Increasing Competition and the Winner's Curse: Evidence from Procurement*

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Abstract

We empirically measure the effects increasing competition on equilibrium bidding in procurement auctions. In common value auctions, the winner's curse counsels more conservative bidding as the number of competitors increases. First, we estimate the structural parameters of an equilibrium bidding model and test for the importance of common value components in bidders' preferences. Second, we use these estimates to measure the effects on increasing competition on both individual bids as well as winning bids (ie. procurement costs).

We analyze bid data from construction procurement auctions run by the New Jersey department of transportation in the years 1989-1997. Our results indicate that the winner's curse is particularly strong in a large subset of these auctions. For the median bidder, the percentage markup increases from 50% with 2 bidders to above 70% with 10 bidders. Furthermore, the median procurement cost is increasing in the number of bidders as competition intensifies: for example, the median costs rise about 30%, as the number of bidders increases from 3 to 6. These results emphasize a policy-relevant setting in which asymmetric information can overturn the common economic wisdom that more competition is always desirable.

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1 Introduction

How does an increase in competition affect equilibrium bidding in an auction? According to the Walrasian analogy of markets as auctions, an increase in the number of bidders should encourage more aggressive bidding, so that in the limit, as the number of bidders becomes arbitrarily large, the imperfectly-competitive auction setting approaches the efficient perfectly-competitive outcome.

While this is true for private value auctions, it may not be true in common value auctions in which the competing bidders are differentially (but incompletely) informed about the value of the object which they are trying to win. A distinctive feature of common-value auctions is the *winner's curse*, an adverse-selection problem which arises because the winner tends to be the bidder with the most overly-optimistic information (or "signal") concerning the object's value. Bidding naively based simply on one's information would lead to negative expected profits, so that in equilibrium, a rational bidder internalizes the winner's curse by bidding less aggressively.[1]

In common value settings, then, an increase in the number of bidders has two counteracting effects on equilibrium bidding behavior. First, the increased competition generally leads to more aggressive bidding, as each bidder tries to maintain her chances of winning against more rivals: we call this the *competitive effect*. Second, the winner's curse becomes more severe as the number of bidders increases, and rational bidders will bid less aggressively in response: we call this the *winner's curse effect*. If the winner's curse effect is large enough, the possibility arises that prices could actually rise as the number of market-makers increases Recently, Bulow and Klemperer (1999), Krishna and Morgan (1997), and Bordley and Harstad (1996) have pointed out this possibility; as these authors note, the winner's curse is a prominent example where asymmetric information can overturn the common economic wisdom that more competition is always desirable.

In this paper, we investigate the empirical importance of these considerations using bid data from construction procurement auctions run by the New Jersey department of transportation in the years 1989–1997. We address two questions in turn. First, we test for the importance of common value components in bidders' preferences. This is important because winner's curse considerations only arise in common value settings. Second, provided

[1] More precisely, in the procurement setting, even if any bidder's signal x is an unbiased estimate of the unknown (but common) project cost w (i.e., $E(x) = w$), conditional on winning the signal is an downwardly-biased (in the case of procurement auctions where the lowest bid wins) estimate of w (i.e., $E(x|\text{win}) < w$). This implies that if a bidder were to naively bid her unbiased signal x, her expected profits would be *negative*.

we find some evidence of common values, we investigate how an increase in the number of competitors affects equilibrium bidding.

Specifically, we quantify two comparative statics which have been the focus of theoretical work. First, does an increase in the number of competitors leads a given bidder to bid more or less aggressively, in equilibrium? Theoretical examples in Smiley (1979) and Matthews (1984) have shown that the result often depends on the parametric assumptions made about the information structure, and we measure the effects given the parameter estimates for our model of equilibrium bidding. Second, and more important for policy purposes, does the winning bid rise or fall as competition increases? Previous theoretical work (eg. Wilson (1977), Milgrom (1979)) have specified limit laws for the winning bid, as the number of bidders grows arbitrarily large. In contrast, we measure these effects for the (often modest) range of bidders which we observe in real-world auctions. To our knowledge, we are the first to address these issues empirically.

The procurement setting is particularly pertinent for the issues raised above. Although exact figures are difficult to come by, McAfee and McMillan (1987) (pg. 3) state that "the national, regional, and local governments in a typical modern market economy together spend between one-quarter and one-third of national income on goods and services [...]; of this amount, perhaps one-half [...] is paid by governments to firms." Many of these payments are for contract work awarded via low-bid auctions identical to that considered here, so that there appear to be important efficiency and revenue lessons to be learned from the results.

Our results show that different types of contracts differ significantly in the degree that private and/or common value components are important. Auctions for highway work contracts are very close to a pure common value auction, while both common value and private value components are important in auctions of bridge repair and road paving contracts. Furthermore, our results indicate that the winner's curse is particularly strong in highway contract auctions. Simulated bid functions show that for the median bidder, the percentage markup increases from 50% with 2 bidders to above 70% with 10 bidders. Furthermore, winning bid simulations indicate that the average procurement cost is strictly increasing in the number of bidders as competition intensifies: for example, the average costs rise about 30%, as the number of bidders increases from 3 to 6. For these auctions, the optimal number of participants (which would minimize expected procurement costs) would be 5. Clearly, then, there are cases where the "law of demand" is violated: an increase in competition leads to *higher* procurement costs.

In the next section we describe the model of equilibrium bidding which we employ in our work. In section 3, we introduce our data on New Jersey department of transportation procurement auctions, and discuss institutional features which affect our model specification, which we introduce in section 3. In section 4 we discuss our estimator. Section 5 contains the empirical results, and in section 6 we discuss the policy implications of our results. Section 7 concludes.

2 Equilibrium bidding in low-price procurement auctions

In our empirical work, we employ a structural approach which allows us to recover bidders' equilibrium strategies. These are required for our investigation into how increasing competition affects equilibrium bidding behavior.[2] We build on the previous literature by considering a model where bidders' valuations have both private and common value components. Such a model seems especially warranted for procurement settings, where uncertainty about future input prices could drive common values but differences in input efficiency across firms could drive private values.[3]

Next, we briefly describe equilibrium bidding behavior in single-unit, low-price procurement auctions.[4] We delay discussion of the specification of contractors' costs until later. (Since much of this section is standard single-unit auction theory, readers who are familiar with this literature may wish to skip to the next section.)

An auction has n risk-neutral contractors (indexed $i = 1, \ldots, n$), each of whom has a cost c_i associated with completing the project, and receives a private signal x_i about c_i. Contractor

[2]The seminal empirical auction papers in the structural vein are by Paarsch (1992) and Laffont, Ossard, and Vuong (1995). Most recently, important progress has been made in the structural estimation of private value auction models (cf.Bajari (1998), Guerre, Perrigne, and Vuong (1999), Li, Perrigne, and Vuong (1998), Deltas and Chakraborty (1997)).

[3]In previous work (Hong and Shum (1997)), we have empirically implemented an equilibrium ascending auction model also allowing for both private and common values. Previously, Bajari (1998) and Pesendorfer and Jofre-Bonet (1999) have modeled procurement auctions in a private-values framework. Furthermore, Hendricks, Pinkse, and Porter (1999) and Bajari (1999) have also considered common value models recently, but they consider pure common value (i.e., "mineral rights") models of competitive bidding. Our model has both common and private values.

In a previous study of procurement auctions, Thiel (1988) concluded that observed bids do indeed reflect winner's curse considerations, but we take the analysis one step further by showing what this implies about equilibrium bids and, more important, equilibrium procurement costs.

[4]The assumption of rational equilibrium bidding which characterizes our approach is potentially at odds with findings in the experimental literature (cf. Kagel and Levin (1986)), which find that bidders only learn to internalize the winner's curse (i.e., bid "rationally") through experience. In the procurement setting, however, the bidders are by and large experienced firms, so we feel our assumption of rational bidding is justified.

i observes only x_i prior to the beginning of the auction, but not any of the costs, c_j, for $j = 1, \ldots, n$, or any of her rivals' signals, x_j, for $j \neq i$.

The contractors' costs and private signals are assumed to be distributed according to the distribution function $\tilde{F}(c_1, \ldots, c_n, x_1, \ldots, x_n; \theta)$ parameterized by the vector of parameters θ, which are the parameters of interest in the estimation process. As we describe in the next section, we consider a specification which allows both common and private value components in bidders' cost functions.[5]

The low-price auction proceeds as follows: observing x_i, contractor i chooses a bid b_i to maximize his expected payoff, given the other contractors' equilibrium behavior:

$$b_i = \text{argmax}_b \mathcal{E}_{x_j, j \neq i} \mathcal{E}_{c_i | x_1, \ldots, x_n} \left[(b - c_i) \mathbf{1} \left(x_j \geq s_{j,n}^{-1}(b), j \neq i \right) | x_i \right]$$

where $s_{i,n}(\cdot), i = 1, \ldots, n)$ denotes the equilibrium bidding strategy (or *bid function*) for contractor i in an n-bidder auction.

We assume that contractors are *symmetric*, in the sense that the joint distribution $\tilde{F}$ is exchangeable with respect to the indexes $1, \ldots, n$.[6] As is standard, we assume that the random variables $(c_1, \ldots, c_n, x_1, \ldots, x_n)$ are affiliated.[7] Given these assumptions, there is a unique pure-strategy Bayesian Nash equilibrium in which each contractor bids according to identical, monotonically increasing strategies (i.e., $s_j(\cdot) \equiv s(\cdot), \ \forall j$).[8]

The first order condition of this maximization problem is (cf. (Milgrom and Weber, 1982,

[5] In the pure *private value* paradigm, $c_i = x_i \ \forall \ i$ (i.e. each bidder knows his true valuation for the object) while in the pure *common value* paradigm $c_i = c, \ \forall \ i$ (i.e. the value of the object is the same to all bidders, but none of the bidders knows the true value of the object; here the individual x_i's are noisy signals of the true but unknown c).

[6] Milgrom and Weber (1982) provided the seminal analysis for the symmetric versions of most of the usual auction forms. However, much of the recent theoretical work has focused on asymmetric cases (Maskin and Riley (1996), Bulow, Huang, and Klemperer (1999), Lebrun (1999)). The extension to asymmetric bidders will resemble the treatment in Campo, Perrigne, and Vuong (1998), and is the focus of ongoing research. In the next section we discuss an explicit test of the symmetry assumption using our data.

[7] cf. Milgrom and Weber (1982). Affiliation roughly implies that large values for some of the variables make the other variables more likely to be large than small. Given two n-vectors $\vec{x}_1$ and $\vec{x}_2$ which are *i.i.d.* realizations from F, let $\bar{x}$ denote the componentwise maximum of $\vec{x}_1$ and $\vec{x}_2$, and $\underline{x}$ the component-wise minimum. Affiliation means that the likelihood of $(\bar{x}, \underline{x})$ is at least as high as that of $(\vec{x}_1, \vec{x}_2)$. In our log-additive specification (described below), we actually make the stronger assumption that $x_1, \ldots, x_n$ are mutually positive correlated.

[8] Symmetry and monotonicity imply that $b_i > b_j \Leftrightarrow s_n(x_i) > s_n(x_j) \Leftrightarrow x_i > x_j$. Analogously, the event that bidder i wins can be simplified: $\mathbf{1}\left(x_j \geq s_{j,n}^{-1}(b_i), j \neq i\right) = \mathbf{1}\left(x_j \geq s_n^{-1}(b_i), j \neq i\right) = \mathbf{1}\left(\min_{j \neq i} x_j \geq x_i\right)$, since $x_i = s_n^{-1}(b_i)$ in equilibrium.

pg. 1107))

$$s_n(x_i) = \frac{s_n'(x_i)\left(1 - F_{-i}(x_i|x_i)\right)}{f_{-i}(x_i|x_i)} = v_n(x_i, x_i), \tag{1}$$

a differential equation which defines the equilibrium bidding strategy $s_n(x_i)$. In this equation, $v_n(\cdot,\cdot)$ is the conditional expectation

$$v_n(x,y) = \mathcal{E}\left[c_i | x_i = x, \min_{j\in[1,n];j\neq i} x_j = y\right],$$

where the expectation is taken over the posterior distribution of c_i given the joint event $\left(x_i = x, \min_{j\in[1,n];j\neq i} x_j = y\right)$, and $f_{-i}(\cdot|\cdot)$ denotes the conditional density of $\min_{j\neq i} x_j$ conditional on x_i. Integrating out this differential equation, the equilibrium bid function can be analytically expressed as (Weber (1983)[pg. 174]):

$$s_n(x_i) = \mathcal{E}_{y_{-i}|x}\left[v_n(y_{-i}, y_{-i}) | y_{-i} > x_i\right] \tag{2}$$

where $y_{-i} \equiv \max_{j\in[1,n];j\neq i} x_i$[9], and the n subscript emphasizes that for a given x_i, the equilibrium bid $s_n(x_i)$ varies for different n.

2.1 Winner's Curse vs. Competitive Effect

The *competitive* and *winner's curse* effects alluded to earlier are distinguishable in equation (1) above. From this equation we see that an equilibrium bid $b_i = s(x_i)$ is governed by two important components: (1) the $\frac{s'(x_i)(1-F_{-i}(x_i|x_i))}{f_{-i}(x_i|x_i)}$ term and (2) the $v_n(x_i, x_i)$ term. Given our affiliation assumptions, an increase in n will increase the second term $v_n(x_i, x_i)$, holding x_i fixed (cf. (Milgrom, 1982, section 6)).[10] This is the winner's curse effect. On the other hand, an increase in n will generally lower the first term, because the $1 - F_{-i}(x_i|x_i)$ portion of the numerator of the first term is essentially the probability of winning the low-bid auction for a given signal x_i, and this probability shrinks to zero as n increases. This is the competitive effect.

[9]Note: $\mathcal{E}_{y_{-i}|x}\left[v_n(y_{-i}, y_{-i})|y_{-i} > x_i\right] = \int_x^\infty v_n(\alpha,\alpha) dF_{y_{-i}|x}(\alpha | y_{-i} > x) = \frac{\int_x^\infty v_n(\alpha,\alpha) dF_{y_{-i}|x}(\alpha)}{1 - F_{y_{-i}|x}(x)}$. Plugging in the identity $F(y) = \exp\left(-\int_y^\infty \frac{f(s)}{F(s)} ds\right)$, one obtains an expression analogous to that given in (Milgrom and Weber, 1982, Theorem 14) (which is for the high-bid auction).

