Joint Bidding in Federal O®shore Oil and Gas Lease Auctions

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Abstract

This paper provides an explanation for why cartels are not observed frequently in mineral-rights auctions even though it was not illegal for them to form. We use the techniques of mechanism design to characterize the e±cient, incentive compatible cartel and show that it can be implemented by a ¯rst-price knockout tournament with information sharing. We show, however, that bidders with the highest signals typically prefer to bid alone rather than join the cartel. We examine bid data from federal o®shore oil and gas auctions for evidence that cartels used bid coordination schemes. We also examine the determinants of joint bidding.

1 Introduction

Collusion is frequently observed in auctions where bidders' willingness to pay are independently distributed (IPV). Examples include highway construction contracts (Porter and Zona, 1993), school milk delivery (Pesendorfer, 1998, Porter and Zona, 1999), and timber auctions (Baldwin et al., 1997). In each of these examples, the heterogeneity in bidder valuations is due primarily to di®erences in costs, which are arguably idiosyncratic to each bidder. The cartel's problem is to devise a mechanism to divide the spoils and select who is going to receive the contract. In doing so, the cartel has to overcome an adverse selection problem: none of its members know how much each of their fellow cartel members is willing to pay for the item being auctioned and each member wants to exploit this private information to argue for a bigger share of the spoils. Graham and Marshall (1987), Mailath and Zemesky (1991), and McAfee and McMillan (1992) have shown that, if an all-inclusive cartel is allowed to use side payments, it can design implementable mechanisms that induces each member to tell the truth and awards the item to the member who has the highest valuation. Furthermore, each cartel member's expected payment conditional on her type exceeds her payo® from noncooperative bidding. Consequently, even after all members realize their types, each prefers the cartel mechanism to bidding noncooperatively in the seller's auction.

This paper studies cartel formation in oil and gas lease auctions where bidders have private signals about an unknown common value. Cramton and Palfrey (1990) and McAfee and McMillan (1992) have observed that, if all members have the same ex post value, then the cartel does not have to worry about who should be awarded the lease and a joint bidding agreement in which all members share equally in the spoils is incentive compatible. Cartel members will have no incentive to misrepresent their information since they share a common goal, which is to bid only when the expected value of the item conditional on the pooled information exceeds the reserve price. The problem with such agreements, however, is that the expected payment to cartel members may not exceed the amounts they can expect to earn (conditional on their information) by bidding alone in the seller's auction. The intuition is simple. A bidder who has obtained a very high signal from a distribution where poor signals are more likely to be drawn may rationally believe it can win the lease by bidding not much above the reserve price. It pays a somewhat higher price to the seller, but it does not have to share the surplus with any other ¯rms.

This intuition may explain the surprisingly low incidence of joint bidding among ¯rms with the highest participation rates in the auctions of federal o®shore oil and gas leases is quite low. Table 1 describes the incidence of solo and joint bidding by seven of the most active bidders: Exxon, Gulf, Mobil, Shell, Standard Oil of California, Standard Oil of Indiana, and Texaco, hereafter called the Big 7. The sample consists of 2227 wildcat tracts o® the coasts of Texas and Louisiana that received bids between 1954 and 1975.[1] The table reveals that solo bidding was the dominant form of bidding for most of the Big 7 ¯rms. Joint bids involving pairs of Big 7 ¯rms represented less than 15% of all bids submitted by these ¯rms. Shell and Exxon rarely bid jointly. Furthermore, if ¯rms bid jointly, they did so almost always in pairs.

Solo bidding does not imply the absence of a cartel. In testimony before Congress, the head of Department of Interior, Darius Gaskins argued that the collusive e®ects of joint ventures should not be measured solely in terms of tracts receiving joint bids. The negotiations to bid jointly could allow partners to coordinate their solo bids. The cartel could, for example, hold a ¯rst-price knockout tournament on each tract in a speci¯c area to determine which partners valued which tracts more highly than others and allocate the tracts accordingly. If this allocation does not achieve an equitable balance among its members, the ¯rm with more wins could agree to bear

[1] In 1975, Congress passed the Energy Policy and Conservation Act that prohibited these seven ¯rms from bidding with each other for OCS leases.

a larger share of the costs of drilling the area or, if oil is discovered and the area unitized, a small share of production. Mechanisms involving side payments could give high types a stronger incentive to participate. Certainly, the potential gains from forming a cartel appear substantial. The stakes are large. The announced reserve price is approximately $100,000 whereas the average winning bid in wildcat auctions for the period 1954-79 was 12.8 million dollars (in 1982 dollars). The cost of drilling an exploratory well was approximately 1.5 million dollars and the risk of loss is signi¯cant, as only 39% of the tracts receiving bids in the period 1954-79 were productive (Porter (1995)). By pooling geological data and expertise in interpreting the data, ¯rms could reduce the risk of buying dry leases and, by pooling ¯nancial resources, they can bid for more leases and diversify away more of the tract-speci¯c uncertainties.

