APPENDIX: Derivation of The Compensation Components

In this appendix we formally derive the compensation components. We consider the labor supply problem of a worker who maximizes a well-behaved twice differentiable utility function defined on annual consumption and leisure, $V(c,l)$.[1] We pick the composite consumption good as the numeraire, set total time equal to 1 and express the minimum (full) expenditure function for an unconstrained worker facing the wage rate w as:

$$\text{(A1)} \qquad e(w,V^*) = \min_{c,h}\left\{c + w(1-h)\,;\, V(c,1-h) \geq V^*\right\}.$$

By construction, when $w = w^*$ the solution to this minimization problem yields the unconstrained equilibrium labor supply of $h = h^* < 1$, consumption level $c = w^*h^*$, and utility $V(c^*,1-h^*) = V^*$.

Next, consider the restricted expenditure function for a worker facing the labor supply constraint $h = \bar{h}$, whose reservation utility level remains at V^*:

$$\text{(A2)} \qquad \bar{e}((1-\bar{h}), w, V^*) = \min_{c}\left\{c + w(1-\bar{h})\,;\, V(c,1-\bar{h}) \geq V^*\right\}.$$

Let $w_1 > w^*$ denote the wage rate which yields utility V^* for $h = \bar{h} < h^*$. The increase in the minimum full expenditure that restores the worker to the reservation utility level is the sum of the change in actual consumption, plus the change in the value of leisure:

$$\text{(A3)} \qquad \bar{e}((1-\bar{h}), w_1, V^*) - e(w^*, V^*) = w_1\bar{h} - w^*h^* + w_1(1-\bar{h}) - w^*(1-h^*) = w_1 - w^*.$$

A second equation that links the quantities of interest can be obtained by defining the virtual (real) wage $\omega(\bar{h}, V^*)$ as the relative price at which the unconstrained worker would choose to supply $\bar{h}$ units of labor. Following the

[1] See Neary and Roberts (1980: 27) for the conditions this imposes on the preference ordering.

derivation in Neary and Roberts (1980: 30), or Deaton and Muelbauer (1981: 1527), we get:

(A4) $$\bar{e}((1-\bar{h}), w_1, V^*) = e(\omega(\bar{h}, V^*), V^*) + (w_1 - \omega(\bar{h}, V^*))(1-\bar{h}).$$

A Taylor series expansion of the right hand side of (A4) around the point $\bar{h} = h^*$ yields:

(A5) $$\bar{e}((1-\bar{h}), w_1, V^*) \cong e(w^*, V^*) + (w_1 - w^*)(1-\bar{h}) + \frac{1}{2\eta}\frac{w^*}{h^*}(\bar{h} - h^*)^2,$$

where $\eta \equiv \left.\frac{\partial h}{\partial w^*}\right|_{h=h^*} \frac{w^*}{h^*} > 0$ is the compensated labor supply elasticity for the unconstrained individual evaluated at the equilibrium number of hours.

To obtain (A5) we also relied on the following:

(i) $w^* = \omega(h^*, V^*)$;

(ii) $\left.\partial\omega(\bar{h}, V^*)/\partial\bar{h}\right|_{\bar{h}=h^*} \cong 1/(\partial h^*/\partial w^*)$;

(iii)From Shephard’s lemma, $\partial e(\omega(\bar{h}, V^*), V^*)/\partial\omega = (1-\bar{h})$;

(iv)The ration $\bar{h}$ and the unconstrained labor supply h^* do not depend on w_1.

Combining (A3) and (A5), our second order approximation to the first compensation component for anticipated risk is obtained as:

(A6) $$\frac{w_1 - w^*}{w^*} \cong \frac{1}{2\eta}\frac{(\bar{h} - h^*)^2}{\bar{h}h^*}.$$

Given the two argument utility function $u(c,l)$, where c is subject to random shocks, Killingsworth (1983: 258) suggests that a measure of relative risk-aversion along the lines of Arrow and Pratt may be constructed as

(A7) $$r = -c\, u_{cc}(c,l)/u_c(c,l).$$

We follow Killingsworth and focus on the variation in consumption, ignoring the induced variation in lesiure. In our case consumption is a function of two random variables, y_0 and y_1. We modify (A7) as

(A8) $r_e = -c\, u_{ee}(c(y_e),l)/u_e(c(y_e),l)$, e = 0,1.

