Community Enforcement with Endogenous Information*

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Abstract

We consider cooperative arrangements in a fixed community where agents may change partners over time and where public communication is possible. Public monitoring and exogenous information flows are absent: any player's action in any period is observed only by the agent himself and his partner in that period. We show that cooperation can be sustained as a sequential equilibrium in such an environment if agents are required to make public and simultaneous announcements about their activities even if such announcements are non-verifiable. This result also holds in the presence of small costs of *information transmission*; however, there may be inefficiencies in such an environment. In the presence of *information processing* costs, cooperation may be difficult to sustain; however, if there are some exogenous probabilities of a change in the environment, cooperation can be sustained even in the presence of (private and unobservable) costs of gathering information.

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1 Introduction

This paper studies interactions between members in a community where agents change partners over time and observability of a given agent's action in any period is limited. We consider environments where the flow of relevant information is generated strategically and non-verifiably by the agents themselves and look at the sustainibility of cooperative arrangements through community enforcement and social reputations.

In developed nations, some information about agents' behaviour and history is available in the form of credit rating agencies etc. In general, however, centralised information flow is a rarity and usually, there are no exogenous mechanisms for credible information flow. A key issue in such a context is the *credibility* of information. The previous literature has by and large assumed exogenous information processes. Also, the studies which do consider the endogeneity of information flows have assumed that there are no costs to generating or receiving information. This seems a strong assumption in economies where such flows are crucial for cooperative arrangements, yet are meagre. This paper makes the natural assumption that information flow is *costly* and examines the implications of these two central features for long-run efficient arrangements.

To model infrequent interaction, we assume random matching within the community. In such environments, the onus of maintaining cooperation through credible punishments shifts somewhat from the individual to the group. We follow Kandori (1992) and Okuno-Fujiwara and Postlewaite (1995), who consider situations of this sort and analyse them with the help of *social norms*. A social norm may be roughly described as "...specification of desirable behaviour together with sanction rules in a community" (Kandori (1992)). Examples of environments where such issues are of critical importance can be found in the work of Greif (1993), Greif, Milgrom and Weingast (1994) etc. where the role of the community in maintaining long-term efficient arrangements is explored in the context of mediaeval trading groups in Europe and Asia.

The main results are as follows. With sufficiently patient players cooperation can be sustained as a sequential equilibrium if people are required to give reports about what they and their partners did every period. The problem of private information is resolved through communication and incentives to tell the truth are generated by punishment of incompatible messages[1]. This result holds also in the presence of small costs of sending messages. However, there is a problem of inefficiency in the following sense: if we restrict the space on which messages can be conditioned, a 'folk' theorem can be established, but in equilibrium, agents would be required to send messages every period even if there were no deviations. We show that introduction of costs of information processing makes a non-trivial difference: the equilibrium described above is not robust to small costs of information processing. To resolve this problem, we introduce the idea that messages, in travelling from the senders to the receivers, can become distorted with a small probability. We show, interestingly enough, that it is this very distortion of information which enables cooperation to be restored in an environment with costly information gathering.

The rest of the paper is organised as follows. Section 2 surveys the literature while Section 3 outlines the basic model. Section 4 proves the benchmark cooperation result and discusses possible inefficiencies in the presence of costs of information transmission. In Section 5, we consider the case where receiving messages is no longer costless. Section 6 concludes. Section 7 contains some proofs not found in other sections.

2 Related Literature

The typical assumption of the literature on *imperfect monitoring* games is that statistical signals are generated by the actions of agents, the distributions of which are common knowledge. In this situation, a distinction needs to be drawn between the literature which assumes that the same signals are publicly observable (see, for example, Abreu, Milgrom and Pearce (1991), Abreu, Pearce and Stacchetti (1986, 1990), Fudenberg and Maskin (1986), Fudenberg, Kreps and Maskin (1990), Fudenberg and Levine (1992), Fudenberg, Levine and Maskin (1994), etc.) and the literature which considers the case of private signals.

In the latter literature, an action by agent i may generate different signals for agents j and k. This complicates the analysis as the private information

[1]This idea is not new. See Postlewaite and Schmeidler (1986) and Ben-Porath and Kahneman (1996) for applications of the same idea.

makes coordination on strategies difficult and may, after certain histories, induce different continuation strategies for different players. Hence the recursive method of Abreu (1988) and Abreu, Pearce and Stacchetti (1990) will no longer be applicable. Within this strand, the earlier papers by Radner (1986), the series of works by Lehrer (see, for example, Lehrer (1990)) and Fudenberg and Levine (1991) assume no discounting or epsilon rationality of players in constructing equilibria.[2]

Compte (1998) and Kandori and Matsushima (1998) look at environments in which all or some players receive potentially different signals about the actions or strategies of other players; the issues considered are whether different actions or deviations by different players can be statistically distinguished, how many people must receive certain signals and what kind of communication can help resolve coordination problems so that punishment threats are credible. The paper closest to ours is the one by Ben-Porath and Kahneman (1996). They consider a situation where a group of agents interact every period and while a player's actions are in general not observable, they are perfectly observable to a subset of agents. They show that in the discounting case, having at least two such outside monitors enables the construction of folk theorems whereas one outside monitor is sufficient in the no-discounting case. We show it is possible to get similar results with only one outside monitor, provided messages are sufficiently detailed. Also, we consider the case of costly information.[3] In Ahn (1997), there is one player who can perfectly observe the actions of all other players whereas other players can observe the action of the perfect observer; the question considered is whether cooperation can be sustained through signalling behaviour of the perfect observer.

