Barriers and the Transition to Modern Growth[¤]

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Abstract

This paper attempts to account for current huge international income disparities by incorporating the fact that modern growth begins at di¤erent points in time for di¤erent countries. This di¤erence is due to di¤erent levels of barriers to capital accumulation. There are three main results in this paper. First, modern growth begins in all countries but sooner in those with lower level of barrier. Second, the path of income di¤erence exhibits an inverted U-shape over time for countries that di¤er only in their levels of the barrier. Third, given the actual beginning dates of modern growth of two countries, the model can account for a signi…cant portion of the observed income di¤erence between them.

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1 Introduction

Long run economic data demonstrates three important development facts. First, countries experienced stagnation for long periods and subsequently enter a modern growth regime, that is, a sustained increase in per capita output, though at di¤erent points in time. This important moment of a country is referred as its "turning point" by Reynolds. Second, the income di¤erences between the early and the later developers exhibits an inverted U-shape over time. Third, some countries with similar turning points have experienced dramatically di¤erent rates of economic development.

On the other hand, evidence shows a persistent factor 30 income di¤erence between rich and poor countries for the period 1960-1985. Such persistency has stimulated a line of research examining the implications of policy di¤erences on income along the balanced growth path among versions of the neoclassical growth model.[1] However, the …rst long run development fact implies that for a country which has su¤ered long stagnation, its current poverty level relative to a early developer is evidently not a phenomenon along the balanced growth path, but at least in part a transitional one. The question, then, is why do countries have di¤erent turning points, and what does this imply for the current income disparity?

In parallel with previous papers, this paper attempts to answer these questions by considering the implication of policy di¤erences. I build on the model of Hansen and Prescott (1999). In their model, there are two technologies with exogenous technological improvement. The Malthusian technology uses land, labor and capital, while the Solow technology uses labor and capital only. The economy is in stagnation when the Solow technology is not used as population growth will cancel out any increase in total output. It reaches its turning point when the Solow technology is used and it starts to grow. It will then asymptotically converge to a Solow balanced growth path.

I extend the Hansen-Prescott model by introducing polices which act as a barrier to discourage capital accumulation in the Solow technology. In contrast to previous models, barriers in my model not only lower the level of income along the balanced growth path but also delay the turning point. Because of this second e¤ect, the income di¤erence between economies with di¤erent barriers displays an inverted U-shape over time, and hence the model can also account for the second development fact. More precisely, there is no income di¤erence before

[1] This literature generally focuses on policies that distort capital accumulation (Mankiw, Romer and Weil(1992), Chari, Kehoe and McGrattan (1996), Parente, Rogerson and Wright (1999)), technology adoption (Parente and Prescott(1994)), and level of total factor productivity (Hall and Jones(1998), Prescott(1998) , and Parente and Prescott(1997)). See McGrattan and Schmit(1998) for a survey of papers on cross country income di¤erences.

the early developer reaches its turning point. Then, the income difference first increases before it decreases to its balanced growth path level. The income disparity generated by this model is shown to be significantly higher than the level along the balanced growth path, and this result is robust to changes in parameters. To account for the third development fact, the model offers a story as follows: two economies initially are subject to the same barrier, and hence the same turning point. Given the actual data on turning points, the sizes of barriers can be determined. After the turning point is reached, the barrier is reduced in one economy. Now, it is not hard to imagine the country with a significant reduction in barriers will experience a development miracle, and their income level with diverge drastically.

I consider two empirical case studies to illustrate the strength of this model as a development model. These case studies are the development experiences of Africa and Japan. In both cases, I am interested in their experiences relative to the UK which is the first country to experience industrial revolution. In the case of Africa, I use the actual difference in turning points between Africa and the UK to determine the relative barrier in Africa. I find that the barrier that can account for their difference in turning points can also account for the path of income difference between them. Moreover, my model predicts relative income in Africa will continue worsen even if its relative barrier remains unchanged. In the case of Japan, I show that its postwar miracle experience is a result of reduction in barriers based on historical evidence. Moreover, I also find that its slowdown during the 70s is not necessary a result of an increase in its relative barrier as argued by Parente and Prescott (1994).

The transition from stagnation to modern growth has only recently received attention in the literature. Becker, Murphy and Tamura (1990) develop a multiple equilibrium model. The transition in their model requires an exogenous change in the return to human capital. Kremer (1993) documents the long run population data and argues that the population growth rate increases at low levels of income and then decreases when income is high enough. Based on these ideas, Goodfriend and McDermott (1995), Galor and Weil (1998), Jones (1999) and Hansen and Prescott (1999) endogenize the transition from stagnation to modern growth. These models differs in several aspects regarding the driving forces of the transition to modern growth and whether such transition is inevitable or not. This paper is closely related to Lucas (1999) which studies the evolution of the relative income distribution by assigning turning points exogenously, and finds that the income inequality exhibits an inverted U-shape. I study the same issue but with the turning point endogenously determined.

The remainder of the paper is organized as follows. Section 2 documents the three long run development facts as a motivation for this paper. Section 3 presents the model. The model's implication for international income differences are studied in section 4. Section 5 discuss the role of the population pro…le in the model. The actual data on turning points are used in section 6 to confront the model with the data. A conclusion is given in section 7.

2 Motivation

This section documents three important long run development facts in the data. (1) All countries experienced stagnation for long periods and subsequently enter the modern growth regime (sustained increase in per capita GDP), though at di¤erent points in time. (2) The income di¤erence between the early developers and the later developers exhibits an inverted U-shape. (3) Some countries with similar turning points have experienced dramatically di¤erent rates of economic development since 1950. The data used in this paper are reported in Lucas (1998).

Figure 1: GDP per Capita for 5 Regions

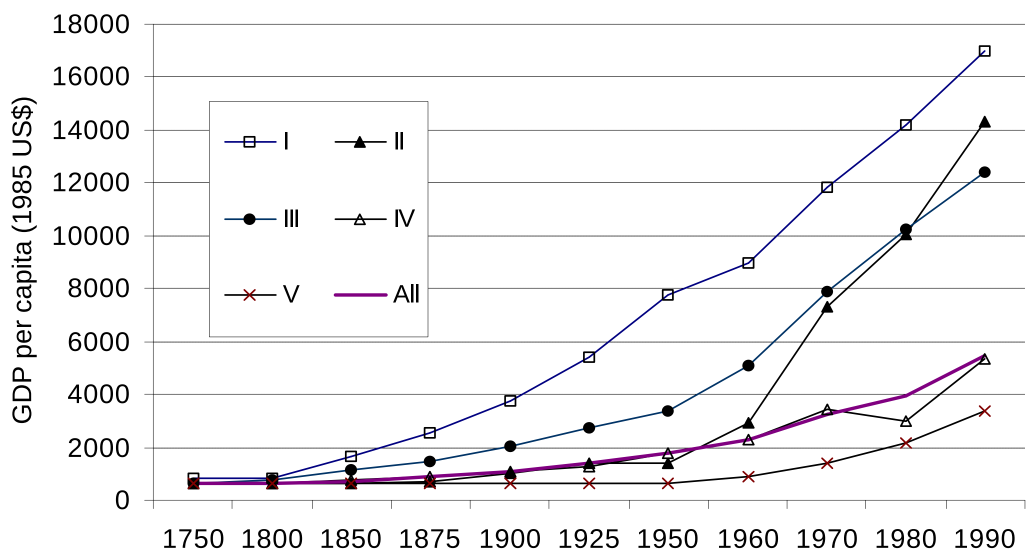
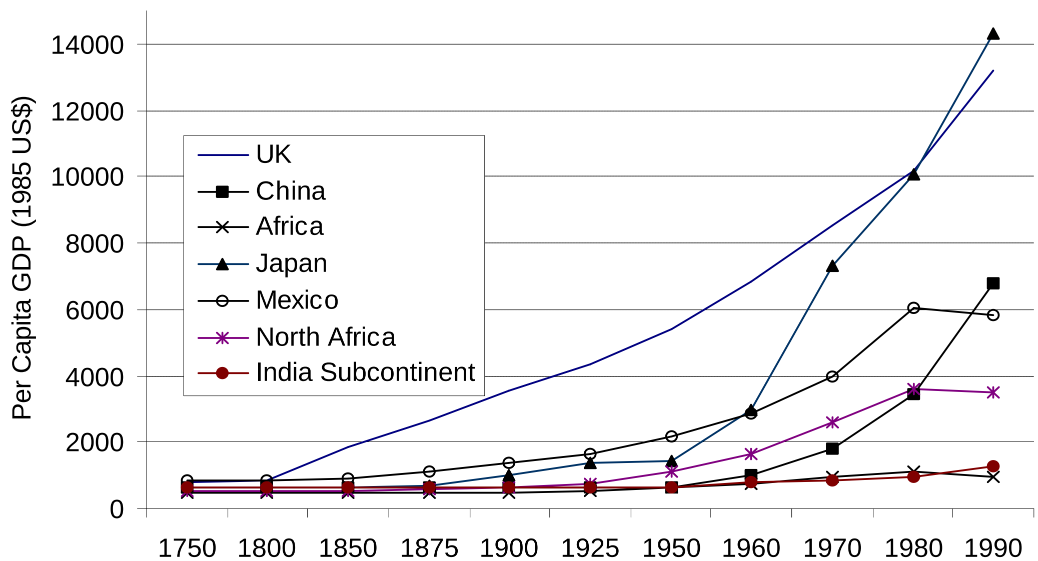


Figure (1) demonstrates that per capita income for all …ve di¤erent regions in the world had been stagnant before the 19th century and started to grow at di¤erent times for di¤erent regions.[2] This stagnation is not because the world experienced no growth in total output but, rather because the increase in population o¤set the increases in output. The Malthusian theory

[2] Region I includes UK, US, Canada, Australia and New Zealand. Japan and Western Europe are region II and III respectively. Region IV includes Latin America, Eastern Europe and Soviet Union. Finally, region V includes Africa and Asia(except Japan).

therefore describes those countries in stagnation very well. However, countries subsequently start to leave this type of stagnation and enter the modern growth regime. For instance, ...gure (2) shows that the turning point (the time at which modern growth begins) for the UK is around 1800 while the turning points for Japan and Africa are around 1900 and for China and India Subcontinent are around 1950.[3],[4]

Figure 2: Turning Points



For the same group of countries, ...gure (3) plots the GDP per capita ratio between the UK and the rest of the group. The picture displays an inverse U-shape over time for most of the countries.[5] This pattern suggests that the income ratio will be much lower when the "poor" country is closer to its balanced growth path. This o¤ers an explanation as to why the income disparity predicted by the balanced growth path approach is much lower than the observed number in the data. The picture also reveals another interesting fact: countries which have the same turning points can have dramatically di¤erent experiences. For example, Japan and Africa both have turning points around 1900 while China and Indian Subcontinent both have turning points around 1950 yet there is substantial divergence. Lucas (1993) documents the