We investigate the cartel mechanism design problem in a simple environment in which each bidder receives an independently and identically distributed signal, and the value of the lease being auctioned depends on her own signal and symmetrically on the signals of the other bidders. Each bidder may weight her own signal more heavily than the signals of others. This bidding environment has been used by Bulow, Huang, and Klemperer (1999) to study \common value" takeovers, by Klemperer (1998) to study the e®ect of small asymmetries on equilibria in ascending price auctions of common value objects, and by Krishna and Morgan (1997) to study the anti-competitive e®ects of joint bidding and bidder restrictions in common value, second-price auctions. We derive necessary and su±cient conditions under which an e±cient, incentive compatible cartel exists. As in the IPV case, it can be implemented by a ¯rst-price knockout tournament in which the high bidder wins the right to bid the reserve price in the seller's auction and the other members divide the high bid equally among themselves.

To determine whether the cartel is enforceable, we compare the optimal cartel payo®s to the equilibrium payo®s from bidding competitively in a ¯rst-price, sealed bid auction. In contrast to the IPV case, we ¯nd that ¯rms with high signals may prefer the competitive mechanism. The reason is closely related to the phenomenon known as the "winner's curse". A ¯rm does not bid in the auction if the value of the lease conditional on its signal being the highest is less than the reserve price. However, it may bid in the cartel's knockout tournament because, even if it has the highest signal, there is a chance that the expected value of the lease after all of the signals are revealed exceeds the reserve price. Since more types bid in the cartel, bidding in the cartel can be more aggressive than in the auction and the expected payment (including the reserve price) higher than in the auction.[2] We show that the

[2] This situation cannot occur in the IPV environment because the participation rule for the auction and the knockout tournament are the same. In both cases, ¯rms bid if and only if their

conditions under which the cartel is not enforceable are likely to be satisfied by oil and gas auctions. We also show that, when it is enforceable, joint bidding typically dominates other sharing mechanisms, at least for the limiting case of pure common values (i.e., all signals receive the same weight).

Our empirical study of cartel formation and behavior employs data from Hendricks, Pinkse, and Porter's (1999) study of bidding on federal oil and gas wildcat lease auctions off the coasts of Texas and Louisania. The novel feature of that data set was the construction of a measure of the number of potential bidders on a tract. based upon who bid in the area and when. Previous empirical studies of joint bidding by Mead (1968), Erickson and Spann (1974), Gaskins and Vann (1976), Mead and Sorenson (1980), Rockwood (1983) and Gilley et al (1985) have documented a positive correlation between the incidence of joint bidding and the value of tracts. These authors typically use the government's estimate of the value of the tract, the winning bid, and/or the number of bids as a measure of ex ante value of the tract. We establish a similar result but use the number of potential bidders and the ex post value of the tracts as proxies for ex ante value. Much of this correlation probably reflects the incentive for firms to find financial partners on tracts where the high bid is likely to quite large. However, it may also reflect the fact that potential bidders are more likely to know who are their competitors (i.e., who intends to bid) on high value tracts.

We examine the latter issue by restricting the sample of auctions to auctions in which the number of potential bidders is fewer than four. These are the set of tracts that are most likely to meet the conditions of the theory. In particular, the number of bidders is small enough that an all-inclusive cartel is feasible, and the bidders are not capital constrained since the tracts are marginal and bids are low. We consider two questions. First, is there any evidence of potential bidders using bid coordination schemes, especially bidders who bid jointly in the sale? Second, is competitive bidding more likely to occur on tracts where ex post values are higher, as predicted by the theory?

The paper is organized as follows. In the next section we characterize optimal cartel mechanisms and discuss implementation. In section 3 we show that the optimal cartel may not be enforceable and describe the class of distributions for which this will be the case. In section 4 we analyze the special case of pure common value. In Section 5 we develop an empirical strategy for identifying cartels and the determinants of joint bidding. The data is presented in Section 5. The econometric results are reported in Section 6. Section 7 provides concluding remarks.

value exceeds the reserve price.