Kandori (1992) and Okuno-Fujiwara and Postlewaite (1995) also consider cooperation sustenance through social norms; however, they assume exogenous information flows. Ghosh and Ray (1996) explore imperfect monitoring games without information flows and consider equilibrium norms whereby agents may endogenously form long-term relationships.[4] However, for their equilibrium to hold, it is crucial that there be a non-trivial proportion of completely myopic players in the community who cannot be distinguished

[2] See Mailath and Morris (1999) for a discussion of 'almost-public' monitoring.

[3] See also Ahn and Suominen (1999) for a discussion of a repeated game between a seller and many buyers with limited information and cooperation through 'word-of-mouth' communication.

[4] See also Datta (1996).

ex-ante from 'rational' players. Sekiguchi (1997) also looks at imperfect monitoring games without information flows and derives approximate folk theorems where information is nearly perfect, though privately observed.

3 The model

There is a finite set of agents $\mathbf{N} = \{1, 2, ..., N\}$[5]. Time is indexed by $t = 0$, 1, 2, ... At the beginning of every date, each player is randomly matched with another player and they play a stage game. There is a common action set $\mathbf{A}$ and symmetric payoff functions for all agents. The stage game is described a payoff function $f : A \times A \rightarrow \mathcal{R}^2$. f is continuous.

Let $f_n(a_n, a_m)$ denote the payoff to player n in the stage game when he has chosen action $a_n \in \mathbf{A}$ while his opponent, player m has chosen action $a_m \in \mathbf{A}$. A minimax point $M^n \in A \times A$ for player n is:

$M^n_m \in \arg \min\limits_{a_m \in A} (\max\limits_{a_n \in A} f_n(a_n, a_m))$

and $M^n_n \in \arg \max\limits_{a_n \in A} f_n(a_n, M^n_m)$.

$M^m \in A \times A$ for player m is defined similarly. The 'mutual minimax' point is (M^m_n, M^n_m). We normalise payoffs such that $f_n(M^n) = 0$.

The set of feasible and individually rational payoffs is $V = \{v \in cof(A \times A) | v > 0\}$, where co denotes the convex hull. Let $x_n = f_n(M^m_n, M^n_m)$ be the payoff to agent n under mutual minimaxing and $\overline{v}_n(a_m) = \max\limits_{a_n \in A} f_n(a_n, a_m)$. x_m and $\overline{v}_m$ are defined similarly. Also, let $\underline{v}_n(a_n) = \min\limits_{a_m \in A} f_n(a_n, a_m)$. Define $\underline{v}_m$ similarly.

After the stage game, at the end of each period, each agent n can make an announcement from a set R_n. The reports are simultaneous and are publicly heard. A strategy for player n, π^n in the repeated game is a sequence of functions $\pi^n = (\pi^n_{1,1}, \pi^n_{1,2}, ..., \pi^n_{t,1}, \pi^n_{t,2}, ...)$, where $\pi^n_{t,1}$ specifies an action as a function of observed actions and messages from previous periods and $\pi^n_{t,2}$ specifies an announcement or message as a function of observed actions and messages from previous periods and the observed actions in the current period.

[5]For simplicity, we assume N to be even. The extension to the case where it is odd is straightforward.

We assume that players maximise the discounted sum of stage game payoffs over an infinite horizon. Throughout, we restrict attention to pure strategies. All agents have a common discount factor $\delta \in (0,1)$. All previous messages are perfectly remembered and the identity of any player's current partner is publicly known.

The equilibria that we characterise will be in terms of norms as discussed in Section 1. We use the notion of sequential equilibrium as a solution concept. In our equilibria, the beliefs are either unimportant or very simple and will satisfy consistency. Extension of sequential rationality to infinite games is straightforward.

4 Cooperation with truthful messages

This section shows that cooperation is sustainable even if information flows through endogenously generated non-verifiable reports of agents. To be precise, we construct a sequential equilibrium such that as the discount factor tends to 1, the set of payoffs under this equilibrium contains the set of individually rational payoffs. For now, assume there are no costs of information flow.

Consider the following norm. The symmetric payoff $f(a,a) = v \in V$, $(a,a) \in A \times A$ is to be sustained. Agents may have one of two labels: 'innocent' or 'guilty'. At the beginning of any period, if (and only if) an agent is guilty, then he is currently on a 'punishment path'. A punishment path is of finite length; we shall consider two possible paths, one of which is longer than the other. The meanings of the labels and the characteristics of the paths will be defined forthwith.

At the beginning of time all agents are innocent. Every period the norm specifies the following behaviour in the stage-game. If two innocent players meet each other they each play a and get v. If at least one of the partners is guilty, they mutually minimax each other and get x.

After the stage game, if at least one of the partners had the label 'guilty' at the beginning of that period, each makes a report. If both partners were innocent, they are both required to make a report if and only if at least one of them deviated at the stage game. A single report coming in from a match between two innocent agents is believed. For any player n, the message space is $R_n = \{(0,0),(0,1),(1,0),(1,1)\}$, where a 0 means "no deviation" and a 1 means "deviation". For player n message (a, b) means that he did a while his partner did b.[6]

If the two partners send different messages,[7] then both become guilty for T' periods (i.e., goes onto a T'-period punishment path) starting in the next period. If they give the same report, then if anyone is indicted by both parties, he becomes guilty starting the next period. If the incriminated player is currently already on a T'-period punishment path, then this path restarts for him. If not, he is going to be considered guilty for T periods starting in the next period, where $T < T'$. The other agent in the match (in the event that only one of them is indicted) remains innocent (if previously innocent) or becomes innocent (if previously guilty). If the reports say both conformed then whoever is currently innocent remains innocent and whoever is guilty continues on the punishment path.

