}'+W_{2j}'\right),
\tag{A33}
$$

which means that there must be at least one sector in at least one country where profits are not maximized. Thus x_w^* must be on the frontier of $X_w\left(\pi\right)$ and since utility maximization implies that all bundles better than x_w^* must be strictly more expensive given world prices this completes the proof of the "only if" part. The "if" part follows step by step the autharky case. ■

Proof of Proposition 4 (sketch). Uniqueness of x_w^* is immediate from Proposition 3. For contradiction suppose that (x_a^*, x_b^*) and (x_a^{**}, x_b^{**}) are distinct country specific equilibrium outputs that generate x_w^*. By strict convexity of $X(\pi_a)$ and $X(\pi_b)$ it is then possible to find $x_w' >> x_w^*$ that is in $X_w(\pi)$, which contradicts that x_w^* is on the frontier of $X_w(\pi)$. Hence, (x_a^*, x_b^*) are unique and uniqueness of $(c_{1j}^*, c_{2j}^*, s_{1j}^*, s_{2j}^*)$ can then be established exactly as in the model without trade. ■

Proof of Proposition 6. For contradiction, suppose that $\pi_a < \pi_b$ and that country a does not import the skill intensive good. Suppose first that the equilibrium is such that both goods are produced by both countries. Since the two goods are consumed at the same ratio in each country we than have $x_{1a}^*/x_{2a}^* \geq x_{1b}^*/x_{2b}^*$ and $(w_{ca}^*, w_{sa}^*) = (w_{cb}^*, w_{sb}^*)$. Inspection of the resuced form profit maximization problem then shows that the factor ratio is the same in both industries. For brevity let $\theta_j^* = \theta(c_{1j}^* + c_{2j}^*, \pi_j)$ and observe that (32), together with equal factor prices imply that $P(\theta_a^*, \pi_a) = P(\theta_b^*, \pi_b)$, which (since $\pi_a < \pi_b$ and P strictly increasing in both arguments) implies that $\theta_a^* > \theta_b^*$, so,

$$c_{1a}^* + c_{2a}^* = \pi_a(1 - F_q(\theta_a^*) < \pi_b(1 - F_q(\theta_b^*) = c_{1b}^* + c_{2b}^*. \tag{A34}$$

Moreover, $F_q(\theta) < F_u(\theta)$ for all $\theta < 1$ since if for some $\theta' < 1$ we have that $F_q(\theta') = F_u(\theta')$, then the monotone likelihood ratio assumption can not hold, so

$$\begin{aligned} s_{1a}^* + s_{2a}^* &= \pi_a F_q(\theta_a^*) + (1-\pi_a) F_q(\theta_a^*) > \pi_b F_q(\theta_a^*) + (1-\pi_b) F_q(\theta_a^*) > \quad \text{(A35)} \\ &> \pi_b F_q(\theta_b^*) + (1-\pi_b) F_q(\theta_b^*) = s_{1b}^* + s_{2b}^* \end{aligned}$$

From constant returns it follows that

$$\frac{s_{1a}^*}{s_{1b}^*} = \frac{c_{1a}^*}{c_{1b}^*} = \frac{x_{1a}^*}{x_{1b}^*} \geq \frac{x_{2a}^*}{x_{2b}^*} = \frac{c_{2a}^*}{c_{2b}^*} = \frac{s_{2a}^*}{s_{2b}^*} \tag{A36}$$

which implies that

$$c_{1a}^* + c_{2a}^* = c_{1b}^* \frac{x_{1a}^*}{x_{1b}^*} + c_{2b}^* \frac{x_{2a}^*}{x_{2b}^*} < c_{1b}^* + c_{2b}^* \Rightarrow \tag{A37}$$

$$0 > c_{1b}^* \left(\frac{x_{1a}^*}{x_{1b}^*} - 1 \right) + c_{2b}^* \left(\frac{x_{2a}^*}{x_{2b}^*} - 1 \right) \tag{A38}$$

$$s_{1a}^* + s_{2a}^* = s_{1b}^* \frac{x_{1a}^*}{x_{1b}^*} + s_{2b}^* \frac{x_{2a}^*}{x_{2b}^*} > s_{1b}^* + s_{2b}^* \Rightarrow \tag{A39}$$