2 Optimal Cartel Mechanisms

The seller sells one lease via a first-price sealed bid auction with a pre-announced reserve price of R. Our primary interest is in the question of whether bidders have incentives to form all-inclusive cartels. Given this focus, and without loss of generality, we simplify the model by assuming that there are only two bidders. Bidders 1 and 2 receive private signals s_1 and s_2 respectively; they are independently and identically distributed on $[0;1]$ according to the cumulative distribution function F with the associated density f. Let $u(s_1; s_2)$ represent the expected value of the lease to bidder 1 given signals s_1 and s_2. Symmetrically, we assume that bidder 2 values the lease at $u(s_2; s_1)$. We assume that u is increasing in both signals and that,

$$s_2 > s_1 \Longrightarrow u(s_2; s_1) > u(s_1; s_2). \tag{1}$$

This condition states that the bidder with higher signal values the lease more than the bidder with lower signal. Later we will study the case of pure common values, that is, when $u(s_2; s_1) = u(s_1; s_2)$ for all $s_1; s_2 \in [0;1]$. The valuation function and distribution functions are common knowledge.

The cartel's objective in designing a mechanism is to maximize the ex ante sum of bidders' profits in the auction. It faces three problems: dividing the collusive surplus, selecting a sole bidder, and sharing information to ensure that the sole bidder pays R to the seller only when it is profitable to do so.

Instead of analyzing the particular mechanisms used by cartels in oil and gas lease auctions, we first study collusive direct revelation mechanisms in which the sole bidder and side-payments are determined as functions of the bidders' reported signals, which we denote as t_1 and t_2. A collusive direct revelation mechanism is a pair $\{P; T\}$ where $P : [0;1]^2 \to [0;1]^2$ and $T : [0;1]^2 \to \mathbb{R}^2$. Here, given reports $(t_1; t_2)$, the probability that bidder 1 obtains the right to bid in the seller's auction is $P_1(t_1; t_2)$ and its expected side-payment is $T_1(t_1; t_2)$. Clearly,

$$P_1(t_1; t_2) + P_2(t_2; t_1) \le 1$$

for all $(t_1; t_2) \in [0;1]^2$. We assume that transfers are feasible if they satisfy

$$T_1(t_1; t_2) + T_2(t_2; t_1) = 0 \tag{2}$$

for every pair of reported signals $(t_1; t_2)$. This requires the cartel to balance its budget ex post.

The main benefit of pooling information is that it allows the cartel to make a more informed bidding decision. Define

$$v(s_1; s_2) = \max[0; u(s_1; s_2) - R].$$

to be bidder 1's payoff in the event that it is selected to be the cartel's bidder. The payoff assumes that bidder 1 learns the reported signal of bidder 2 and pays the seller the reserve price R if and only if its expected value of the object conditional on its own signal and the reported signal of bidder 2 exceeds R. We refer to this property as bidding efficiency.

Efficiency also imposes a second condition, namely, that the bidder with the highest signal should be selected as the cartel's bidder. More precisely, for any $i = 1, 2$ and $s_i, s_j \in [0, 1]$,

$$P_i(s_i, s_j) = 1 \text{ if and only if } s_i > s_j. \tag{3}$$

This requirement is the standard ex post allocation efficiency condition.

A mechanism is efficient if it satisfies both bidding efficiency and ex post allocation efficiency. Define b as the solution to the equation $u(b, b) = R$ and define

$$\theta(s) = \inf\{x : u(s, x) \geq R\}.$$

Note that $\theta(b) = b$. Efficient mechanisms imply that $P_i(s_i, s_j) = 1$ if $s_i > b$ and $s_i > s_j > \theta(s_i)$; it is equal to zero otherwise.

For $i = 1, 2$, define

$$\pi_i(s_i, t_i) = E_x[P_i(t_i, x)v(s_i, x) + T_i(t_i, x)] \tag{4}$$

where $s_i, t_i \in [0, 1]$. Denote $\pi_i(s_i, s_i)$ by $\pi_i(s_i)$. A cartel mechanism $\{P, T\}$ is incentive compatible if for $i = 1, 2$, and for all $s_i, t_i \in [0, 1]$,

$$\pi_i(s_i) \geq \pi_i(s_i, t_i). \tag{5}$$

A cartel mechanism is called optimal if it is incentive compatible and efficient.

