Risk Sharing through Labor Contracts - Risk Aversion, Market Incompleteness and Employment

Sunanda Roy*

December 1, 1999

Abstract

Labor contracts are a way of sharing idiosyncratic production risks between entrepreneurs and workers, especially when such risks are too complex for contingent contracts to be written on them. So it is important to understand how equilibrium employment and wages are affected by risk related factors, such as risk aversion of entrepreneurs and workers, risk sharing opportunities in the economy etc. The paper develops a general equilibrium model with several sectors of production which are subject to idiosyncratic productivity shocks, two inputs - labor and capital - and stock markets which diversify sectoral risks but not completely. We prove the existence of equilibrium for this general model. The model is then parameterized by CRRA utility functions. We prove that the equilibrium employment levels vary inversely with the coefficient of relative risk aversion of agents under certain conditions. Numerical simulations show that over a range of the coefficient employment levels are higher when markets are complete than when they are not. A substantive implication of the comparative static results is that a low paying, productively less efficient alternative to working for private firms may be desirable as an insurance instrument.

*The author wishes to acknowledge a deep debt of gratitude to Michael Magill and Martine Quinzii for their help and suggestions at every stage of this work. Helpful comments from Fernando Zapatero, and some participants at the North American Summer Meetings and the Latin American Meetings of the Econometric Society, 1999, are also gratefully acknowledged.

1 Introduction

Economists agree that labor contracts are a way of sharing idiosyncratic production risks between entrepreneurs and workers, especially when such risks are too complex for insurance contracts to be written on them and traded in organized markets. It is therefore important to understand how (economy wide) equilibrium employment and wages are influenced by risk related factors, in particular, risk aversion of entrepreneurs and workers and risk sharing opportunities elsewhere in the economy, such as in the asset markets.

The paper develops a general equilibrium model to address this issue. The model has several sectors of production which are subject to idiosyncratic productivity shocks, two inputs - labor and capital - and stock markets which help diversify sectoral risks although not completely. We first prove the existence of equilibrium for this general model. Having parameterized the model by CRRA utility functions, we then prove that *under certain conditions*, the equilibrium employment levels vary inversely with the coefficient of relative risk aversion of agents - a non obvious result in a general model with many risky assets of which labor is one.[1] The method that is proposed to prove this result is easily extendible to other types of utility functions from the HARA class, for which closed form solutions for demand functions are easy to find. We choose to present the result only for the CRRA type because of its wide applicability in Macroeconomics and Finance. We then compare results from numerical simulations of this parameterized model and those from a comparable benchmark model in which a complete set of Arrow securities are traded. It is seen that over a range of the risk aversion parameter, employment levels are higher when markets are complete. An explanation is provided for this.

The above results have a practical implication which may be regarded as one of the substantive motivation behind this paper. Workers in this model have an outside alternative to working for private entrepreneurs. This option is productively less efficient from the macro point of view and pays less than the latter. As the relative risk aversion goes up so does the use of this option. Also, over a certain range of the risk parameter, this option is less attractive when markets are complete than when they are not. The paper thus describes a role for a productively inefficient, low paying activity as an insurance

[1] In fact the theorem cannot be proved to be true for another risky asset, namely physical capital.

instrument.

One can think of many examples of such outside options in real economies. Within the context of a developing economy, for example, household production (cottage industry) or self cultivation of land with the help of family labor can be such an alternative to wage labor in the manufacturing sector. Government financed unemployment doles, in developed economies also fits the description. A third example would be working in a less productive state owned firm in an erstwhile command/regulated economy which is trying to privatize. We choose the last example to think of the outside option in this paper, partly because of the current interest in privatization issues and partly because transition/deregulating economies are good examples of incomplete insurance markets. Thus the paper supplements other attempts[2] to explain the slow pace of privatization[3] by providing a rationale for state enterprises from a risk sharing point of view.

The Implicit Contract models were among the earliest attempts to point out the importance of labor contracts as risk sharing devices (see Rosen (1998) for a survey of this literature). The absence of asset markets however, make these essentially partial equilibrium approaches to the issue. Some later Real Business Cycle models with contractual labor (Boldrin and Horvath (1995), Gomme and Greenwood (1995)) remove this limitation. However, the assumption of a representative agent and market completeness simplifies many of the problems associated with risk sharing. The model in this paper is closer in spirit to Dreze's (1991) CAPM model with labor contracts. The CAPM assumption, which for the purposes of this paper is restrictive, is removed and laborers are assumed to have sector specific skills (unlike in Dreze) which make them suitable for employment in only one sector at a time. We consider specificity of labor to be a more reasonable assumption as it explains why labor may be subject to idiosyncratic risks in the first place.

In the model below there are several productive activities producing the same good (income) which we call sectors. Sectors differ from each other in their risk profiles only. State firms have historically organized production up to the point at which our story starts. To increase productive efficiency the

[2] see Arabadjiev (1999) for a survey. Some other explanations are job search costs (Atkeson and Kehoe, 1996), training costs (Arabadjiev, 1999), political compromises (Dewatripont and Roland, (1992), Fernandez and Rodrik (1991) etc.

[3] see Ramamurti and Vernon (1991), Cook and Kirkpatrick (1988), and Commander and Coricelli (1995) for evidence which suggest that the public - private balance has not been dramatically altered in favor of the private in most of these economies, particularly in large scale manufacturing sectors.

government allows risk averse private entrepreneurs to organize production. State firms continue to act as extensions of a benevolent government and employ the residual labor supply not working for private firms. There are positive costs of working for private firms (because of retraining to acquire more skills or maybe because households have to work harder) and zero costs of working for state firms.

Workers and private entrepreneurs have sector specific skills which expose them to sectoral shocks. The extent to which these risks can be shared through the wage contract depends on the labor market structure. Two extreme scenarios may be potentially considered - (i) *Competitive* under which there are innumerable workers and private entrepreneurs in each sector. Competition among firms and workers ensure that wages are equal to marginal product in each state of Nature in equilibrium (Section 2 explains how this may be achieved under incomplete markets). Both parties are exposed to sectoral risks in equal measure under this scenario. (ii) *Monopsonist* under which there is one private firm in each sector. The firm acts as the wage leader by taking the worker's optimal labor supply response into account to decide on the optimal wage contract. In this paper as a first cut and as mathematically the more tractable case, the competitive structure is assumed (for a discussion of the other structure see Roy (1999)).

State firms by contrast to private firms always pay their employees an average output (averaged across sectors) per worker in each state. State employees are thus protected against sectoral but not aggregate shocks. Sectoral shocks can be partially diversified by trading in the financial markets which are not complete. The model below is that of a stock market economy with equities as the only assets. The number of equities (sectors) is less than the number of states, which make asset markets incomplete.

Section 2 lays down the details of the model. Section 3 proves the existence of an equilibrium. Proving existence of an equilibrium in a production model with incomplete markets is difficult in general because the market subspace may be influenced by the action of the agents. In this, we are helped by the competitive assumption for labor markets. The concept of a no-arbitrage equilibrium (NAE) common in exchange based finance models[4] is extended for a production economy to rewrite the Stock

[4] See Magill and Quinzii (1996) for a comprehensive discussion.

Market Equilibrium as a constrained Arrow-Debreu Equilibrium. The method of Debreu (1959) is then adapted to prove existence. The comparative statics results in section 4.1 are proved using the monotonicity properties of the first order conditions of agents and the fixed point technique - an adaptation of a methodology now well known in the literature, following the work of Milgrom and Roberts (1994), Milgrom and Shannon (1994), Villas Boas (1997) and others. Finally in Section 4.2 we report the simulation results on employment under incomplete vis.a vis. complete markets and conclude.

2 The Model

There are two periods 0 and 1, and J sectors of production indexed by $j = 1 \ldots J$. Production in each sector is organized by state and privately owned firms. Production decisions (i.e. employment and investment decisions) are made at date 0. The actual production takes place at date 1. At date 1, Nature subjects each sector j to a total productivity shock η_j^s with probability ρ_s. All sectors produce the same good (income) and differ only in their risk profiles $\boldsymbol{\eta}_j = \{\eta_j^s\}$. Shocks are multiplicative. The production function of private firms in sector j is given by $\boldsymbol{y}_j^p = \{y_j^p(s)\} = \{\eta_j^s f^j(l_j^p, k_j)\}$ where l and k stand for labor and capital.

Private and government firms in any sector are subject to the same productivity shocks but have different state independent production functions. In particular government firms operate with an exogenous and historically given stock of capital.[5] The production function of the state firm in sector j is $\boldsymbol{y}_j^g = \{y_j^g(s)\} = \{\eta_j^s g^j(l_j^g)\}$.

All functions are assumed to be continuous and differentiable. The production functions satisfy,

Assumption 1 *1.* $f^j(l_j, 0) = f^j(0, k_j) = 0$, *Both inputs are essential.*

2. f^j *is strictly concave and* $f_l^j > 0$, $f_k^j > 0$

3. $f^j(l_j, k_j)$ *is linear homogeneous*

4. $f_l^j(0, k_j) = \infty$, $f_l^j(l_j, 0) = 0$, $f_k^j(l_j, 0) = \infty$, $f_k^j(0, k_j) = 0$ *(Inada conditions)*

[5] The state firm's investment decision is not modelled here.

5. $g^j(0) = 0, \;\; g^{j\prime} > 0$

Note that no concavity/convexity assumption is made about g_j at this stage. Such assumptions will be made in Section 4 when they become necessary to prove the comparative static results. The model is interesting only when for k_j above a critical minimum, the level of output in private firms is sufficiently higher than that in state firms given the same employment levels in both such that private firms are able to pay their workers more than the state firms in equilibrium. Two reasons suggested for the lower productivity of workers in state firms and assumed in the model are - firstly, the state firms operate with a fixed and outdated capital stock, and secondly, workers in state firms lack incentives to put in quality effort because of a free rider problem involved in the government wage contract. So,

Assumption 2 *There exists k_j^c such that for* $k_j > k_j^c, \;\; g^j(l_j) < f^j(l_j, k_j)$

The state independent utility function $u(c)$ is assumed to be identical for workers and entrepreneurs with,

Assumption 3 $u'(c) > 0, \;\; u''(c) < 0, \;\; u'(0) \to \infty$

Private firms are initially (at date 0) created and owned by the entrepreneurs. Labor and entrepreneurship are sector specific, which means that each household has the skills to work and each entrepreneur the leadership to organize production in one sector only. There are however numerous identical entrepreneurs and households in each sector j. With regard to labor and stock markets, this implies that entrepreneurs and households perceive their private actions as not influencing the market wage rates or the security pay-off structures. In other words they make private decisions taking the market wage contracts and the market subspace as given. The households and entrepreneurs are said to be having *competitive price perceptions* in both the labor and stock markets when this is the case (see Magill and Quinzii, 1996).

Entrepreneurs maximize expected utility as consumers and total dividends (output minus costs) from production as producers. As initial owners of firms, entrepreneurs make capital investment and employment decisions, k_j and l_j respectively, for their firms at date 0. Capital investment can be financed by selling ownership shares of firm j to households and other entrepreneurs. Trading equities is also a way of sharing sectoral production risks. These are assumed to be the only assets in the economy. Introducing a bond into the model does not change any of the results qualitatively. We shall however need to discuss this issue again in Section 4.2. Labor is hired at date 0 for date 1 and paid a contract $\boldsymbol{W}_j = \{W_j^s\}_{s=1}^S$. Employment levels are thus not state contingent but wages are.

Agent's actions l_j and k_j influence the dividend payments of the jth representative firm. However, agents do not perceive this as causing the market subspace to change and hence their date 1 income streams from securities (in particular from the share of the jth representative firm) to be affected. One way to explain this is to think that a household or an entrepreneur in buying a share of the jth representative firm is actually investing the amount on the income stream offered by the industry. He can always change his portfolio for the numerous firms within the industry to take care of any changes in the dividend stream offered by a single firm without changing his portfolio for the industry.

Let $\boldsymbol{V}$ represent the dividend streams of the sectors, V_s^j the sth row, jth column element, and $\boldsymbol{V}_s$ the sth row of the matrix. Under a competitive market structure in sector j the representative firm makes *zero profits* in equilibrium. This means that output net of wage costs are paid out as dividends to the shareholders by way of return on capital invested in the firm. Another way to think about this is that the firm's budget constraint (revenue $\geq$ wage costs $+$ dividends) must bind in equilibrium. It must not be left with a surplus. Thus in equilibrium,

$$\boldsymbol{V} = \{\boldsymbol{y}_j^p - \boldsymbol{W}^j l_j\}_{j=1}^J \tag{1}$$

Let us define $\boldsymbol{k} = \{k_j\}_{j=1}^J$, $\boldsymbol{l} = \{l_j\}_{j=1}^J$, and $\boldsymbol{W} = \{\boldsymbol{W}^j\}_{j=1}^J$ the vector of private wage contracts. Then in equilibrium, $\boldsymbol{V} = \boldsymbol{V}(\boldsymbol{k}, \boldsymbol{l}, \boldsymbol{W})$ i.e. the dividend streams depend on the actions chosen by the agents. Since production functions are constant returns to scale, they can be written as $\boldsymbol{y}_j^p = \boldsymbol{\eta}^j f_j(\frac{k_j}{l_j}, 1) l_j$. Hence $\boldsymbol{V}(.)$ can *also* be written as $\boldsymbol{V} = \boldsymbol{V}(\frac{\boldsymbol{k}}{\boldsymbol{l}}, \boldsymbol{l}, \boldsymbol{W})$ in equilibrium, where $\frac{\boldsymbol{k}}{\boldsymbol{l}} = \{\frac{k_j}{l_j}\}_{j=1}^J$. We shall choose either representation according as whichever is more convenient.

We are now ready to discuss the optimal programs of the representative household and entrepreneur. For convenience of notation we shall regard date 0 also as another state, namely, the state 0 and $s \in \{0, 1 \ldots S\}$.