If messages come in reporting deviations in different matches in the same period, they are all ignored. If a new deviation is reported, anyone currently on a punishment path is forgiven and the new deviator is punished. If a currently guilty player is consistently reported[8] to have conformed to the norm in any period, then the number of periods left on the punishment path reduces by 1, unless that period is the last for that person on the current punishment path, in which case the person becomes innocent from the next period onward. In the event of indifference, agents are assumed to follow the norm. Finally, after the action stage, a players' belief is that any player not observed by him has not deviated.

We are now ready to state our first result. We shall show that cooperation can be sustained in the long run within the community by using the norm or collection of strategies described above. In addition, even though messages are non-verifiable, when messages are sent, they are truthful. In equilibrium, players cooperate and messages are not sent. The intuition is simple. Innocent partners do not have an incentive to report anything when both have conformed. Since inconsistent messages are punished heavily, players will tell the truth, given simultaneous reporting.[9] Once it is shown that

[6] Hence a report $(1, 0)$, for example, means that player n deviated while his partner conformed to the norm.

[7] For example, when n and m are matched, n reports $(0, 1)$ while m reports $(0, 1)$.

[8] By 'consistently reported' we mean that both partners gave the same message.

[9] Our norm takes care of the following perverse situation; an agent who is on a long punishment path deviates at the action stage and then reports the truth to get onto a shorter punishment path. This is not feasible as the longer punishment path just restarts

announcements will indeed be truthful, it is straightforward to show that if people are sufficiently patient, cooperation can be sustained. Hence players remain innocent and messages are never sent.

Proposition 1 *The norm described above constitutes a sequential equilibrium and hence, any $v \in V$ can be sustained as a long run equilibrium payoff as long as people are sufficiently patient, i.e., as long as $\delta \in (\delta_*, 1)$, for some $\delta_* \in (0,1)$.*

The main idea is that as long as deviation payoffs are bounded, one-shot gains don't matter in the long run if people are patient enough. And if punishments are sufficiently severe, people conform. This threat of renewed punishment is also sufficient to induce people to report truthfully, even though no one in the economy can monitor truthfulness. Hence we endogenise the information structure of Kandori (1992) and Okuno-Fujiwara and Postlewaite (1995). Further, we show if there's only *one* other person in the economy who can observe the actions of a given agent (such that the latter also observes the former), then even with random matching, community sanctions can work to maintain efficient agreements. Finally, the norm described above satisfies all the criteria (simplicity, local information processing, straightforwardness, global stability and independence of detail) laid down by Kandori (1992).

4.1 Costly information transmission

We now introduce costs of transmitting information into the benchmark environment. For the time being assume there are no costs of processing information. Every time an agent sends a report, he incurs a cost $c_t > 0$,[10] which is small in comparison to the payoffs the community is trying to sustain in equilibrium. The question is, will the incentive to cooperate and to send truthful messages be preserved in this new environment?

Notice, whether an agent is sending a message or not is public information. Consider the following modification of the norm. When reports come in, if an agent in a partnership with at least one guilty player has *not* made

for such an agent.

[10] The subscript in c_t stands for 'transmission' and should not be confused with time.

an announcement, he goes onto a punishment path of length $T'' > T'$ regardless of other deviations that may have occurred. If both partners in such a match are silent then both are sent onto a T''-period punishment path. In matches between two innocent people, if a single report comes in, the agent *not* sending a report is sent onto a T''-period path. If people in different matches deviate from sending messages then all such deviations are ignored. If an agent is currently on a T''-period path and if any punishment has to restart for him, then he goes on to a T''-period path again. The rest of the norm remains as before.

Thus, we have added a new tier to the hierarchy of punishments described earlier. The relationship between the hierarchies and the prescribed behaviour at the stage game and the report stage remain the same as earlier. It is easy to see that this modified norm is capable of sustaining cooperation as in the benchmark case if c_t is small: in equilibrium, agents cooperate at the stage game. Moreover, off the equilibrium path, truthful messages are always generated by agents in matches where at least one partner is guilty. Similarly, in matches between two innocent agents, if one of them deviates at the action stage, this is reported by both. However, in the event that two such agents cooperate at the action stage, they have no incentives to send in any reports. We thus have the following:

Corollary 1 *In an environment with small costs of information transmission c_t, the modified norm described above can sustain cooperation as a sequential equilibrium as long as people are sufficiently patient. Moreover, this equilibrium is efficient in the sense that in equilibrium, messages are not sent and hence, agents do not have to incur the cost c_t.*

4.2 Inefficiency

The efficiency result above relied on the fact that messages were allowed to depend on observed behaviour at the stage game as well as the 'state' of the partners, i.e., whether the partners were innocent or guilty at the beginning of the period. Suppose, however, that messages cannot be conditioned on the states. Can our norm sustain cooperation as a sequential equilibrium?

Clearly, in this case, the norm must be such that, for all matches, partners are supposed to report truthfully if there has been a deviation at the stage game and remain silent otherwise. Suppose that single reports coming in from a match are believed.[11] Consider partners n and m with the former innocent and the latter guilty. Imagine both conformed at the stage game. m, believing n will keep silent in accordance with the norm, has an incentive to send a message saying n deviated as then his punishment will stop. Of course, m will have incur cost c_t; since it is small, this will be outweighed by the benefit of becoming innocent. Hence, a profitable deviation exists for m.

