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# The Performance of Forecast-Based Monetary Policy Rules under Model Uncertainty

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# 1 Introduction

In recent years there has been a surge of quantitative studies evaluating alternative strategies for monetary policy in a wide variety of macroeconomic models.[1] The strategies considered in this literature include rules for setting the policy instrument—usually a short-term nominal interest rate—in response to recent outcomes for a limited number of key variables, for example output and inflation such as the well-known Taylor (1993) rule, rules for setting the interest rate in response to a forecast of inflation, and so-called "forecast targeting" rules that assign a loss function to deviations of the forecast from target. While *outcome-based rules* express the interest rate as an explicit function of available information, *forecast-based rules* are equilibrium relations which require a forecasting model in order to generate an interest rate prescription. Thus, the success of a specific forecast-based rule in actual practice is likely to depend on the accuracy of the underlying forecasting model. The quantitative evaluations in the literature, however, have typically restricted attention to the authors' preferred macroeconomic model, and may have advocated rules that perform well in that specific model but would perform quite poorly in other models.

In this paper, we compare the performance of outcome- and forecast-based rules in four different macro-econometric models of the U.S. economy and investigate directly how robust these rules are to model uncertainty. In doing so we build on earlier work regarding the robustness of simple versus complicated outcome-based rules in Levin, Wieland and Williams (1999). The four models that we consider are the Fuhrer-Moore (1995) model (henceforth referred to as the FM model), the MSR model of Orphanides and Wieland (1998), Taylor's (1993b) Multi-Country Model (henceforth TAYMCM) and the FRB staff model (cf. Brayton et al. 1997a). All four models incorporate the assumptions of rational expectations[2]

[1]Recent contributions to this literature include among others Ball (1999), Batini and Haldane (1999), Black et al. (1998), Fuhrer (1997), Laxton, Isard and Eliasson (1999), Lowe and Ellis (1997), McCallum and Nelson (1998), Orphanides (1999), Orphanides, Small, Wieland and Wilcox (1998), Rudebusch and Svensson (1999), Rotemberg and Woodford (1997), (1999), Tetlow and von zur Muehlen (1998) and Williams (1999).

[2]Policy rule analysis using traditional backward-looking models is particularly prone to the Lucas Critique (1976), but to the extent that our models are interpreted as reduced-form specifications of optimizing models it applies to them too.

short-run nominal inertia, and long-run monetary neutrality, but differ in many other respects, including the dynamics of output and inflation, the degree of disaggregation, model size and estimation period.

As far as forecast-based rules are concerned, we focus on instrument rules, which set the interest rate as an explicit function of the model-based inflation forecast. Of course, deriving this forecast also requires an assumption regarding future policy. Both, constant-interest rate as well rule-consistent forecasts have been considered in the literature. In the following we focus on rule-consistent forecasts, which are the optimal forecast within the confines of each model. We do not study another type of forecast-based rule, which has been discussed in the literature under the name of "forecast targeting rules".[3]

Forecast-based instrument rules seem to have received considerable attention at several inflation-targeting central banks that have adopted an explicit inflation-targeting framework. For example, researchers at the Bank of England[4], the Bank of Canada[5] and the Reserve Bank of Australia[6] have advocated such forecast-based interest rate rules. Specific parameterizations of inflation-forecast-based instrument rules are being used both at the Bank of Canada and the Reserve Bank of New Zealand[7] in economic forecasting and policy analysis. Recent work by Clarida, Gali and Gertler(1997), (1998) and Orphanides (1998) suggests that an interest rate rule that responds to inflation and output forecasts as well as the lagged interest rate provides a good description of federals funds rate behavior in the U.S.A. over the 1980s and 1990s.

The potential advantages of forecast-based interest rate rules, and in particular rules that respond exclusively to an inflation forecast are summarized nicely by Batini and Hal-

[3]The distinction between these two types of forecast-based rules is made very clearly in Rudebusch and Svensson (1999) and Svensson (1999). "Forecast targeting" rules in the terminology of Rudebusch and Svensson assign a loss function over deviations of the forecast from target. For a given model, such a "targeting" rule defines an optimal interest rate reaction function, which sets the interest rate in response to all relevant state variables of the model. In this respect, it is similar to the complicated outcome-based rules we investigated in Levin, Wieland and Williams (1999), albeit with a different loss function.

[4]See Haldane (1995), Batini and Haldane (1999) and Batini and Nelson (1999))

[5]See Black, Macklem and Rose (1998) and Amano, Coletti and Macklem (1999)

[6]See de Brouwer and Ellis (1998)

[7]See Coletti, Hunt, Rose and Tetlow (1996) and Black et al.(1997) respectively.

dane (1998) from the Bank of England, who note that these rules are lag-, information- and output-encompassing. In other words, forecast-based rules are preemptive and can account for policy transmission lags (*lag-encompassing*), by responding to forecasts of inflation these rules may well embody all relevant information on the state of the economy (*information-encompassing*), and given an appropriate combination of response coefficient and forecast horizon these rules may be effective in stabilizing both, inflation and output (*output-encompassing*). Furthermore, proponents of forecast-based interest rate rules have argued that they are more robust than outcome-based interest rate rules such as the Taylor rule, because the inflation forecast will also take into account information regarding the structure of the economy (see Isard, Laxton and Eliasson (1999)).

We reconsider these proposed advantages of forecast-based interest rate rules in our four models using the same methodology as in Levin, Wieland and Williams (1999).[8] We assume the objective of policy is to minimize the weighted sum of the unconditional variances of the inflation rate and the output gap (the percent deviation of GDP from its potential level). In addition, we allow interest rate volatility to enter into the policymakers' optimization problem. The funds rate is set according to a time-invariant policy rule. For a given model and a particular class of policy rules, we first determine the region of the parameter space for which the policy rules generate unique equilibria. Then, we compute the policy frontier for this class of rules, which traces out the best obtainable outcomes in terms of inflation, output, and funds rate volatility. We refer to the policy rules underlying such a frontier as "optimal" in the sense that these rules represent solutions to the specified constrained optimization problem. Finally, we evaluate robustness to model uncertainty by taking the rules that perform well in one model and assessing their performance in each of the other three models. We consider both, the case where the rules make use of rule- and model-

[8]In the past, this type of policy rule analysis using rational expectations models was hampered by the computational cost in solving and computing moments of models with more than a small number of equations. Analysis in large-scale models was generally limited to comparing a small set of policy regimes as in Bryant (1989), Bryant (1993), and Taylor (1993b). Increases in computer speed and the development of efficient solution algorithms have made the computation of optimal policies in large linear rational expectations models feasible.

consistent forecasts, and the case where the forecast is based on an the wrong model of the economy.

The remainder of this paper proceeds as follows. Section 2 provides a brief description of the four models. Section 3 reviews alternative forecast-based interest rate rules from the literature and presents the macroeconomic outcomes resulting from each of these rules in our four models. The methodology for evaluating the performance of policy rules is outlined in section 4. Section 5 compares the performance and properties of optimized simple forecast-based and outcome-based policy rules. Section 6 proves an assessment of the robustness of these rules under model uncertainty. Conclusions then follow. Two appendices provide a detailed summary of the structure of the four different models and a mathematical description of the evaluation methodology.

## 2 The Dependence of Forecasts on Model Structure

The four models that we consider in this paper share some important similarities. For example, in each model expectations in financial, labor and goods markets are formed in a forward-looking (rational) manner taking into account the policy rule pursued by the central bank. Furthermore, all models are characterized by short-run nominal inertia, typically due to some form of wage or price contracts, which induces short-run real effects of monetary policy. In the long-run however, monetary policy is neutral and all four models incorporate a long-run vertical Phillips curve.

The four models differ in important ways, however, making them useful tools for evaluating the robustness of alternative monetary policy rules. The specification of wage and price dynamics; the level of aggregation for expenditures, prices, employment and the external sector; the forward-looking elements of the expenditure block as well as the estimation period and methodology all differ across the four models. These differences are summarized in **Table 1** [to be added] and discussed in further detail in **Appendix 1** of this paper.

As a consequence of differences in specification and parameter estimates, the behavior of output and inflation in the four models differs substantially even when monetary policy is assumed to follow the same rule in each model. To illustrate these differences, we compute the unconditional autocorrelations of output and inflation as well as the impulse responses of these two variables to an interest rate shock in each model assuming a common benchmark policy rule. The rule we use for this purpose is an interest rate reaction function, which was estimated using quarterly U.S. data over the sample period 1980Q1 to 1998Q4:[9]

$$r_t = -\underset{(.31)}{.28} + \underset{(.061)}{.755}\, r_{t-1} + \underset{(.111)}{.603}\, \pi_t^{(4)} + \underset{(.254)}{1.176}\, y_t - \underset{(.234)}{.972}\, y_{t-1} + u_{i,t} \tag{1}$$

$$\bar{R}^2 = .930,\ DW = 2.50$$

where $r_t$ is the federal funds rate, $\pi_t^{(4)}$ is the four-quarter moving average of the inflation rate, and $y_t$ is the current output gap.

With interest rate behavior determined by this reaction function, we compute the dynamic properties inflation and output in each model using the solution methods described in Section 3 below. The inflation autocorrelograms are depicted the in the upper left panel of **Figure 1**. Inflation is highly persistent in the FM and MSR models, a feature of the overlapping relative wage contract that Fuhrer and Moore (1995) have emphasized. The FRB model exhibits somewhat less inflation persistence, but by far the least degree of inflation persistence is found in TAYMCM, in which the inflation autocorrelogram falls below zero after only four quarters. Even when combined with some inertia in price markups, the staggered nominal wage contract specification in TAYMCM delivers relatively low inflation persistence. The right panel of **Figure 1** depicts the output gap autocorrelograms for each model. In the FM model, the output gap is extremely persistent and displays some "over-shooting" in that the autocorrelation turns negative after five years. The FRB model output gap displays considerably less persistence and slightly more over-shooting than the FM model. In the MSR and TAYMCM model, which use similar aggregate demand specifications, the degree of output gap persistence relatively low. Given these differences in serial correlation patterns, inflation and output forecasts generated in the four models will differ substantially even under the same policy rule.

[9] This rule is close to the one used in Levin, Wieland and Williams (1999) and estimated by Orphanides and Wieland (1998). We have simply extended the estimation period by two years up to the end of 1998.

In designing a forecast-based interest rate rule that performs well, the time horizon for the relevant forecast is a key policy parameter, which depends crucially on the transmission lags of monetary policy. The impulse responses of output and inflation to a one-percentage-point interest rate shock to the estimated reaction function shown in the lower panels of **Figure 1** indicate the differences in policy transmission lags across our four models. The lag until policy has its maximum effect, the magnitude of this effect, and its persistence differ quite a bit across the models. As shown in the lower right panel, the maximum effect of the increase in interest rates on output in TAYMCM and MSR occurs relatively quickly, in about three quarters, after the original policy shock. Output then returns close to potential by the end of the second year. The maximum effect is much larger in the FRB and TAYMCM models than in the FM and MSR models. There is a slight overshooting effect in MSR in the third year. In FM and FRB the maximum effect on output is only achieved by the middle of the second year. While the output response is of similar persistence, the magnitude of the effect is more than twice as big in the FRB model. The time lag until the maximum effect of interest rate changes is felt on inflation, is, not surprisingly, a bit longer than in the case of the output gap. As shown in the lower left panel of **Figure 1**, the maximum effect on inflation in the TAYMCM and MSR models is achieved in the fourth quarter, while it takes 6 and 10 quarters respectively in the FM and FRB models. The effect on inflation in the latter two models is also more persistent.

These differences in inflation and output dynamics and the impact of policy changes across models suggest that the four models are well-suited to assess the robustness of alternative forecast-based rules under model uncertainty. Forecasts of inflation and output and estimates of the impact of policy changes on these variables will be quite different in each model even starting from the same state of the economy.