Weber's expression is perhaps the most intuitive, especially in comparison with the equilibrium bid strategy for the second-price auction, which can be expressed as $\mathcal{E}_{y_{-i}}\left[v(y_{-i}, y_{-i})|y_{-i} = x_i\right]$.

[10]This is only true for common value models, but not for private vale models. In ongoing work (Haile, Hong, and Shum (2000)), we are using this insight as the basis of a formal test for the presence of common value components, based upon recent developments by Guerre, Perrigne, and Vuong (1999) and Hendricks, Pinkse, and Porter (1999) in the nonparametric estimation of auction models.

In private value auctions, the winner's curse effect is absent, so *ceteris paribus* we should expect the markups to decrease as the number of participants increases, due solely to the competitive effect. In auctions where bidders' costs have both common and private value components (as is the case in the model we employ), it is unclear which effect will dominate: this is an empirical question, which can only be answered once one has estimates of the structural parameters in bidders' preferences.

Next, we describe the particular procurement setting which we study in this paper, and discuss issues related to applying the model of equilibrium bidding described in this section to this empirical setting.

3 New Jersey Department of Transportation Construction Services Procurement Auctions

In order to examine empirically the questions we have raised, we collected a dataset of bids submitted in procurement contract auctions conducted by the New Jersey department of transportation (NJDOT) in the years 1989-1997. Over this period, the NJDOT conducted 1018 low-price sealed bid auctions of contracts to procure various services. For our empirical work, we focus on auctions for the three types of jobs: highway work (worktype A in what follows), bridge construction and maintenance (worktype B), and road paving (worktype C), which together account for over half of the contracts auctioned during the sample period. See table 1 for summary statistics for these auctions.

Clearly, the variation in the average winning bid across types of contracts indicate that the jobs proscribed in these contracts are markedly different. For that reason, we estimate separate parameters for each type of contarct in our empirical work.

Table 1: Breakdown of auctions by job type

	Worktype	#auctions	Avg. winning bid (1989$, mil)	Stdev., winning bid	Total bidders	Total winners
2	Grading & Paving	150	0.97	1.11	109	33
3	Bridge construc./repair	194	1.48	1.82	171	74
4	General highway	423	4.97	9.61	274	121

Summary statistics Table 2 presents statistics on the observed bids, broken down according to the number of bidders in the auction. Most auctions have between 3-7 bidders. Importantly, there is a generally increasing (but by no means monotonically so) trend between number of bidders and both submitted and winning bids. For example, the average bid in the worktype A auctions rises from about \$1.8 million in 4-bidder auctions to over \$10 million, in 10-bidder auctions. While this is consistent with the hypothesis that the winner's curse (which leads to more cautious — higher — bids) dominates the increased competition effect (which leads to more aggressive — lower — bids), a more likely and non-strategic explanation is that these auctions are characterized by selective participation and contract heterogeneity, so that larger projects (which, from a purely cost perspective, command larger bids) attract more bidders. This suggests that controlling for contract heterogeneity is crucial to measuring the effects of the winner's curse, and we discuss this issue further below.

Before proceeding to discuss specification issues, two remarks about the equilibrium bidding model we have just described are in order.

3.1 Contractor reimbursement schemes: is competitive bidding appropriate?

The contracts offered by the department of transportation are characterized by contractor reimbursement guidelines which have both *cost-plus* (government assumption of all cost overruns above the bid) and *fixed price* (contractor assumption of all cost overruns above the bid) features. Specifically, a contract specifies L tasks, and an associated L-vector of quantities q, which must be performed by the contractor.[11] The contractor essentially "bids" a L-vector of prices p at which it is willing to perform the required tasks.[12] Generally, the government assumes all cost increases due to unexpected increases in job quantities q, and prices out these cost increases at the prices bid by the contractors. This is the *cost-plus* aspect of these procurement contracts. On the other hand, the contractor is bound to supply its services at its submitted prices p; losses arising from unanticipated cost increases are *not* borne by the government. This is the *fixed-price* aspect of the contract.

This cost-plus aspect of these contracts raises several concerns. First, the perfect insurance offered by a cost-plus contract could potentially lead to moral hazard problems since a contractor has little incentive to keep costs low by either taking cost-saving measures or working at optimal productivity levels. However, as McAfee and McMillan (1987) (see also McAfee and McMillan (1986)) point out, such post-contractual opportunism will not affect

[11] This is a simplification, but given that we have data only on the total bids submitted, this example's purposes is solely illustrative.

[12] Note that the contractor is free to bid p above actual cost (or actual expected cost) in order to earn a margin on the contract.

Table 2: Summary statistics on bids

Worktype	Number of bidders	#auctions	Average bid (1989$; mils)	Std. dev.	Median bid (1989$; mils)	Average winning bid
A	2	12	5.894	14.954	1.482	5.601
A	3	31	1.692	2.000	1.042	1.520
A	4	46	1.843	1.919	1.124	1.605
A	5	51	3.380	5.223	1.204	3.015
A	6	58	4.513	8.310	1.369	3.982
A	7	46	4.435	9.751	1.406	3.526
A	8	40	6.365	12.567	3.000	5.229
A	9	39	8.658	15.438	4.016	6.640
A	10	22	10.612	12.828	4.745	9.256
A	11	20	15.087	30.432	2.951	11.471
A	12	17	11.704	10.935	7.739	10.263
A	13	12	10.652	14.879	3.767	8.984
A	14	8	10.523	12.264	4.229	9.350
A	15	8	9.274	10.338	4.885	8.274
A	16	3	2.506	1.053	2.477	1.544
A	17	4	9.999	5.029	9.830	8.583
B	2	12	1.265	0.651	1.171	1.167
B	3	7	1.577	1.244	1.368	1.432
B	4	24	1.672	0.495	1.193	1.386
B	5	12	1.049	0.730	0.999	0.819
B	6	23	1.566	1.265	1.175	1.286
B	7	23	1.278	1.120	0.968	1.001
B	8	16	2.644	3.496	1.064	2.241
B	9	19	1.755	1.439	1.266	1.349
B	10	14	1.480	1.245	1.065	1.141
B	11	7	1.149	0.811	1.010	0.888
B	12	10	1.932	1.899	1.019	1.398
B	13	13	2.261	2.457	1.063	1.869
B	14	5	1.784	1.496	0.728	1.462
B	15	3	8.472	6.032	6.953	6.873
B	16	5	2.658	2.816	1.254	1.952
B	17	6	3.056	3.825	1.352	2.415
C	2	9	0.737	0.418	0.679	0.664
C	3	21	0.629	0.308	0.650	0.546
C	4	23	1.086	0.624	0.968	0.910
C	5	34	1.566	2.266	1.068	1.347
C	6	27	0.826	0.674	0.543	0.705
C	7	19	1.469	1.011	1.320	1.273
C	8	8	0.961	0.608	0.963	0.789
C	9	4	2.424	0.939	2.045	2.059
C	10	5	1.087	0.518	0.951	0.955
C	11	4	1.164	0.801	1.056	0.844
C	12	1	1.231	0.098	1.221	1.105
C	14	1	0.152	0.057	0.138	0.094
C	15	1	0.453	0.049	0.448	0.374

the bidding process *per se*; if the bidding is relatively competitive (an assumption which is maintained through this analysis), any rents arising from such opportunism should be competed away in equilibrium.

Second, and more troubling, contractors have no incentive to submit cost-based bids in an auction for cost-plus contracts for which their eventual payoff is independent of their costs; instead, each contractor has an incentive to bid as low as possible, in order to maximize the probability of winning. This result is potentially problemmatic for our analysis, which focuses on (the non-degenerate case of) equilibrium bids which as are functions of firms' underlying costs.

However, we believe our assumption of competitive bidding to be justified considering certain institutional considerations and patterns in the data. First, repeated interactions may render reputational effects important in this procurement setting. Many of the contractors in these auctions bid on many contracts over time, and likely derive a large part of their revenues from doing contract work for the state.[13] The NJDOT maintains a list of "pre-qualified contractors" which all firms must be on in order to be eligible to bid; given that the government observes *ex-post* compensation from all contracts, it is likely that firms who are judged to have acted opportunistically will be struck off the list. The potential loss of future bidding eligibility may counteract contractors' incentives to submit bids which are non-indicative of their costs. Secondly, contractors' costs are monitored on a fairly regular basis (every few weeks, from conversations with NJDOT officials). The original submitted bid must already indicate clearly projected materials costs, labor costs, and labor hours required. Deviations from these estimates must be rigorously justified. [14]

Most convincingly, however, the raw data seems to support the competitive bidding hypothesis. For a subset of the auctions we study, we were also able to obtain data on the *ex-post* compensation. The regression equation of (log) compensation[15] on (log) winning

[13]In our data, we observe 421 distinct bidders in our dataset, and each firm submits bids in an average of around 15.86 auctions in our dataset. Successful bidders are even more active: in our sample, firms which are awarded at least one contract bid in an average of 29.43 auctions.

[14]The standard text by Halpin and Woodhead (1998) contains examples of typical cost-plus contracts (appendix E), as well as a description of cost control/monitoring techniques which are widely used in practice (chapter 14).

[15]Note that we never observe a firm's actual costs: the basis for firms' compensation are the per-unit costs submitted as part of the bid, which presumably already include a margin above actual costs.

bids is:

$$\log(\text{compensation}) = \begin{pmatrix} 0.0080 \\ (0.226) \end{pmatrix} + \begin{pmatrix} 1.0158 \\ (0.0166) \end{pmatrix} \log(\text{winning bid}) \qquad (3)$$
$$R^2 = .928, \ N = 291$$

indicating no systematic overruns (insignificant constant) and a strong correlation between compensation and the bid (as is consistent with competitive bidding). In fact, cost *underruns* — which are inconsistent with any post-contractual opportunism scenario — occurred in 132/291 of these auctions. In any case, there is no evidence of systematic underbidding, which would be reflected in systematic overruns.[16]

3.2 Collusion and capacity constraints: is symmetry assumption appropriate?

Collusion is a chronic concern in procurement settings. Generally, collusion is one reason which could lead to *ex ante* asymmetries among the contractors, which our symmetric model rules out. Recently, Bajari (1998) and Pesendorfer and Jofre-Bonet (1999) have focused on other sources of asymmetries across contractors due to, respectively, geographical location and capacity constraints.[17]

We exploit the panel nature of our dataset (the availability of observations of bids by the same contractor submitted across different auctions) to explicitly test the symmetry hypothesis. More precisely, under our assumption of *ex-ante* symmetry, each contractor i in an n-bidder auction identically wins the auction with probability $\frac{1}{n}$. Let $T_{n,i}$ denote the number of n-bidder auctions in which contractor i bids, in our dataset; if these auctions are independent over time (which we also assume), the expected number of n-bidder auctions that contractor i wins is $\tilde{W}_{n,i} \equiv \frac{T_{n,i}}{n}$. If $W_{n,i}$ denotes the actual number of n-bidder auctions won by contractor i, then $X_{n,i} \equiv \left(W_{n,i} - \frac{T_{n,i}}{n}\right)$ is approximately distributed normal with zero mean and variance $\frac{T_{n,i}(n-1)}{n^2}$.

For each type of contract, then, let $\bar{X}$ denote the $X_{n,i}$'s averaged across contractors i and across the number of bidders n. By a standard non-*iid* central limit theorem, the normalized average

$$Z \equiv \frac{\bar{X}}{\sqrt{\text{Var}\left(\bar{X}\right)}} \xrightarrow{d} N(0,1)$$

[16] In the context of the bidding example above, however, under-runs occur chiefly because a job has required less time or materials than the contractor originally anticipated in submitting its bid.

[17] They have also both focused on independent private value model, which simplify the analysis of bidder asymmetries greatly.

under the null. Table (3) contains Z's and the associated p-value, for each of the three types of contracts we include in our analysis. Remarkably, in each case, we certainly would not want to reject the null of symmetry. Furthermore, the same result holds even when we restrict the sample to observations where $T_{n,i} \geq 5$ (since the normal approximation of a binomial random variable is generally valid for a large number of trials).[18]

Table 3: Test for symmetry

Worktype	#obs[a]	$Z \equiv \frac{\bar{X}}{\sqrt{\text{Var}(\bar{X})}}$	p-value
Across all observations			
2	363	-0.0000169	0.99999
3	793	-0.0000058	>0.99999
4	1190	-0.0000027	>0.99999
For observations where $\mathbf{M}_{n,i} \geq 5$			
2	42	-0.0002378	0.99810
3	53	0.0021586	0.99828
4	178	-0.0001771	0.99986

[a]Each observation is a pair (n, i) where the datum is $X_{n,i} \equiv \left(W_{n,i} - \frac{T_{n,i}}{n}\right)$.

Despite the convincing nature of these statistical results, we would like to raise an important caveat. Our test has most power against alternatives which imply that the *ex-ante* probability of winning differs across contractors: these include capacity constraints and geographical asymmetries[19], as well as *asymmetric* bid-rotation schemes where some cartel members are allowed to win more often than others. We have tested and cannot reject the hypothesis that, *ex ante*, the probability of winning any auction is identical across all participants.[20]

[18]Although not reported here, the same results obtain when we further restrict $T_{n,i} \geq 10$. In this case, however, there are not many observations left.

[19]Note that our symmetric model allows fully for asymmetries may not exist at other junctures of the bidding process. For example, our model (and these results) do not rule out the possibility that capacity constrained contractors choose not to participate in particular auctions. What we test is that the bidders who choose to participate in a particular auction are *ex-ante* symmetric. Our results imply that capacity constraints and geographic differences may affect the decision to participate more than the amount to bid.

[20]On the other hand, this hypothesis is indistinguishable from a perfectly symmetric bid rotation collusion scheme (similar to that considered by Porter and Zona (1993)) involving a full cartel where the non-winning bidders submit "phantom bids" which have no probability of winning. In such a case, and assuming an n-bidder cartel, each contractor wins $\frac{1}{n}$ of the auctions; if there are T auctions, then, each bidder wins a fraction $\frac{T}{n}$ of them, which is observationally equivalent to an environment where each bidder faces an identical probability of winning. However, the large number of bidders vying for each type of contract during the sample period (cf. column 5 of table (1)) would appear to render full cartels very difficult.