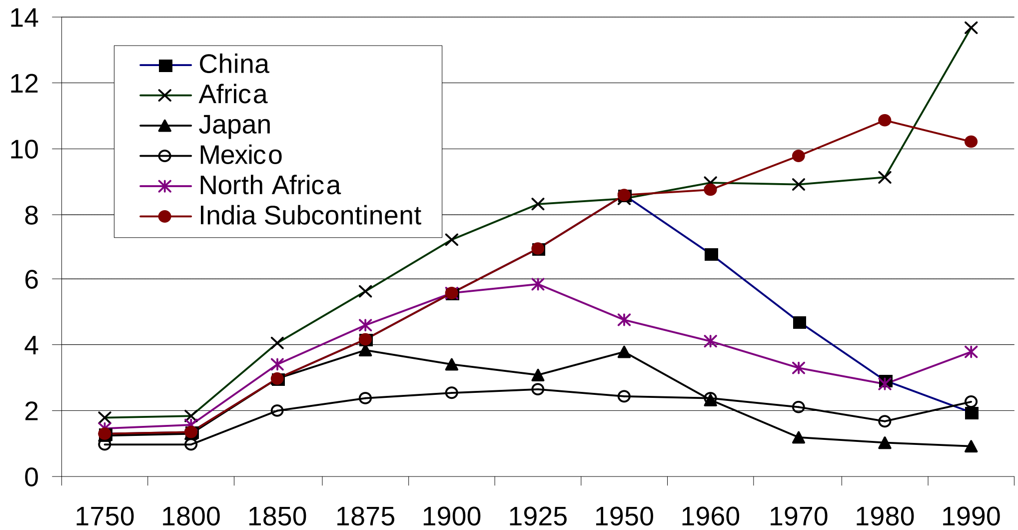
[3] Africa includes all of Africa except Morocco, Algeria, Tunisia, Libya and Egypt. Indian Subcontinent includes Pakistan, India, Bangladesh, Sri Lanka, Nepal and Bhutan.

[4] These turning points are suggested by Reynold (1985).

[5] The increasing income disparity has also been documented by Pritchett (1997).

same result for two very similar economies, South Korea and Philippines.[6] Both countries had about the same GDP per capita in 1960, yet the growth rate for Korea was about 3 times that of Philippines for the period 1960-1988.

Figure 3: Relative GDP per Capita ($\frac{y_{Uk}}{y_i}$)



The message from the data is: in order to understand the current income differences, the fact that countries have different turning points must not be overlooked. This raises both analytical and empirical questions. Why do countries have different turning points? How important is this in explaining the current income differences? The answer suggested by this paper are: (1) countries have different turning points because of differences in policies that affect capital accumulation. (2) Differences in turning points are quantitatively important in accounting for current income differences.

To proceed, I need a model that can determine the turning point endogenously. I choose to use the model by Hansen and Prescott (1999) as asymptotically the model behaves the same as a standard Solow model which has replicated the modern rich countries fairly well.

[6] As documented in Lucas (1993), the population, labor force, fraction of population in the main city, fraction of labor in agriculture, and level of education are all similar for both countries in 1960.

3 The Model

Following the existing literature, I use barriers to capital accumulation as an explanation for why countries are poor and, in the context of this paper, why modern growth begins later. Barrier can take the form of taxes on investment goods, corruption or other institution factors that increase the relative price of investment goods, which in turn discourages capital accumulation. There are many ways to model it. I assume it takes the form of reducing the efficiency of transforming forgone consumption goods into usable capital goods for next period.

3.1 The Economy

Technology Output in this economy can be produced using two exogenously growing technologies, the Malthus technology and the Solow technology. The Malthus technology features constant return to scale in capital, labor and land. On the other hand, the Solow technology features constant return to scale in capital and labor only. The two production functions are as follows:

$$Y_{mt} = A_m \gamma_m^t K_{mt}^{\phi} N_{mt}^{\mu} L_{mt}^{1-\mu-\phi} \tag{1}$$

$$Y_{st} = A_s \gamma_s^t K_{st}^{\theta} N_{st}^{1-\theta} \tag{2}$$

where K_{it}, N_{it} and L_{it} denote capital, labor and land used in technology i at time $t = 0, 1, \ldots$; $\gamma_m > 1$ and $\gamma_s > 1$ are the growth rate of the total factor productivity (TFP) for the Malthus and Solow technology respectively.

Physical capital is assumed to depreciate completely each period.[7] Land is a fixed factor. Output of the two sectors are identical, and can be used either for consumption or investment. Hence, feasibility requires:

$$C_t + X_{mt} + X_{st} = Y_{mt} + Y_{st}$$

where C_t is aggregate consumption, while X_{mt} and X_{st} are the aggregate investment in the Malthus and Solow capital in period t.

Firms in each sector are assumed to behave competitively and rent all factors of production from households. A representative firm in sector j takes the wage rate and rental rates for capital and land as given, and chooses labor, capital and land input to maximize profits.

$$\underset{N_{jt}, K_{jt}, L_{jt}}{\text{Max}} \; Y_{jt} - w_t N_{jt} - r_{Kjt} K_{jt} - r_{Lt} L_{jt} \qquad j = m, s$$

[7] In the quantitative work carried out later a period will be interpreted to be 35 years, so this assumption is empirically reasonable.

$$s:t:(1) \text{ and } (2)$$

Household Sector The population structure is that of a two period lived overlapping generations model with endogenous fertility. In the beginning of each period, the current old agents give birth to young agents. The number of children they have depends on their living standard when young. Letting N_t be the number of young agents in period t, and c_{1t} be the consumption level for young agents in period t, the population dynamics are given by:

$$N_{t+1} = g(c_{1t})N_t$$

where $g(:)$ is an exogenous function that will be speci…ed in more detail when the model is calibrated.

In period 0, there are $N_{¡1}$ old agents and N_0 young agents. Each initial old agent is endowed with $\frac{K_0}{N_{¡1}}$ units of capital and $\frac{L}{N_{¡1}}$ units of land. Young agents are endowed with one unit of time, which they supply inelastically. Old agents are assumed to be unable to work. Young agents make a consumption-saving decision by deciding how much land and capital to purchase. An old agent receives income from renting land and capital to …rms and by selling land to the next generation.[8] The barrier, ¼; is modelled as discouraging young agent from investing in Solow capital as follow. For every unit of consumption good that a young agent give up, he can get one unit of Malthus capital by investing in Malthus sector, or $\frac{1}{¼}$ unit of Solow capital by investing in Solow sector. In equilibrium, ¼ will be the relative price of Solow capital goods to consumption goods. In my international income comparison, ¼ is the main parameter varies across countries.

For each generation t, a young agent's lifetime utility maximization problem is:

$$\underset{c_{1t+1};c_{2t+1}}{Max} \; u(c_{1t}) + \text{¯}u(c_{2t+1})$$

$$s:t: \; c_{1t} + x_{mt} + x_{st} + q_t l_{t+1} = w_t \tag{3}$$

$$c_{2t+1} = r_{kmt+1}x_{mt} + r_{kst+1}\frac{x_{st}}{¼} + (q_{t+1} + r_{Lt+1})l_{t+1} \tag{4}$$

where c_{2t+1} is an old agent's consumption in period t + 1, x_t and l_{t+1} are the young agent's investment in capital and land respectively, and q_t is the price of land in period t: Assume a CRRA utility function, $u(c) = \frac{c^{1¡¾}¡1}{1¡¾}$

[8] More generally, if capital did not depreciate completely, the old agent would also sell capital to next generation.

3.2 Competitive Equilibrium

Given $\pi; N_0; K_0$ and L, the total land of the economy, a competitive equilibrium for this economy consists of sequences for $t \geq 0$ of prices $\{q_t; w_t; r_{Kt}; r_{Lt}\}$; firm allocations, $\{K_{mt;}\ K_{st}; N_{mt}; N_{st}; L_{mt}; Y_{mt}; Y_{st}\}$; and household allocations, $\{c_{1t}; c_{2t+1}; x_{mt}; x_{st}; l_{t+1}\}$; such that:

1. Given the sequence of prices, household allocations solve the household's utility maximization problem
2. Given the sequence of prices, firm allocations solve the firm's profit maximization problem
3. All markets clear:

$$Y_{mt} + Y_{st} = N_t c_{1t} + N_{t-1} c_{2t} + N_t x_t$$

$$N_{mt} + N_{st} = N_t$$

$$K_{mt} + K_{st} = K_t$$

$$L_{mt} = L = N_{t-1} l_t$$

where N_t and K_t denotes the aggregate labor and capital in this economy.

4. The following laws of motion hold:

$$K_{mt+1} = N_t x_{mt}$$

$$K_{st+1} = N_t \frac{x_{st}}{\pi}$$

$$N_{t+1} = g(c_{1t}) N_t$$

3.3 Dynamics of the Model

The first issue I address concerns under what circumstances the two technologies will be operated. Because land is always supplied inelastically, it is easy to see that in equilibrium it is always profitable to operate the Malthus technology[9]. This, however, is not necessarily true for the Solow technology. However, I will show that for sufficiently high TFP in the Solow

[9] Suppose $r_{Lt}; r_{kt}$ and w_t are equilibrium prices such that the Malthus technology is not operated. Then since land can only be used in the Malthus technology, there is an excess supply of land, which implies that these prices cannot be an equilibrium.

technology, it will also be operated. When the Solow technology is not operated, I call this the Malthus-only economy. When the Solow technology is used, I say that the economy is in transition.

I now proceed as follows. First, I characterize the Malthus-only economy. Second, I find the condition for the Solow technology to be operated. Third, I describe the asymptotic behavior of the economy.

3.3.1 Malthus-only Economy

When the Solow technology is not used, the firm's optimization problem implies

$$r_{Kt} = \phi A_m \gamma_m^t K_{mt}^{\phi - 1} N_{mt}^{\mu} L_{mt}^{1-\mu-\phi} = \phi \frac{y_{mt}}{k_{mt}} \quad (5)$$

$$w_t = \mu A_m \gamma_m^t K_{mt}^{\phi} N_{mt}^{\mu-1} L_{mt}^{1-\mu-\phi} = \mu y_{mt} \quad (6)$$

$$r_{Lt} = (1 - \phi - \mu) A_m \gamma_m^t K_{mt}^{\phi} N_{mt}^{\mu} L_{mt}^{-\mu-\phi} = (1 - \phi - \mu) Y_{mt} \quad (7)$$

where y_{mt} and k_{mt} are the output and capital per worker in this Malthus-only economy.