Making reporting mandatory in every period gets around this problem. Failure to report attracts the heaviest punishment, followed by inconsistent messages, and so on. The rest of the norm remains the same. It is easy to see that cooperation can be sustained as a sequential equilibrium. All agents send in reports and they prefer to tell the truth when sending in reports. Finally, if they are patient enough, they prefer to cooperate at the stage game.

Notice, however, that agents are required to make announcements always, even if everyone is cooperating. This is what we refer to as an inefficiency in this paper. The key features which give us this negative result are the facts that sending a message is a simultaneous and one-shot process and that punishment strategies are conditioned on agents' reports about themselves. We can sum up the arguments as follows:

Corollary 2 *The norm described above can sustain cooperation as a sequential equilibrium if players are sufficiently patient. However, there is an inefficiency in the sense that agents have to communicate and thus have to incur the costs of sending reports every period.*

5 Costly information processing

Now consider the possibility of people having to expend resources to *receive* information. For simplicity, assume there are no costs of sending reports.

[11] Similar arguments as the ones presented below can be constructed for alternative reasonable beliefs conditional on the receipt of a single report from a match.

Suppose, to hear messages, agents have to incur a cost $c_p > 0$ after the stage game and before messages are generated[12] and that once the cost is incurred, all messages can be heard. c_p is small in compared to the equilibrium payoffs. The question is: are the norms described earlier robust to such small costs of information processing?

5.1 A Failure of Cooperation

Consider the previous norm, modified as follows. In addition to the earlier strategies, people are also required to hear messages: failure to do so results in punishment. Note that if any agent's partner could observe his investment decision then, by a suitable modification of messages and punishment contingencies, the norm described above could be enforced. This can be achieved by requiring all agents to simultaneously report, in addition to actions carried out during the stage game, whether he and his partner incurred the cost or not. Punishments based on incompatibility will ensure that one-shot savings of c_p will be wiped out if people are sufficiently patient.

On the other hand, suppose that incurring the cost is a completely private activity. Then, communication-based equilibria will fail to support efficient arrangements as sequential equilibria. We continue to make the same stationarity and symmetry assumptions as before.[13] Let U denote the set of payoffs from the one-shot Nash equilibria of the stage game. The report stage assumptions remain the same as before.

Define a set C_p of sequential equilibria which have the following features:

(i) Along any equilibrium path an agent is required to play $a \in A$ designed to yield gross payoff $v > c_p$, $v \notin U$. This is common knowledge.

(ii) Deviation by agent n is reported immediately by himself and his partner. Along the equilibrium path, reports are truthful, i.e., if n has not deviated, then he cannot be reported to have deviated. If a player is reported to have deviated, he is considered guilty and goes onto a punishment path. Multiple reports in at the same time are ignored.

[12] What is critical is that the investment has to be made *before* a player knows whether he has received a message or not.

[13] The arguments below are actually more general and can be extended to accomodate non-stationarity, asymmetry, more general matching rules, observability etc.

(iii) Cooperation is sustained by communication and stage-game strategies are conditioned *only* on reports received in the past and the initial history whereas message-game strategies are conditioned on past reports as well as observed behaviour. If two partners in period t are both currently innocent in accordance with past reports both play a.

Clearly, the earlier equilibria are members of C_p (with $c_p = 0$). For any $e \in C_p$, all agents cooperate always along the equilibrium path by (i). By (ii) actions are truthfully reported and deviations are punished. Moreover, cooperation is sustained entirely by communication by (iii): if there were no information flows, agents would have the incentive to deviate knowing they would not be punished and hence (i) would be violated.

Consider an agent n in period t. Suppose all $i \in \mathbf{N}$ are innocent at t and n and his partner played a, the prescribed action, in t. Suppose n is contemplating whether to incur the cost or not, given that everyone else is hearing messages. Since n is currently innocent and so is everyone else, and no deviations have occurred or are expected, everyone is expected to remain innocent in the future, in equilibrium.

A one-period deviation gives payoff $\sum_{\tau=t+1}^{\infty} \delta^{\tau}(v - c_p)$. Else, if he hears all messages in all periods including t, his payoff is $-c_p + \sum_{\tau=t+1}^{\infty} \delta^{\tau}(v - c_p)$.

Since $\delta < 1$ and $c_p > 0$ he doesn't incur the cost: by symmetry, neither does anyone else. Thus messages aren't heard and deviations, even if reported, cannot be punished. Thus cooperation cannot be sustained by any such equilibrium. We thus have the following:

Proposition 2 *If $c_p > 0$, $C_p = \phi$ for any $\delta < 1$.*

Thus for $c_p = 0$, there exist equilibria in C_p which can sustain any payoff dominating the individually rational payoffs in the infinitely repeated game. But for *any* positive c_p, C_p is empty. The following is then immediate:

Corollary 3 *The set of payoffs for any $e \in C_p$ is discontinuous at $c_p = 0$.*

This result shows that communication, which resolved the uncertainty stemming from imperfect observability, removes all uncertainty in equilibrium. Since agents expect the equilibrium path to be sustained with probability one, one-shot saving of the processing cost becomes optimal. We now turn to the question of whether we can design a mechanism which can sustain cooperation and yet be robust to costs of processing information.