Given an optimal choice of k_j (which he makes as a producer), the jth representative entrepreneur is restricted to the following budget set as a consumer:

$$\boldsymbol{B}^j(\boldsymbol{W}^j, \boldsymbol{Q}, \boldsymbol{V}) = \left\{ \boldsymbol{x}^j = (x_0^j, x_1^j..x_S^j) \in R_+^{S+1} \;\middle|\; \begin{array}{lll} x_0^j & \leq & e_0^j + (1-\delta_j^j)Q_j - \sum_{\substack{i=1 \\ i\neq j}}^J \delta_i^j Q_i - k_j \\ x_s^j & \leq & e_s^j + \sum_{i=1}^J \delta_i^j V_s^i \\ & & \forall s \in \{1, \ldots S\} \end{array} \right\} \quad (2)$$

where $\boldsymbol{x}^j$ represents consumption, $\boldsymbol{e}^j = \{e_s^j\}_{s=0}^S$ initial endowment, $\boldsymbol{\delta}^j = \{\delta_i^j\}_{i=1}^J$ ownership shares of representative firms in other sectors, $\boldsymbol{Q} = \{Q_j\}_{j=1}^J$ the price of full ownership of firm j.

Define $\boldsymbol{\Pi}_e^j(\boldsymbol{x}^j) = (\Pi_{0e}^j(\boldsymbol{x}^j), \boldsymbol{\Pi1}_e^j(\boldsymbol{x}^j))$ as the jth entrepreneur's vector of personal valuations of income streams - i.e. his present value vector. Assuming the wage contract $\boldsymbol{W}_j$, the security prices $\boldsymbol{Q}$, and the market subspace or $< \boldsymbol{V} >$ to be given, the entrepreneur j,

1. chooses k_j and l_j^d, to maximize

$$\boldsymbol{\Pi1}_e^j(\boldsymbol{x}^j)(\boldsymbol{y}_j^p - \boldsymbol{W}^j l_j^d) - \Pi_{0e}^j(\boldsymbol{x}^j)k_j$$

as a producer. Since entrepreneurs are initial owners of their firms, production projects are evaluated using their personal valuation vectors.

2. given his choice of k_j above, chooses $\boldsymbol{x}^j$ and $\boldsymbol{\delta}^j$ to maximize

$$U(\boldsymbol{x}^j) = u(x_0^j) + \sum_{s=1}^S \rho_s u(x_s^j)$$

subject to Equation 2, as a consumer.

Since labor is sector specific, the index j can also be used for the representative worker/household working in sector j. The jth representative household has 1 unit of labour which it distributes between the private and the government firms. Working for a government firm is costless and working for a private firm is costly for the laborer, either because they have to invest to acquire more skills or because they have to put in harder efforts. The cost of supplying labour to the private firm is $c_j(l_j)$ where $c_j'(0) = 0$, $c_j'(l_j) > 0$ for $l_j > 0$ and $c_j'' > 0$. We also assume that $c_j'(0) = 0$ which ensures that certain relevant sets are bounded, in Section 3 where we discuss the existence of equilibrium.

Since it is costless for households to work for state firms, it is optimal for them to supply any residual labour to the government. So,

$$l_j^p + l_j^g = 1$$

So from now on, $l_j^p = l_j, \;\; l_j^p = 1 - l_j$.

The jth household's budget set is given by,

$$\boldsymbol{M}^j(\boldsymbol{W}^j, \boldsymbol{G}, \boldsymbol{Q}, \boldsymbol{V}) = \left\{ \boldsymbol{m}^j = (m_0^j, m_1^j .. m_s^j) \in R_+^{S+1} \;\middle|\; \begin{array}{lll} m_0^j & \leq & \omega_0^j - \sum_{i=1}^J \theta_i^j Q_i \\ m_s^j & \leq & \omega_s^j + W_s^j l_j + (1-l_j) G_s \\ & + & \sum_{i=1}^J \theta_i^j V_s^i \\ & & \forall s \in \{1, \ldots S\} \end{array} \right\} \tag{3}$$

where $\boldsymbol{m}^j$ is consumption of household, $\boldsymbol{\omega}^j = \{\omega_s^j\}_{s=0}^S$ initial endowment, $\boldsymbol{G} = \{G_s\}_{s=1}^S$ the state wage contract, and $\boldsymbol{\theta}^j = \{\theta_i^j\}_{i=1}^J$, ownership shares of private firms.

Household j chooses $\boldsymbol{m}^j$, l_j, and $\boldsymbol{\theta}^j$ assuming the private and state wage contracts, $\boldsymbol{W}^j$ and $\boldsymbol{G}$, security prices $\boldsymbol{Q}$ and the $< \boldsymbol{V} >$ to be given, to maximize,

$$U(\boldsymbol{m}^j, l_j) = u(m_0^j) + \sum_{s=1}^S \rho_s u(m_s^j) - c_j(l_j)$$

subject to Equation 3.

We need to discuss the characteristics of the competitive equilibrium labor contracts at this point. There is no enforcement mechanism for the private wage contracts in the model. This means that there is no penalty for the households or firms to renege on a wage contract at date 1. Since there are many firms in each sector, it is therefore possible for a worker in sector j to join one firm at date 0 and leave it to work for another at date 1 for a higher wage. Similarly a firm can attract workers from a competitor by paying higher wages at date 1. It is clear that incentives to renege on a contract drawn at date 0 will generally exist if at date 1 Nature moves before the agents and announces a state. We want however to look at an equilibrium in which at date 1 workers and entrepreneurs do not have any incentive to switch parties with whom they have drawn contracts at date 0. Under this equilibrium all entrepreneurs in any individual sector j must therefore offer the same wage contract to any worker in this sector. Of the set of profit maximizing wage contracts, under the competitive assumption, there is only one which is robust with respect to parties reneging on their date 0 contracts in this sense. We argue in Section 3.2 that this is also the equilibrium wage contract which is robust with respect to opening of spot markets although in our model there are no spot markets for labor.

There are no enforcement problems with the state wage contracts. Wages paid to the employees are the only expenses of the government and the revenue from the state firms its only income. In equilibrium the two must balance. We assume that the Government is interested in maximizing the welfare of its employees all of whom are given the same weights in its objective function. So the government firms distribute the total revenue that is generated from all their activities equally among all the employees by way of wages. Thus the wage profile in the state firms is,

$$\boldsymbol{G}(\boldsymbol{l}) = \frac{\sum_{l=1}^{J} \eta_j^s g^j (1 - l_j)}{\sum_{l=1}^{J} (1 - l_j)} \tag{4}$$

2.1 Stock Market Equilibrium

Let $\boldsymbol{x} = \{\boldsymbol{x}^j\}_{j=1}^{J}$, $\boldsymbol{m} = \{\boldsymbol{m}^j\}_{j=1}^{J}$ represent the consumption allocations, and $\boldsymbol{\theta} = \{\boldsymbol{\theta}^j\}_{j=1}^{J}$, $\boldsymbol{\delta} = \{\boldsymbol{\delta}^j\}_{j=1}^{J}$ the portfolio allocations. Then,

Definition 1 *A Stock Market Equilibrium (SME) with state firms is a 4-tuple* $((\bar{\boldsymbol{x}}, (\bar{\boldsymbol{m}}, \bar{\boldsymbol{l}})), (\bar{\boldsymbol{k}}, \bar{\boldsymbol{l}}^d), (\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\delta}}), (\bar{\boldsymbol{W}}, \bar{\boldsymbol{G}}, \bar{\boldsymbol{V}}, \bar{\boldsymbol{Q}}))$ *of consumption and labor supply plans of entrepreneurs and households, production plans of entrepreneurs, portfolio plans of households and entrepreneurs, private and state wage contracts, dividends and security prices such that,*

1. *For each representative entrepreneur* j*,*

$$(\bar{\boldsymbol{x}^j}, \bar{\boldsymbol{\delta}^j}) = \textit{argmax}\{U(\boldsymbol{x}^j\} \ \textit{ and } \ (\bar{\boldsymbol{x}^j}, \bar{\boldsymbol{\delta}^j}, \bar{k_j}) \in \boldsymbol{B}^j(\bar{\boldsymbol{W}}^j, \bar{\boldsymbol{Q}}, \bar{\boldsymbol{V}})$$

$$(\bar{k_j}, \bar{l_j^d}) = \textit{argmax}\boldsymbol{\Pi}^{\boldsymbol{1}}{}_e^j(\bar{\boldsymbol{x}^j})(\boldsymbol{y}_j^p - \bar{\boldsymbol{W}}^j l_j) - \Pi_{0e}^j(\bar{\boldsymbol{x}^j})k_j,$$

2. *For each representative household* j*,*

$$(\bar{\boldsymbol{m}^j}, \bar{\boldsymbol{\theta}^j}, \bar{l_j}) = \textit{argmax}\{U(\boldsymbol{m}^j, l_j)\} \ \textit{ and } \ (\bar{\boldsymbol{m}^j}, \bar{\boldsymbol{\theta}^j}, \bar{l_j}) \in \boldsymbol{M}^j(\bar{\boldsymbol{W}}^j, \bar{\boldsymbol{G}}, \bar{\boldsymbol{Q}}, \bar{\boldsymbol{V}})$$

3. *At date 1, workers and entrepreneurs have no incentives to switch parties with whom they have drawn contracts at date 0.*

4. *Firms in each sector j make zero profits.*

$$\bar{\boldsymbol{V}} = \{\boldsymbol{y}_j^p(\bar{k}_j, \bar{l}_j) - \bar{\boldsymbol{W}}^j \bar{l}_j\}_{j=1}^J$$

5. *Labor markets clear*

$$\begin{aligned} \bar{\boldsymbol{l}}^d &= \bar{\boldsymbol{l}} \\ \bar{\boldsymbol{G}} &= \boldsymbol{G}(\bar{\boldsymbol{l}}) \end{aligned}$$

6. *Equity markets clear,*

$$\sum_{i=1}^J \bar{\delta}_i^j + \sum_{i=1}^J \bar{\theta}_i^j = 1, \forall j \in \{1, \ldots J\}$$

3 Existence of the Stock Market Equilibrium

To prove the existence of a SME we proceed along the following steps.

We first define in Section 3.1 the concept of a normalized No-Arbitrage Equilibrium (NAE) which is a *constrained Arrow-Debreu Equilibrium*. By this is meant that under this equilibrium the agents are allowed to trade in a complete set of contingent goods as in an Arrow-Debreu set up. But all of them excepting one are allowed to trade only those commodity bundles which lie on a subspace of the whole commodity space. This concept, which has been used before to prove existence of equilibrium in exchange based financial models (see Magill and Quinzii (1996)) needs some explanation. When there are no arbitrage opportunities in financial markets, security prices in equilibrium are equal to the present value of their income streams. When markets are incomplete the present value vectors of agents generally differ in equilibrium. However they are unanimous in their evaluation of the income stream of a marketed security. Thus the present value vector of any agent at his equilibrium consumption can be used to evaluate the income stream of a marketed security. We can then use these *no-arbitrage pricing* equations to eliminate the demand functions for securities $\boldsymbol{\theta}, \boldsymbol{\delta}$ and security prices $\boldsymbol{Q}$, from the description of a Stock Market Equilibrium and replace these by demand functions for goods $\boldsymbol{x}, \boldsymbol{m}$ and state prices which can be chosen to be the present value vector of any agent in equilibrium. The Stock Market Equilibrium looks very much like a Contingent Market Equilibrium with these substitutions. However, as we pointed out, the present value vector of any agent can be chosen to represent the state price vector under this equilibrium. And so equilibrium state prices are not uniquely defined. The usual convention in the literature is to normalize state prices in equilibrium by choosing agent 1's present value vector to be the equilibrium state price vector. His budget set then becomes an *unconstrained Arrow-Debreu* budget set.

Section 3.1 adapts the concept of the normalized NAE to a production model with labor. Section 3.2 proves the existence of a NAE for our model. Then in Section 3.3 we show that a SME is equivalent to a normalized NAE thus defined. Hence as a normalized NAE exists, so does a SME.

3.1 No Arbitrage Equilibrium (NAE)

Let $\boldsymbol{\pi} = \{\pi_s\}_{s=0}^{S}$ denote the state price vector and $\boldsymbol{\pi^1} = \{\pi_s\}_{s=1}^{S}$ and $\boldsymbol{\Pi}_h^j(\boldsymbol{m}^j) = \{\Pi_{h0}^j(\boldsymbol{m}^j), \boldsymbol{\Pi^1}_h^j(\boldsymbol{m}^j)\}$ the present value vector of household j. Also let us denote by $\boldsymbol{m_1}^j$ and $\boldsymbol{x_1}^j$ the date 1 consumption vectors. When there are no arbitrage opportunities in the financial markets, there exists a $\boldsymbol{\pi} \in R_+^{S+1}$ such that,

$$\pi_0 \boldsymbol{Q} = \boldsymbol{\pi^1 V} \tag{5}$$

We can use these equations to eliminate the security prices from the the budget constraints of the jth household and write these as,

$$\begin{aligned}
\pi_0(m_0^j - \omega_0^j) + \sum_{s=1}^{S} \pi_s(m_s^j - \omega_s^j - W_s^j l_j - (1-l_j)G_s) &= 0 \\
m_s^j - \omega_s^j - W_s^j l_j - (1-l_j)G_s &= \boldsymbol{V}_s \boldsymbol{\theta}^j, \ \forall s \in \{1, \dots S\}
\end{aligned}$$

The date 0 budget constraint in the above expression is the Arrow-Debreu contingent market budget set. Since the jth household is free to choose any portfolio (short sales are allowed) the date 1 constraints merely imply that the "net trade" vector (demand minus endowments minus earnings from production) must lie in the market subspace $< \boldsymbol{V} >$. Thus the date 1 budget constraints of the households can be written without any explicit reference to the portfolio variables $\boldsymbol{\theta}^j$ and the NA budget set for household j can be written as,

$$\boldsymbol{M}^j_{na}(\boldsymbol{\pi}, \boldsymbol{W}^j, \boldsymbol{G}, \boldsymbol{V}) = \left\{ \boldsymbol{m}^j \in R^{S+1}_+ \;\middle|\; \begin{array}{lcc} \pi_0(m^j_0 - \omega^j_0) & & \\ + \sum_{s=1}^S \pi_s(m^j_s - \omega^j_s - W^j_s l_j - (1-l_j)G_s) & = & 0 \\ \{m^j_s - \omega^j_s - W^j_s l_j - (1-l_j)G_s\} & \in & < \boldsymbol{V} > \end{array} \right\}$$

which is thus a constrained Arrow-Debreu budget set with constraints on date 1 trade. The no-arbitrage budget set for the jth entrepreneur can be written as,

$$\boldsymbol{B}^j_{na}(\boldsymbol{\pi}, \boldsymbol{W}^j, \boldsymbol{V}) = \left\{ \boldsymbol{x}^j \in R^{S+1}_+ \;\middle|\; \begin{array}{lcl} \pi_0(x^j_0 + k_j - e^j_0) + \sum_{s=1}^S \pi_s(x^j_s - e^j_s) & = & \boldsymbol{\pi}^1 \boldsymbol{V}^j(\boldsymbol{k}, \boldsymbol{l}, \boldsymbol{W}) \\ \{x^j_s - W^j_s\} & \in & < \boldsymbol{V} > \end{array} \right\}$$

It is to be noted at this point that in exchange based financial models $< \boldsymbol{V} >$ is a fixed subspace of R^{S+1}_+ because security pay-offs are exogenous. In production models the market subspace is endogenous.

