

## *Rotating Savings and Credit Associations as Insurance*

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### Abstract:

Recent theoretical research on rotating savings and credit associations (Roscas) suggests that identical individuals prefer a random to a bidding Rosca when participants save for a lumpy durable or an investment good. Here, in contrast, under the assumption that participants are risk averse and that their incomes are stochastic and independent, it is shown that a random Rosca is not advantageous, while participation in a bidding Rosca improves ex ante expected utility if temporal risk aversion is less pronounced than static risk aversion. When information on individual incomes is private, fixed contributions to a bidding Rosca help to mitigate the problem of information asymmetries. When information on incomes is public, a lack of enforceability of variable contributions may explain the existence of Roscas instead of more efficient insurance arrangements.

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## 1 Introduction

The rotating savings and credit association (Rosca), an informal financial institution observed around the world, has attracted considerable theoretical and empirical research recently. Roscas are popular among high as well as low income households<sup>1</sup> and flourish in economic settings where formal financial institutions seem to fail to meet the needs of a large fraction of the population. In general terms, a Rosca can be defined as ‘a voluntary grouping of individuals who agree to contribute financially at each of a set of uniformly-spaced dates towards the creation of a fund, which will then be allotted in accordance with some prearranged principle to each member of the group in turn’ (Calomiris and Rajarman, 1998). Once a member has received a fund, also called pot, she is excluded from the allotment of future pots until the Rosca ends. In a so-called random Rosca, a lot determines each period’s ‘winner’ of the pot. In a bidding Rosca, an auction is staged among the members who have not yet received a pot. The highest bid wins the pot and the amount the winner pays is distributed among the members or added to future pots. In a third, empirically relevant, allocation mechanism, the decision on each period’s allocation of the pot is left to the Rosca organizer.<sup>2</sup>

The name suggests that Roscas serve as a financial intermediary by transforming the bundled savings of a group into what might be considered a loan to one Rosca participant in each period. The theoretical literature on Roscas has entirely focussed on participants with non-stochastic incomes. Kuo (1993) analyses bidding Roscas when individuals differ in that they discount future consumption with distinct discount factors. These are drawn from a common distribution and are private knowledge. Moreover, in every period each participant is

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<sup>1</sup> While Levenson and Besley (1996) find that participation is highest among high income households in Taiwan, Handa and Kirton (1999) report that, in Jamaica, low income households are most likely to join a Rosca.

<sup>2</sup> In Handa and Kirton’s (1999) sample, 53 percent of the Roscas operated in this way.

assigned a new discount factor. Assuming that all participants share the same beliefs about the distribution of other participants' discount factors, the author derives Bayes-Nash equilibrium bidding strategies. In Kovsted and Lyk-Jensen (1999), each participant can engage in an investment project and has limited access to outside credit. The revenues of the projects differ among participants. The revenue yielded by each participant's project is his private information, but all participants have the same beliefs about the distribution of revenues of the other participants' projects. Deriving Bayes-Nash equilibrium bidding strategies in a bidding Rosca, the authors find that when either outside credit is not too costly, or the distribution of revenues is sufficiently widely dispersed, a bidding Rosca is preferred to a random Rosca. Besley et al. (1993, 1995) assume that participants do not have access to outside credit and join a Rosca to finance a durable good whose costs require saving for more than one-period. If participants have identical preferences and incomes, a random Rosca is preferred to the bidding arrangement. If, however, participants are sufficiently heterogeneous, a bidding Rosca can be preferred to a random Rosca. In both of the former two papers, the bidding arrangement provides a mechanism to allocate pots earlier to participants who have a higher willingness to pay and can therefore be advantageous if participants are not identical.

In many economic settings where Roscas are found, individuals are exposed to both idiosyncratic and aggregate risks. Examples are farmers' uncertainty about harvests, employment uncertainty among casual labourers and individual illness when no health insurance is available. There is a body of empirical evidence that, when participants are exposed to risk, Roscas can serve as an insurance mechanism (Calomiris and Rajaraman, 1998). In the approaches taken by Besley et al. (1993) and Kovsted and Lyk-Jensen (1999), the outcome of the bidding Rosca remains unaltered no matter if the auctions for all future pots are staged at the beginning of the Rosca or if the auction for each period's pot takes place in

that same period. Calomiris and Rajaraman (1998), however, find that, except for one case<sup>3</sup>, all of the empirical literature reports Rosca arrangements where bidding is concurrent with the allocation of pots. For an actual Rosca in an Indian city, they calculate the implicit interest rate for the funds each participant received from the Rosca. For each period, this rate depends on the remaining duration of the Rosca as well as the value of the winning bid. In contrast to the predictions of the models of Besley et al. (1993) and Kovsted and Lyk-Jensen (1999), they find that winning bids do not decrease steadily from period to period and that the implicit rate of interest fluctuates significantly without any obvious trend. Calomiris and Rajaraman (1998) conclude that, at least for their sample Rosca, deterministic models do not capture the essential features. Instead, they stress the role of Roscas as an insurance mechanism by allocating each period's pot to the bidder who has suffered the most severe shock. Moreover, if several members suffer a severe shock in the same period, bidding compensates those who do not win said period's pot. Besley et al. (1993) argue that Roscas are not suited for insuring against risk because the fund can be obtained only once while shocks might occur several times during the duration of the Rosca. Empirical evidence, however, shows that many individuals are members of several Roscas or hold more than one share in the same Rosca, thus being entitled to bid for more than one pot.<sup>4</sup> Of course, Roscas cannot effectively insure against aggregate shocks when participants belong to an economically and socially homogenous group like small farmers in a village whose harvests depend on the weather to a large extent. But even there, as Townsend's (1994) results suggest, a variety of mechanisms appear to be at work in providing substantial insurance against idiosyncratic risks like illness or death of farm animals.

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<sup>3</sup> This is Campbell and Ahn (1962) for Korea.

<sup>4</sup> Handa and Kirton (1999) report that, in their sample of 1000 Jamaican households, respondents joined on average 1.4 Roscas during the year prior to the interview and held 1.3 shares in any Rosca they joined.

The aim of this study is twofold: I analyze how bidding Roscas function under the following assumptions. Participants are risk-averse and use funds from the Rosca entirely for consumption, each participant's income being stochastic. For most of the analysis, I assume, moreover, that participants cannot observe other participants' incomes, but all share the same beliefs about the distribution from which the incomes are drawn. In addition, the case of public information on incomes is also briefly considered. There are neither credit nor savings opportunities outside the Rosca. Second, with the results thus derived, it is shown how Roscas can partly solve the problem of insurance against idiosyncratic risks when no formal insurance is available.

## 2 Optimal Ex-ante Insurance among Individuals

As point of departure for the analysis of insurance in the absence of market institutions, this section outlines the problem of optimal insurance when individual income is private knowledge. To keep the analysis simple, assume that two identical individuals in period zero are confronted with stochastic incomes  $Y_i$ ,  $i = 1, 2$  in period one which are drawn from a distribution  $F$  on some domain  $I$ . There are neither savings nor borrowing opportunities. Each agent evaluates period one consumption with a Bernoulli utility function  $v(y_i)$  where  $v' > 0$  and  $v'' < 0$  for all  $y_i$ .<sup>5</sup> First assume that the realizations  $y_1$  and  $y_2$  are observed by both agents and that they can make binding commitments, i.e., in period zero, they agree on a menu that assigns a (possibly negative) transfer from agent  $j$  to agent  $i$ ,  $t(y_i, y_j)$  say, to each possible pair of period one incomes. Restricting attention to symmetric rules for  $t$ , i.e.  $t(y_j, y_i) = -t(y_i, y_j)$ , the task is to maximize ex ante expected utility given by

$$E[v(Y_j - t(Y_j, Y_i))] = E[v(Y_i + t(Y_i, Y_j))] = \int_I \int_I v(y_i + t(y_i, y_j)) dF(y_i) dF(y_j), \quad i, j = 1, 2, \quad i \neq j.$$

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<sup>5</sup> Throughout the paper, random variables are denoted by capital letters, while lower case letters represent particular values random variables assume.

It is easily shown that under these assumptions  $t^*(y_i, y_j) = (y_j - y_i)/2$  and thus  $E[v(Y_i + t^*(Y_i, Y_j))] = E[v((Y_i + Y_j)/2)]$  constitutes the optimal arrangement.

The question is whether this ex ante optimum can also be achieved when each agent only observes his own income but both agents know  $F$ . In the terminology of mechanism design theory, we now seek a mechanism that implements  $t^*$  in Bayes-Nash strategies. In period zero the agents agree on some rule  $g(b_i, b_j)$  that determines the transfer  $t$  in period one where  $b_i$  is an announcement  $i$  makes after observing  $y_i$ . Obviously,  $b$  is a strategic variable and depends on  $g$ . In a symmetric equilibrium, both agents play  $b^*(y)$ , which must satisfy

$$(2.1) \quad b^*(y) = \underset{b}{\operatorname{argmax}} E[v(y + g(b, b^*(Y)))] \text{ for all } y.$$

If  $g^*$  is to implement  $t^*$ , we must have

$$(2.2) \quad g^*(b^*(y_i), b^*(y_j)) = (y_j - y_i)/2.$$

Thus

$$(2.3) \quad g^*(b, b^*(y)) = h^*(b) + y/2 \text{ for some function } h^*.$$

Substituting  $g^*$  for  $g$  in (1) and using (3), the  $b^*$  corresponding to  $g^*$  must satisfy

$$(2.4) \quad b^*(y) = \underset{b}{\operatorname{argmax}} E[v(y + h^*(b) + Y/2)] \text{ for all } y.$$

It is immediately seen that the maximizing  $b$  of the RHS of (2.4) is independent of  $y$ , which contradicts (2.2). Thus, even if agents can make binding commitments, the ex ante first-best outcome cannot be achieved when individual income is private information. As long as there are no outside savings or borrowing opportunities, the above example can be generalized easily to the situation where agents make an arrangement for more than one future period.