## 3 Alternative Forecast-Based Interest Rate Rules

In this paper, we follow the literature on forecast-based interest rate rules and focus primarily on rules which respond to the forecasts of inflation and the output gap, and allow for some degree of interest-rate smoothing. This class of forecast-based rules *(FB rules)* is defined by

$$r_t = \rho r_{t-1} + (1-\rho)(r^* + E_t\pi_{t+J}) + \alpha(E_t\pi_{t+J} - \pi^*) + \beta E_t y_{t+K}. \tag{2}$$

These rules imply that the short-term nominal interest rate $r_t$ (the federal funds rate in the case of the U.S.) is determined as a linear function of the lagged interest rate $r_{t-1}$, the J-period ahead forecast of inflation $\pi_{t+J}$ (annualized) and the K-period ahead forecast of the output gap $y_{t+K}$. $r^*$ denotes the unconditional mean of the equilibrium real interest rate and $\pi^*$ is the inflation target. We consider two different measures for inflation that have been used in the existing literature on forecast-based rules: the four-quarter change in the price level $\pi^{(4)}$ and the annualized one-quarter change in the price level $\pi^{(1)}$. Through most of this paper we assume that forecasts of inflation and output are formed using the model itself with the policy rule in operation, that is, forecasts are rule- and model-consistent. By assuming away systematic forecast errors, we believe, we are erring on the side of favoring forecast-based rules. Later, we consider cases where the forecasts are generated using a misspecified model. We do not consider rules where forecasts are based on an exogenous path for the interest rate as in Rudebusch and Svensson (1999).

Before investigating rules that are optimal in our models, however, we will consider a number of forecast-based interest rate rules that have received attention in the literature, and particularly at inflation-targeting central banks. These rules are summarized in **Table 2**. Most but not all of these rules are nested in specification (2). The rules differ with regard to the forecast horizons, the degree of partial adjustment, the policy instrument variable, the measure of inflation and the measure of the resource gap if it is included at all.

The first two rules are from Batini and Haldane (1999), who analyzed rules in a small forward-looking open-economy model of the Bank of England that is calibrated to match U.K. data. The first rule (BH-1) is the inflation-forecast-targeting rule that they use as a benchmark for comparison. It uses an 8-quarter horizon forecast of the quarterly inflation rate, a low degree of partial adjustment, and defines the instrument as the short-term real interest rate. The second rule (BH-2), which responds much more aggressively to a shorter-horizon inflation forecast, was found to be particularly effective in stabilizing both output and inflation in the model used by Batini and Haldane. The next two rules are taken from Amano, Coletti and Macklem (1999) from the Bank of Canada. In this case the policy instrument is defined in terms of the difference of the short-term and long-term nominal interest rates. Both rules respond to an 8-quarter-ahead forecast of the four-quarter inflation rate with no partial adjustment. The first rule, (ACM-1), is apparently used regularly as a reference rule in the Bank of Canada's Quarterly Model. ACM-2 is a more aggressive rule that is found particularly effective at stabilizing output and inflation in Amano et al. (1999).

The next three rules use shorter forecast horizons of 1 year or less on inflation and include a response to the output or unemployment gap. The first rule (BE-1) is taken from de Brouwer and Ellis (1998), who found this rule near-efficient in a small model of the Australian economy. The other two rules are from Isard, Laxton and Eliasson (1999), who found them to perform well in a small moderately-nonlinear model of the U.S. economy with a time-varying NAIRU. The following three rules,(RS-1, RS-2, RS-3), are due to Rudebusch and Svensson (1999) and quite similar to the BH-rules, except that the instrument is the nominal short rate. Rudebusch and Svensson found that these rules, which use inflation forecast horizons of 8, 12 and 16 quarters and allow for partial adjustment, performed very well compared to a range of alternative rules in a small back-ward-looking model of the U.S. economy.

The remaining three rules, that are due to Clarida, Gali and Gertler (1998) and Orphanides (1998), have been estimated based on U.S. data for the 1980s and 1990s rather than optimized in some model. The estimation for CGG-1 and CGG-2 is 87:3 to 96:4 and 82:3 to 96:4, respectively. The estimate obtained by Orphanides (1998) is based on real-time data on Federal Reserve staff forecasts of inflation and output gaps. All three estimates suggest that interest-rate setting in the 1980s and 1990s in the U.S. is well described by a systematic response to inflation and output forecasts of a horizon of four quarters or less and allowing for some degree of partial adjustment. For comparison, Table 2 also reports some outcome-based rules, such as the rules due to Taylor (1993) and Henderson-McKibbin (1993) as well as two simple rules that we found to be optimal in the FRB model and robust across the MSR, FM and TAYMCM models.

Table 3 summarizes the macroeconomic outcomes in terms of the standard deviations of the output gap, the quarterly inflation rate and the change of the federal funds rate in the four different models. While each of the outcome-based rules yields a unique rational expectations equilibrium in all four models, this is not always the case for the forecast-based rules. As can be seen from Table 3 many of the rules that use longer-horizon forecasts such as ACM-1, ACM-2, and the RS rules induce indeterminacy and thus multiple equilibria, both in the MSR and the TAYMCM model.[10] This result provides a first clear warning sign against the use of rules with forecast horizons that are significantly beyond one year. The only model in which all rules deliver unique stable rational expectations equilibria is the FM model, which, as noted in section 2, exhibits the highest degree of output and inflation persistence. We analyze in detail the stability conditions for forecast-based rules in the four models in section 5.

The various rules deliver very different outcomes in terms of output, inflation and interest rate variability. The differences are related to the choice of forecast horizon, the inclusion or exclusion of a resource gap, the choice of inflation measure, the degree of partial adjustment, and the parameters of the rules. For the remainder of the paper, we systematically examine the characteristics of efficient forecast-based rules in the four models and compare their performance and robustness to those of outcome-based rules.

[10]Results from the FRB model are still incomplete. The FRB model includes a number of highly persistent variables–stocks of capital goods and housing, government debt, and net foreign assets–absent from the other three models (and nearly all rational expectations models used for policy rule evaluation). These variables interact with forecast-based policy rules in a way that in some cases causes apparently reasonable policy rules to yield multiple equilibria. This does not occur in the case of outcome-based rules. We are currently investigating this issue.

# 4 Methodology for Evaluating Policy Rules

We evaluate the performance of alternative policy rules using policy frontiers, which summarize the best attainable outcomes in terms of output and inflation variability for a given class of policy rules. We present the policy frontiers associated with forecast-based and outcome-based rules in inflation-output variability space with each curve corresponding to a particular constraint on the variability of the first-difference of the funds rate. Throughout this analysis, we only consider policy rules that generate a unique stationary rational expectations solution. As the discussion of alternative forecast-based rules in the preceding section indicated, not all rules are associated with a unique equilibrium. There are cases which lead to explosive behavior and therefore no equilibrium exists, and there are cases with multiple equilibria. For this reason, we explore a wide range of values for the response coefficients $(\rho, \alpha, \beta)$ and forecast horizons $(J, K)$ to identify regions of instability, before computing optimal policy rules.

In evaluating a specific policy rule in anyone of the four models, we proceed as in Fuhrer (1997) by computing the reduced form representation of the saddle point solution of the model and then evaluating an analytic expression for the unconditional second moments of the model variables. A detailed mathematical description of this methodology is provided in **Appendix 2**. For linear models, this approach yields accurate results far more efficiently than simulation-based methods.[11] We compute the unique stationary rational expectations solution of these models using the Anderson and Moore (1985) implementation of the Blanchard and Kahn (1980) method. Based on the reduced form of this solution we then compute the unconditional contemporaneous covariance matrix of the endogenous variables iteratively using a version of the doubling algorithm described in Hansen and Sargent (1997).

For a given functional form of the interest rate rule, we assume that the parameters of

[11] To take advantage of these methods, we have constructed a linearized version of TAYMCM and a log-linear version of the FRB model. We have found that these approximations have negligible effects on the relevant dynamic properties of the two models.

the rule (denoted by $H$) are chosen to solve the following optimization problem:

$$\begin{aligned} \underset{H}{\text{Min}} \quad & \lambda Var(y_t) + (1-\lambda) Var(\pi_t) \\ \text{s.t.} \quad & \tilde{x} \;=\; P(H)\tilde{x}_{t-1} \;+\; Q(H)e_t \\ \text{and} \quad & Var(\Delta r_t) \leq k^2 \end{aligned} \tag{3}$$

where $y_t$ indicates the output gap, $\pi_t$ indicates the inflation rate, and $Var(s)$ indicates the unconditional variance of variable $s$. The weight $\lambda \in [0, 1]$ reflects the policymaker's preference for minimizing output volatility relative to inflation volatility. For the remainder of the paper, we use the letter $V$ to denote the square-root of the objective function for a given policy rule, and $V^*$ to denote the square-root of the minimized objective, that is, the constrained optimal value of $V$. The first set of constraints represents the reduced form of the respective rational expectations model with vector of state variables $\tilde{x}$ and the vector of shocks $e_t$. The reduced-form parameter matrices denoted by P(.) and Q(.) are functions of the response coefficients, $H$, of the monetary policy rule. In addition, we constrain the level of interest rate volatility by imposing the upper bound $k$ on the standard deviation of the first-difference of the federal funds rate. The benchmark value of $k$ is set equal to the funds rate variability under the estimated policy rule given in equation (1).[12]

To compute the policy frontier of each model for a particular functional form of the interest rate rule, we determine the parameter values in the rule which maximize the objective function for a set of values of the weight $\lambda$ over the range zero to unity. Thus, for a given form of the interest rate rule, the policy frontier of each model traces out the best obtainable combinations of output and inflation volatility, subject to the upper bound on funds rate variability. Henceforth, when we construct a policy frontier subject to the benchmark constraint that funds rate variability does not exceed that of the estimated rule, we refer to the resulting policy frontier as an E-frontier.

The constraint on interest volatility plays an important role in our analysis. All four

[12] This benchmark value of $k$ differs across the four models, in part because each model has been estimated over a different sample period and as a result generates a different amount of funds rate volatility for the same policy rule.

models exhibit a tradeoff between interest rate variability on the one side and inflation and output variability on the other side, even at levels of interest rate variability significantly above those implied by estimated policy rules or observed in the data. That is, the variability of output and inflation can be reduced by using highly aggressive rules, but such rules also induce wild fluctuations in interest rates. However, in the case of optimized simple rules which incorporate some degree of partial adjustment ($\rho > 0$) the improvement in output and inflation stabilization that is possible is rather small.[13] Furthermore, linear policy rules which generate highly volatile interest rates are often not implementable in practice, because they would frequently prescribe violations of the non-negativity constraint on the federal funds rate.[14] Finally, the hypothesized invariance of the estimated model parameters to changes in policy rules is unlikely to hold true under policies that are so dramatically different (in terms of funds rate volatility) from those seen during the sample periods over which the models were estimated. For these reasons we use the above-mentioned constrained in interest rate volatility in deriving policy frontiers and focus our attention on rules that feature relatively moderate levels of interest rate volatility similar to what has been observed in the U.S. economy. We then analyze the sensitivity of our results to changes in interest rate variability.

# 5 The Performance of Forecast-Based Rules

## 5.1 Stability Analysis

One concern regarding policy rules is that they may lead indeterminacy (multiple equilibria) or explosiveness (no equilibrium). In our past work on outcome-based rules, we found that, for reasonable values of $\rho$ and $\beta$, a value of $\alpha$ slightly above zero was sufficient to guarantee a unique rational expectations solution in all four models. The results reported in **Table 3**

[13]See also the discussion in Levin, Wieland and Williams (1999) and Sack and Wieland (1999).

[14]In principle, we could analyze non-linear rules that incorporate this lower bound on interest rates (see Orphanides and Wieland (1998) and Reifschneider and Williams (1999)), but doing so would substantially increase the computational costs of our analysis.

suggest that this is not the case for all forecast-based rules.