4 Specification and Estimation

4.1 Specification details: the Wilson log-additive model

Next, we describe the specific parameterization of contractors' costs which we employ. We follow Wilson (1998) in choosing a log-additive form for the cost function c_i.[21] In particular, we focus on a *symmetric* version of Wilson's log-additive model. Contractor i's cost c_i is assumed to take the form

$$c_i = a_i \times w,$$

where a_i is a bidder i's private cost from undertaking the project (which could includes differences in labor efficiency between firms), and w is an unknown cost component which is common across all bidders (including, for example, uncertainty in future materials costs). In other words, c_i is the product of a common value (w) part and a private value (a_i) part.[22]

We assume that w and the a_i's are independently log-normally distributed: letting $\tilde{w} \equiv \log w$ denote the natural log of w, and $\tilde{a}_i \equiv \log a_i$, then

$$\begin{aligned} \tilde{w} &= m + \epsilon_w \sim N(m,\ \sigma_w^2) \\ \tilde{a}_i &= \bar{a} + \epsilon_{a_i} \sim N(\bar{a},\ \sigma_a^2). \end{aligned}$$

Each bidder is assumed to have a noisy signal of her cost of fulfilling the contract terms, x_i, which has the form

$$x_i = c_i \times e_i.$$

Here x_i is a contractor i's noisy estimate of the unknown cost c_i. $e_i = \exp\{s_i \xi_i\}$ in which ξ_i is an (unobserved) error term that has a normal distribution with mean 0 and variance 1. If we let $\tilde{c}_i \equiv \log c_i$ and $\tilde{x}_i \equiv \log x_i$, then conditional on $\tilde{c}_i$, $\tilde{x}_i = \tilde{c}_i + \epsilon_{e_i} \sim N(\tilde{c}_i,\ \sigma_e^2)$.[23]

σ_w, σ_a, and σ_e are parameters to be estimated. Since m and $\bar{a}$ always appear together as a sum in this manner, we will not be able to estimate both parameters, but just their sum

[21] See also Wilson (1983), in which a similar specification was considered for a symmetric first-price auction.

[22] Given that c_i represents firm i's costs, there is a natural interpretation of the common component w as an index of unknown future input prices, and a_i as a "quantity index" of inputs (where the amount of each input required depends on firms' efficiency levels). Standard assumptions (cf. (Varian, 1992, ch. 9)) on the production technology enable one to aggregate the inner product of vectors of inputs and input prices as the product of a single price and quantity index.

[23] Note that bidder i observes only one signal x_i, and is not assumed to be able to distinguish between its two components a_i and $w \times e_i$. Wilson (1998) allows for this by assuming a diffuse prior assumption on the common value w (i.e., $\sigma_w \to \infty$).

$\mu \equiv m + \bar{a}$.[24]

The relative magnitudes of the σ's are indicative as to the relative importance of common and/or private value components in bidders' preferences. As $\sigma_e \to 0$, bidders' uncertainty about a common component to costs disappears, and the model resembles a pure (correlated) private values model. As $\sigma_a \to 0$, the importance of the idiosyncratic component in bidders' valuations falls, implying a pure common value model. As $\sigma_w \to 0$, bidders' uncertainty about the common component w disappears, making the model a correlated private value model, but one in which bidders' imperfectly observe their private value (since $\sigma_e > 0$).[25]

4.2 Estimation approach: monotone quantile estimator

Equation 2 shows how assumptions on the joint distribution $F(x_1, v_1, \ldots, x_n, v_n; \theta)$ induces a joint distribution for the equilibrium bids, which we observe. We include θ explicitly as an argument in $s(x_i, \theta)$ to emphasize this dependence. Given that the distribution of $s_n(x;\theta)$ is likely to be quite asymmetric, even if we assume any individual x to be symmetrically distributed[26], we estimate the parameters via quantile restrictions, which try to match the "shapes" of the distributions of the observed bids and the $s_n(x;\theta)$. As is well-known, quantile estimators are more robust to outliers in the data than estimators based on matching the centered moments.

Two insights drive our estimation procedure. First, the quantiles of a distribution $F(x)$ are invariant to monotonic transformations of the random variable x. Second, for our symmetric first-price auction, the equilibrium bidding strategies $s_{M_i}(x;\theta)$ are monotonic transformations of the unobserved signals $x \sim F(x;\theta)$, where $F(\cdots)$ denotes the marginal distribution for a single signal.[27]

In particular, $q_{\tau_k}^{M_i}(\theta)$, the τ_k-th quantile of the equilibrium bid function for the i-th auction is just $s_{M_i}(x_{\tau_k}, \theta)$, the equilibrium bid function evaluated at x_{τ_k}, the τ_k-th quantile of the

[24] m, the mean of the prior distribution of w, could potentially include a "cost-padding" component which represents the bidders' common opportunities to engage in cost-inflation activities while undertaking the project. In this way, we accommodate moral hazard issues which are otherwise absent from our analysis. We feel that this is adequate since, as McAfee and McMillan (1987) note, equal cost-padding (or "shirking") opportunities across bidders will simply shift up bidders' costs by an equal amount, and not affect equilibrium bidding.

[25] In short, this is a private value model where bidders don't observe their private value model; it can be turned into a standard PV model where we redefine the private value as $\mathcal{E}c_i|x_i$.

[26] Recall that $\log x$ is normally distributed.

[27] For our log-normal model, $x_{\tau_k} = \exp\left(\sigma\Phi^{-1}(\tau_k) + \mu\right)$, where $\Phi(\cdot)$ is the standard normal CDF, and μ and σ are, respectively, the mean and variance of $\log x_{\tau_k}$.

marginal distribution $F_{M_i}(x;\theta)$.[28]

Our estimator $\hat{\theta}$ minimizes the quantile objective function:

$$Q(\theta) = \sum_{i=1}^{T} \sum_{j=1}^{M_i} \sum_{k=1}^{K} \rho_{\tau_k} \left(b_{ij} - q_k^{M_i} \left(x_{\tau_k}; \theta \right) \right)$$

where the function $\rho_{\tau_k}(\cdot)$ is defined as

$$\rho_{\tau_k}(x) = (\tau_k - 1(x \leq 0))\, x.$$

There are two important features of this estimator. First, it relieves (dramatically) the computational burden associated with simulating the moments of the equilibrium bid distribution. Second, this estimator has a formal appeal since the quantile restrictions follow wholly (and *solely*) from the theoretical result that the bidding strategies are monotonic transformations of the signals in equilibrium. Such an estimation approach is potentially very useful in other incomplete information settings where the equilibrium strategies (or the "policy functions") are monotonic transformations of the unobserved types: for example, nonlinear pricing (and, more generally, mechanism design) models appear very well-suited to this approach.[29]

5 Estimation results

Table 4 presents estimates of the four parameters of the benchmark model. In the discussion below, standard errors are enclosed in square brackets ($[\cdots]$).

Recall that the model resembles a private value model as $\sigma_e \to 0$, and resembles a pure common value model as $\sigma_a \to 0$. The small estimated σ_a for all three types of contracts (0.051 [0.219] for worktype A; 0.006 [2.593] for worktype B; and 0.023 [0.153] for worktype

[28] The same insight appears implicit in the War of Attrition example considered in Milgrom and Weber (1985). More recently, Athey (1998) derives general conditions under which monotonic pure strategy equilibria will obtain in games of incomplete information.

Powell (1984) used the same intuition to derive a distribution-free least-absolute deviations (LAD) estimator for censored linear regression models: in his case, the monotonic censoring operation preserved the quantiles between the distribution of the additive error term and the (censored) dependent variable. (Manski, 1994, section 4.4) labels these "quantile independent monotone models".

[29] In these models, the policy function $p(x)$ is often constrained to be monotonic in the type x in order to be implementable (i.e., satisfy incentive compatibility). See (Fudenberg and Tirole, 1991. 257ff.). More recently, Florens, Protopescu, and Richard (1997) have also developed a general estimation methodology for games of incomplete information, which requires inverting the equilibrium mapping of types to actions for each given set of parameter values. Our approach avoids this by exploiting the monotonicity of the equilibrium strategies and the invariance of distribution quantiles to monotonic transformations.

C) suggests that all these auctions can be described as predominantly pure common value auctions.[30]

Table 4: Parameter estimates: baseline specification
Standard errors in parentheses

Worktype: Parameter:	A	B	C
σ_w	0.371 (0.163)	1.338 (0.386)	0.542 (0.016)
σ_a	0.051 (0.219)	0.006 (2.593)	0.023 (0.153)
σ_e	0.642 (0.244)	0.161 (0.059)	0.614 (0.004)
μ	-0.288 (0.332)	0.069 (2.024)	-0.105 (0.014)
Simulation draws:			
R_1[a]	50	50	50
R_2[b]	100	100	100
R_3[c]	50	50	50

[a]Number of simulation draws used in calculating quantiles of equilibrium bid $s_n(x;\theta)$
[b]Number of simulation draws used in calculating $v_n(x,x;\theta)$
[c]Number of simulation draws used in calculating $s_n(x;\theta)$

While striking, this finding is not wholly surprising, given the upward trend in both average and median bids observed in the raw data (cf. table (2)). As discussed earlier, this upward trend can be explained, abstracting away from all else, as indicative of the less aggressive bidding caused by the winner's curse, which only occurs in common value auctions. However, a more likely and non-strategic explanation is that these auctions are characterized by selective participation and contract heterogeneity, so that larger projects (which, from a purely cost perspective, command larger bids) attract more bidders. This suggests that controlling for contract heterogeneity is crucial to measuring the effects of the winner's curse, and next we explore specifications which explicitly accommodate this possibility by allowing the (parameters which characterize the) distribution of signals in auction i to depend explicitly on M_i, the number of participants in auction i.[31]

[30]In general, given the parameter restrictions that σ_a, σ_w, σ_e are all ≥ 0, it is not always straightforward to test whether any of the σ's is equal to zero, the lower bound of the parameter space. However, nonstandard tests need be employed only when testing joint hypotheses that two or more of the σ's are equal to zero (cf. Andrews (1998), Wolak (1989)); a standard one-sided t-test is valid for univariate tests.

[31]While this is admittedly reduced-form, it is sufficient for addressing the questions at hand. An explicitly structural model of contractors' participation decisions is beyond the scope of this paper, and would not aid in measuring the effects of an increase in the number of competitors on equilibrium bidding.

Table 5: Parameter estimates
Control for selective participation: parameterize $\mu = \mu_0 + \mu_1 * n + \mu_2 * n^2$

Worktype:	A	B	C
Parameter:			
σ_w	1.495 (0.406)	0.944 (0.054)	0.905 (0.077)
σ_a	0.126 (0.197)	1.649 (0.247)	0.978 (0.032)
σ_e	0.685 (0.205)	0.376 (0.640)	0.362 (0.211)
μ_0	-1.712 (0.924)	-0.361 (0.388)	-1.656 (0.510)
μ_1	0.353 (0.308)	0.097 (0.148)	0.421 (0.177)
μ_2	-0.018 (0.017)	-0.004 (0.010)	-0.026 (0.014)
Simulation draws:			
R_1 [a]	50	50	50
R_2 [b]	100	100	100
R_3 [c]	50	50	50

[a] Number of simulation draws used in calculating quantiles of equilibrium bid $s_n(x;\theta)$
[b] Number of simulation draws used in calculating $v_n(x,x;\theta)$
[c] Number of simulation draws used in calculating $s_n(x;\theta)$

5.1 Robustness check: controlling for selective participation

First, we allow μ_i, the mean of the log-costs c_i in auction i, to have a quadratic trend in M_i, the number of participants in auction i:

$$\mu_i = \mu_0 + \mu_1 * <_i + \mu_2 * M_i^2 \tag{4}$$

and estimate μ_0, μ_1, and μ_2 as parameters. The results from these specifications are given in table 5.

Allowing the means to differ depending on the number of bidders does lead to changes in the parameter estimates for the important σ parameters. While we continue to reject the importance of private values in the worktype A auctions (0.126 [0.197]), we can no longer do so for the auctions of the other types of contracts. The point estimate for σ_a in the auctions of worktype B and C contracts are much larger than before (1.649 [0.247] for worktype B, and 0.978 [0.032] for worktype C). From these results, we conclude that auctions for highway work contracts are very close to a pure common value auction, while both common value and private value are important in auctions of bridge repair and paving contracts. These findings support our *a priori* expectations, since the generally longer duration of highway work contracts imply a greater impact of uncertainty concerning future input prices, which we believe to be the main driver of common values.

The positive estimates of μ_1 for all three worktypes are consistent with the hypothesis that auctions with more bidders feature larger contracts; correspondingly, the negative estimate for the quadratic coefficient μ_2 for these worktypes indicates that this positive relationship only holds at smaller values of M_i.

5.2 Robustness check: Random effects for contract heterogeneity

While the preceding specification is still restrictive because all contracts of the same worktype and with the same number of bidders are still assumed to be homogeneous, we next explicitly allow each contract to be heterogeneous by employing random effects in the specification.

More precisely, we assume that μ_i, the median of the signal distribution for auction i, is drawn from a normal distribution with mean γ_i and standard deviation σ_{γ_i}, and independent across auctions. Furthermore, we parameterize γ and σ_γ as a function of the number of bidders M_i in auction i:

$$\gamma_i = \gamma_0 + \gamma_1 * M_i + \gamma_2 * M_i^2$$
$$\sigma_{\gamma_i} = \exp\left(\gamma_3 + \gamma_4 * M_i + \gamma_5 * M_i^2\right)$$

where $(\gamma_0 - \gamma_5)$ are parameters to be estimated. Note that this specification allows μ_i (the median signal for auction i) to be a *random* (across auctions, and from the econometrician's point of view) function of the number of bidders M_i. In the previous specification (equation (4)), the median bid μ_i is a deterministic function of M_i (and the same across all auctions with the same M_i).

Table (6) contains parameter estimates from this specification. Instead of reporting estimates of $\gamma_1, \ldots, \gamma_5$ directly, we report the more easily interpretable estimates of γ_M and σ_{γ_M}, for $M = 2, \ldots, 15$.

Qualitatively, the above result that worktype A auctions can be described as almost pure common value auctions, while both private and commonvalues are important for the other types of contracts, remains with these results, although the magnitudes for the σ's are generally smaller across all three worktypes. Below, we will graphically consider the qualitative differences in these magnitudes in terms of predicted bids.