When markets are incomplete i.e dim$(\boldsymbol{V}) < S$, the no-arbitrage price equations 5 cannot uniquely solve for the state price vector given the equilibrium security prices. There are thus in general many state price vectors associated with a no-arbitrage equilibrium allocation.[6] To uniquely define the state prices in equilibrium we shall follow the usual convention and choose the present-value vector of the first household in equilibrium to represent them. This assumption converts the first household's budget set into a non-constrained Arrow-Debreu budget set. The definition of this normalized No-Arbitrage Equilibrium for our production model is,

Definition 2 *A normalized NAE for a stock market economy with state firms is a 3-tuple* $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{m}}, \bar{\boldsymbol{l}}), (\bar{\boldsymbol{k}}, \bar{\boldsymbol{l}}^d),$ $(\bar{\boldsymbol{W}}, \bar{\boldsymbol{\pi}}, \bar{\boldsymbol{G}}, \bar{\boldsymbol{V}}))$ *of consumption and labor supply plans of entrepreneurs and households, production plans of entrepreneurs, and contracts and state prices, such that,*

1. *For household 1,*

$$(\bar{\boldsymbol{m}}^1, \bar{l}_1) \in \textit{argmax}\{U(\boldsymbol{m}^1, l_1) \;|\; (\bar{\boldsymbol{m}}^1, \bar{l}_1) \;\; \textit{satisfies}$$

[6] see Magill and Quinzii, 1996

$$\bar{\pi}_0(m_0^1 - \omega_0^1) + \sum_{s=1}^{S} \bar{\pi}_s(m_s^1 - \omega_s^1 - \bar{W}_s^1 l_1 - (1 - l_1)\bar{G}_s) = 0)\}$$

2. *For all other households $j \in \{2, \ldots J\}$*

$$(\bar{\boldsymbol{m}}^j, \bar{l}_j) \in \textit{argmax}\{U(\boldsymbol{m}^j, l_j) \ | \ (\bar{\boldsymbol{m}}^j, \bar{l}_j) \in \boldsymbol{M}_{na}^j(\bar{\boldsymbol{\pi}}, \bar{\boldsymbol{W}}^j, \bar{\boldsymbol{G}}, \bar{\boldsymbol{V}}$$

3. *For entrepreneurs $j \in \{1, \ldots J\}$*

$$(\bar{\boldsymbol{x}}^j) \in \textit{argmax}\{U(x^j) \ | \ (\bar{\boldsymbol{x}}^j, \bar{k}_j) \in \boldsymbol{B}_{na}^j(\bar{\boldsymbol{\pi}}, \bar{\boldsymbol{W}}^j, \bar{\boldsymbol{V}}\}$$

4. $$(\bar{k}_j, \bar{l}_j^d) = \textit{argmax}\{\bar{\boldsymbol{\pi}}^{\boldsymbol{1}}(\boldsymbol{y}_j^p - \bar{\boldsymbol{W}}^j l_j) - \bar{\pi}_0 k_j\}$$

5. *At date 1, workers and entrepreneurs have no incentives to switch parties with whom they have drawn contracts at date 0.*

6. *Firms in each sector j make zero profits.*

$$\bar{\boldsymbol{V}} = \{\boldsymbol{y}_j^p(\bar{k}_j, \bar{l}_j) - \bar{\boldsymbol{W}}^j \bar{l}_j\}_{j=1}^J$$

7. *Labor markets clear*

$$\begin{aligned} \bar{\boldsymbol{l}}^d &= \bar{\boldsymbol{l}} \\ \bar{\boldsymbol{G}} &= \boldsymbol{G}(\bar{\boldsymbol{l}}) \end{aligned}$$

8. *Markets clear at dates 0 and 1.*

$$\begin{aligned} \textstyle\sum_{j=1}^J (\bar{m}_0^j + \bar{x}_0^j + \bar{k}_j - \omega_0^j - e_0^j) &= 0 \\ \textstyle\sum_{j=1}^J (\bar{\boldsymbol{m}}_{\boldsymbol{1}}^j + \bar{\boldsymbol{x}}_{\boldsymbol{1}}^j - \boldsymbol{\omega}_{\boldsymbol{1}}^j - \boldsymbol{e}_{\boldsymbol{1}}^j - \boldsymbol{y}_j^p(\bar{k}_j, \bar{l}_j) - \boldsymbol{y}_j^g(1 - \bar{l}_j)) &= 0 \end{aligned}$$

It should be noted that in Part 4 of Definition 2 we use state prices rather than the personal present value vector of entrepreneur j to define the profit function of his firm. We show in the subsection below that this is valid because the expression $(\boldsymbol{y}_j^p - \bar{\boldsymbol{W}}^j l_j)$ belongs to the market subspace in equilibrium.

3.2 Existence of a normalized NAE

We now derive the set of equations that describe a NAE for a stock market economy.

Let $\mathcal{P} = \{\boldsymbol{\pi}\}$ denote the set of state price vectors.

Since utility functions are strictly concave and the NAE budget set of household 1 is convex, the household 1's demand and labor supply as functions of state prices and contracts are given by,

$$(\boldsymbol{m}^1(\boldsymbol{\pi}, \boldsymbol{W}^1, \boldsymbol{G}), l_1(\boldsymbol{\pi}, \boldsymbol{W}^1, \boldsymbol{G})) = \text{argmax}\{U(\boldsymbol{m}^1, l_1) \\ \mid \ \pi_0(m_0^1 - \omega_0^1) + \sum_{s=1}^{S} \pi_s(m_s^1 - \omega_s^1 - W_s^1 l_1 - (1 - l_1)G_s) = 0\}$$

Since $\boldsymbol{M}_{na}^1(.)$ is continuous in $(\boldsymbol{\pi}, \boldsymbol{W}^1, \boldsymbol{G})$ the functions $\boldsymbol{m}^1(.)$ and $l_1(.)$ which are the set of maximal elements in $M_{na}^1(.)$ are continuous.

The budget sets for all other households $j \in \{2, \ldots J\}$ are constrained Arrow-Debreu and is the intersection of the unconstrained budget set and $< \boldsymbol{V} >$ which is given to the individual agents under competitive price perceptions. The constrained budget set is thus an intersection of the unconstrained Arrow-Debreu budget set and a given subspace of R_+^{S+1} and hence is convex. Thus the demand and labor supply of household $j, j \neq 1$ are functions defined by,

$$\begin{aligned}(\boldsymbol{m}^j(\boldsymbol{\pi}, \boldsymbol{W}^j, \boldsymbol{G}, \boldsymbol{V}), l_j(\boldsymbol{\pi}, \boldsymbol{W}^j, \boldsymbol{G}, \boldsymbol{V})) \;&=\; \text{argmax}\{U(\boldsymbol{m}^j, l_j) \\ &\mid \quad \pi_0(m_0^j - w_0^j) + \sum_{s=1}^{S} \pi_s(m_s^j - \omega_s^j - W_s^j l_j - (1 - l_j)G_s) = 0 \\ &\quad\;\; \{m_s^j - \omega_s^j - W_s^j l_j - (1 - l_j)G_s\} \in < \boldsymbol{V} >\} \end{aligned} \tag{6}$$

The household's demand and supply functions are continuous because the budget sets are continuous in $\boldsymbol{\pi}, \boldsymbol{W}^j, \boldsymbol{G}$ and the dividend streams $\boldsymbol{V}^j$.

Given his optimal choice of k_j, the entrepreneur j's utility maximization program yields his demand functions.

$$\boldsymbol{x}^j(\boldsymbol{\pi}, \boldsymbol{W}^j, \boldsymbol{V}) \;=\; \text{argmax}\{U(x^j) \ \mid \ \pi_0(x_0^j + k_j - e_0^j)$$

$$+\sum_{s=1}^{S}\pi_s(x_s^j - e_s^j) - \boldsymbol{\pi}^1\boldsymbol{V}^j(\boldsymbol{k},\boldsymbol{l},\boldsymbol{W}) = 0$$
$$\{x_s^j - e_s^j\} \in < \boldsymbol{V} >\} \tag{7}$$

These functions are continuous because the budget sets are continuous in $\boldsymbol{\pi}, \boldsymbol{W}^j$ and $\boldsymbol{V}^j$. The profit maximization program yields the optimal capital/labor ratio and the first order condition that the optimal wage contract must satisfy.

$$\sum_{s=1}^{S}\pi_s\eta_j^s f_k^j(\frac{k_j}{l_j^d}) - \pi_0 = 0 \tag{8}$$
$$\sum_{s=1}^{S}\pi_s(\eta_j^s f_l^j(\frac{k_j}{l_j^d}) - W_s^j) = 0 \tag{9}$$

As defined, the NAE wage contract must be such that parties to it must not have the incentive to switch at date 1. We now claim that the *only* contract which maximizes net earnings for any entrepreneur in sector j, and which is robust with respect to such incentives is,

$$\boldsymbol{W}^j = \boldsymbol{\eta}^j f_l^j(\frac{k_j}{l_j^d}) \tag{10}$$

It is easy to see that if all firms in sector j pay this there will be no incentives either for the workers or for the firms to withdraw from an existing contract. To understand why this is the *only* contract with this feature, note firstly that all firms in sector j must pay the same contract so that the incentives to withdraw do not exist. Combine this with the feature of competitive markets which allow for free entry and exit of firms and households and our claim is true.

Equation 8 yields the optimal capital/labor ratio employed as a function of state prices,

$$\frac{k_j}{l_j^d} = \kappa_j(\boldsymbol{\pi}) \tag{11}$$

Combining this with the labor market clearing condition $l_j = l_j^d$, i.e. assuming that the entrepreneur decides to employ all the labor that is offered at given state prices, Equation 11 alternatively yields the optimal capital stock k_j of firm j as a function of state prices and labor supply.

$$k_j = \kappa_j(\boldsymbol{\pi})l_j(.) \tag{12}$$

Equation 10 yields the optimal wage contract as a continuous function of the capital/labor ratio employed by the firm j or after substitution of Equation 11 into Equation 10, as a function of $\boldsymbol{\pi}$.

$$\boldsymbol{W}^j = \tilde{\boldsymbol{W}}^j(\frac{k_j}{l_j}) = \tilde{\boldsymbol{W}}^j(\kappa(\boldsymbol{\pi})) = \boldsymbol{W}^j(\boldsymbol{\pi}) \tag{13}$$

REMARK 1: Since production functions are constant returns to scale, $(\boldsymbol{y}_j^p - \bar{\boldsymbol{W}}^j l_j) = \boldsymbol{\eta}^j f_k^j(\frac{k_j}{l_j^d})$ where $f_k^j(\frac{k_j}{l_j^d})$ is a scaler, under competitive assumptions and optimal behavior of entrepreneurs. Also note that $\boldsymbol{\eta}^j$ is marketed in equilibrium since $< \bar{\boldsymbol{V}} > = < \boldsymbol{\eta} >$. Thus all agents agree on the equilibrium valuation of $(\boldsymbol{y}_j^p - \bar{\boldsymbol{W}}^j l_j)$. This means that $\bar{\boldsymbol{\pi}}^{\mathbf{1}}(\boldsymbol{y}_j^p - \bar{\boldsymbol{W}}^j l_j) - \bar{\pi}_0 k_j = \boldsymbol{\Pi}^{\mathbf{1}}{}_e^j(\bar{\boldsymbol{x}}^j)(\boldsymbol{y}_j^p - \bar{\boldsymbol{W}}^j l_j) - \Pi_{e0}^j(\bar{\boldsymbol{x}}^j)k_j$. Hence to define the profit function of the representative firm in sector j, (in Definition 2), using state prices or using the personal present value vector of the entrepreneur j are equivalent.

The aggregate excess demand functions are defined by,

$$\begin{aligned}
&\sum_{j=1}^{J} \boldsymbol{m}^j(\boldsymbol{\pi}, \boldsymbol{W}^j, \boldsymbol{G}, \boldsymbol{V}) + \boldsymbol{x}^j(\boldsymbol{\pi}, \boldsymbol{W}^j, \boldsymbol{V}) - \boldsymbol{y}_j^p(k_j, l_j) - \boldsymbol{y}_j^g(1 - l_j) - \boldsymbol{\omega}^j - \boldsymbol{e}^j \\
&= \sum_{j=1}^{J} \boldsymbol{m}^j(\boldsymbol{\pi}, \boldsymbol{W}^j, \boldsymbol{G}, \boldsymbol{V}) + \boldsymbol{x}^j(\boldsymbol{\pi}, \boldsymbol{W}^j, \boldsymbol{V}) - \boldsymbol{y}_j^p(k_j(\boldsymbol{\pi}, l_j(\boldsymbol{\pi}, \boldsymbol{W}^j, \boldsymbol{G}, \boldsymbol{V})), l_j(.)) \\
&\qquad - \boldsymbol{y}_j^g(1 - l_j(.)) - \boldsymbol{\omega}^j - \boldsymbol{e}^j \\
&= \boldsymbol{Z}(\boldsymbol{\pi}, \boldsymbol{W}, \boldsymbol{G}, \boldsymbol{V})
\end{aligned} \tag{14}$$

where $\boldsymbol{Z}(\boldsymbol{\pi}, \boldsymbol{W}, \boldsymbol{G}, \boldsymbol{V}) = \{Z_s(\boldsymbol{\pi}, \boldsymbol{W}, \boldsymbol{G}, \boldsymbol{V})\}_{s=1}^S$. The aggregate excess demand functions are continuous.