The existence of multiple equilibria in FM, MSR, and TAYMCM depends on the forecast horizon and the parameters of the policy rule, with shorter forecast horizons reducing the regions of instability. **Figure 2** shows stability regions for the four models, assuming there is no response to the output gap ($\beta = 0$). For this figure and the next two, the policy rules are assumed to respond to the four-quarter change in the price level, $\pi^{(4)}$. For FM, MSR, and TAYMCM, the dark shaded regions indicate the combinations of $\rho$ and $\alpha$ where multiple equilibria exist when the forecast horizon for inflation is 20 quarters. The other contours indicate the lower bounds of the regions of multiple equilibria for shorter forecast horizons (indicated by the number on the chart). For example, in the FM model, the combination of a forecast horizon of 16 quarters, $\rho = 0$, and $\alpha = 20$ is stable, but yields multiple equilibria if $\alpha$ exceeds about 21. For these three models, the basic pattern of the contours is the same: long-horizon inflation forecasts are only stable if the inflation response parameter is small or the gradual adjustment parameter is large. As noted, the analysis of the stability properties of FRB under forecast-based results is still underway and results reported in this figure and the next two figures on FRB should be disregarded until further analysis is completed.

The regions of instability in FM, MSR, and TAYMCM generally shrink the more policy responds to the output gap. **Figure 3** repeats the same analysis as in the previous figure, but with the coefficient on the output gap set to 2. In the MSR and TAYMCM models, the regions of instability have moved up and to the left, relative to the case of no response to the output gap. In these models, adding a response to the output gap aids stability. For the FM model, the regions of instability changed, but no clear general improvement in stability resulted. **Figure 3** repeats the same analysis as in the previous figure, but with the coefficient on the output gap set raised to 5. The regions of instability in the FM, MSR, and TAYMCM models have vanished for values of $\alpha$ below 3, even with a forecast horizon of 20 quarters.

## 5.2 Optimized Forecast-Based and Outcome-Based Rules

In Levin, Wieland and Williams (1999) we found that simple outcome-based rules *(OB Rules)* with optimized response parameters, that is rules of the type of (2) with the forecast horizons $J$ and $K$ constrained at zero, and $\rho$, $\alpha$ and $\beta$ chosen optimally according to the minimization problem defined by (4), perform very well in terms of stabilizing output, inflation and interest rates in our four models. In fact, complicated rules which respond to all observable state variables, yield only small stabilization gains over such 3-parameter rules and tend to be less robust to model uncertainty than simple rules. These earlier results form a useful benchmark for comparison for the investigation of forecast-based rule in this paper. **Table 4** provides a summary of the characteristics and performance of optimized outcome-based rules in the four models. For each model, we show five different rules that were optimized for the following values of $\lambda = \{0, 0.25, 0.5, 0.75, 1\}$. In every case the relevant frontier is an E-frontier, that is, the variability of the change of the interest rate for any of these rules is no larger than in the case of the estimated rule (1)in the respective model. In all four models the optimized rules incorporate a high degree of partial adjustment. In three out of four models, the coefficient $\rho$ is close to unity. The parameter $\alpha$ which measures the response to inflation deviations from target, typically decreases, and the coefficient $\beta$ increases with $\lambda$ as output stability is weighted relatively more heavily than inflation stability in the objective function. Because we restrict attention to policy rules that generate a unique stationary rational expectations equilibrium, the unconditional variance of inflation is finite even in the case where reducing inflation variability is not an objective ($\lambda = 1$).

To assess the potential advantages of forecast-based rules, in particular their lag-, output- and information-encompassing features we have constructed the policy frontier for rules in which, both, the response coefficients $\rho$, $\alpha$ and $\beta$ and the forecast horizons $J$ and $K$ for the inflation and output forecasts are chosen optimally.

**Figure 5** shows the policy frontiers in each of the four models. In each case, the solid line depicts the frontier for forecast-based rules while the dashed line corresponds to the frontier for outcome-based rules. As expected the frontier is convex to the origin, with truncated vertical and horizontal asymptotes as the objective function in (4) switches from exclusive concern about stabilizing inflation ($\lambda = 0$) towards exclusive concern about stabilizing output ($\lambda = 1$). As can be seen from the four panels in **Figure 5**, optimized forecast-based interest rules yield only very small improvements over simple outcome-based rules in our four models. In each model, the policy frontier associated with forecast-based is rules is only slightly closer to the origin. Evidently, the advertised gains from basing policy on forecasts are oversold; at least in the context of optimal rules in these four models.[15]

Each panel of **Figure 3** also indicates the relative performance of the estimated rule, denoted by the letter $E$. Because the estimated rule generates the same amount of funds rate volatility, comparison of the estimated rule to the policy frontier is straightforward. The estimated rule performs appreciably worse than the frontier rules, despite the fact that the estimated rule incorporates an additional variable (the lagged output gap) compared to the optimized outcome-based rules. As discussed in Levin et al. (1999), the performance under the estimated rule is substantially better than the policy frontier that would be achieved by 2-parameter rules (including Taylor's rule), which do not allow for any degree of interest-rate smoothing ($\rho = 0$). The optimal value of $\rho$, however, turns out to be higher than the estimated value of 0.76 in equation (1), in fact, in three out of the four models it is close to unity.[16]

Given the results in the literature on forecast-based rules, it may seem surprising that we find so little improvement over optimized outcome-based rules. Since other contributions literature have focussed on rules that use the forecast of the quarterly inflation rate $\pi^{(1)}$ rather than the four-quarter moving average $\pi^{(4)}$, we have repeated the above analysis with the quarterly rate as the relevant inflation measure in the rule. Again however, we find

[15] Note this result holds also true in some backward-looking models, e.g. Rudebusch and Svensson (1999), where they find small gains from optimized FB rules relative to optimized simple OB rules.

[16] For a more general discussion of interest rate smoothing and optimal monetary policy see Sack and Wieland (1999).

no major improvement in output and inflation stabilization as we compare outcome- and forecast-based interest rate rules.

We have also investigated to what extent our results are sensitive to the constraint on interest rate variability. So far, we have repeated the above analysis in the FM and MSR models with an upper bound on the standard deviation of the federal funds rate that is twice the value associated with the E-frontier. We find that this allows small improvements in stabilization performance under both, forecast-based and outcome-based rules, but difference between two types of rules remains very small.

These results imply that two of the proposed advantages of forecast-based rules, namely their *lag-encompassing* and *information-encompassing* characteristics are quantitatively unimportant in the four rational expectations model that we consider. First, while much of the literature on inflation forecast targeting has emphasized how important it is to pick the forecast horizon optimally in light of the estimated policy transmission lag, we find little benefit from choosing a forecast horizon greater than zero. This is the case, even though we are careful to allow the optimal forecast horizon to differ for each of the variables and for each different weight in the objective function. Second, as noted in the literature, rule- and model-consistent inflation and output forecasts introduce information on all observable state variables, on the future path of monetary policy, and on the structure of the model into the interest rate rule. Nevertheless, our results show that this information induces no major quantitative improvement in stabilization performance beyond what is achievable by efficient simple rules that respond to recent output, inflation and interest rate outcomes alone. To obtain a better understanding of these results, it is helpful to take a closer look at the characteristics of the forecast-based rules that are optimal in our models.

### 5.3 The Characteristics of Optimized Forecast-Based Rules

***The Optimal Forecast Horizon.*** In all four models, we find that the optimal forecast horizon for FB rules using the 4-quarter change of the price level as measure of inflation, ranges from 0 to 4 quarters. There is no support for horizons greater than one year. As noted above the optimal forecast horizon for both inflation and output is chosen separately and may differ for alternative weights on output and inflation variability in the objective function (4). This is illustrated by the bar charts in **Figure 6** for the case of the FM model. The top panel of **Figure 6** considers the choice of forecast horizons for a policymaker who focuses exclusively on inflation variability ($\lambda = 0$), that is, an "inflation nutter" in the terminology of Mervyn King (1997). There are five groups of bars in different shades of grey, each group associated with a different horizon $J$ of the inflation forecast, ($J = \{0, 1, 2, 3, 4\}$), and each bar within a group associated with a different horizon $K$ of the output gap forecast, ($K = \{0, 1, 2, 3\}$). The height of each bar corresponds to the standard deviation of inflation under a rule with those forecast horizons and optimized response coefficients. This bar chart illustrates that the optimal combination of forecasts is a 1-quarter-ahead forecast of the four-quarter moving average of inflation and the current value of the output gap. Of course, as shown in the bottom panel of **Figure 6** this combination depends on the relative weight $\lambda$. Once some weight is put on reducing output variability ($\lambda = 0.25$), the optimal combination of forecasts in the FM model turns out to be a 4-quarter-ahead forecast of the output gap and the current value of inflation.[17]

**Table 5** reports the optimal forecast horizons and response parameters for rules that use the four-quarter moving average of inflation, $\pi^{(4)}$, as the relevant inflation measure, and correspond to the forecast-based rules frontiers in **Figure 5**. **Table 6** reports on rules that use the quarterly inflation rate, $\pi^{(1)}$. Outcomes for five values of the the weight $\lambda$ are shown. Recall, that in all cases the variability of the change of the interest rate is constrained to be no larger than in the case of the estimated rule (1)in the respective model.

As far as the rules using $\pi^{(4)}$ are concerned, in most cases the best choice regarding inflation is to respond to its current value not a forecast. Exceptions include the TAYMCM model, where it tends to be optimal to respond to the two-quarter-ahead inflation forecast and cases in the FM, MSR and FRB models when all the weight in the objective function

[17]Note that the height of the bars in lower panel of **Figure 6** represents the value of the square root of the objective function, which in this case depends on both, inflation and output volatility.

is put either on inflation or output variability respectively. As to output forecasts, the optimal horizon is one quarter in TAYMCM and usually also in MSR, two quarters in the FRB model, and four quarters in the FM model (with the exception of the extreme cases, $\lambda = 0$ and $\lambda = 1$).

As shown in **Table 6** the results regarding the optimal forecast horizon for rules that use the quarterly inflation rate $\pi^{(1)}$ differ somewhat form the $\pi^{(4)}$ rules. In several cases we find longer forecast horizons to be optimal. For example, in the case of the FM model, the optimal output gap forecast to be used in the rule looks 7 quarters ahead. In the other four models however, the optimal forecast horizons for output and inflation remain between 0 and 4 quarters ahead. A comparison of the last two columns in **Table 6** shows that forecast-based rules that use the quarterly inflation perform just as well as forecast-based rules which use the four-quarter moving average of inflation. Differences in the expected losses are negligible.

***The Optimal Response Coefficients.*** Comparing the results for forecast-based rules in **Tables 5** and **6** to the results for outcome-based rules in **Table 4**, we find that the optimal value of partial adjustment parameter $\rho$ is larger in forecast- than in outcome-based rules. For example, in the case of the FM model, $\rho$ tends to be close to unity for forecast-based rules that use the four-quarter moving average as the relevant measure of inflation. In the other models the optimal value is even above unity, typically between 1.1 and 1.3. The optimal partial adjustment coefficients for $\pi^{(1)}$ rules tend to be even larger than for $\pi^{(4)}$ rules. Of course, in a model with backward-looking or adaptive expectations such rules would lead to explosive behavior as emphasized for example in Rudebusch and Svensson 1999). However, our finding of optimal values for $\rho$ above unity coincides with similar findings in other rational expectations models such as Rotemberg and Woodford (1999).

The optimal parameters on output and inflation in forecast-based rules tend to be quite a bit larger than in outcome-based rules, even though the constraint on interest-rate variability in the optimization exercise is the same. That the response to a forecast of, say, inflation, is larger than to the current observation results from its tendency to revert back to steady state. Except for the case when $\lambda = 0$ optimal forecast-based rules feature a nontrivial positive coefficient on the output gap.

The last two columns in **Table 5** compare the expected loss under the best forecast-based rule (with $\pi^{(4)}$) to the loss under the best outcome-based rules for each model and each of the five values of the weight $\lambda$. Not surprisingly, given the frontiers shown in **Figure 3**, the expected loss for forecast-based rules is never much below the loss for outcome-based rules. To the extent that improvements are possible they are below 10% of the loss under outcome-based rules.

***Inflation Forecast-Based Rules and Output.*** In the results presented above, optimal forecast-based rules possess an economically significant response to the output gap, except when the objective is to stabilize only inflation ($\lambda = 0$). A topic that has received much attention in the literature on forecast-based rules concerns the need to include the output gap explicitly in the interest rate rule. It has been argued that a rule, which responds exclusively to the inflation forecast with a suitable choice of horizon may be effective at stabilizing, both, output and inflation, even without an explicit response to the output gap. To assess the potential stabilization loss that would result from excluding the output gap from the rule, we replicate the optimization exercise, but this time constrain the coefficient $\beta$ on the output gap to zero.