That is, to characterize aversion towards unemployment and employment risk, we rely on two sets of derivatives with respect to consumption, viewed in turn as a function of the random y_0 and y_1. We evaluate the derivatives in question at the means of y_0 and y_1 to get:

(A9) $$\rho_0 = c\varphi\left(\frac{\overline{V}_{11}}{\overline{V}_1}c + 2\right),$$

(A10) $$\rho_1 = c\varphi\left(\frac{\overline{V}_{11}}{\overline{V}_1}(c - w_3) + 2\right) = \rho_0 - c\left(\frac{\overline{V}_{11}}{\overline{V}_1}w_3\right),$$

where $c = w_3(\pi_1 - \alpha\varphi)$, and $\overline{V}_1$, $\overline{V}_{11}$ denote the first and second derivatives of $V(.)$ with respect to consumption, evaluated at the fully compensated rationed equilibrium $\{w_3\pi_1 - b\varphi, \pi_0\}$. It follows that $\rho_1 > 0$ and $\rho_1 > \rho_0$.

To derive the compensation components associated with unanticipated risk, we ignore the variation in leisure, rely on a Taylor series approximation to random utility $V(\frac{w_3 y_1 - b}{y_0 + y_1}, \frac{y_0}{y_0 + y_1})$ at the point $y_0 = \mu_0$ and $y_1 = \mu_1$, and take expectations to obtain:

(A11) $$EV(\frac{w_3 y_1 - b}{y_0 + y_1}, \frac{y_0}{y_0 + y_1}) \cong V(w_3\pi_1 - b\varphi_1, \pi_0)$$
$$+ \tfrac{1}{2}\varphi^2\overline{V}_1 c\left(\frac{\overline{V}_{11}}{\overline{V}_1}c + 2\right)\sigma_0^2 + \tfrac{1}{2}\varphi^2\overline{V}_1(c - w_3)\left(\frac{\overline{V}_{11}}{\overline{V}_1}(c - w_3) + 2\right)\sigma_1^2.$$

The equilibrium condition that relates w_3 and w_2 is:

(A12) $$EV\left(\frac{w_3 y_1 - b}{y_0 + y_1}, \quad \frac{y_0}{y_0 + y_1}\right) = V\left(\frac{w_2 \mu_1 - b}{\mu_0 + \mu_1}, \quad \frac{\mu_0}{\mu_0 + \mu_1}\right)$$

$$= V(w_2\pi_1 - b\varphi,\ \pi_0) \equiv \overline{V}\ .$$

Before exploiting the equality of (A11) and (A12), we express the first term on the right hand side of (A11) as:

(A13) $$V(w_3\pi_1 - b\varphi,\ \pi_0) = V(w_2\pi_1 - \alpha w_2\varphi + (w_3 - w_2)(\pi_1 - \alpha\varphi),\ \pi_0) \equiv \overline{V}^{+},$$

and note the fact that for small $(w_3-w_2)(\pi_1-\alpha\varphi)$,

(A14) $$(\overline{V}^{+} - \overline{V}\ )/(w_3 - w_2)(\pi_1 - \alpha\varphi) \cong \overline{V}_1\ .$$

With this simplification in hand, manipulation yields:

(A15) $$\frac{w_3 - w_2}{w^*} \cong \frac{1}{2}\frac{\rho_0}{w^*}\frac{1}{(\mu_1 - \alpha)}\sigma_0^2 + \frac{1}{2}\frac{\rho_1}{w^*}\frac{(\mu_0 + \alpha)}{(\mu_1 - \alpha)^2}\sigma_1^2\ .$$