Define $\boldsymbol{\kappa}(\boldsymbol{\pi})\boldsymbol{l} = \{\kappa_j(\boldsymbol{\pi})l_j\}_{j=1}^J$, $\boldsymbol{l}(\boldsymbol{\pi}, \boldsymbol{W}, \boldsymbol{G}, \boldsymbol{V}) = \{l_j(\boldsymbol{\pi}, \boldsymbol{W}, \boldsymbol{G}, \boldsymbol{V})\}_{j=1}^J$, and $\boldsymbol{W}(\boldsymbol{\pi}) = \{\boldsymbol{W}^j(\boldsymbol{\pi})\}_{j=1}^J$.

Then a No-Arbitrage Equilibrium is a 6-tuple $(\bar{\boldsymbol{\pi}}, \bar{\boldsymbol{l}}, \bar{\boldsymbol{k}}, \bar{\boldsymbol{W}}, \bar{\boldsymbol{G}}, \bar{\boldsymbol{V}})$ such that

$$\boldsymbol{Z}(\bar{\boldsymbol{\pi}}, \bar{\boldsymbol{W}}, \bar{\boldsymbol{G}}, \bar{\boldsymbol{V}}) \quad \leq \quad 0 \tag{15}$$

$$
\begin{aligned}
\bar{\boldsymbol{l}} - \boldsymbol{l}(\bar{\boldsymbol{\pi}}, \bar{\boldsymbol{W}}, \bar{\boldsymbol{G}}, \bar{\boldsymbol{V}}) &= 0 && (16)\\
\bar{\boldsymbol{k}} - \boldsymbol{\kappa}(\bar{\boldsymbol{\pi}})\bar{\boldsymbol{l}} &= 0 && (17)\\
\bar{\boldsymbol{W}} - \boldsymbol{W}(\bar{\boldsymbol{\pi}}) &= 0 && (18)\\
\bar{\boldsymbol{G}} - \boldsymbol{G}(\bar{\boldsymbol{l}}) &= 0 && (19)\\
\bar{\boldsymbol{V}} - \boldsymbol{V}(\bar{\boldsymbol{k}}, \bar{l}, \bar{W}) &= 0 && (20)
\end{aligned}
$$

The excess demand functions defined above satisfies Walras' Law. There are in effect four types of "agents" in the stock market economy whose budget constraints must be satisfied given their optimal choices, even when the economy is not in equilibrium. These are the workers, the entrepreneurs, the government and the firms. Thus so long as the government has a balanced budget Equation 4 must be true for any $\boldsymbol{l}$. Similarly the firm's output must be at least as large as the sum of wages and dividends for any $\boldsymbol{W}$, $\boldsymbol{l}$ and $\boldsymbol{k}$. Thus Walras' Law in this model implies that for any $(\boldsymbol{\pi}, \boldsymbol{W}, \boldsymbol{k}, \boldsymbol{l})$,

$$
\sum_{s=0}^{S} \pi_s Z_s(\boldsymbol{\pi}, \boldsymbol{W}, \boldsymbol{G}(\boldsymbol{l}), \boldsymbol{V}(\boldsymbol{k}, \boldsymbol{l}, \boldsymbol{W}) + \pi_0(\sum_{j=1}^{J} k_j) \leq 0
$$

$$
\Rightarrow \sum_{s=0}^{S} \pi_s Z_s(\boldsymbol{\pi}, \boldsymbol{W}, \boldsymbol{G}(\boldsymbol{l}), \boldsymbol{V}(\boldsymbol{k}, \boldsymbol{l}, \boldsymbol{W}) \leq 0
$$

since capital and state prices are always assumed to be non-negative in our model.

To prove the existence of a NAE, we shall work with a reduced form of the system described by Equations 15- 20. Equations 17, 18 and 20 can be used to eliminate $\boldsymbol{W}, \boldsymbol{V}$ and $\boldsymbol{k}$ from the set. The reduced form set of equations which determine the NAE $(\bar{\boldsymbol{\pi}}, \bar{\boldsymbol{l}}, \bar{\boldsymbol{G}})$ are given by,

$$
\begin{aligned}
\tilde{\boldsymbol{Z}}(\bar{\boldsymbol{\pi}}, \bar{\boldsymbol{l}}, \bar{\boldsymbol{G}}) &\leq 0 && (21)\\
\bar{\boldsymbol{l}} - \tilde{\boldsymbol{l}}(\bar{\boldsymbol{\pi}}, \bar{\boldsymbol{G}}) &= 0 && (22)\\
\bar{\boldsymbol{G}} - \boldsymbol{G}(\bar{\boldsymbol{l}}) &= 0 && (23)
\end{aligned}
$$

Since the budget sets of the agents are invariant with respect to a scaler multiple of $\boldsymbol{\pi}$, the excess demand functions are homogeneous of degree zero in state prices. Thus we can choose an appropriate normalization for $\boldsymbol{\pi}$. We choose $\mathcal{P}$ such that,

$$\mathcal{P} = \{\boldsymbol{\pi} \in R_+^{S+1} \left| \sum_{s=0}^{S} \pi_s = 1 \right.\}$$

$\mathcal{P}$ is compact and convex.

As total available labor in each sector has been normalized to 1, $\boldsymbol{l} \in \mathcal{L} = [0,1]^J$ which is compact and convex. Since markets have to clear at date 0, there is an upper bound of $\sum_{j=1}^{J}(\omega_0^j + e_0^j)$ for each k_j. Thus production functions for each sector is bounded above which places a lower bound on excess demand functions. Next note that $\boldsymbol{G}(\boldsymbol{l})$ has an upper bound given by $\boldsymbol{l} = \{0, 0 \ldots 0\}$ so that $\mathcal{G} = \{\mathcal{G}\}$ is a closed cube of length $\boldsymbol{G}_{max} = \boldsymbol{G}(\mathbf{0})$ in R_+^S.

REMARKS 2 We could have reduced the set of equations further by eliminating $\boldsymbol{l}$ and $\boldsymbol{G}$ from above and defined the equilibrium on the set of prices $\boldsymbol{\pi}$ only. However for the comparative statics propositions in section 4, looking at the fixed point (equilibrium) in the larger space $\mathcal{L} \times \mathcal{P} \times \mathcal{G}$ becomes a convenient tool. Because employment levels appear explicitly in the fixed point vector under this representation, we are able to derive comparative statics of employment levels across different equilibria by comparing the fixed point vectors directly.

Proposition 1 *There exists a normalized NAE for the stock market economy.*

Proof:

We use a standard technique in the literature (see Varian, 1986).

Define $\Omega = \mathcal{L} \times \mathcal{P} \times \mathcal{G}$ which is compact and convex since component sets are. Define the function,

$$\mu(\boldsymbol{\pi}, \boldsymbol{l}, \boldsymbol{G}) = \frac{\boldsymbol{\pi} + \max(\mathbf{0}, \boldsymbol{Z}(\boldsymbol{\pi}, \boldsymbol{l}, \boldsymbol{G}))}{1 + \sum_{s=0}^{S} \max(0, Z_s(\boldsymbol{\pi}, \boldsymbol{l}, \boldsymbol{G}))} \tag{24}$$

$\mu(\boldsymbol{\pi}, \boldsymbol{l}, \boldsymbol{G})$ maps $\mathcal{P}$ into itself and is continuous.

Define the function, $\psi(\omega = (\boldsymbol{l}, \boldsymbol{\pi}, \boldsymbol{G})) = (\tilde{\boldsymbol{l}}(\boldsymbol{\pi}, \boldsymbol{G}), \mu(\boldsymbol{\pi}, \boldsymbol{l}, \boldsymbol{G}), \boldsymbol{G}(\boldsymbol{l}))$ from Ω to itself. ψ is continuous. Since Ω is compact and convex, all conditions of Brower's theorem are satisfied. Hence ψ has a fixed point, $\omega^* = (\boldsymbol{l}^*, \boldsymbol{\pi}^*, \boldsymbol{G}^*)$. It is straightforward to show that this is an equilibrium (see Varian). In particular, if consumption in every state is *desirable* the equilibrium is interior in $\boldsymbol{\pi}$. Δ

3.3 Equivalence of normalized NAE and SME

We need to prove that,

Lemma 1 *(i) If* $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{m}}, \bar{\boldsymbol{l}}), (\bar{\boldsymbol{k}}, \bar{\boldsymbol{l}}^d), (\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\delta}}), (\bar{\boldsymbol{W}}, \bar{\boldsymbol{G}}, \bar{\boldsymbol{Q}}, \bar{\boldsymbol{V}}))$ *is a SME, and if* $\bar{\boldsymbol{\Pi}}_h^1$ *is agent 1's present value vector under this equilibrium, then* $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{m}}, \bar{\boldsymbol{l}}), (\bar{\boldsymbol{k}}, \bar{\boldsymbol{l}}^d), (\bar{\boldsymbol{W}}, \bar{\boldsymbol{G}}, \bar{\boldsymbol{\Pi}}_h^1)$ *is a normalized NAE.*

(ii) If $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{m}}, \bar{\boldsymbol{l}}), (\bar{\boldsymbol{k}}, \bar{\boldsymbol{l}}^d), (\bar{\boldsymbol{W}}, \bar{\boldsymbol{G}}, \bar{\boldsymbol{\pi}}, \bar{\boldsymbol{V}}))$ *is a normalized NAE then there exist portfolios* $(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\delta}})$ *and security prices* $\bar{\boldsymbol{Q}} = \bar{\boldsymbol{\pi}}^1 V(\bar{\boldsymbol{k}}, \bar{\boldsymbol{l}}, \bar{\boldsymbol{W}})$ *such that* $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{m}}, \bar{\boldsymbol{l}}), (\bar{\boldsymbol{k}}, \bar{\boldsymbol{l}}^d), (\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\delta}}), (\bar{\boldsymbol{W}}, \bar{\boldsymbol{G}}, \bar{\boldsymbol{Q}}))$ *is a SME.*

Proof:

(i) Note first that under a SME (from the first order conditions of the entrepreneurs) the equilibrium private wage contract in sector j is also equal to the marginal product of labor at each state, i.e. $\boldsymbol{W}^j = \boldsymbol{\eta}^j f_l^j(\frac{k_j}{l_j^d})$ (as under a NAE). Since $\bar{\boldsymbol{\Pi}}_h^1$ and $\bar{\boldsymbol{V}}$ are in equilibrium we have $\bar{\boldsymbol{\Pi}}_h^1 = \boldsymbol{\Pi}_h^1(\bar{\boldsymbol{m}}^1, \bar{l}_1)$ and $\boldsymbol{V}(\bar{\boldsymbol{k}}, \bar{\boldsymbol{l}}, \bar{\boldsymbol{W}}) = \bar{\boldsymbol{V}}$. Since $\bar{\boldsymbol{m}}^1 - \omega^1 - \bar{\boldsymbol{W}}^1 \bar{l}_1 - (1 - \bar{l}_1)\bar{\boldsymbol{G}} = \bar{\boldsymbol{V}}\bar{\boldsymbol{\theta}}^1$ and from the first order conditions of the 1st household $\bar{\boldsymbol{\Pi}}_h^1[-\bar{\boldsymbol{Q}}, \bar{V}]^T = 0$ then $\bar{\Pi}_{h0}^1(\bar{m}_0^1 - \omega_0^1) + \sum_{s=1}^S \bar{\Pi}_{hs}^1(\bar{m}_s^1 - \omega_s^1 - \bar{W}_s^1 \bar{l}_1 - (1 - \bar{l}_1)\bar{G}_s) = 0$. Therefore $\bar{\boldsymbol{m}}^1 \in \boldsymbol{M}_{na}^1(\bar{\boldsymbol{\Pi}}_h^1, \bar{\boldsymbol{W}}^1, \bar{\boldsymbol{G}})$. Since $\boldsymbol{\Pi}_h^1(\bar{\boldsymbol{m}}^1, \bar{l}_1) = \bar{\boldsymbol{\Pi}}_h^1$, the first order conditions for maximizing $U(\boldsymbol{m}^1, l_1)$ over $\boldsymbol{M}_{na}^1(\bar{\boldsymbol{\Pi}}^1, \bar{\boldsymbol{W}}^1, \bar{\boldsymbol{G}})$ are satisfied at $(\bar{\boldsymbol{m}}^1, \bar{l}_1)$. At private and state wage contracts $(\bar{\boldsymbol{W}}, \bar{\boldsymbol{G}})$, $\boldsymbol{M}_{na}^j(\bar{\boldsymbol{\Pi}}_h^1, \bar{\boldsymbol{W}}^j, \bar{\boldsymbol{G}}, \bar{\boldsymbol{V}}) = \boldsymbol{M}^j(\bar{\boldsymbol{W}}^j, \bar{\boldsymbol{G}}, \bar{\boldsymbol{Q}}, \bar{\boldsymbol{V}})$ for households $j \in \{2, \ldots J\}$ so for the households $(\bar{\boldsymbol{m}}^j, \bar{l}_j)$ are optimal in $\boldsymbol{M}_{na}^j$. Since the profit function for entrepreneur j, $\boldsymbol{\Pi}_e^j(\bar{\boldsymbol{x}}^j)(\boldsymbol{y}_j^p - \bar{\boldsymbol{W}}^j l_j)$ is identical to $\bar{\boldsymbol{\Pi}}_h^1(\boldsymbol{y}_j^p - \bar{\boldsymbol{W}}^j l_j)$ by no-arbitrage and the fact that $(\boldsymbol{y}_j^p - \bar{\boldsymbol{W}}^j l_j)$ is a marketed security for $\bar{\boldsymbol{W}}^j$, the pair $(\bar{k}_j, \bar{l}_j^d)$ maximizes profits for entrepreneur j under state prices $\bar{\boldsymbol{\Pi}}_h^1$. For $k_j = \bar{k}_j$, $\boldsymbol{B}_{na}^j(\bar{\boldsymbol{\Pi}}_h^1, \bar{\boldsymbol{W}}^j, \bar{\boldsymbol{V}}) = \boldsymbol{B}^j(\bar{\boldsymbol{W}}^j, \bar{\boldsymbol{Q}}, \bar{\boldsymbol{V}})$ for entrepreneurs $j \in \{1, \ldots J\}$. So for the entrepreneurs $(\bar{\boldsymbol{x}}^j)$ are optimal in $\boldsymbol{B}_{na}^j$. Since $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{m}})$ are clearly feasible $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{m}}, \bar{\boldsymbol{l}}), (\bar{\boldsymbol{k}}, \bar{\boldsymbol{l}}^d), (\bar{\boldsymbol{W}}, \bar{\boldsymbol{\Pi}}_h^1, \bar{\boldsymbol{G}}))$ is a NAE.

