

Model Uncertainty and Liquidity

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Abstract

We investigate the dynamic portfolio problem of a market-maker for a derivative security whose preferences exhibit uncertainty aversion (Knightian uncertainty). The Choquet-expected utility implied by such preference is used to capture the feature that the trader is uncertain about which model should be used. The prices that emerge from the model are similar to standard models and have the feature that as uncertainty is removed, the derivative prices converge to standard prices. However, the optimal changes in the agent's portfolio that results from the option position are quite different than the standard hedge position. It is this feature that links uncertainty with market liquidity.

Extended Abstract

One of the puzzles from the August 1998 Russian debt default and the subsequent collapse of Long Term Capital Management was that during the crisis market liquidity in emerging market debt dried up. Anecdotal evidence suggests that people were unable to trade emerging market debt, and even some corporate bonds, at any price. The drop in the yield of treasury bonds and the rise in the credit spread has been called a “flight to quality.” In this paper we

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investigate how a market collapse (lack of liquidity and trade) can result from uncertainty about the “model” of the world. For example, banks that trade derivatives typically assume some model for the stochastic process for an underlying security (e.g., in Black and Scholes (1973), this is a log-normality assumption). In the context of this assumed model they price a derivative contract and hedge their position. In addition, banks typically use an ad hoc heuristic to manage the risk that the assumed model is not a complete or accurate characterization. This often takes the form of “stress testing” or “value at risk” calculations.

In the paper we capture “model uncertainty” formally in preferences that exhibit uncertainty aversion (often described as Knightian uncertainty). These preferences allow risk (the outcome of a coin toss) and uncertainty (the unknown probability that the coin falls heads) to enter separately. This distinction, introduced by Knight (1921) is documented in the Ellsberg Paradox (Ellsberg (1961)). Gilboa and Schmeidler (1989) developed the axiomatic structure for such preferences showing that an aversion to uncertainty can be represented by a Choquet expected utility that calculates expected utility by minimizing over the multiple possible prior beliefs

Specifically in our paper, we look at the dynamic portfolio problem under Knightian uncertainty of a derivative-market maker. In contrast to the representative agent approach taken in Epstein and Wang (1994) and (1995), we explicitly solve the portfolio problem of an uncertainty-averse agent. From this optimal portfolio, we consider the effect of adding a small long or short position in a derivative (e.g., a call option). We define the price of the derivative contract as the wealth needed to compensate the agent for the change in terminal consumption caused by the derivative. By considering both short and long positions, we construct a bid-ask spread caused entirely by the model uncertainty. The prices that emerge from the model are similar to standard models and have the appealing feature that as uncertainty is removed (the set of prior beliefs converges to a singleton), the derivative prices converge to standard prices. However, the optimal change in the agent’s portfolio

that results from the option position are quite different than the standard hedge position. For example in the standard Black-Scholes set-up, the amount of stock used to hedge a single call option lies between zero and one share. In contrast, under Knightian uncertainty, the “hedge” portfolio can be much larger than one or even negative. It is this feature that leads us to explore how model-uncertainty and liquidity are linked. Future work includes an investigation of how market trade can act to reduce or increase the amount of uncertainty.

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