One can look for a balanced growth path in the Malthus-only economy. To do this, I need to put some restrictions on the population growth function $g(\cdot)$: As the model is motivated to reproduce the fact that output per worker is stagnant before the industrial revolution, $g(c_{1t})$ is chosen such that the population growth rate is the same as the growth rate of output along the balanced growth path in the Malthus-only economy. I now show that the population growth function $g(\cdot)$ can be chosen to ensure this. Letting $\hat{y}_m$ and $\hat{k}_m$ be the stagnant levels of output and capital per worker respectively, the Malthus production function implies:

$$\hat{y}_m = A_m \gamma_m^t \hat{k}_m^{\phi} \left(\frac{L}{N_t}\right)^{1-\mu-\phi}$$

Thus, along the Malthus-only balanced growth path, output per worker is stagnant if

$$g(\hat{c}_{1m}) = \gamma_m^{1/(1-\phi-\mu)}$$

and

$$g(c_1) > g(c_{1m}) \quad \forall c_1 \in [c_{1m}, c_{1m} + \epsilon] \text{ where } \epsilon > 0$$

which I henceforth assume.

Under this restriction, equations (1), (5), (6) and (7) together with the market clearing conditions imply that, along a Malthus-only balanced growth path, aggregate output, capital, the price of land and the rental rate of land all grow at the same rate as does population. The wage rate, the rental rate of capital, output per capita, capital per capita, and consumption of the young and old are all constant.

3.3.2 Transition

Given N_0; I can choose K_0 such that the economy begins on the Malthus-only balanced growth path in period 0.[10] Then one can determine when the Solow technology will be used.

Proposition 1 Assume the economy is on the Malthus-only balanced growth path in period 0. The Solow technology is used when

$$t > \frac{\ln \frac{A_m}{A_s} + \ln B_0 + \theta \ln \pi}{\ln \gamma_s} \tag{8}$$

where $B_0 = (\frac{\phi}{\theta})^{\theta}(\frac{1}{1-\theta})^{1-\theta} N_0^{\mu-(1-\theta)} K_0^{\phi-\theta} L^{1-\mu-\phi}$

Proof. First note that if the Solow technology were to be used, profit maximization implies that the capital to labor ratio would be:

$$\frac{K_{st}}{N_{st}} = \frac{\theta w_t}{(1-\theta) r_t}$$

The profit function for a firm in the Solow sector in period t is:

$$\Psi(r_{kmt}; w_t) = \max_{K_{st}; N_{st}} A_s \gamma_s^t K_{st}^{\theta} N_{st}^{1-\theta} - r_{kt} K_{st} - w_t N_{st}$$

which is equivalent to:

$$\Psi(r_{kmt}; w_t) = \max_{N_{st}} A_s \gamma_s^t (\frac{\theta w_t}{(1-\theta) r_{kt}})^{\theta} N_{st} - \frac{w_t N_{st}}{1-\theta}$$

If both technologies were used, household utility maximization implies marginal rates of return to land and the two capitals must be the same for household to be indifferent:

$$\frac{q_{t+1} + r_{Lt+1}}{q_t} = \frac{r_{kst+1}}{\pi} = r_{kmt+1}$$

which implies

$$\Psi(r_{kmt}; w_t) = \max_{N_{st}} A_{st} (\frac{\theta w_t}{(1-\theta)\pi r_{kmt}})^{\theta} N_{st} - \frac{w_t N_{st}}{1-\theta}$$

Let $\hat{r}_m$ and $\hat{w}_m$ be the constant rental rate of capital and wage along the Malthus balanced growth path, it is profitable to start operating the Solow technology if $\Psi(\hat{r}_m; \hat{w}_m)$ is positive,

$$A_s \gamma_s^t > (\frac{\pi \hat{r}_m}{\theta})^{\theta} (\frac{\hat{w}_m}{1-\theta})^{1-\theta}$$

[10] One can solve for the capital per output ratio along the Malthus-only balanced growth path: $\frac{K_m}{Y_m} = \frac{1+\beta-\mu-\sqrt{(1+\beta-\mu)^2-4\mu\phi\beta(1+\beta)}}{2(1+\beta)\gamma_m^{1/(1-\mu-\phi)}} = v_m$ which implies $K_0 = [N_0^{\mu} L^{1-\phi-\mu} v_m]^{1/(1-\phi)}$

By assumption, the economy is on the Malthus-only balanced growth path in period 0,

$$A_s \gamma_s^t > \pi^{\theta} A_m B_0$$

It follows that the Solow technology is first used in period t_π^*, where t_π^* is the minimum integer that satisfies:

$$t > \frac{\ln \frac{A_m}{A_s} + \ln B_0 + \theta \ln \pi}{\ln \gamma_s}$$

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Once the Solow technology is used, the output per worker starts to grow and this is precisely when modern growth begins. In what follows I will refer to t_π^* as the turning point. Note that the Solow technology is used independently of the relative size of γ_m and γ_s: Since the right hand side of the equation (8) is just a constant, the Solow technology will be used at some point as long as $\gamma_s > 1$. Therefore, the model predicts that modern growth is inevitable in all countries, but that the time at which it begins depends on the level of barrier, relative level of total factor productivity of the two technologies, input shares and initial quality of land, labor force and capital. Note that the population growth function will not affect the turning point as it only takes effect after consumption exceeds the level along the Malthus balanced growth path, at which point the economy has already passed its turning point.

To characterize the equilibrium when both technologies are used, profit maximization conditions imply:

$$\frac{r_{kst}}{\pi} = \theta A_{st} K_{st}^{\theta-1} N_{st}^{1-\theta} = \phi A_{mt} K_{mt}^{\phi-1} N_{mt}^{\mu} L_{mt}^{1-\mu-\phi} = r_{kmt} \quad (9)$$

$$w_t = (1-\theta) A_{st} K_{st}^{\theta} N_{st}^{-\theta} = \mu A_{mt} K_{mt}^{\phi} N_{mt}^{\mu-1} L_{mt}^{1-\mu-\phi} \quad (10)$$

$$r_{Lt} = (1-\phi-\mu) A_{mt} K_{mt}^{\phi} N_{mt}^{\mu} L_{mt}^{-\mu-\phi} \quad (11)$$

Note that, as implied by (9) and (10), when both technologies are operated, marginal products are equated across technologies. This, together with the market clearing conditions, determines the fraction of labor and capital being allocated to each sector (see appendix 1).

3.3.3 Solow-only Economy

Assume now that the Solow technology has a higher growth rate of TFP ($\gamma_s \geq \gamma_m$). As already noted, this condition is not necessary for the Solow technology to be used. However, given $\gamma_s \geq \gamma_m$ and $g(c_{1t}) \to g$, one can show that the fraction of labor and capital devoted to the Malthus sector will converge to zero (see appendix 1).[11] Equation (11) then implies that the rental rate of land relative to the price of output will also converge to zero.[12] Hence, asymptotically, the economy behaves the same as a standard Solow growth economy and will converge to a balanced growth path. In particular, the firm's problem implies:

$$r_{kt} = \theta A_s \gamma_s^t K_{st}^{\theta-1} N_{st}^{1-\theta} = \theta \frac{\hat{y}_{\pi st}}{\hat{k}_{\pi st}} \tag{12}$$

$$w_t = (1-\theta) A_s \gamma_s^t K_{st}^{\theta} N_{st}^{-\theta} = (1-\theta)\hat{y}_{\pi st} \tag{13}$$

where $\hat{y}_{\pi st}$ and $\hat{k}_{\pi st}$ are the output and capital per worker along the asymptotic Solow balanced growth path for an economy with barrier π.

Along the asymptotic balanced growth path, output and capital per worker grow at the same constant rate. The Solow technology production function implies:

$$\hat{y}_{\pi st} = A_s \gamma_s^t \hat{k}_{\pi st}^{\theta}$$

Thus, both output and capital per worker grow at the rate $(\gamma_s^{1/(1-\theta)} - 1)$ along the asymptotic balanced growth path. Equations (2), (7), (12) and (13), together with the market clearing conditions then imply that output per worker, capital per worker, consumption per young and old, and wage all grow at the rate $(\gamma_s^{1/(1-\theta)} - 1)$.

The dynamics of the model, therefore, capture what the rich countries have experienced so far. The economy starts off with stagnant output per worker, modern growth then begin with increase in labor being allocated to the industrial sector. Finally, the economy converge to a balanced growth path.

4 International Income Differences

Having understood the dynamics of the model for one economy, next, I turn my attention to international income differences. To use the model to account for international income differences, I consider two identical economies except their levels of barriers.

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[11] Note that these are sufficient conditions. The fraction of labor allocated to the Malthus sector converges to zero in the computer simulations even when $\gamma_s < \gamma_m$ as long as $\gamma_s > 1$.

[12] A test for this result should be compared to the value of farmland in the data, as land in this model is only used for the Malthus sector. Hansen and Prescott (1999) documents that value of farmland relative to the value of GNP has declined from 88% in 1870 to 9 % in 1990.

4.1 Analytical Results

With the CRRA utility function, the ratio of output per worker for two economies along the asymptotic balanced growth paths can be derived.

Proposition 2 Assume $\gamma_s \geq \gamma_m$ and $g(.) \to g$. Consider two identical economies except their levels of barrier, let $\hat{y}_{\pi_i st}$ denote the output per worker along the asymptotic Solow-only balanced growth path for country i. Then

$$\frac{\hat{y}_{\pi_1 st}}{\hat{y}_{\pi_2 st}} = \left(\frac{\pi_2}{\pi_1}\right)^{\theta/(1-\theta)} \tag{14}$$

The proof consists essentially of showing that the ratio of the rental rate of Solow capital in the two economies is equal to $\frac{\pi_1}{\pi_2}$.(See appendix 2) This income ratio is the same as that of the standard one sector barrier model.[13]

The interesting point of this model, however, is its implications for different turning points as a result of different level of barriers. Proposition 1 implies two main analytical results.

Lemma 3 Industrial revolution is inevitable in both economies which means there is no absolute poverty trap.

[13] By standard barrier model, we mean the following:

There is a representative infinitely-lived agent with preferences

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $0 < \beta < 1$ and c_t is agent's consumption in period t.

The production function is:

$$Y_t = A\gamma^t K_t^{\theta} N_t^{1-\theta}$$

where γ is the total factor productivity growth, K_t, and N_t are capital and labor inputs at period t.

The law of motion for capital is:

$$K_{t+1} = (1-\delta)K_t + \frac{X_t}{\pi}$$

where δ is depreciation rate for capital, and X_t is aggregate investment at period t. Feasibility requires:

$$C_t + X_t = Y_t$$

where C_t is aggregate consumption at period t.