As for the unobserved heterogeneity parameters, the estimates indicate that for the worktype A contracts, the means of the heterogeneity distribution grow larger as the number of bidders increases, supporting the selective participation hypothesis (i.e., that larger con-

Table 6: Parameter estimates
Robustness Check: Random effects

Worktype:	A	B	C
Parameter:			
σ_w	1.061 (0.074)	0.265 (0.023)	0.149 (0.075)
σ_a	0.295 (0.259)	0.405 (0.033)	0.549 (0.090)
σ_e	0.760 (0.055)	0.473 (0.052)	0.259 (0.237)
γ_2	-1.533 (0.066)	-0.059 (0.029)	-1.114 (0.069)
γ_3	-1.444 (0.075)	-0.209 (0.032)	-0.847 (0.070)
γ_4	-1.360 (0.091)	-0.329 (0.037)	-0.624 (0.071)
γ_5	-1.282 (0.109)	-0.420 (0.042)	-0.444 (0.074)
γ_6	-1.210 (0.128)	-0.481 (0.047)	-0.308 (0.076)
γ_7	-1.144 (0.147)	-0.513 (0.052)	-0.216 (0.079)
γ_8	-1.084 (0.164)	-0.516 (0.058)	-0.167 (0.082)
γ_9	-1.030 (0.179)	-0.489 (0.063)	-0.162 (0.085)
γ_{10}	-0.983 (0.193)	-0.432 (0.069)	-0.201 (0.088)
γ_{11}	-0.941 (0.206)	-0.346 (0.075)	-0.283 (0.091)
γ_{12}	-0.905 (0.216)	-0.231 (0.082)	-0.409 (0.094)
γ_{13}	-0.875 (0.225)	-0.086 (0.089)	-0.578 (0.097)
γ_{14}	-0.852 (0.233)	0.088 (0.097)	-0.791 (0.101)
γ_{15}	-0.834 (0.239)	0.291 (0.106)	-1.048 (0.105)
σ_{γ_2}	0.162 (0.017)	0.438 (0.014)	0.520 (0.020)
σ_{γ_3}	0.174 (0.014)	0.437 (0.010)	0.511 (0.013)
σ_{γ_4}	0.188 (0.011)	0.440 (0.006)	0.502 (0.008)
σ_{γ_5}	0.204 (0.008)	0.445 (0.005)	0.493 (0.005)
σ_{γ_6}	0.222 (0.006)	0.453 (0.005)	0.485 (0.005)
σ_{γ_7}	0.241 (0.005)	0.464 (0.006)	0.477 (0.006)
σ_{γ_8}	0.264 (0.005)	0.478 (0.007)	0.469 (0.007)
σ_{γ_9}	0.289 (0.006)	0.496 (0.007)	0.462 (0.007)
$\sigma_{\gamma_{10}}$	0.318 (0.007)	0.518 (0.007)	0.455 (0.008)
$\sigma_{\gamma_{11}}$	0.351 (0.007)	0.544 (0.007)	0.448 (0.010)
$\sigma_{\gamma_{12}}$	0.388 (0.008)	0.575 (0.007)	0.442 (0.014)
$\sigma_{\gamma_{13}}$	0.431 (0.010)	0.612 (0.009)	0.436 (0.020)
$\sigma_{\gamma_{14}}$	0.479 (0.016)	0.655 (0.014)	0.430 (0.027)
$\sigma_{\gamma_{15}}$	0.535 (0.025)	0.706 (0.021)	0.425 (0.035)
#contracts	406	177	150
Simulation draws:			
R_1[a]	50	50	50
R_2[b]	100	100	100
R_3[c]	50	50	50

[a]Number of simulation draws used in calculating quantiles of equilibrium bid $s_n(x;\theta)$
[b]Number of simulation draws used in calculating $v_n(x,x;\theta)$
[c]Number of simulation draws used in calculating $s_n(x;\theta)$

tracts attract more contractors). However, the increasing values of σ_{γ_M} suggest that there is a greater degree of heterogeneity in auctions attracting a larger number of bidders.

Table 7: Contract-specific covariates: Highway-work contracts

Variable	N	Mean	Std Dev	Minimum	Maximum
Total Auctions	415				
NY[a]	410	7588.57	737.28	6234.50	8967.49
PHIL[b]	410	6005.94	521.88	5083.90	7311.37
TRAFFIC[c]	364	27996.41	21227.55	0	97235.00
GATEWAY[d]	364	0.56	0.50	0	1.00
SKYLANDS[e]	364	0.35	0.48	0	1.00
SHORE[f]	364	0.24	0.43	0	1.00
DELAWARE[g]	364	0.49	0.50	0	1.00
SOUTH[h]	364	0.26	0.44	0	1.00

[a]Construction Cost Index (CCI) for New York area, in 1913$. Source: ENR (1990-1997).
[b]Construction Cost Index (CCI) for Philadelphia area, in 1913$. Source: ENR (1990-1997).
[c]Weekday traffic volume of road being repaired, in both directions. (Source: NJDOT)
[d]GATEWAY=1 if road lies (partly) in Bergen, Hudson, Middlesex, Passaic, Union, Essex counties.
[e]SKYLANDS=1 if road lies (partly) in Hunterdon, Morris, Somerset, Sussex, Warren counties.
[f]SHORE=1 if road lies (partly) in Ocean or Monmouth counties.
[g]DELAWARE=1 if road lies (partly) in Camden, Gloucestor, Salem, Burlington, Mercer counties.
[h]SOUTH=1 if road lies (partly) in Atlantic, Cumberland, Cape May counties.

Using these covariates, we parameterize μ_i as:

$$\mu_i = \mu_0 + \mu_1 * M_i + \mu_2 * M_i^2 + \mu_3 * \left(\frac{1}{2} * \text{NY+PHIL}\right) +$$
$$\mu_4 * \text{TRAFFIC} + (\mu_5 \; \mu_6 \; \mu_7 \; \mu_8 \; \mu_9) \begin{pmatrix} \text{GATEWAY} \\ \text{SKYLANDS} \\ \text{SHORE} \\ \text{DELAWARE} \\ \text{SOUTH} \end{pmatrix}.$$

While the results are qualitatively similar for worktype B, they are generally reversed for worktype C. Encouragingly, these results correspond to the patterns observed in the raw data, as given in columns 4 and 5 of table (2), suggesting that we are, indeed, adequately controlling for contract heterogeneity.

5.3 Robustness check: Accounting for contract-specific heterogeneity

In our last robustness check, we control for observed heterogeneity across contracts by parameterizing μ_i, the median of the signal distribution for auction i, as a function not only of M_i, the number of bidders in auction i, but also the covariates specific to auction i. The two most important dimensions of heterogeneity are size differences across contracts of a given worktype, and changes in input costs across time which also exogenously affect bids for a contract. Since our bid data do not allow us to determine the per-unit costs submitted by the bidders, adequately controlling for size heterogeneity is particularly important.

Table 8: Parameter estimates: accounting for observed contract heterogeneity
Standard errors in parentheses.
Estimated for worktype 4 contracts only.

Parameter:	
σ_w	1.043 (0.410)
σ_a	0.436 (1.087)
σ_e	6.671 (7.099)
μ_0	-1.680 (0.996)
μ_1	0.234 (0.069)
μ_2	-.0561 (0.006)
COST[a] (μ_3)	3.624E-4 (1.554E-3)
TRAFFIC (μ_4)	-1.639E-5 (5.137E-5)
GATEWAY (μ_5)	-0.0019 (0.0150)
SKYLANDS (μ_6)	-0.0041 (0.0207)
SHORE (μ_7)	-0.0055 (0.0176)
DELAWARE (μ_8)	-0.0051 (0.0159)
SOUTH (μ_9)	-0.0098 (0.0166)
#contracts	356
Simulation draws:	
R_2[b]	100

[a]For definitions of covariates, see table 7. COST $= \frac{1}{2}$(NY + PHIL).
[b]Number of simulation draws used in calculating $v_n(x, x; \theta)$

We were only able to obtain contract-specific covariates for the worktype A contracts. This is because many of these contracts specified roads upon which work was to be done. We obtained contract-specific covariates which were characteristics of these roads, as extarcted from a database maintained by the NJDOT.[32] Table 7 below summarizes the covariates. They are: TRAFFIC, a measure of the weekday traffic volume (in both directions) of the road being repaired; and geographic dummies (GATEWAY, SKYLANDS, SHORE, DELAWARE, SOUTH) which describe the geographic location of the road. Furthermore, we also obtained construction cost indices corresponding to the month in which a particular contract was auctioned from the trade publication *Engineering News-Record* (NY is the index for New York City, and PHIL is the index for Philadelphia; in our specification, we use an average of the two).

Since we were only able to obtain covariates for the worktype A contracts, this specification was only estimated on those auctions. The results are reported in table (8). While the magnitudes and signs of the parameters common across all the specifications remain quite stable to the incorporation of covariates, it is noteworthy that the μ_1 (the coefficient on the linear trend in the number of bidders) is now much smaller in magnitude (0.234 [0.0669]) This decreased importance of n in predicting the magnitude of the bids suggests that the covariates are in fact controlling for some dimension of contract heterogeneity which lead to selective participation on the part of the bidders. The estimates for the σ's are qualitatively stable across all the specifications, except that the estimate of σ_e, which measures the noisiness of bidders' signals, has grown in magnitude (6.671 [7.099]). This implies that contractors' cost signals are very noisy, which tends to reinforce our finding of a strong winner's curse effect in these auctions (as we discuss below).

Strikingly, none of the covariates enter significantly in the specification, and for that reason we do not draw any results from them except that their presence does appear to affect the magnitudes of the other parameters. We conclude by noting that the main qualitative result obtained in the earlier specification for the worktype A auctions — namely that they are essentially pure common value auctions — appears robust to the inclusion of covariates. In what follows, then, we focus on the results from tables (5) and (6), which were obtained for all three types of contracts.

5.4 Model Fit

In a discussion in section C of the appendix, we note that variation in the number of bidders is needed to identify the parameters of the model, given observations of the bids. By the allowing the bid distribution to differ across the number of bidders, however, we rely on the parametric restrictions of our log-normal model to identify the parameters rather than

[32] This database can be searched on-line at `www.state.nj.us/transportation/count/search/search1.htm`.

variation in the data. Therefore, before proceeding, we check that our chosen parametric specifcation indeed fits the observed data.

In figure (1), we plot the actual and predicted median bids, by worktype and across different number of bidders. The predicted median bids were calculated from the estimation results reported in tables (5) and (6).[33] In general, the predicted and actual bids correspond reasonably closely, with the table (6) results fitting noticeably better.

The "hump" from 8–10 bidders in the actual worktype A auctions are missed by both specifications, but the predicted values are close at a smaller number of bidders. For the worktype B contracts, the table (6) fit remarkably close for auctions with less than 10 bidders. For the worktype C contracts, the table (6) fit reasonably well, across the whole range of cost signals.

In general, the fit of the model to the actual bids is reasonably close, and is valid for the bid simulations which we undertake in the next section to address the effects of increasing competition on equilibrium bidding.

6 Increasing competition and equilibrium bidding

Next, we turn our attention to what our results imply about the two comparative statics in the number of bidders n which have occupied the past theoretical literature.

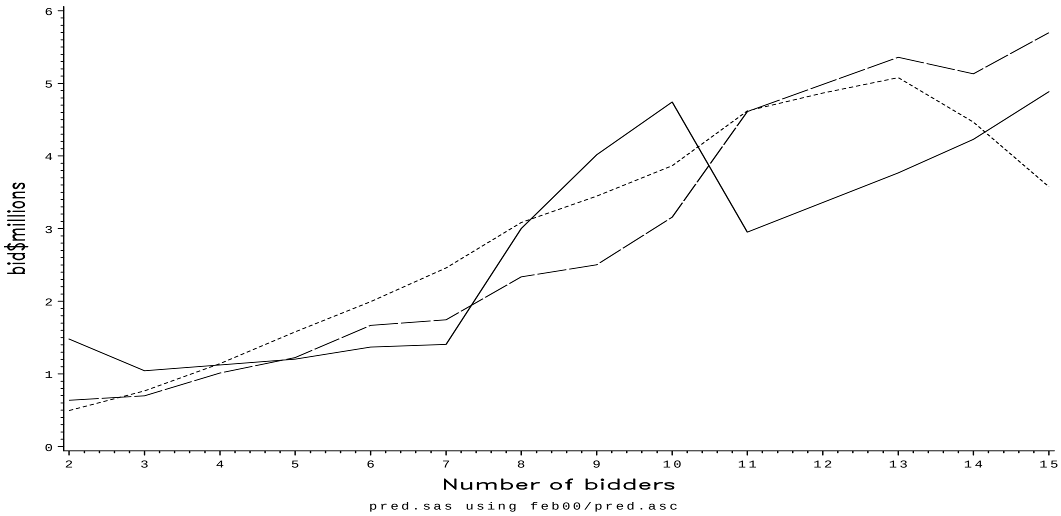
6.1 Increasing competition and individual bids

First, we examine how individual contractor bids would be potentially affected by increases in the number of competitors, i.e., whether $s_n(x)$ is increasing or decreasing in n, fixing the signal x. We are not aware of any general results for this comparative static. Previously, (Smiley, 1979, chap. 3) and Matthews (1984) have shown that the sign can go either way, depending on the parametric assumptions: Smiley (1979) gave examples of multiplicative bid functions where $s_n(x)$ decreases in n (i.e., more aggressive bidding), whereas Matthews (1984) focuses on the uniform distribution where $s_n(x)$ increases in n (i.e., less aggressive bidding). Since our framework is more complicated than that considered by these authors, we treat this as an open question which we address empirically.[34]

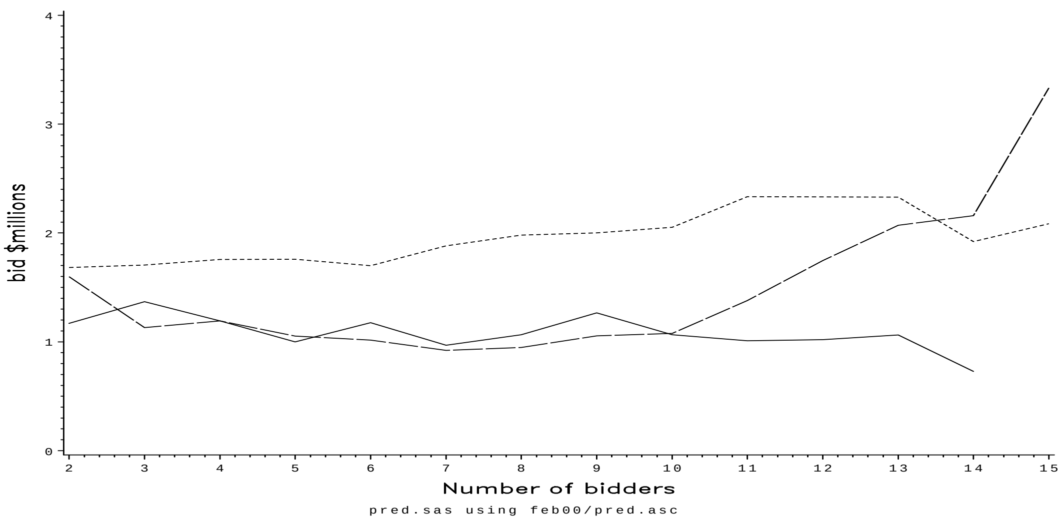
[33] For the table (6) results, we evaluate the bid functions at the mean random effect, i.e., assuming $\mu_i = \gamma_i$.