(ii) $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{m}}, \bar{\boldsymbol{l}}), (\bar{\boldsymbol{k}}, \bar{\boldsymbol{l}}^d), (\bar{\boldsymbol{W}}, \bar{\boldsymbol{\pi}}, \bar{\boldsymbol{G}}))$ is a normalized NAE. Define $\boldsymbol{V}(\bar{\boldsymbol{k}}, \bar{\boldsymbol{l}}, \bar{\boldsymbol{W}}) = \bar{\boldsymbol{V}}$. Then the equilibrium present value vector of the 1st household is given by $\bar{\boldsymbol{\Pi}}_h^1 = \boldsymbol{\Pi}_h^1(\bar{\boldsymbol{m}}^1, \bar{l}_1) = \bar{\boldsymbol{\pi}}$. Define $\bar{\boldsymbol{Q}} = \frac{\bar{\boldsymbol{\Pi}}_{h1}^1}{\bar{\Pi}_0^1}\bar{\boldsymbol{V}}$, and $\bar{\boldsymbol{\theta}}^j$ as solutions of $(\bar{\boldsymbol{m}}_1^j - \boldsymbol{\omega_1}^j - \bar{\boldsymbol{W}}^j \bar{l}_j - (1 - \bar{l}_j)\bar{\boldsymbol{G}}) = \bar{\boldsymbol{V}}\boldsymbol{\theta}^j$ for households $j \in \{2, \ldots J\}$ and $\bar{\boldsymbol{\delta}}^j$ as solutions of $(\bar{\boldsymbol{x}}_1^j - \boldsymbol{e_1}^j) = \bar{\boldsymbol{V}}\boldsymbol{\delta}^j$ for entrepreneurs $j \in \{1, \ldots J\}$. Define $\bar{\boldsymbol{\theta}}^1 = 1 - \sum_{j=2}^J \bar{\boldsymbol{\theta}}^j - \sum_{j=1}^J \bar{\boldsymbol{\delta}}^j$. Then the market clearing conditions for date 1, $\sum_{j=1}^J (\bar{\boldsymbol{m}}_{\boldsymbol{1}}^j + \bar{\boldsymbol{x}}_{\boldsymbol{1}}^j - \boldsymbol{\omega_1}^j - \boldsymbol{e_1}^j - \boldsymbol{y}_j^p(\bar{k}_j, \bar{l}_j) - \boldsymbol{y}_j^g(1 - \bar{l}_j) = 0$ implies that $(\bar{\boldsymbol{m}}^1, \bar{l}_1)$ satisfy the 1st household's date 1 budget constraints, i.e. $\bar{\boldsymbol{m_1}}^1 - \boldsymbol{\omega_1}^1 - \bar{\boldsymbol{W}}^1 \bar{l}_1 - (1-\bar{l}_1)\bar{\boldsymbol{G}} = \bar{\boldsymbol{V}}\bar{\boldsymbol{\theta}}^1$. Since $\bar{\boldsymbol{\Pi}}_h^1 = \boldsymbol{\Pi}_h^1(\bar{\boldsymbol{m}}^1, \bar{l}_1)$, $(\bar{\boldsymbol{m}}^1, \bar{l}_1, \bar{\boldsymbol{\theta}}^1)$ satisfies the FOC's of the 1st household and is utility maximizing over $M^1(\bar{\boldsymbol{W}}^1, \bar{\boldsymbol{G}}, \bar{\boldsymbol{Q}}, \bar{\boldsymbol{V}})$. For all other households, the NA budget sets are identical to the SM budget sets with the variables defined as above. So that $(\bar{\boldsymbol{m}}^j, \bar{l}_j)$ are utility maximizing for the respective households. Since $\bar{\boldsymbol{\pi}}(\boldsymbol{y}_j^p - \bar{\boldsymbol{W}}^j l_j) = \bar{\boldsymbol{\Pi}}_e^j(\bar{\boldsymbol{x}}^j)(\boldsymbol{y}_j^p - \bar{\boldsymbol{W}}^j l_j)$ by REMARK 1, the pair $(\bar{k}_j, \bar{l}_j^d)$ maximizes the profit function $\boldsymbol{\Pi}_e^j(\bar{\boldsymbol{x}}^j)(\boldsymbol{y}_j^p - \bar{\boldsymbol{W}}^j l_j)$. Given $k_j = \bar{k}_j$ the SM budget sets of the entrepreneurs are identical to the NA budget sets, so that $\bar{\boldsymbol{x}}^j)$ are optimal for the entrepreneurs. Since $(\bar{\boldsymbol{m}}^j, \bar{\boldsymbol{x}}^j)$ are feasible as well, $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{m}}, \bar{\boldsymbol{l}}), (\bar{\boldsymbol{k}}, \bar{\boldsymbol{l}}^d), (\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\delta}}), (\bar{\boldsymbol{W}}, \bar{\boldsymbol{G}}, \bar{\boldsymbol{Q}}))$ is an SME. Δ

We are now ready to combine Proposition 1 and Lemma 1 to prove the existence of a SME.

Proposition 2 *A Stock Market Equilibrium (Definition 1) exists.*

Proof: Follows from Lemma 1 and Proposition 1. Δ

4 Comparative Statics of Employment

4.1 Employment and Relative Risk Aversion

When the utility function is CRRA, the solutions of Equations 21 to 23 are defined for a given value of β, the coefficient of relative risk aversion. In this section we ask the question - Are private employment levels *in equilibrium* adversely affected by a rise in β, *assuming that the labor supply function itself is adversely affected* by such a rise? Appendix 1, discusses the conditions under which the labor supply function $\tilde{\boldsymbol{l}}(\boldsymbol{\pi}, \boldsymbol{G}, \beta)$ diminishes with respect to β. The answer to the question above is non-obvious because equilibrium $\boldsymbol{l}$ depends on $\boldsymbol{\pi}$ and $\boldsymbol{G}$. In fact clear answers can be given only for special cases. Section 4.1 (Propositions 3, 4 and 5) discusses these situations. Finally in Section 4.2 numerical simulations of the Stock Market and Benchmark (described below) economies with Cobb-Douglas production functions are discussed. These show that for a certain range of β, private employment levels are higher when markets are complete than when they are not. Although we are unable to prove this result mathematically, an intuition is provided.

To prove the comparative static results we first of all need $\boldsymbol{G}(\boldsymbol{l})$ to be *monotonic* in each l_j i.e. we need either,

$$\eta_j^s g^{j'} \leq \sum_{j=1}^{J} (\frac{\eta_j^s g^j}{1-l_j})(\frac{1-l_j}{\sum_{j=1}^{J}(1-l_j)}), \ \forall \ j = 1 \ldots J, \ \forall \ s = 1 \ldots S$$

or

$$\eta_j^s g^{j'} \geq \sum_{j=1}^{J} (\frac{\eta_j^s g^j}{1-l_j})(\frac{1-l_j}{\sum_{j=1}^{J}(1-l_j)}), \ \forall \ j = 1 \ldots J, \ \forall \ s = 1 \ldots S$$

The first condition is usually satisfied if g_j's are concave. It implies that state wages increase (or stay constant) as the number of state employees decrease because there are fewer people to share the pie with. The second inequality which is likely to be satisfied for sufficiently convex g_j's is however not an implausible scenario either. Since state firms operate with historically fixed capital stocks which had absorbed the entire labor force till date 0, these firms are likely to have increasing returns to labor over the range of available labor at date 1. The second inequality requires that the marginal product of labor in each sector be sufficiently high (compared to the average of the average products) so that as labor is transferred from state firms to private firms the per capita wages in the former do not increase. For the rest of the section we assume that $\boldsymbol{G}(\boldsymbol{l})$ is increasing in each l_j.

We assume all the equilibria to be interior in $\boldsymbol{\pi}$ and $\boldsymbol{l}$ in this section, in partisular $\boldsymbol{l}^* < \mathbf{1}$.

To study how the equilibrium l_j changes with respect to a change in β we require that the functions $\tilde{\boldsymbol{l}}(\boldsymbol{\pi}, \boldsymbol{G}, \beta)$ be monotone in its arguments. The first order condition of household $j, j \neq 1$ (i.e. for households having constrained budget sets), with respect to l_j,is,

$$\sum_{s=1}^{S} \frac{\pi_s}{\pi_0} u'(m_0^j)(W_s^j - G_s) + \sum_{s=1}^{S} (u'(m_s^j) - \frac{\pi_s}{\pi_0} u'(m_0^j))(W_s^j - G_s) - c'(l_j) = 0 \tag{25}$$

For $j = 1$, this condition is

$$\sum_{s=1}^{S} \frac{\pi_s}{\pi_0} u'(m_0^1)(W_s^1 - G_s) - c'(l_1) = 0 \tag{26}$$

The left hand side of the above equations is the marginal value of the extra income that household j makes from working for private rather than state firms. We denote this as VMG for short. The right hand side is the marginal cost of labor supply to private firms. For an interior equilibrium the two must be equal. The next step is to substitute for W_s^j, m_0^j and m_s^j on the left hand sides to get l_j as a function of $\boldsymbol{\pi}$ and $\boldsymbol{G}$ only.

Note that since $\boldsymbol{W}^j = \boldsymbol{\eta}^j \kappa_j(\boldsymbol{\pi})$ under competition and profit maximization by firms, the market subspace $< \boldsymbol{V} >$ is equal to $< \boldsymbol{\eta} >$[7]. Since $\boldsymbol{G}$ is a linear combination of $\{\boldsymbol{\eta}_j\}$, we have $(\boldsymbol{W}^j - \boldsymbol{G}) \in < \boldsymbol{V} >$. Since for marketed securities, $(u'(m_s^j) - \frac{\pi_s}{\pi_0} u'(m_0^j)) = 0$ from the first order conditions of household j, Equation 24 can be written as,

$$\sum_{s=1}^{S} \frac{\pi_s}{\pi_0} u'(m_0^j)(W_s^j(\boldsymbol{\pi}) - G_s) - c'(l_j) = 0 \tag{27}$$

Thus the first order conditions of all households with respect to labor have the same form whether they have constrained or unconstrained budget sets.

We next attempt to eliminate m_0^j from equation 26. For CRRA $u(c)$, it is possible to get closed form expression for m_0^j as a function of $\frac{\boldsymbol{\pi}^1}{\pi_0}$ and l_j, given β (see Appendix) *provided the household is unconstrained*. For the *constrained* agents, since individual demand functions for securities are not defined, it is not possible to eliminate the $\boldsymbol{\theta}^j$ terms from the expressions for m_0^j. To proceed with the comparative statics, we therefore make use of (without proving) the following property of a normalized NAE (see Magill and Quinzii, 1996).

Property of normalized NAE : *If* $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{m}}, \bar{\boldsymbol{l}}), (\bar{\boldsymbol{k}}, \bar{\boldsymbol{l}}^d), (\bar{\boldsymbol{W}}, \bar{\boldsymbol{G}}, \bar{\boldsymbol{\Pi}}_h^1, \bar{\boldsymbol{V}})$ *is a normalized NAE with household 1 as the unconstrained agent, then* $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{m}}, \bar{\boldsymbol{l}}), (\bar{\boldsymbol{k}}, \bar{\boldsymbol{l}}^d), (\bar{\boldsymbol{W}}, \bar{\boldsymbol{G}}, \bar{\boldsymbol{\Pi}}_h^j, \bar{\boldsymbol{V}})$ *is a normalized NAE for any household* j *as the unconstrained agent.*

What this implies is that the allocations of a particular NAE are invariant with respect to the choice of normalizing agent. Hence the *comparative statics* of the fixed point $(\boldsymbol{l}^*, \boldsymbol{\pi}^*, \boldsymbol{G}^*)$ with respect to β are also *invariant* with respect to the choice of the unconstrained agent i.e. the results will not change depending on our choice of the normalizing agent. At an equilibrium, any agent can therefore be assumed without loss of generality to be the unconstrained agent.

Substituting for m_0^j (from Appendix), Equation 26 yields l_j as an implicit function $\tilde{l}_j(.)$ of $\boldsymbol{\pi}$ and $\boldsymbol{G}$, in particular of $\frac{\boldsymbol{\pi}^1}{\pi_0}$ and $\boldsymbol{G}$. By the implicit function theorem $\frac{\partial \tilde{l}_j}{\partial \frac{\pi_s}{\pi_0}}$ is equal to and has the same sign as $-(\frac{\partial VMG}{\partial \frac{\pi_s}{\pi_0}} / \frac{\partial VMG}{\partial l_j})$.

[7] This equality is true outside equilibrium, so long as firms are optimizing and labor markets are competitive

m_0^j is increasing in l_j as long as $(W_s^j - G_s)$ is positive (see Appendix 5.2). Since $u'(m_0^j)$ is diminishing in m_0^j, $\frac{\partial VMG}{\partial l_j}$ is negative. Hence the sign of $\frac{\partial VMG}{\partial \frac{\pi_s}{\pi_0}}$ is decisive.