Except for the extreme case where output stabilization receives no weight in the loss function, rules that do not respond to the output gap perform significantly worse than rules which do include a response to the output gap: We find no support for the *output-encompassing* characteristic of inflation-forecast-based rules. **Table 7** reports the outcomes from the experiment of optimally choosing $\alpha, \rho, J$, and $K$, subject to the constraint $\beta = 0$. The comparison of the last two columns indicates that the expected loss under inflation-forecast based rules that do not include a response to the output gap is generally more than 20% larger than under the best comparable outcome-based rule in all four models. The exceptions typically occur when the weight on output stabilization in the objective function is very small or nonexistent.

Optimal forecast-based rules that are constrained to have no response to the output gap are in most cases characterized by inflation forecast horizons generally much longer than those of optimal forecast-based rules that do respond to the output gap. When the output gap is excluded from the rule, the optimal forecast horizon in the FM model is from 2–5 years, with the latter figure an imposed upper bound on the analysis. In the TAYMCM, FRB, and MSR models the optimal forecast horizon exceeds one year for some weights on inflation stabilization.

# 6 The Robustness of Outcome- and Forecast-Based Rules

To evaluate the extent to which a specific class of policy rules are robust to model uncertainty we take rules from the associated policy frontier of one model, and evaluate the performance of these rules in each of the other three models. For comparison, we evaluate both the robustness of forecast-based as well as outcome-based rules.

***Simple Outcome-Based Rules.*** **Table 8** documents the robustness of outcome-based rules. The results confirm our earlier finding in Levin et al (1999) that simple outcome-based rules are remarkably robust.[18] The table is organized as follows. The first five rows consider efficient outcome-based rules from the Fuhrer-Moore model and assess their performance in the other three models, first in the FRB model, then in the MSR model, and finally in TAYMCM. Each of the rows is associated with a different value of the weight $\lambda$ in the objective function, $\lambda \in [0, 0.25, 0.5, 0.75, 1]$.

The first set of three columns summarizes the performance of outcome-based rules from FM in FRB. The first among these three columns with the heading $\sigma(\Delta r)$ shows the standard deviation of the first-difference of the funds rate that obtains under these rules in the FRB model. The second column with the heading $V_{OB}$ shows the expected loss resulting from these rules in FRB. Finally, the third column with the heading $V^*_{OB}$ reports the expected loss under the best possible outcome-based rule in the FRB model given the

[18]Note the only change in this exercise concerns the constraint on federal funds rate volatility that enforces a standard deviation of the funds rate no larger than under the estimated policy rule. This constraint has changed slightly since we have re-estimated that rule including two additional years worth of data.

same value of $\lambda$ and using the standard deviation of the first-difference of the funds rate reported in the first column as a constraint in the optimization exercise. In other words, the difference between the two expected losses measures the deterioration in performance that occurs when the policymaker believes the Fuhrer-Moore model to be an appropriate representation of the true economy and picks an efficient policy rule accordingly, while the true economy in fact corresponds to the FRB model.

Looking further at the performance of efficient outcome-based rules from the FM model in MSR and TAYMCM, we conclude that these rules are quite robust to model uncertainty. Except for two cases the deterioration in performance due to a mistaken belief regarding the correct model of the economy always remains below 10%. A similar evaluation of efficient outcome-based rules from FRB, MSR and TAYMCM respectively, that is summarized in the remaining three sets of five rows in **Table 8** indicates that efficient outcome-based rules from those models, are also quite robust to model uncertainty. In other words, these rules perform surprisingly well compared to the best possible outcome-based rules in the respective model chosen to represent the true economy.

***Forecast-Based Rules.*** In order to assess the robustness of forecast-based rules some additional issues need to be considered. While outcome-based rules simply respond to observed data, the forecasts entering forecast-based rules require a model of the economy. Regarding robustness, a forecast-based rule that is efficient in one model may perform badly in another model, either because the response coefficients and forecast horizon are suboptimal, or, because the forecast itself is based on the wrong model. In a first step, we consider the case where the rule is taking from one model (the "policymaker's model") and simulated in another model (the "true economy model"), but the forecast that enters the rule is still generated from the "true economy model" model. In a second step, we investigate the consequences from using model-inconsistent forecasts. In other words, we simulate the rule that was optimized for the "policymaker's model" in the "true economy model" assuming that the forecasts that enter the rule are generated by the "policymaker's model" based on the data that is observed in the "true economy model".

***With Model-Consistent Forecasts.*** **Table 9** reports summary results on the robustness of forecast-based rules which use the four-quarter moving average as the relevant measure of inflation, ($\pi^{(4)}$) and use model- and rule-consistent forecasts. The results in the table are organized in exactly the same manner as the results regarding outcome-based rules in **Table 8**. Of particular interest again is the difference between the column titled $V_{FB4}$ and the column titled $V_{OB}$. The two columns compare the loss under a forecast-based rule with coefficients and forecast horizon that were optimized in the wrong model to the best possible outcome-based rule in the true model. Overall, efficient forecast-based rules from each model tend to perform slightly worse in the other three models than was the case for the outcome-based rules discussed previously in **Table 8**. There are a few more cases where the expected loss is more than 10% above the best comparable outcome-based rule in the true model and in a few cases even 20% above that. Examples, for which forecast-based rules are somewhat less robust than outcome-based rules, include FM rules in the MSR model, MSR rules in the FM model, and FRB rules in the TAYMCM. Nevertheless, in most cases the performance of the forecast-based rules is still relatively close to the outcome-based rules benchmark. Overall, these results suggest that optimized rules with model-consistent forecasts are reasonably robust to model uncertainty within our four models. In the end this may not be too surprising since the optimal forecast horizon in our models is very short. Thus efficient forecast-based rules are not that different from the efficient outcome-based rules, which we found to be robust.

We also assess the robustness of forecast-based rules that use the quarterly inflation rate as relevant inflation measure. As noted earlier this alternative has been considered in several contributions to the literature on forecast-based rules. **Table 10** summarizes the outcome of this investigation. $\pi^(1)$ rules turn out to be somewhat less robust than $\pi^(4)$ rules. In several cases the expected loss is more than 20% above the outcome-based benchmark rule. In fact, there are even some cases where no unique rational expectations equilibrium exists. For example, in the case for efficient forecast-based rules from the Fuhrer-Moore model, when the true economy is the MSR model. The rules which lack robustness use a long-term (at least for our models), that is 7-quarter-ahead, output forecast.

***With Model-Inconsistent Forecasts.*** The above robustness exercise, in which the policymaker uses an inefficient rule that was optimized in the wrong model, but still generates the correct forecast using the "true" model, may be biased in favor of forecast-based rules. The alternative assumption is that the forecast is also generated using the wrong model. Our strategy for evaluating robustness of forecast-based rules in this case proceeds as follows. First, we solve the model, which takes the role of "policymaker model", for its reduced form given the policy rule optimized for this model. This reduced form includes the reduced-form equations determining the forecasts of inflation and output, which enter the policy rule, as a function of the models state variables. We then add these reduced-form forecast equations to the model, which plays the role of "true economy". The inflation and output forecasts in the policy rule are then computed based on the forecast taken from the "policymaker's model" and the data generated in the "true economy" model.

Since the above procedure requires that the state variables from the "policymaker model" also appear in the "true economy" model, we start by considering the smallest one, that is the FM model, as "policymaker model". Table 11 reports the outcomes that we obtain when we evaluate optimized FB4 rules from FM in the MSR model, using the FM model to generate the forecast that enters the rule. In this case, we find that the FM rules perform noticeably worse in the MSR model than the best MSR rule which generates a comparable degree of interest rate variability. For the different values of the weight on output- versus inflation variability in the objective function, $\lambda$ the performance deterioration is between 8 and 18 percent. However, comparing these results with the outcomes reported in **Table 9**, we find that the loss in performance is of the same order of magnitude as in the case where we evaluated FM rules in the MSR model using model-consistent forecasts. We obtained similar results when applying the optimal rules and forecasts from FM to the FRB model (not reported). According to these two examples, model misspecification in forecasting does not appear to significantly reduce the performance of optimal forecast-based rules of horizons of one year or less. Presumably, such misspecification causes greater harm for policy rules with forecast horizons of 2 or more years. We are currently investigating this further.

# 7 Conclusion

Our analysis yields several conclusions regarding the potential advantages and drawbacks of forecast-based interest rate rules. First, in all four models, we find that forecast-based rules yield at best only small benefits in stabilizing inflation, output and interest rates relative to optimized outcome-based rules that respond to inflation, the output gap and the lagged interest rate. This is even true in the two large-scale models, which contain literally hundreds of state variables and allow for significant lags until the maximum effect of a policy change on the economy is felt. Thus, as far as the potential advantages of forecast-based rules are concerned, neither the lag- nor the information-encompassing feature turn out to be of quantitative importance. Second, even if output stabilization only receives a small weight in the policy objective, constraining the policy rule not to respond directly to output causes a significant deterioration in performance. Only if minimizing inflation variability is the sole policy objective, that is if the policymaker is an "inflation nutter" in the terminology of King (1997), we find pure inflation-forecast-based rules to be efficient. Third, we find that rules which are associated with forecast horizons of two or more years, often are not associated with a stable unique rational expectations equilibrium in our four models. In these cases we find the possibility of multiple equilibria due to self-fulfilling expectations. Outcome-based rules however, are generally associated with unique equilibria in our models, as log as they imply a nominal interest rate response to inflation of more than one for one in the long run.

Regarding the robustness properties of forecast-based rules, we obtain mixed results. Rules which are based on a rule- and model-consistent forecasts and involve response coefficients and a forecast horizon that have been optimized in one of our models, usually also perform reasonably well in the other three models. However, the forecast horizon is typically fairly short, between one and four quarters ahead, and the relevant measure of inflation is a moving average over four quarters. Forecast-based rules that respond to expectations of inflation more than one year into the future, which includes many rules proposed in the literature, generally are not robust to model uncertainty, owing to the sharp differences in output and inflation persistence in the four models we consider. By considering forecasts that are model-consistent we have biased our analysis in favor of forecast-based rules. Finally, we have evaluated the additional risk, which arises due to the possibility that the forecast itself is generated by an incorrectly specified model. We conclude that proposals that advocate interest rate rules which respond to inflation forecasts alone with a forecast horizon of more than one year should be treated with significant caution.

## Appendix 1: A Comparison of the Four Models

This appendix provides a more detailed description of similarities and differences between the four macroeconomic models, in particular regarding the specification of aggregate demand, aggregate supply and the foreign sector. As discussed previously the basic features of each model, including the level of aggregation, the specification of wage and price dynamics, the forward-looking elements of the expenditure block, and the sample period have been summarized in **Table 1**. The behavioral equations of the FM model were estimated using FIML, while a combination of OLS, 2SLS, and GMM were used in estimating the parameters of the other three models.

**Aggregate Demand.** The FM model represents aggregate spending by a single reduced-form equation corresponding to an IS curve. The current output gap depends on its lagged values over the past two quarters and the lagged value of the long-term real interest rate, which is defined as a weighted average of ex ante short-term real interest rates with maturity equivalent to a 30-year coupon bond. The parameter estimates are taken from Fuhrer (1997a). The FM model does not explicitly include trade variables or exchange rates; instead, net exports (and the relationship between real interest and real exchange rates) are implicitly incorporated in the IS curve equation.

The MSR model disaggregates real spending into five components: private consumption, fixed investment, inventory investment, net exports, and government purchases. The aggregate demand components exhibit partial adjustment to their respective equilibrium levels, measured as shares of potential GDP. Equilibrium consumption is a function of permanent income (discounted 10equilibrium fixed investment is a function of output growth and the real bond rate, and equilibrium inventory investment depends only on output growth. Equilibrium government purchases are a constant share of GDP. Net exports are assumed to be fixed in the simulations reported here.