5.2 Restoring Cooperation

The reason cooperation breaks down is that, in equilibrium, messages lose their value. Thus, restoring cooperation, requires some uncertainty about future behaviour. For example, suppose a common randomising device is available to all agents which yields an idiosyncratic signal every period after the stage game and that, along the equilibrium path, actions at the stage game are contingent on the actual realisations, which are known only to the partners. Then messages could be modified to report not only behaviour at the stage game but also the realisation of the randomising device. This would restore value to hearing messages.

Here, we consider an alternative formulation of *exogenous* uncertainty to resolve the problem. The idea is that in imperfet information environments, there may be garbling of messages with small probabilities. Specifically, we assume that given any profile of messages generated at the end of a period, there may be a garbling of messages such that one (and only one) agent's 'state' may change at the beginning of the next period with a small probability p'. Independence across agents implies that the probability of such a state change for a given agent n is $\frac{p'}{N} = p$.[14]

Consider the benchmark norm and assume that all punishments last for T periods[15]. Also, the norm requires that all players incur the cost every period. For any two agents n and m who have been matched in a period t, there are 16 possible reports at the end of the period coming from the set $R_n \times R_m$. Agent n can be in one of two possible states at the beginning of period t: 'innocent' or, 'guilty' with t being the $\overline{T}^{th}$ period of his current punishment, $1 \leq \overline{T} < T$. The case where he is on the last period of his punishment will prove to be analytically equivalent to the innocence state. The following scheme shows the state-transition rule; the state n is supposed to be in period $t+1$ given the state in period t and the messages at the end of the period from the partnership with no garbling of messages.

[14]This p should be distinguished from the subsript in c_p, where the 'p' stands for 'processing'. Hopefully, the context should clear any confusion.

[15]This reduces computation. All results extend to the more general case.

1. Innocent today with reports being either $\{(0,0),(0,0)\}$ or $\{(0,1),(1,0)\}$ means innocent tomorrow.

2. As in (1), with reports being anything else means guilty for T periods tomorrow.

3. Currently guilty, t being the $\overline{T}^{th}$ period of the punishment path with reports $\{(0,0),(0,0)\}$ means guilty tomorrow with $t+1$ being the $(\overline{T}+1)^{th}$ period of the path..

4. As in (3), with reports $\{(0,1),(1,0)\}$ means innocent tomorrow.

5. As in (3), with reports being anything else means guilty for T periods tomorrow.

We are now ready to identify what we mean by an exogenous change of state. Because of the garbling of the messages, the state of agent n, instead of being what it is supposed to be at time $t+1$, becomes changed by the following scheme. The numbering follows the same order as for the state-transition rule above.

1. Guilty for T periods tomorrow.

2. Innocent tomorrow.

3. Guilty for T periods tomorrow.

4. Guilty tomorrow with $t+1$ being the $(\overline{T}+1)^{th}$ period of the path.

5. Guilty tomorrow with $t+1$ being the $(\overline{T}+1)^{th}$ period of the path.

A player can hear all messages generated at the end of a period if and only if he incurs the cost in that period. If he hears the messages, he shall also be aware of what his own and his partner's states are the next period and hence what actions to play in accordance with the norm. If not, he will be unaware of any possible state changes and hence may end up playing the wrong action. Of course, if he makes this mistake (which occurs with a small probability) then with a high probability he will be punished from the next period. It is this which restores cooperation with everyone incurring the cost of processing information. If the costs are low enough, then the one-period gain from saving this cost will be outweighed by the expected punishment resulting from playing the wrong action. We thus have the following proposition:

Proposition 3 *Let $V' = \{v \in V | v - c_p > 0\}$. The norm described above constitutes a sequential equilibrium and hence, given any target payoff $v' \in V'$, agents take action a, $f(a,a) = (v', v')$ and incur the cost c_p every period as long as people are sufficiently patient, i.e., as long as $\delta \in (\delta_*, 1)$, for some $\delta_* \in (0,1)$, provided $c_p \in [0, c_p^*)$, for some c_p^* small and $p \in (p_c, p_*)$, for some p_* small and $p_c > 0$. Moreover, long-run average payoffs converge to v' as p and c_p go to 0.*

6 Conclusion

We studied the maintenance of cooperation in communities with pairwise interactions. The role of communication turns out to be crucial in devising methods to ensure that deviations from social norms are punished. We showed that folk theorems can be proved even if there is limited monitoring in the economy and information flows are endogenous and non-verifiable. Moreover, we showed cooperation can be sustained, under some circumstances, even if gathering or sending information are costly and private activities.

It has been observed that size of a group and the ease of information flow within a group are related. The argument is that information should flow more smoothly in smaller communities. Of course, in smaller communities each individual has more to lose from punishing deviators. In our model, information flows are meagre but this difficulty is not related to group size. In this context, it is useful to compare the results here with the *contagion equilibria* of Kandori (1992).[16] A nice feature of such strategies is that they do not require communication. However, they require some restrictions on the stage game payoffs and are not very useful with large populations. Also, in general, there is a lack of robustness to noise.[17] With uncertainty, it is difficult to sustain cooperation. In both mechanisms, along the equilibrium path, cooperation always occurs and messages are never sent. There is thus some observational equivalence between them. However, it can be easily shown that for the same game, the equilibrium strategies considered in this paper can sustain cooperation with a smaller discount factor. To an extent

[16] See also Ellison (1994) and Harrington (1995).