It may be convenient to refer to figure 1 at this stage. The intersection of downward sloping VMG and upward rising MC gives $\tilde{l}_j(.)$ as a function of $\boldsymbol{\pi}$ and $\boldsymbol{G}$.

We need to discuss how the function VMG behave with respect to $\frac{\boldsymbol{\pi}^1}{\pi_0}$. From the foc of the entrepreneurs Equations 8 to 13, W_s^j is increasing in each of the relative prices $\frac{\pi_s}{\pi_0}$. Thus $\frac{\pi_s}{\pi_0}(W_s^j - G_s)$ is increasing in each $\frac{\pi_s}{\pi_0}$. $u'(m_0^j)$ however behaves in a more complicated way.

A rise in $\frac{\pi_s}{\pi_0}$ for any s has three effects on m_0^j as given by the expression in Appendix. (i) Current consumption becomes cheaper relative to future consumption causing m_s^j to be substituted by m_0^j. This is the usual positive substitution effect. (ii) Value of income in state s is higher relative to value of income at date 0. (iii) Real income at state s rises because of rise in capital-output ratio. The last two causes m_s^j to move up relatively to m_0^j. These are negative income effects. For $\beta \leq 1$ the substitution effect is stronger with m_0^j positively and $u'(m_0^j)$ negatively related to each $\frac{\pi_s}{\pi_0}$. When $\beta > 1$, however, income effects may be stronger with m_0^j diminishing and $u'(m_0^j)$ increasing in $\frac{\pi_s}{\pi_0}$.

Denote by $\ell_j(\boldsymbol{\pi}, \boldsymbol{G}) = \sum_{s=1}^{S} \frac{\pi_s}{\pi_0} u'(m_0^j(\boldsymbol{\pi}, \boldsymbol{G})(W_s^j(\boldsymbol{\pi}) - G_s)$, the marginal gains from l_j. We have two cases.

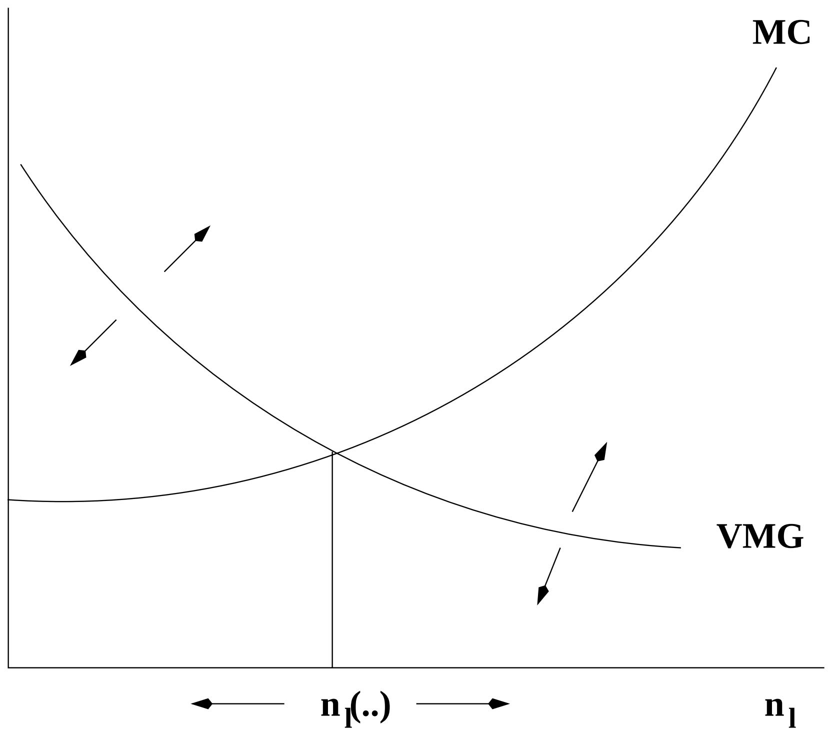
(A) *Strong income effects*: m_0^j decreases and $u'(m_0^j)$ increases with each $\frac{\pi_s}{\pi_0}$. $\ell_j(\boldsymbol{\pi}, \boldsymbol{G})$ therefore unambiguously rises with each $\frac{\pi_s}{\pi_0}$. This implies that $\tilde{l}_j(\boldsymbol{\pi}, \boldsymbol{G})$ is *monotone increasing* in each $\frac{\pi_s}{\pi_0}$ or alternatively monotone decreasing in $\frac{\pi_0}{\pi_s}$ everything else constant.

(B) *Strong substitution effects*: m_0^j increases and $u'(m_0^j)$ decreases with each $\frac{\pi_s}{\pi_0}$. Then there are conflicting effects on labor supply. Differentiating $\ell_j(\boldsymbol{\pi}, \boldsymbol{G})$ with respect to $\frac{\pi_s}{\pi_0}$ and simplifying, the sign of the derivative is ≥ 0 or ≤ 0 according as,

$$\sum_{t=1}^{S} u'(m_0^j)(W_t^j - G_t)(\varepsilon_s^j - \beta \epsilon_{0s}^j) + u'(m_0^j)(W_s^j - G_s) \geq 0 \ \ or \leq 0. \tag{28}$$

where ϵ_{0s}^j is the elasticity of m_0^j and ε_s^j the elasticity of the marginal product of labor, $f_l^j(\kappa_j(\boldsymbol{\pi}))$ with respect to $\frac{\pi_s}{\pi_0}$. Since $\epsilon_{0s}^j > 0$, the magnitude and sign of the following expression determines the sign of the derivative.

Figure 1: Optimal labor supply to private firms



$$\begin{aligned} VMG &= \sum_{s=1}^{S} \frac{\pi_s}{\pi_0} u'(m_0^j)(W_s^j - G_s) \\ MC &= c'(l_j) \end{aligned}$$

$$(\varepsilon_s^j - \beta\epsilon_{0s}^j) = \varepsilon_s^j - \beta\{\varepsilon_s^j(\frac{\pi_s}{\pi_0})\sum_{t=1}^{S} W_t^j + (\frac{\pi_s}{\pi_0})(W_s^j l_j + (1-l_j)G_s)\} - (1-\beta)(\rho_s)^{1/\beta}(\frac{\pi_s}{\pi_0})^{(1-1/\beta)} \quad (29)$$

We shall look at cases when this expression is negative and sufficiently large so that $\tilde{l}_j(\boldsymbol{\pi}, \boldsymbol{G})$ is *monotone decreasing* in each $\frac{\pi_s}{\pi_0}$.

Define

$$\psi_1(\boldsymbol{l}, \boldsymbol{\pi}, \boldsymbol{G}) = (\tilde{\boldsymbol{l}}(\boldsymbol{\pi}, \boldsymbol{G}, \beta_1), \mu(\boldsymbol{\pi}, \boldsymbol{l}, \boldsymbol{G}, \beta_1), \boldsymbol{G(l)})$$

and

$$\psi_2(\boldsymbol{l}, \boldsymbol{\pi}, \boldsymbol{G}) = (\tilde{\boldsymbol{l}}(\boldsymbol{\pi}, \boldsymbol{G}, \beta_2), \mu(\boldsymbol{\pi}, \boldsymbol{l}, \boldsymbol{G}, \beta_2), \boldsymbol{G(l)})$$

where $\beta_1 > \beta_2$. Let $(\boldsymbol{l}^*, \boldsymbol{\pi}^*, \boldsymbol{G}^*)$ be a fixed point of ψ_2. We are now ready to prove the following propositions.

Proposition 3 *Given, income effects are strong, $\tilde{\boldsymbol{l}}(\boldsymbol{\pi}, \boldsymbol{G}, \beta)$ is strictly monotone increasing in each $\frac{\pi_s}{\pi_0}$ and $\boldsymbol{G(l)}$ is monotone increasing in each l_j, $\psi_1(\boldsymbol{l}, \boldsymbol{\pi}, \boldsymbol{G})$ has a fixed point $(\boldsymbol{l}^{**}, \boldsymbol{\pi}^{**}, \boldsymbol{G}^{**})$ such that $l_j^{**} < l_j^*$ for some j provided $Z_0(\boldsymbol{\pi}', \boldsymbol{l}', \beta_1) \geq (\frac{\pi_0^*}{\pi_s^*})Z_s(\boldsymbol{\pi}', \boldsymbol{l}', \beta_1)$ for all s, for $\frac{\pi_0'}{\pi_s'} \geq \frac{\pi_0^*}{\pi_s^*}$ and $\boldsymbol{l}' \leq \boldsymbol{l}^*$.*[8]

Proof :

Define $\bar{l}_j = \tilde{l}_j(\boldsymbol{\pi}^*, 0, \beta_2)$ and $\bar{\boldsymbol{G}} = \boldsymbol{G}(\bar{\boldsymbol{l}})$. Therefore $\bar{l}_j > l_j^* = \tilde{l}_j(\boldsymbol{\pi}^*, \boldsymbol{G}^*, \beta_2)$ and $\bar{\boldsymbol{G}} > \boldsymbol{G}^* = \boldsymbol{G}(\boldsymbol{l}^*)$ given that $\tilde{\boldsymbol{l}}(\boldsymbol{\pi}, \boldsymbol{G}, \beta)$ is monotone decreasing in each G_s and $\boldsymbol{G(l)}$ is monotone increasing in each l_j.

Define the set $\mathcal{L}' \times \mathcal{P}' \times \mathcal{G}'$ where $\mathcal{L}' = \{\boldsymbol{l}; l_j \leq \bar{l}_j\}, \mathcal{G}' = \{\boldsymbol{G}; \boldsymbol{G} \leq \bar{\boldsymbol{G}}\}$ and $\mathcal{P}' = \{\boldsymbol{\pi}; \sum_{s=0}^{S} \pi_s = 1, \pi_0 \geq \pi_0^*, \pi_s \leq \pi_s^* \ \forall s = 1 \ldots S\}$. Define the partial order $\succeq$ on $\mathcal{P}$ as $\boldsymbol{\pi}' \succeq \boldsymbol{\pi}$ iff $\pi_0' \geq \pi_0$ and $\pi_s' \leq \pi_s$ for all $s = 1 \ldots S$ with $\succ$ for at least one strict inequality and "=" otherwise.

[8] If all agents have demand functions of the form given in Appendix 2, excess demand functions are independent of $\boldsymbol{G}$ which cancels out under aggregation. When markets are incomplete closed form solutions of demand functions for all but one agent are difficult to derive.

Define the function $\psi_1^{'}(\boldsymbol{l}^{'},\boldsymbol{\pi}^{'},\boldsymbol{G}^{'}) = \psi_1(\boldsymbol{l}^{'},\boldsymbol{\pi}^{'},\boldsymbol{G}^{'})$ for $(\boldsymbol{l}',\boldsymbol{\pi}',\boldsymbol{G}') \in \mathcal{L}^{'} \times \mathcal{P}^{'} \times \mathcal{G}^{'}$.

Since $\tilde{l}_j(\boldsymbol{\pi},\boldsymbol{G},\beta)$ is diminishing in each $\frac{\pi_0}{\pi_s}$, G_s and β we have $\tilde{l}_j(\boldsymbol{\pi}^{'},\boldsymbol{G}^{'},\beta_1) < \tilde{l}_j(\boldsymbol{\pi}^{'},0,\beta_1) < \tilde{l}_j(\boldsymbol{\pi}^{*},0,\beta_2) = \bar{l}_j$. $\boldsymbol{G}(\boldsymbol{l}^{'}) \leq \boldsymbol{G}(\bar{\boldsymbol{l}}) = \bar{\boldsymbol{G}}$ since $\boldsymbol{l}^{'} \leq \bar{\boldsymbol{l}}$.

$$\begin{aligned}
\frac{\mu_0(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1)}{\mu_s(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1)} &= \frac{\pi_0^{'} + \max(0, Z_0(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1))}{\pi_s^{'} + \max(0, Z_s(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1))} \\
&= (\frac{\pi_0^{'}}{\pi_s^{'}})\frac{\pi_s^{'}}{\pi_s^{'} + \max(0, Z_s(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1))} \\
&\quad +(\frac{\max(0, Z_0(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1)}{\max(0, Z_s(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1))})\frac{\max(0, Z_s(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1))}{\pi_s^{'} + \max(0, Z_s(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1))} \\
&\geq (\frac{\pi_0^{*}}{\pi_s^{*}})\frac{\pi_s^{'}}{\pi_s^{'} + \max(0, Z_s(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1))} + (\frac{\pi_0^{*}}{\pi_s^{*}})\frac{\max(0, Z_s(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1))}{\pi_s^{'} + \max(0, Z_s(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1))}
\end{aligned}$$

since $(\frac{\pi_0^{'}}{\pi_s^{'}}) \geq (\frac{\pi_0^{*}}{\pi_s^{*}})$ and $Z_0(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1) \geq (\frac{\pi_0^{*}}{\pi_s^{*}})Z_s(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1)$ for all s, which implies $(\frac{\max(0,Z_0(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1)}{\max(0,Z_s(\boldsymbol{\pi}^{'},\boldsymbol{l}^{'},\beta_1))}) \geq (\frac{\pi_0^{*}}{\pi_s^{*}})$.

Therefore $\psi_1^{'}$ maps $\mathcal{L}^{'} \times \mathcal{P}^{'} \times \mathcal{G}^{'}$ into itself. The latter is compact and convex. Hence $\psi_1^{'}$ satisfies all conditions of the Brower's theorem and has a fixed point $((\boldsymbol{Z}^{**},\boldsymbol{l}^{**}),(\boldsymbol{\pi}^{**},\boldsymbol{G}^{**}))$ in $\mathcal{L}^{'} \times \mathcal{P}^{'} \times \mathcal{G}^{'}$. This is a solution for Equations 21 to 23 and hence a NAE for $\beta = \beta_1$.