$$0 < s_{1b}^* \left(\frac{x_{1a}^*}{x_{1b}^*} - 1 \right) + s_{2b}^* \left(\frac{x_{2a}^*}{x_{2b}^*} - 1 \right) < \tag{A40}$$

$$< s_{1b}^* \left(\frac{x_{1a}^*}{x_{1b}^*} - 1 \right) - s_{2b}^* \frac{c_{1b}^*}{c_{2b}^*} \left(\frac{x_{1a}^*}{x_{1b}^*} - 1 \right) \tag{A41}$$

All factor inputs are positive, so to satisfy (A40) one coefficient must be strictly positive and sombining with (A36) we have that $\left(\frac{x_{1a}^*}{x_{1b}^*} - 1\right) > 0$, hence

$$0 < s_{1b}^* - s_{2b}^* \frac{c_{1b}^*}{c_{2b}^*} \Leftrightarrow s_{2b}^* \frac{c_{1b}^*}{c_{2b}^*} < s_{1b}^* \Rightarrow \frac{c_{1b}^*}{s_{1b}^*} < \frac{c_{2b}^*}{s_{2b}^*}, \tag{A42}$$

which is a contradiction since the factor intensity assumption is violated.

There are a few possibilities left to rule out for cases when factor price equalization doesn't hold. First it may be that country a only produces good 1 and country 2 only produces good 2. Here (A34) and (A35) gives an immediate violation of the factor intensity assumption. Next, country a may only produce good 1 while the production is diversified in the other country. The first 3 equalities in (A36) still applies, so

$$\begin{aligned} c_{1a}^* &= c_{1b}^* \frac{x_{1a}^*}{x_{1b}^*} < c_{1b}^* + c_{2b}^* \Rightarrow c_{1b}^* \left(\frac{x_{1a}^*}{x_{1b}^*} - 1 \right) < c_{2b}^* \\ s_{1a}^* &= s_{1b}^* \frac{x_{1a}^*}{x_{1b}^*} > s_{1b}^* + s_{2b}^* \Rightarrow s_{1b}^* \left(\frac{x_{1a}^*}{x_{1b}^*} - 1 \right) > s_{2b}^* \end{aligned}$$

All inputs in country b are strictly positive, so $\left(\frac{x_{1a}^*}{x_{1b}^*} - 1\right) > 0$, which leads to a violation of the factor intensity assumption. The final possibility is when country a is diversified and country 2 only produces good 2 and the argument follows the same line of reasoning as the previous argument. ■

Proof of Lemma 4. For brevity we drop the country index j and let $p_1 = p(\pi), p_1' = p(\pi')$, $p_2 = p_2' = 1$, $c = c_j(\pi), c' = c_j(\pi'), c_i = c_{ij}(\pi)$ for $i = 1, 2$, $c_i' = c_{ij}(\pi')$ and so on. Since (c_i, s_i) is a solution to the profit maximization problem given prices (p_i, w_c, w_s) and since (c_i', s_i') is a solution to the profit maximization problem given prices (p_i', w_c', w_s') it follows that

$$\begin{aligned} p_i y^i(c_i, s_i) - w_c c_i - w_s s_i &\geq p_i y^i(c_i', s_i') - w_c c_i' - w_s s_i' \\ p_i' y^i(c_i', s_i') - w_c' c_i' - w_s' s_i' &\geq p_i' y^i(c_i, s_i) - w_c' c_i - w_s' s_i \end{aligned}$$

for $i = 1, 2$ and using strict quasi-concavity of y^i is is easy to show that if $\frac{c_i}{s_i} \neq \frac{c_i'}{s_i'}$, then

$$\begin{aligned} p_i y^i(c_i, s_i) - w_c c_i - w_s s_i &> p_i y^i(c_i', s_i') - w_c c_i' - w_s s_i' \text{ and} \\ p_i' y^i(c_i', s_i') - w_c' c_i' - w_s' s_i' &> p_i' y^i(c_i, s_i) - w_c' c_i - w_s' s_i \end{aligned}$$