[34] Both Smiley and Matthews consider only the pure common value (or "mineral rights") model, whereas our model has both common and private value components. Note that both authors also focus on hig-bid auctions, where more aggressive bidding implies that $s_n(x)$ increases in n, and vice versa. The interpretations are reversed for our low-bid procurement auctions.

For worktype A contracts



For worktype B contracts



For worktype C contracts

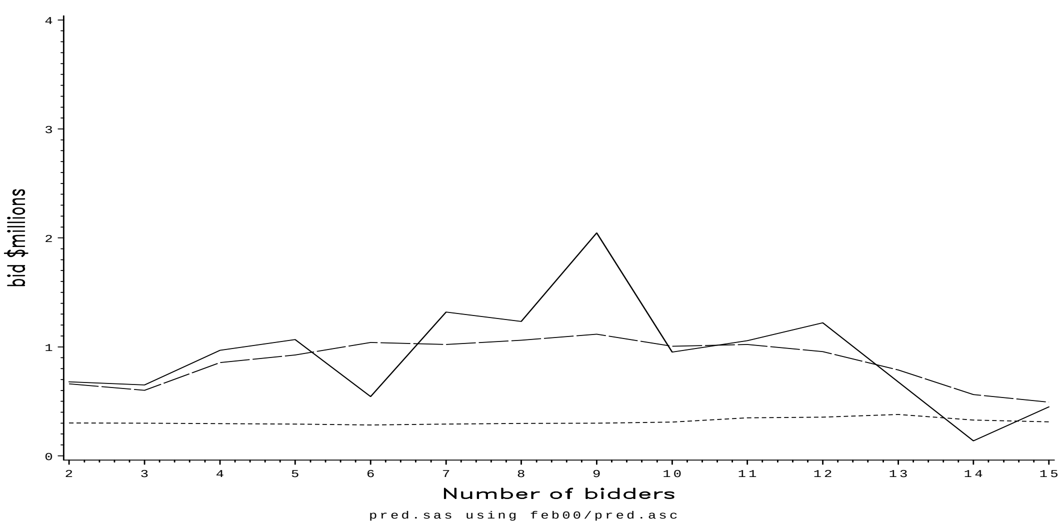
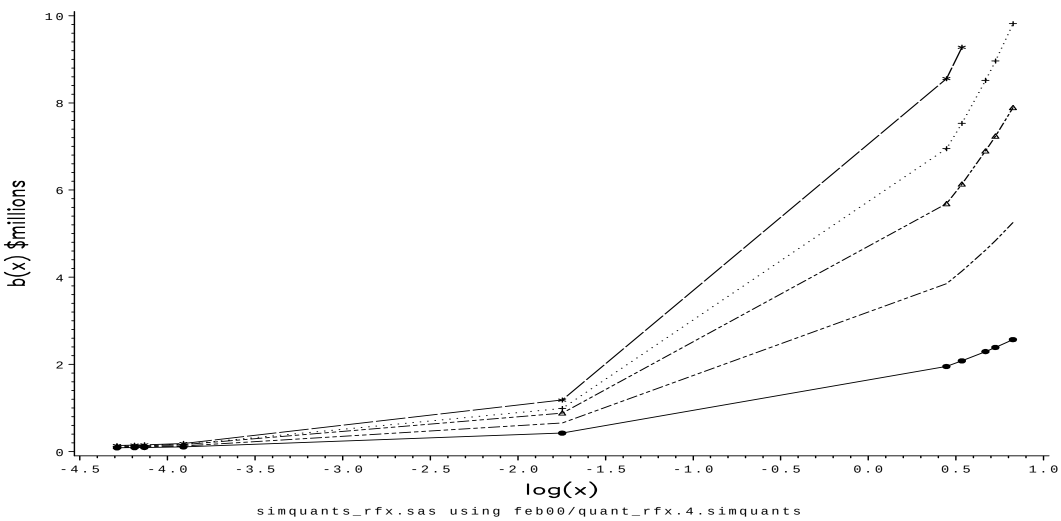


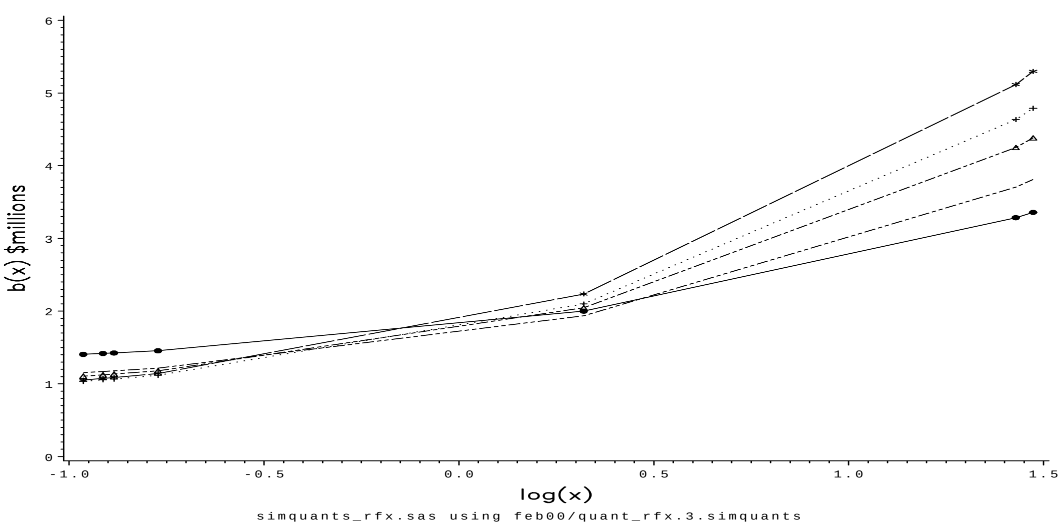
Figure 1: Predicted vs. actual bid functions: using table (5) and table (6) results

Legend: Actual median bid (from table (2); solid line)
Predicted median bid using table (5) results (dotted line)
Predicted median bid using table (6) results (dashed line)

For worktype A results



For worktype B results



For worktype C results

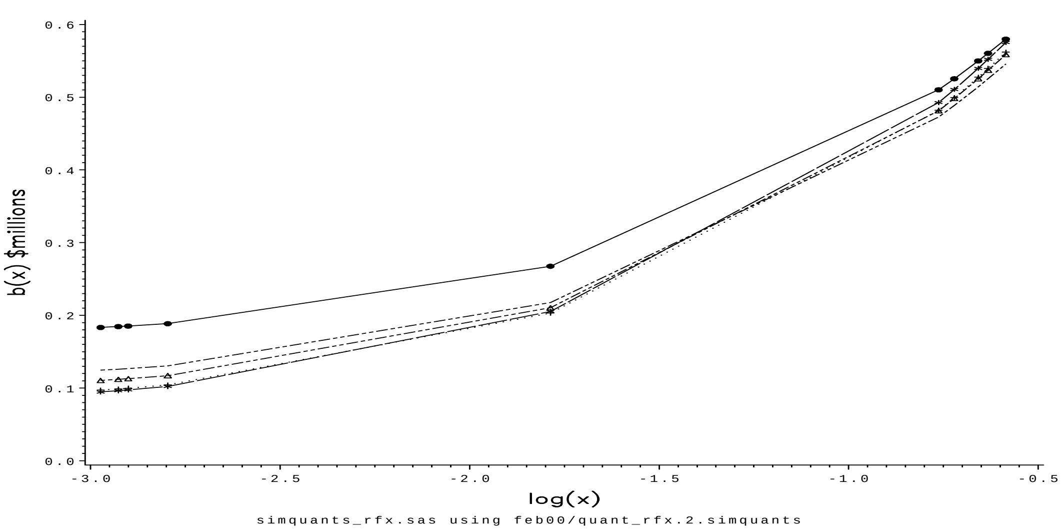
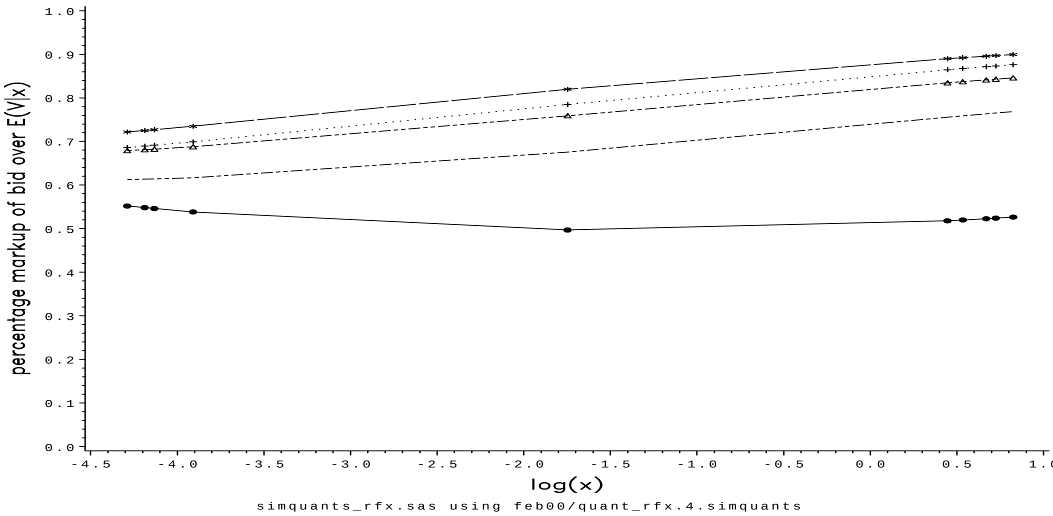


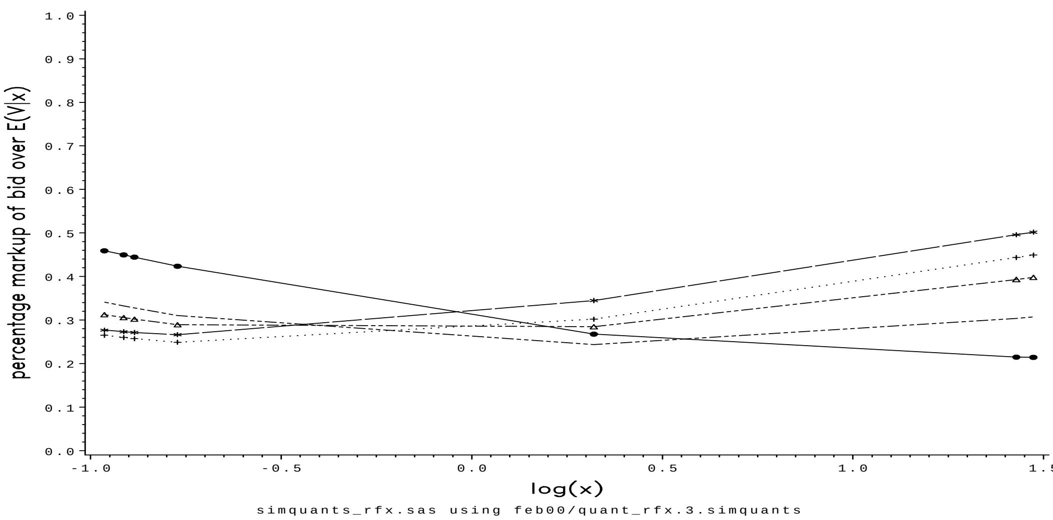
Figure 2: Equilibrium bid functions: simulated using table 6 results

Legend: –•–•– 2 bidders – — – – — 4 bidders △- — –△– 6 bidders · · + · · + · 8 bidders —-*—*—* 10 bidders

For worktype A results



For worktype B results



For worktype C results

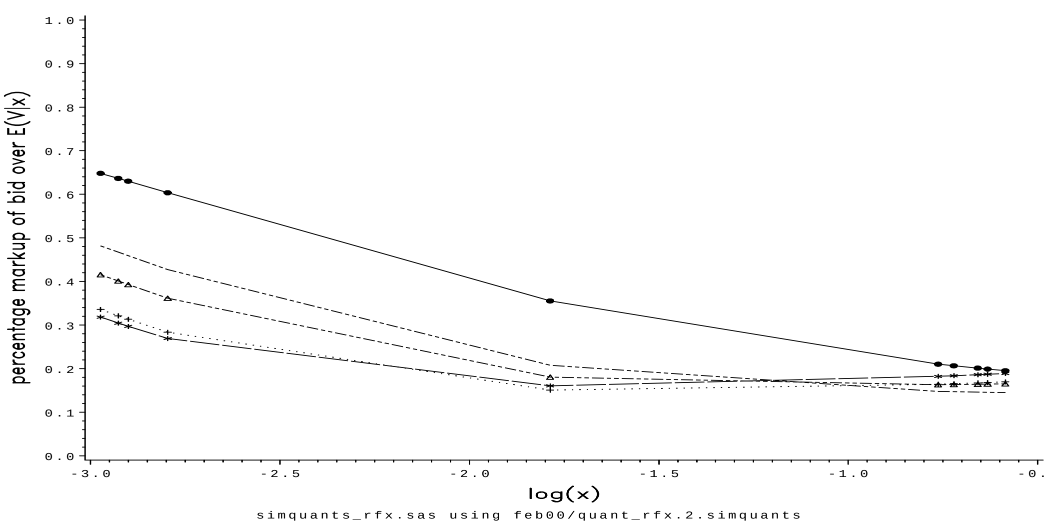


Figure 3: Bidder markups: simulated using table 6 results

Markups are given by $\frac{b-\mathcal{E}(c|x;\theta)}{b}$.

Legend: –•–•– 2 bidders – — – – — 4 bidders △- — –△– 6 bidders · · + · · + · 8 bidders —-*—*—* 10 bidders

Equilibrium bid functions Next, we simulate the equilibrium bidding function $s_n(x;\theta)$ function via formula (2) for the results from table (6). The graphs are shown in figure 2. Clearly, the shapes of the graphs vary noticeably over the three worktypes. In order to isolate the pure effect of the winner's curse, we zero out the coefficients attached to the number of bidders (i.e., $\gamma_1, \gamma_2, \gamma_4, \gamma_5$) in these simulations.

Our results indicated that the worktype A auctions can be considered pure common value auctions; for this reason, the equilibrium bid function for this case is increasing in the number of bidders n; for instance, the equilibrium bid function evaluated at the -1.712, the mean value of $\log(x)$, rises from about \$0.5 million with two bidders to over \$1 million in ten-bidder auctions, a 200% percent increase; the magnitude of increase is greater at larger values of $\log(x)$. This indicates that the winner's curse effect strongly dominates the competitive effect.

On the other hand, the graphs for worktypes B and C reflect the finding that privates values are important in these auctions. At low values of x, the equilibrium bids are falling in n, as we would expect in pure private value auctions. At large levels of x, however, some of the bid functions cross, indicating that the winner's curse seems to be more important in this range of x. For example, at $\log(x)$=-0.5, the worktype B equilibrium bid function for $n = 10$ lies below that for $n = 6$ (indicating that the competitive effect dominates), but at $\log(x) = 1.0$ the bid function for $n = 10$ has crossed over the $n = 6$ bid function. This makes sense upon examination of equation 1: for a given n, the probability of winning is smaller as signals get larger, which implies that as n increases the competitive effect will be relatively weaker upon bidders with larger signals. Intuitively, winning an auction with a less optimistic signal (i.e., higher x) conveys worse news than winning with a more optimistic signal (i.e., lower x).

Equilibrium markups These results are clearly illustrated in terms of "markups" of the equilibrium bids in excess of expected costs which we may have expected naive participants to bid for these contracts. Using our parameter estimates, we calculate the markup $\frac{b(x_i) - E[c_i|x_i]}{b(x_i)}$, where $E[c_i|x_i]$, the expected project cost to bidder i based just upon his signal x_i, is taken to be a "naive" estimate which bidder i might have bid were he incognizant about the winner's curse. The markups are plotted in figure 3.

For the worktype A auctions, which are essentially pure common value auctions, the simulated markup is increasing in the n, across the entire range of x's. The markups themselves are quite large in magnitude: they increase from roughly 50% with 2 bidders to around

70% with 10 bidders. This increase is reversed in part in the worktype B and C auctions. In particularly, the worktype C auction markups are generally decreasing for all n, across practically the entire range of signals.

The difference in the magnitudes of the markups between the worktype A and worktype B/C auctions is in large part attributable to winner's curse considerations. For the worktype B and C auctions, there is an important private value element, so that the markups (at least in the lower range of the signals, when winner's curse considerations are not so important) are symptomatic of "market power" which recedes as the number of bidders increases. In the worktype A auctions, which have a negligible private value element, the markups arises to counter the adverse selection associated with winning the auction, analogous to the "unfair" premia that insurance companies must levy in order to break even due to the adverse selection of sicker patients into the insurance market.

This point is illustrated in table (9), where equilibrium markups for the median bidder were calculated at different values of σ_e, which parameterizes the noisiness of contractors' signals regarding their unknown costs, with a larger σ_e corresponding to greater uncertainty. The markups were calculated after reducing σ_e by one-half (column 4) and then by 90% (column 5). Note that the reduction in uncertainty reduces the equilibrium markups for the worktype A auctions dramatically (for 6-bidder auctions, the markup falls from over 75% to 54% when $sigma_e$ is cut in half, and down to just 28% when $sigma_e$ is at one-tenth of its estimated value). This illustrates how a reduction in uncertainty reduces the winner's curse, and therefore the "premia" which bidders demand in order to participate in this market.

For worktype B and C auctions, however, a reduction in markups is not obviously apparent, except when the number of bidders grows large. This emphasizes the point that when private value components in costs are important, winner's curse considerations are not so important in auctions with few bidders, so that equilibrium markups in these cases are attributable in large part to "market power". When the number of bidders increases, however, winner's curse effects become more important, and a reduction in uncertainty brings about a fall in equilibrium markups just as in the worktype A case.

In summary, the most remarkable implication of these simulation is that the winner's curse is indeed very strong in the worktype A auctions. Next, we explicitly explore what these effects imply about government procurement costs.

Table 9: Equilibrium markups: the effect of reduction in uncertainty

Worktype	# bidders	Markup: σ_e	Markup: $0.5*\sigma_e$	Markup: $0.1*\sigma_e$
A	2	0.497	0.397	0.292
A	3	0.562	0.418	0.261
A	4	0.675	0.481	0.274
A	5	0.711	0.507	0.277
A	6	0.759	0.541	0.288
A	7	0.783	0.543	0.267
A	8	0.785	0.545	0.260
A	9	0.814	0.560	0.257
A	10	0.820	0.567	0.256
A	11	0.852	0.597	0.272
A	12	0.852	0.599	0.268
A	13	0.867	0.613	0.268
A	14	0.875	0.623	0.269
A	15	0.887	0.633	0.271
B	2	0.268	0.304	0.317
B	3	0.230	0.245	0.243
B	4	0.244	0.243	0.229
B	5	0.259	0.244	0.220
B	6	0.284	0.256	0.223
B	7	0.291	0.245	0.201
B	8	0.302	0.244	0.191
B	9	0.322	0.252	0.191
B	10	0.345	0.264	0.195
B	11	0.365	0.277	0.204
B	12	0.383	0.286	0.207
B	13	0.392	0.286	0.201
B	14	0.399	0.286	0.197
B	15	0.413	0.293	0.198
C	2	0.356	0.375	0.382
C	3	0.245	0.257	0.261
C	4	0.208	0.215	0.218
C	5	0.187	0.191	0.192
C	6	0.181	0.182	0.182
C	7	0.161	0.157	0.155
C	8	0.151	0.144	0.141
C	9	0.152	0.143	0.138
C	10	0.161	0.149	0.144
C	11	0.165	0.151	0.145
C	12	0.171	0.156	0.149
C	13	0.166	0.148	0.140
C	14	0.166	0.145	0.136
C	15	0.171	0.148	0.138

6.2 Increasing competition and project procurement costs

In procurement, common wisdom dictates that increasing the number of contractors would lower project costs. This is true for private value models but, given the results above (for worktype A at least), one questions whether this is a wise policy for reducing project costs in common value auctions. Next, we present results from simulations of the winning bid in auctions in which the number of competitors is varied. We do this for the results in table 6.

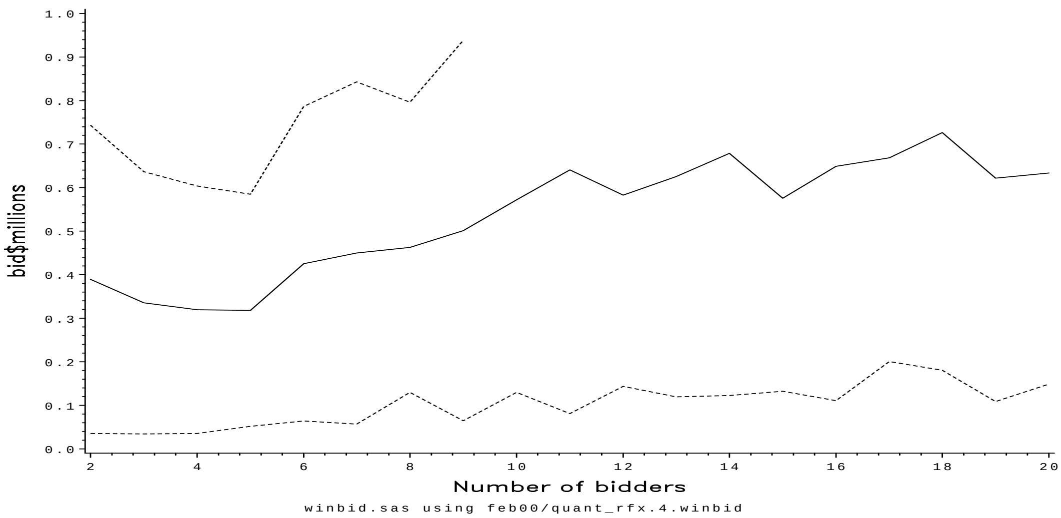
For the worktype A results, the average of the simulated winning bids is generally increasing in n, indicating that procurement costs would rise if the government invites more competition. The median winning bid rises from about \$0.35 million with two bidders to \$0.55 million once 10 bidders are involved, but stabilizes in this range for $n > 10$. For $n = 6$, the mode of the distribution of n for the actual worktype A auctions (cf. table (2)), these results suggest that the government could reduce procurement costs by about 25% by restricting participation to just four contractors. Given that the average contract outlay is just under \$5 million, this potential savings could be very large. Furthermore, these simulations indicate that the "optimal" number of participants (which would minimize expected procurement costs) would be 5, which is only one less than the mode of the empirical distribution of n for the worktype A auctions, which lies at 6 (cf. table (2)). Our results suggest that the government could lower expected procurement costs by about 25% by reducing n from 6 to 5 which, considering the average contract outlay of about \$5 million, constitute substantial savings

Opposite results are obtained for the worktype B and C results. For the worktype B results, which indicate a strong private value component, the average winning bids fall quickly in n, from an average bid of \$1.8 million with 2 bidders to about \$1.3 million, with 10 bidders. This drop also appears in the worktype C results, but is much less precipitous. For these two types of contarcts, then, increasing competition would indeed lower procurement costs, and it is optimal for the government to invite as many tenders as possible.

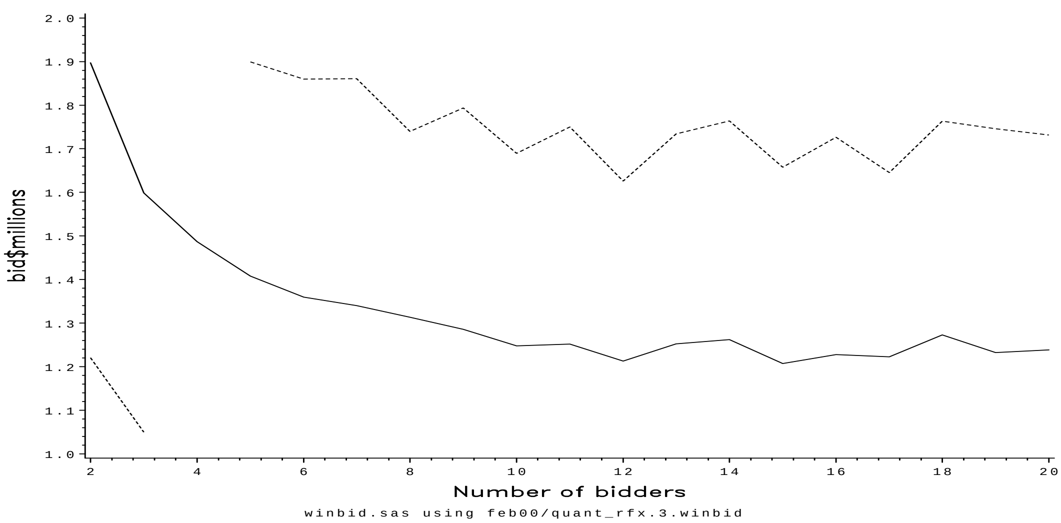
For the pure common value case, there have been some well-known results concerning the limiting behavior of the winning bid. Wilson (1977) and Milgrom (1979) provided sufficient conditions for strong and weak (respectively) convergence of the winning bid, conditional on the unknown common value w, to w, as n grows large.[35]

[35] Matthews (1984) provides an intuitive illustration of these limit arguments, for the specific case where the distribution of the signal x conditional on the common value w is uniform in the interval $[0, w]$. More recently, Pesendorfer and Swinkels (1997) have shown that these conditions specified by Wilson (1977) and Milgrom (1979) can be weakened if one allows for multiple-unit auctions.

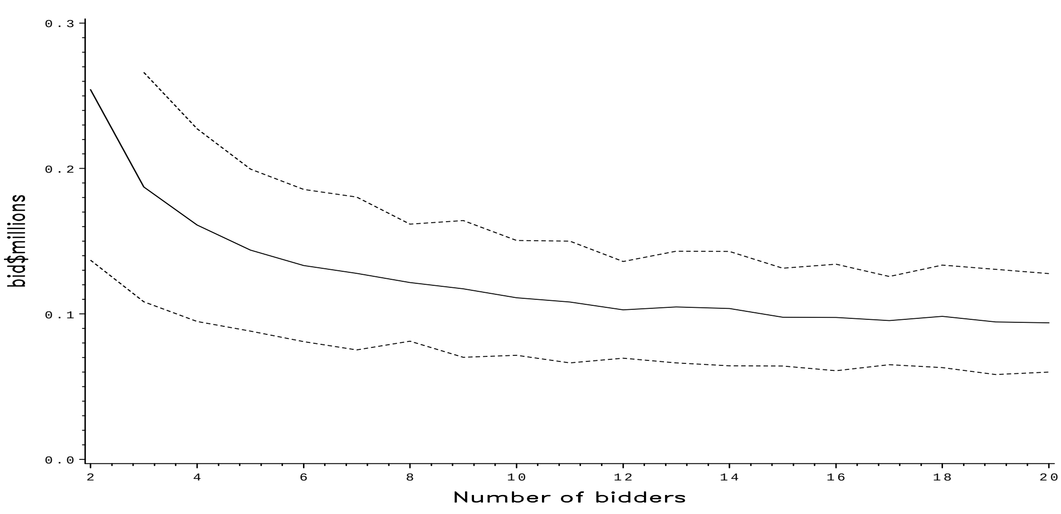
For worktype A results



For worktype B results



For worktype C results



Legend: Average winning bid in solid line
+2stdev and -2stdev in dashed lines

The simulated winning bids for the (almost-) common value worktype A auctions (top panel of figure 4) indicate that, for the range of n which we observe in the data, the average winning bid is increasing at smaller values of n. This does not necessarily contradict the theoretical limit results, however. First, Wilson's condition that the lower bound of the support of $x|w$ be strictly decreasing in w is not satisfied for the log-normal model we employ. Second, even if the less stringent conditions of Milgrom (basically that there exists a signal x such that the likelihood ratio $\frac{f(x|w)}{f(x|w')}$ shrinks to zero in the limit for $w' < w$) are satisfied for our assumed functional forms, the limit results do not preclude the winning bid from diverging for small values of n.[36]

Note that these results do *not* imply that bidders' expected profits from an auction are rising in n, since expected profits depend not only on the magnitude of the winning bid but also on the probability of winning the auction. While bidders may well expect that the winning bid is rising in n due to the winner's curse, they also realize that the probability of winning falls in n, for any given signal x.

7 Conclusions

We empirically measure the effects of the winner's curse on equilibrium bidding in procurement auctions. In common value auctions, the winner's curse is an adverse selection problem which, in equilibrium, counsels more conservative bidding as the number of competitors increases. FIrst, we estimate the structural parameters of an equilibrium bidding models and test for the importance of common value components in bidders' preferences. Second, we use these estimates to simulate hypothetical equilibrium bidding strategies as we increase the number of auction participants. We measure the effects on increasing competition on both individual bids as well as winning bids (ie. procurement costs).

We analyze bid data from construction procurement auctions run by the New Jersey department of transportation in the years 1989-1997. Our results show that different types of contracts differ significantly in the degree that private and/or common value components are important, and these have contrasting implications on the effects of increasing competition on equilibrium bids and expected government procurement outlays. Auctions for highway work contracts are very close to a pure common value auction, while both common value and private value are important in auctions of bridge repair contracts. Furthermore, our results indicate that the winner's curse is particularly strong in these highway contract

[36]In additional (unreported) simulations which were run for much values of n up to 150, a downward trend in winning bids did eventually set in.

auctions. Simulated bid functions show that for the median bidder, the percentage markup increases from 50% with 2 bidders to above 70% with 10 bidders. Furthermore, winning bid simulations indicate that the average procurement cost is strictly increasing in the number of bidders as competition intensifies: for example, the median costs rise about 30%, as the number of bidders is increases from 3 to 6. These results emphasize how asymmetric information can overturn the common economic wisdom that more competition is always desirable, and have potentially important policy implications.

Methodologically, we have estimated a model which allows bidders' latent valuations for a contract to have both common and private value components. To our knowledge, this is the first empirical implementation of a model with such flexible bidder preferences.

The obvious policy implication here is that governments may wish to restrict entry, or favor "negotiations" over auctions (cf. Bulow and Klemperer (1996)) when the winner's curse is particularly strong. But in practice, this may not be always feasible, since government procurement agencies try to reduce the possibility of collusion among contractors by inviting more tenders (i.e., increasing competition).[37] Our findings, while not directly addressing these issues, do contain a striking implication: in situations where the winner's curse is so severe as to lead to *higher* procurement costs as the number of bidders increases (as in the worktype 4 results in table 4), municipal authorites may actually prefer to *allow* collusion, since in a common value setting the informational pooling that arises from bidder discussions may defuse the winner's curse effects. This is one potential justification for why the US government allowed joint bidding in the Outer Continential Shelf offshore lease auctions from their inception in the 1950s until the mid-1970s (cf. Hendricks and Porter (1992), Hendricks and Porter (1996)). The potential benefits of such restrictions on competition have been noted previously in the theoretical literature by, among others, Bulow and Klemperer (1999). In ongoing research we are exploring this possibility empirically, extending the approach in Campo, Perrigne, and Vuong (1998)) to model bidding between joint and non-joint bidders in a common-value environment.

[37]For example, a recent tender of auto-towing contracts in Toronto was scrapped due to low levels of participation while municipal staff were "instructed [...] to come back with suggestions on how more companies might be able to participate" (*Toronto Star* (1999)). In general, collusion and bid-rigging seem rampant in procurement settings (see Pesendorfer (1998), Baldwin, Marshall, and Richard (1997), and Porter and Zona (1993) for studies of several instances).

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A Details on estimation

For the Wilson log-normal specification, one can derive an analytic form for the conditional expectation $\mathcal{E}(c_i|x_1,\dots,x_n)$. Before deriving this analytic form, we introduce some notation. In this model, the vector $(c_1,\dots,c_n,x_1,\dots,x_n)$ is joint-lognormally distributed. In logs, then, the vector $(\tilde{c}_1,\dots,\tilde{c}_n,\tilde{x}_1,\dots,\tilde{x}_n)$ is distributed as jointly normal with identical means μ for all elements and variance-covariance matrix $\hat{\Sigma}=((\Sigma,\Sigma_{12}),(\Sigma_{12}',\Sigma^*))'$.[38] Furthermore, given the above log-normality assumptions, the conditional expectation functions for c_i take the following form:

$$E[c_i \mid x_1,\dots,x_n]=\exp\left(E(\tilde{c}_i \mid \tilde{x}_1,\dots,\tilde{x}_n)+\frac{1}{2}Var(\tilde{c}_i \mid \tilde{x}_1,\dots,\tilde{x}_n)\right), \tag{5}$$

for $i=1,\dots,n$.

Next, we denote the marginal variance-covariance matrix of $(\tilde{c}_1,\tilde{x}_1,\dots,\tilde{x}_n)$ by

$$\Sigma_i \equiv \begin{pmatrix} \sigma_c^2 & \sigma_c^{*\prime} \\ \sigma_c^* & \Sigma^* \end{pmatrix} \quad \text{also let} \quad x' \equiv (x_1,\dots,x_n)$$

where $\sigma_c^2=\sigma_w^2+\sigma_a^2$ is the variance of c_i. Then, using the conditional mean and variance of joint normal random variables (see, for example, Amemiya (1985), pg. 3):

$$E(\tilde{c}_i \mid \tilde{x})=\mu+\sigma_c^*\Sigma^{*-1}(\tilde{x}-\mu) \tag{6}$$

and

$$Var(\tilde{c}_i \mid \tilde{x})=\sigma_c^2-\sigma_c^{*\prime}\Sigma^{*-1}\sigma_c^*. \tag{7}$$

Expressions (6) and (7) can be plugged into equation (5). Given parameter estimates, the conditional expectation (5) can be explicitly evaluated for every vector of log-signals $(\tilde{x}_1,\dots,\tilde{x}_n)$.[39]

A.1 Asymptotic distribution for quantile estimator

Pakes and Pollard (1989) derive a general asymptotic theory for estimators obtained by maximizing simulated objective functions.[40] At this point, we utilize expressions for the variance-covariance matrices of the estimators which do not take account of the possible simulation bias due to using a finite number of draws. The implicit assumption, then, is that the number of simulation draws used in the various stages of

[38] Explicit formulas for the elements in the matrix $\hat{\Sigma}$ can be derived from the information structure of the model.

[39] Consequently, the desired conditional expectations can be obtained by simulation, viz:

$$v(x,x)=\mathcal{E}\left(c_i|x_i=x,\min_{j\neq i}x_j=x\right)=$$

$$\underbrace{\int\cdots\int}_{x_k\geq x,\ \forall k=3,\dots,n} \mathcal{E}(c_1|x_1,\dots,x_n)\ dF(x_3,\dots,x_n|x_1=x,x_2=x,x_k\geq x,k=3,\dots,n;\theta) \tag{8}$$

where F here denotes the conditional distribution of the signals $x_3,\dots,x_n$, conditional on $x_1=x_2=x$. Given symmetry, there is no loss of generality in focusing on the pair of signals x_1 and x_2.

[40] See the survey by Stern (1997) for additional details on simulation methodologies and asymptotics.

computation (which we denoted R_1, R_2, and R_3) increase faster than $\sqrt{T}$, the rate at which $\hat{\theta}$ converges to the true $\hat{\theta}_0$.

In what follow, we assume that θ is L-dimensional. Out quantile estimator minimizes the quantile objective function, reproduced here as

$$Q(\theta) = \sum_{i=1}^{T} \sum_{j=1}^{M_i} \sum_{k=1}^{K} \rho_{\tau_k} \left(b_{ij} - q_k^{M_i}(\theta) \right)$$

where M_i denotes the number of bidders in auction i, $q_k^{M_i}(\theta)$ denotes the kth quantile of bids for an M_i-bidder auction, and $\rho_{\tau_k}(\cdot)$ is defined as

$$\rho_{\tau_k}(x) = (\tau_k - 1(x \leq 0))\, x.$$

At $\hat{\theta}^{SQ}$, the SQ estimator, the approximate first order condition must hold:

$$\frac{1}{\sqrt{T}} \sum_{i=1}^{T} \sum_{j=1}^{M_i} \sum_{k=1}^{K} \left(\tau_k - 1\left(b_{ij} \leq q_k^{M_i}\left(\hat{\theta}^{SQ}\right)\right)\right) \frac{\partial q_k^{M_i}\left(\hat{\theta}^{SQ}\right)}{\partial \theta} = o_p(1)$$

It can be shown (cf. (Gourieroux and Monfort, 1995, chap. 8.5.2)) that

$$\sqrt{T}\left(\hat{\theta}^{SQ} - \theta\right) = -A_T^{-1} B_T + o_p(1)$$

where

$$B_T = \frac{1}{\sqrt{T}} \sum_{i=1}^{T} \sum_{j=1}^{M_i} \sum_{k=1}^{K} \left(\tau_k - 1\left(b_{ij} \leq q_k^{M_i}(\theta_0)\right)\right) \frac{\partial q_k^{M_i}(\theta_0)}{\partial \theta}$$

and

$$A_T = \frac{1}{T} \sum_{i=1}^{T} \sum_{j=1}^{M_i} \sum_{k=1}^{K} f_{\tau_k}^{M_i}\left(q_k^{M_i}(\theta_0)\right) \frac{q_k^{M_i}(\theta_0)}{\partial \theta} \frac{q_k^{M_i}(\theta_0)}{\partial \theta'}'$$

where $f_{\tau_k}^{M_i}$ is the density of the distribution of the marginal distribution of the random variable $s_{M_i}(x;\theta)$ in a M_i bidder auction at the τ_kth quantile, where x denotes the signal.

We estimate the A_T matrix using a finite difference approximation for the density $f_{\tau_k}^{M_i}$ as well as the gradient $\frac{q_k^{M_i}(\theta_0)}{\partial \theta}$. In this way, the (d_1, d_2) element of the LxL A_T matrix will be

$$\hat{A}_T^{rs} = \frac{1}{4Th_T^2} \sum_{i=1}^{T} \sum_{j=1}^{M_i} \sum_{k=1}^{K} \left[1\left(b_{ij} \leq q_k^n\left(\hat{\theta}^{SQ} + h_T e_{d_1}\right)\right) - 1\left(b_{ij} \leq q_k^n\left(\hat{\theta}^{SQ} - h_T e_{d_1}\right)\right)\right] * \left[q_k^n\left(\hat{\theta}^{SQ} + h_T e_{d_2}\right) - q_k^n\left(\hat{\theta}^{SQ} - h_T e_{d_2}\right)\right] \tag{9}$$

where e_d is a L-vector with 1 in the dth position and zeros otherwise, and h_T is the perturbation factor for the finite-difference approximation.

Via a central-limit theorem (using independence over auctions i), the L-vector B_T has an asymptotic distribution $N(0, V)$. A consistent estimator of V is

$$\hat{V}_T = \frac{1}{T} \sum_{i=1}^{T} \hat{m}_i(b_i)\, \hat{m}_i(b_i)' \tag{10}$$

where

$$\hat{m}_i(b_i) = \sum_{k=1}^{K}\sum_{j=1}^{M_i}\left(\tau_k - 1\left(b_{ij} \le q_k^{M_i}\left(\hat{\theta}^{SQ}\right)\right)\right)\frac{\partial q_k^{M_i}\left(\hat{\theta}^{SQ}\right)}{\partial\theta}.$$

The gradient vector $\frac{\partial q_k^{M_i}(\hat{\theta}^{SQ})}{\partial\theta}$ can likewise be evaluated using finite difference methods.

Based on equations 9 and 10, the asymptotic variance-covariance matrix for $\hat{\theta}^{SQ}$ can be approximated by

$$\frac{1}{T}\hat{A}_T^{-1}\hat{V}_T\hat{A}_T^{-1}.$$

B Simulation details

1. Simulating the required conditional expectation functions Given the analytic expressions for the conditional expectations (5), the next step is to calculate the conditional expectations (8). Recall that the vector of log-signals $(\tilde{x}_1, \ldots, \tilde{x}_n)$ is jointly-normal with mean vector $M^* = (\mu^*, \ldots, \mu^*)$ and variance covariance matrix Σ^*. Let $\tilde{x}_{3+}$ denote the sub-vector of log-signals $\tilde{x}_3, \ldots, \tilde{x}_n$, and $\tilde{x}_{+2}$ denote the vector of log-signals $\tilde{x}_1, \tilde{x}_2$. Then

$$\begin{pmatrix} \tilde{x}_{3+} \\ \tilde{x}_{+2} \end{pmatrix} \sim \text{normal}\left(\begin{bmatrix} \mu \\ \mu \end{bmatrix}, \begin{bmatrix} \Sigma_{3+}^* & \Sigma_{3+,+2}^{*\prime} \\ \Sigma_{3+,+2}^* & \Sigma_{+2} \end{bmatrix}\right)$$

where the elements of the mean vector and variance-covariance matrix can be determined from the information structure of the model.