Now consider a two-period bidding Rosca. In period zero the two agents make an arrangement to pay a stipulated amount  $m$  into a pot, both in period one and two. In the first period, the agents bid for pot one. Assuming that the price  $b^\circ$  the winner of this auction has to pay gets equally distributed among the two participants and that agents bid with identical bidding functions  $b(y)$ , we would expect the agent with the higher need for funds to submit the

winning bid. In our model, ‘higher need’ is equivalent to ‘lower income’. If  $b^\circ < 2m$ , then the agent with the lower income in period one, say agent one, receives a net transfer from the other agent. If the bidding functions are such that it is always true that the agent with lower income in period one receives a net transfer from the other agent, then we have an ex ante improvement from joining the Rosca when the utility contribution of period two consumption is neglected. In period two, however, agent one has to pay a net transfer of  $m$  to the other agent. Whether this payment constitutes a transfer from the better to the worse off depends on the particular values of period two incomes. Adding this effect to the improvement of ex ante utility the period one transfer has, the question is whether the expectation over the sum of these two effects still lets the participation in a Rosca appear advantageous. Towards this end, the functioning of bidding Roscas needs to be analyzed first.

### 3 Bidding Procedures and Equilibrium Bid Functions

To set out the analytical framework, assume that participant  $i$  evaluates period one and two consumption levels  $c_{i1}$  and  $c_{i2}$  with a bivariate von-Neuman-Morgenstern utility function,  $u(c_{i1}, c_{i2})$ <sup>6</sup> which is strictly increasing and concave in each argument, and that, in period  $t$ , her income is drawn from a distribution characterized by the smooth distribution function  $F_t$  on domain  $I_t = [y_{lt}, y_{ut}]$ . All  $y_{it}$ ,  $i, t = 1, 2$  are assumed to be independently and identically distributed according to  $F_t$ . We thus allow for seasonal variations in the income generating process. The participants have access to neither credit nor savings opportunities outside the Rosca. Although this is a very restrictive assumption, it is not wholly implausible considering that, in many parts of the world, Roscas are observed primarily among women.<sup>7</sup> If they are not the heads of their respective household, they might not have control over money that is not invested in some fixed scheme because heads of households may have different, likely shorter

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<sup>6</sup> For ease of exposition, I restrict attention to two-period Roscas.

<sup>7</sup> See, for example, Adams and Canavesi de Saherno (1989) for Roscas in Bolivia.

sighted, ideas about how to use the money.<sup>8</sup> In this section it is further assumed that every agent participates in only one Rosca and that the contribution to the Rosca each member makes every period,  $m$  has been agreed upon beforehand and can be considered fixed.

If any partial derivative of  $u$  satisfies a lower Inada condition, define  $\bar{c}_i = \max_{c_j} \{x: \lim_{c_i \downarrow x} u_i(c_1, c_2) = \infty, j \neq i\}$ , where  $u_i(x_1, x_2) \equiv \frac{\partial u(x_1, x_2)}{\partial x_i}$ . To avoid technical complications, we require  $y_{it} - m > \bar{c}_i$ .

In the literature, a variety of arrangements have been observed when it comes to the auctioning of the pot. The main differences are the type of auction staged and the rule that distributes the winning bid among the other participants. As to the latter, the most important issue is whether the winning bid is distributed among the active participants only or if all participants receive a fraction. In both cases, the distribution occurs equally. Since this paper concentrates on two-period Roscas where only one auction takes place, this difference does not matter for the present analysis. Further, throughout this study, I neglect any problems of enforceability of contributions to the Rosca by members who have received a pot before and thus are only left with obligations. It is assumed that defaulting on contributions results in exclusion from future Roscas and that the disutility therefrom is prohibitively high.<sup>9</sup> Another important empirical feature, the remuneration of the Rosca organizer is also excluded from this analysis.

The two predominant types of auctions encountered are the so-called first-price sealed

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<sup>8</sup> Anderson and Baland (1999) find strong support for this claim in their sample of low income households in Nairobi. Thomas (1993) reports that income in the hands of women tends to increase the share of the household budget spent on health, education and housing as well as improvements in child health.

<sup>9</sup> There is sufficient empirical evidence in support of this assumption. See, among others, Calomiris and Rajaraman (1998).



bid auction and the oral English auction. In the former, each active member<sup>10</sup> submits a closed envelope with her bid. In a meeting, the envelopes are opened and the highest bid receives the pot at the price of her bid submitted. Equivalently, each active member communicates her bid to the organizer of the Rosca privately who then allocates the pot to the participant with the highest bid.

In an oral English auction, the active participants of the Rosca meet and submit successive oral bids until only one bidder, the winner, remains. We might ideally think of an oral English auction as a so called button auction where each bidder presses a button in front of him as the standing bid continuously increases. A bidder drops out of the bidding process once she releases the button. The auction is over once there is only one bidder pressing her button (see Matthews, 1990). She receives the pot at a price equal to the standing bid at the moment the last bidder dropped out. For the derivation of bidding equilibria in the oral English auction, it is useful to consider a second price sealed bid auction. In this auction, as in the first price sealed bid auction, the active participants submit their bids in sealed envelopes. The highest bid wins but this time the winner does not pay his own bid, but the second highest bid. Although this type of auction is not reported in any of the Rosca literature, its equilibrium bidding strategy is the same as in the oral English auction. In the button auction, each bidder's problem is to decide when to release the button. Suppose, however, that each agent releases her button at a standing bid equal to her bid in the second price sealed bid auction. If participants follow this rule, the payoffs to all participants are equal in the second price sealed bid and the English auction. In the language of game theory, the reduced normal form games corresponding to the second price sealed bid and the oral English auction are identical. Thus they are strategically equivalent, which implies that the equilibrium in the second price sealed

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<sup>10</sup> Throughout this paper, those participants who have not yet received a pot are referred to as 'active'.

bid auction is also the equilibrium of the oral English auction.<sup>11</sup>

Roscas with an oral English auction are prevalent in rural settings where the meetings each period also have a socializing function. The members typically belong to the same social group, e.g. caste (see, e.g., Bouman, 1979). In such cases, it is likely that the participants are fairly well informed about each other's incomes. In urban settings, in contrast, while mostly still belonging to the same social group, participants frequently do not know each other outside the Rosca. Often, the Rosca is administered by a professional organizer (see, e.g., Kumar, 1991). Consequently, the members know little about the other participants' incomes. Therefore the following analysis is limited to the following cases. First we consider Roscas with oral English auctions under public information on incomes. Secondly we analyze Roscas with first price sealed bid and oral English auctions under private information on incomes.

#### *A Public Information on Incomes, Oral English Auction*

In an oral English auction, if agent two drops out of the bidding process first, agent one's consumption in period one is given by  $y_1 - m + (2m - b_2 + b_2/2)$  where  $y_1 - m$  is his period one income minus his contribution to the Rosca and  $(2m - b_2 + b_2/2)$  is the pot he receives minus the standing bid at which agent two drops out, plus half of this bid that is redistributed to him. If he is a period one winner, his period two consumption is  $y - m$  where  $y$  is agent one's period two income. Accordingly, his expected utility is  $\tilde{u}(y_1 + m - b_2/2, Y - m)$ , where  $\tilde{u}(\cdot, X) \equiv E_2[u(\cdot, X)] = \int_{y_{12}}^{y_{12}^*} u(\cdot, x) dF_2(x)$ . If, on the other hand, agent one drops out of the bidding process first, his expected utility is given by  $\tilde{u}(y_1 - m + b_1/2, Y + m)$ , where  $b_1$  is the standing bid at which he drops out.

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<sup>11</sup> This result is in line with the standard literature on auctions, see, e.g., Matthews, 1990.

What is a participant's maximum willingness to pay for the period-one pot,  $b^0$  say? Obviously, at a price of  $b^0$ , he attains the same level of utility no matter whether he receives the pot or not. Formally,

$$(3.1) \quad b^0(y) \equiv \{b: \tilde{u}(y - m + b/2, Y + m) = \tilde{u}(y + m - b/2, Y - m)\}.$$

I shall argue that  $b^0$  corresponds to a bidder's value in a standard (not a Rosca) auction with symmetric independent private value (SIPV) bidders.<sup>12</sup> In such auctions, by definition, a bidder is indifferent between winning and not winning the item auctioned when she has to pay a price equal to her true value. This definition applies to  $b^0(\cdot)$  in the present case. By (3.1), a bidder with first period income  $y$  is indifferent between receiving pot one or not at a price of  $b^0(y)$ .

In the empirical literature on the role of Roscas as event insurance, it is observed that the bidder with the most urgent current need submits the highest bid to auction the pot (see Calomiris and Rajaraman, 1998). For the present model, this gives rise to

*Assumption 1:  $b^0$  is strictly decreasing in period-one income, formally*

$$\frac{db^0(y)}{dy} = 2 \frac{\tilde{u}_1(y + m - b^0(y)/2, Y - m) - \tilde{u}_1(y - m + b^0(y)/2, Y + m)}{\tilde{u}_1(y + m - b^0(y)/2, Y - m) + \tilde{u}_1(y - m + b^0(y)/2, Y + m)} < 0 \text{ for all } y.$$

Assumption 1 states that we exclude decision makers whose maximum willingness to pay for pot one is not strictly decreasing in period-one income. With this at hand, we can characterize the bidding equilibrium for the present case.

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<sup>12</sup> In a standard SIPV bidder auction, there is one seller who owns a single, indivisible item and  $n$  buyers. Each bidder knows  $n$  and his own valuation (or value, in short) for the item, which is the maximum amount he would be willing to pay for the item, but none of the other bidders' values. The values are identically, independently distributed (see Matthews, 1990). It is further assumed that the seller cannot set a minimum price.

*Proposition 1: If Assumption 1 holds and incomes are publicly observed, the unique Nash equilibrium of an oral English auction implies that the bidder with the lower period one income,  $j$  say, wins the pot and pays  $b^0(y_j) - \varepsilon$  with some small  $\varepsilon$ .*

Proof: For  $j$ , it is a dominant strategy not to release his button before  $b^0(y_j)$  while for the other bidder,  $i$  say,  $b^0(y_j) - \varepsilon$  is a best reply to  $j$ 's playing  $b^0(y_j)$  because, as long as  $j$  wins pot one,  $i$ 's utility is increasing in the winning bid. QED

Such 'crafty' bidding practices are reported in Bouman (1979) where those in most urgent need of funds are bid up higher than the actual needs of other participants would require.