Now let x_i and x_i' denote the equilibrium outputs $x_{ij}(\pi)$ (equal to $y^i(c_i, s_i)$), add and rearrange to get

$$\sum_i (p_i - p_i')(x_i - x_i') \geq (w_c - w_c')(c - c') + (w_s - w_s')(s - s')$$

and the proof is then complete by observing that $p_2 = p_2' = 1$ by choice of unit of account. ■

Proof of Proposition 6. If $x_{ij}(\pi), x_{ij}(\pi') > 0$ for all i, j Proposition 5 implies that for both i we have that

$$\begin{aligned} \frac{\partial y^2 (r_{2a}(\pi), 1)}{\partial s} &= w_s(\pi) = \frac{\partial y^2 (r_{2b}(\pi), 1)}{\partial s} \\ \frac{\partial y^2 (r_{2a}(\pi), 1)}{\partial c} &= w_c(\pi) = \frac{\partial y^2 (r_{2b}(\pi), 1)}{\partial c}. \end{aligned} \tag{A43}$$

Moreover, the equilibrium wage schemes are also such that the technical rate of substitution of factors are equalized across sectors, i.e.,

$$\frac{\frac{\partial y^1(r_{1j}(\pi),1)}{\partial c}}{\frac{\partial y^1(r_{1j}(\pi),1)}{\partial s}} = \frac{\frac{\partial y^2(r_{2j}(\pi),1)}{\partial c}}{\frac{\partial y^2(r_{2j}(\pi),1)}{\partial s}}. \tag{A44}$$

The same equalities hold also for π'. This implies that:

1. If $w_c(\pi) \leq w_c(\pi')$, then $r_{2j}(\pi) \geq r_{2j}(\pi')$ for $j = a, b \Rightarrow w_s(\pi) \geq w_s(\pi')$ (from (A43))
2. Similarly, if $w_s(\pi) \geq w_s(\pi')$, then then $r_{2j}(\pi) \geq r_{2j}(\pi')$ for $j = a, b \Rightarrow w_c(\pi) \leq w_c(\pi')$ (from (A43))
3. If $r_{2j}(\pi) \geq r_{2j}(\pi')$ then $r_{1j}(\pi) \geq r_{1j}(\pi')$ (from (A44))

Hence, a failure of the result could only happen if $w_c(\pi) \leq w_c(\pi')$ and $w_s(\pi) \geq w_s(\pi')$ and $r_{ij}(\pi) \geq r_{ij}(\pi')$ for all i, j if this would be the case. But from the characterization of the equilibrium wage scheme we have that

$$P(\theta_j(\pi), \pi) \frac{\partial y^i (r_{ij}(\pi), 1)}{\partial c} = \frac{\partial y^i (r_{ij}(\pi), 1)}{\partial s} \tag{A45}$$

for all i, j and similarly for π', so

$$P(\theta_j(\pi), \pi) = \frac{\frac{\partial y^i(r_{ij}(\pi),1)}{\partial s}}{\frac{\partial y^i(r_{ij}(\pi),1)}{\partial c}} \geq \frac{\frac{\partial y^i(r_{ij}(\pi'),1)}{\partial s}}{\frac{\partial y^i(r_{ij}(\pi'),1)}{\partial c}} = P\left(\theta_j\left(\pi'\right), \pi'\right). \tag{A46}$$

Since P is strictly increasing in both arguments this means that $\theta_a(\pi) > \theta_a(\pi')$ and $\theta_b(\pi) \geq \theta_b(\pi')$ and this means that we can sign the changes in total efficient inputs of the two factors as

$$\begin{aligned} c_j(\pi) &= \pi_j(1 - F_q(\theta_j(\pi))) \leq \pi_j'(1 - F_q(\theta_j(\pi'))) = c_j\left(\pi'\right), \\ s_j(\pi) &= \pi_j F_{\pi_j}(\theta_j(\pi)) \geq \pi_j' F_{\pi_j'}(\theta_j\left(\pi'\right)) = s_j\left(\pi'\right) \end{aligned} \tag{A47}$$

with strict inequalities for $j = a$ and the second inequality follows since $F_\pi(\theta) = \pi F_q(\theta) + (1-\pi) F_u(\theta)$ is decreasing in π by the monotone likelihood ratio property and increasing in θ since it is a cumulative distribution function (same calculation as in the proof of Proposition 6). Adding the inequalities in (A47) we get that

$$\begin{aligned} c_w(\pi) &\equiv c_a(\pi) + c_b(\pi) < c_a(\pi') + c_b(\pi') \equiv c_w(\pi') \\ s_w(\pi) &\equiv s_a(\pi) + s_b(\pi) > s_a(\pi') + s_b(\pi') \equiv s_w(\pi') \end{aligned} \tag{A48}$$