Lemma 4 The relationship between their turning points $t^{¤}_{¼_1}$ and $t^{¤}_{¼_2}$ is as follow:

$$t^{¤}_{¼_2} = µ\frac{\ln \left(\frac{¼_2}{¼_1} \right)}{\ln °_s} + t^{¤}_{¼_1} \qquad (15)$$

Thus, the turning point for a economy with a barrier ¼ times the other happens $µ\frac{\ln ¼}{\ln °_s}$ periods later. Note that a higher capital share for the Solow technology not only increases the income di¤erence along the Solow balanced growth path, but also increases the delay of the turning point for given value of $°_s$. The intuition is as follow. The turning point is reached when the Solow technology is used which implies the investment in Solow capital is positive. On the other hand, the e¤ect of the barrier is to reduce investment in Solow capital. As µ increases, the role of capital in the Solow technology becomes more important. Thus, given the TFP growth rate for the Solow technology, a given size of barrier causes longer delay in turning point when µ is increased.

4.2 Quantitative Results

4.2.1 Calibration and Computation

The economy with barrier equal to one is calibrated to match the development experience of England before 1800 and the postwar development experience of the industrialized countries. The year 1800 is taken as the time at which modern growth begins for the English economy, and will map to my endogenously determined variable $t^{¤}_1$: A period in this economy is interpreted to be 35 years in real time, which as noted earlier, justi…es the assumption that capital fully depreciates after one period. Agents in this economy will therefore live for 70 years and work for the …rst 35 years of their life-span. The postwar period will therefore be interpreted as $t^{¤} + 5$ in my model. The initial conditions, $A_m; A_s; L$ and N_0 are set to be one arbitrarily. Given N_0, K_0 is chosen such that the economy is initially on the Malthus-only economy.[14] As the calibration strategy is the same as Hansen and Prescott (1998), I will only brie‡y describe what they did. Basically, the population growth rate for the pre-1800 period in the UK is used to calibrate the productivity growth rate of the Malthus technology, and the relationship between the population growth rate and the GDP per capita for the industrial economies is used to calibrate the population growth function $g(:)$: Finally, the postwar economic development of the industrial economies is used to calibrate the productivity growth rate of the Solow

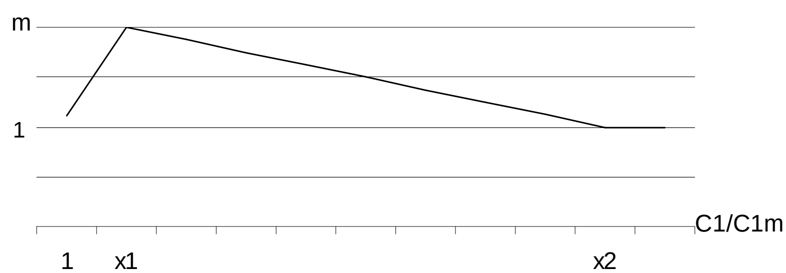
[14] See footnote (11).

technology and the discount factor. To summarize, the parameters values are:

¾	µ	¹	Á	$°_m$	$°_s$	¯
1	0.4	0.6	0.1	1.03	1.52	1

A general pattern in the long run population data presented in Lucas (1998) can be summarized by the following $g(c_1)$ function which is also similar to …gure II in Kremer (1993). The population growth function is calibrated to the following with $x_1 = 2$; $x_2 = 18$ and $m = 2$ where $m = 2$ corresponds to a 2% average annual population growth rate.

Figure 4: Population Growth Function



The main issue in solving for the equilibrium in this model is to …nd the equilibrium price of land. Given $L; N_0$ and K_0, the equilibrium price for land is solved using the shooting algorithm described in Hansen and Prescott (1999).

4.2.2 Results

With the same calibrated parameters, I then simulate another economy with barrier equal to 4 as a benchmark case. Jones (1994) studies the Summer and Hetson data set and …nds that the maximum relative machinery price to that of the US for the period 1960-85 is equal to 4. More recently, using the same data set, Restuccia and Urrutia (2000) construct a panel for the relative price of aggregate investment to consumption over the period 1960-85. They found that the relative price di¤erences across countries are large. In particular, the ratio between the average of the top and bottom …ve percent of the distribution of relative prices is 11.3 in 1960 and 6.5 in 1985. Therefore, I will also report the results of using higher values of ¼ later in the section.

Figures (5) to (9) summarize the quantitative results for the case of ¼ equal to 4. Figures (5) and (6) show that while the UK starts to allocate its labor and capital inputs into the Solow sector, the Solow technology is still inactive in the distorted economy. The …rst development fact is replicated in …gure (7). It demonstrates that output per worker is stagnant and starts to grow in 1800 for the UK and in 1870 for the distorted economy. The model predicts that in 1975, output per worker for the UK is 18 times higher than its level in 1765 while it's only 7 times higher for the distorted economy.

The model also captures the second development fact. Figure (8) plots the corresponding ratio of output per worker between the two countries. The model predicts that relative output per worker will increase from one to a maximum of 3.2 before declining to its Solow-only balanced growth path level of 2.5. This shape of the income di¤erence closely resembles the data in …gure (3). Moreover, a bigger income di¤erence is obtained (a 26 percent increase) relative to the balanced growth path level.

Figure 5: Fraction of Labor Allocated to the Malthus Sector

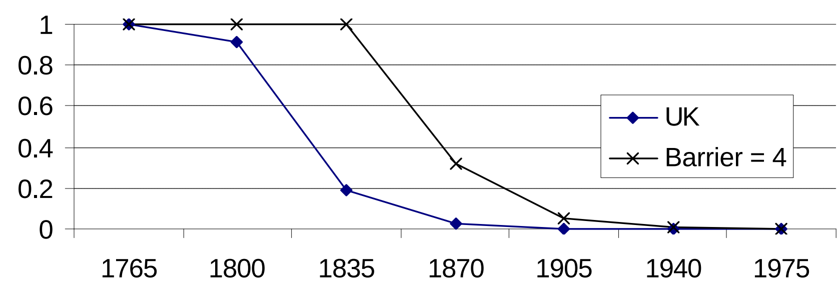


Figure 6: Fraction of Capital Allocated to the Malthus Sector

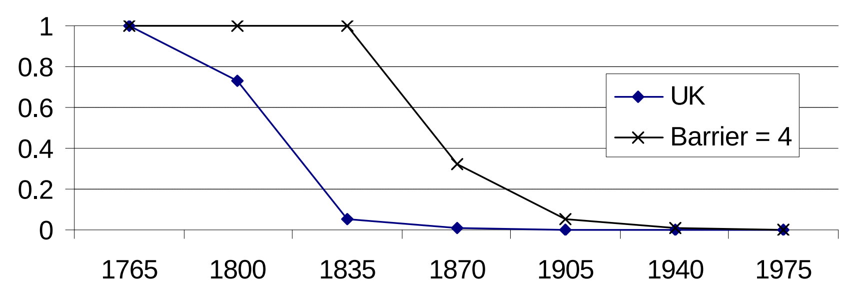


Figure 7: Normalized Output per Worker

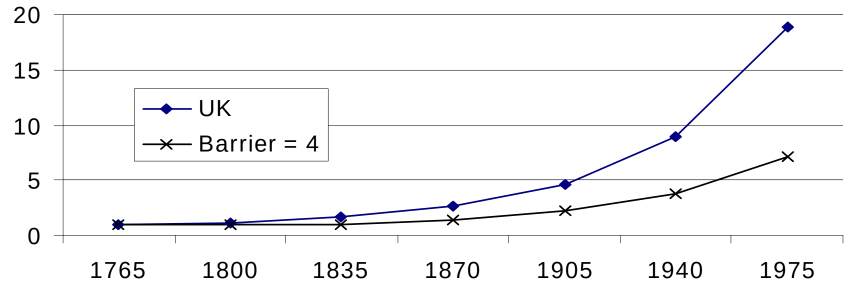


Figure 8: Relative Output per Worker

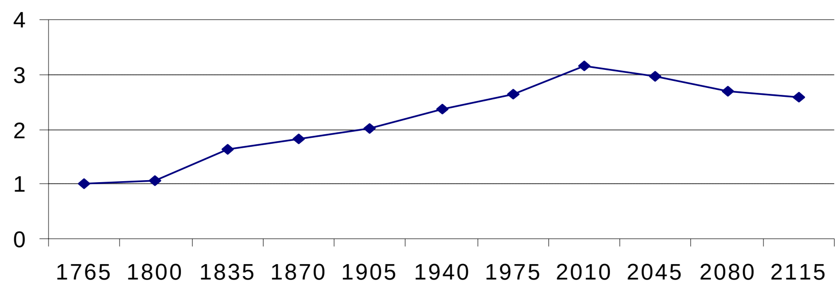


Figure 9: Growth rate of Output per Worker

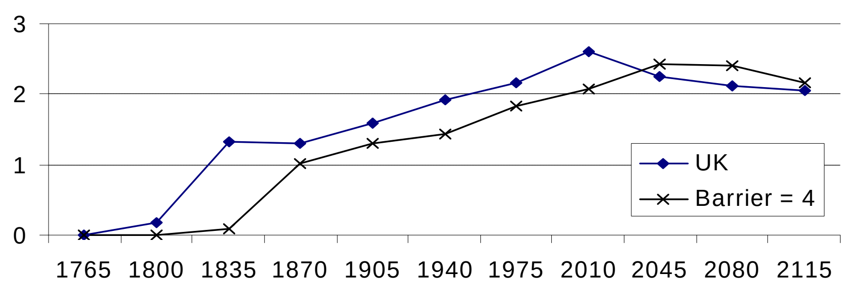


Figure (9) displays the average annual growth rate of output per worker for these two economies. It is interesting to note that the growth rate is not monotone as it is in the standard Solow growth model. In particular, the growth rate first increases and then decreases until it reaches its balanced growth path level. The increasing growth rate is precisely what motivated Romer (1986) to study increasing return to scale. It is interesting that this model can produce such an outcome with two constant return to scale technologies. Given this shape of growth rate, the inverted U-shape income difference in figure (8) becomes clear. As the barrier delay the turning point for the distorted economy, the growth rate for the undistorted economy starts to increase first and is therefore always higher than that of the distorted economy until it reaches its maximum. Thus, the income difference increases during this period. The maximum income difference is reached when the undistorted economy reaches its maximum growth rate. Then, the income difference starts to decrease when the growth rate of the undistorted economy decreases while the growth rate of the distorted economy is still increasing. When the growth rates for both economies decrease to their balanced growth path level, the income difference also decreases to its balanced growth path level.

Within the context of this model, income differences across countries can be decomposed into the differences along the balanced growth path and the timing of transition. Table (1) reports the results of allowing higher levels of barrier on these two components. It shows that as the level of barrier increases, the maximum income difference is increasingly higher than the level along the balanced growth path. This is partly due to the increased delay of modern growth. For example, when the level of barrier is increased from 8 to 16, the delay in modern growth increases from 2 to 3 periods. Thus, the percentage increased in the income difference rises from 33% to 40%.