The TMCM model disaggregates IS components further; for example, spending on fixed investment is separated into three components: equipment, nonresidential structures, and residential construction. The specification of these equations is very similar to that of the more aggregated equations in the MSR model. In TMCM, imports follow partial adjustment to an equilibrium level that depends on U.S. income and the relative price of imports, while exports display partial adjustment to an equilibrium level that depends on foreign output and the relative price of exports. Uncovered interest rate parity determines each bilateral exchange rate (up to a time-varying risk premium); e.g., the expected one-period-ahead percent change in the DM/USDollar exchange rate equals the current difference between U.S. and German short-term interest rates.

The FRB model features about the same level of aggregation as TMCM for private spending but divides government spending into six components, each of which follows a simple reduced-form equation that includes a counter-cyclical term. The specification of most non-trade private spending equations follows Tinsley's (1993) generalized adjustment cost model.

Each component has a specific flow or stock equilibrium condition; for example, equilibrium aggregate consumption is proportional to wealth. Households and businesses adjust their spending in each category according to the solution of a quadratic adjustment cost problem. The resulting spending decision rules are specified as forward-looking error correction equations: the current growth of each spending variable depends on up to three of its own lagged values and on expected future growth in equilibrium spending, and responds negatively to the lagged percent deviation between actual and equilibrium spending levels. Exports and non-oil imports are specified as error-correction processes with long-run income and price elasticities set equal to unity. Uncovered interest rate parity determines the multilateral exchange rate, subject to a sovereign risk premium that moves with the U.S. net external asset position.

**Aggregate Supply.** In FM, MSR, and TMCM, the aggregate wage rate is determined by overlapping wage contracts. In particular, the aggregate wage is defined to be the weighted average of current and three lagged values of the contract wage rate. TMCM follows the specification in Taylor (1980), where the current nominal contract wage is determined as a weighted average of expected nominal contract wages, adjusted for the expected state of the economy over the life of the contract. FM and MSR use the overlapping real contract wage specification proposed by Buiter and Jewitt (1981, 1989) and implemented by Fuhrer and Moore (1995), in which the real contract wage, that is the contract wage deflated by the aggregate wage, is determined as a weighted average of expected real contract wages, adjusted for the expected average output gap over the life of the contract.

In FM and MSR, the aggregate price level is a constant markup over the aggregate wage rate. In contrast, the output price in TMCM follows a backward-looking error-correction specification: current output price inflation depends positively on its own lagged value, on current wage inflation, and on lagged import price inflation, and responds negatively (with a coefficient of -0.2) to the lagged percent deviation of the actual price level from equilibrium. Import prices error-correct slowly to an equilibrium level equal to a constant markup over a weighted average of foreign prices converted to dollars. This partial adjustment of import and output prices imparts somewhat more persistence to output price inflation than would result from staggered nominal wages alone.

The FRB model explicitly models potential output as a function of the labor force, crude energy use, and a composite capital stock, using a three-factor Cobb-Douglas production technology. The equilibrium output price is a markup over a weighted average of the productivity-adjusted wage rate and the domestic energy price. The specification of wage and price dynamics follows the generalized adjustment cost framework used in the FRB IS block. Wage inflation depends on lagged wage inflation over the previous three quarters, as well as expected future growth in prices and productivity, and a weighted average of expected future unemployment rates. Price inflation depends on its own lagged values over the past two quarters, as well as expected future changes in equilibrium prices and expected future unemployment rates. In addition, both wages and prices error-correct to their respective equilibrium levels. As in the other models, a vertical long-run Phillips curve is imposed in estimation.

Unlike the other three models, the FRB model contains a detailed accounting of various categories of income, taxes, and stocks, an explicit treatment of labor markets, and endogenous determination of potential output. Long-run equilibrium in the FRB model is of the stock-flow type; the income tax rate and real exchange rate risk premium adjust over time to bring government and foreign debt-to-GDP ratios back to specified (constant) levels.

**Foreign Sector.** Neither FM nor MSR explicitly include foreign variables; in contrast, both TMCM and the full FRB staff model include detailed treatments of foreign variables. TMCM features estimated equations for demand components and wages and prices for the other G-7 countries at about the level of aggregation of the U.S. sector. The full FRB staff model includes a total of 12 sectors (countries or regions) which encompass the entire global economy. Because of the size of the model, the cost of solving and computing the moments of the full FRB model is prohibitive. Previous investigations using TMCM suggest that the characteristics of optimal U.S. monetary policies are not greatly affected by the precise specification of the foreign sector (see Levin, Wieland and Williams (1999)). Based on these results and the benefits of reduced computational cost, we replaced the full set of equations describing foreign countries in the FRB staff model with two simple reduced form equations for foreign output and prices.

## Appendix 2: Our Methodology for Computing Policy Frontiers of Linear Rational Expectations Models

In this appendix, we indicate the methods used to solve each model and obtain its unconditional moments for a specific interest rate rule. Then we specify the objective function and constraints faced by the monetary policymaker, and describe how to obtain the optimal response parameters for a given type of rule.

**Analyzing a Specific Rule.** Our analysis not only considers the forecast-based rules in equation (1) but allows for a wide variety of rules, in which the federal funds rate may depend on its own lagged values as well as the current, lagged, and expected future values of other model variables. In general, the interest rate rule can be expressed as follows:

$$r_t = \sum_{j=1}^{l} H_j^1 r_{t-j} + \sum_{j=1}^{m} H_j^2 E_t z_{t+j} + \sum_{j=1}^{n} H_j^3 z_{t-j} \tag{4}$$

where $r_t$ is the federal funds rate, and the vector $z$ is a set of model variables that enter the interest rate reaction function. The lagged funds rate coefficients are given by $H_j^1 (j = 1 \text{ to } l)$, while the coefficients on other model variables are given by the vectors $H_j^2 (j = 1 \text{ to } m)$ and $H_j^3 (j = 0 \text{ to } n)$. Henceforth we will refer to the combined set of coefficients by the vector $H = \{H^1, H^2, H^3\}$. After discussing how to compute the moments of each model for a specific value of $H$, we will consider the problem of determining the optimal value of $H$ for a given choice of the elements of $z$ and the lead and lag orders $l$, $m$, and $n$.

As in Fuhrer (1997), we analyze the performance of a specified policy rule in each model by computing the reduced form representation of the saddle point solution and then evaluating an analytic expression for the unconditional second moments of the model variables. For linear models, this approach yields accurate results far more efficiently than simulation-based methods. To take advantage of these methods, we have constructed a linearized version of TMCM and a log-linear version of the FRB model; however, these approximations have negligible effects on the relevant dynamic properties of the two models.

Thus, each of the four models can be written in the following form:

$$\sum_{j=1}^{M} A_j E_t x_{t+j} + \sum_{j=1}^{N} A_j x_{t-j} + C e_t \;=\; 0 \tag{5}$$

where x is the vector of all model variables, and e is a vector of serially uncorrelated disturbances with mean zero and finite covariance matrix $\Omega$ . The interest rate reaction function comprises a single row of equation (4), while the remaining rows contain the structural equations of the model. Thus, the parameters of the interest rate rule are contained in one row of the coefficient matrices $A_j (j = 1 \text{ to } M)$ and $B_j (j = 0 \text{ to } N)$, while this row of $C$ is identically equal to zero.

We compute the unique stationary rational expectations solution to equation (4) using the Anderson and Moore (1985) implementation of the Blanchard and Kahn (1980) method, modified to take advantage of sparse matrix functions. The reduced form of this solution can be expressed as follows:

$$x_t \;=\; \sum_{j=1}^{N} D_j(H) x_{t-j} + F(H) e_t \tag{6}$$

where the reduced-form coefficient matrices $D_j (j = 1 \text{ to } N)$ and $F$ depend on the monetary policy parameters $H$ as well as the structural parameters of the model. By defining the vector $\tilde{x_t} = (x_{t-1}, ...., x_{t-N})$, we can express this solution in companion form:

$$\tilde{x}_t \;=\; P(H)\tilde{x}_{t-1} + Q(H) e_t \tag{7}$$

Then the unconditional contemporaneous covariance matrix for $\tilde{x}_t$, denoted by $V_0$, is given by:

$$V_0 \;=\; \sum_{j=0}^{\infty} P^j Q \Omega Q' P'^j \tag{8}$$

Using the implicit expression $V_0 \;=\; P V_0 P + Q \Omega Q$ , we compute $V_0$ iteratively using the doubling algorithm described in Hansen and Sargent (1997), modified to take advantage of sparse matrix functions. Given $V_o$, the autocovariance matrices of $\tilde{x}_t$ are readily computed using the relationship $V_j = P^j V_0$ .

**The Optimization Problem.** For a given functional form of the interest rate rule, we assume that the interest rate rule is chosen to solve the following optimization problem:

$$\begin{array}{ll} \underset{H}{\text{Min}} & \lambda Var(y_t) + (1-\lambda) Var(\pi_t) \\ \text{s.t.} & \tilde{x} \;=\; P(H)\tilde{x}_{t-1} \;+\; Q(H) e_t \\ \text{and} & Var(\Delta r_t) \leq k^2 \end{array} \tag{9}$$

where $y_t$ indicates the output gap, $\pi_t$ indicates the inflation rate, and $Var(s)$ indicates the unconditional variance of variable $s$. The weight $\lambda \in [0, 1]$ reflects the policymaker's preference for minimizing output volatility relative to inflation volatility. We constrain the level of interest rate volatility by imposing the upper bound $k$ on the standard deviation of the first-difference of the federal funds rate; as discussed below, the benchmark value of $k$ is set equal to the funds rate volatility under the estimated policy rule given in equation (2). Finally, throughout our analysis, we only consider policy rules that generate a unique stationary rational expectations solution.

To compute the policy frontier of each model for a particular functional form of the interest rate rule, we determine the parameters of this rule which maximize the objective function for each value of l over the range zero to unity. Thus, for a given form of the interest rate rule, the policy frontier of each model traces out the best obtainable combinations of output and inflation volatility, subject to the upper bound on funds rate volatility. This approach differs slightly from that commonly found in the literature, in which interest rate volatility is incorporated into the objective function and each policy frontier is drawn using a different weight on interest rate volatility. The standard approach combines information about model-imposed constraints on policy with policymakers' preferences regarding funds rate volatility, whereas we prefer to maintain a strict distinction between the policymaker's preferences and the constraints implied by the model.

To obtain a benchmark value of $k$ for each model, we obtain the rational expectations solution generated by the estimated policy rule in equation (2), and then we compute the standard deviation of the one-quarter change in the funds rate associated with this rule. It should be noted that this benchmark value of $k$ differs across the four models, in part because each model has been estimated over a different sample period and as a result generates a different amount of funds rate volatility for the same policy rule. For example, the moments for the FRB and TMCM models depend in part on shocks from the 1970's, a period of substantial economic turbulence, while the shocks for the FM and MSR models are from the relatively tranquil 1980's and early 1990's. Henceforth, when we construct a policy frontier subject to the benchmark constraint that funds rate volatility does not exceed that of the estimated rule, we refer to the resulting policy frontier as an E-frontier.

For a particular functional form of the interest rate rule, we determine the policy frontier by solving the optimization problem in equation (8) for a range of values of the objective function weight $\lambda$. For a given value of $\lambda$, we start with an initial guess for the rule parameters, obtain the reduced-form solution matrices $G$ and $H$, compute the unconditional moments, and calculate the value of the objective function; then a hill-climbing algorithm is applied which iteratively updates the parameter vector until an optimum is obtained.

Thus, to determine a single policy frontier, it is necessary to compute hundreds or even thousands of rational expectations solutions at alternative values of the policy rule parameters. Given our objective of performing a systematic analysis of policy frontiers for a wide range of functional forms of the interest rate rule, it is essential to make use of the highly efficient solution algorithms outlined above. On a Sun Ultra Enterprise 3000 computer about as fast as an Intel Pentium II 300 Mhz computer only a few CPU seconds are needed to solve and compute the moments of a small-scale model like FM or MSR, while solving a large-scale macroeconometric model like TMCM or the FRB model requires about five CPU minutes.

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**Table 1: Comparison of Model Specifications**

TO BE ADDED

For a detailed comparison see appendix 1.