Hence, in spite of all factor ratios being higher in the equilibrium with the lower π it must be that the aggregate use of skilled labor is lower in the equilibrium with lower π and that the aggregate use of unskilled labor is higher. We now let $c_{iw}(\pi) = c_{ia}(\pi) + c_{ib}(\pi)$ denote the sector specific use of complex labor in the world and let $s_{iw}(\pi)$ be defined in the same way and note that $c_{iw}(\pi)/s_i(\pi)$ $= r_{iw}(\pi) = r_{ia}(\pi) = r_{ib}(\pi)$ and similarly for π'. We have that

$$s_w(\pi') = \frac{c_{1w}(\pi')}{r_{1w}(\pi')} + \frac{c_{2w}(\pi')}{r_{2w}(\pi')} = c_{1w}(\pi')\left(\frac{1}{r_{1w}(\pi')} - \frac{1}{r_{2w}(\pi')}\right) + \frac{c_w(\pi')}{r_{2w}(\pi')}, \tag{A49}$$

and, since $r_{iw}(\pi) \geq r_{iw}(\pi')$ for $i = 1, 2$,

$$\begin{aligned} s_w(\pi) &= s_{1w}(\pi) + s_{2w}(\pi) = \frac{c_{1w}(\pi)}{r_{1w}(\pi)} + \frac{c_{2w}(\pi)}{r_{2w}(\pi)} \leq \\ &\leq \frac{c_{1w}(\pi)}{r_{1w}(\pi')} + \frac{c_{2w}(\pi)}{r_{2w}(\pi')} = \frac{c_{1w}(\pi)}{r_{1w}(\pi')} + \frac{(c_w(\pi) - c_{1w}(\pi))}{r_{2w}(\pi')} = \\ &= c_{1w}(\pi)\left(\frac{1}{r_{1w}(\pi')} - \frac{1}{r_{2w}(\pi')}\right) + \frac{c_w(\pi)}{r_{2w}(\pi')}. \end{aligned} \tag{A50}$$

Hence by combining (A49) and (A50) we get that

$$0 < s_w(\pi) - s_w(\pi') \leq \left(c_{1w}(\pi) - c_{1w}(\pi')\right)\left(\frac{1}{r_{1w}(\pi')} - \frac{1}{r_{2w}(\pi')}\right) + \frac{c_w(\pi) - c_w(\pi')}{r_{2w}(\pi')}. \tag{A51}$$

Since $c_w(\pi) < c_w(\pi')$ and, by the factor intensity assumption, $r_{1w}(\pi') > r_{2w}(\pi')$ it follows that a necessary condition for (A51) to hold is that $c_{1w}(\pi) < c_{1w}(\pi')$. Similar algebra also reveals that

$$\begin{aligned} s_w(\pi') &= \frac{c_{1w}(\pi')}{r_{1w}(\pi')} + \frac{c_{2w}(\pi')}{r_{2w}(\pi')} = c_{2w}(\pi')\left(\frac{1}{r_{2w}(\pi')} - \frac{1}{r_{1w}(\pi')}\right) + \frac{c_w(\pi')}{r_{1w}(\pi')} \\ s_w(\pi) &= \frac{c_{1w}(\pi)}{r_{1w}(\pi)} + \frac{c_{2w}(\pi)}{r_{2w}(\pi)} \leq c_{2w}(\pi)\left(\frac{1}{r_{2w}(\pi')} - \frac{1}{r_{1w}(\pi')}\right) + \frac{c_w(\pi)}{r_{1w}(\pi')}, \end{aligned} \tag{A52}$$