[17] Ellison (1994) shows that contagion equilibria are not too fragile. However, the equilibrium he constructs is not globally stable.

this results from the assumption that all messages can be heard publicly. A possible area of future research would be to investigate how contagion equilibria and communications based equilibria compare when messages are no longer fully public, as in Ahn and Suominen (1999).

Finally, in our model, costly information processing led to a failure of cooperation as there was no value to hearing messages in equilibrium. The presence of exogenous uncertainty helped resolve the problem. It would be interesting to see whether such problems could be tackled by the creation of endogenous equilibrium uncertainty.

7 Proofs

Proof of Proposition 1. First of all, if everyone always follows the norm, clearly payoff v can be sustained. We therefore have to check whether it is in the interest of everyone to follow the norm. Since we are using a Nash equilibrium concept, we shall check if it is in the interest of any arbitrary player to follow the norm, given that everyone else is following it. For simplicity, we shall assume that the number of agents is large[18].

We first show that, given any set of actions during the stage game, an agent sends truthful messages. Suppose n and m, two arbitrary agents, have been matched in some period t.

A) n is innocent while m is currently being punished. The case where both are currently guilty is similar.

i) Suppose they both follow the norm during the stage game.

Given m's strategy of reporting of $(0,0)$, if n says anything else, a T'-period punishment starts for him. Hence he prefers to report $(0,0)$. Similarly, m he prefers to say $(0,0)$, as his punishment path then reduces by a period.

ii) Suppose n has deviated during the stage game, while m has not.

Given m's report of $(0,1)$, if n reports $(1,0)$ a T-period punishment path starts for him. If he reports anything else, a T'-period punishment path starts. Similarly, if m prefers to report $(0,1)$, as he is then forgiven.[19]

iii) Suppose m has deviated during the stage game, while n has not.

[18] This assumption simplifies calculations and is otherwise unimportant.

[19] This way of generating strict incentives is, to the best of our knowledge, new. However, it does make punishments (somewhat) history-dependent. Also, sanctions are not inefficient in equilibrium. For alternate approaches, see Kandori and Matsushima (1998)

Given m's strategy of reporting $(1,0)$, if n prefers to reports $(0,1)$, as he then remains innocent. If m is currently on a T'-period punishment path, he reports $(1,0)$ by indifference. If he is on a T-period punishment path, he also prefers to report $(1,0)$ as otherwise he goes onto a T'-period path.

B) Both n and m are currently innocent.

i) Suppose they both follow the norm during the stage game. Clearly, neither has any individual incentive to send a report.

ii) Suppose n has deviated during the stage game, while m has not. The case where m deviated while n did not is symmetric.

Given m's strategy of reporting $(0,1)$, if n reports $(1,0)$, a T-period punishment path starts for him. If he reports anything else, a T'-period punishment path starts. Hence, he reports $(1,0)$. Similar arguments show that m also prefers to report $(0,1)$.

Hence, we see that irrespective of the history, messages are generated truthfully. We now check that, given that messages will be truthful, the prescribed stage game actions are indeed incentive-compatible.

Suppose agent n is currently guilty. Suppose also agent n is currently on a T-period punishment path.

If he conforms, he gets at least $x + \delta x + ... + \delta^{T-1}x + \frac{\delta^T v}{1-\delta}$.

If he deviates, he gets at most $0 + \delta x + ... + \delta^T x + \frac{\delta^{T+1} v}{1-\delta}$.

We choose T such that $x\frac{1-\delta^T}{1-\delta} + \frac{\delta^T v}{1-\delta} > 0$.

Clearly, he is strictly better off conforming for any $\delta < 1$.

Also, the same argument holds if n is currently on a T'-period punishment path.

If agent n is currently innocent, by conforming, he gets $\frac{v}{1-\delta}$.

By deviating, he gets at most $\overline{v}_n + \delta x + ... + \delta^T x + \frac{\delta^{T+1} v}{1-\delta}$.

Therefore, in order for the norm to be an equilibrium, it has to be the case that

$$\delta(1-\delta^T)x + \delta^{T+1}v + (1-\delta)\overline{v}_n \leq v$$

If we take $\delta \to 1$, holding δ^T constant such that $\delta^T < 1$, the L.H.S. becomes $0 < (1-\delta^T)x + \delta^T v < v$.

Hence, n is strictly better off conforming if δ is high enough.

Thus the norm is an equilibrium. Q.E.D. ■

and the discussion of revelation constraints and potential inefficiencies in Compte (1998). Their approaches are in general not relevant for matching models.

Proof of Proposition 3. The following notation will be useful.

Let $v = v' - c_p$.

Let V_I denote the lifetime expected payoffs for a person who is currently innocent, $V_{\overline{T}}$ denote the same for a person who is currently on the $\overline{T}^{th}$ period of his punishment path (i.e., has $T - (\overline{T} - 1)$ periods left for his sentence), $1 \leq \overline{T} < T$ and V_T denote the same for a person who is currently on the last period of his punishment path. All the above payoffs are at the beginning of the period.

In equilibrium, $V_I = v + \delta\{(1-p)V_I + pV_1\} = \frac{v + p\delta V_1}{1-(1-p)\delta}$,

$V_T = x - c_p + \delta\{(1-p)V_I + pV_1\} = \frac{(1-p)\delta v + p\delta V_1}{1-(1-p)\delta} + (x - c_p)$,

and $V_{\overline{T}} = x - c_p + \delta\{(1-p)V_{\overline{T}+1} + pV_1\}$

$= (x - c_p)\{1 + (1-p)\delta + ... + (1-p)^{T-\overline{T}}\delta^{T-\overline{T}}\} + \frac{(1-p)^{T-\overline{T}+1}\delta^{T-\overline{T}+1} v + p\delta V_1}{1-(1-p)\delta}$

Thus, $V_1 = \frac{(1-p)^T\delta^T v}{1-\delta} + (x - c_p)\frac{\{1+(1-p)\delta+...+(1-p)^{T-1}\delta^{T-1}\}}{1-\delta}\{1 - (1-p)\delta\}$

Therefore, $V_{\overline{T}+1} - V_{\overline{T}} = (1-p)^{T-\overline{T}}\delta^{T-\overline{T}}(v - x + c_p) > 0$,

and $V_I - V_T = v - x + c_p > 0$.

Now, $V_I(p=0) = \frac{v}{1-\delta}$ and $\left(\frac{\partial V_I}{\partial p}\right)_{p=0} = \frac{-(v-x+c_p)\delta(1-\delta^T)}{(1-\delta)^2}$.

Similarly, $V_{\overline{T}}(p=0) = \frac{\delta^{T-\overline{T}+1} v}{1-\delta} + (x - c_p)\{1 + \delta + ... + \delta^{T-\overline{T}}\}$

and $\left(\frac{\partial V_{\overline{T}}}{\partial p}\right)_{p=0} = \frac{-v\delta^{T-\overline{T}+1}}{(1-\delta)^2}\{\delta + (T - \overline{T} + 1)(1-\delta) - \delta^{\overline{T}}\} + (x - c_p)\frac{\delta(1-\delta^T)}{(1-\delta)^2} + (x - c_p)\{\frac{(T-\overline{T}+1)\delta^{(T-\overline{T}+1)}}{1-\delta} - \delta\frac{1-\delta^{(T-\overline{T}+1)}}{(1-\delta)^2}\}$

Finally, $V_1(p=0) = \frac{\delta^T v}{1-\delta} + (x - c_p)\{1 + \delta + ... + \delta^{T-1}\}$

and $\left(\frac{\partial V_1}{\partial p}\right)_{p=0} = \frac{-v\delta^T}{(1-\delta)^2}T(1-\delta) + (x - c_p)\frac{\delta(1-\delta^T)}{(1-\delta)^2} + (x - c_p)\{\frac{T\delta^T}{1-\delta} - \delta\frac{1-\delta^T}{(1-\delta)^2}\}$

With the above results, the rest of the proof proceeds in the following steps. Suppose n and m are two agents who have been matched in some period. Any agent takes his decisions assuming all other agents are following the norm. We consider one-shot deviations only. In Steps 1 through 3 below, we shall consider 6 cases. To save space, we enumerate them now.

i) n is currently innocent and both conformed at the stage game.

ii) As in (i); n deviated at the stage game while m did not.

iii) As in (i); n did not deviate at the stage game while m did.

iv) n is currently guilty and is on the $\overline{T}^{th}$ period of his punishment path. Both conformed at the stage game.

v) As in (iv); n deviated at the stage game while m did not.

vi) As in (v); m deviated at the stage game while n did not.

Step 1: Suppose the action stage is over and that n has incurred processing cost. In this step, we prove that it is optimal for him to make truthful announcements.

i) Given m's report of $(0,0)$, if n reports $(0,0)$, his payoff is $\delta\{(1-p)V_I + pV_1\} = A_1 + c_p$.

Otherwise, his payoff is $\delta\{(1-p)V_1 + pV_I\}$. Since $V_1 < V_I$ and p small, he reports the truth.

ii) Given m's report of $(0,0)$, whatever n reports, his payoff is $\delta\{(1-p)V_1 + pV_I\} = A_2 + c_p$. By weak dominance, he reports the truth.

iii) Same as (i) and hence n sends truthful messages.

iv) Given m's report of $(0,0)$, if n reports $(0,0)$, his payoff is $\delta\{(1-p)V_{\overline{T}+1} + pV_1\} = A_4 + c_p$.

Otherwise, his payoff is $\delta\{(1-p)V_1 + pV_{\overline{T}+1}\}$. Since $V_{\overline{T}+1} > V_1$ and p small, he reports the truth.

v) Given m's report of $(0,1)$, whatever n reports, his payoff is $\delta\{(1-p)V_1 + pV_{\overline{T}+1}\} = A_5 + c_p$. Hence, he reports the truth.

vi) Given m's report of $(1,0)$, if n reports $(0,1)$, his payoff is $\delta\{(1-p)V_I + pV_{\overline{T}+1}\} = A_6 + c_p$.

Otherwise, his payoff is $\delta\{(1-p)V_1 + pV_{\overline{T}+1}\}$. Since $V_1 < V_I$ he reports the truth.

Therefore, regardless of the history, if n incurs the cost of processing information, he sends truthful messages.

Step 2: Suppose the action stage is over and n has *not* incurred the cost. Since he does not know exactly what his and his partner's states will be next period, we have to specify what actions he will play next period. Here, we assume that he plays the action which is optimal conditional on the states not changing and then check if he will generate truthful messages.[20]

i) If n reports $(0,0)$, his payoff is at least $\delta[(1-2p)v + 2p\{\underline{v}_n - c_p\}] + \delta^2[(1-2p)\{(1-p)V_I + pV_1\} + 2p\{(1-p)V_1 + pV_I\}] = B_1$.

Otherwise, his payoff is at most $\delta[(1-2p)(x-c_p) + p\{\overline{v}_n - c_p\} + p(x - c_p)] + \delta^2[(1-p)\{(1-p)V_2 + pV_1\} + p\{(1-p)V_1 + pV_2\}]$.

Since p small, $V_I > V_2$ and $v > 0$, he reports the truth.

[20] Alternative specifications yield the same result. Moreover, this seems to be the reasonable specification given the small probability of state changes.

ii) Whatever n reports his payoff is $\delta[(1-2p)(x-c_p)+p\{\overline{v}_n-c_p\}+p(x-c_p)]+\delta^2[(1-p)\{(1-p)V_2+pV_1\}+p\{(1-p)V_1+pV_2\}]=B_2$.

Thus, he reports the truth.

iii) Same as (i).

iv) If n reports $(0,0)$, his payoff is

$\delta\{(1-2p)(x-c_p)+p(x-c_p)+p(x-c_p)\}+\delta^2\{(1-p)V_{\overline{T}+2}+pV_1\}=B_4$.

Otherwise, his payoff is

$\delta\{(1-2p)(x-c_p)+p(x-c_p)+p(x-c_p)\}+\delta^2\{(1-p)V_1+pV_{\overline{T}+2}\}$.

He chooses the truth as p is small and $V_1<V_{\overline{T}+2}$.

v) Whatever n reports his payoff is

$\delta\{(1-2p)(x-c_p)+p(x-c_p)+p(x-c_p)\}+\delta^2\{(1-p)V_1+pV_{\overline{T}+2}\}=B_5$.

Hence, he reveals the truth.

vi) If n reports $(0,1)$, his payoff is $\delta[(1-2p)v+2p\{\underline{v}_n-c_p\}]+\delta^2[(1-2p)\{(1-p)V_I+pV_1\}+2p\{(1-p)V_1+pV_I\}]=B_6$.

Otherwise, his payoff is $\delta\{(1-2p)(x-c_p)+2p(x-c_p)\}+\delta^2\{(1-p)V_2+pV_1\}$. Since p small, $v>0$ and $V_I>V_2$, he reveals the truth.

Hence, n reports truthfully in all cases.

Step 3: Hence, we see that messages are generated truthfully regardless of whether costs are incurred or not and regardless of the initial state of the players or actions at the stage game. Now we check that, given truthful messages, the costs are indeed incurred in equilibrium.

i) If n incurs the cost his payoff is A_1 where $A_1(p=0)=-c_p+\frac{\delta v}{1-\delta}$.

Also, $\frac{\partial A_1}{\partial p}=\delta\frac{\partial V_I}{\partial p}-\delta V_I-\delta p\frac{\partial V_I}{\partial p}+\delta V_1+\delta p\frac{\partial V_1}{\partial p}$

If, on the other hand, he does not incur the cost, his payoff is B_1 where $B_1(p=0)=\frac{\delta v}{1-\delta}$

and $\frac{\partial B_1}{\partial p}=-2\delta\{v'-\underline{v}_n\}+\frac{\partial V_I}{\partial p}\delta^2(1-3p+4p^2)-\delta^2V_I(3-8p)+\frac{\partial V_I}{\partial p}\delta^2p(3-4p)+\delta^2V_1(3-8p)$

Therefore, at $c_p=0$

$\left(\frac{\partial A_1}{\partial p}\right)_{p=0}-\left(\frac{\partial B_1}{\partial p}\right)_{p=0}=2\delta\{v'-\underline{v}_n\}+\frac{(v-x)\delta(1-\delta^T)}{(1-\delta)^2}(3\delta-2\delta^2-1)>0.$

for δ close to 1. Since A_1, B_1, $\frac{\partial A_1}{\partial p}$ and $\frac{\partial B_1}{\partial p}$are continuous in p and c_p, for sufficiently small c_p, we can find p small such that $A_1>B_1$.

Hence, people will incur the cost.

Cases (ii) through (vi) are omitted as almost identical tedious algebra shows that it is indeed optimal to incur the cost of processing information given that messages are generated truthfully, regardless of the actions at the stage game.[21]

Step 4: So, the only thing left to check is whether taking the actions specified by the norm are indeed optimal, given that everyone sends truthful messages and that everyone incurs the cost and hears the messages.

a) n and m are both currently innocent. If n deviates he gets $\overline{v}_n - c_p + \delta\{(1-p)V_1 + pV_I\}$.

If he does not, he gets $v' - c_p + \delta\{(1-p)V_I + pV_1\}$.

Thus, he does not deviate if $\delta(1-2p)(V_I - V_1) + v' - \overline{v}_n > 0$. Since p is small, this is true as $\delta \to 1$, holding δ^T constant.

b) n is currently guilty and is on the $\overline{T}^{th}$ period of his punishment path, $1 \leq \overline{T} < T$.

If he deviates, he gets $-c_p + \delta\{(1-p)V_1 + pV_{\overline{T}+1}\}$.

If he does not, he gets $x - c_p + \delta\{(1-p)V_{\overline{T}+1} + pV_1\}$. He follows the norm if $\delta(1-2p)(V_{\overline{T}+1} - V_1) + x > 0$, which is true as p small as $\delta \to 1$, holding δ^T constant.

Now, notice that since the norm is an equilibrium, expected lifetime (normalised) payoffs of a currently innocent person is V_I which converges to v' as p and c_p go to 0. Further, as p goes to 0 a currently innocent person expects to get back to innocent status with probability 1. Hence, our norm sustains cooperation as a sequential equilibrium. Q.E.D. ∎

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