To characterize the set of optimal mechanisms, we begin with a standard characterization lemma on incentive compatible mechanisms.

Lemma 1: A cartel mechanism $\{P, T\}$ is incentive compatible if and only if for $i = 1, 2$, and for any $s, t \in [0, 1]$,

$$\frac{d\pi_i(s)}{ds} = E_x[P_i(s, x)v_1(s, x)] \tag{6}$$

and

$$E_x[(\partial P_i(t, x)/\partial t)v_1(s, x)] \geq 0, \tag{7}$$

where v_1 is the partial derivative of v with respect to its first argument.

Combining the incentive compatibility with the efficiency requirement yields the following characterization.

Theorem 1: Any efficient, incentive compatible mechanism generates the following expected profits to a bidder with signal s

$$\pi^C(s) = \pi_0 + \int_{\theta(s)}^{s} v(s;x)dF(x) - \int_{b}^{s} v(x;x)dF(x) \tag{8}$$

for $s > b$ and is equal to π_0 otherwise, where

$$\pi_0 = \int_{b}^{1} v(x;x)[1 - F(x)]dF(x).$$

Note that a bidder whose signal is below b earns a positive expected payment even though it will never bid in the seller's auction. If it has the highest signal, the lease is not worth R and if they have the lowest signal, the other cartel member is selected as the cartel bidder. The expected payment is strictly increasing in s for signals above b since there is some chance that a bidder with such a signal will be selected and still find the lease is worth R.

We turn next to the question of implementation. Suppose the cartel uses a first-price knockout tournament with information sharing. Each member submits a bid to the ring center. The member with the highest bid is awarded the right to bid R in the seller's auction and pays its bid to the "losing" member. This is the mechanism analyzed by McAfee and McMillan (1992) in the case of independent private values. In our context, we need to add an extra feature, namely, that the members commit to revealing their signals to each other so that the high bidder knows whether or not it should bid in the seller's auction. It can be shown that the bidding equilibrium in the knockout auction is given by:

$$B^K(s) = \frac{\int_b^s [u(x;x) - R]F(x)dF(x)}{F(s)^2}$$

for $s > b$ and $B^K(s) = 0$ for $s \leq b$. The expected payoff is given in (8).

Theorem 2: Any efficient, incentive compatible cartel mechanism can be implemented by a first-price knockout auction with information sharing.

Theorem 2 extends the result obtained by McAfee and McMillan (1992) for the case of independent private values model to common value models with independent signals.

3 Cartel Enforceability

What is the competitive alternative that determines whether firms want to joint the cartel? One approach to specify an extensive form game in which bidders first decide whether to join the cartel or not and if not, bid competitively in the seller's auction. The strength of this approach is that it deals explicitly with the "leakage" problem that arises when bidders draw inferences about each other's signal based upon their cartel participation decisions. When there are more than two bidders, it also allows for the possibility of cartels containing only some of the bidders. The difficulty is that, if at least one bidder chooses not to join the cartel, the subsequent bidding game involves asymmetric distributions, and the equilibrium for such auctions needs to be computed numerically. As a result, a general characterization of equilibrium payoffs is not easily obtained. Moreover, the nature of the payoffs will depend critically on details about how firms make their cartel participation decision. As a practical matter, the noise in the firms' decisions to bid on tracts and in the cartel formation process makes it unlikely that firms will draw sharp inferences about each other's from their cartel participation decisions.

In this paper we ignore the potential "leakage" issue and model the competitive alternative as a first-price sealed bid auction played under symmetric information. It follows from Milgrom and Weber (1982) that the (symmetric) equilibrium bidding function for a bidder with signal s is given by

$$B^D(s) = R + \frac{\int_a^s [u(x;x) - R]dF(x)}{F(s)}; \tag{9}$$

where a solves the equation

$$E_x[u(a;x)|x \leq a] = R; \tag{10}$$

and where the expectation is taken with respect to x whose cumulative distribution function is $F(x)$. The assumptions on u imply a unique interior solution to (10). Taking into account the boundary values, we formally define a as

$$a = \inf\{a' : E_x[u(a';x)|x \leq a'] \geq R\}.$$

The monotonicity assumption on u implies that $b < a$ as long as $R > 0$. Thus, there is a range of signals where a bidder does not bid in the competitive auction but bids in the cartel mechanism.