Table 1: Relative output per worker with capital share equal to 0.4

Barrier	Delay	Ratio (BGP)	Maximum ratio	Percent Increased
2	1	1.6	1.8	18%
4	2	2.5	3.2	26%
8	2	4	5.3	33%
16	3	6.3	8.8	40%
32	4	10	14.1	41%
64	4	16	23	44%

To address the factor 30 income differences in the data, table (2) reports the corresponding combination of capita shares and barriers that can generate maximum differences of this magnitude. It shows that by considering different turning points, the required size of the barrier to generate a factor 30 income difference is reduced by 40 percent given a capital share equal to 0.4. The reduction holds true for other levels of capital share as well.

Table 2: Combinations of capital shares and barrier for factor 30 income differences

Capital Share	Delay	Barrier (BGP)	Barrier (Transition)	Percent Reduced
0.33	4	900	500	44%
0.4	4	164	100	39%
0.45	4	64	40	37%
0.5	4	30	18	40%
0.55	3	16	10	38%
0.6	3	10	6.5	35%

Finally, I consider the exercise of increasing capital shares holding the level of barrier fixed. Note that, to be consistent with my calibration procedure, γ_s and b have to be adjusted when θ is increased. Note, therefore, that increasing θ need not necessarily increase the delay in modern growth as noted in earlier section. Given the level of barrier equal to 4, table (3) illustrates that the maximum income difference is increasing higher than the level along the balanced growth path for capital's shares ranging from 0.33 to 0.6. In particular, when capital share is equal to 0.5, the delay in modern growth increases the income differences by 45%.

Table 3: Relative output per worker with barrier equal to 4

Capital Share	Delay	Ratio (BGP)	Maximum ratio	Pecent Increased
0.33	1	2.0	2.3	15%
0.4	2	2.5	3.2	26%
0.45	2	3.1	4.1	31%
0.5	2	4	5.8	45%
0.55	2	5.4	8.1	50%
0.6	2	8	13	63%

Alternatively, some have argued that some countries are poor because there are barriers that deter technology adoption which therefore lower the level of the level of total factor productivity in the production function. Thus, an alternative way to incorporate barriers into this model is through reducing the level of TFP in the Solow sector, i.e.

$$Y_s = \frac{A_{st}}{\pi_2} K_{st}^{\theta} N_{st}^{1-\theta}$$

At a general level, these two types of models are isomorphic, in the sense that I can choose barrier parameters such that they imply the same output per worker ratio along the balanced growth path for the two models. In particular, if

$$¼_2 = ¼_1^{µ}$$

where $¼_1$ and $¼_2$ are the barriers to capital accumulation and technology adoption respectively. Then, the delay in turning point implied by these two models are the same and same quantitative results apply.[15]

5 Population Pro…le

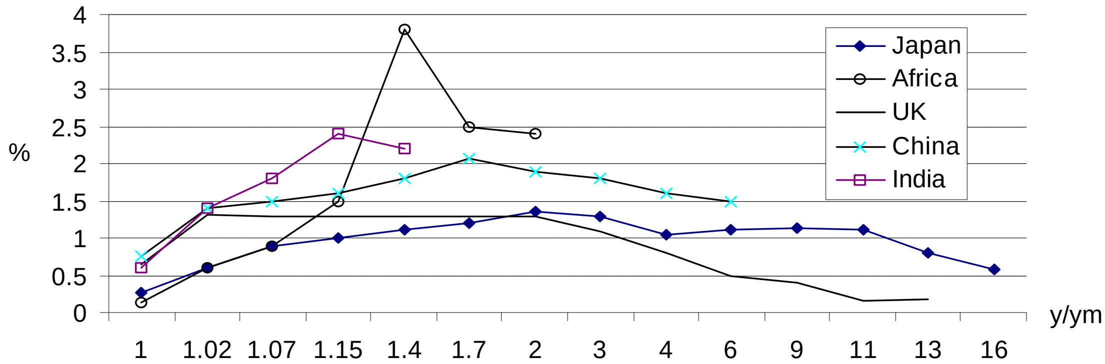
In this model, the shape of the population pro…le is calibrated using the relationship between population growth rate and GDP per capita for industrialized countries. This shape of population pro…le is summarized by three parameters, $x_1; x_2$ and m: It implies that population growth per period …rst increases linearly from its Malthusian level to m when consumption per young is x_1 times its Malthusian level, then decreases from m to one when consumption per young is x_2 times its Malthusian level. In the computer simulation, I have assumed the population pro…le is the same for both distorted and undistorted economies. My focus there is to study the e¤ect of barrier holding other things constant. I …nd that the income di¤erence between these two economies …rst increases, reaching a maximum equal to 3.2 (when barrier = 4), then decreases to its balanced growth path level. In the sensitivity analysis (appendix 3), I …nd that this result is sensitive to the change in m. In particular, it shows that when maximum population growth rate is increased from 2% to 3% (m = 2 to m = 2:8) for both economies, the maximum income di¤erence is increased from 3.2 to 3.5, a nearly 10% increase. In view of this, it is of interest to see what the data has to say for the population pro…les for a broader range of countries.

As shown in …gure (10), the data suggests that the shape of the population pro…le is similar across countries but the peaks are very di¤erent. More precisely, the late developers have much higher peaks than the early developers.

A question one may ask is why does the population growth rate increase during the early development stage of an economy? One may think that this is solely due to the decline in

[15] The rental rate of capital will be very di¤erent for these two models because in the …rst model the barrier increases the relative price of capital while in the other it only works through TFP. In particular, if $r_{ks}(¼_1)$ and $r_{ks}(¼_2)$ are the rental rate of capital under these two models, one can show $r_{ks}(¼_1)=¼_1 = r_{ks}(¼_2) = r_{ks}$ where r_{ks} satis…es $r_{ks}^{\frac{1}{¾}}$ ¡ $\frac{µg}{(1¡µ)} r_{ks}^{\frac{1}{¾}¡1}$ ¡ $\frac{µg}{1¡µ}^{-¡\frac{1}{¾}} = 0$ However, if one could rede…ne the e¤ective capital to be $¼k_s$, this di¤erence could be removed.

Figure 10: Average Annual Population Growth Rate

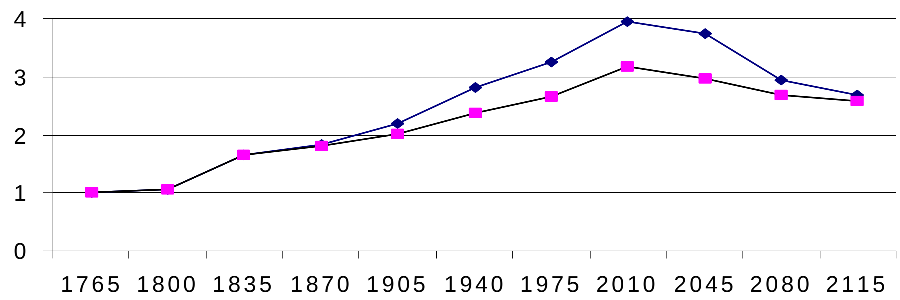


mortality rate. However, as Coale (1979) has documented for the case of Europe, and Dyson and Murphy (1985) have documented for the case of many other countries, the total fertility rate was also increasing during this period. This increase in the total fertility rate can be decomposed into changes in marriage behavior and changes in martial fertility. Wrigley and Scho…eld (1981) provide evidence that in England, the marriage rate increased and age at …rst marriage decreased during the initial stage of industrialization. Evidence from the demography literature suggests that marital fertility was increasing during the early development stage and that this increase is mainly due to changes in postpartum sexual abstinence and duration of breast-feeding.[16] In addition, Livi-Bacci (1997) shows that mortality levels at the early development stage in developing countries are more or less the same as European mortality rates. However, the fertility rates in developing countries considerably exceed European rates. Hence, available literature suggests that the di¤erence in the peaks of population pro…les in …gure (10) is due mainly to di¤erential fertility rates. Cultural, religious and policy di¤erences that a¤ect the fertility decision are important for understanding …gure (10) while understanding what accounts for these di¤erences is of interest in its own right, I will simply take these di¤erences as exogenous.

Given this di¤erence in population pro…les, I now ask what is the implication of the model if I allow the distorted economy to also have a population pro…le with a bigger m? Speci…cally, the maximum growth rate for the distorted economy (¼ = 4) is increased to 3% compare to the 2% of the undistorted economy. As argued before, this change in the maximum growth rate will not a¤ect the turning point. The result on income di¤erences is shown in …gure (11).

[16] See Dyson and Murphy (1985).

Figure 11: Relative Output per Worker (Di¤erent Population Pro…les)



The lower line in …gure (11) is the same as my …gure (8) which plots the income di¤erence between two economies that are identical except the level of their barriers. The upper line corresponds to case in which the maximum population growth rate in the distorted economy is higher than the undistorted economy. The maximum income di¤erence is increased from 3.2 to 4 which is a 25% increase. Moreover, the income di¤erences from 1940 to 2045 have all been increased by more than 20%. Therefore, the model con…rms our intuition that the di¤erence in population pro…les between the early and the later developer is important in accounting for their income di¤erences.

6 Applications

In this section, I consider two case studies to illustrate the strength of my model. These two cases are Japan and Africa. They demonstrate two interesting and important development facts: (1) the wide income disparities observed across countries and (2) the miracle experience of those who are initially among the bottom. In the case study of Africa, I show that size of the barrier that accounts for the delay in the turning point can also account for the long run pattern and range of income disparities. In the case study of Japan, I …nd that the model can generate both the miracle and subsequent slowdown in growth of income.

6.1 Application I: Africa

The long run data presented in Lucas (1998) shows that the UK's income is only two times higher than Africa in 1750 but increases to 14 times in 1990. Moreover, it also indicates that the turning point for Africa is around 1900 as suggested by Reynolds (1985). In this section, I address the following issue: can barriers that account for the di¤erence in the turning points between two economies also account for the whole path of their income disparities?