Note that $((\boldsymbol{Z}^{**},\boldsymbol{l}^{**}),(\boldsymbol{\pi}^{**},\boldsymbol{G}^{**})) \neq ((\boldsymbol{Z}^{*},\boldsymbol{l}^{*}),(\boldsymbol{\pi}^{*},\boldsymbol{G}^{*}))$ where, "=" denotes componentwise equality. If this was true then $\boldsymbol{\pi}^{**} = \boldsymbol{\pi}^{*}$, $\boldsymbol{G}^{**} = \boldsymbol{G}^{*}$ and $\boldsymbol{l}^{**} = \tilde{\boldsymbol{l}}(\boldsymbol{\pi}^{**},\boldsymbol{G}^{**},\beta_1) = \boldsymbol{l}^{*} = \tilde{\boldsymbol{l}}(\boldsymbol{\pi}^{*},\boldsymbol{G}^{*},\beta_2) = \tilde{\boldsymbol{l}}(\boldsymbol{\pi}^{**},\boldsymbol{G}^{**},\beta_2)$ which is false because $\tilde{\boldsymbol{l}}(.)$ is strictly monotone decreasing in β. We therefore have the following cases.

Case 1: $\boldsymbol{\pi}^{**} = \boldsymbol{\pi}^{*}$, but $\boldsymbol{l}^{**} \neq \boldsymbol{l}^{*}$ and $\boldsymbol{G}^{**} \neq \boldsymbol{G}^{*}$ (these have to be true together since $\boldsymbol{G}(\boldsymbol{l})$ is strictly monotone in l_j.

Suppose $\boldsymbol{l}^{**} \geq \boldsymbol{l}^{*}$ which implies $\tilde{\boldsymbol{l}}(\boldsymbol{\pi}^{**},\boldsymbol{G}^{**},\beta_1) \geq \tilde{\boldsymbol{l}}(\boldsymbol{\pi}^{**},\boldsymbol{G}^{*},\beta_2)$. Therefore it must not be that $\boldsymbol{G}^{**} > \boldsymbol{G}^{*}$. But $\boldsymbol{G}^{**} = \boldsymbol{G}(\boldsymbol{l}^{**}) > \boldsymbol{G}(\boldsymbol{l}^{*}) = \boldsymbol{G}^{*}$ since $\boldsymbol{G}(\boldsymbol{l})$ is increasing in l_j. Hence we have a contradiction. Therefore $l_j^{**} < l_j^{*}$ for some j.

Case 2: $\boldsymbol{\pi}^{**} \succ \boldsymbol{\pi}^{*}$.

Suppose $\boldsymbol{l}^{**} \geq \boldsymbol{l}^{*}$ which implies $\tilde{\boldsymbol{l}}(\boldsymbol{\pi}^{**},\boldsymbol{G}^{**},\beta_1) \geq \tilde{\boldsymbol{l}}(\boldsymbol{\pi}^{*},\boldsymbol{G}^{*},\beta_2)$. Since $\tilde{\boldsymbol{l}}(.)$ is strictly monotone decreasing in $\boldsymbol{\pi}$ under the order $\succeq$, and $\boldsymbol{\pi}^{**} \succ \boldsymbol{\pi}^*$ it must not be that $\boldsymbol{G}^{**} > \boldsymbol{G}^*$. But once again, $\boldsymbol{G}^{**} = \boldsymbol{G}(\boldsymbol{l}^{**}) > \boldsymbol{G}(\boldsymbol{l}^*) = \boldsymbol{G}^*$ since $\boldsymbol{G}(\boldsymbol{l})$ is increasing in l_j, which is a contradiction. Therefore $l_j^{**} < l_j^*$ for some j. Δ

Proposition 3 implies that if excess demand functions satisfy the conditions above in the neighborhood of an equilibrium, private employment levels in some sectors will locally decrease(increase) as the coefficient of relative risk aversion increases(decreases). The condition imposed on the excess demand functions need explanation.

REMARKS

3. We know that $Z_0(\boldsymbol{\pi}^*, \boldsymbol{l}^*, \beta_2) = Z_s(\boldsymbol{\pi}^*, \boldsymbol{l}^*, \beta_2) = 0$ for all $s = 1 \ldots S$. If $\frac{m_0^j}{m_s^j}$ is strictly increasing in β (see Appendix 1) for all the agents, - i.e. if a rise in relative risk aversion increases date 0 consumption relative to date 1 consumption at all states - we will have $Z_0(\boldsymbol{\pi}^*, \boldsymbol{l}^*, \beta_1) > Z_s(\boldsymbol{\pi}^*, \boldsymbol{l}^*, \beta_1)$ for all s. Since income effects are strong m_0^j is increasing in $\frac{\pi_0}{\pi_s}$ for all agents. What the condition requires is that the combined effects of a rise in β and in $\frac{\pi_0}{\pi_s}$ be sufficient to counter the substitution effect and the negative effect of a lower l_j on $\frac{Z_0(.)}{Z_s(.)}$.

4. It is possible that for some i, $l_i(\boldsymbol{\pi}^{**}, \boldsymbol{G}^{**}, \beta_1) > l_i(\boldsymbol{\pi}^{**}, \boldsymbol{G}^{**}, \beta_2)$. We can write this inequality as,

$$l_i(\boldsymbol{\pi}^{**}, \boldsymbol{G}^{**}, \beta_1) - l_i(\boldsymbol{\pi}^{**}, \boldsymbol{G}^{*}, \beta_1) > l_i(\boldsymbol{\pi}^{*}, \boldsymbol{G}^{*}, \beta_2) - l_i(\boldsymbol{\pi}^{**}, \boldsymbol{G}^{*}, \beta_1)$$

The left hand side depends on the overall productivity of state firms and the marginal returns to labor in these. The right hand side depends on the marginal productivity of labor in private firms and how sensitive this is to changes in relative state prices (i.e. how sensitive the ratio k_j/l_j is to changes in relative state prices). The more productively efficient the private firms are relative to state firms, the less likely that this inequality will be satisfied. Hence the larger the number of sectors that will be adversely affected by a rise in relative risk aversion.

The next proposition describes a situation in which substitution effects are stronger and the above result holds. Date 0 consumption for every agent now is a decreasing function of $\frac{\pi_0}{\pi_s}$ or alternatively increasing function of $\frac{\pi_s}{\pi_0}$.

Proposition 4 *Given, $\tilde{\boldsymbol{l}}(\boldsymbol{\pi}, \boldsymbol{G}, \beta)$ is strictly monotone decreasing in each $\frac{\pi_s}{\pi_0}$ and $\boldsymbol{G}(\boldsymbol{l})$ is monotone increasing in each l_j, $\psi_1(\boldsymbol{l}, \boldsymbol{\pi}, \boldsymbol{G})$ has a fixed point $(\boldsymbol{l}^{**}, \boldsymbol{\pi}^{**}, \boldsymbol{G}^{**})$ such that $l_j^{**} < l_j^*$ for some j provided*

$Z_0(\boldsymbol{\pi}', \boldsymbol{l}', \beta_1) \leq (\frac{\pi_0^*}{\pi_s^*}) Z_s(\boldsymbol{\pi}', \boldsymbol{l}', \beta_1)$ *for all* s, *for* $\frac{\pi_0'}{\pi_s'} \leq \frac{\pi_0^*}{\pi_s^*}$ *and* $\boldsymbol{l}' \leq \boldsymbol{l}^*$.

Proof:

Define $\bar{\boldsymbol{l}}$ and $\bar{\boldsymbol{G}}$ as before. Define the sets $\mathcal{P}'$ and $\mathcal{G}'$ as before. Define $\mathcal{P}' = \{\boldsymbol{\pi}; \sum_{s=0}^{S} \pi_s = 1, \pi_0 \leq \pi_0^*, \pi_s \geq \pi_s^* \ \forall s = 1 \ldots S\}$ and the partial order $\succeq$ on $\mathcal{P}$ as $\boldsymbol{\pi}' \succeq \boldsymbol{\pi}$ iff $\pi_0' \leq \pi_0$ and $\pi_s' \geq \pi_s$ for all $s = 1 \ldots S$ with $\succ$ for at least one strict inequality and "=" otherwise.

Also as before, define the function $\psi_1'(\boldsymbol{l}', \boldsymbol{\pi}', \boldsymbol{G}') = \psi_1(\boldsymbol{l}', \boldsymbol{\pi}', \boldsymbol{G}')$ for $(\boldsymbol{l}', \boldsymbol{\pi}', \boldsymbol{G}') \in \mathcal{L}' \times \mathcal{P}' \times \mathcal{G}'$.

We note that,

$$\frac{\mu_0(\boldsymbol{\pi}', \boldsymbol{l}', \beta_1)}{\mu_s(\boldsymbol{\pi}', \boldsymbol{l}', \beta_1)} \leq (\frac{\pi_0^*}{\pi_s^*}) \frac{\pi_s'}{\pi_s' + \max(0, Z_s(\boldsymbol{\pi}', \boldsymbol{l}', \beta_1))} + (\frac{\pi_0^*}{\pi_s^*}) \frac{\max(0, Z_s(\boldsymbol{\pi}', \boldsymbol{l}', \beta_1))}{\pi_s' + \max(0, Z_s(\boldsymbol{\pi}', \boldsymbol{l}', \beta_1))}$$

since $(\frac{\pi_0'}{\pi_s'}) \leq (\frac{\pi_0^*}{\pi_s^*})$ and $Z_0(\boldsymbol{\pi}', \boldsymbol{l}', \beta_1) \leq (\frac{\pi_0^*}{\pi_s^*}) Z_s(\boldsymbol{\pi}', \boldsymbol{l}', \beta_1)$ for all s, which implies $(\frac{\max(0, Z_0(\boldsymbol{\pi}', \boldsymbol{l}', \beta_1))}{\max(0, Z_s(\boldsymbol{\pi}', \boldsymbol{l}', \beta_1))}) \leq (\frac{\pi_0^*}{\pi_s^*})$.

We follow all the remaining steps and prove the proposition. Δ.

REMARKS

5. For the conditions of Proposition 4 to be satisfied we need a rise in β to increase consumption in state s relative to consumption at date 0 (i.e. m_s^j / m_0^j), sufficiently. Appendix 1 discusses when this can be the case.

6. It is *necessary but not sufficient* for the comparative statics result above that the foc of the households with respect to labor satisfy $\tilde{l}_j(\boldsymbol{\pi}, \boldsymbol{G}, \beta_1) > \tilde{l}_j(\boldsymbol{\pi}, \boldsymbol{G}, \beta_2)$ for any $(\boldsymbol{\pi}, \boldsymbol{G})$. There are other risky assets in the model such as k_j for which this monotonicity property cannot be shown. The monotonicity property for labor supply can be easily shown to be true in this model because utility functions are separable in income and labor. The rise in the rate of relative risk aversion affects the marginal utility of income but not the marginal disutility or costs of labor. Separability however is not necessary for this property. What we need for the monotonicity property to hold, is that the coefficient of relative risk aversion affect marginal utility of income more than the marginal disutility of labor.

7. As we pointed out in the introduction, the above method of deriving the comparative static results can be easily extended to other types of utility functions in the HARA class, for which closed form solutions for the demand functions can be found. We need to know the latter to check for monotonicity properties of the relevant functions.

4.2 Employment and Market Incompleteness

A benchmark economy

Stock markets are a way of sharing idiosyncratic production risks when such risks are too complex for agents to write and exchange contingent contracts on these. It is however useful to compare such an economy with an idealized one in which a complete set of such contracts are traded. In such an economy entrepreneurs have no incentives to sell ownership shares of their firms and so are sole proprietors. We end this section by formally defining the equilibrium for this benchmark economy in which a complete set of Arrow securities are traded.

The budget set of the jth entrepreneur is:

$$\boldsymbol{B}^j_{cm}(\boldsymbol{P},\boldsymbol{W}^j) = \left\{ \boldsymbol{x}^j \in R^{S+1}_+ \;\middle|\; \begin{array}{lcl} x^j_0 & \leq & e^j_0 - k_j - \sum_{s=1}^S p_s \xi^j_s \\ x^j_s & \leq & e^j_s + Y^p_j(s) - W^j_s l_j + \xi^j_s \end{array} \right\} \tag{30}$$

where $\boldsymbol{P} = \{p_s\}_{s=1}^S$ represents the prices of Arrow securities, $\boldsymbol{\xi}^j = \{\xi^j_s\}_{s=1}^S$ represents quantities of Arrow securities purchased.

The budget set of the jth household is:

$$\boldsymbol{M}^j_{cm}(\boldsymbol{P},\boldsymbol{W}^j,\boldsymbol{G}) = \left\{ \boldsymbol{m}^j \in R^{S+1}_+ \;\middle|\; \begin{array}{lcl} m^j_0 & \leq & w^j_0 - \sum_{s=1}^S p_s \zeta^j_s \\ m^j_s & \leq & W^j_s + W^j_s l_j + (1-l_j)G_s + \zeta^j_s \end{array} \right\} \tag{31}$$

where $\boldsymbol{\zeta}^j = \{\zeta^j_s\}_{s=1}^S$ represents quantities of Arrow securities purchased. Let $\boldsymbol{\xi} = \{\boldsymbol{\xi}^j\}_{j=1}^J$ and $\boldsymbol{\zeta} = \{\boldsymbol{\zeta}^j\}_{j=1}^J$. Then,

Definition 3 *A Complete Markets Equilibrium (CME) with state firms is a 4-tuple* $((\hat{\boldsymbol{x}},\hat{\boldsymbol{m}},\hat{\boldsymbol{l}}),(\hat{\boldsymbol{k}},\hat{\boldsymbol{l}}^d),$ $(\hat{\boldsymbol{\xi}},\hat{\boldsymbol{\zeta}}),(\hat{\boldsymbol{W}},\boldsymbol{G}(\hat{\boldsymbol{l}}),\hat{\boldsymbol{P}})$ *of consumption plans of entrepreneurs and consumption and labor supply plans of households, production plans of entrepreneurs, portfolios of households and entrepreneurs, private and state wage contracts and state prices, such that:*

1. *For each entrepreneur* j,

$$(\hat{\boldsymbol{x}^j},\hat{\boldsymbol{\xi}^j}) = \textit{argmax}\{U(\boldsymbol{x}^j) \;|\; (\hat{\boldsymbol{x}^j},\hat{\boldsymbol{\xi}^j}) \in \boldsymbol{B}^j_{cm}(\bar{\boldsymbol{P}},\bar{\boldsymbol{W}^j})\}$$

$$(\hat{k_j},\hat{l^d_j}) = \textit{argmax}\boldsymbol{\Pi^1}^j_e(\hat{\boldsymbol{x}^j})(\boldsymbol{y}^p_j - \hat{\boldsymbol{W}}^j l_j) - \Pi^j_{0e}(\hat{\boldsymbol{x}^j})k_j,$$

2. *For each household j,*

$$(\hat{\boldsymbol{m}^j}, \hat{\boldsymbol{\zeta}^j}, \hat{l}_j) = \textit{argmax}\{U(\boldsymbol{m}^j, l_j) \mid (\hat{\boldsymbol{m}^j}, \hat{\boldsymbol{\zeta}^j}, \hat{l}_j) \in \boldsymbol{M}^j_{cm}(\hat{\boldsymbol{P}}, \hat{\boldsymbol{W}}^j, \hat{\boldsymbol{G}})\}$$

3. *At date 1, workers and entrepreneurs have no incentives to switch parties with whom they have drawn contracts at date 0.*

4. *Labor markets clear*

$$\begin{aligned} \hat{\boldsymbol{l}}^d &= \hat{\boldsymbol{l}} \\ \hat{\boldsymbol{G}} &= \boldsymbol{G}(\hat{\boldsymbol{l}}) \end{aligned}$$

5. *Markets for Arrow Securities clear,*

$$\sum_{j=1}^{J}(\hat{\boldsymbol{\xi}^j} + \hat{\boldsymbol{\zeta}^j}) = 0, \forall s \in \{1, \ldots S\}$$

Section 3 shows that a SME is a constrained Arrow-Debreu equilibrium, because the budget sets of the agents can be converted into constrained Arrow-Debreu budget sets. By contrast, those in the Benchmark model are unconstrained Arrow-Debreu budget sets. Thus the first order conditions in the two models, particularly for the employment levels, are not directly comparable in a way that was possible in the previous section. It is possible however to get some general intuition about the relative employment levels in the two economies on the basis of some numerical examples.