**Table 2: Alternative Forecast-Based Interest Rate Rules**

**Generalized Interest Rate Rule**

$$rate_t = \rho rate_{t-1} + (1-\rho)(r^* + E_t\pi_{t+I}) + \alpha(E_t\pi_{t+J} - \pi^*) + \beta(E_t gap_{t+K}))$$

| Authors | Instrument | Partial Adj. | | Inflation | | | Resource Gap | | |
|---|---|---|---|---|---|---|---|---|---|
| | *rate* | $\rho$ | $I$ | *measure* | $\alpha$ | $J$ | *measure* | $\beta$ | $K$ |
| Batini & Haldane (1999) | | | | | | | | | |
| BH-1 | $r_t - E_t\pi_{t+1}$ | .5 | 1 | quarterly | .5 | 8 | - | - | - |
| BH-2 | $r_t - E_t\pi_{t+1}$ | .5 | 1 | quarterly | 5.0 | 5 | - | - | - |
| Amano, Coletti & Macklem (1999) | | | | | | | | | |
| ACM-1 | $r_t - r_t^l$ | - | - | 4-quart.ave. | 3.0 | 8 | - | - | - |
| ACM-2 | $r_t - r_t^l$ | - | - | 4-quart.ave. | 9.0 | 8 | - | - | - |
| de Brouwer & Ellis (1998) | | | | | | | | | |
| BE-1 | $r_t$ | 0 | 4 | 4-quart.ave. | 2.8 | 4 | output | 1.0 | 4 |
| Isard, Laxton & Eliasson (1999) | | | | | | | | | |
| ILE-1 | $r_t$ | 0 | 4 | 4-quart.ave. | 0.8 | 0 | unemp. | .3 | 0 |
| ILE-1 | $r_t$ | 0 | 4 | 4-quart.ave. | 1.5 | 3 | unemp. | .0 | 0 |
| Rudebusch & Svensson (1999) | | | | | | | | | |
| RS-1 | $r_t$ | 0.62 | 8 | 4-quart.ave. | 1.97 | 8 | - | - | - |
| RS-2 | $r_t$ | 0.71 | 12 | 4-quart.ave. | 3.31 | 12 | - | - | - |
| RS-3 | $r_t$ | 0.47 | 16 | 4-quart.ave. | 7.8 | 16 | - | - | - |
| Clarida, Gali & Gertler (1998) | | | | | | | | | |
| CGR-1 | $r_t$ | 0.67 | 4 | 4-quart.ave. | 0.32 | 4 | output | .026 | 0 |
| CGR-2 | $r_t$ | 0.84 | 4 | 4-quart.ave. | 0.27 | 4 | output | .026 | 0 |
| Orphanides (1998) | | | | | | | | | |
| AO-1 | $r_t$ | 0.67 | 4 | 4-quart.ave. | 0.21 | 4 | output | .1 | 4 |
| Memo: Outcome-Based Rules | | | | | | | | | |
| Taylor (1993) (TAY) | $r_t$ | 0 | 0 | 4-quart.ave. | 0.5 | 0 | output | 0.5 | 0 |
| Henderson & McKibbin (1993) (HM) | $r_t$ | 0 | 0 | 4-quart.ave. | 1 | 0 | output | 2 | 0 |
| Levin, Wieland & Williams(1993) | | | | | | | | | |
| LWW-1 | $r_t$ | 1 | 0 | 4-quart.ave. | 1.3 | 0 | output | 0.6 | 0 |
| LWW-2 | $r_t$ | 1 | 0 | 4-quart.ave. | 0.8 | 0 | output | 1.0 | 0 |
| Estimated Rule (see (1)) (EST) | | | | | | | | | |

## Table 3: Performance of Alternative Forecast-Based (FB) Rules

| Rule | FM | | | MSR | | | FRB | | | TAYMCM | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\sigma_y$ | $\sigma_\pi$ | $\sigma_{\Delta r}$ | $\sigma_y$ | $\sigma_\pi$ | $\sigma_{\Delta r}$ | $\sigma_y$ | $\sigma_\pi$ | $\sigma_{\Delta r}$ | $\sigma_y$ | $\sigma_\pi$ | $\sigma_{\Delta r}$ |
| BH-1 | 2.79 | 2.68 | 1.15 | 1.15 | 0.67 | 0.27 | - | - | - | 3.44 | 3.27 | 1.39 |
| BH-2 | 2.88 | 2.55 | 1.23 | 1.16 | 0.64 | 0.28 | 6.89 | 1.60 | .44 | 3.27 | 3.01 | 1.41 |
| ACM-1 | 2.66 | 2.45 | 2.16 | m.e. | m.e. | m.e. | - | - | - | m.e. | m.e. | m.e. |
| ACM-2 | 3.08 | 2.05 | 4.72 | m.e. | m.e. | m.e. | - | - | - | m.e. | m.e. | m.e. |
| BE-1 | 2.71 | 2.30 | 2.76 | 1.23 | 0.48 | 0.47 | 3.55 | 1.53 | 2.30 | 2.56 | 2.12 | 2.78 |
| ILE-1 | 2.90 | 2.51 | 1.07 | 1.11 | 0.65 | 0.25 | 4.37 | 1.55 | .58 | 3.10 | 2.76 | 0.76 |
| ILE-2 | 3.03 | 2.30 | 2.13 | 1.20 | 0.55 | 0.46 | 5.80 | 1.41 | 1.30 | 2.91 | 2.50 | 2.12 |
| RS-1 | 2.98 | 2.15 | 1.40 | m.e. | m.e. | m.e. | 5.27 | 1.34 | 0.79 | m.e. | m.e. | m.e. |
| RS-2 | 2.86 | 2.17 | 1.28 | m.e. | m.e. | m.e. | 4.96 | 1.26 | 1.00 | m.e. | m.e. | m.e. |
| RS-3 | 2.66 | 2.29 | 1.78 | m.e. | m.e. | m.e. | 4.97 | 1.25 | 1.85 | m.e. | m.e. | m.e. |
| CGG-1 | 2.96 | 2.45 | 0.71 | 1.12 | 0.61 | 0.15 | 5.48 | 1.56 | 0.39 | 3.25 | 2.71 | 0.76 |
| CGG-2 | 3.15 | 2.22 | 0.57 | 1.11 | 0.51 | 0.11 | 4.81 | 1.46 | .26 | 3.19 | 2.27 | 0.50 |
| AO-1 | 2.75 | 2.73 | 0.61 | 1.11 | 0.70 | 0.14 | - | - | - | 3.39 | 2.96 | 0.73 |
| Memo: Outcome-Based (OB) Rules | | | | | | | | | | | | |
| TAY | 2.68 | 2.63 | 0.75 | 0.99 | 0.70 | 0.30 | 2.92 | 1.86 | 0.90 | 2.89 | 2.58 | 1.58 |
| HM | 2.08 | 2.81 | 1.50 | 0.73 | 0.66 | 0.89 | 1.75 | 1.89 | 2.00 | 2.09 | 2.04 | 4.17 |
| LWW-1 | 3.78 | 1.85 | 1.97 | 0.84 | 0.40 | 0.34 | 2.12 | 1.46 | 1.22 | 2.33 | 1.73 | 1.71 |
| LWW-2 | 2.37 | 2.45 | 1.83 | 0.58 | 0.53 | 0.48 | 1.41 | 1.65 | 1.22 | 1.95 | 1.79 | 2.01 |
| EST | 2.85 | 2.23 | 0.94 | 0.99 | 0.49 | 0.61 | 2.75 | 1.57 | 1.24 | 2.59 | 2.04 | 2.63 |

**Notes:** Results from FRB incomplete, pending futher analysis of stability conditions under forecast-based policy rules.

**Table 4: Characteristics and Performance of Optimal OB Rules**

| Model | Weight | Parameters | | | Performance | | | |
|---|---|---|---|---|---|---|---|---|
| | $\lambda$ | $\rho$ | $\alpha$ | $\beta$ | $\sigma_{\Delta r}$ | $\sigma_y$ | $\sigma_\pi$ | $V_{OB}^*$ |
| | 0.00 | .94 | .52 | .11 | .937 | 1.85 | 4.17 | 1.85 |
| | 0.25 | .88 | .54 | .37 | .937 | 2.13 | 2.78 | 2.32 |
| FM | 0.50 | .87 | .39 | .54 | .937 | 2.50 | 2.21 | 2.37 |
| | 0.75 | .87 | .23 | .61 | .937 | 2.94 | 1.88 | 2.19 |
| | 1.00 | .83 | .01 | .65 | .937 | 5.66 | 1.58 | 1.58 |
| | 0.00 | .89 | 1.41 | .06 | 1.24 | 1.30 | 4.59 | 1.30 |
| | 0.25 | 1.02 | .98 | .92 | 1.24 | 1.59 | 1.55 | 1.58 |
| FRB | 0.50 | 1.02 | .59 | 1.08 | 1.24 | 1.72 | 1.27 | 1.51 |
| | 0.75 | 1.02 | .33 | 1.13 | 1.24 | 1.86 | 1.15 | 1.36 |
| | 1.00 | 1.01 | .05 | 1.15 | 1.24 | 2.31 | 1.08 | 1.08 |
| | 0.00 | .96 | 4.14 | .02 | .606 | .21 | 2.42 | .21 |
| | 0.25 | 1.09 | 1.91 | 1.36 | .606 | .45 | .67 | .51 |
| MSR | 0.50 | 1.06 | 1.06 | 1.36 | .606 | .53 | .55 | .54 |
| | 0.75 | 1.03 | .59 | 1.36 | .606 | .63 | .49 | .53 |
| | 1.00 | .97 | .04 | 1.34 | .606 | 1.16 | .44 | .44 |
| | 0.00 | .92 | 2.77 | .26 | 2.63 | 1.66 | 3.23 | 1.66 |
| | 0.25 | .95 | 1.05 | 1.41 | 2.63 | 1.79 | 1.82 | 1.80 |
| TAYMCM | 0.50 | .95 | .42 | 1.49 | 2.63 | 1.85 | 1.72 | 1.78 |
| | 0.75 | .94 | .14 | 1.51 | 2.63 | 1.87 | 1.70 | 1.74 |
| | 1.00 | .94 | .01 | 1.52 | 2.63 | 1.89 | 1.69 | 1.69 |

**Notes:** $V_{OB}^*$: objective function value for optimal OB rule.

## Table 5: Characteristics and Performance of Optimal FB4 Rules

| Model | $\lambda$ | Parameters | | | | | Performance | | | | Best OB Rule |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $J$ | $K$ | $\rho$ | $\alpha$ | $\beta$ | $\sigma_{\Delta r}$ | $\sigma_y$ | $\sigma_\pi$ | $V^*_{FB4}$ | $V^*_{OB}$ |
| | 0.00 | 1 | 0 | .96 | .51 | .10 | .937 | 1.84 | 4.17 | 1.84 | 1.85 |
| | 0.25 | 0 | 4 | .97 | .86 | .68 | .937 | 2.13 | 2.78 | 2.31 | 2.32 |
| FM | 0.50 | 0 | 4 | 1.00 | .67 | .98 | .937 | 2.48 | 2.21 | 2.35 | 2.37 |
| | 0.75 | 0 | 4 | 1.02 | .43 | 1.11 | .937 | 2.91 | 1.88 | 2.19 | 2.19 |
| | 1.00 | 4 | 0 | .84 | .01 | .66 | .937 | 7.13 | 1.58 | 1.58 | 1.59 |
| | 0.00 | 5 | 4 | 1.40 | 7.68 | .05 | 1.24 | 1.23 | 4.88 | 1.23 | 1.30 |
| | 0.25 | 0 | 2 | 1.16 | 1.63 | 1.46 | 1.24 | 1.57 | 1.46 | 1.54 | 1.58 |
| FRB | 0.50 | 0 | 2 | 1.19 | 1.21 | 1.97 | 1.24 | 1.68 | 1.20 | 1.46 | 1.51 |
| | 0.75 | 0 | 2 | 1.19 | .74 | 2.16 | 1.24 | 1.81 | 1.08 | 1.30 | 1.36 |
| | 1.00 | 0 | 2 | 1.20 | .01 | 2.27 | 1.24 | 3.99 | 1.00 | 1.00 | 1.08 |
| | 0.00 | 0 | 0 | .96 | 4.14 | .02 | .606 | .21 | 2.42 | .21 | .21 |
| | 0.25 | 0 | 1 | 1.25 | 2.91 | 1.92 | .606 | .44 | .66 | .51 | .51 |
| MSR | 0.50 | 0 | 1 | 1.22 | 1.71 | 2.01 | .606 | .52 | .55 | .54 | .54 |
| | 0.75 | 0 | 1 | 1.19 | .99 | 2.03 | .606 | .61 | .49 | .52 | .53 |
| | 1.00 | 3 | 0 | .97 | .05 | 1.35 | .606 | 1.17 | .44 | .44 | .44 |
| | 0.00 | 2 | 0 | 1.04 | .90 | .11 | 2.63 | 1.63 | 3.21 | 1.63 | 1.66 |
| | 0.25 | 2 | 0 | .97 | .33 | 1.28 | 2.63 | 1.78 | 1.83 | 1.80 | 1.80 |
| TAYMCM | 0.50 | 1 | 1 | 1.31 | .38 | 4.93 | 2.63 | 1.85 | 1.70 | 1.78 | 1.78 |
| | 0.75 | 1 | 1 | 1.33 | .21 | 5.10 | 2.63 | 1.87 | 1.69 | 1.73 | 1.74 |
| | 1.00 | 2 | 1 | 1.35 | .10 | 5.20 | 2.63 | 1.90 | 1.68 | 1.68 | 1.69 |

**Notes:** $V^*_{FB4}$: objective function value for the optimal FB4 rule.
$V^*_{OB}$: objective function value for the best comparable OB rule (with the same constraint on interest rate variability).
**Bold**: value of objective is 10-20% greater than that of the best comparable OB rule.
**Bold and Underlined**: value of objective is more than 20% greater than that of the best comparable OB rule.