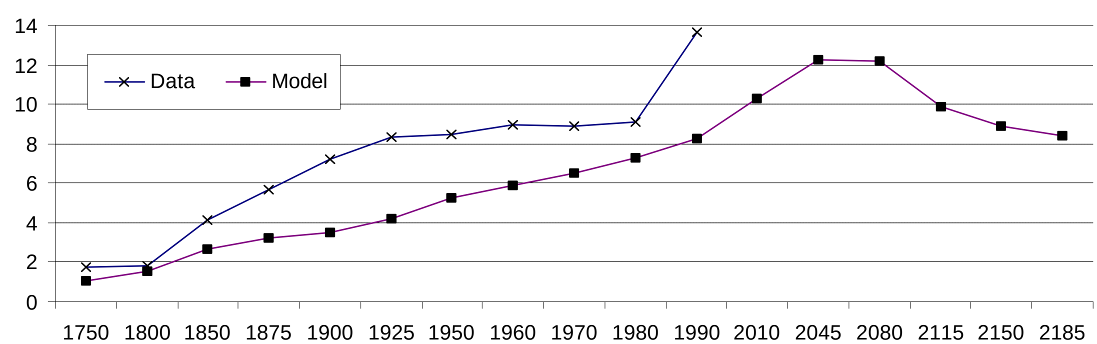
We learn from …gure (8) that this model can generate the pattern of increasing income di¤erence for a given level of barrier in Africa. More importantly, the model shows that the size of this barrier can be determined from the turning points of the UK and Africa as:

$$\pi = \gamma_s^{\frac{t^{*}_{Africa} - t^{*}_{UK}}{\theta}}$$

Assume that the values of all parameters except π are the same for the UK and the Africa. Speci…cally, $\theta = 0.5$ and $\gamma_s = 1.43$. The value of θ is larger than the value in section 4. This is in accordance with many authors, e.g. Parente and Prescott, who have argued that capital share should be higher than the canonical values because of the mismeasurement issue.[17] Then the barrier in Africa relative to the UK must be between 5 and 9 to generate a three-period delay. In what follows I assume that $\pi = 8$ for Africa.

With this size of the barrier, the model predicts that the UK's income relative to Africa will reach a maximum of 12 in 2045 and then decrease to its balanced growth path level of 8. I use linear interpolation between the periods in the model to compare the model with the data. Figure (12) shows that the model replicates the increasing trend of the income di¤erence between the UK and Africa and accounts for around 70% of the income di¤erence in 1970 and 1980. This results is impressive. It basically says if we know the di¤erence in the turning points, we can obtain a close replica of the world income distribution and make predictions about future income disparities.

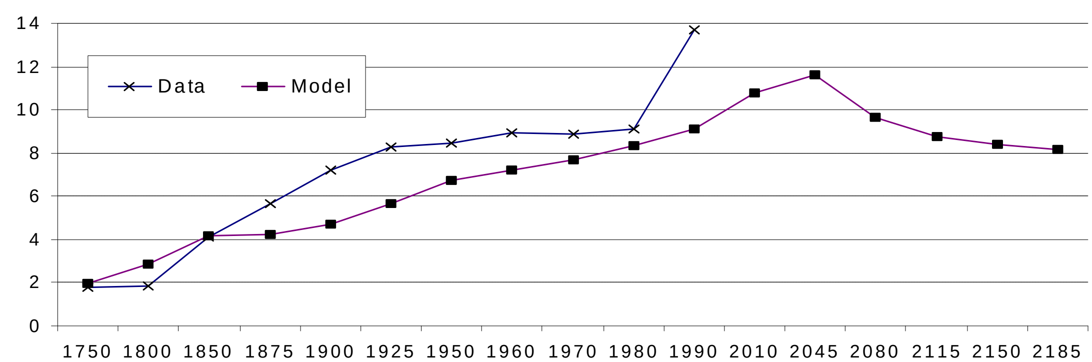
Figure 12: The UK's Income Relative to Africa



[17] The choice of θ will mainly a¤ect the level of π but not the main results.

The above calculation assumed no differences between the UK and Africa other than the barrier. In what follows I analyze how incorporating other sources of heterogeneity may improve on the model's predictions. The first element I consider is initial conditions. As mentioned earlier, even before the turning point of the UK, output per capita in the UK was almost double it's corresponding value in Africa. Assuming Africa and the UK the same capital to output ratio, input shares and exogenous Malthus technology, the ratio of their outputs per capita along the Malthus balanced growth path in the model is equal to $(\frac{l_{UK}}{l_{Africa}})^{(1-\mu-\phi)/(1-\phi)}$ where l denotes land per worker. I then choose the relative value of land per worker in the two economies to match the initial income difference.[18] Figure (13) shows the model's prediction for this scenario. With this adjustment in initial conditions, the model now accounts for around 90% of the income difference in 1970 and 1980.

Figure 13: The UK's Income Relative to Africa (Adjust Initial Condition)

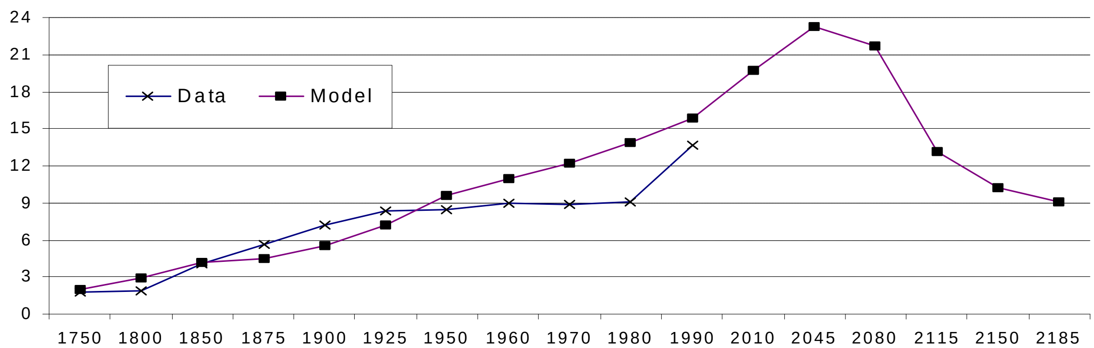


The second source of heterogeneity that I consider is differences in population profiles. As seen in figure (10) the population profiles for Africa and the UK are quite different. Specifically, Africa had a maximum population growth rate of 4% whereas the UK had a maximum level of 1.5%. In order to assess the impact of these differences, I set $m = 4$ for Africa according to the calibration for the population profile described in section 4.2.1. Figure (14) shows that the model implies a much higher income difference for the period 1960-1990. Moreover, the model predicts that if the level of the barrier in Africa relative to the UK remains unchanged, the UK's income relative to Africa will increase to 24 in 2045 before decreasing towards its balanced growth path level.

To sum up, the case of Africa illustrates some interesting predictions of the model. First,

[18] Of course it is not literally land per person that matters, but rather efficiency units of land per person from the perspective of the technology.

Figure 14: The UK's Income Relative to Africa (Adjust Population Profile)



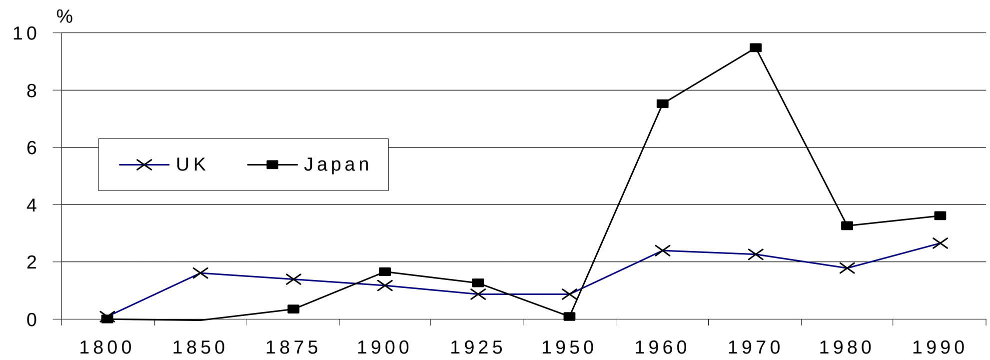
the barrier that accounts for the delay in the turning point for Africa relative to the UK can also account for 70% of the current income difference. Second, in contrast with the standard balanced growth path approach, the model predicts that income disparities between Africa and the UK will continue worsen even if relative barriers are unchanged. Last but not the least, the high peaked population profile in Africa implies the current income difference will be doubled in fifty years.

6.2 Application II: Japan

Japan is an interesting case study because of its distinctive miracle experience. Modern growth began in Japan around the end of 19th century, 100 years later than the UK. However, Japan's GDP per capita exceeds that of the UK in 1990, only 90 years after its period of modern growth began. This rapid rate of catch up can be seen in its soaring growth rate for the postwar period. As shown in figure (15), its GDP per capita growth rate is 7.5% for 1950-60 and 9.5% for 1960-70, compared to a 2.5% for the UK for 1950-70. Subsequently, however, the growth rate in Japan dropped to 3.5% for 1970-90.

Parente and Prescott (1994) provide one way to account for Japan's growth experience in the postwar period using the standard balanced growth path approach. They show that the miracle in Japan corresponds to a reduction in the size of the barrier in Japan to be less than that of the US, while the slowdown is associated with an increase in the size of barrier in Japan to be greater than that of the US. In other words, Japan is converging to three different balanced growth paths corresponding to the period before the miracle, during the miracle, and the slowdown after the miracle.

Figure 15: Per Capita GDP Growth Rate in the Data



Instead of studying this postwar development as an isolated experience, I look at it as a part of the long run economic development of Japan. I …nd that the slowdown of the Japanese economy after its miracle can be obtained without increasing its level of the barrier. The di¤erence in our results highlight the key di¤erence of my approach and the standard balanced growth path approach in accounting for international income di¤erence.

As Japan also experienced a three-period delay compared to the UK, I assume the barrier in Japan is equal to 8 along its Malthus-only balanced growth path. The historical record suggests two episodes that signi…cantly lowered barriers in Japan. They are the Meiji Restoration in 1868 which ended Shogunate Japan, and the postwar economic and institution reforms. According to Yamamura (1977), the new Meiji government adopted policy to encourage the absorption and dissemination of western technologies and skills, and help the growth of the private industries. In particular, the fraction of workers employed in industry by both private and public …rms have increased signi…cantly in 1907. Postwar Japan underwent many major reforms such as introducing numerous tax-exemptions or tax-reliefs for investment; industry-…nancing program; allowing the purchase of new foreign patents; dissolving the zaibatsu system[19] and the deconcentration of many zaibatsu subsidiaries; and trade liberalization.[20] According to Ohkawa and Rosovsky (1963), these reforms led to a steep rise in the rate of private investment, a rapid decline in agriculture sector, an acceleration of the introduction new technologies, and a 38% productivity growth in the manufacturing sector.

[19] The "zaibatsu" is referred to a relatively small number of family-dominated company systems holding assets through large segments of the Japanese economy. These groups had become a major force in Japanese economic and political life before the World War II.

[20] There are many sources for these reforms. For examples, Tsuru (1961), Ohkawa and Rosovsky (1963) and Rotwein (1964).