#### *B Private Information on Incomes, First Price Sealed Bid Auction*

In a first price sealed bid auction, bids are submitted after both agents have observed their respective period one incomes. If agent one bids higher than agent two, his consumption in period one is given by  $y_1 - m + (2m - b_1 + b_1/2)$  where  $y_1 - m$  is his period one income minus his contribution to the Rosca and  $(2m - b_1 + b_1/2)$  is the pot he receives minus his bid he has to pay as the winner, plus half of this bid that is redistributed to him. If he is a period one winner, his period two consumption is  $y - m$ . Accordingly, his expected utility is  $\tilde{u}(y_1 + m - b_1 / 2, Y - m)$ . The probability of this event is  $P(b_1 > B_2)$ . If, on the other hand, agent one submits a lower bid than agent two, his expected utility is  $E[\tilde{u}(y_1 - m + B_2 / 2, Y + m) | B_2 > b_1]$ , the probability of this event being  $P(b_1 < B_2)$ . Note that it is assumed that bids range over an interval of the real line, so that the probability of identical bids is zero. To derive the Bayes-Nash equilibrium bidding strategy, substitute  $b(Y_2)$  for  $B_2$  where  $b'(\cdot) < 0$ . Thus, agent one assumes that the other participant follows some strategy according to which she submits higher bids the lower her period one income is. Since  $b(\cdot)$  is assumed to be smooth and strictly decreasing, the inverse function  $b^{-1}(\cdot)$  is defined and

continuously differentiable. Consequently, agent one's interim expected utility before submitting his bid is given by

$$(3.2) \quad E[U^1 | y_1] = \tilde{u}(y_1 + m - b_1 / 2, Y - m)(1 - F(b^{-1}(b_1))) \\ + E[\tilde{u}(y_1 - m + b(Y_2) / 2, Y + m) | b(Y_2) > b_1] F(b^{-1}(b_1)),^{13}$$

where the task is to maximize (3.2) over  $b_1$ . Equating the derivative of (3.2) with respect to  $b_1$  to zero and substituting  $b(y_1)$  for  $b_1$ , we obtain the Bayes-Nash equilibrium:

$$(3.3) \quad -F(y_1) \frac{d}{dx} E[\tilde{u}(y_1 - m + b(Y_2) / 2, Y + m) | Y_2 < x]_{|x=y_1} = -\tilde{u}'_1(y_1 + m - b(y_1) / 2, Y - m) b'(y_1)(1 - F(y_1)) \\ + \{ E[\tilde{u}(y_1 - m + b(Y_2) / 2, Y + m) | Y_2 < y_1] - \tilde{u}(y_1 + m - b(y_1) / 2, Y - m) \} f(y_1).$$

Equation (3.3) states that the marginal loss of expected utility from pretending to have a slightly different  $y_1$  than that actually realized, equals the marginal gain in expected utility from doing so. Thus  $b(\cdot)$  is such that for any  $x \neq y_1$  submitting  $b(x)$  is not optimal when one's true income is  $y_1$ . The term on the LHS represents the loss of expected utility when pretending  $y_1 + dy_1$  instead of  $y_1$  arising from an expected lower winning bid of agent two which, conditional on agent two winning the pot, implies a higher net transfer from agent one to agent two. The first term on the RHS of (3.3) is the utility gain resulting from paying a lower price for pot one in the case agent one wins, while the term in braces is the change in expected utility arising from the change in the probability of winning the pot in period one.

Rearranging (3.2) gives the differential equation for  $b(\cdot)$ .

*Proposition 2:*

*If Assumption 1 and conditions (i) and (ii) from Appendix 1 hold and incomes are privately observed, then*

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<sup>13</sup> For notational convenience, we write  $F(\cdot)$  instead  $F_1(\cdot)$  and  $y_k$  instead of  $y_{k1}$ ,  $k = l, u$ , throughout the paper.

(i) the symmetric Bayes-Nash equilibrium of a first price sealed bid auction is characterized by

$$(3.4) \quad b'(y_1) = 2 \frac{f(y_1)}{1-F(y_1)} \left[ \frac{\tilde{u}(y_1 - m + b(y_1)/2, Y + m) - \tilde{u}(y_1 + m - b(y_1)/2, Y - m)}{\tilde{u}_1(y_1 + m - b(y_1)/2, Y - m)} \right],$$

$$b(y_u) = b^0(y_u),$$

(ii) bidders underbid, i.e.  $b(y) < b^0(y)$  for all  $y < y_u$ ,

(iii) bids are strictly decreasing in income, i.e.  $b'(y) < 0$  for all  $y$ .

Proof:

(i) Necessity follows from (3.3), sufficiency from (i) and (ii) of Appendix 1.<sup>14</sup>

(ii) and (iii) Applying L'Hôpital's rule,<sup>15</sup> we find that

$$0 > b'(y_u) = 2 \frac{\tilde{u}_1(y_u + m - b(y_u)/2, Y - m) - \tilde{u}_1(y_u - m + b(y_u)/2, Y + m)}{2 \tilde{u}_1(y_u + m - b(y_u)/2, Y - m) + \tilde{u}_1(y_u - m + b(y_u)/2, Y + m)}$$

$$> 2 \frac{\tilde{u}_1(y_u + m - b(y_u)/2, Y - m) - \tilde{u}_1(y_u - m + b(y_u)/2, Y + m)}{\tilde{u}_1(y_u + m - b(y_u)/2, Y - m) + \tilde{u}_1(y_u - m + b(y_u)/2, Y + m)} = b^{0'}(y_u).$$

Thus  $b(y_u - dy) < b^0(y_u - dy)$  for some  $dy > 0$ . Further, if  $b(y) < b^0(y)$  it follows from (i) of Proposition 2 together with Assumption 1 that  $b'(y) < 0$ . To show that  $b(y_u - dy) < b^0(y_u - dy)$  for some  $dy > 0$  implies  $b(y) < b^0(y)$  for all  $y$ , assume that for some  $y_0 < y_u$   $b(y_0) = b^0(y_0)$ . This implies  $b'(y_0) = 0 > b^{0'}(y_0)$  and thus  $b(y_0 - dy) < b^0(y_0 - dy)$  for some positive  $dy$ . Thus  $b(y)$  and  $b^0(y)$  do not intersect for  $y < y_u$ .

To prove that there are no symmetric equilibria with increasing bidding strategies,  $b^+(y)$  say, notice that (i) and (ii) of Appendix 1 are sufficient to show that any strategy  $b^+$  satisfying a

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<sup>14</sup> Note that Appendix 1 only gives sufficient conditions for  $b$  being a local maximizer of (3.2). For sufficient conditions for  $b$  being a global maximizer, see the remarks in Appendix 1.

<sup>15</sup> Note that the point  $(y_u, b^0(y_u))$  constitutes a singularity for the differential equation (3.4).

first order condition analogous to (3.3) constitutes a local minimum of the bidder's resulting interim expected utility analogous to (3.2). QED

*C. Private Information on Incomes, Oral English Auction*

If agent one bids lower than agent two, his expected utility is  $\tilde{u}(y_1 - m + b_1 / 2, Y + m)$ . The probability of this event is  $P(b_1 < B_2)$ . If, on the other hand, agent one submits a higher bid than agent two, his expected utility is  $E[\tilde{u}(y_1 + m - B_2 / 2, Y - m) | B_2 < b_1]$ , the probability of this event being  $P(b_1 > B_2)$ . To derive the Bayes-Nash equilibrium bidding strategy, substitute  $b(Y_2)$  for  $B_2$  where  $b'(\cdot) < 0$ . Consequently, agent one's interim expected utility before submitting his bid is given by

$$(3.5) \quad E[U^2 | y_1] = \tilde{u}(y_1 - m + b_1 / 2, Y + m) F(b^{-1}(b_1)) \\ + E[\tilde{u}(y_1 + m - b(Y_2) / 2, Y - m) | b(Y_2) < b_1] (1 - F(b^{-1}(b_1))),$$

Equating the derivative of this with respect to  $b_1$  to zero and substituting  $b(y_1)$  for  $b_1$ , we obtain the Bayes-Nash equilibrium:

$$-(1 - F(y_1)) \frac{d}{dx} E[\tilde{u}(y_1 + m - b_s(Y_2) / 2, Y - m) | Y_2 > x]_{x=y_1} = -\tilde{u}_1(y_1 - m + b_s(y_1) / 2, Y + m) b_s'(y_1) \\ F(y_1) \\ (3.6) \quad + \{ E[\tilde{u}(y_1 + m - b_s(Y_2) / 2, Y - m) | Y_2 > y_1] - \tilde{u}(y_1 - m + b_s(y_1) / 2, Y + m) \} f(y_1).$$

The subscript  $s$  indicates that (3.6) characterizes the equilibrium of a second price sealed bid auction. Equation (3.6) states that the marginal loss of expected utility from pretending to have realized a slightly different  $y_1$  than is actually the case equals the marginal gain in expected utility from doing so. The interpretation of the terms in (3.6) is analogous to (3.3). Rearranging (3.6) yields the differential equation for  $b_s(\cdot)$ .

*Proposition 3:*

*If Assumption 1 and conditions (iii) and (iv) from Appendix 1 hold and incomes are privately observed, then*

(i) *the symmetric Bayes-Nash equilibrium of an oral English auction is characterized by*

$$(3.7) \quad b_s'(y_1) = 2 \frac{f(y_1)}{F(y_1)} \frac{\tilde{u}(y_1 + m - b_s(y_1)/2, Y - m) - \tilde{u}(y_1 - m + b_s(y_1)/2, Y + m)}{\tilde{u}_1(y_1 - m + b_s(y_1)/2, Y + m)},$$

$$b(y_1) = b^0(y_1),$$

(ii) *bidders overbid, i.e.  $b(y) > b^0(y)$  for all  $y > y_b$ ,*

(iii) *bids are strictly decreasing in income, i.e.  $b_s'(y) < 0$  for all  $y$ .*

(i) Necessity follows from (3.6), sufficiency from (iii) and (vi) of Appendix 1.

(ii) and (iii) The proof of  $b_s'(y) < 0$  for all  $y$  is analogous to that of  $b'(y) < 0$ . One first shows that  $0 > b_s'(y_1) > b^0'(y_1)$ . Secondly,  $b_s(y) > b^0(y)$  together with Assumption 1 implies  $b_s'(y) < 0$ . Finally  $b_s(y_0) = b^0(y_0)$  for some  $y_0$  would imply  $b_s'(y_0) > b^0'(y_0)$  and thus  $b_s(\cdot)$  and  $b^0(\cdot)$  do not intersect for any  $y > y_1$ .