implying that

$$0 < s_w(\pi) - s_w(\pi') \leq \left(c_{2w}(\pi) - c_{2w}(\pi')\right)\left(\frac{1}{r_{2w}(\pi')} - \frac{1}{r_{1w}(\pi')}\right) + \frac{c_w(\pi) - c_w(\pi')}{r_{1w}(\pi')}, \tag{A53}$$

so $c_{2w}(\pi) > c_{2w}(\pi')$ is also necessary under the hypothesis of the claim. In the same spirit we have that

$$\begin{aligned} c_w(\pi') &= \left(r_{1w}(\pi') - r_{2w}(\pi')\right) s_{1w}(\pi') + r_2(\pi') s_w(\pi') \\ c_w(\pi) &\geq \left(r_{1w}(\pi') - r_{2w}(\pi')\right) s_{1w}(\pi) + r_2(\pi') s_w(\pi), \end{aligned} \tag{A54}$$

so that

$$0 > c_w(\pi) - c_w(\pi') \geq \left(r_{1w}(\pi') - r_{2w}(\pi')\right)\left(s_{1w}(\pi) - s_{1w}(\pi')\right) + r_2(\pi')\left(s_w(\pi) - s_w(\pi')\right), \tag{A55}$$

which implies that $s_{1w}(\pi) < s_{1w}(\pi')$. Finally, we proceed in the same way to show that

$$\begin{aligned} c_w(\pi') &= \left(r_{2w}(\pi') - r_{1w}(\pi')\right) s_{2w}(\pi') + r_1(\pi') s_w(\pi') \\ c_w(\pi) &\geq \left(r_{2w}(\pi') - r_{1w}(\pi')\right) s_{2w}(\pi) + r_1(\pi') s_w(\pi), \end{aligned} \tag{A56}$$

which can be combined as

$$0 > c_w(\pi) - c_w(\pi') \geq \left(r_{2w}(\pi') - r_{1w}(\pi')\right)\left(s_{2w}(\pi) - s_{2w}(\pi')\right) + r_1(\pi')\left(s_w(\pi) - s_w(\pi')\right) \tag{A57}$$

implying that $s_{2w}(\pi) > s_{2w}(\pi')$. Hence, since both inputs in industry 1 (2) are higher (lower) in the equilibrium with investments according to π' than with π it follows that

$$\begin{aligned} x_{1w}(\pi) &= y^1\left(c_{1w}(\pi), s_{1w}(\pi)\right) < y^1\left(c_{1w}(\pi'), s_{1w}(\pi')\right) = x_{1w}(\pi') \\ x_{2w}(\pi) &= y^2\left(c_{2w}(\pi), s_{2w}(\pi)\right) > y^2\left(c_{2w}(\pi'), s_{2w}(\pi')\right) = x_{2w}(\pi'). \end{aligned} \tag{A58}$$

To finish up the proof of the result we note that $x_{1w}(\pi) < x_{1w}(\pi')$ and $x_{2w}(\pi) > x_{2w}(\pi')$ implies that $\frac{x_{1w}(\pi)}{x_{2w}(\pi)} < \frac{x_{1w}(\pi')}{x_{2w}(\pi')}$, which in turn implies that $p(\pi) > p(\pi')$ from optimal consumer behavior. Moreover, with factor price equalization Lemma 7 implies that

$$\begin{aligned} 0 > \underbrace{\left(p(\pi) - p(\pi')\right)}_{>0}\underbrace{\left(x_{1w}(\pi) - x_{1w}(\pi')\right)}_{<0} &\geq \left(w_c(\pi) - w_c(\pi')\right)\underbrace{\left(c_w(\pi) - c_w(\pi')\right)}_{<0} \\ &\quad + \left(w_s(\pi) - w_s(\pi')\right)\underbrace{\left(s_w(\pi) - s_w(\pi')\right)}_{>0}. \end{aligned} \tag{A59}$$

We showed above that under our assumption that the proposition fails we must have that $w_c(\pi) \leq w_c(\pi')$ and $w_s(\pi) \geq w_s(\pi')$ implying that the right hand side is weakly positive, which is a contradiction. ■

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