Equilibrium profit to the bidder with signal s in the seller's auction is given by

$$\pi^D(s) = \int_0^s [u(s;x) - R]dF(x) - \int_a^s [u(x;x) - R]dF(x) \tag{11}$$

for $s > a$ and it is equal to zero otherwise. A cartel mechanism is called enforceable if

$$\pi_i(s_i) \geq \pi^D(s_i);$$

for all $s_i \in [0,1]$ and $i = 1,2$. The next theorem provides a necessary and sufficient condition under which the efficient, incentive compatible cartel mechanism is enforceable.

Theorem 3: Any efficient, incentive compatible cartel mechanism is enforceable if and only if $\pi^C(1) \geq \pi^D(1)$.

When bidders draw signals between b and a, they do not bid in the competitive auction. This can give rise to an inefficient outcome: if both bidders have such signals, then neither will bid even though the expected value of the tract conditional on the signals drawn exceeds the reserve price. The cost of this inefficiency causes $\pi^D(s)$ to increase at faster than $\pi^C(s)$. Therefore, if the two curves intersect, they can intersect only once, and it suffices to compare the profits of the cartel and competitive mechanisms for the highest type.

The inequality condition in Theorem 3 can be equivalently written as

$$\pi_0 \geq \int_0^{\mu(1)} [u(1,x) - R]dF(x) + \int_b^a [u(x,x) - R]dF(x). \tag{12}$$

Suppose $R \leq u(1,0)$ so that $\mu(1) = 0$. Then condition (12) can be rewritten as

$$\int_b^1 [u(x,x) - R][1 - F(x)]dF(x) \geq \int_b^a [u(x,x) - R]dF(x). \tag{13}$$

When the reserve price is zero, $a = b$. This condition also holds when valuations are private. This leads to the following corollary.

Corollary 1: Suppose either $R = 0$, or $u(s_1,s_2) = U(s_1)$ and $U(1) \geq R$. Then any efficient, incentive compatible cartel mechanism is enforceable.

To illustrate how and why condition (13) may be violated, we provide the following class of examples:

$$u(s_1,s_2) = s_1 + s_2, \quad R \leq 1, \quad \text{and} \quad F(s) = s^\beta, \quad \text{where} \quad \beta > 0.$$

Distributions with higher values of β have less mass on low signals. The unconditional expected tract value is $E(V) = 2\beta/(1+\beta)$, which is increasing in β. The critical cutoff values are

$$b = \frac{1}{2}R, \quad a = \frac{1+\beta}{1+2\beta}R.$$

Note that the difference $a - b$ decreases with β. The probability of observing at least one bid in the seller's auction is $1 - F(a)^2$, which is increasing in β. The average value of the leases that receive at least one bid is given by

$$E(V | s_1 > a, \text{or } s_2 > a) = \frac{2\beta}{1+\beta}\frac{1 - a^{1+2\beta}}{1 - a^{2\beta}}.$$

It can be verified that $E(V|s_1 > a, \text{or } s_2 > a)/E(V)$ decreases with β.

Simple calculation shows that there is a monotonically increasing function $R = g(\beta)$ with $g(0) = 0$ such that inequality (13) is satisfied if and only if $R \leq g(\beta)$. Thus, given any positive reserve price R, if the distribution is sufficiently skewed toward low types (i.e., low value of β), the cartel is not enforceable since some high types will prefer the competitive mechanism. Interestingly, in this example, the signal at which the bidder is indifferent is always below R.

In the case of $R = 1$ and $\beta = 1/2$, we provide the following details. The conditional expected value is given by

$$E_x[u(a, x)|x \leq a] = \int_0^s (s + x - R)\,dF(x)/F(s) = \frac{4}{3}s - 1.$$

It follows that the cutoff point above which the bidder submits a positive amount is

$$a = 3/4.$$

The equilibrium bidding function is given by

$$B^D(s) = \frac{2}{3}s + \frac{\sqrt{3}}{4\sqrt{s}}, \quad s \geq 3/4;$$

The equilibrium expected payoff for a bidder with signal s from the noncooperative bidding is given by

$$\pi^D(s) = \frac{2}{3}s^{3/2} - \frac{\sqrt{3}}{4}, \quad s \geq 3/4.$$

In the case of the first-price knockout auction with information sharing, note that $u(b, b) = R$ implies that $b = 1/2$, which is the cutoff point above which the bidder submits a positive amount. The equilibrium bidding function in this case can be derived as

$$B^K(s) = \frac{s^2 - s + 1/4}{2s}, \quad s \geq 1/2.$$

The equilibrium expected payoff is

$$\pi^K(s) = \frac{2}{3}s^{3/2} + \frac{2}{3}(1 - s)^{3/2} - \frac{11}{24}$$

for all $s \in [1/2, 1]$. It can be verified that the two curves intersect at $s = 0.887$.

4 Pure Common Value

In this section, we study the polar case of a pure common-values model with independent signals. In this case it does not matter what allocation rule is adopted by the cartel since all bidders value the lease identically given the same information. Unfortunately, if $P_i(s_i; s_j)$ is not uniquely determined by the condition of allocation efficiency, the set of incentive compatible, efficient cartel mechanisms is very large and difficult to characterize. We will focus primarily on a comparison of two mechanisms: the first-price knockout tournament with information sharing and the equal sharing (or joint bidding) mechanism with information sharing. The objective is to identify the circumstances under which a cartel may prefer joint bidding to the knockout auctions. Throughout this section, we shall assume that $u(s; t) = s + t$; where s denotes bidder 1's signal and t denotes bidder 2's signal. The substantive restriction here is additivity, which rules out complementaries between the signals. With arbitrary distribution functions, signals can always be relabeled to make bidder valuations linear in the signals.

The interim payoff under equal sharing is given by

$$\pi^E(s) = \frac{1}{2}\int_{\theta(s)}^{1} \mathrm{Max}[s + t - R; 0]dF$$

where $\theta(s) = \mathrm{Max}[0; R - s]$, $s \in [0; 1]$. Both mechanisms are efficient and incentive compatible. They also satisfy ex post budget balance. Thus, even though they distribute the spoils differently across types, the ex ante expected payoffs under the two mechanisms are the same, that is, $E_s\pi^E(s) = E_s\pi^C(s)$: This implies that the two curves have to intersect at least once.

Theorem 4: Suppose $u(s; t) = s + t$ for all $s; t \in [0; 1]$: Then π^C and π^E intersect at least once, but at most twice.

We now compare the two payoffs for the highest type. Using simple calculations, we can verify that $\pi^C(1) > \pi^E(1)$ if and only if

$$\frac{1}{2}\int_{0}^{1} F(x)dx < \int_{\theta(1)}^{1} F(x)^2dx;$$

where $\theta(1) = \mathrm{Max}[0; R - 1]$. The inequality is satisfied for a fairly large class of distribution functions. For instance, it can be easily verified that if $F(x) = x^{\beta}$, for $\beta > 0$, then the inequality always holds. In general, the inequality holds when the distribution has more mass at lower types.

Given this class of distributions, Theorem 4 implies that there are only two possibilities: either the two profit curves intersect once, in which case, the lower types prefer equal sharing to the knockout auction, or the two curves intersect twice, in which case the intermediate types prefer equal sharing to the knockout auction. In both cases, the high types prefer the knockout tournament since it tends to reward them relatively more the equal-sharing mechanism.

The main result of the previous section is the high types on marginal tracts will reject a cartel that uses a knockout tournament. The main result of this section is that high types prefer the knockout tournament to joint bidding. Combining these two results, we obtain the prediction that joint bidding is likely to occur in a common value environments when neither bidder has a very high signal.

5 Empirical Strategy

To be written. Much of the material will be based on a previous draft of this paper (Hendricks and Porter, 1997).

6 Appendix

Proof of Theorem 1: Efficiency implies that $P_i(s_i; s_j) = 1$ if $s_i > b$ and $s_i > s_j > \theta(s_i)$ and equal to 0 otherwise. It then follows from (6) that

$$\frac{d\pi_i(s)}{ds} = \int_{\theta(s)}^{s} v_1(s; x)dF(x)$$

for $s \geq b$ and 0 otherwise. Integrating the above equation yields

$$\pi_i(s) = \pi_0 + \int_b^s \int_{\theta(y)}^{y} v_1(y; x)dF(x)dy$$

for $s \geq b$, where π_0 is a constant. Changing integration order in the above expression yields

$$\pi_i(s) = \pi_0 + K(s) - \int_b^s v(x; x)dF(x);$$

where

$$K(s) = \int_{\theta(s)}^{s} v(s; x)dF(x):$$

To determine π_0, note that, from (4),

$$\pi_i(s) = E_x[P_i(s; x)v(s; x)] + E_x T_i(s; x):$$

It follows that

$$E_x T_i(s;x) = \pi_i(s) - K(s)$$

for $s \geq b$ and is equal to $\pi_i(s)$ if $s \leq b$. Thus,

$$\begin{aligned} E_{(s;x)} T_i(s;x) &= E_s \pi_i(s) - \int_b^{\infty} K(s)\,dF(s) \\ &= \pi_0 - \int_b^{\infty} \int_b^{s} v(x;x)\,dF(x)\,dF(s). \end{aligned}$$

Using integration by parts, we obtain

$$E_{(s;x)} T_i(s;x) = \pi_0 - \int_b^{\infty} v(x;x)[1 - F(x)]\,dF(x).$$

The budget balance condition (2) and symmetry together imply $E_{(s;x)} T_i(s;x) = 0$. It follows that

$$\pi_0 = \int_b^{\infty} v(x;x)[1 - F(x)]\,dF(x).$$

Proof of Theorem 2: We first derive the equilibrium bid function in a first-price knockout auction with information sharing. Let $B(s)$ be a symmetric equilibrium bidding function. If a bidder with signal s submits a bid $B(w)$, its expected profit is

$$\pi(w;s) = \int_0^{w} [v(s;x) - B(w)]\,dF(x) + \int_w^{\infty} B(x)\,dF(x).$$

The bidder chooses w to maximize its expected profit. The first-order condition yields

$$\frac{dB(s)}{ds} = [v(s;s) - 2B(s)]\frac{f(s)}{F(s)}$$

which, together with $B(b) = 0$, yields a solution

$$B^K(s) = \frac{\int_b^s v(x;x)F(x)\,dF(x)}{F(s)^2}$$

for $s > b$ and $B^K(s) = 0$ for $s \leq b$. It can be verified that the second-order condition for the bidder's optimization problem is satisfied. Given the monotonicity of the bidding function and information sharing among bidders after bidding, the allocation outcome from the first-price knockout auction is the same as that in the efficient, incentive compatible cartel mechanism.

Next, we show that the interim transfers are also the same between the two mechanisms. It suffices to show that bidders receive the same profits. Indeed, the equilibrium profit from the knockout auction is

$$\pi^K(s) = \int_0^s [v(s;x) - B^K(s)]dF(x) + \int_s^1 B^K(x)dF(x)$$

for $s > b$ and equal to $\int_b^1 B^K(x)dF(x)$ for $s \leq b$. Integrating by parts yields

$$\int_b^1 B^K(x)dF(x) = \int_b^1 v(x;x)[1 - F(x)]dF(x)$$

which is π_0 as in Theorem 1. Note that for $s > b$,

$$\begin{aligned} \frac{d\pi^K(s)}{ds} &= \int_{\theta(s)}^s v_1(s;x)dF(x) \\ &= \frac{d\pi^C(s)}{ds}. \end{aligned}$$

It follows that $\pi^K(s) = \pi^C(s)$ for all $s \in [0;1]$.

Proof of Theorem 3: First notice that the profit for a bidder with signal s from the seller's auction, $\pi^D(s)$, is equal to zero for $s \leq a$, and strictly increasing in s for $s \geq a$. Moreover, for $s > a$;

$$\frac{d\pi^D(s)}{ds} = \int_0^s u_1(s;x)dF(x).$$

On the other hand, by Theorem 1, the profit for a bidder with signal s from an efficient, incentive cartel mechanism, $\pi^C(s)$, is a positive constant when $s \leq b$, and strictly increasing in s for $s \geq b$. Furthermore, for $s > b$;

$$\frac{d\pi^C(s)}{ds} = \int_{\theta(s)}^s u_1(s;x)dF(x).$$

It follows that, for any $s > b$,

$$\frac{d\pi^D(s)}{ds} \geq \frac{d\pi^C(s)}{ds}.$$

Therefore, $\pi^C(s) \geq \pi^D(s); \forall s \in [0;1]$ if and only if $\pi^C(1) \geq \pi^D(1)$. The claim follows.