Figure 16: Relative Prices in Japan

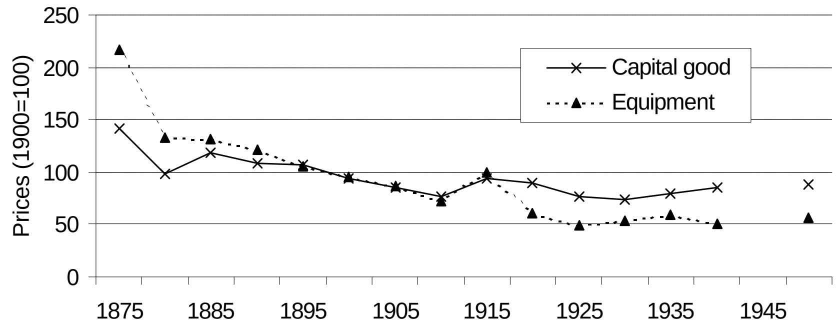
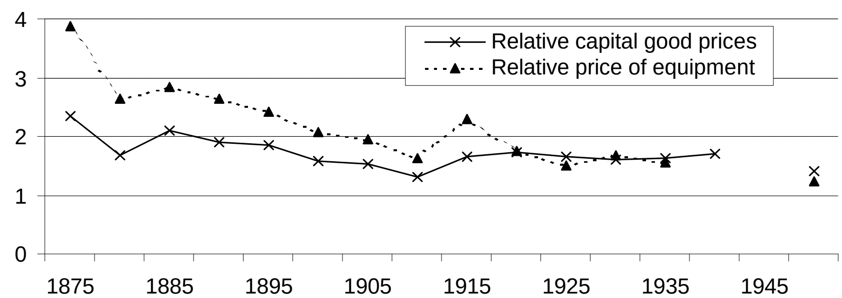


Figure 17: Ratio of Relative Prices (Japan/UK)



These reductions in barriers are also consistent with the data reported in Collins and Williamson (1999) and Jones (1994). The plots in ...gures (16) and (17) are based on the data in tables (1a) – (2b) of Collins and Williamson (1999). Figure (16) illustrates two consistent facts for Japan during the period 1750 – 1950. First, the price of capital goods relative to consumer goods decreased drastically between 1875/79 to 1880/84 and remained fairly stable since then. Second, the price of equipment relative to consumer goods fell by 63 percent between 1875/79 to 1880/84 and continued to fall steadily during the period 1880 – 1950. Figure (17) plots the ratio of these relative prices between Japan and UK. The ratio of the relative price of capital goods in Japan to the UK dropped drastically between 1875/79 and 1880/84 and remained fairly stable until 1945. Similarly, the relative price of equipment in Japan to the UK dropped by 32% between 1875/79 and 1880/84, then fell steadily to a ratio of 2 in 1910, and remained fairly stable until 1945. This evidence supports the argument that the barrier in Japan was reduced after the Meiji Restoration. According to Collins and Williamson, the relative price of equipment in Japan is 1.9 times that of the US in 1950. For the period 1960-1985, ...gures in Jones (1994) demonstrate that the relative price of equipment in Japan relative to the UK is equal to 0.6. Therefore, the data also supports a further reduction in barrier for the postwar period.

In view of these facts, we carry out the following calculation to simulate the experiences of Japan. The size of barrier is set equal to 8 initially. In 1905 the size of the barrier is reduced by half. As ...gure (17) shows the ratio of relative equipment price between UK and Japan is reduced by half in 1905. While I am not limiting my interpretation of the barrier to this one dimension, I think this magnitude of reduction is at least a useful benchmark. Finally, consistent with the evidence in the Jones (1994), I assume the barrier is reduced to 0.6 for the postwar period. These changes in the size of barrier are assumed to be unexpected for the household.

Figure 18: The UK's Income Relative to Japan

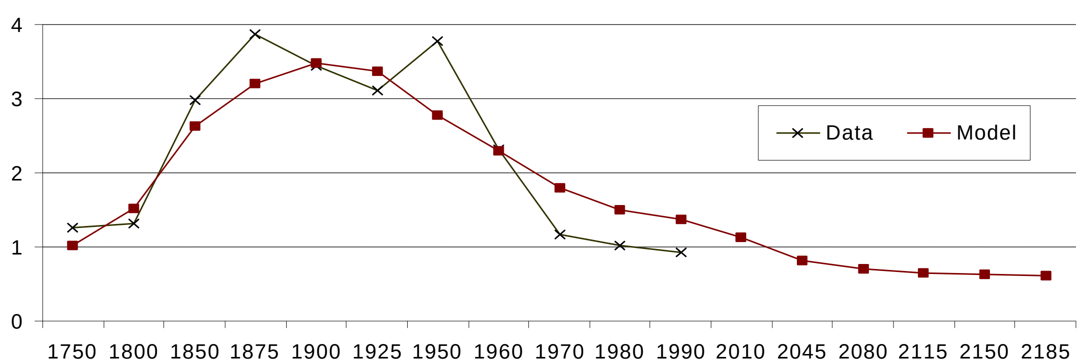
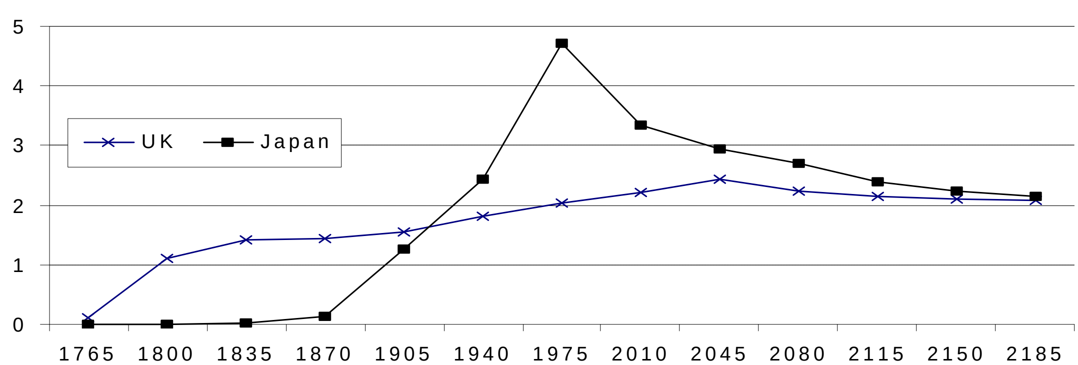


Figure (18) shows the model's predictions. As seen, the model predicts that Japan will eventually catch up with the UK. There are two interesting points to note in ...gure (18). Firstly, the income di¤erence for the period 1875 to 1940 is fairly stable even if the barrier is reduced by half in 1905. This is because the model predicts an inverted U-shape (see ...gure (8)) for the time path of income di¤erences for a given level of ¼: Therefore, if the barrier is reduced before the maximum income di¤erence is reached, it will only cause the income di¤erence to increase at a smaller rate but not necessarily reduce it. This is an interesting property of the model and is consistent with the ...nding of Restuccia and Urrutia (2000) that the range of the relative price of investment is decreasing for the period 1960-85 while the magnitude of income di¤erences is not.

Secondly, it replicates the slowdown in GDP per capita growth rate in the data. To illustrate this point more precisely, I plot the average annual growth rate of output per capita in the model in …gure (19). It displays both the miracle and the slowdown.[21] As mentioned earlier, for these to happen in the standard barrier model, the size of barrier has to be decreased and then increased. In this model, we have seen that both the miracle and the slowdown can be obtained without increasing the level of barrier. This di¤erence implies that two di¤erent interpretations for the slowdown in Japanese economy. According to the standard barrier model, Japan was converging to a di¤erent balanced growth path with a higher level of barrier compared to the balanced growth path where miracle happened. According to my model, however, both the slowdown and miracle are along the same development path. Intuitively, when barrier is reduced, the economy jumps to a di¤erent development path. Thus a signi…cant enough reduction will generate a miracle. After the jump, the economy will grow according to the new development path.

Figure 19: Per Capita Output Growth Rate in the Model



The model is consistent with the interpretation that the Japanese miracle is a result of the economic reforms after the Meiji Restoration and World War II. In particular, the values of ¼ I choose have a clear interpretation in Japanese economic history. In contrast to Parente and Prescott (1994), I do not need to resort to an increase in the level of the barrier to generate the economic slowdown in Japan.

I close this section with a remark. Reynolds (1985) documents that turning points for many countries have been associated with major political reform. In the context of this model, political reform (a permanent reduction in the level of barrier) is not necessary to generate a turning point, as shown in proposition 1. However, it can speed up the process of shifting

[21] The removal of barrier can only partly replicate the postwar miracle of Japan as the destruction of the capital during the war is also an important factor.

input from the Malthus sector to the Solow sector. Moreover, as in the standard barrier model, it moves the economy to a higher balanced growth path. As shown by Japan's example, the political reform increases the growth rate significantly.

7 Conclusion

Recent papers have interpreted the current factor 30 income differences as a balanced growth path result. This paper takes another perspective. I argue that this magnitude of income differences is mainly due to the fact that poor countries have only recently entered the modern growth regime. Taking this into account, this model generates a much larger income difference compare to the balanced growth path approach for given level of barriers. It also replicates the three long run development facts observed in the data, namely, (1) countries have all experienced a period of stagnation and subsequently enter the modern growth regime at different points in time, (2) the long run income differences exhibit an inverse U-shaped, and (3) countries with similar turning points can have dramatically different development experiences. I find interesting results in the two case studies. The case of Africa demonstrates that the barrier that can account for the delay in the turning point can also account for the path of the income differences. The case of Japan illustrates how the model can generate the postwar miracle and slowdown along the same development path.

There still remain other interesting questions. We have seen from figure (10) that population profiles are very different across countries, especially between those developed earlier and those developed recently. While the population profiles of these countries do not affect their turning points, I have shown in section 5 that they have significant effects on the path of the income differences between these countries. In this model, the difference in the population profiles across countries are treated as exogenous while endogenizing this difference is certainly an interesting step. Doepke (1999) endogenizes the fertility dynamics for the Hansen-Prescott model I consider here. By assuming countries have the same population growth rate at their common turning point, he finds that differences in child labor restrictions and education subsidies can account for the differences in the speed of the fertility decline. However, the difference in the peak of the population growth rate in figure (10) cannot be addressed in his model.

This model abstracts from the fact that home production (non-market sector) plays an important role in the developing countries. Parente, Rogerson and Wright (2000) extend the standard barrier model to include home production. They find that the measured income disparity along the balanced growth path increases significantly if market and home produced goods are close substitutes and the capital share of the home production technology is small.

Incorporating home production in this model is expected to work in a similar way as in their model.