**Table 6: Characteristics and Performance of Optimal FB1 Rules**

| Model | $\lambda$ | Parameters | | | | | Performance | | | | Best FB4 Rule |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $J$ | $K$ | $\rho$ | $\alpha$ | $\beta$ | $\sigma_{\Delta r}$ | $\sigma_y$ | $\sigma_\pi$ | $V^*_{FB1}$ | $V^*_{FB4}$ |
| | 0.00 | 0 | 0 | .97 | .45 | .10 | .937 | 1.85 | 4.15 | 1.85 | 1.84 |
| | 0.25 | 0 | 7 | 1.27 | 1.95 | 2.07 | .937 | 2.14 | 2.79 | 2.32 | 2.31 |
| FM | 0.50 | 0 | 7 | 1.35 | 2.11 | 3.66 | .937 | 2.49 | 2.24 | 2.37 | 2.35 |
| | 0.75 | 0 | 7 | 1.60 | 1.97 | 5.66 | .937 | 2.94 | 1.89 | 2.20 | 2.19 |
| | 1.00 | 2 | 0 | .84 | .01 | .66 | .937 | 6.80 | 1.58 | 1.58 | 1.58 |
| | 0.00 | 2 | 0 | 1.19 | 3.73 | .03 | 1.24 | 1.23 | 4.88 | 1.23 | 1.23 |
| | 0.25 | 1 | 3 | 1.37 | 2.85 | 2.42 | 1.24 | 1.56 | 1.49 | 1.54 | 1.54 |
| FRB | 0.50 | 1 | 3 | 1.42 | 2.14 | 3.46 | 1.24 | 1.70 | 1.20 | 1.47 | 1.46 |
| | 0.75 | 1 | 3 | 1.41 | 1.32 | 3.74 | 1.24 | 1.83 | 1.08 | 1.31 | 1.30 |
| | 1.00 | 3 | 2 | 1.19 | .01 | 2.26 | 1.24 | 4.72 | 1.00 | 1.00 | 1.00 |
| | 0.00 | 0 | 0 | .92 | 2.22 | -.09 | .606 | .22 | 2.39 | .22 | .21 |
| | 0.25 | 1 | 1 | 1.30 | 3.14 | 2.21 | .606 | .46 | .67 | .52 | .51 |
| MSR | 0.50 | 1 | 1 | 1.26 | 1.76 | 2.20 | .606 | .55 | .55 | .55 | .54 |
| | 0.75 | 1 | 1 | 1.22 | 2.00 | 2.14 | .606 | .64 | .49 | .53 | .52 |
| | 1.00 | 2 | 0 | .97 | .05 | 1.35 | .606 | 1.17 | .44 | .44 | .44 |
| | 0.00 | 1 | 1 | 1.16 | 4.39 | .21 | 2.63 | 1.63 | 3.25 | 1.63 | 1.63 |
| | 0.25 | 1 | 0 | .98 | 1.32 | 1.27 | 2.63 | 1.79 | 1.82 | 1.80 | 1.80 |
| TAYMCM | 0.50 | 1 | 0 | .96 | .55 | 1.43 | 2.63 | 1.85 | 1.71 | 1.78 | 1.78 |
| | 0.75 | 1 | 1 | 1.33 | .77 | 5.00 | 2.63 | 1.88 | 1.69 | 1.74 | 1.73 |
| | 1.00 | 0 | 1 | 1.36 | .33 | 5.25 | 2.63 | 1.90 | 1.68 | 1.68 | 1.68 |

**Notes:** $V^*_{FB1}$: objective function value for the optimal FB1 rule.
$V^*_{FB4}$: objective function value for the best comparable FB4 rule.
**Bold**: value of objective is 10-20% greater than that of the best comparable FB4 rule.
**Bold and Underlined**: value of objective is more than 20% greater than that of the best comparable FB4 rule.

**Table 7: Characteristics and Performance of FB4 Rules where $\beta = 0$**

| Model | $\lambda$ | Parameters | | | Performance | | | | Best OB Rule |
|---|---|---|---|---|---|---|---|---|---|
| | | $J$ | $\rho$ | $\alpha$ | $\sigma_{\Delta r}$ | $\sigma_y$ | $\sigma_\pi$ | $V^*_{FB4(\beta=0)}$ | $V^*_{OB}$ |
| | 0.00 | 9 | 1.21 | 2.34 | .937 | 1.85 | 4.12 | 1.85 | 1.85 |
| | 0.25 | 18 | 1.28 | 19.90 | .937 | 2.08 | 2.99 | 2.34 | 2.32 |
| FM | 0.50 | 18 | .77 | 4.82 | .937 | 2.34 | 2.63 | 2.49 | 2.37 |
| | 0.75 | 20* | .61 | 3.88 | .937 | 2.61 | 2.46 | **2.50** | 2.19 |
| | 1.00 | 20* | .33 | 1.99 | .937 | 3.42 | 2.37 | **<u>2.37</u>** | 1.59 |
| | 0.00 | 5 | 1.37 | 7.44 | 1.24 | 1.23 | 5.00 | 1.23 | 1.28 |
| | 0.25 | 6 | .89 | 4.16 | 1.24 | 1.26 | 4.73 | **<u>2.60</u>** | 1.58 |
| FRB | 0.50 | 6 | .88 | 4.08 | 1.24 | 1.26 | 4.72 | **<u>3.46</u>** | 1.51 |
| | 0.75 | 6 | .87 | 4.04 | 1.24 | 1.27 | 4.72 | **<u>4.14</u>** | 1.36 |
| | 1.00 | 6 | .87 | 4.04 | 1.24 | 1.27 | 4.72 | **<u>4.72</u>** | 1.08 |
| | 0.00 | 0 | .95 | 4.00 | .606 | .21 | 2.44 | .21 | .21 |
| | 0.25 | 5 | -.06 | 5.23 | .606 | .47 | 1.27 | **<u>.76</u>** | .51 |
| MSR | 0.50 | 4 | -.38 | 4.55 | .606 | .59 | 1.18 | **<u>.93</u>** | .54 |
| | 0.75 | 4 | -.52 | 4.19 | .606 | .71 | 1.14 | **<u>1.05</u>** | .53 |
| | 1.00 | 3 | -.62 | 3.57 | .606 | .98 | 1.12 | **<u>1.12</u>** | .44 |
| | 0.00 | 3 | 1.14 | 4.92 | 2.63 | 1.63 | 3.22 | 1.63 | 1.66 |
| | 0.25 | 3 | .73 | 3.40 | 2.63 | 1.72 | 2.68 | **2.00** | 1.80 |
| TAYMCM | 0.50 | 3 | .58 | 3.00 | 2.63 | 1.80 | **<u>2.58</u>** | 2.22 | 1.78 |
| | 0.75 | 6 | .50 | 7.92 | 2.63 | 1.91 | 2.52 | **<u>2.38</u>** | 1.74 |
| | 1.00 | 6 | .45 | 7.36 | 2.63 | 1.93 | 2.52 | **<u>2.52</u>** | 1.69 |

**Notes:** * Value of J constrained by imposed upper bound.
$V^*_{FB4(\beta=0)}$: objective function value for the optimal FB4 rule, with $\beta$ constrained at zero.
$V^*_{OB}$: objective function value for the best comparable OB rule with no constraint on $\beta$.
**Bold**: value of objective is 10-20% greater than that of the best comparable OB rule.
**<u>Bold and Underlined</u>**: value of objective is more than 20% greater than that of the best comparable OB rule.

## Table 8: Robustness of Optimal OB Rules

| OB Rule From | $\lambda$ | in FRB $\sigma_{\Delta r}$ | in FRB $V_{OB}$ | in FRB $V^*_{OB}$ | in MSR $\sigma_{\Delta r}$ | in MSR $V_{OB}$ | in MSR $V^*_{OB}$ | in TAYMCM $\sigma_{\Delta r}$ | in TAYMCM $V_{OB}$ | in TAYMCM $V^*_{OB}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | .53 | 1.39 | 1.33 | .13 | **.36** | .31 | .74 | 1.79 | 1.78 |
| | 0.25 | .72 | 1.68 | 1.60 | .24 | .57 | .54 | 1.09 | 1.95 | 1.92 |
| FM | 0.50 | .79 | 1.65 | 1.54 | .32 | .61 | .58 | 1.29 | 2.00 | 1.95 |
| | 0.75 | .84 | 1.54 | 1.41 | .35 | .61 | .57 | 1.35 | 2.03 | 1.98 |
| | 1.00 | .80 | **1.37** | 1.19 | .37 | .56 | .51 | 1.39 | 2.10 | 1.99 |
| **OB Rule From** | **$\lambda$** | **in FM $\sigma_{\Delta r}$** | **in FM $V_{OB}$** | **in FM $V^*_{OB}$** | **in MSR $\sigma_{\Delta r}$** | **in MSR $V_{OB}$** | **in MSR $V^*_{OB}$** | **in TAYMCM $\sigma_{\Delta r}$** | **in TAYMCM $V_{OB}$** | **in TAYMCM $V^*_{OB}$** |
| | 0.00 | 2.11 | 1.68 | 1.65 | .24 | .27 | .26 | 1.57 | 1.72 | 1.71 |
| | 0.25 | 1.90 | 2.38 | 2.30 | .46 | .53 | .52 | 1.95 | 1.84 | 1.83 |
| FRB | 0.50 | 1.91 | 2.49 | 2.32 | .51 | .56 | .55 | 2.06 | 1.85 | 1.84 |
| | 0.75 | 1.93 | 2.28 | 2.10 | .52 | .54 | .54 | 2.09 | 1.84 | 1.82 |
| | 1.00 | 1.96 | <u>**1.71**</u> | 1.35 | .53 | .46 | .46 | 2.08 | 1.82 | 1.81 |
| **OB Rule From** | **$\lambda$** | **in FM $\sigma_{\Delta r}$** | **in FM $V_{OB}$** | **in FM $V^*_{OB}$** | **in FRB $\sigma_{\Delta r}$** | **in FRB $V_{OB}$** | **in FRB $V^*_{OB}$** | **in TAYMCM $\sigma_{\Delta r}$** | **in TAYMCM $V_{OB}$** | **in TAYMCM $V^*_{OB}$** |
| | 0.00 | 6.16 | **1.64** | 1.48 | 3.72 | 1.30 | 1.26 | 3.98 | 1.66 | 1.63 |
| | 0.25 | 3.19 | 2.47 | 2.30 | 1.85 | 1.59 | 1.57 | 1.94 | 1.84 | 1.80 |
| MSR | 0.50 | 2.52 | 2.47 | 2.31 | 1.53 | 1.52 | 1.52 | 1.67 | 1.84 | 1.79 |
| | 0.75 | 2.30 | 2.25 | 2.08 | 1.42 | 1.36 | 1.34 | 1.59 | 1.81 | 1.86 |
| | 1.00 | 2.12 | **1.55** | 1.32 | 1.42 | 1.05 | 1.05 | 2.39 | 1.74 | 1.74 |
| **OB Rule From** | **$\lambda$** | **in FM $\sigma_{\Delta r}$** | **in FM $V_{OB}$** | **in FM $V^*_{OB}$** | **in FRB $\sigma_{\Delta r}$** | **in FRB $V_{OB}$** | **in FRB $V^*_{OB}$** | **in MSR $\sigma_{\Delta r}$** | **in MSR $V_{OB}$** | **in MSR $V^*_{OB}$** |
| | 0.00 | 3.68 | 1.54 | 1.55 | 2.36 | 1.29 | 1.48 | .31 | .27 | .24 |
| | 0.25 | 2.25 | 2.42 | 2.30 | 2.41 | 1.59 | 1.57 | .64 | .54 | .51 |
| TAYMCM | 0.50 | 2.19 | **2.59** | 2.31 | 2.44 | 1.54 | 1.50 | .66 | .59 | .54 |
| | 0.75 | 2.23 | <u>**2.52**</u> | 2.09 | 2.49 | 1.41 | 1.34 | .67 | **.59** | .52 |
| | 1.00 | 2.27 | **1.47** | 1.31 | 2.54 | 1.04 | 1.03 | .67 | .43 | .43 |