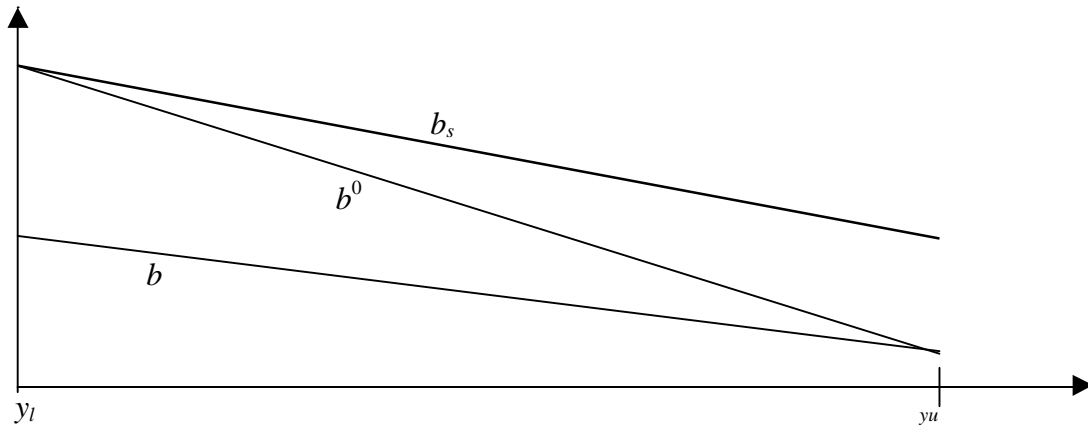
To show that there are no symmetric equilibria with increasing bidding strategies, an argument analogous to that in the proof of Proposition 1 can be applied. QED

#### *D Discussion*

Figure 1 illustrates that in a first price sealed bid auction, bidders underbid relative to their true valuation while they overbid in an English auction. Concerning the first price auction, this result is in line with the equilibrium bidding behaviour in a standard SIPV bidder auction. Suppose a bidder submits  $b^0(y)$  when  $y$  is her period one income. In this case, winning the pot does not make her any better off. If she submits a slightly lower bid, however, her probability of winning decreases slightly, but if she wins she improves her situation. (3.3) and (3.4) together with (3.6) show that, in equilibrium, the gains from underbidding more than outweigh the corresponding losses. For a bidder with income  $y_u$ , however, there is no sense in underbidding because he loses the pot with probability one. Thus he will submit a bid equal to  $b^0(y_u)$ , which is exactly what the boundary condition (3.4) states.



**Figure 1. Equilibrium bidding strategies in a two-period Rosca.**



Turning to the second price auction, the present result of overbidding is in marked contrast to the equilibrium behaviour in a standard SIPV bidder auction, where bidding one's true value is a dominant strategy. Suppose agent  $i$  bids  $b^0(y_i)$ . If agent  $j$  submits a higher bid,  $i$  is not any better off than if she had won the pot at a price of  $b^0(y_i)$ . Bidding slightly more than  $b^0(y_i)$ , however, improves her situation if  $j$  wins. On the other hand, by bidding  $b^0(y_i) + dy$ , she takes the chance of winning the pot at a price higher than her valuation with positive probability. (3.9) and (3.10) together with (3.6), however, show that, in equilibrium, the gains from overbidding exceed the losses except for a bidder with income  $y_l$ , who wins the pot with probability one. Thus  $b_s(y_l) = b^0(y_l)$ . The key lesson from this is that, contrary to the standard SIPV bidder second price auction, bidding in a Rosca is always strategic and equilibria in dominant strategies fail to exist. The reason for this arises from the fact that, in the terminology of Kovsted and Lyk-Jensen (1999), in a Rosca auction, the seller is internalised in the group of bidders. As a consequence, the loser of a Rosca auction is not left with the same economic situation as before the beginning of the auction, but rather receives a gain from the share of the winning bid that is distributed to him.

Another interesting feature is the relationship between the rate of time preference and bidding. If participants have utility functions of the form  $u(x_1, x_2) = v_1(x_1) + \beta v_2(x_2)$  where  $\beta$  is a discount factor, it is easily shown that

$$\frac{db^0}{d\beta}(y) = 2 \frac{E[v_2(Y-m) - v_2(Y+m)]}{v_1'(y+m-b^0(y)/2) + v_1'(y-m+b^0(y)/2)} < 0 \text{ for all } y,$$

$\frac{db'}{d\beta}(y) > 0$  and  $\frac{db'_s}{d\beta}(y) < 0$  for all  $y$ . Thus high discounting of future consumption goes

together with unambiguously higher bids for pot one. This comes as no surprise, as individuals who care less about future consumption are less concerned about a possible obligation to pay a net transfer of  $m$  one period later than receiving the pot in period one.

#### 4 The Design of Equivalent Rosca Auctions

In this section it is shown that participants' payoffs are independent not only of how the winning bid is distributed among participants, but also, more surprisingly, of the amount of the contribution participants make to the Rosca in period one.

Assume that participants have agreed on redistributing a fraction of  $(1-\delta)$  of the amount the winner of pot one has to pay back to the winner, where  $0 < \delta \leq 1$ . Then the winner's consumption in period one is  $(y_1 - m) + (2m - b + (1-\delta)b)$  where  $b$  is the amount he has to pay. For the loser, period one consumption is given by  $(y_1 - m) + \delta b$ . Thus, in equilibrium, expected utility at the interim stage is given by

$$(4.1) \quad E[U | y_1, \delta] = E[\tilde{u}(y_1 - m + \delta b(Y_2, y_1, \delta), Y + m) | Y_2 < y_1] F(y_1) \\ + E[\tilde{u}(y_1 + m - \delta b(y_1, Y_2, \delta), Y - m) | Y_2 > y_1] (1 - F(y_1)),$$

where  $E[U | y_1]$  in (4.1) comprises both the English and the first price auction considered in the previous section.  $b(\cdot, \cdot, \delta)$  indicates that bidding now appears to depend on the value of  $\delta$ . Substituting  $b(\cdot, \cdot, 1/2)/(2\delta)$  for  $b(\cdot, \cdot, \delta)$ , however, transforms (4.1) to interim expected utility under the rule that distributes the price paid for pot one equally among participants. Thus for

any  $\delta$  and any of the auction regimes considered previously, participants will receive the same payoffs in equilibrium. This means that, theoretically, the participants would not have to decide on the value of  $\delta$  until the very start of the auction, no matter what incomes they observe.

What payoffs occur if, in period zero, participants agree not to pay in any contributions in period one, but only in period two, and stage an auction for pot two in period one? If the said auction is held as an English auction, participants will receive the same payoffs as in a usual Rosca where they stage a first price auction for pot one. The payoffs of a usual Rosca with an English auction, on the other hand, equal those from a Rosca with no contributions in period one when a first price auction for pot two is staged in period one. To see this, note that, in equilibrium, interim expected utility in the latter case is given by

$$(4.2) \quad E[U_{\Delta}^1 | y_1] = \tilde{u}(y_1 - \Delta(y_1), Y + m) F(y_1) + E[\tilde{u}(y_1 + \Delta(Y_2), Y - m) | Y_2 > y_1] (1 - F(y_1)),$$

where  $\Delta(\cdot)$  represents the bidding strategy played in the first price auction in period one. Recall that equilibrium interim expected utility in a usual Rosca with a second price auction in the first period is

$$(4.3) \quad E[U_{b_s}^2 | y_1] = \tilde{u}(y_1 - m + b_s(y_1) / 2, Y + m) F(y_1) \\ + E[\tilde{u}(y_1 + m - b_s(Y_2) / 2, Y - m) | Y_2 > y_1] (1 - F(y_1)).$$

Now, for any  $\hat{b}_s(\cdot)$ , define the one to one mapping

$$(4.4) \quad \hat{\Delta}(\cdot) = m - \hat{b}_s(\cdot) / 2.$$

For any pair of bidding strategies  $(\hat{b}_{s_1}(\cdot), \hat{b}_{s_2}(\cdot))$  the agents can play in the usual Rosca second price auction, the pair  $(\hat{\Delta}_1(\cdot), \hat{\Delta}_2(\cdot))$  in a first price auction for pot two in period one yields identical payoffs for all  $y_1$ . Consequently, the equilibrium in the latter arrangement will involve  $\Delta(\cdot) = m - b_s(\cdot) / 2$ . The proof of the payoff equivalence of a usual Rosca with a first price auction and a Rosca without period one contributions and a second price auction for pot two in period one is analogous. There, the equilibrium involves  $\Delta_s(\cdot) = m - b(\cdot) / 2$ .

These results show that, in a usual Rosca, the net transfer the loser of pot one ('she') pays to the winner can literally be interpreted as her bid for a net transfer of  $m$  one-period ahead. It further highlights the crucial importance of the fixed amount period two transfer as an incentive for participants to pay an ex ante utility increasing transfer in period one.

## 5 Preferences for Risk Bearing and Preferences for Roscas

With the results of the previous two sections in hand, we can now ask the question: how do preferences for risk-bearing influence the decision to participate in a bidding or a random Rosca? To answer this question, we shall make use of the concept of temporal risk aversion, which was first defined by Richard (1975) as follows: a decision maker is said to be multivariate risk averse if, for any pair  $(x, y)$ ,  $u_{12}(x, y) < 0$  and multivariate risk seeking if  $u_{12}(x, y) > 0$ . The case of  $u_{12}(x, y) = 0$  is defined as multivariate risk neutrality. If  $u$ 's arguments refer to consumption at two points in time, 'multivariate' may be replaced by 'temporal' (see Ingersoll, 1987). This concept can be illustrated as follows: Consider two lotteries  $L_1$  and  $L_2$  that are both resolved in period zero.  $L_1$  involves a consumption level of  $x$  in both periods with probability 0.5 and a consumption level of  $y$  in both periods with probability 0.5.  $L_2$  involves a consumption level of  $x$  in period one and  $y$  in period two with probability 0.5, and  $y$  in period one and  $x$  in period two with probability 0.5. A temporal risk averse decision maker prefers  $L_1$  to  $L_2$ , while a temporal risk seeking decision maker prefers  $L_2$  to  $L_1$  for any pair  $(x, y)$ . Thus, loosely speaking, a temporal risk seeking agent has a preference for lotteries whose payoffs are positively correlated over time while a temporal risk averse agent prefers negatively correlated payoffs.<sup>16</sup>

### A *Random Roscas*

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<sup>16</sup> Ronn (1988) argues that for a temporal risk averse agent, consumption levels in any two periods are 'substitutes through time' while they are complements for a temporal risk seeker.