A numerical example

We consider a model with 5 states of Nature which are equiprobable and two sectors of production, hence 2 households and 2 entrepreneurs. There are effectively 4 “assets” for each household - 2 equities, labor for private firms and labor for state firms. The “degree” of market incompleteness (no. of assets relative to no. of states of Nature) is thus not very severe for households. (How would a riskless bond change things?)

The state independent production function is Cobb-Douglas, $Al_j^{\alpha} k_j^{1-\alpha}$. Both sectors have identical linear cost function $c.l_j$. The state firm’s production function is $(1 - l_j)^{\tau}$ where, the parameters $A, \alpha, c,$ andτ are all chosen so that (i) interior solutions in employment levels exist and (ii) government wages are less than private wages in every state.

The linearity of the cost function of labor keeps the model simple for the purpose of getting numerical solutions. This leads to a problem however since our model allows for short sales in equities. As wages and dividend streams for any firm are collinear in equilibrium (both are scaler multiples of the risk profile of that sector), with a linear cost function of labor the two assets for household j - l_j and θ_j^j - end up having very similar payoff structures. If we allow for short sales in the numerical examples below, each of the households end up shortselling over 30% of the firms that they work for, in equilibrium. This boosts the equilibrium employment levels in the private firms considerably. What happens is that household j shortsells θ_j^j at date 0, sells more labor to firm j and pays dividends to other shareholders at date 1 from its labor earnings. Short sales of this magnitude is clearly not realistic. To keep the model simple for numerical solutions and at the same time meaningful, we therefore impose a no short sales restriction in equilibrium. Since the whole point of this section is to form some idea about the effect of market incompleteness on employment based on a stylized model such a step is not unjustified.

The productivity shocks are,

$$\begin{aligned} \eta_1 &= \{1, 6, 6, 1, 1\} \\ \eta_2 &= \{4, 1, 1, 2, 2\} \end{aligned}$$

Qualitatively, there are 4 levels of shocks - 6 represents very high, 4 high, 2 medium and 1 very low. Thus sector 1 has higher variability and mean in productivity compared to sector 2. The sectoral shocks are negatively correlated. Aggregate risk (measured by standard deviation of aggregate production) is less than the sum of individual sectoral risks. This allows equities in the Stock Market model and the Arrow securities in the Benchmark model to perform well in hedging sectoral risks. An alternative scenario is discussed below (Table 2) in which the shock pattern of sector 2 is altered to yield positive correlation between sectors. Both equities and Arrow-securities perform less well in hedging. As we shall see gains from market completion are also lower over the relevant range.

Lastly, the risk aversion coefficients are chosen keeping in mind the usual conventions in the RBC literature.

Observations and Explanations

Table 1: Comparing SME and CME Equilibria when shocks are negatively correlated

Sector 1								
β	$\bar{n_1}$	$\hat{n_1}$	$\bar{k_1}$	$\hat{k_1}$	$\sigma(\boldsymbol{W}^{1^{-}} - G)$	$\sigma(\boldsymbol{W}^{1^{\wedge}} - G)$	$\sigma(\bar{Y_1^p})$	$\sigma(\hat{Y_1^p})$
0.8	0.81	0.84	31	46	96	119	198	256
0.9	0.51	0.53	26	40	117	145	150	194
1.0	0.33	0.34	24	35	143	177	119	152
1.1	0.22	0.23	23	33	175	216	99	124
1.3	0.11	0.11	22	30	258	317	73	88
1.6	0.05	0.04	22	29	445	549	52	59
Sector 2								
β	$\bar{n_2}$	$\hat{n_2}$	$\bar{k_2}$	$\hat{k_2}$	$\sigma(W^{2^{-}} - G)$	$\sigma(W^{2^{\wedge}} - G)$	$\sigma(\bar{Y_2^p})$	$\sigma(\hat{Y_2^p})$
0.8	0.50	0.65	15	32	39	51	48	82
0.9	0.36	0.45	17	31	50	62	45	70
1.0	0.26	0.31	18	30	63	77	41	60
1.1	0.19	0.22	19	30	80	95	37	52
1.3	0.10	0.11	22	32	126	145	32	41
1.6	0.04	0.05	28	36	242	268	26	31

β	0.8	0.9	1.0	1.1	1.3	1.6
$\sigma(\bar{Y})$	167	122	94	77	55	39
$\sigma(\hat{Y})$	204	151	117	93	65	43

$\bar{l_j}$ = employment under a SME, $\hat{l_j}$ = employment under a CME, $\bar{k_j}$ = physical investment under a SME, $\hat{k_j}$ = physical investment under a CME, $\sigma(W_j\bar{\ } - G)$ = standard deviation of wage differentials under SME, $\sigma(W_j\hat{\ } - G)$ = standard deviation of wage differentials under CME, $\sigma(\bar{Y_j^p})$ = standard deviation of sectoral output under SME, $\sigma(\hat{Y_j^p})$ = standard deviation of sectoral output under CME, $\sigma(\bar{Y})$ = standard deviation of aggregate output under SME, $\sigma(\hat{Y})$ = standard deviation of aggregate output under CME.

Table 2: Positively correlated shocks

β	$\bar{n_1}$	$\hat{n_1}$	$\bar{n_2}$	$\hat{n_2}$	$\bar{k_1}$	$\hat{k_1}$	$\bar{k_2}$	$\hat{k_2}$
0.8	0.91	0.91	0.45	0.55	34	52	12	25
0.9	0.57	0.56	0.36	0.40	28	44	14	26
1.0	0.38	0.36	0.27	0.29	25	39	16	27
1.1	0.26	0.24	0.20	0.21	24	35	17	28
1.3	0.13	0.12	0.12	0.11	23	32	21	31

1. CME employment levels are higher than SME employment levels only over a certain range of the risk aversion parameter. Complete markets allow entrepreneurs and workers to diversify idiosyncratic risks more (than is possible with incomplete markets) and hence has a positive effect on input supply. However as the activity levels in individual sectors go up agents are also exposed to greater risks (as shocks are multiplicative). These cannot be completely diversified away even with complete markets because of the presence of aggregate risks. There is thus a trade off between risk diversification and generation of additional risks in the Benchmark model. When risk aversion is not too high the trade off is resolved in favor of higher employment in a CME.

2. For reasons explained in the previous paragraph the gains from market completion are higher when shocks are negatively correlated than when they are positively so (aggregate risks increase by larger amounts as the activity levels go up). Table 2 presents summary of the case $\eta_2 = \{1, 4, 1, 2, 2\}$, so that shocks are positively correlated. As we see, differences in employment levels are lower compared to the previous case. Also the band over which these gains are positive is smaller.

Conclusion

The paper develops a general equilibrium model with incomplete asset markets to discuss the influence of two risk related factors on labor contracts - namely risk aversion and market incompleteness. We have proved the existence of equilibrium for this general model assuming competition in the labor market. The comparative statics of risk aversion has been proved analytically for CRRA utility and

Cobb-Douglas production functions. As we pointed out, the method shown can be extended relatively easily for many other utility functions of the HARA class for which closed form solutions for demand functions are available. The comparative statics of market incompleteness is difficult to prove analytically because the first order equations are not directly comparable. However an intuition has been given for the numerical results. The model and main results of the paper has a practical implication. Low paying, productively inefficient outside options for workers may be attractive from the risk sharing point of view. The policy implications of this observation has not been explored in this paper and is a subject for future research. The paper also does not address the economically more interesting but mathematically less tractable case(s) of non-competitive labor markets. An extreme example (and therefore a first cut) of a non-competitive situation is discussed in another paper (Roy, 1999).

5 Appendix

5.1

The foc of the jth household is given by,

$$\sum_{s=1}^{S} \frac{\pi_s}{\pi_0} u'(m_0^j)(W_s^j(\boldsymbol{\pi}) - G_s) - c'(l_j) = 0 \tag{32}$$

where W_s^j is a function of $\{\frac{\pi_s}{\pi_0}\}_{s=1}^S$ and m_0^j is function of $\{\frac{\pi_s}{\pi_0}\}_{s=1}^S$ and β.

$$\frac{\partial(u'(m_0^j(\beta,.))}{\partial\beta} = (m_0^j(\beta,.))^{-\beta}(-\ln m_0^j(\beta,.) - \beta\frac{m_0^{j'}(\beta,.)}{m_0^j(\beta,.)}) \tag{33}$$

where $m_0^{j'}(\beta,.)$ represents the partial derivative with respect to β.

$$\frac{\partial(u'(m_0^j(\beta,.))}{\partial\beta} \leq 0, \text{ iff } -\beta\frac{m_0^{j'}(\beta,.)}{m_0^j(\beta,.)} \leq \ln m_0^j(\beta,.)$$

which is clearly satisfied when $m_0^{j'}(\beta,.) \geq 0$. When $m_0^{j'}(\beta,.) < 0$, the inequality requires,

$$\left|\frac{\partial \ln m_0^j(\beta,.)}{\partial\beta}\right| \leq \frac{\ln m_0^j(\beta,.)}{\beta}$$

We shall assume this inequality to be strictly satisfied because then $\tilde{l}_j(\boldsymbol{\pi}, \boldsymbol{G}, \beta)$ is strictly decreasing in β.

The sign of $m_0^{j'}(\beta, .)$ depends on the behavior of the individual terms $(\frac{\rho_s \pi_0}{\pi_s})^{1/\beta}$.

Since $m_s^j = \rho_s m_0^j \frac{\boldsymbol{\pi}_0}{\boldsymbol{\pi}_s}$ (see Appendix 5.2 below), we have,

$$\frac{\partial}{\partial \beta}(\frac{m_s^j}{m_0^j}) \geq 0 \text{ iff } (\frac{\rho_s \pi_0}{\pi_s})^{1/\beta} \leq 1, \ < 0 \text{ otherwise}$$

Whether m_0^j and $(\frac{m_s^j}{m_0^j})$ increases with β or not (the signs of these are crucial for the comparative statics propositions) in the neighborhood of the fixed point $(\boldsymbol{l}^*, \boldsymbol{\pi}^*, \boldsymbol{G}^*)$ depends on the magnitudes of $\{\pi_s^*\}_{s=0}^S$.

5.2

For CRRA utility functions, from the foc's of household l, the demand functions can be shown to be,

$$\begin{aligned} m_0^j &= \frac{w_0^j + \sum_{s=1}^S \frac{\boldsymbol{\pi}_s}{\boldsymbol{\pi}_0}(W_s^j + W_s^j l_j + (1-l_j)G_s)}{1 + \sum_{s=1}^S (\rho_s)^{\frac{1}{\beta}} (\frac{\boldsymbol{\pi}_s}{\boldsymbol{\pi}_0})^{1-\frac{1}{\beta}}} \\ m_s^j &= (\rho_s)^{\frac{1}{\beta}} (\frac{\boldsymbol{\pi}_0}{\boldsymbol{\pi}_s})^{\frac{1}{\beta}} m_0^j \end{aligned}$$

when $\beta \neq 1$. When $\beta = 1$, these are given by,

$$\begin{aligned} m_0^j &= \frac{1}{2}(w_0^j + \sum_{s=1}^S \frac{\boldsymbol{\pi}_s}{\boldsymbol{\pi}_0}(W_s^j + W_s^j l_j + (1-l_j)G_s) \\ m_s^j &= \rho_s m_0^j \frac{\boldsymbol{\pi}_0}{\boldsymbol{\pi}_s} \end{aligned}$$

Since production functions are Cobb-Douglas $W_s^j l_j = \alpha Y_j^p(s, l_j, k_j)$. If G_s is approximately constant with respect to l_j and wage differentials between private and public firms $(W_s^j - G_s)$ is always positive, m_0^j is positively related to l_j.

When $\beta = 1$, clearly m_0^j is increasing in $\frac{\boldsymbol{\pi}_s}{\boldsymbol{\pi}_0}$. When $\beta < 1$, a rise in $\frac{\boldsymbol{\pi}_s}{\boldsymbol{\pi}_0}$ reduces the denominator and increases the numerator. So m_0^j increases. When $\beta > 1$, both the numerator and the denominator move in the same direction, so the difference in the rate of increase of the two will determine the direction. Thus the rate of growth of the numerator which is equal to state income in s relative to total income, will determine whether m_0^j increases or not.

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