Another interesting extension is to allow life expectancy to vary with income. Life expectancy is assumed to be constant in this model, while in the data, there is significant improvement in life expectancy over time for every age level. It will be interesting to incorporate the idea that the improvement in life expectancy at young age encourages investment in human capital while that of the old age encourages investment in physical capital. This intuition is supported by the finding in McGrattan and Schmitz (1998). They find strong correlation between GDP per worker and capital to output ratio, GDP per worker and primary school enrollment, and GDP per worker and secondary school enrollment in 1985 using data from Summer and Hetson (1991) and Barro and Lee (1993). When life expectancy is assumed to depend on income, the model can generate differences in life expectancy, education and investment between the poor and the rich countries using barrier as the only source of heterogeneity between them.

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Appendix 1

Given $q_{t¡1}; N_t, L$, and H_t ´ $N_{t¡1}(w_{t¡1}$ ¡ $c_{1t¡1})$ ¡ $q_{t¡1}L$ and the fact that Solow technology is used, the fraction of labor and capital input allocated to each sectors can be determined as follow. Use …rm pro…t maximization condition and the household utility maximization conditions, I have

$$\frac{¹K_{mt}}{ÁN_{mt}} = \frac{¼(1 ¡ µ)K_{st}}{µN_{st}}$$

Now use the market clearing conditions; K_{mt} and K_{st} can be determined as functions of H_t and m_t ´ $\frac{N_{mt}}{N_t}$

$$K_{mt} = \frac{Á(1 ¡ µ)H_t m_t}{µ¹(1 ¡ m_t) + (1 ¡ µ)Ám_t}$$

$$K_{st} = \frac{µ¹(H_t=¼)(1 ¡ m_t)}{µ¹(1 ¡ m_t) + (1 ¡ µ)Ám_t}$$

Thus, the fraction of labor allocated to the Malthus sector satis…es the following implicit function:

$$g(m_t; H_t; N_t) ´ ¼r_{kmt}(m_t; H_t; N_t) ¡ r_{kst}(m_t; H_t; N_t) = 0$$

It is easy to show that $g(:)$ has the same sign as $f(m_t; H_t; N_t)$ which is de…ned as follows:

$$f(m_t) = ¼^µ D L^{1¡¹¡Á}[µ¹ ¡ (µ¹ ¡ (1 ¡ µ)Á)m_t]^{µ¡Á} D ¡ \frac{A_s}{A_m}(\frac{°_s}{°_m})^t m_t^{1¡Á¡¹} H_t^{µ¡Á}$$

where $D = Á^{Á}¹^{1¡µ}µ^{¡µ}(1 ¡ µ)^{Á¡1}$and the function $f(:)$ has the following properties:

$$f(0) = ¼^µ D L^{1¡¹¡Á}(µ¹)^{µ¡Á} N_t^{¹¡(1¡µ)} > 0$$

$$f0(m_t) = ¼^µ(µ ¡ Á)(¡¹µ + Á(1 ¡ µ))A_{1t} ¡ (1 ¡ Á ¡ ¹)A_{2t} < 0$$

where $A_{1t} > 0$ and $A_{2t} > 0$ if $m_t > 0$; and

$$f(1) = ¼^µ D L^{1¡¹¡Á}[(1 ¡ µ)Á]^{µ¡Á} N_t^{¹¡(1¡µ)} ¡ \frac{A_s}{A_m}(\frac{°_s}{°_m})^t H_t^{µ¡Á} < 0$$

The last property follows from the condition for the Solow technology to be used. One set of su¢cient condition for $f(:)$ to be strictly decreasing are:

1. Capital share in Solow technology is at least as big as that of Malthus technology $(µ > Á)$
2. Labor share in Malthus technology is at least as big as that of Solow technology $(¹ ¸ 1 ¡ µ)$
3. Land share in Malthus technology is greater than zero. $(1 ¡ Á ¡ ¹ > 0)$

Given that $f(0)$ is strictly positive, $f(1) < 0$ and $f(:)$ is strictly decreasing, there is an unique $m_t < 1$ solves $f(m_t) = 0$. Note that if $°_s ¸ °_m$ and $g(c_{1t})$ ¡! g, I must have m_t converge to zero.

Appendix 2.

This appendix shows that relative output per worker converge to $\left(\frac{\pi_1}{\pi_2}\right)^{\theta/(1-\theta)}$. With the CRRA utility function $u(.) = \frac{c^{1-\sigma}-1}{1-\sigma}$, the optimal solutions are interior solutions and satisfy

$$\frac{1}{\beta}\left(\frac{c_{2t+1}}{c_{1t}}\right)^{\sigma} \equiv \frac{u'(c_{1t})}{\beta u'(c_{2t+1})} = \frac{(q_{t+1} + r_{Lt+1})}{q_t} = \frac{r_{kst+1}}{\pi}$$

$$\frac{q_{t+1} + r_{Lt+1}}{q_t} = \frac{r_{kst+1}}{\pi}$$

Using the budget constraints, one can solve:

$$c_{1t} = \frac{w}{1 + \beta^{-1/\sigma}(\frac{r_{kst+1}}{\pi})^{1/\sigma - 1}}$$

$$c_{2t+1} = (\frac{\beta r_{kt+1}}{\pi})^{1/\sigma} w/[1 + \beta^{-1/\sigma}(\frac{r_{kst+1}}{\pi})^{1/\sigma - 1}]$$

As argued before, the price of land converges to zero as the economy converges to the Solow balanced growth path. From the budget constraint,

$$x_t = \frac{c_{2t+1}}{(r_{kst+1}/\pi)}$$

Let γ be the asymptotic growth rate of capital per worker along the Solow balanced growth path. The total value of investment x_t can also be derived from the firm's profit maximization condition and the condition $g(c_{1t}) \to g$,

$$x_t = \frac{\theta w_t}{(1-\theta)(r_{kst+1}/\pi)}\gamma g$$

Let $\hat{r}_{k\pi}, \hat{y}_{\pi st}, \hat{k}_{\pi t}$ and $\hat{w}_{\pi t}$ be the constant rental rate of capital, output per worker, capital per worker and wage along the balanced growth path respectively for a country with barrier π,

$$(\frac{\hat{r}_k}{\pi})^{1/\sigma} - \frac{\theta\gamma g}{1-\theta}(\frac{\hat{r}_k}{\pi})^{1/\sigma - 1} - \frac{\theta\gamma g}{1-\theta}\beta^{-\frac{1}{\sigma}} = 0$$

which implies $\frac{\hat{r}_{k\pi}}{\pi}$ is independent of π. Thus

$$\frac{\hat{r}_{k\pi_1}}{\hat{r}_{k\pi_2}} = \frac{\pi_1}{\pi_2}$$

Now use the production function, firm profit maximization condition, I have

$$\hat{y}_{\pi st} = A_s \gamma_s^t (\frac{\theta}{1-\theta}\frac{\hat{w}_{\pi t}}{\hat{r}_{k\pi}})^{\theta}$$

Finally use the condition that $\hat{w}_{st} = (1-\theta)\hat{y}_{st}$, I have

$$\frac{\hat{y}_{\pi_1 st}}{\hat{y}_{\pi_2 st}} = \left(\frac{\pi_2}{\pi_1}\right)^{\theta/(1-\theta)}$$

Appendix 3. Sensitivity Analysis

I examine the robustness of the shape of …gure (8) with respect to changes in parameters of the model. These parameters are initial population, initial capital stock, quality of land, initial TFP levels for the Malthus and Solow technologies, input shares for Malthus technology, population growth rate along the Malthus balanced growth path, and the population growth function $g(c_1)$.

Initial Conditions Figures (20) and (21) demonstrate that doubling initial population, initial capital, quality of land and $\frac{A_m}{A_s}$ all have insigni…cant e¤ects on the shape of the income di¤erence curve.

Input Shares of the Malthus Technology Conditioning on the fact that the input shares does not a¤ect the turning points, changing both the capital and land shares of the Malthus technology have an insigni…cant e¤ect on the income di¤erence. This is not surprising given …gure (5); the economy is almost in a Solow-only economy three periods after modern growth begins. Therefore input shares of the Malthus technology are not important in determining the income di¤erence along the transition path.

Population Growth Rate Along the Malthus Balanced Growth Path Doubling the population growth rate along the Malthus balanced growth path from 0.3 percent to 0.6 percent will increase °$_m$ from 1.03 to 1.07. This will not have an e¤ect on the turning point according to the equation (8). Moreover, °$_m$ does not enter into $g(c_1)$ when consumption is more than double its Malthus steady level. And, …gure (7) illustrates that consumption is doubled two periods after the transition. Therefore, °$_m$ is insigni…cant in determining the income di¤erence once modern growth begins.

Population Dynamics I check the robustness of shape of income di¤erence by varying x_1, x_2 and m. Figures (22) and (23) show that both x1 and x2 have an insigni…cant e¤ect on the maximum income di¤erence but m has a signi…cant e¤ect. By increasing the maximum annual population growth rate from 2% to 3% ($m = 2$ to $m = 2{:}81$), the maximum income di¤erence is increased from 3.2 to 3.5 (a nearly 10% increase).

Figure 20:

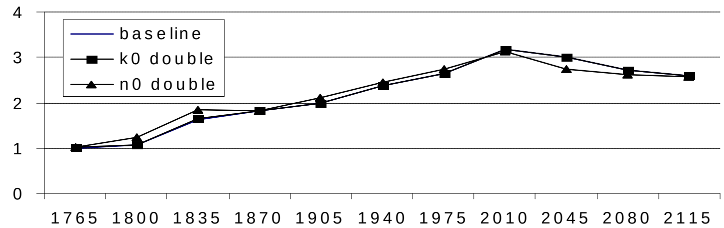

Figure 21:

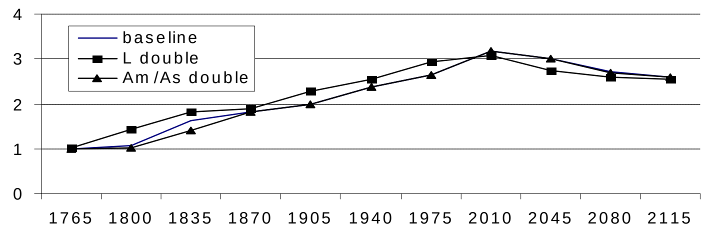

Figure 22:

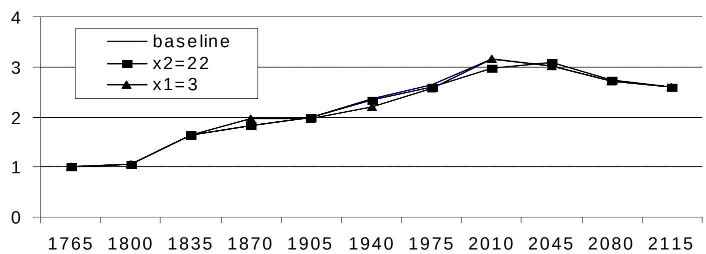

Figure 23: