Environmental Policy and Growth when Inputs are Differentiated in Pollution Intensities*

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Abstract

I show that environmental policy affects the cross sectoral allocation of demand and output if intermediate goods are differentiated according to their pollution intensity. When innovations are environmental friendly, a tax on emissions skews demand towards new goods, which are the most productive. In this case along a balanced growth path the tax on emissions is increasing to keep the market share of innovations constant. Furthermore, comparing balanced growth paths I find that an increase in the burden of green taxes lowers output on impact but increases the rate of growth of the economy, because it fosters innovation.

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1 Introduction

This paper proposes a positive theory of environmental policy, namely on the effects of taxing emissions of pollutants on economic growth and the accumulation of pollution. We adopt the analytical framework of the schumpeterian theory of growth, where incentives to engage in productivity enhancing activities (hereafter R&D) are disentangled and highlighted (Romer 1990, Aghion and Howitt 1992, Grossman and Helpman 1991). We extend the vertical innovations hypothesis (as that of Aghion and Howitt 1998, ch.3) by assuming that capital goods are characterized by a productivity level and a pollution intensity. Capital goods are thus potentially differentiated along two dimensions.[1] The flow of services that they provide are called intermediate goods. Their operating cost is composed of the rental price of capital and the tax burden on emissions. The relative weight of these components can be managed by the environmental policy-maker, and affects incentives to invest and the pace of growth if it varies across different capital goods.

As a first step we consider the case when new goods are characterized by both higher productivity and lower pollution intensity than existing goods, in a mechanic way. That is to say that we do not consider the cost linked to the reduction in emissions intensity. It is clearly an abrupt assumption, but it allows us to focus on the most favorable case when environmental policy can foster the pace of economic growth. We show that when capital goods are differentiated in emissions intensities: the green tax increases along a balanced growth path; the level of aggregate output is reduced on impact by an increase in the level of the green tax burden; the rate of growth increases with the level of green tax burden. A marginal increase in the green tax burden depresses both aggregate output (thus wages) and the prospective profits to innovators, and thus the value of innovations. Nevertheless, the negative impact is stronger on costs (the wage) than on returns to R&D, and this activity increases at equilibrium. This change can foster the pace of economic growth, or at least counter the negative effect of environmental regulation.

Among economists there is a shared support for a more ambitious environmental policy.[2] Mainstream theory invokes public intervention in the absence

[1] This is not a vintage capital model because capital goods can be scraped at no cost. Capital is putty-putty, yet each machine has fixed characteristics. Designs of these machines are putty-clay objects, and are doomed to become obsolete.

[2] The "Economists' Statement on Climate Change" issued on 1/3/97, sponsored by Redefining Progress was endorsed by over 2000 economists, with first signatures by Arrow, Jorgenson, Krugman, Nordhaus and Solow. It states "[..] we believe [..] that preventive steps are justified. [..] there are many potential policies to reduce GHG emissions for which the total benefits outweigh the total costs. [..] these measures may in fact improve U.S. productivity in

of property rights on natural resources, or in the presence of externalities from resource harvesting. Environmental regulation can increase welfare when society attaches an amenity value to the environmental quality. Although there is consensus among economists that this policy is costly at least from an accounting point of view, there is substantial disagreement on the significance of these costs.[3] It is symptomatic of this situation the debate that followed the publication by Porter of an article on *Scientific American* suggesting that the U.S. industry's competitiveness was not threatened but actually fostered by environmental regulation.[4] As restated in Porter and van der Linde (1995) the argument is that environmental policy imposes new constraints on firms and triggers innovations, which might well outweigh the short-run direct costs of regulation. This free-lunch hypothesis has attracted many criticisms.[5] It is indeed difficult to maintain that environmental regulation can solve those problems internal to firms, which make them myopic with respect to returns on R&D investment.

Taking a different step, Xepapadeas and de Zeeuw (1999) show that environmental policy can improve average productivity, in a partial equilibrium world with rational agents. They consider a vintage capital model, where new vintages are both more productive and less polluting. On the demand-for-capital side of the economy, an increase in environmental taxation induces firms to change the composition of their capital stock, reducing its average age and increasing its average productivity. This process still leaves firms worst off than in the absence of taxes, and thus differs from the one argued by Porter and van der Linde. The theory presented in the following sections incorporates these mechanisms in a general dynamic equilibrium model, where the supply-of-capital side of the economy is endogenous on both its quality (designs of innovations) and quantity (saving-investment decision) dimensions.[6]

Two main lines of argument have been advanced in the literature to justify

the longer run. [..] The U.S. and other nations can most efficiently implement their climate policies through market mechanisms, such as carbon taxes or the auction of emissions permits." The Statement is available at *www.rprogress.org/pubs/ecstat.html.* See also Krugman "Earth in the Balance Sheet: Economists go for the Green.", *The Dismal Science* 4/17/97, at *www.mit.edu/krugman/www/green.html.*

[3] Out of a large body of studies quantifying these costs, we signal Gray (1987), Hazilla and Koop (1990), Jorgenson and Wilcoxen (1992). For an early survey see Nordhaus (1991).

[4] Porter (1991) "America's Green Strategy", *Scientific American*, 264, 168.

[5] Porter and van der Linde (1995) review a number of case studies supporting their argument. Jaffe et al. (1995) provide empirical evidence which does not contradict the Porter-hypothesis. See Palmer et al. (1995) for criticisms.

[6] In spite of the similarity between the approach of this paper with that of Xepapadeas and de Zeeuw, we would like to point out that the set up of the model was constructed and the main result found before reading their interesting article. In this sense these two papers reflect independent research efforts.

possible positive effects of environmental policy on economic growth. A first wave of articles includes environmental quality along with emissions as a factor of production (Bovenberg and Smulders 1995, Smulders and Gradus 1996, Rosendahl 1996). Increasing the green tax reduces emissions and improves environmental quality. If as a result the marginal product to reproducible factors is increased, equilibrium investment increases and the economy expands at a faster rhythm. The effect is reinforced when the improved quality of the environment allows it to absorb a larger flow of emissions at steady state (Smulders, 1995), when there are externalities in abatement activity (Michel, 1993), or when the green tax revenue allows the authorities to cut growth-depressing distortionary taxes (Bovenberg and Mooij, 1997).

The second approach builds on the schumpeterian theory of endogenous growth (Hung et al. 1993, Verdier 1993, and Elbasha and Roe 1996). In these papers the green tax depresses the level of activity in the sector which is intensive in the factor of production employed in R&D, so favoring this activity. However all intermediate goods share the same output-pollution intensity at the equilibria in these economies.[7] This is not the case in the economy we present here. Instead in the theory that we propose the growth enhancing effect of environmental policy disappears as soon as intermediate goods are not differentiated in pollution intensities. Actually, here green taxes can have a positive impact on growth only if they skew demand across sectors towards most recent goods, which are cleaner but also more productive.

This paper follows previous research by Grimaud and Ricci (1999) and therefore refers to their results for comparison.[8] They analyze schumpeterian growth models where emissions are an input in the final sector of the economy, independent of the mix of capital goods employed in the last stage of production. Final sector firms choose the emissions intensity in response to the relative price of pollutants, that is the "green" tax. Formally, the emissions intensity of output is a control variable. However, conceptually the emissions intensity is a technological variable. It is therefore awkward to treat it as a control variable within schumpeterian growth theory. The latter takes fully into account the stock na-

[7] Hung et al. (1993) allow R&D firms to develop either clean or dirty goods, but characterize the equilibria where all goods are either of one or the other type. Verdier (1993) is less demanding, and allows R&D firms to choose the pollution intensity of new goods out of a continuum. Yet, pollution intensity is treated as a control variable, and as a consequence the R&D firm's problem is stationary and one pollution intensity is chosen for each level of green taxes. Thus Verdier resorts to the natural assumption of symmetry with respect to pollution intensity (which however implies that the green tax rate has never changed since the beginning of times, when the first good was introduced). So pollution intensity is unifrom in Verdier.

[8] This article can be downloaded at *www.feem.it*, or can be obtained from the authors upon request.

ture of technology, and the costs associated to its improvement. If knowledge in environmental friendly technology accumulates over time, the reduction in the pollution intensity of aggregate output is a smooth process resulting from the adoption of the state of the art knowledge by innovators. In this case old intermediate goods will be relatively dirtier than new ones and this is the crucial asymmetry driving our results.

The next section presents the main assumptions about the economy and the functioning of the environment. Section 3 analyzes balanced growth paths, and contains the first two results. The growth fostering effect of environmental policy is derived and explained in section 4. Section 5 extends the model to allow for a trade-off between environmental protection and economic growth. The complex dynamic effects of environmental policy are there discussed. We conclude in the last section. Most proofs are contained in the appendix.

2 The model economy

We extend the schumpeterian model of endogenous growth to consider that production emits pollutants, which accumulate according to a simplified natural law. We distinguish three stages of production. First, labor is competitively engaged in research and development (R&D) activities aimed at designing higher-quality intermediate goods. Successful innovations are characterized by higher productivity and, possibly, by lower pollution intensity. Second, designs are protected by patents, so that intermediate goods are supplied under monopoly power. These goods are produced employing capital. We assume a continuum of intermediate goods. They are combined with labor in the final sector to produce an homogeneous good, which can be consumed or invested. The competitive final sector firms pay factors of production -labor and intermediate goods- plus a tax per unit of emissions resulting from the use of intermediate goods.

In this section we first present the production functions of the final and intermediate sectors, and the environment. Then we look at the behavior of the agents: the final sector, a representative intermediate good monopolist, the R&D sector, the consumers and the government.

2.1 Production and the environment

Final output is produced employing labor and a continuum of intermediate inputs according to the production function:

$$Y_\tau = (1-n)^{1-\alpha} \int_0^1 Z_{j\tau} A_{j\tau} x_{j\tau}^\alpha dj \tag{1}$$

where $\alpha \in (0,1)$; labor supply is fixed and normalized to unit mass; a share $(1-n)$ of labor is employed in production and n in R&D activities; $A_{j\tau}$ is the productivity index and $Z_{j\tau}$ the pollution intensity index of intermediate good $j \in [0,1]$ at date τ. Intermediate goods are produced employing capital, according to:

$$x_{j\tau} = \frac{K_{j\tau}}{A_{j\tau}} \tag{2}$$

Thus, intermediate goods are services from capital goods, and the more productive the good the higher its capital intensity.

Aggregate emissions, P, result from the employment of services from capital goods according to their pollution intensities as follows:

$$P_\tau = \int_0^1 Z_{j\tau}^{1/\alpha\beta} K_{j\tau} dj \tag{3}$$

with $\beta \in (0,1)$. Substituting for K from (3) into (2) we see that intermediate good $j \in [0,1]$ is produced out of emissions:

$$x_{j\tau} = \frac{P_{j\tau}}{Z_{j\tau}^{1/\alpha\beta} A_{j\tau}}$$

(3) can also be written as:

$$Z_{j\tau} = \left(\frac{P_{j\tau}}{K_{j\tau}}\right)^{\alpha\beta}$$

Thus $Z_{j\tau}$ is a measure of the emissions-capital ratio characteristic of good $j \in [0,1]$ at a given date τ. This means that for any given technology Z_j, substitution of capital for emissions cannot take place, e.g. in response to a shift in their relative price. The only way substitution can take place is by introducing a new technology in sector j, say $Z_j^{'}$. The emissions-capital ratio is reduced if the new technology satisfies $Z_j^{'} < Z_j$. But we assume that a new technology can be introduced only if R&D investment takes place. At the industry or firm level substitution is therefore costly and discontinuous over time.

Emissions are implicit inputs that are combined with intermediate inputs according to their pollution intensity. Substituting for $Z_{j\tau}$ above and $x_{j\tau}$ from (2) into (1), we can write the final sector production function as:

$$Y_\tau = \int_0^1 \left[(1-n)A_{j\tau}\right]^{1-\alpha} \left[P_{j\tau}^{\beta} K_{j\tau}^{1-\beta}\right]^{\alpha} dj$$

We can think of equipment machines which are employed in the process of production. Labor is required to operate them, and their use implies some pollution. The productivity of labor, and dirtiness of the process of production depends on the design of the machines.

The stock of pollution is increased by emissions and recovers according to a natural process of assimilation. We assume:

$$\dot{S}_\tau = P_\tau - dS_\tau$$

2.2 The final sector

We introduce a tax per unit of emission, h, to price this input. Normalizing the price of the homogeneous final good to unity and denoting by w wages and p the price of intermediate inputs, instantaneous profits of the fictitious competitive final firm are:[9]

$$\begin{aligned}\psi_\tau &= (1-n_\tau)^{1-\alpha}\int_0^1 Z_{j\tau}A_{j\tau}x_{j\tau}^{\alpha}dj - w_\tau(1-n_\tau) \\ &\quad -\int_0^1 p_{j\tau}x_{j\tau}dj - h_\tau\int_0^1 Z_{j\tau}^{1/\alpha\beta}A_{j\tau}x_{j\tau}dj\end{aligned}$$

Therefore the (inverse) demand for labor from the final sector is given by:

$$w_\tau = (1-\alpha)(1-n_\tau)^{-\alpha}\int_0^1 Z_{j\tau}A_{j\tau}x_{j\tau}^{\alpha}dj \tag{4}$$

and the (inverse) demand for intermediate inputs is given by, $\forall j \in [0,1]$:

$$p_{j\tau} = Z_{j\tau}A_{j\tau}[\alpha(1-n_\tau)^{1-\alpha}x_{j\tau}^{\alpha-1} - h_\tau Z_{j\tau}^{1/\alpha\beta-1}] \tag{5}$$

Thus demand for labor is not directly affected by green taxes, while the demand schedule for intermediate goods shifts downwards in response to an increase in the green tax. Furthermore, this depressing effect of green taxes on the demand for intermediate inputs is not uniform across sectors. Specifically it is stronger the "dirtier" the intermediate good (that is the greater its Z_j).

2.3 The intermediate goods monopolists

Consider the problem of the monopolist in sector j characterized by technology (A_j, Z_j). It rents from households A_j units of capital per unit produced and therefore maximizes instantaneous profits $\Pi_{j\tau} = (p_{j\tau} - r_\tau A_j)x_{j\tau}$, where r denotes the rate of return on savings, i.e. the cost of capital faced by intermediate goods producers. Substituting for the demand from final sector derived above,

[9] We consider one fictitious final sector firm which acts as a price taker. As the sector is competitive and the production function is constant returns to scale with respect to labor and intermediate goods, the number of firms is irrelevant.

(5), and proceeding for maximization, we obtain partial equilibrium sales, the pricing rule and profits of the monopolist in sector j:

$$\hat{x}_{j\tau} = (1-n_\tau)\left(\frac{\alpha^2 Z_j}{r_\tau + h_\tau Z_j^{1/\alpha\beta}}\right)^{\frac{1}{1-\alpha}} \tag{6}$$

$$\hat{p}_{j\tau} = A_j\left[\frac{r_\tau}{\alpha} + \frac{1-\alpha}{\alpha}h_\tau Z_j^{1/\alpha\beta}\right]$$

$$\hat{\Pi}_{j\tau} = A_j\frac{1-\alpha}{\alpha}[r_\tau + h_\tau Z_j^{1/\alpha\beta}]\hat{x}_{j\tau} \tag{7}$$

In what follows we refer to the term $[r_\tau + h_\tau Z_j^{1/\alpha\beta}]$ as the "gross" marginal cost of firm j, and denote it by m_j.[10] Notice that the burden of green taxes is shared between the final sector and the monopoly. Thus green taxes depress sales and profits, and more so the dirtier the good (the higher the Z_j).

2.4 The R&D stage

Any firm in the competitive R&D sector targets improvements on one particular intermediate good. The R&D activity is modeled as a Poisson process with instantaneous arrival rate λn, where n is the mass of labor employed in R&D and $\lambda > 0$ is a productivity parameter. Each innovation improves the quality of the intermediate good on both dimensions, A and Z. Namely, every innovation allows the patent holder to produce the intermediate good characterized by the leading edge technology, that is the highest of all A's, denoted by $\bar{A}$, and the lowest of all Z's, denoted by $\underline{Z}$, at the date of arrival of the innovation. Each innovation contributes marginally to the improvement of the state of knowledge concerning the technology on the two dimensions. To take into account this intersectoral-and-intertemporal spillover we assume that the rates of growth of the two dimensions of the leading edge technology are proportional to the flow of innovations λn:

$$\dot{\bar{A}}_\tau = \lambda n_\tau \bar{A}_\tau \ln\gamma \qquad \gamma > 1 \tag{8}$$

[10] Results would not change if the green taxes were levied on the intermediate good producers, at rate h. Remember that $x_{j\tau} = \frac{K_{j\tau}}{A_{j\tau}} = \frac{P_{j\tau}}{A_{j\tau}Z_{j\tau}^{1/\alpha\beta}}$, so that to produce one unit of good it is necessary to employ $A_{j\tau}$ units of K, and $A_{j\tau}Z_{j\tau}^{1/\alpha\beta}$ units of emissions. The marginal cost is therefore $A_{j\tau}(r_\tau + h_\tau Z_{j\tau}^{1/\alpha\beta})$. The demand would be $p_{j\tau} = Z_{j\tau}A_{j\tau}\alpha(1-n_\tau)^{1-\alpha}x_{j\tau}^{\alpha-1}$. Then the quantity sold and profits would match excatly those obtained in the text, and only the price charged would change to: $\hat{p}_{j\tau} = A_{j\tau}(r_\tau + h_\tau Z_{j\tau}^{1/\alpha\beta})/\alpha$.

$$\dot{\underline{Z}}_\tau = \lambda n_\tau \underline{Z}_\tau \ln \zeta \qquad \zeta \in (0,1] \tag{9}$$

As a first step in the research agenda we assume the parameter ζ to be exogenous. It is an important simplification because it implies that innovations are both more productive and cleaner (if $\zeta < 1$), but that the degree of cleanliness targeted as represented by ζ has no impact on the cost of the R&D program.[11] In section 5 we extend the model to link the size of ζ to the other economic variables in the simplest possible fashion. We will suppose that ζ is a function of green tax revenues, $\zeta = f(h_\tau P_\tau)$, with $f' < 0$, $f(0) = 1$. This is the case if green tax revenues finance public environmental friendly research, which allows entrants to develop goods with lower pollution intensities at no additional cost. Then the rate of accumulation of environmental friendly technological knowledge results from the interaction between the flow of innovations (λn) and of public research activity ($hP \rightarrow \zeta$) according to (9).

Free entry in R&D ensures that at equilibrium the following arbitrage condition holds:

$$n_\tau \in (0,1) \quad \Rightarrow \quad w_\tau = \lambda V_\tau \tag{10}$$

where V_τ is the value of an innovation arrived at date τ. The arbitrage condition states that if R&D activity takes place at all, then its marginal cost equals its expected marginal return.

The innovator can sell the patent to the highest bidder. Since she faces an infinity of potential buyers, she will be able to extract all the present value of the expected stream of profits. The value of an innovation is therefore:

$$V_\tau = \int_\tau^\infty e^{-\int_\tau^t r_s ds} e^{-\lambda \int_\tau^t n_s ds} \hat{\Pi}_t(\bar{A}_\tau, \underline{Z}_\tau) dt$$

where $\hat{\Pi}_t(\bar{A}_\tau, \underline{Z}_\tau)$ stands for the instantaneous profits at date t of a monopoly characterized by the technological parameters $(\bar{A}_\tau, \underline{Z}_\tau)$. The first discount factor takes into account the opportunity cost, thus the return on savings. The second discount factor is the probability of survival of the monopoly, which takes into

[11] We could model an explicit cost of designing cleaner intermediate goods at the R&D stage. We may think that if researchers spend time to design a cleaner product, then it takes longer to accomplish an innovation of given productivity, or that the productivity gain will be lower. To take into account these potential trade-offs I should endogenize the choice of ζ. The idea is that if at the R&D stage an improvement in Z is targeted (that is if ζ is chosen smaller than unity) then the arrival rate is reduced and R&D programs will last longer on average. For instance I can introduce a function $\phi(\zeta)$ such that $\phi(1) = 1$, and $\phi' > 0$, then say that the arrival rate is $\lambda\phi(\zeta)n_T$. Verdier (1993) considers a similar case, where the cost of the R&D program is increasing in the size of the improvement targeted. However this feature would complicate the model, and I first wish to analyze how the model behaves when Z is a technological parameter rather than a control variable.

account the fact that an innovation on the same sector can arrive, destroying the value of the patent.

2.5 Consumers and the government

The representative consumer chooses the paths of consumption and savings to maximize the present value stream of instantaneous isoelastic utilities, subject to a dynamic budget constraint:[12]

$$\max_{\{c\}_0^\infty} \int_0^\infty e^{-\rho\tau} \left(\frac{c_\tau^{1-\varepsilon}}{1-\varepsilon} \right) d\tau$$

$$\dot{W} = w_\tau + r_\tau W_\tau - c_\tau + T_\tau$$

Where W is financial wealth and T are transfers from the government. The solution links the rate of growth of consumption to the rate of return on savings and preference parameters, according to the Ramsey rule:

$$g_c = \frac{r_\tau - \rho}{\varepsilon} \tag{11}$$

where g_i denotes the rate of growth of variable i. To complete the problem we have to impose the no-Ponzi game condition:

$$\lim_{\tau\to\infty} e^{-\int_0^\tau r_s ds} W_\tau = 0$$

Finally, we need to impose a budget constraint on the government. To be simple but without loss of generality, we assume that the budget is held balanced at any given date, by rebating the green tax revenue to the representative household:[13]

$$T_\tau = h_\tau P_\tau$$

3 Balanced growth path analysis

Along a balanced growth path, n must be constant for $\bar{A}$ and $\underline{Z}$ to grow at constant rates from (8), (9). Furthermore, the law of motion of capital, $\dot{K}_\tau = Y_\tau - c_\tau$, implies that capital, output and consumption grow at the common rate, $g = g_c$. Finally, the Ramsey rule (11) we have that g is constant only if r is constant. Hence, we obtain the following:

[12] There is no need to consider the effect of pollution on utility in as much we do not deal with the normative analysis of environmental policy in this paper.

[13] In section 5 we assume that the green tax revenue is spent to finance public research on environmental friendly technology, and transfers to households will be nil.

Proposition 1 *There exists a balanced growth path if the green tax increases according to the following policy rule:*

$$g_h = -\frac{g_{\underline{Z}}}{\alpha\beta} = \frac{-\ln\zeta}{\alpha\beta}\lambda n \tag{PR}$$

Along this path output growth is function of n and ζ according to:

$$g = \left(\ln\gamma + \frac{\ln\zeta}{1-\alpha}\right)\lambda n = G\lambda n \tag{12}$$

where we have defined the growth factor $G = \ln\gamma + \ln\zeta/(1-\alpha)$. Therefore, growth is positive only if:

$$\zeta \in \left(\gamma^{\alpha-1}, 1\right] \tag{a}$$

Proof. First, we compute the value of an innovation using (7) and (6):

$$\begin{aligned} V_\tau &= \frac{1-\alpha}{\alpha}\bar{A}_\tau\left(\alpha^2\underline{Z}_\tau\right)^{\frac{1}{1-\alpha}}(1-n)\int_\tau^\infty e^{-(r+\lambda n)(t-\tau)}\left[r+h_t\underline{Z}_\tau^{1/\alpha\beta}\right]^{\frac{-\alpha}{1-\alpha}}dt \\ &= \bar{\Pi}_\tau\int_\tau^\infty e^{-(r+\lambda n)(t-\tau)}\left[\frac{r+h_\tau\underline{Z}_\tau^{1/\alpha\beta}}{r+h_t\underline{Z}_\tau^{1/\alpha\beta}}\right]^{\frac{\alpha}{1-\alpha}}dt \\ &= \bar{\Pi}_\tau\int_\tau^\infty e^{-(r+\lambda n)(t-\tau)}\left(\frac{\bar{m}_\tau}{m_t}\right)^{\frac{\alpha}{1-\alpha}}dt \end{aligned} \tag{13}$$

Where $\bar{\Pi}_\tau$ and $\bar{m}_\tau$ denote initial profits and gross marginal cost of an innovator at date τ, and m_t denotes the gross marginal cost at any future date $t > \tau$ of the firm innovating at date τ. The latter increases over time, and thus profits are crowded-out, if and only if the green tax, h, increases. The integral in the first expression is constant if the gross marginal cost of the leading edge monopolist, $\bar{m}_\tau$, is constant, that is if $h_\tau\underline{Z}_\tau^{1/\alpha\beta}$ is constant. This is ensured by policy rule (PR).

For n to be constant the arbitrage condition (10) must hold at all times for the equilibrium level of n. Under policy (PR) the value of patents (proportional to the right-end-side of (10)) grows at the same rate as the initial profit of innovators, that is: $g_V = g_{\bar{A}} - \frac{1}{1-\alpha}g_{\underline{Z}}$. The left-hand-side of (10) is increasing with the wage. From (4), the latter is the marginal product of labor in the final sector, i.e. $w_\tau = (1-\alpha)Y_\tau/(1-n)$, so that it increases at the rate of output growth, hence $g_w = g$ for n constant. Logdifferentiating the arbitrage condition (10), we obtain the following relationship:

$$g = g_w = g_V = g_{\bar{A}} - \frac{1}{1-\alpha}g_{\underline{Z}}$$

(12) is derived using (8) and (9). ■

Along a balanced growth path, the green tax increases at a constant rate if innovations are environmental friendly. This is the case when the emissions-capital ratio of innovations is lower than that of the goods they replace, i.e. when

$\zeta < 1$. To understand this result suppose that the green tax is held constant although innovations are environmental friendly. In this case the weight of the green tax burden over gross marginal cost for innovations would fall over time. Then innovations would become increasingly competitive relative to existing intermediate goods. As a result the share of innovations in the market for intermediate goods would increase progressively. This is clearly incompatible with the concept of balanced growth.[14]

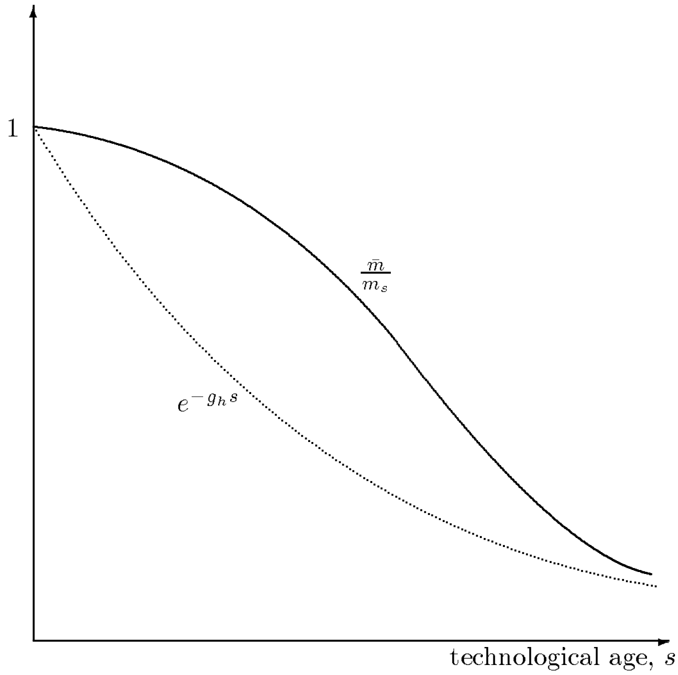

Figure 1: Competitiveness-loss or "green crowding-out" effect.

The crucial feature of environmental policy in this economy is that it affects the relative gross marginal costs across different intermediate goods. Under policy rule (PR) the gross marginal cost of the leading edge good is constant and equal to $\bar{m} = r + h\underline{Z}^{1/\alpha\beta}$ (where we omit the subscript τ since both terms are constant). Then the gross marginal cost increases with the age of the technology,

[14] As the market share of innovations increases, so does the value of innovations (which is forward looking) relative to the cost of innovation (which reflects the current cross-sectoral distribution of market shares). Then the incentive to engage in R&D grows faster than its cost, and R&D activity intensifies over time (n grows).

so that at some later date $t > \tau$ it is equal to $m_t = r + h_t \underline{Z}_\tau^{1/\alpha\beta}$, with $h_t = e^{g_h(t-\tau)} h_\tau$. Therefore the distribution of intermediate goods according to their relative technological age is characterized by the ratio of the gross marginal cost of a sector of technological age s relative to the leading edge sector:

$$\frac{\bar{m}}{m_s} = \frac{r + h\underline{Z}^{1/\alpha\beta}}{r + e^{g_h s} h\underline{Z}^{1/\alpha\beta}}$$

Older technologies are less competitive than new ones because of their relative dirtiness, implying a larger burden of green taxes. The ratio would indeed be constant in the absence of environmental friendly technological progress (i.e. $\zeta = 1$ and $g_h = 0$). This effect of environmental policy is called the "green crowding-out" effect, because the policy reduces the competitiveness of aging technologies and crowds out their profit generating capacity. The loss of competitiveness follows the path illustrated in figure 1. We see that the loss accelerates its rhythm and eventually, when most of the gross marginal cost is represented by the green tax burden, it proceeds at rate g_h.

3.1 The aggregate economy.

Let us first express aggregate variables in terms of the leading edge sector output, $\bar{x}_\tau$.[15] We can compute aggregate demand for capital from the intermediate goods sector by integrating over the space of goods the rearranged the production function (2), to obtain:

$$K_\tau = \bar{A}_\tau \bar{x}_\tau \Gamma \tag{14}$$

where:[16]

$$\Gamma = \int_0^1 \frac{A_{j\tau} x_{j\tau}}{\bar{A}_\tau \bar{x}_\tau} dj = \lambda n \int_0^\infty e^{-(1+G)\lambda n s} \left(\frac{\bar{m}}{m_s} \right)^{\frac{1}{1-\alpha}} ds \tag{15}$$

Similarly, the flow of aggregate emissions of pollutants can be written as:

$$P_\tau = \underline{Z}_\tau^{1/\alpha\beta} \bar{A}_\tau \bar{x}_\tau \Lambda \tag{16}$$

where:[17]

$$\Lambda = \int_0^1 \frac{Z_{j\tau}^{1/\alpha\beta} A_{j\tau} x_{j\tau}}{\underline{Z}_\tau^{1/\alpha\beta} \bar{A}_\tau \bar{x}_\tau} dj = \lambda n \int_0^\infty e^{-\left(1+G+\frac{\ln\zeta}{\alpha\beta}\right)\lambda n s} \left(\frac{\bar{m}}{m_s} \right)^{\frac{1}{1-\alpha}} ds \tag{17}$$

[15] Aggregate variables are actually average variables in this economy, since we have normalized the mass of sectors to unity.
[16] This expression for Γ is derived in appendix 7.1.
[17] The function Λ is computed in appendix 7.1.

Finally, output can be computed as:

$$Y_\tau = (1-n)^{1-\alpha} \underline{Z}_\tau \bar{A}_\tau \bar{x}_\tau^\alpha \Delta \tag{18}$$

where: [18]

$$\Delta = \int_0^1 \frac{Z_{j\tau} A_{j\tau} x_{j\tau}^\alpha}{\underline{Z}_\tau \bar{A}_\tau \bar{x}_\tau^\alpha} dj = \lambda n \int_0^\infty e^{-(1+G)\lambda n s} \left(\frac{\bar{m}}{m_s} \right)^{\frac{\alpha}{1-\alpha}} ds \tag{19}$$

Functions Γ, Λ and Δ are constant along a balanced growth path, and are characterized by the following properties:

1. $\Gamma < 1, \ \forall \zeta$
2. $\Lambda > \Gamma, \ \forall \zeta < 1$
3. $\Lambda > \Delta, \ \forall \zeta < 1$
4. $\Delta \Gamma^{-\alpha} < (1+G)^{\alpha-1} < 1, \ \forall \zeta < 1$
5. $\Gamma = \Lambda = \Delta = \int_0^1 A_{j\tau}/\bar{A}_\tau dj < 1, \ \text{iff } \zeta = 1$

Proof : These properties are proved in appendix 7.1.

Substituting for $\bar{x}_\tau$ from (14) into (16) we can express aggregate emissions as:

$$P_\tau = \underline{Z}_\tau^{1/\alpha\beta} K_\tau \frac{\Lambda}{\Gamma} \tag{20}$$

Property 2 implies:

$$P_\tau \geq \underline{Z}_\tau^{1/\alpha\beta} K_\tau$$

which means that aggregate emissions are higher that if all capital was of the less polluting kind. Equation (20) also implies:

$$\underline{Z}_\tau = \left(\frac{P_\tau \Gamma}{K_\tau \Lambda} \right)^{\alpha\beta} \tag{21}$$

Thus the leading edge pollution intensity, $\underline{Z}$, also measures the pollution intensity of aggregate output. When $\zeta < 1$ the pollution intensity declines continuously for the economy as a whole, although it is fixed for any given intermediate good.

Next, substituting for $\bar{x}_\tau$ using (14) into (18), we obtain the following expression for aggregate output:

[18] See appendix 7.1 for the derivation of Δ.

$$Y_\tau = \underline{Z}_\tau \left[(1-n)\bar{A}_\tau\right]^{1-\alpha} K_\tau^\alpha \frac{\Delta}{\Gamma^\alpha} \tag{22}$$

This representation of the aggregate production function is very similar to the one considered by Stokey (1998), Aghion and Howitt (1998, ch.5) and Grimaud and Ricci (1999). That is:

$$Y_\tau = z_\tau \left[(1-n)A_\tau\right]^{1-\alpha} K^\alpha$$

where $A_\tau = \int_0^1 A_j dj$ is the index of average productivity and the variable z represents the pollution intensity in the final output sector. There are two differences here with respect to these previous models. First the measure of pollution intensity of output, is here considered as a state variable ($\underline{Z}$), while it was a control variable (z) in previous settings. The "equivalent" variable in the present model would be the average pollution intensity $Z = \int_0^1 Z_j dj \geq \underline{Z}$ (with equality holding only if $\zeta = 1$ by definition). Second, the index of average productivity enters the production function in a simple way, while in (22) we have the term $\bar{A}_\tau^{1-\alpha}\Delta\Gamma^{-\alpha}$ (equal to $A_\tau^{1-\alpha}$ only if $\zeta = 1$ by properties 4 and 5). Thus for uniform pollution intensities we obtain a representation of the aggregate production function which is "equivalent" to the one used by Stokey (1998), Aghion and Howitt (1998) and Grimaud and Ricci (1999). However the pollution intensity index would still be a state variable rather than a control variable. This means that reductions in the pollution intensity of output can occur only with time and imply an additional dimension of differentiation between intermediate goods. The difference between the previous models and the present one is thus entirely due to differentiation in pollution intensities.

Finally, substituting for $\underline{Z}$ from (21) into (22), we can write emissions explicitly as inputs into the production function:

$$Y_\tau = \Delta \left[(1-n)\bar{A}_\tau\right]^{1-\alpha} \left[\left(\frac{P_\tau}{\Lambda}\right)^\beta \left(\frac{K_\tau}{\Gamma}\right)^{1-\beta}\right]^\alpha \tag{23}$$

It is clear then that the lower are the emissions inputs the lower is output. Moreover, equation (23) shows that emissions are combined with services from capital goods, with unitary elasticity of substitution, and then this composite intermediate good is combined with labor.[19]

[19] It may be interesting to compare this representation with the one considered by Stokey (1998), Aghion and Howitt (1998) and Grimaud and Ricci (1999):

$$Y_\tau = P_\tau^{\alpha\beta} \left[((1-n)A_\tau)^{1-\alpha} K_\tau^\alpha\right]^{1-\alpha\beta} \tag{S}$$

Proposition 2 *For a given amount of labor, capital, emissions inputs and level of labor productivity, if intermediate goods are sold uniformly aggregate output is lower the more environmental friendly are innovations (the lower ζ).*

Proof. Using the methodology of appendix 7.1, for uniform sales $X(z_j) = 1$ $\forall j \in [0,1]$ we have $\Gamma = (1+\ln\gamma)^{-1}$, $\Delta = (1+\ln\gamma+\ln\zeta)^{-1}$, $\Lambda = (1+\ln\gamma+\ln\zeta/\alpha\beta)^{-1}$. We also evaluate average labor productivity as $A_\tau = \bar{A}_\tau \int_0^1 A_j/\bar{A}_\tau dj = \bar{A}_\tau \lambda n \int_\tau^\infty e^{-\lambda n(s-\tau)} e^{-\lambda n \ln\gamma(s-\tau)} ds = \bar{A}_\tau (1+\ln\gamma)^{-1}$. We use these expressions to substitute for $\bar{A}_\tau$, Γ, Δ and Λ in (23) and write output as:

$$Y_\tau = [(1-n)A_\tau]^{1-\alpha} \left(P_\tau^\beta K_\tau^{1-\beta}\right)^\alpha f(\zeta)g(\zeta)$$

where $f(\zeta) = \Delta/\Gamma = (1+\ln\gamma)/(1+\ln\gamma+\ln\zeta)$ and $g(\zeta) = [\Gamma/\Lambda]^{\alpha\beta} = [(1+\ln\gamma+\ln\zeta/\alpha\beta)/(1+\ln\gamma)]^{\alpha\beta}$.

For uniform pollution intensities ($\zeta = 1$) $f(\zeta)g(\zeta) = 1$. Then:

$$\frac{\partial f(\zeta)g(\zeta)}{\partial(-\zeta)} = \frac{f(\zeta)g(\zeta)}{\partial\zeta}[\Delta - \Lambda] \leq 0$$

with strict inequality $\forall\zeta < 1$. Thus with uniform sales of intermediate goods, output is maximum, given the inputs, when all goods use the same amount of emissions. ■

This result is due to the fact that the total productivity of an intermediate good does not depend only on its implicit labor productivity parameter, A, but also on the quantity of pollution inputs associated to its use. For any given distribution of labor productivity parameters across goods, the distribution of their total productivity is lower and flatter the more environmental friendly are innovations (the lower ζ).

Conjecture *The equilibrium aggregate output is larger than the one obtained employing uniformly intermediate goods.*

The tax on emissions biases sales in favor of new (cleaner) goods and against old (dirtier) goods. Then aggregating across sectors the weight of the most productive goods is increased and that of the less productive reduced, and the output gap outlined in proposition 2 is reduced (if not reversed).

In this case labor is combined with services from capital goods first, and then their joint product with emissions. Notice that in (23) the elasticity of output with respect to emissions, $\alpha\beta$, is equal, and with respect to capital inputs, $\alpha(1-\beta)$, is smaller, and the share of labor, $1-\alpha$, larger than in (S). This implies that the investment in R&D, n, necessary to sustain any couple of targets (g_Y, g_P) is lower in economies represented by (23) than in those reflected in (S). From (S) we have $g_Y = \alpha\beta g_P + (1-\alpha)(1-\alpha\beta)g_{\bar{A}} + \alpha(1-\alpha\beta)g_K$; and from (23) $g_Y = \alpha\beta g_P + (1-\alpha)g_A + \alpha(1-\beta)g_K$. Taking into account $g_Y = g_K$ and $g_{\bar{A}} = g_A$, and imposing that the two economies follow paths characterized by the same growth rates (g_Y, g_P), it is immediate to obtain that the g_A (thus the n) implied by (23) is smaller than the one implied by (S) whenever $g_Y > g_P$. This last assumption is common to all of these analysis, as pollution intensity of output is never allowed to increase.

To conclude on the aggregate picture of the economy we find that the green tax revenue (thus transfers to households) grow at the same rate as output. In fact, the green tax revenue grows at rate $g_h + g_P$ which equals g from (20) under policy (PR).

3.2 Equilibrium

The dynamic general equilibrium is determined when workers are indifferent between working in final sector firms and in R&D firms, using the labor market clearing condition, the R&D arbitrage condition and labor demand from the final sector. The equilibrium level of R&D employment, n, is obtained by equating the marginal product of labor in the final sector (4) with the expected marginal return to R&D from (10):

$$(1-\alpha)(1-n)^{-\alpha}\underline{Z}_{\tau}\bar{A}_{\tau}\bar{x}_{\tau}^{\alpha}\Delta = \lambda V_{\tau}$$

Substituting the value of innovations as expressed in (13), and simplifying the condition is:

$$(1-n)^{-\alpha}\underline{Z}_{\tau}\bar{x}_{\tau}^{\alpha}\Delta = \frac{\lambda}{\alpha}\left[r + h\underline{Z}^{1/\alpha\beta}\right]\bar{x}_{\tau}\int_{\tau}^{\infty} e^{-(r+\lambda n)(t-\tau)}\left(\frac{\bar{m}_{\tau}}{m_{t-\tau}}\right)^{\frac{\alpha}{1-\alpha}} dt \quad \text{(E)}$$

Figure 2 depicts the left-hand-side of equation (E) at a given date τ as an upward sloping, and the right-hand-side as a downward sloping schedule in the (n,value) space.[20] The equilibrium level of R&D activity, n^*, is determined as the intersection of the two schedules. From (12) we also know that $g^* = G\lambda n^*$.

The left-hand-side is proportional to the marginal product of labor in the final sector. Due to diminishing returns, the marginal product of labor in the final sector tends to infinity as all labor is employed in R&D (as n tends to unity). The right-hand-side is proportional to the value of an innovation. The first factor before the integral, is proportional to the initial instantaneous profit of the innovator (see 13). An increase in n has two negative effects on this profit rate (see 7). First, lower labor inputs in the final sector reduce the marginal product of intermediate inputs, and thus depress demand (from 5). Second, the faster the growth rate, the higher the interest rate (from 11) and, *ceteris paribus*, the greater the gross marginal cost of intermediate goods. Also the

[20] Both the LHS and the RHS in figure 2 shift downwards over time. However they cross at a constant level of n. To see this divide (E) by $\bar{x}$, substitute $\bar{x}^{1-\alpha}$using (6) to get:

$$\frac{\Delta}{\alpha(1-n)} = \lambda\int_{\tau}^{\infty} e^{-(r+\lambda n)(t-\tau)}\left(\frac{\bar{m}_{\tau}}{m_{t-\tau}}\right)^{\frac{\alpha}{1-\alpha}} dt$$

which defines a unique value of n.

integral on the right-hand-side of (E) is strictly decreasing in n. This is because both the discount factor and the survival probability are reduced. Furthermore the competitiveness loss proceeds at a faster pace, because the greater is the rate of innovation, the faster the green-tax will be increasing, according to policy rule (PR). The expected flow of profits is therefore crowded out at a faster rate.

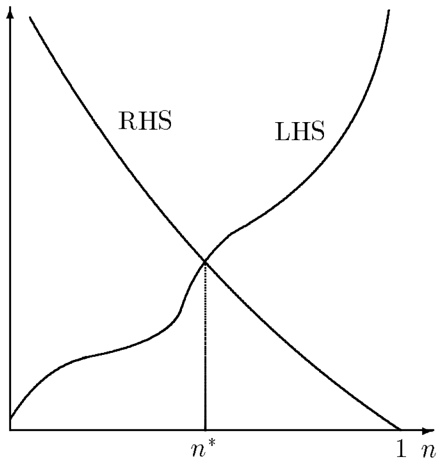

Figure 2: The equilibrium condition.

It follows that, if at $n = 0$, the expected return from R&D is larger than the cost, there exists a unique equilibrium level of R&D activity, n^*, and growth rate, g^*.

Finally we can define implicitly the equilibrium level of R&D employment by substituting for $\bar{x}_\tau$ with (6), using the definition of Δ, (19), and simplifying:

$$n = \left[1 + \frac{1}{\alpha} \frac{\int_0^\infty e^{-(G\lambda n + \lambda n)s} \left(\frac{\bar{m}}{m_s} \right)^{\frac{\alpha}{1-\alpha}} ds}{\int_0^\infty e^{-(r+\lambda n)t} \left(\frac{\bar{m}}{m_t} \right)^{\frac{\alpha}{1-\alpha}} dt} \right]^{-1} \in (0,1)$$

4 The impact of environmental policy

Under policy rule (PR) $g_h = -g_{\underline{Z}}/\alpha\beta = -\lambda n \ln \zeta / \alpha\beta$ always, so that the policy tool of interest is the level of the green tax levied on the leading edge technology: $h\underline{Z}^{1/\alpha\beta}$. We find the following result.

Proposition 3 *Along a balanced growth path a marginal increase in the green* *tax burden levied on innovations, $h\underline{Z}^{1/\alpha\beta}$, reduces the level of aggregate output:*

$$\frac{\partial Y}{\partial h\underline{Z}^{1/\alpha\beta}} < 0$$

This negative impact is greater the more environmental friendly are innovations, i.e. the lower is ζ.

Proof. The proof consists in differentiating output as given by (18) and analyze the cases $\zeta = 1$ and $\zeta < 1$. See appendix 7.2. ■

The result is not surprising as we have seen from (23) that emissions are inputs in the aggregate production function. Thus the higher their relative price, the lower will be their employment and the lower aggregate output. This direct input effect is active even in the absence of differentiation in the pollution intensities. However when there is differentiation the impact is magnified because the policy change affects older (i.e. dirtier) goods more heavily. In other words the increase in the green tax burden shifts downwards the competitiveness loss schedule, and older sectors suffer from a greater loss in demand.

This asymmetric impact of green taxes across intermediate goods leads us to the main result of the paper.

Proposition 4 *Along a balanced growth path a marginal increase in the green tax burden levied on innovations, $h\underline{Z}^{1/\alpha\beta}$, increases R&D investment if innovations are environmental friendly. That is:*

$$\frac{\partial n^*}{\partial h\underline{Z}^{1/\alpha\beta}} > 0 \qquad \text{if } \zeta < 1$$

This implies that the rate of growth of the economy, $g^* = G\lambda n^*$ from (12), is increasing in $h\underline{Z}^{1/\alpha\beta}$.

Proof. To prove the result we show that, at the original equilibrium level of n, the left-hand-side of the equilibrium condition (E) falls more than its right-hand-side. These shifts of the schedules depicted in figure 2 result in a new equilibrium level of R&D employment, n, higher than the original one. The proof is contained in appendix 7.3. ■

An increase in the tax burden has a negative impact on the equilibrium value of innovations but also on equilibrium wages, thus on the cost of R&D. A larger burden of green taxes reduces the marginal product of intermediate inputs. This depressing impact on demand reduces directly the innovator's prospective profits. The larger burden of taxation, however, also translates into lower aggregate output and lower demand for labor from the final sector. *A priori* the net effect on the equilibrium level of R&D activity is ambiguous. We have established that the impact is stronger on the final sector's labor demand schedule than on the value of innovations when innovations are environmental friendly. In this case, the fall in wages outweighs the fall in the value of innovations. As a result R&D activity increases at equilibrium.

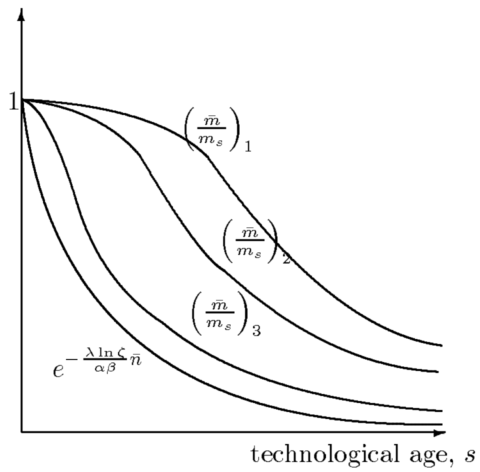

Figure 3: The green tax burden and competitiveness-loss: $\left(h\underline{Z}^{1/\alpha\beta}\right)_1 < \left(h\underline{Z}^{1/\alpha\beta}\right)_2 < \left(h\underline{Z}^{1/\alpha\beta}\right)_3$

An increase in the green tax burden "punishes" relatively more older technologies, or in other words skews demand in favor or relatively modern intermediate goods, because these are cleaner. This distortionary effect of taxation improves incentives to engage in innovative activities. The equilibrium condition depends on the shape of the competitiveness-loss schedule. In fact, on the left-hand-side there are wages, which are proportional to output. At any given point in time output is computed looking backwards to the distribution of sales and technological parameters across intermediate goods in use. For a given evolution of technologies, the competitiveness-loss schedule affects equilibrium sales across goods. From the definition of function Δ, (19), we know that this is an average of the competitiveness-loss function computed with exponential density of rate $\lambda n + g$. On the right-hand-side of the equilibrium condition, instead, there is the present value of expected profit stream accruing to innovators. This is computed looking forward to the expected evolution of profits, which depends on the expected pace of competitiveness-loss. Along a balanced growth path this is just the same schedule as the one prevailing in the cross-section of intermediate goods in use. As it can be seen from (13), the value of patents too is an average of the competitiveness-loss function. However this is now computed with exponential density of rate $\lambda n + r$. Since the negative impact of an increase in the green tax burden falls more heavily on older sectors (or on future profits for innovators), $r > g$ implies that the loss matters more for the computation of output and wages than for the computation of the present value stream of expected profits ($r > g$ is necessary for the no-Ponzi game condition to hold, see end of appendix 7.3).

The growth enhancing effect of environmental policy is however stronger for low levels of the green tax burden than for high ones. An increase in the green tax burden levied on the leading edge sector increases the relative share of this burden over the gross marginal cost, $[h\underline{Z}^{1/\alpha\beta}/(r + h\underline{Z}^{1/\alpha\beta})]$, for all sectors, but not in a uniform fashion. Such an increase shifts downwards the competitiveness-loss schedule and, as shown in figure 3, this distortionary impact of the green tax burden fades gradually away with the tax level. In the limit case of an infinite burden the distortion is at its maximum and cannot be improved upon. To understand this statement consider an economy where all capital income is subsidized at one-hundred percent rate, that is one where the gross marginal cost of monopolies consists exclusively of the green tax burden. In this situation the green crowding-out effect, or the loss of competitiveness, proceeds at rate g_h. Then the integrals in Δ and in V_τ can be solved explicitly to show that the equilibrium condition is independent of the value of $h\underline{Z}^{1/\alpha\beta}$.[21] As a result the equilibrium level of R&D is itself independent of the green tax burden. This means that an increase in the green tax burden levied on the leading edge sector depresses output and wages exactly by the same amount as the value of innovations, leaving the incentives to engage in innovative activities unchanged. The reason for this result is that the policy shock does not affect the shape of the competitiveness-loss schedule. This is the crucial point: the policy has no impact on R&D activity if it leaves unaltered the cross sectoral

[21]The procedure is as follows. Set $r = \mathbf{0}$ in the gross marginal cost, so that $\bar{m}/m_t = e^{-g_h t}$. Then using (13) and (7) we compute: $V_\tau = \left[\bar{A}_\tau h\underline{Z}^{1/\alpha\beta}\bar{x}_\tau (1-\alpha)/\alpha\right] / [\lambda n + r - \ln\zeta\lambda n/[\beta(1-\alpha)]]$. To evaluate wages $w = (1-\alpha)Y/(1-n)$, we can solve explicitly Δ and use (18) to get $w_\tau = \left[(1-\alpha)(1-n)^{-\alpha}\bar{A}_\tau \underline{Z}_\tau \bar{x}_\tau^{\alpha}\right] / [1 + G - \ln\zeta/[\beta(1-\alpha)]]$. Insert these values in the equilibrium condition $w = \lambda V$. Simplify and finally use the Ramsey rule (11) to write the rate of interest as function of the growth rate: $r = \rho + \varepsilon G\lambda n$. We have:

$$\bar{n} = \frac{\alpha - \rho/\{\lambda[1 + G - \ln\zeta/[\beta(1-\alpha)]]\}}{\alpha + \{1 + \varepsilon G - \ln\zeta/[\beta(1-\alpha)]\}/\{1 + G - \ln\zeta/[\beta(1-\alpha)]\}}$$

(and intertemporal) distribution of relative demand and profits.[22]

The main lesson that we want to retain from this result is that environmental policies are in general not neutral with respect to the cross sectoral distribution of demand. Green taxes skew demand towards cleaner intermediate goods. If innovations are environmental friendly, environmental policy fosters innovative activities and leads to faster growth in productivity (given ζ).

5 The growth-environment trade-off

So far we have assumed that innovations are environmental friendly and that this kind of technological improvement was a by-product of the R&D process. The extent to which innovations are environmental friendly was measured by the exogenous parameter ζ. This independence of ζ from the environmental policy rules out the main force generating a trade-off between the preservation of environmental quality and the rate of growth of the economy.[23] Moreover, there seems not to be clear evidence of continuous reduction in the pollution intensity of capital goods, at least in the absence of an active environmental policy. In this section we endogenize the pace of environmental friendly technological progress, by linking the value of ζ to environmental policy. We assume that the rate at which the leading edge pollution intensity is reduced, ζ, is a function of the green tax revenue, thus of the level of taxes on emissions. Such a relationship would hold for instance if the green tax revenue is spent to finance public research on environmental friendly technology. This knowledge would be available to private R&D laboratories which could apply the state of the art pollution intensity to their innovations. We adopt this kind of hypothesis.

In appendix 7.4 we analyze the microeconomic problem of R&D firms targeting improvements in pollution intensity. We show that there exists a critical level of the green tax, h^*, beyond which innovators choose to embody the en-

[22] Grimaud and Ricci (1999) find that the level of green taxes has no impact on the growth rate of the economy. Their result is due to the fact that in the economy they analyze the cross sectoral relative demand and profits depend only on the distribution of productivity parameters, and are independent of the level of green taxes.

[23] According to the aggregate production function (23), emissions are an input to production. Taking logarithms and differentiating that expression we obtain:

$$[1-\alpha(1-\beta)]\, g = (1-\alpha) g_{\bar{A}} + \alpha\beta g_P$$

Thus, *ceteris paribus* any reduction in the rate of growth of emissions inputs tends to depress the growth rate of production. If we want to reduce the growth rate of emissions below that of output leaving the latter unchanged, we must increase the rate of growth of productivity, by investing more in R&D. Yet, we have assumed until now that increasing the rate of productivity growth would automatically speed up the process of reduction in pollution intensity. In this sense the conflict has been solved by assumption.

vironmental friendly technology into the design of their intermediate goods, and that the desired as well as the feasible improvements are increasing in the share of green tax revenue in output, which is proportional to $h\underline{Z}^{1/\alpha\beta}$. That is, endogenizing ζ, we obtain:

$$\frac{\partial \zeta}{\partial h\underline{Z}^{1/\alpha\beta}} < 0 \qquad \forall h \geq h^*$$

To study the effect of environmental policy on the growth rates of the economy and of emissions of pollutants, let us first recall their expressions:

$$g = \left(\ln \gamma + \frac{\ln \zeta}{1-\alpha} \right) \lambda n \tag{12}$$

$$g_P = \left(\ln \gamma + \frac{\ln \zeta}{1-\alpha} + \frac{\ln \zeta}{\alpha\beta} \right) \lambda n \tag{24}$$

where (24) is obtained from (20), using (9). The sign of these growth rates depends only on the extent of environmental friendly technological progress, ζ, since n is positive. We have the following partition:

$$\begin{array}{llllll} g > 0 & \text{and} & g_P > 0 & \text{if} & \zeta \in I^c \equiv \left] \gamma^{\frac{(\alpha-1)\alpha\beta}{\alpha\beta+(1-\alpha)}}, 1 \right] \\ g > 0 & \text{and} & g_P < 0 & \text{if} & \zeta \in I \equiv \left] \gamma^{\alpha-1}, \gamma^{\frac{(\alpha-1)\alpha\beta}{\alpha\beta+(1-\alpha)}} \right] \\ g < 0 & \text{and} & g_P < 0 & \text{if} & \zeta \in \left] 0, \gamma^{\alpha-1} \right] \end{array}$$

Both intervals I and I^c widen with the knowledge spillover concerning the productivity dimension of goods, γ. When emissions do not matter in final production (i.e. when α or β or both are nil) $I^c = \emptyset$, since there is no reason to increase emissions. When labor and productivity are irrelevant for production, (i.e. $\alpha = 1$) emissions cannot be substituted with labor productivity, A. As a result $I = I^c = \emptyset$ and a sustained reduction in aggregate emissions implies a reduction in output. What we want to retain is that for any $\alpha < 1$ interval I is non-empty. That is to say that there exists some range of ζ, which can be targeted by policy-makers, for which the balanced growth path is characterized by economic growth ($g > 0$) and a continuous reduction in the aggregate flow of emissions ($g_P < 0$), implying complete cleaning of the environment ($g_S < 0$).

Differentiating the growth rates with respect to the green tax burden levied on the leading edge sector, we have:

$$\frac{\partial g}{\partial h\underline{Z}^{1+\mu}} = \frac{1}{(1-\alpha)} \frac{1}{\zeta} \frac{\partial \zeta}{\partial h\underline{Z}^{1/\alpha\beta}} \lambda n + G\lambda \frac{\partial n}{\partial h\underline{Z}^{1/\alpha\beta}}$$

$$\frac{\partial g_P}{\partial h\underline{Z}^{1+\mu}}=\left(\frac{1}{1-\alpha}+\frac{1}{\alpha\beta}\right)\frac{1}{\zeta}\frac{\partial\zeta}{\partial h\underline{Z}^{1/\alpha\beta}}\lambda n+\left(G+\frac{\ln\zeta}{\alpha\beta}\right)\lambda\frac{\partial n}{\partial h\underline{Z}^{1/\alpha\beta}}$$

From these expressions we can see the complexity of the impact of environmental policy. For low levels of the green tax ($h < h^*$) the policy has no impact on growth rates, since $\partial\zeta/\partial h\underline{Z}^{1/\alpha\beta}=0$, which implies uniform pollution intensities and therefore $\partial n/\partial h\underline{Z}^{1/\alpha\beta}=0$ from proposition 4. If instead the tax increase induces innovators to adopt cleaner technologies $\partial\zeta/\partial h\underline{Z}^{1/\alpha\beta}<0$, emissions grow at a lower rate, and this slows the growth rate of output because emissions are an input to production. Yet, this direct impact of the green tax burden on the growth rates through ζ is stronger on the rate of growth of emissions than on the rate of growth of the economy. On the other hand, as goods are differentiated in their pollution intensities, the policy shock also affects R&D investment, $\partial n/\partial h\underline{Z}^{1/\alpha\beta}\neq 0$. This indirect effect is stronger on the growth rate of output than on that of emissions. Thus, if $\partial n/\partial h\underline{Z}^{1/\alpha\beta}<0$ there is a clear trade-off between economic growth and environmental quality, since the policy measures apt to reduce the flow of pollution also reduce the equilibrium rate of growth of the economy. Nevertheless we find that this needs not be the case.

Proposition 5 *There are configurations of parameters that give rise to win-win environmental policy. This is defined as an increase in the green tax burden levied on innovations, $h\underline{Z}^{1/\alpha\beta}$, which leads to faster output growth and slower growth of emissions. It is most likely the lower the elasticity of intertemporal substitution.*

Proof. The proof consists in showing, along the lines of proof 4, that the effect on the equilibrium level of R&D can be positive, and that it is larger the greater is ε. Therefore, for any $\partial\zeta/\partial h\underline{Z}^{1/\alpha\beta}$, there exists some value of ε for which the result is true. See appendix 7.5. ■

The statement is not made more precise because no explicit solution for $\partial n/\partial h\underline{Z}^{1/\alpha\beta}$ can be derived, and the choice of the shape of $\partial\zeta/\partial h\underline{Z}^{1/\alpha\beta}$ could appear no less arbitrary than the sentence itself.

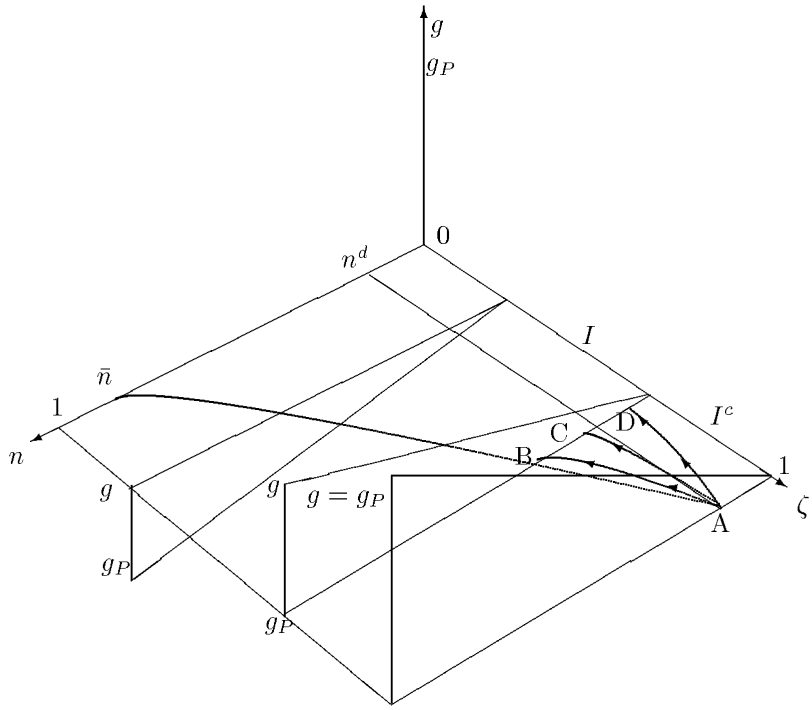

Figure 4: Trade-off versus win-win.

Figure 4 summarizes the possible outcomes. n^d denotes the level of R&D activity prevailing when pollution intensities are uniform, at $\zeta = 1$; the schedule $\bar{n}A$ represents the maximum level of n which could be attained by choosing freely ζ and $h\underline{Z}^{1/\alpha\beta}$.[24] Until the green tax is below h^* the balanced growth path of the economy is at point A, where $g = g_P > 0$.[25] Now suppose that the authorities increase taxation beyond this threshold level, and goods become differentiated according to their pollution intensities. Any further increase in the green tax burden would imply a reduction in ζ. However it can either generate a fall (as along trajectory AD) or an increase (as along trajectories AB and AC) in

[24] $\bar{n}$ is derived in footnote 21 and represents the equilibrium level of n when the whole gross marginal cost is given by the green tax burden. In figure 4 it is drawn as a decreasing function of ζ. For this to be true it is sufficient that $\varepsilon > [1 - \alpha(1-\beta)]/\beta > 1$. That $\bar{n} = n^d$ at $\zeta = 1$ results from proposition 4.

[25] Verdier (1993) has the opposite result that green taxes may enhance growth only when they are sufficiently low. In this case they reduce the cost of R&D because they depress wages, but they are not high enough to induce firms to adopt cleaner technologies. Thus there is no additional cost on the R&D program.

the equilibrium level of R&D. Along the balanced growth path represented by points B, C and D the green tax burden is such that the aggregate flow of emissions is constant, and the stock of pollution is stabilized. The three points are distinguished by the relative importance of the reaction of the equilibrium level of R&D to green tax policy. So in D, the growth rate and the R&D intensities are lower than in the *laisser-faire* regime (point A). At point C the R&D effort is higher, but the growth rate still lower. In the case of point B, however, the R&D has increased so much that the economy grows faster.

One final remark concerns the relationship between the growth rates of the economy, g, and of the green tax rate, g_h. Grimaud and Ricci (1999) derive a causal relationship, stating that the faster grows the green tax, the slower is the pace of economic growth. In the present model economy, a negative correlation between g_h and g may be observed, though there is no direct causal relationship. Consider again the example on figure 4, where g_h is measured by the vertical distance between g and g_P. Then a negative correlation can be found when comparing economies in A with economies in C or D. However a positive correlation would be observed when comparing economies at A with those at B, or economies at B, C and D among each other.

Although in this section we have not obtained a clear-cut result, we believe that it represents a realistic generalization of our result in section 4, on the growth enhancing virtues of environmental policy.

6 Conclusions

We have studied the effect of environmental policy in an economy where intermediate goods are differentiated on two dimensions: productivity and pollution intensity. New intermediate goods with higher productivity are introduced by profit seeking agents, the R&D firms. Demand for and profits generated by new goods are higher than those of older goods. Then a tax on emissions affects demand for intermediate goods in a heterogeneous manner if goods are differentiated in pollution intensities. If innovations are environmental friendly an emission tax skews demand in favor of new goods. The tax depresses both output and the value of innovations, but the distortionary impact favors innovations in relative terms. As a result green taxes foster R&D activity and growth. We also show that with environmental friendly innovations, the policy maker increases the tax on emissions in order to keep the market share of new goods constant and carry the economy along a balanced growth path.

In the last section we have allowed environmental policy to affect the extent to which innovations are environmental friendly. We have given a more complete picture of the complex reactions of the economic system to a change in the green tax burden. We have shown that there are a number of countervailing effects in play, and that overall the result can be an increase in the rate of growth of the economy. The intuition behind this insight is the same as in the simplified model: though environmental policy represents an additional burden on economic activity, it discriminates in favor of cleaner products, and these are also the newest and most productive. Thus environmental policy can foster innovative activities, and lessen the depressing impact of green taxes on growth, even to the point of reverting the whole outcome.

Overall a tougher environmental policy depresses the level of output instantaneously but increases its long-run growth rate. It is natural to ask the normative question: at what level should the green tax be set? This normative question has been explored by the author and will soon be added to this paper. First of all we need to introduce pollution in the utility function to make sense of normative issues. Then we use three policy tools: the green tax to price pollution inputs; a subsidy to the purchase of intermediate goods to deal with market power; a subsidy to R&D to match private and social returns to this activity. With all these tools, each can be specialized to its objective. In particular the R&D subsidy is chosen to target the optimal investment in R&D. We find a dichotomy in the choice of the optimal ζ and green tax, h. The optimal ζ depends only on the productivity growth opportunities and preferences concerning environmental quality. Instead the optimal level of the green tax depends only on the technological role of emissions in production. We find that, with separable and isoelastic preferences, a path along which the flow of pollution decreases and the green tax increases is optimal if the intertemporal elasticity of substitution is smaller than unity ($\varepsilon > 1$).

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7 Appendix

7.1 Functions Γ, Λ and Δ

Let us define :

- $a_j = \frac{A_j}{\bar{A}} \in (0,1]$, an index measuring the productivity gap of good j relative to the leading edge technology at a given date τ. The greater a_j, the smaller the gap. Ex-ante this is a stochastic variable;

- $z_j = \frac{\underline{Z}}{Z_j} \in (0,1]$, measuring the (inverse) gap in pollution intensity of good j relative to the leading edge technology at a given date τ. The greater z_j, the smaller the gap. This too is a stochastic variable.

- $M_j = \frac{\bar{m}}{m_j} \in (0,1]$,the ratio of the gross marginal cost in sector j relative to that prevailing in the leading edge sector, at any given date τ. By the definition of m we have:

$$M(z_j) = \frac{r + h\underline{Z}^{1/\alpha\beta}}{r + h\underline{Z}^{1/\alpha\beta} z_{j,\tau}^{-1/\alpha\beta}}$$

  implying:

$$M(1) = 1 \qquad \text{and} \qquad M' > 0$$

- The sales of any good j relative to the leading edge sector can be expressed as:

$$X(z_{j\tau}) = \frac{\hat{x}_{j\tau}}{\bar{x}_\tau} = \left(\frac{r + h\underline{Z}^{1/\alpha\beta}}{z_{j\tau}\left[r + h\underline{Z}^{1/\alpha\beta} z_{j,\tau}^{-1/\alpha\beta}\right]} \right)^{\frac{1}{1-\alpha}} = \left(\frac{M_{j\tau}}{z_{j\tau}}\right)^{\frac{1}{1-\alpha}}$$

- We define:

$$\Gamma = \int_0^1 a_j X(z_j) dj$$

  The integral has no mathematical sense because a and z are distributed stochastically (and a priori discontinuously) over the space of goods $[0,1]$. However at any date τ and along a balanced growth path, it is possible to reshuffle the goods by order of decreasing technological gap. In other words the integral can be evaluated by ordering goods according to their technological age s. Any technology is adopted by a mass λn of firms. Out of this initial mass, only a proportion $e^{-\lambda n s}$ of those with age s survives at date τ. Furthermore we know that the productivity gap for firms of age s is: $\frac{\bar{A}_{\tau-s}}{\bar{A}_\tau} = e^{-g_{\bar{A}}s} = e^{-\lambda n \ln \gamma s}$; and the gap for pollution intensity is: $\frac{\underline{Z}_\tau}{\underline{Z}_{\tau-s}} = e^{g_{\underline{Z}}s} = e^{\lambda n \ln \zeta s}$. Finally we know that under policy rule (PR) the older technology will sell less, according the shape of the competitiveness-loss function, $\bar{m}/m_s$. We therefore have:

$$\begin{aligned}\Gamma &= \int_0^1 a_j z_j^{\frac{-1}{1-\alpha}} \left(M(z_j)\right)^{\frac{1}{1-\alpha}} dj \\ &= \int_0^\infty \lambda n e^{-\lambda n s} \frac{\bar{A}_{\tau-s}}{\bar{A}_\tau} \left(\frac{\underline{Z}_{\tau-s}}{\underline{Z}_\tau}\right)^{\frac{1}{1-\alpha}} \left(\frac{\bar{m}}{m_s}\right)^{\frac{1}{1-\alpha}} ds \\ &= \lambda n \int_0^\infty e^{-\left(\lambda n+\lambda n \ln\gamma+\frac{\lambda n \ln\zeta}{1-\alpha}\right)s} \left(\frac{r+h\underline{Z}^{1/\alpha\beta}}{r+e^{g_h s}h\underline{Z}^{1/\alpha\beta}}\right)^{\frac{1}{1-\alpha}} ds \\ &= \lambda n \int_0^\infty e^{-(1+G)\lambda n s} \left(\frac{\bar{m}}{m_s}\right)^{\frac{1}{1-\alpha}} ds\end{aligned}$$

- We define:

$$\Lambda = \int_0^1 z_j^{-1/\alpha\beta} a_j X(z_j) dj$$

and proceeding as above:

$$\begin{aligned}\Lambda &= \lambda n \int_0^\infty e^{-(1+G)\lambda n s} e^{-g_{\underline{Z}} s/\alpha\beta} \left(\frac{\bar{m}}{m_s}\right)^{\frac{1}{1-\alpha}} ds \\ &= \lambda n \int_0^\infty e^{-(1+G)\lambda n s} e^{g_h s} \left(\frac{\bar{m}}{m_s}\right)^{\frac{1}{1-\alpha}} ds \\ &= \lambda n \int_0^\infty e^{-\left(1+G+\frac{\ln\zeta}{\alpha\beta}\right)\lambda n s} \left(\frac{\bar{m}}{m_s}\right)^{\frac{1}{1-\alpha}} ds\end{aligned}$$

- We define:

$$\Delta = \int_0^1 \frac{a_j}{z_j} [X(z_j)]^\alpha dj$$

and again:

$$\begin{aligned}\Delta &= \lambda n \int_0^\infty e^{-(\lambda n+\lambda n \ln\gamma+\lambda n \ln\zeta)s} \left(\frac{\underline{Z}_{\tau-s}}{\underline{Z}_\tau}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\bar{m}}{m_s}\right)^{\frac{\alpha}{1-\alpha}} ds \\ &= \lambda n \int_0^\infty e^{-(1+G)\lambda n s} \left(\frac{\bar{m}}{m_s}\right)^{\frac{\alpha}{1-\alpha}} ds\end{aligned}$$

We have the following properties :

Property 1 : $\Gamma < 1$,
From the definition of Γ, of a and of X, we have that:

$$A_j \hat{x}_j < \bar{A}\bar{x} \quad \forall j \in [0,1] \quad \text{but one} \quad \Rightarrow \quad \Gamma < 1$$

Consider any sector of age $s > 0$, using (6) we have:

$$
\begin{aligned}
A_{s\tau}\hat{x}_{s\tau} &= \bar{A}_{\tau-s}\hat{x}_{s\tau} \\
&= \bar{A}_{\tau}e^{-g_{\bar{A}}s}(1-n)\left(\frac{\alpha^2 \underline{Z}_{\tau}e^{-g_{\underline{Z}}s}}{r+e^{g_h s}h\underline{Z}^{1/\alpha\beta}}\right)^{\frac{1}{1-\alpha}} \\
&= e^{-G\lambda n s}\bar{A}_{\tau}(1-n)\left(\frac{\alpha^2 \underline{Z}_{\tau}}{r+e^{g_h s}h\underline{Z}^{1/\alpha\beta}}\right)^{\frac{1}{1-\alpha}} \\
&< e^{-G\lambda n s}\bar{A}_{\tau}(1-n)\left(\frac{\alpha^2 \underline{Z}_{\tau}}{r+h\underline{Z}^{1/\alpha\beta}}\right)^{\frac{1}{1-\alpha}} \\
&< \bar{A}_{\tau}\bar{x}_{\tau}
\end{aligned}
$$

Property 2 : $\Lambda > \Gamma$, since $z_j^{-1/\alpha\beta} \in [1,\infty)$;

Property 3 : $\Lambda > \Delta$,
Indeed :

$$
\begin{aligned}
\Delta &= \int_0^1 \frac{a_j}{z_j}[X(z_j)]^{\alpha}dj \\
&= \int_0^1 a_j z_j^{\frac{-1}{1-\alpha}}\left(\frac{\bar{m}}{m_j}\right)^{\frac{1}{1-\alpha}}\left(\frac{r+h\underline{Z}^{1/\alpha\beta}z_j^{-1/\alpha\beta}}{r+h\underline{Z}^{1/\alpha\beta}}\right)dj \\
&= \frac{r}{r+h\underline{Z}^{1/\alpha\beta}}\Gamma + \frac{h\underline{Z}^{1/\alpha\beta}}{r+h\underline{Z}^{1/\alpha\beta}}\Lambda
\end{aligned}
$$

Thus : $\Delta - \Lambda = \frac{r}{r+h\underline{Z}^{1/\alpha\beta}}(\Gamma - \Lambda) < 0$, by property 2;

Property 4 : $\Delta\Gamma^{-\alpha} \leq (1+G)^{1-\alpha} < 1$, with $G = \ln\gamma + \ln\zeta/(1-\alpha)$,
Indeed by Jensen's inequality:

$$
\begin{aligned}
[\Delta(1+G)]^{\frac{1}{\alpha}} &= \left\{\lambda n \int_0^{\infty}(1+G)e^{-(1+G)\lambda n s}\left[\left(\frac{\bar{m}}{m_s}\right)^{\frac{1}{1-\alpha}}\right]^{\alpha}ds\right\}^{\frac{1}{\alpha}} \leq \\
&\leq \lambda n \int_0^{\infty}(1+G)e^{-(1+G)\lambda n s}\left(\frac{\bar{m}}{m_s}\right)^{\frac{1}{1-\alpha}}ds = \Gamma(1+G)
\end{aligned}
$$

Property 5 : $\Gamma = \Delta = \Lambda = \int_0^1 a_j dj$ if $\zeta = 1$

If there is no differentiation in pollution intensities, that is if $\zeta = 1$, then $z_j = 1$ $\forall j \in [0,1]$, and $g_h = 0$ by policy rule (PR) so that $X(z_j) = 1$ $\forall j \in [0,1]$. The result is immediate from the definitions of Γ, Λ and Δ.

7.2 Proof of proposition 3

To prove the proposition, we differentiate output given by (18) with respect to $h\underline{Z}^{1/\alpha\beta}$. Through the computation we make use of the equilibrium sales of intermediate goods, given by (6), the definitions of Δ and Λ given by (19) and (17), and the first definitions in appendix 7.1. We have:

$$\begin{aligned}
\frac{\partial Y}{\partial h\underline{Z}^{1/\alpha\beta}} &= (1-n)^{1-\alpha}\underline{Z}_\tau \bar{A}_\tau \bar{x}_\tau^\alpha \left[\frac{\partial \Delta}{\partial h\underline{Z}^{1/\alpha\beta}} + \frac{\alpha\Delta}{\bar{x}_\tau}\frac{\partial \bar{x}_\tau}{\partial h\underline{Z}^{1/\alpha\beta}}\right] \qquad \text{(B)}\\
&= \frac{\alpha(1-n)^{1-\alpha}\underline{Z}_\tau \bar{A}_\tau \bar{x}_\tau^\alpha}{(1-\alpha)\left[r+h\underline{Z}^{1/\alpha\beta}\right]}\left[\int_0^1 \frac{a_j}{z_j}\left(\frac{\hat{x}_{j\tau}}{\bar{x}_\tau}\right)^\alpha \frac{(1-z_j^{-1/\alpha\beta})r}{\left[r+h\underline{Z}^{1/\alpha\beta}z_j^{-1/\alpha\beta}\right]}dj - \Delta\right]\\
&= \frac{\bar{A}_\tau \bar{x}_\tau}{\alpha(1-\alpha)}\int_0^1 \frac{a_j}{z_j}\left(\frac{\hat{x}_{j\tau}}{\bar{x}_\tau}\right)^\alpha \left[\frac{\left(1-z_j^{-1/\alpha\beta}\right)r}{\left[r+h\underline{Z}^{1/\alpha\beta}z_j^{-1/\alpha\beta}\right]} - 1\right] dj\\
&= \frac{-\bar{A}_\tau \bar{x}_\tau}{\alpha(1-\alpha)}\int_0^1 z_j^{-1/\alpha\beta} a_j \left(\frac{\hat{x}_{j\tau}}{\bar{x}_\tau}\right) dj = \frac{-\bar{A}_\tau \bar{x}_\tau \Lambda}{\alpha(1-\alpha)} < 0
\end{aligned}$$

The fall in output is lower when pollution intensities are uniform across sectors, $\zeta = 1$. In this case $\partial\Delta/\partial h\underline{Z}^{1/\alpha\beta} = 0$ according to property 5, so that the first term in brackets on the second line is nil. The latter is negative whenever $\zeta < 1$, since $z_j^{-1/\alpha\beta} > 1$ for all goods j but one.

7.3 Proof of proposition 4

To prove the result we show that, at the original equilibrium level of n, the left-hand-side (LHS) of the equilibrium condition (E) falls more than the right-hand-side (RHS). These shifts of the schedules depicted in figure 2 result in a new equilibrium level of R&D employment, n, higher than the original one.

The LHS of (E) can also be written as $Y_\tau/\left[(1-n)\bar{A}_\tau\right]$ and therefore falls along with output. Using (B) from appendix 7.2 we obtain:

$$\frac{\partial LHS}{\partial h\underline{Z}^{1/\alpha\beta}} = \frac{-\bar{x}_\tau \Lambda}{\alpha(1-\alpha)(1-n)} \qquad \text{(L)}$$

The RHS of (E) is equal to $\lambda V_\tau/\left[\bar{A}_\tau(1-\alpha)\right]$. We can first compute the impact of a marginal change in $h\underline{Z}^{1/\alpha\beta}$ on the value of innovations V_τ as expressed in the first line of (13). This is:

$$\begin{aligned}
\frac{\partial V_\tau}{\partial h\underline{Z}^{1/\alpha\beta}} &= \frac{1-\alpha}{\alpha}\bar{A}_\tau\left(\alpha^2\underline{Z}_\tau\right)^{\frac{1}{1-\alpha}}(1-n)\\
&\quad \int_0^\infty e^{-(r+\lambda n)t}\left(\frac{-\alpha}{1-\alpha}\right)e^{g_h t}\left[r+e^{g_h t}h\underline{Z}^{1/\alpha\beta}\right]^{\frac{-\alpha}{1-\alpha}-1}dt\\
&= -\bar{x}_\tau \bar{A}_\tau \int_0^\infty e^{-(r+\lambda n)t}e^{g_h t}\left(\frac{\bar{m}}{m_t}\right)^{\frac{1}{1-\alpha}}dt < 0
\end{aligned}$$

And therefore:

$$\frac{\partial RHS}{\partial h\underline{Z}^{1/\alpha\beta}} = -\frac{\lambda\bar{x}_\tau}{(1-\alpha)}\int_0^\infty e^{-(r+\lambda n)t}e^{g_h t}\left(\frac{\bar{m}}{m_t}\right)^{\frac{1}{1-\alpha}}dt \qquad \text{(R)}$$

The increase in $h\underline{Z}^{1/\alpha\beta}$ reduces more the LHS than the RHS if (L) is smaller than (R). Rearranging using the definition of Λ (17) and (12) :

$$\frac{n}{\alpha(1-n)} > \frac{\int_0^\infty e^{-(r+\lambda n)t} e^{g_h t} \left(\frac{\bar{m}}{m_t}\right)^{\frac{1}{1-\alpha}} dt}{\int_0^\infty e^{-(g+\lambda n)s} e^{g_h s} \left(\frac{\bar{m}}{m_s}\right)^{\frac{1}{1-\alpha}} ds}$$

We can substitute the left-hand-side of this last inequality by its value prevailing at the original equilibrium, given by the equilibrium condition (E), which rearranged gives:

$$\frac{n}{\alpha(1-n)} = \frac{\int_0^\infty e^{-(r+\lambda n)t} \left(\frac{\bar{m}}{m_t}\right)^{\frac{\alpha}{1-\alpha}} dt}{\Delta/\lambda n} = \frac{\int_0^\infty e^{-(r+\lambda n)t} \left(\frac{\bar{m}}{m_t}\right)^{\frac{\alpha}{1-\alpha}} dt}{\int_0^\infty e^{-(g+\lambda n)s} \left(\frac{\bar{m}}{m_s}\right)^{\frac{\alpha}{1-\alpha}} ds}$$

That the LHS of (E) falls more than the RHS if (L) is smaller than (R) implies:

$$\frac{\int_0^\infty e^{-(g+\lambda n)s} e^{g_h s} \left(\frac{\bar{m}}{m_s}\right)^{\frac{1}{1-\alpha}} ds}{\int_0^\infty e^{-(g+\lambda n)s} \left(\frac{\bar{m}}{m_s}\right)^{\frac{\alpha}{1-\alpha}} ds} > \frac{\int_0^\infty e^{-(r+\lambda n)t} e^{g_h t} \left(\frac{\bar{m}}{m_t}\right)^{\frac{1}{1-\alpha}} dt}{\int_0^\infty e^{-(r+\lambda n)t} \left(\frac{\bar{m}}{m_t}\right)^{\frac{\alpha}{1-\alpha}} dt} \tag{I}$$

The two sides of (I) are equal if $\zeta = 1$ since this implies $\bar{m}/m_t = 1\ \forall t$ and $g_h = 0$. The difference between the two sides of the inequality consists of the discount rate of the integrands. We now show that the ratio of the two integrals is decreasing in the discount rate. It is easy to show that $\forall\ t$ (or s) > 0 :

$$e^{g_h t} \left(\frac{\bar{m}}{m_t}\right)^{\frac{1}{1-\alpha}} > \left(\frac{\bar{m}}{m_t}\right)^{\frac{\alpha}{1-\alpha}}$$

Let us denote the ratio of the two functions by Ψ:

$$\Psi(t) = e^{g_h t} \frac{r + h\underline{Z}^{1/\alpha\beta}}{r + e^{g_h t} h\underline{Z}^{1/\alpha\beta}}$$

Then we have that Ψ is increasing for $r > 0$:

$$\frac{\partial \Psi(t)}{\partial t} \propto 1 - \frac{e^{g_h t} h\underline{Z}^{1/\alpha\beta}}{r + e^{g_h t} h\underline{Z}^{1/\alpha\beta}} > 0$$

Now consider a generic discount rate δ, and redefine the ratio of the integrals in (I) as:

$$\begin{aligned} f(\delta) &= \frac{\int_0^\infty e^{-\delta t} e^{g_h t} \left(\frac{\bar{m}}{m_t}\right)^{\frac{1}{1-\alpha}} dt}{\int_0^\infty e^{-\delta t} \left(\frac{\bar{m}}{m_t}\right)^{\frac{\alpha}{1-\alpha}} dt} \\ &= \frac{\int_0^\infty e^{-\delta t} \left(\frac{\bar{m}}{m_t}\right)^{\frac{\alpha}{1-\alpha}} \Psi(t) dt}{\int_0^\infty e^{-\delta t} \left(\frac{\bar{m}}{m_t}\right)^{\frac{\alpha}{1-\alpha}} dt} \\ &= \int_0^\infty \Omega(t,\delta)\Psi(t) dt \end{aligned}$$

where:

$$\Omega(t,\delta) = \frac{e^{-\delta t}\left(\frac{\bar{m}}{m_t}\right)^{\frac{\alpha}{1-\alpha}}}{\int_0^\infty e^{-\delta u}\left(\frac{\bar{m}}{m_u}\right)^{\frac{\alpha}{1-\alpha}} du}$$

is a normalized weight function, characterized by:

$$\frac{\partial\Omega}{\partial\delta} \propto \int_0^\infty u e^{-\delta u}\left(\frac{\bar{m}}{m_u}\right)^{\frac{\alpha}{1-\alpha}} du - t\int_0^\infty e^{-\delta u}\left(\frac{\bar{m}}{m_u}\right)^{\frac{\alpha}{1-\alpha}} du$$

Therefore $\exists$

$$\tilde{t} = \frac{\int_0^\infty u e^{-\delta u}\left(\frac{\bar{m}}{m_u}\right)^{\frac{\alpha}{1-\alpha}} du}{\int_0^\infty e^{-\delta u}\left(\frac{\bar{m}}{m_u}\right)^{\frac{\alpha}{1-\alpha}} du}$$

such that:

$$\frac{\partial\Omega}{\partial\delta} > 0 \quad \forall t < \tilde{t} \quad \text{and} \quad \frac{\partial\Omega}{\partial\delta} < 0 \quad \forall t > \tilde{t}$$

This means that an increase in the discount rate δ shifts the weight from high values of t to low values of t. Since $\Psi(t)$ is increasing, this implies that:

$$\frac{\partial f(\delta)}{\partial\delta} < 0$$

Inequality (I) holds as long as the discount rate on its left-hand-side, $g + \lambda n$, is lower than the one on its right-hand-side, $r + \lambda n$, that is whenever $r > g$. Notice that this is always the case at equilibrium for the no-Ponzi game condition to hold. This condition states that the present value of households' financial wealth, W, is nil asymptotically. Along a balanced growth path W grows with income, so:

$$\lim_{t\to\infty} e^{-rt}W_t = \lim_{t\to\infty} e^{-(r-g)t}W_0 = 0 \qquad \Leftrightarrow \qquad r > g$$

We have established that in the neighborhood of the equilibrium defined by (E): $r > g$ $\Rightarrow$ (I) holds $\Rightarrow \partial LHS_E/\partial h\underline{Z}^{1/\alpha\beta} < \partial RHS_E/\partial h\underline{Z}^{1/\alpha\beta} < 0 \Rightarrow \partial n^*/\partial h\underline{Z}^{1/\alpha\beta} > 0$.

7.4 The micro background to endogenize ζ

Suppose first that the monopolist can freely choose the pollution intensity at any point in time within the interval $[Z^l, Z^h]$. This constraint is represented by the curve $Z^h Z^l$ in figure 5. The lower bound is the state of the art environmental friendly technological knowledge prevailing at a given point in time, and is taken as given by the monopolist. It is assumed decreasing in the green tax, because a greater tax generates higher fiscal revenues, which allow the government to finance a more ambitious environmental friendly research agenda. The upper bound Z^h is the level of pollution intensity historically inherited. We assume that the pollution intensity cannot be increased. This is crucial because emissions enter as inputs in a Cobb-Douglas production function and have nil marginal cost. Thus, emissions would tend to infinity as the green tax approaches zero. Our assumption means that there is an upper bound beyond which the marginal product of emissions falls to zero, and this seems to us a reasonable assumption. Then we can differentiate instantaneous profits (7) with respect to Z_j, to obtain:

$$\frac{\partial \hat{\Pi}_j}{\partial(-Z_j)} = \frac{A_j}{\alpha Z_j}(1-n)\left(\frac{\alpha^2 Z_j}{r + h_\tau Z_j^{1/\alpha\beta}}\right)^{\frac{1}{1-\alpha}} \left[\frac{1}{\beta} h_\tau Z_j^{1/\alpha\beta} - (r + h_\tau Z_j^{1/\alpha\beta})\right]$$

Thus a reduction in Z has two opposite effects. First, when emissions are subject to taxation, the lower the Z the higher the demand for the intermediate good, hence the greater the profit. Second, a reduction in the pollution intensity implies a decline in the marginal product of the intermediate good in the final sector, because it reduces directly the emissions inputs associated to it, and therefore depresses profits. Notice that when green taxes are nil, i.e. $h = 0$, profits are increasing in Z, and therefore the optimal choice is Z^h. The unconstrained maximum is given by:

$$Z_j^u = \left(\frac{\beta}{1-\beta}\frac{r}{h_\tau}\right)^{\alpha\beta}$$

The best policy would be: Z^h if $Z^u \geq Z^h$, Z^u if $Z^u \in \left]Z^l, Z^h\right[$, Z^l if $Z^u \leq Z^l$. Unsurprisingly, Z^u is decreasing in the level of the green tax, h, as drawn on figure 5. It is interesting that the value of Z^u can be combined with the pollution intensity index $Z_j = (P_j/K_j)^{\alpha\beta}$, to hand:

$$\frac{h_\tau P_j}{rK_j} = \frac{\beta}{1-\beta}$$

Remember that β is the share of emissions, and $1-\beta$ the share of capital in the production function of the composite good, which was discussed upon the presentation of the aggregate production function (23). Therefore the total cost of producing the composite intermediate good is shared, at Z^u, between capital income and the green tax burden according to the elasticity of the composite good output to the two inputs.

So far we have considered the static problem of a monopolist who can alter the pollution intensity of the good at any moment. In our set-up R&D firms have to choose the pollution intensity of their blueprints, which then represents a constraint for the patent holder. They do so in order to maximize the value of patents. Differentiating the first line of equation (13) with respect to $\underline{Z}$ (which is now the choice variable), we have:

$$\frac{\partial V_\tau}{\partial(-Z)} = \int_\tau^\infty e^{-(r+\lambda n)(t-\tau)} \frac{\bar{A}_\tau}{\alpha Z}(1-n)\left(\frac{\alpha^2 Z}{r + h_t Z^{1/\alpha\beta}}\right)^{\frac{1}{1-\alpha}} \left[\frac{1-\beta}{\beta} h_t Z^{1/\alpha\beta} - r\right] dt$$

Notice that this derivative is positive at Z^u, at least in the region where $Z^u < Z^h$, since in this case the integrand starts at zero and then turns positive as the green tax increases. This implies that the optimal choice is certainly a smaller Z in the dynamic context than in the flexible, static framework. The reason is that the best Z takes into account the future savings on the green tax burden, which are increasing at the rate of growth of green taxes, g_h. Therefore the greater $g_h = -\ln\zeta\lambda n/\alpha\beta$, the smaller the choice of Z. Instead, the greater the discount rate $r + \lambda n$ the higher the preferred Z. We denote by Z^c this solution and draw it on figure 5.

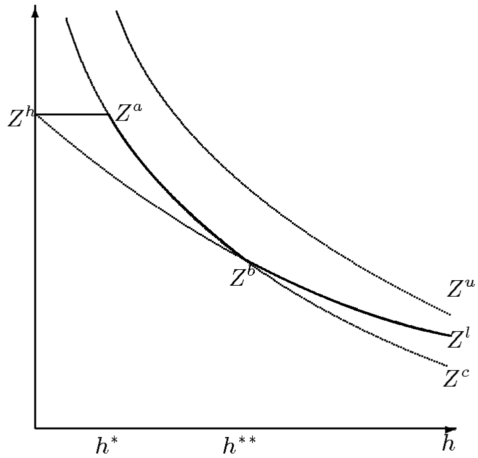

Figure 5: The monopolist and R&D target of pollution intensity.

The solution is represented by the $Z^h Z^a Z^b Z^l$ locus on figure 5, that is the choice of the firm will be: Z^h until $Z^c > Z^h$, i.e. $\forall h \in [0, h^*]$, Z^c $\forall h \in]h^*, h^{**}[$, and Z^l $\forall h \geq h^{**}$. The critical tax h^{**} may not exist. What matters to us is that the improvement in pollution intensity is an increasing function of the green tax, at least for green taxes greater than h^*. This last level is such that the ratio of the green tax burden to gross marginal cost is smaller than the share of emissions in the production of the composite good, at the inherited pollution intensity.[26] If this is the case then the green tax does not provide sufficient incentives to reduce the pollution intensity in the static problem.

We conclude that $\zeta = 1$, and pollution intensity is uniform across sectors, until $h \leq h^*$. Beyond this level, innovators choose to embody the environmental friendly technology into the design of their intermediate goods. And the desired as well as the feasible improvements will grow with the green tax, for any given level of inherited pollution intensity. This behavior translates into an aggregate negative relationship between ζ and the green tax burden $h\underline{Z}^{1/\alpha\beta}$, in short to:

$$\frac{\partial \zeta}{\partial h \underline{Z}^{1/\alpha\beta}} < 0$$

[26]Indeed at h^* we have $Z^h = Z^c < Z^u$, that is:

$$\frac{h^* \left(Z^h\right)^{1/\alpha\beta}}{r + h^* \left(Z^h\right)^{1/\alpha\beta}} < \beta$$

7.5 Proof of proposition 5

Let us study the long run effect of a marginal change in $h\underline{Z}^{1/\alpha\beta}$ on the equilibrium equation (E)[27]. The impact on the LHS, leaving n constant, is:

$$\frac{\partial LHS}{\partial h\underline{Z}^{1/\alpha\beta}} = -(1-n)^{-\alpha}\underline{Z}\bar{x}^{\alpha}\lambda n \int_0^{\infty} e^{-(G\lambda n+\lambda n)s}\left(\frac{\bar{m}}{m_s}\right)^{\frac{\alpha}{1-\alpha}} \cdot \Big[\underbrace{G'\lambda ns}_{1} + \frac{\alpha}{1-\alpha}\frac{1}{m_s}\Big[\underbrace{\varepsilon G'\lambda n}_{2} + \underbrace{e^{g_h s}}_{3}\Big(1+\underbrace{g_h' h\underline{Z}^{1/\alpha\beta}s}_{4}\Big)\Big]\Big]ds$$

with $G' = (\lambda n/\zeta)\,\partial\zeta/\partial h\underline{Z}^{1/\alpha\beta}$, and $g_h' = -(\lambda n/\zeta\alpha\beta)\,\partial\zeta/\partial h\underline{Z}^{1/\alpha\beta}$. And the equivalent for the value of patents is:

$$\frac{\partial RHS}{\partial h\underline{Z}^{1/\alpha\beta}} = -\frac{\lambda\bar{x}\bar{m}}{\alpha}\int_0^{\infty} e^{-(r+\lambda n)t}\left(\frac{\bar{m}}{m_t}\right)^{\frac{\alpha}{1-\alpha}} \cdot \Big[\underbrace{\varepsilon G'\lambda nt}_{1} + \frac{\alpha}{1-\alpha}\frac{1}{m_t}\Big[\underbrace{\varepsilon G'\lambda n}_{2} + \underbrace{e^{g_h t}}_{3}\Big(1+\underbrace{g_h' h\underline{Z}^{1/\alpha\beta}t}_{4}\Big)\Big]\Big]dt$$

In both cases we can distinguish four effects:

1. The aggregation (or discount) rate effect: the increase in tax burden reduces the growth factor, G, and thus reduces the aggregation rate for the wage, and the discount rate on the side of the value of innovations (according to ε since $r = \rho + \varepsilon G\lambda n$); this increases ceteris paribus both wages and the value of patent, though more (less) the latter than the former if $\varepsilon > 1$ ($\varepsilon < 1$).

2. The competitiveness relaxing effect: the reduction in the rate of interest, reduces the marginal cost for all monopolies, lowering ceteris paribus the relative weight of the green tax burden; this effect implies a flatter competitiveness loss schedule.

3. The direct competitiveness loss effect: the greater initial green tax burden increases ceteris paribus its weight relative to other costs of production, for all goods; this was the only effect considered in section 4, when the impact on ζ was ruled out.

[27]For the immediate impact on the level of R&D, we must consider only the direct competitiveness loss effect on the wage aggregator (the left-hand-side of (E)). Then we find that the R&D intensity increases if:

$$\int_0^{\infty}\Omega(g)\Psi(t)dt > \int_0^{\infty}\Omega(r)\Psi(t)dt + h\underline{Z}^{1/\alpha\beta}\int_0^{\infty} g_h' t\Omega(r)\Psi(t)dt - -(-G')\lambda n\varepsilon\int_0^{\infty}\Omega(r)\left(\frac{1-\alpha}{\alpha}t + \frac{1}{m_t}\right)dt$$

From proof 4 we know that the left hand side is larger than the first term on the right hand side. Thus a sufficient condition for the inequality to hold is that the third term be larger than the second, on the right hand side. If this is the case, then n shoots up upon arrival of the policy innovations.

4. The faster green crowding-out effect: the higher speed of environmental friendly technological progress (the lower ζ) requires a faster growth of the green tax rate at equilibrium, and therefore a faster process of crowding-out, or policy-induced technological obsolescence.

The last three effects run through the impact of the change in the green tax burden on the competitiveness loss schedule. They are hence symmetric on the two sides of the arbitrage condition. As we will show next, overall these effects depress both wages and the value of innovations, but more so the former than the latter. Instead, the aggregation and discounting effects are asymmetric, so that we are unable to establish clearly what is their net contribution the change in n.
Following the steps of the proof to proposition 4, we find that the equilibrium R&D activity is increased by the policy shock, i.e. $\partial n / \partial h \underline{Z}^{1/\alpha\beta} > 0$ if:

$$\begin{aligned}&(-G')\lambda n \int_0^\infty t\left(\varepsilon\Omega(r) - \Omega(g)\right) dt \\ &+\tfrac{\alpha}{1-\alpha}\varepsilon(-G')\lambda n \int_0^\infty \tfrac{1}{m_t}\left(\Omega(r) - \Omega(g)\right) dt \\ &+\tfrac{\alpha}{1-\alpha}\int_0^\infty \tfrac{e^{g_h t}}{m_t}\left(1 + g_h' h \underline{Z}^{1/\alpha\beta} t\right)\left[\Omega(g) - \Omega(r)\right] dt \quad > 0\end{aligned}$$

The normalized weight function Ω, defined in proof 4, is such that $\Omega(r)$ discounts more late events than $\Omega(g)$. Thus the integral on the last line is positive, because the direct competitiveness loss and faster green crowding-out effects are increasing functions of t. The integral on the second line is positive, because $1/m_t$ is decreasing. The whole second line is positive since $G' < 0$. Instead the first line compares the aggregation and the discounting effects, and its sign is therefore ambiguous. The line is negative for $\varepsilon = 1$ but is increasing with ε, so that it turns positive beyond some values of ε. For $\partial n / \partial h \underline{Z}^{1/\alpha\beta} > 0$ it is sufficient that the sum of this term with the two other lines be positive, thus we can state that there exists some level of ε such that this is the case.