Then, again using the multivariate normal conditional expectation formulas:

$$\tilde{x}_{3+}|\tilde{x}_{+2} \sim \text{normal}\left(\mu + \Sigma_{3+,+2}^{*}{}'\Sigma_{+2}^{-1}(\tilde{x}_{+2} - \mu)\ , \Sigma_{3+}^* - \Sigma_{3+,+2}^{*}{}'\Sigma_{+2}^{-1}\Sigma_{3+,+2}^*\right) \tag{11}$$

Thus, given a vector of log-signals $(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)$, we can take S draws of $\tilde{x}_{3+}$ according to the conditional distribution (11) and simulate the integral in (8). In the results below, we utilized the GHK algorithm for drawing from truncated multivariate distributions. Essentially, this algorithm "importance samples" recursively from the truncated region, one dimension at a time, and allows for smooth (in θ) simulation of the required conditional expectation $v_n(x, x; \theta)$. While this simulated estimate of the conditional expectation is biased for a finite number of draws, some Monte Carlo evidence (cf. Keane (1994), McFadden and Ruud (1995)) has demonstrated that this bias is small in practice.[41] Let R_2 denote the number of simulation draws used in simulating $v_n(x, x; \theta)$.

2. Simulating the equilibrium bids We also employ simulation to evaluate the equilibrium bid $s_n(x; \theta)$ for a given signal x which, as formula 2 shows, can also be expressed as a conditional expectation, taken over the distribution of the maximum element among a vector $(x_2, \ldots, x_n)$ generated from $F(x_2, \ldots, x_n|x_1; \theta)$. Given the difficulty in expressing the distribution of this maximum In order to draw from the distribution of the maximum element, we use a GHK simulator (cf. (Gourieroux and Monfort, 1996. 98–100)) to draw vectors $(x_2^r, \ldots, x_n^r)$ from the truncated multivariate normal distribution $F(x_2, \ldots, x_n|x_1, x_j > x_1, j =$

[41] We used a version of the GHK code for multivariate normal distribution available from V. Hajivassiliou's website: `www.lse.ac.uk/vassilis`.

$2, \ldots, n; \theta)$, where $r = 1, \ldots, R_3$ indexes the simulation draws, and R_3 denotes the total number of simulation draws. Let y^r denote $\max_{j=1,\ldots,n}\{x_2^r, \ldots, x_n^r\}$. Then a simulator of the equilibrium bid is

$$s_n(x_i;\theta) \approx \frac{1}{T_{x_i}} \frac{1}{R_3} \sum_{r=1}^{R_3} v(y^r, y^r; \theta)\omega^r \tag{12}$$

where ω^r is the GHK "importance sampling weight" corresponding to simulation draw r[42], and $T_{x_i} \equiv \frac{1}{r}\sum_r \omega^r$ is an estimate of the multivariate-normal truncation probability
$\text{Prob}\,(x_2 > x_1, x_3 > x_1, \ldots, x_n > x_1 | x_1 = x_i; \theta)$.

C Identification

Variation in the data Our large auction dataset contains variation along four dimensions: bidders, worktypes, and number of participants.

Bidders: Since we focus on a symmetric auction model, we make the assumption that bidders are homogeneous. Therefore we do not exploit the fact that we observe an identical bidder across many bidding situations. Such variation will be important once we investigate the asymmetric case, which is the topic of ongoing research.

Worktypes: In the current version, we assume that all contracts for a specific worktype are homogeneous, and estimate different θ's for each worktype. Therefore, we do not exploit observations on contracts related to different worktypes to help identify common parameters. One reason we did so was to minimize the computational burden involved in simulating the equilibrium bids. But in future extensions we plan to include more contract-specific covariates (year and seasonal dummies to capture secular time effects).

Number of participants: While we assume that all contracts for a given worktype are homogeneous and that all bidders contending for a given contracts are ex ante identical, there will still be heterogeneity in bidding behavior across contracts of a given worktype due to variation in the number of participants. For a given bidder, equilibrium bidding strategies in the first-price auction model described in the previous section will differ depending on the number of rival bidders: therefore, the distributions of the equilibrium bids will not be identical across auctions with different number of participants, even among all contracts of the same worktype. This is an important source of variation which we use to identify the parameters.[43]

Shape of $\mathbf{s_n(x;\theta)}$ *function* It is in general difficult in very nonlinear models such as this to be very explicit about what types of variation in the data serve to identify particular parameters. Therefore we tackle the identification issues by proceeding in the opposite direction: we simulate our model to investigate how changes in the parameter values affect the $s_n(x;\theta)$ function, the moments and quantiles of which form

[42] cf. (Gourieroux and Monfort, 1996, eq. 5.14). Note that each ω^r, $r = 1, \ldots, R_3$ is an unbiased estimate of the multivariate-normal truncation probability $\text{Prob}\,(x_2 > x_1, x_3 > x_1, \ldots, x_n > x_1 | x_1 = x_i; \theta)$.

[43] Given the sizeable variation in the number of bidders observed in the auction dataset, even for contracts of a given worktype, the possibility arises that bidders may not be aware of the number of rivals when they submit their bid. Hendricks, Pinkse, and Porter (1999) derive the equilibrium bidding strategies for a symmetric affiliated value auction, and we plan to explore this extension. However, once we assume that bidders are not aware of the number of participants, we will not be able to use this variation to identify the parameters, and will probably need to make stronger assumptions along another dimension.

the basis of our estimation strategy. This exercise will alert us to parameters which are badly identified as those which spark no change in the (simulated) $s_n(x;\theta)$ function.[44]

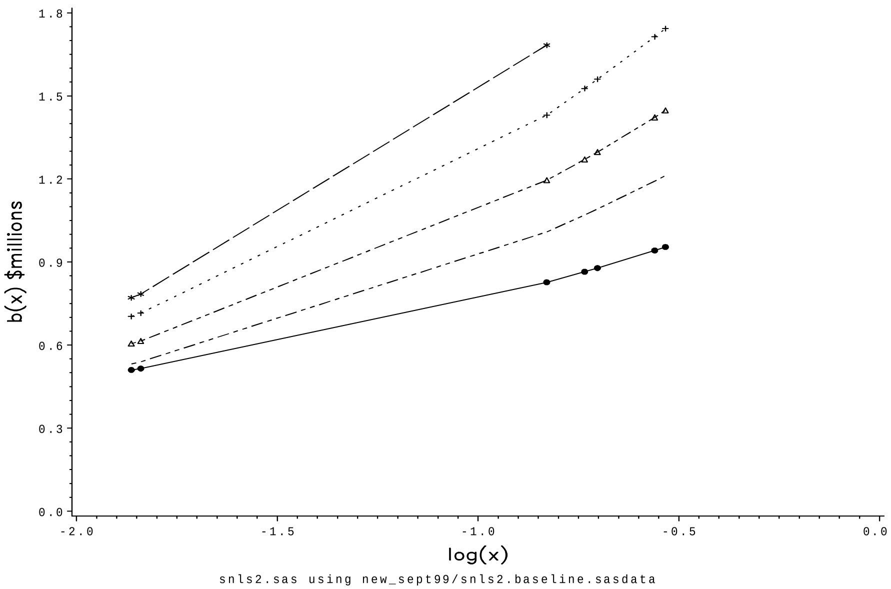
Graphs of $s_n(x;\theta)$ for alternate values of the parameters θ are given in figure 5. The benchmark values for the parameters $(\mu, \sigma_w, \sigma_a, \sigma_e)$ were $(-1, 1, 1, 1)$, and the graphs of $s_n(x;\theta)$ for these values, for 2,4,6,8 and 10 participants, is shown in the upper left-hand panel of figure 5. The axis scales on all the graphs in figure 5 are held constant, for easy comparison. Note that the graphs are upward-sloping, as expected. More significantly, note that, for any given x, $s_n(x;\theta)$ is higher for larger number of participants: this demonstrates that for these parameters, the winner's curse (which encourages more cautious bidding as n increases) dominates the competitive effect (which encourages more aggressive bidding).

Next, we explore how changes in the parameter values, relative to the benchmark case, affect $s_n(x;\theta)$. First, the lower right hand graph shows what happens to $s_n(x;\theta)$ in the pure private value case, as σ_e is set to 0. In this case, the equilibrium bid functions are decreasing in n, for a given x: as expected, in private-value auctions when the winner's curse is absent, bidding becomes more agressive as n increases.

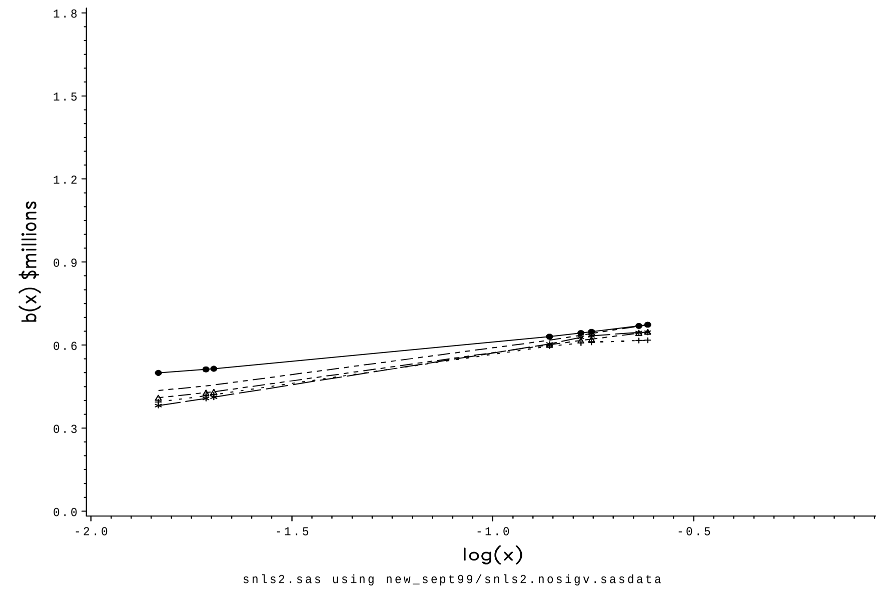
A similar pattern results if we let $\sigma_w = 0$, as shown in the top right-hand side graph. While this outcome doesn't correspond to any of the "standard" auction specifications (is it essentially a independent private values model where bidders have imperfect information about their private values), it can be transformed into a standard IPV model by redefining the private value $\tilde{x}_x$ as $E[c_i|x_i]$. Finally, in the pure common value case (lower left hand side graph), in which σ_a is taken to zero, the $s_n(x;\theta)$ graphs resemble the benchmark graphs, but differ in magnitude.

In short, these graphs have demonstrated that perturbations in the basic parameters in the model do lead to changes in the shapes and/or magnitudes of the $s_n(x;\theta)$ graphs, which lead us to believe that, from a computational point of view, the parameters are indeed identified and estimable.

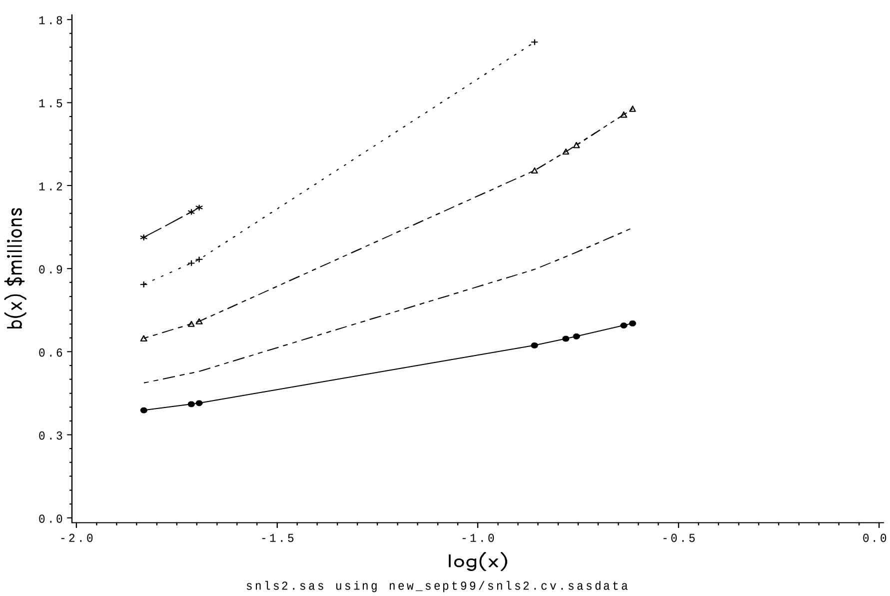
[44]Moreover, from a computational point of view, the parameters of the model, even if well-identified, are readily estimable only if changes in each parameter have some degree of independent effect on $s_n(x;\theta)$. The results from this exercise also draw attention to potential computational difficulties we might face in identifying the parameters.



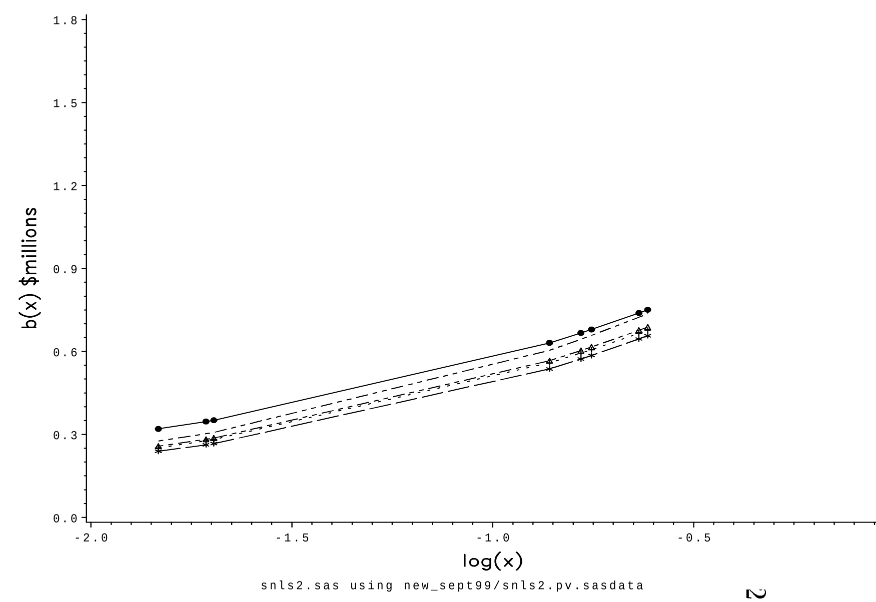
Baseline case: Parameters=(-1 1 1 1)



Parameters=(-1 0 1 1)



Parameters=(-1 1 0 1): Pure common value



Parameters=(-1 1 1 0): Pure private value

Figure 5: Plots of EV(x) for different parameter values

Legend: –●–●– 2 bidders – — – – — 4 bidders △- — –△– 6 bidders · · + · · + · 8 bidders —-*—*—* 10 bidders