Proof of Theorem 4: It suffices to show that the curves intersect at most twice. Let $\Delta(s) = \pi^C(s) - \pi^E(s)$. Consider first the case $R \leq 1$. It follows that

$$\Delta'(s) = \frac{1}{2}F(R - s) - \frac{1}{2} < 0$$

for $s \in [0, R/2]$ and

$$\Delta'(s) = F(s) - \frac{1}{2}F(\theta(s)) - \frac{1}{2}$$

for $s \in [R/2, 1]$. Thus, $\Delta(s)$ is strictly decreasing in $[0, R/2]$, so it intersects the horizontal axis at most once. Moreover, $\Delta(s)$ is strictly convex in $[R/2, 1]$ so it intersects the horizontal axis at most twice. If $\Delta(s)$ intersects the horizontal axis in $[0, R/2]$, it follows from

$$\Delta'(s) = \frac{1}{2}F(R/2) - \frac{1}{2} < 0$$

that $\Delta(s)$ cannot intersect the horizontal axis more than once. The case of $R > 1$ is similar. The claim follows.

7 References

Baldwin, L., R. Marshall, and J-F. Richard (1997), "Bidder collusion at forest service timber sales," Journal of Political Economy, 105, 657-699.

Bulow, J., M. Huang, and P. Klemperer (1999), "Toeholds and takeovers," Journal of Political Economy.

Cramton, P., R. Gibbons, and P. Klemperer (1987), "Dissolving partnership efficiently," Econometrica, 55(3), 615-632.

Cramton, P. and T. Palfrey (1990), "Cartel enforcement with uncertainty about costs," International Economic Review, 31(1), 17-47.

Erickson, E.W. and R.M. Spann (1974), "An analysis of the competitive effects of joint ventures in the bidding for tracts in OCS Offshore Lease Sales," in Hearings before the Special Subcommittee on Integrated Oil Operations of the Senate Committee on Interior and Insular Affairs, Market Performance and Competition in the Oil Industry, U.S. Government Printing Office, Washington, D.C., 1974.

Gaskins, D. and B. Vann (1976), "Joint buying and the seller's return: the case of OCS Lease Sales," in Energy Industry Investigation, Part 1, Hearings before the Subcommittee on Monopolies and Commercial Law of the Committee on the Judiciary, House of Representatives, Ninety-Fourth Congress, U.S. Government Printing House, Washington, D.C., 207-235.

Gilley, O.W., G.V. Karels, and R. Lyon (1985), \Joint ventures and O®shore Oil lease sales, " Economic Inquiry, 24, 321-339.

Graham, D. and R. Marshall (1987), \Collusive bidder behavior at single-object second-price and English auctions," Journal of Political Economy, 95(6), 1217-1239.

Hendricks, K. and R. Porter (1992), \Joint bidding in the Federal OCS auctions," American Economic Review, 82, 506-511

Hendricks, K., J. Pinkse, and R. Porter (1999), \Empirical implications of equilibrium bidding in ¯rst-price, symmetric, common value auctions," working paper .

Klemperer P. (1998), \Auctions with almost common values: The `Wallet Game' and its applications," European Economic Review, 42, 757-769.

Krishna, V. and J. Morgan (1997), \(Anti-) competitive e®ects of joint bidding and bidder restrictions," working paper.

McAfee and McMillan (1992), \Bidding rings," American Economic Review, 82(3), 579-599.

Mailath, G. and P. Zemesky (1991), \Collusion in second price auctions with heterogeneous bidders," Games and Economic Behavior, 3(4), 467-503.

Mead, W. (1968), \The competitive signi¯cance of joint venture," Antitrust Bulletin, 12, 819-849.

Mead, W. and P.E. Sorensen (1980), \Competition and performance in OCS Oil and Gas Lease and lease development, 1954-1969," Final Report to U.S. Geological Survey, Reston, Virginia, Contract No. 14-08-0001-16552.

Milgrom and Weber (1982), \A theory of auctions and competitive bidding," Econometrica, 50, 1089-1122.

Pesendorfer, W. (1998).

Porter, R. (1995), \The role of information in U.S. O®shore Oil and Gas lease auctions," Econometrica, 63, 1-27.

Porter, R. and D. Zona (1993), \Detection of bid rigging in procurement auctions," Journal of Political Economy, 101, 518-538.

Porter, R. and D. Zona (1999), \Ohio school milk markets: An analysis of bidding," RAND Journal of Economics.

Rockwood, A. (1983), \The impact of joint ventures on the market for OCS Oil and Gas Leases," Journal of Industrial Economics, 31, 453-466.