A random Rosca does exactly the latter. While uncorrelated without the Rosca, consumption levels of random Rosca participants are negatively correlated. To make this argument rigorous, we write a random Rosca participant's ex ante expected utility as

$$(5.1) \quad E[U^R] = E_{Y_1, Y_2} [u(Y_1 + m, Y_2 - m) + u(Y_1 - m, Y_2 + m)] / 2,$$

where  $E_{Y_1, Y_2} [\cdot]$  indicates that expectation is taken both over  $Y_1$  and  $Y_2$ . For the sake of analytical tractability, we concentrate on Roscas with an infinitesimally small contribution  $m$ . Evaluating the derivative of (5.1) w.r.t.  $m$  at  $m = 0$  yields zero, while the second derivative

$$(5.2) \quad \frac{d^2 E[U^R]}{dm^2} \Big|_{m=0} = 2E[u_{11}(Y_1, Y_2) + u_{22}(Y_1, Y_2) - 2u_{12}(Y_1, Y_2)].$$

It is seen that if  $u_{12}$  is positive, equation (5.2) is negative and thus not participating in a random Rosca is the optimal decision. If, however,  $u_{12} < 0$ , the case is ambiguous. The question then is whether the effect of temporal risk aversion arising from the negative cross derivative outweighs the effect of static risk aversion arising from the concavity of  $u$  in each argument.<sup>17</sup>

Formally, similar to Ronn, 1988, define the coefficients of static and temporal risk aversion

$$RA_t(x_1, X_2) \equiv -\frac{\tilde{u}_{tt}(x_1, X_2)}{\tilde{u}_t(x_1, X_2)} \quad \text{and} \quad TRA_{kt}(x_1, X_2) \equiv -\frac{\tilde{u}_{kt}(x_1, X_2)}{\tilde{u}_t(x_1, X_2)},$$

respectively, to rewrite (5.2) as

$$(5.3) \quad \frac{d^2 E[U^R]}{dm^2} \Big|_{m=0} = 2E[u_1(Y_1, Y_2)(TRA_{21}(Y_1, Y_2) - RA_1(Y_1, Y_2)) + u_2(Y_1, Y_2)(TRA_{12}(Y_1, Y_2) - RA_2(Y_1, Y_2))].$$

Defining autarky as not participating in a Rosca, we thus have

*Proposition 4: If  $TRA_{21}(y_1, y_2) \leq RA_1(y_1, y_2)$  and  $TRA_{12}(y_1, y_2) \leq RA_2(y_1, y_2)$  for all  $y_1, y_2$ , then autarky is preferred to participation in a random Rosca with a small contribution  $m$ .*

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<sup>17</sup> Notice, however, that  $-u_{12} > -u_{11} - u_{22}$  for all  $x_1, x_2$  implies that  $(d^2 x_2 / dx_1^2) < 0$  whenever  $(dx_2 / dx_1) = 1$ , i.e.  $u$ 's indifference curves are not convex.

A borderline case arises when  $u(x_1, x_2) = v(x_1 + x_2)$  for some strictly increasing and concave function  $v$ .<sup>18</sup> Then  $TRA_{tk} = RA_t = RA_k$  and such individuals are indifferent between participating in a random Rosca or not. Although a certain degree of temporal risk aversion seems plausible for individuals whose consumption is not well above the subsistence level, it is rather unlikely that any individual in this situation would improve her ex ante expected utility by joining a Random Rosca.<sup>19</sup>

### *B Public Information on Incomes, Oral English Auction*

If Information on incomes is public and an English auction is staged in period one, it follows from the results of section 3 that ex ante expected utility in equilibrium is given by

$$(5.4) \quad E[U^3] = E_{Y_1} [\tilde{u}(Y_1 + \Delta^0(Y_1), Y - m)(1 - F(Y_1))] + E_{Y_2} [\tilde{u}(Y_1 - \Delta^0(Y_2), Y + m) | Y_2 < Y_1] F(Y_1),$$

where  $\Delta^0(y) \equiv m - b^0(y)/2$ . For simplicity, it is assumed here that the winner of pot one pays  $b^0(y)$  instead of  $b^0(y) - \varepsilon$ . Integrating by parts and employing a change of variable gives

$$(5.5) \quad E[U^3] = E_{Y_1, Y_2} [\tilde{u}(Y_1 + \Delta^0(Y_1), Y - m) + \tilde{u}(Y_2 - \Delta^0(Y_1), Y + m) | Y_1 < Y_2].$$

Defining  $\omega(y, \rho) \equiv \frac{\tilde{u}_2(y, Y) \tilde{u}_1(\rho, Y)}{\tilde{u}_1(y, Y) \tilde{u}_2(\rho, Y)}$  and evaluating the derivative of (5.5) w.r.t.  $m$  at  $m = 0$

yields

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<sup>18</sup> If  $v(x) = x$ , this is the case of risk neutral agents who do not discount future consumption.

<sup>19</sup> Only few studies have addressed the relationship between static and temporal risk aversion empirically, none of them in the context of a developing country. In a data set of US consumers, however, Epstein and Zin (1991) find a statistically significant positive intertemporal elasticity of substitution which, in their framework, implies that static risk aversion is more pronounced than temporal risk aversion.

*Proposition 5:* Assume that information on incomes is public, an English auction is staged and Assumption 1 holds. Then a sufficient condition for participation in such a Rosca is

$$(5.6) \quad [\alpha(y_i, \rho) RA_1(\rho, Y) - TRA_{12}(\rho, Y)] \geq 0 \text{ for all } y_u \geq \rho \geq y_l.$$

Proof: See Appendix 2, Section A.

Notice that, for arbitrarily small  $m$ , Assumption 1 implies that  $RA_1(\rho, Y) > TRA_{12}(\rho, Y)$  for all  $y_u \geq \rho \geq y_l$  and that  $\alpha(y, \rho)$  is strictly decreasing in  $\rho$ .<sup>20</sup> Since  $\alpha(y, \rho) = 1$  whenever  $y = \rho$ , it is clear that Assumption 1 alone is not sufficient for (5.6) to hold. It is obvious, however, that individuals whose static risk aversion is sufficiently more pronounced than their temporal risk aversion, participate in such a Rosca.

### *C Private Information on Incomes, First Price Sealed Bid Auction*

Turning to the case of private information on incomes, equilibrium ex ante expected utility when a first price auction is staged can be written as

$$(5.7) \quad E[U^1] = E_{y_1, y_2}[\tilde{u}(Y_1 + \Delta_s(Y_1), Y - m) + \tilde{u}(Y_2 - \Delta_s(Y_1), Y + m) | Y_1 < Y_2],$$

which is equivalent to (5.5) with  $\Delta_s$  substituted for  $\Delta^0$ . We thus obtain

*Proposition 6:* Assume that information on incomes is private, a first price sealed bid auction is staged and Assumption 1 holds. Then

(i) a sufficient condition for participation in such a Rosca is

$$(5.8) \quad [\alpha(y_i, \rho) RA_1(\rho, Y) - TRA_{12}(\rho, Y)] \geq 0 \text{ for all } y_u \geq \rho \geq y_l.$$

(ii) for arbitrarily small  $m$ , expected ex ante utility from participation in such a Rosca is as least as high as expected ex ante utility from participation in a bidding Rosca under public information on incomes when an oral English auction is staged.

Proof: See Appendix 2, Section C.

Notice that (5.8) and (5.6) are equivalent. Part (ii) of Proposition 6 says that, for certain preferences, participation in a bidding Rosca under private information on incomes might be advantageous, while, for the same preferences, this might not be the case if information on incomes is public.

*D Private Information on Incomes, Oral English Auction*

Proceeding as in the previous subsection, equilibrium ex ante expected utility when an oral English auction is staged can be written as

$$(5.9) \quad E[U^2] = E_{Y_1, Y_2} [\tilde{u}(Y_1 + \Delta(Y_2), Y - m) + \tilde{u}(Y_2 - \Delta(Y_2), Y + m) | Y_1 < Y_2].$$

We thus obtain

*Proposition 7:* Assume that information on incomes is private, an oral English auction is staged and Assumption 1 holds. Then a sufficient condition for participation in such a Rosca is

$$(5.10) \quad [\omega(y_l, \rho) RA_1(\rho, Y) - TRA_2(\rho, Y)] \geq 0 \text{ for all } y_u \geq \rho \geq y_l.$$

Proof: See Appendix 2, Section D.

*E Discussion*

For both types of auctions, temporal risk seeking and moderately temporal risk averse agents seek to participate in a bidding Rosca. When  $m$  is close to zero and temporal and static risk preferences are uniform in the sense that temporal are smaller than static coefficients of risk aversion for all possible realizations of period-one income, then the set of preferences inducing participation in a random Rosca does not intersect with the set of preferences inducing

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<sup>20</sup> See Appendix 2, Section B for a proof.



participation in bidding Roscas with strictly decreasing equilibrium bidding strategies. For preferences whose coefficient of temporal risk aversion is uniformly higher than the coefficient of static period one risk aversion, participation in bidding Roscas with strictly increasing equilibrium bidding strategies can be favourable. All of the qualitative empirical evidence (see, e.g., Calomiris and Rajaraman, 1998), however, suggests that such bidding behaviour does not occur in reality, and it is therefore not analyzed in this paper.

## 6 The Contributions in a Bidding Rosca

In section 3, I assumed  $m$  as fixed. The participants' problem, however, is to determine the optimal value of  $m$ ,  $m^*$  say, before starting a Rosca, thus maximizing ex ante expected utility  $E[U^k]$ ,  $k=1, 2, 3$  over  $m$ . In general, this problem has no explicit solution. One can, however, extend some familiar results about decision makers with constant relative (CARA) and constant absolute risk aversion (CRRA) to the question of optimal contributions to a Rosca.

If agents are temporal risk neutral, their utility function can always be written in an additively separable form, i.e.  $u(x_1, x_2) = v_1(x_1) + v_2(x_2)$  (see Richard, 1975). For CARA, we consider utility functions of the form  $u(x_1, x_2) = v(x_1) + \beta v(x_2)$  with  $v(y) = -Exp[-ay]$  and  $\beta \leq 1$ . In this case (5.9) evaluated at  $m^*$  becomes

$$(6.1) \quad E[U^1] = - \int_{y_1}^{y_1} \int_{y_2}^{y_2} \{v(y_1)v(m^* - b(y_1)/2) + v(y_2)v(-m^* + b(y_1)/2)\} dF(y_1)dF(y_2) - (\beta/2)\tilde{v}(Y)\{v(m^*) + v(-m^*)\},$$

where I have used  $v(x_1 + x_2) = -v(x_1) v(x_2)$  and  $\tilde{v}(X) \equiv \int_{y_{12}}^{y_{u2}} v(x)dF_2(x)$ . If the range of each

period's income random variable is shifted upwards by say  $\alpha$  dollars, we find that the bidding strategy remains unaltered and the resulting ex ante expected utility is  $E[U_\alpha^1] = -v(\alpha)E[U_0^1]$ .

Thus, as expected,  $m^*$  is independent of  $\alpha$ .

Turning to constant relative risk aversion, we are interested in period felicity functions of the form  $v(y) = (y^a - 1)/a$ . Multiplying the income variable by  $\alpha$  and evaluating at the optimum contribution to the Rosca,  $m_\alpha^*$ , we obtain

$$(6.2) \quad E[U_\alpha^1] =$$

$$\int_{y_1}^{y_1} \int_{y_1}^{y_2} \{v(\alpha y_1 + m_\alpha^* - b_\alpha(y_1)/2) + v(\alpha y_2 - m_\alpha^* + b_\alpha(y_1)/2)\} dF(y_1) dF(y_2) + (\beta/2)[\tilde{v}(\alpha Y + m_\alpha^*) + \tilde{v}(\alpha Y - m_\alpha^*)],$$

where  $b_\alpha$  is the equilibrium bidding strategy corresponding to  $m_\alpha^*$ . It can be shown that, evaluated at  $m_\alpha = \alpha m_1^*$ ,  $E[U_\alpha^1] = \alpha^a E[U_1^1] + (1+\beta)v(\alpha)$  where  $b_\alpha = \alpha b_1$ . Thus, as expected, if  $m_1^*$  maximizes  $E[U_1^1]$ , then  $m_\alpha = \alpha m_1^*$  maximizes  $E[U_\alpha^1]$ . In both the CARA and the CRRA cases, the proof for Roscas with an English auction under both private and public information on incomes is analogous.

To conclude this section, we consider a numerical example where  $u(x_1, x_2) = \log(x_1) + \beta \log(x_2)$  and income within each period is uniformly distributed on the interval  $[1, 2]$ . If there is no discounting, i.e.  $\beta = 1$ , the optimum contribution is 0.077 if a first price auction is staged and 0.083 if the Rosca involves an English auction. If information on incomes is public and there is an English auction for pot one, 0.075 obtains. For strong discounting, that is  $\beta = 0.5$ , the corresponding values are 0.104, 0.109 and 0.096, respectively.

## 7 Ex-ante, Interim and Ex-post Considerations

Section 5 discussed which preferences induce participation in bidding or random Roscas based on ex ante expected utility. This section focuses on the interim and ex post stages. The former is most conveniently analyzed graphically. Consider the following zero sum situation after incomes in period one have been revealed, where the first agent's utility is  $\tilde{u}(y_{1+t_1}, Y_{t_2})$  and the second agent one's  $\tilde{u}(y_{2-t_1}, Y_{-t_2})$ . Their indifference curves can be illustrated in the  $t_1$ - $t_2$ -plane, where the origin represents autarky. It can be shown that when agents are temporal risk seeking or when temporal risk aversion is moderate, agent one's indifference curves are

convex to the origin while those of agent two are concave. Further, if  $y_1 = y_2$ , they have the same slope, and if  $y_i < y_j$ , then  $i$ 's indifference curve is steeper than  $j$ 's at the origin. Without loss of generality, let  $y_1 < y_2$ . The situation is depicted in figure 2. Agent one's preferred set is to the north-east while agent two's is to the south-west.

**Figure 2. Rosca allocations at the interim stage.**

Figure 2a.

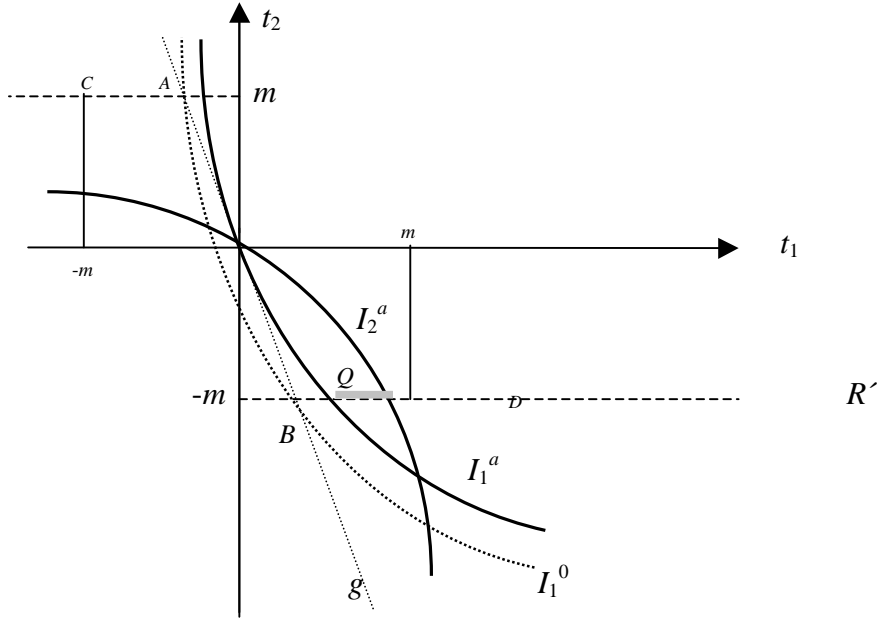
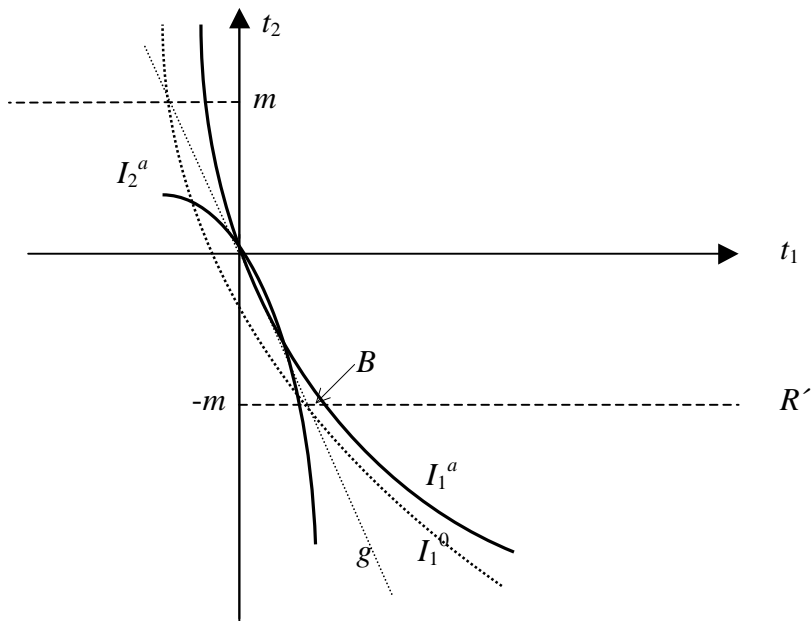


Figure 2b.



If there is a Rosca with contributions  $m$ , then the set of feasible allocations is limited to  $R = \{(t_1, t_2): t_2=m \text{ or } t_2=-m\}$ . A random Rosca allocates the pots such that  $C$  and  $D$  of figure 2a occur with equal probabilities. In the case of a bidding Rosca with private information, since both types of auctions considered in this paper yield the outcome that pot one is allocated to the participant with the lower income in period one and that bids are always smaller than  $2m$ , the actual outcome will always lie on one of the two broken horizontal lines. In the present example, it is the one marked  $R'$ , since  $y_1 < y_2$ . Figure 2a depicts a situation where period one incomes are such that, in principle, both participants can improve their interim expected utility through the Rosca. In particular, the core is given by that segment of  $R'$  that lies in the lens bordered by  $I_1^a$  and  $I_2^a$ , which is denoted by  $Q$ . In contrast, Figure 2b depicts a situation where  $y_1$  and  $y_2$  are not far enough apart from each other to provide gains from the Rosca with contribution  $m$ . It is immediately clear that in this latter case, at the interim stage, both participants can be worse off through participation in the Rosca than under autarky.

Does a bidding Rosca always imply a Pareto improvement if incomes are sufficiently different as depicted in figure 2a? The answer is: not always. Consider a first price auction for pot one. It was shown in section 4 that the resulting payoffs are equivalent to those resulting from a Rosca with no contributions in period one and a second price auction for pot two staged in period one. First consider the role of  $\Delta^0$  in the context of figure 2. Agent one's indifference curve corresponding to  $\Delta^0(y_1)$  is  $I_1^0$  because  $-t_1^A = t_1^B = \Delta^0(y_1)$ . Graphically,  $I_1^0$  is found to be that indifference curve which cuts a straight line through the origin (denoted by  $g$ ) on both of the broken lines simultaneously. Since, in the present example, agent one's bid is the one that determines the actual transfer  $\Delta$  and  $\Delta_s(y) > \Delta^0(y)$  except at  $y_u$ , it follows that  $\Delta_s(y_1) > t_1^B$ . However,  $\Delta_s(y_1)$  need not fall into the set  $Q$ . If  $y_1$  is relatively low, overbidding is high and the participants might end up to the right of  $Q$  involving an improvement for agent one ('she') but a deterioration for agent two. If, however, her period one income is high,

overbidding is small, and the equilibrium payoffs may lie somewhere to the left of  $I_1^a$ . With an English auction for pot one, the case is analogous. Now  $t_1 = \Delta(y_2)$ , is the resulting transfer and again, equilibrium payoffs may involve a Pareto improvement or not.

To complete the analysis, we consider the ex post stage. It is clear that, as in the interim stage, both participants can be worse off than under autarky. Assume, for simplicity, that they earn  $y_i^*$ , the certainty equivalent to period two income  $Y$  given consumption in the first period, i.e.  $\tilde{u}(y_i+t_i, Y-m) = u(y_i+t_i, y_i^*-m)$ . Then the results from the interim case carry over directly to the ex post stage. These findings are in marked contrast to Kovsted and Lyk-Jensen (1999) where, ex post, all members attain a higher level of utility than under autarky, essentially because their agents face deterministic incomes and the earlier access to an investment good through funds from the Rosca unambiguously increases every participant's utility.

If information on incomes is public and there is an English auction, will the participant with the higher period-one income always improve his situation at the interim stage? In both figure 2a and 2b, the Nash equilibrium outcome is approximately  $B$ , because agent one ( $y_1 < y_2$ ) is driven down to approximately  $\Delta^0(y_1)$ . It is thus clear that, compared to autarky, agent one's interim situation always deteriorates. Thus, the ex ante attractiveness of participating in a Rosca under these circumstances arises solely from those cases where a prospective participant achieves the higher period-one income. But even then, an improvement is not certain. While agent two unambiguously improves his position in situations like in figure 2a, where there is scope for a Pareto improvement, in general, his interim and ex post utility might be lower than under autarky. Such a case is depicted in figure 2b.

## 8 Conclusion

Roscas can offer insurance for homogenous, risk averse individuals with stochastic incomes who do not have access to credit. It has been established that, under the assumptions set out

above, bidding Roscas offer advantages for a wide class of participants' preferences, namely when temporal risk aversion is less pronounced than static risk aversion. Under this assumption, participation in a random Rosca does not occur. If, on the other hand, temporal is stronger than static risk aversion, participation in a random Rosca increases expected utility, while participation in a bidding Roscas can be advantageous. Compared to first-best insurance contracts that can be arranged when individuals observe their contract partner's income, Roscas impose severe restrictions on the set of feasible allocations among participants within each period, arising from a fixed transfer in the last period and strategic behaviour of bidders in prior periods. By doing this, however, they stimulate a net transfer from the better to the worse off each time a pot is auctioned and thereby overcome information asymmetries.

When incomes are public knowledge, more efficient insurance arrangements are available than a Rosca. If, however, commitments involving variable contributions in the future cannot be enforced, Roscas may also be observed because their key feature is a fixed contribution in each period, an escape from which is only possible by default. In such situations, the results derived here suggest that participation in a bidding Rosca is advantageous to individuals whose static risk aversion is stronger than temporal risk aversion. In this paper, it has been shown that equilibrium bidding likely causes outcomes that are inferior to the situation in which incomes are private information. In this connection it should be noted that the analysis is restricted to a one-shot game. Should participants decide to start a new Rosca once one is finished, there would be a repeated game, and socially more favourable forms of bidding might be observed, with participants not complying with such a norm being excluded from future Roscas.

In Besley et al. (1993) and Kovsted and Lyk-Jensen (1999), it is proved that, for a group of homogenous individuals, a random Rosca is always preferred. In contrast, the results derived here suggest that, if reasonable restrictions on preferences are imposed, a bidding

Rosca is preferred, because it can allocate funds to those with the most urgent current need. This is, in principle, a similar effect to that observed in the former two studies when individuals are heterogeneous. There, however, heterogeneity is a permanent individual characteristic and bidding serves to accommodate those with the highest willingness to pay first, which in turn generates a gain for the other members through the distribution of the winning bid. In the model presented in this paper, individuals are identical *ex ante* and it is individual-specific uncertainty that generates potential gains from intertemporal trade.

The predictions of the present model better explain the transactions observed in many actual Roscas, where neither the net transfers to the recipients of pots increase steadily with the number of rounds played, nor does the implied interest rate for such funds remain constant or decrease monotonically. Both of these quantities fluctuate significantly in the model presented here, although, even in the absence of savings opportunities outside the Rosca, observed transfers to recipients of pots will increase on average if there is a positive rate of time preference. This is fully in accordance with empirical observation, such as in Calomiris and Rajaraman (1999).

Many of the results derived in this paper carry over to Roscas with more than two participants. If individuals are engaged in several Roscas simultaneously whose participants do not wholly overlap, their bidding strategies, as well as the outcomes, will change. Further analysis is needed to clarify what constitutes an intertemporally optimal portfolio of Rosca shares and how the resulting outcome compares to the benchmark case of a complete set of markets for Arrow-Debreu securities.

## **Appendix 1**

This appendix discusses conditions that ensure that (3.1) and (3.7) assume local maxima when evaluated at  $b$  and  $b_s$ , respectively. Evaluating the second derivative of (3.1) at  $b(y)$  gives

$$(A1.1) \quad \frac{d^2 E[U^1 | y_1]}{db_1^2} \Big|_{b_1=b(y_1)} = f(y_1) \tilde{u}_{i1}(y_1 + \Delta_s(y_1), Y - m) \\ * \left( \frac{\tilde{u}_1(y_1 + \Delta_s(y_1), Y - m) - \tilde{u}_1(y_1 - \Delta_s(y_1), Y + m)}{\tilde{u}_{i1}(y_1 + \Delta_s(y_1), Y - m)} - \frac{\tilde{u}(y_1 + \Delta_s(y_1), Y - m) - \tilde{u}(y_1 - \Delta_s(y_1), Y + m)}{\tilde{u}_1(y_1 + \Delta_s(y_1), Y - m)} \right).$$

It has been shown in Section 3 that the second fraction in brackets is positive. Thus, to ensure that (A1.1) is negative, the first fraction in brackets must be bigger than the second one.

Writing the differences in the numerators of the fractions in (A1.1) as line integrals,

$$\tilde{u}_i(y_1 + \Delta_s(y_1), Y - m) - \tilde{u}_i(y_1 - \Delta_s(y_1), Y + m) \\ = \int_{-1}^{+1} \Delta(y_1) \tilde{u}_{i1}(y_1 + \rho \Delta_s(y_1), Y - \rho m) - m \tilde{u}_{i2}(y_1 + \rho \Delta_s(y_1), Y - \rho m) d\rho, \quad i = 1, 2,$$

and using the coefficients of static and temporal risk aversion as defined in section 5 allows us to rewrite (A1.1) as

$$(A1.2) \quad \frac{d^2 E[U^1 | y_1]}{db_1^2} \Big|_{b_1=b(y_1)} = f(y_1) \\ * \left\{ \Delta_s(y_1) \int_{-1}^{+1} \tilde{u}_1(y_1 + \rho \Delta_s(y_1), Y - \rho m) [RA_1(y_1 + \Delta_s(y_1), Y - m) - RA_1(y_1 + \rho \Delta_s(y_1), Y - \rho m)] d\rho \right. \\ \left. + m \int_{-1}^{+1} \tilde{u}_2(y_1 + \rho \Delta_s(y_1), Y - \rho m) [TRA_{12}(y_1 + \rho \Delta_s(y_1), Y - \rho m) - RA_1(y_1 + \Delta_s(y_1), Y - m)] d\rho \right\}.$$

Since  $u_1$  and  $u_2$  are strictly positive by assumption, sufficient conditions for the negativity of (A1.2) are

- (i)  $RA_1(y_1 + \Delta_s(y_1), Y - m) \leq RA_1(y_1 + \rho \Delta_s(y_1), Y - \rho m)$  for all  $\rho \in [-1, 1]$
- (ii)  $TRA_{12}(y_1 + \rho \Delta_s(y_1), Y - \rho m) \leq RA_1(y_1 + \Delta_s(y_1), Y - m)$  for all  $\rho \in [-1, 1]$ .

If the utility function exhibits utility independence (see Richard, 1975) it follows that the coefficients of static and temporal risk aversion depend on period one consumption only. In this case, (i) and (ii) respectively become

- (i)'  $RA_1(y_1 + \Delta_s(y_1)) \leq RA_1(y_1 + \rho \Delta_s(y_1))$  for all  $\rho \in [-1, 1]$



(ii)'  $TRA_{12}(y_1 + \rho\Delta_s(y_1)) \leq RA_1(y_1 + \Delta_s(y_1))$  for all  $\rho \in [-1, 1]$ .

(i)' is implied by decreasing absolute risk aversion (DARA) for period one decisions while (ii)' holds if temporal risk aversion is less pronounced than static period one risk aversion.

For the second price auction, we evaluate the second derivative of (3.7) at  $b_s(y_1)$ :

$$(A1.3) \quad \frac{d^2 E[U^2 | y_1]}{db_1^2} \Big|_{b_1=b_s(y_1)} = f(y_1) \tilde{u}_{11}(y_1 - \Delta(y_1), Y + m) \\ * \left( \frac{\tilde{u}_1(y_1 + \Delta(y_1), Y - m) - \tilde{u}_1(y_1 - \Delta(y_1), Y + m)}{\tilde{u}_{11}(y_1 - \Delta(y_1), Y + m)} - \frac{\tilde{u}(y_1 + \Delta(y_1), Y - m) - \tilde{u}(y_1 - \Delta(y_1), Y + m)}{\tilde{u}_1(y_1 - \Delta_s(y_1), Y + m)} \right).$$

As shown in section 3, the second fraction in brackets is negative. Further, if  $u_{12} \geq 0$ , then  $\tilde{u}_1(y_1 + \Delta(y_1), Y - m) - \tilde{u}_1(y_1 - \Delta(y_1), Y + m) \leq 0$ . Thus, for the second price auction, temporal risk neutrality or temporal risk preference ensures a local maximum. If, on the other hand,  $u_{12} \leq 0$ , the following sufficient conditions can be obtained.

(iii)  $RA_1(y_1 - \Delta(y_1), Y + m) \leq RA_1(y_1 + \rho\Delta(y_1), Y - \rho m)$  for all  $\rho \in [-1, 1]$

(iv)  $TRA_{12}(y_1 + \rho\Delta(y_1), Y - \rho m) \leq RA_1(y_1 - \Delta(y_1), Y + m)$  for all  $\rho \in [-1, 1]$

If utility independence holds,

(iii)'  $RA_1(y_1 - \Delta(y_1)) \leq RA_1(y_1 + \rho\Delta(y_1))$  for all  $\rho \in [-1, 1]$

(iv)'  $TRA_{12}(y_1 + \rho\Delta(y_1)) \leq RA_1(y_1 - \Delta(y_1))$  for all  $\rho \in [-1, 1]$ .

(iii)' is implied by non-decreasing absolute risk aversion while, as in (ii)', (iv)' holds if temporal risk aversion is less pronounced than static risk aversion.

If preferences are such that a ' $\geq$ ' obtains instead of ' $\leq$ ' in (i) and (ii) [(iii) and (iv)] simultaneously, then  $b(y)$  [ $b_s(y)$ ] is a local minimum. In such cases, however, an increasing Bayes-Nash equilibrium bidding strategy  $b^\#(y)$  [ $b_s^\#(y)$ ] exists that maximizes interim expected utility.

Sufficient conditions for pseudoconcavity of (3.1) and (3.7) in  $b_1$  which ensures global maxima, can be derived along the same lines as for the local maxima. The content of the

resulting conditions is essentially the same as that of (i)-(iv), although the notation becomes considerably messier.

## Appendix 2

### A Proof of Proposition 5

Another way to write equation (5.5) is

$$(A2.1) \quad E[U^3] = \int_{y_1}^{y_u} \int_{y_l}^{y_2} \left\{ \tilde{u}(y_1 + \Delta^0(y_1), Y - m) + \tilde{u}(y_2 - \Delta^0(y_1), Y + m) \right\} dF(y_1) dF(y_2).$$

Taking the derivative of (A2.1) w.r.t.  $m$  and evaluating the result at  $m = 0$  yields

$$\begin{aligned} (A2.2) \quad \frac{dE[U^3]}{dm} \Big|_{m=0} &= \int_{y_1}^{y_u} \int_{y_l}^{y_2} \left( \left\{ \tilde{u}_1(y_1, Y) - \tilde{u}_1(y_2, Y) \right\} \frac{\partial \Delta^0(y_1)}{\partial m} + \left\{ \tilde{u}_2(y_2, Y) - \tilde{u}_2(y_1, Y) \right\} \right) dF(y_1) dF(y_2) \\ &= \int_{y_l}^{y_u} \int_{y_l}^{y_2} \int_{y_l}^{y_2} \left\{ \tilde{u}_{12}(\rho, Y) - \tilde{u}_{11}(\rho, Y) \frac{\partial \Delta^0(y_1)}{\partial m} \right\} d\rho dF(y_1) dF(y_2) \\ &= \int_{y_l}^{y_u} \int_{y_l}^{y_2} \int_{y_l}^{y_2} \tilde{u}_2(\rho, Y) \left\{ \frac{\tilde{u}_2(y_1, Y)}{\tilde{u}_1(y_1, Y)} \frac{\tilde{u}_1(\rho, Y)}{\tilde{u}_2(\rho, Y)} RA_1(\rho, Y) - TRA_{12}(\rho, Y) \right\} d\rho dF(y_1) dF(y_2) \\ &= \int_{y_l}^{y_u} \int_{y_l}^{y_2} \int_{y_l}^{y_2} \tilde{u}_2(\rho, Y) \left\{ \omega(y_1, \rho) RA_1(\rho, Y) - TRA_{12}(\rho, Y) \right\} d\rho dF(y_1) dF(y_2) \\ &> \int_{y_l}^{y_u} \int_{y_l}^{y_2} \int_{y_l}^{y_2} \tilde{u}_2(\rho, Y) \left\{ \omega(y_1, \rho) RA_1(\rho, Y) - TRA_{12}(\rho, Y) \right\} d\rho dF(y_1) dF(y_2). \end{aligned}$$

The third equality follows from the fact that, at  $m = 0$ ,  $\frac{\partial \Delta^0(y)}{\partial m} = \frac{\tilde{u}_2(y, Y)}{\tilde{u}_1(y, Y)}$ . The inequality

follows from the fact that, given the infinitesimal version of Assumption 1 holds,  $\omega(y, \rho)$  is strictly increasing in  $y$ . See Section B of this appendix. QED

### B

For  $m = 0$ ,  $b^0(y) = 0$  for all  $y$ . Thus for an arbitrarily small  $m$ , Assumption 1 translates into

$$(A2.3) \quad \frac{\partial^2 b^0(y)}{\partial y \partial m} = 2 \frac{\tilde{u}_2(y, Y)}{\tilde{u}_1(y, Y)} \{RA_1(y, Y) - TRA_{12}(y, Y)\} > 0 \text{ for all } y,$$

which implies  $RA_1(y, Y) > TRA_{12}(y, Y)$  for all  $y$ . Further,

$$\frac{\partial \omega(y, \rho)}{\partial \rho} \equiv \omega(y, \rho) \{TRA_{12}(\rho, Y) - RA_1(\rho, Y)\},$$

which is always negative if  $RA_1(y, Y) > TRA_{12}(y, Y)$  for all  $y$ , and

$$\frac{\partial \omega(y, \rho)}{\partial y} \equiv \omega(y, \rho) \{RA_1(y, Y) - TRA_{12}(y, Y)\}$$

which is always positive if  $RA_1(y, Y) > TRA_{12}(y, Y)$  for all  $y$ .

### C Proof of Proposition 6 (i) and (ii)

Write equation (5.7) as

$$(A2.3) \quad E[U^1] = \int_{y_l}^{y_u} \int_{y_l}^{y_2} \{\tilde{u}(y_1 + \Delta_s(y_1), Y - m) + \tilde{u}(y_2 - \Delta_s(y_1), Y + m)\} dF(y_1) dF(y_2).$$

Evaluating the derivative of (A2.3) w.r.t.  $m$  at  $m = 0$  and proceeding as in (A2.2) yields

$$(A2.4) \quad \frac{dE[U^1]}{dm} \Big|_{m=0} = \int_{y_l}^{y_u} \int_{y_l}^{y_2} \int_{y_l}^{y_2} \left\{ \tilde{u}_{12}(\rho, Y) - \tilde{u}_{11}(\rho, Y) \frac{\partial \Delta_s(y_1)}{\partial m} \right\} d\rho dF(y_1) dF(y_2) \\ > \int_{y_l}^{y_u} \int_{y_l}^{y_2} \int_{y_l}^{y_2} \left\{ \tilde{u}_{12}(\rho, Y) - \tilde{u}_{11}(\rho, Y) \frac{\partial \Delta^0(y_1)}{\partial m} \right\} d\rho dF(y_1) dF(y_2) = \frac{dE[U^3]}{dm} \Big|_{m=0},$$

where the inequality follows from  $\frac{\partial \Delta_s(y)}{\partial m} > \frac{\partial \Delta^0(y)}{\partial m}$  for all  $y < y_u$ . To see this, notice that, at

$m = 0$ ,  $\Delta^0(y) = \Delta_s(y) = 0$  for all  $y$ . Further, recall that, as a consequence of Proposition 2,

$\Delta_s(y) > \Delta^0(y)$  for all  $m > 0$  and all  $y < y_u$ . Thus, at  $m = 0$ ,  $\frac{\partial \Delta_s(y)}{\partial m} > \frac{\partial \Delta^0(y)}{\partial m}$  for all  $y < y_u$ .

To ensure that, for arbitrarily small  $m$ ,  $b(y)$  is in fact an optimal strategy in the Bayes-

Nash sense, optimality of  $b(y)$  requires  $\frac{d^2 E[U^1 | y_1]}{db_1^2} \Big|_{b_1=b(y_1)}$  from (3.2) to be negative.<sup>21</sup> Since

<sup>21</sup> As in Appendix 1, this requirement only ensures local, not global optimality of  $b(y)$ .

$\frac{d^2 E[U^1|y_1]}{db_1^2}$  , evaluated at  $m=0$ , is equal to zero, for an infinitesimally small  $m$ ,

optimality of  $b(y)$  thus requires that

$$(A2.5) \quad \frac{\partial^3 E[U^1|y_1]}{\partial m \partial b_1^2} \Big|_{b_1=b(y_1), m=0} = 2 f(y_1) \tilde{u}_2(y_1, Y) [TRA_{12}(y_1, Y) - RA_1(y_1, Y)] < 0 \text{ for all } y_1,$$

which is satisfied when  $TRA_{12}(y_1, Y) < RA_1(y_1, Y)$  for all  $y_1$ . But this is exactly what the infinitesimal version of Assumption 1 says [see equation (A2.3)]. QED

#### D Proof of Proposition 7

Writing equation (5.9) as

$$\begin{aligned} \frac{dE[U^2]}{dm} \Big|_{m=0} &= \int_{y_l}^{y_u} \int_{y_l}^{y_2} \left( \{ \tilde{u}_1(y_1, Y) - \tilde{u}_1(y_2, Y) \} \frac{\partial \Delta(y_2)}{\partial m} + \{ \tilde{u}_2(y_2, Y) - \tilde{u}_2(y_1, Y) \} \right) dF(y_1) dF(y_2) \\ &> \int_{y_l}^{y_u} \int_{y_l}^{y_2} \left( \{ \tilde{u}_1(y_1, Y) - \tilde{u}_1(y_2, Y) \} \frac{\partial \Delta^0(y_l)}{\partial m} + \{ \tilde{u}_2(y_2, Y) - \tilde{u}_2(y_1, Y) \} \right) dF(y_1) dF(y_2) \\ &= \int_{y_l}^{y_u} \int_{y_l}^{y_2} \int_{y_l}^{y_2} \tilde{u}_2(\rho, Y) \{ \omega(y_l, \rho) RA_1(\rho, Y) - TRA_{12}(\rho, Y) \} d\rho dF(y_1) dF(y_2). \end{aligned}$$

The inequality follows from the fact that, at  $m=0$ ,  $\frac{\partial \Delta(y)}{\partial m} > \frac{\partial \Delta^0(y_l)}{\partial m}$  for all  $y > y_l$ . To see this,

notice that  $\Delta(y) > \Delta^0(y_l)$  for all  $y > y_l$  and  $m > 0$  (see Proposition 3). Further, at  $m=0$ ,

$\Delta^0(y) = \Delta(y) = 0$  for all  $y$ . Thus, at  $m=0$ ,  $\frac{\partial \Delta(y)}{\partial m} > \frac{\partial \Delta^0(y_l)}{\partial m}$  for all  $y > y_l$ .

Ensuring that, for arbitrarily small  $m$ ,  $b_s(y)$  is in fact an optimal strategy in the Bayes-Nash sense, yields the same expression as equation (A2.5). QED

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