**Notes:** $V_{OB}$: objective function value for the OB rule optimized in the "wrong" model.
$V^*_{OB}$: objective function value for the best possible OB rule in the "true" model.
**Bold**: value of objective is 10-20% greater than that of the reference rule.
<u>**Bold and Underlined**</u>: value of objective is more than 20% greater than that of the reference rule.

## Table 9: Robustness of Optimal FB4 Rules

| FB4 Rule From | $\lambda$ | in FRB $\sigma_{\Delta r}$ | in FRB $V_{FB4}$ | in FRB $V_{OB}^*$ | in MSR $\sigma_{\Delta r}$ | in MSR $V_{FB4}$ | in MSR $V_{OB}^*$ | in TAYMCM $\sigma_{\Delta r}$ | in TAYMCM $V_{FB4}$ | in TAYMCM $V_{OB}^*$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | .50 | 1.38 | 1.33 | .13 | **.36** | .31 | .70 | 1.79 | 1.79 |
| | 0.25 | .90 | 1.63 | 1.59 | .19 | **.62** | .55 | .75 | 2.09 | 1.98 |
| FM | 0.50 | .88 | 1.60 | 1.54 | .19 | **.72** | .61 | .60 | 2.29 | 2.17 |
| | 0.75 | .84 | 1.51 | 1.41 | .19 | **<u>.78</u>** | .62 | .49 | 2.45 | 2.35 |
| | 1.00 | .84 | **1.39** | 1.19 | .37 | **.56** | .51 | 1.42 | 2.09 | 1.99 |

| FB4 Rule From | $\lambda$ | in FM $\sigma_{\Delta r}$ | in FM $V_{FB4}$ | in FM $V_{OB}^*$ | in MSR $\sigma_{\Delta r}$ | in MSR $V_{FB4}$ | in MSR $V_{OB}^*$ | in TAYMCM $\sigma_{\Delta r}$ | in TAYMCM $V_{FB4}$ | in TAYMCM $V_{OB}^*$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 4.30 | 1.76 | 1.61 | .32 | .26 | .24 | 1.43 | 1.73 | 1.71 |
| | 0.25 | 2.04 | 2.36 | 2.30 | .38 | .53 | .53 | 1.07 | 1.94 | 1.92 |
| FRB | 0.50 | 2.23 | 2.41 | 2.31 | .42 | .57 | .56 | 1.01 | 2.02 | 2.02 |
| | 0.75 | 2.30 | 2.23 | 2.08 | .44 | .57 | .55 | .99 | 2.08 | 2.09 |
| | 1.00 | 2.39 | **<u>1.71</u>** | 1.29 | .44 | .52 | .48 | 1.02 | 2.14 | 2.14 |

| FB4 Rule From | $\lambda$ | in FM $\sigma_{\Delta r}$ | in FM $V_{FB4}$ | in FM $V_{OB}^*$ | in FRB $\sigma_{\Delta r}$ | in FRB $V_{FB4}$ | in FRB $V_{OB}^*$ | in TAYMCM $\sigma_{\Delta r}$ | in TAYMCM $V_{FB4}$ | in TAYMCM $V_{OB}^*$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 6.16 | **1.64** | 1.48 | 3.72 | 1.30 | 1.26 | 3.98 | 1.66 | 1.63 |
| | 0.25 | 3.83 | **2.59** | 2.30 | 1.85 | 1.57 | 1.57 | 1.94 | 1.88 | 1.84 |
| MSR | 0.50 | 3.23 | **2.61** | 2.30 | 1.53 | 1.49 | 1.52 | 1.67 | 1.92 | 1.89 |
| | 0.75 | 3.02 | **2.41** | 2.05 | 1.42 | 1.32 | 1.34 | 1.59 | 1.92 | 1.92 |
| | 1.00 | 2.11 | **1.55** | 1.33 | 1.42 | 1.05 | 1.05 | 2.39 | 1.74 | 1.74 |

| FB4 Rule From | $\lambda$ | in FM $\sigma_{\Delta r}$ | in FM $V_{FB4}$ | in FM $V_{OB}^*$ | in FRB $\sigma_{\Delta r}$ | in FRB $V_{FB4}$ | in FRB $V_{OB}^*$ | in MSR $\sigma_{\Delta r}$ | in MSR $V_{FB4}$ | in MSR $V_{OB}^*$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 5.05 | 1.60 | 1.49 | 2.35 | 1.25 | 1.37 | .48 | .24 | .22 |
| | 0.25 | 4.25 | **2.53** | 2.30 | 2.65 | 1.69 | 1.56 | .63 | .56 | .51 |
| TAYMCM | 0.50 | 5.94 | **2.71** | 2.27 | 2.48 | 1.50 | 1.47 | 1.07 | .57 | .52 |
| | 0.75 | 6.16 | **<u>2.43</u>** | 2.00 | 2.52 | 1.28 | 1.29 | 1.09 | .50 | .49 |
| | 1.00 | 6.41 | **<u>2.02</u>** | 1.12 | 2.54 | .93 | .93 | 1.09 | .38 | .37 |

**Notes:** $V_{FB4}$: objective function value for the FB4 rule optimized in the "wrong" model. $V_{OB}^*$: objective function value for the best-possible outcome-based rule in the "true" model.
**Bold**: value of objective is 10-20% greater than that of the reference rule.
**<u>Bold and Underlined</u>**: value of objective is more than 20% greater than that of the reference rule.

## Table 10: Robustness of Optimal FB1 Rules

| FB1 Rule From | $\lambda$ | in FRB $\sigma_{\Delta r}$ | in FRB $V_{FB1}$ | in FRB $V^*_{OB}$ | in MSR $\sigma_{\Delta r}$ | in MSR $V_{FB1}$ | in MSR $V^*_{OB}$ | in TAYMCM $\sigma_{\Delta r}$ | in TAYMCM $V_{FB1}$ | in TAYMCM $V^*_{OB}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | .68 | **1.40** | 1.32 | .14 | **<u>.37</u>** | .30 | .96 | 1.79 | 1.75 |
| | 0.25 | 1.57 | **1.75** | 1.58 | .14 | **<u>.78</u>** | .57 | 2.07 | **<u>2.27</u>** | 1.83 |
| FM | 0.50 | **1.53** | 1.75 | 1.50 | - | **<u>m.e.</u>** | - | 1.89 | **<u>2.54</u>** | 1.86 |
| | 0.75 | 1.16 | **1.63** | 1.37 | - | **<u>m.e.</u>** | - | 1.17 | **<u>2.65</u>** | 2.03 |
| | 1.00 | .84 | 1.40 | 1.19 | .37 | .56 | .51 | 1.42 | 2.08 | 1.98 |

| FB1 Rule From | $\lambda$ | in FM $\sigma_{\Delta r}$ | in FM $V_{FB1}$ | in FM $V^*_{OB}$ | in MSR $\sigma_{\Delta r}$ | in MSR $V_{FB1}$ | in MSR $V^*_{OB}$ | in TAYMCM $\sigma_{\Delta r}$ | in TAYMCM $V_{FB1}$ | in TAYMCM $V^*_{OB}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 4.22 | **<u>1.90</u>** | 1.52 | .40 | .25 | .23 | 1.74 | 1.70 | 1.70 |
| | 0.25 | 2.34 | 2.39 | 2.30 | .30 | .55 | .53 | 1.34 | 1.92 | 1.89 |
| FRB | 0.50 | 2.29 | 2.43 | 2.31 | .35 | .61 | .57 | 1.12 | 2.03 | 1.99 |
| | 0.75 | 2.29 | 2.22 | 2.08 | .36 | .62 | .57 | .96 | 2.13 | 2.10 |
| | 1.00 | 2.44 | **<u>1.73</u>** | 1.29 | .44 | .52 | .48 | .99 | 2.14 | 2.15 |

| FB1 Rule From | $\lambda$ | in FM $\sigma_{\Delta r}$ | in FM $V_{FB1}$ | in FM $V^*_{OB}$ | in FRB $\sigma_{\Delta r}$ | in FRB $V_{FB1}$ | in FRB $V^*_{OB}$ | in TAYMCM $\sigma_{\Delta r}$ | in TAYMCM $V_{FB1}$ | in TAYMCM $V^*_{OB}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 3.68 | **1.70** | 1.54 | 3.01 | 1.35 | 1.41 | 3.96 | 1.68 | 1.63 |
| | 0.25 | 3.54 | **2.66** | 2.30 | 1.79 | 1.60 | 1.57 | 2.28 | 1.83 | 1.82 |
| MSR | 0.50 | 3.49 | **2.69** | 2.29 | 1.52 | 1.52 | 1.50 | 1.91 | 1.88 | 1.86 |
| | 0.75 | 3.11 | **<u>2.48</u>** | 2.05 | 1.44 | 1.35 | 1.35 | 1.74 | 1.89 | 1.89 |
| | 1.00 | 2.11 | **1.55** | 1.33 | 1.42 | 1.05 | 1.05 | 2.39 | 1.74 | 1.74 |

| FB1 Rule From | $\lambda$ | in FM $\sigma_{\Delta r}$ | in FM $V_{FB1}$ | in FM $V^*_{OB}$ | in FRB $\sigma_{\Delta r}$ | in FRB $V_{FB1}$ | in FRB $V^*_{OB}$ | in MSR $\sigma_{\Delta r}$ | in MSR $V_{FB1}$ | in MSR $V^*_{OB}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 5.94 | **1.72** | 1.45 | 2.02 | 1.24 | 1.50 | .45 | .25 | .23 |
| | 0.25 | 2.52 | 2.40 | 2.30 | 1.59 | 1.62 | 1.57 | .58 | .54 | .52 |
| TAYMCM | 0.50 | 2.13 | 2.51 | 2.32 | 1.54 | 1.55 | 1.50 | .63 | .58 | .54 |
| | 0.75 | 6.07 | **<u>2.62</u>** | 2.00 | 2.49 | 1.33 | 1.29 | 1.06 | **.54** | .49 |
| | 1.00 | 6.51 | **<u>2.05</u>** | 1.05 | 2.55 | .93 | .92 | 1.09 | .38 | .37 |

**Notes:** $V_{FB1}$: objective function value for the FB1 rule optimized in the "wrong" model.
$V^*_{OB}$: objective function value for the best-possible outcome-based rule in the "true" model.
**Bold**: value of objective is 10-20% greater than that of the reference rule.
**<u>Bold and Underlined</u>**: value of objective is more than 20% greater than that of the reference rule.

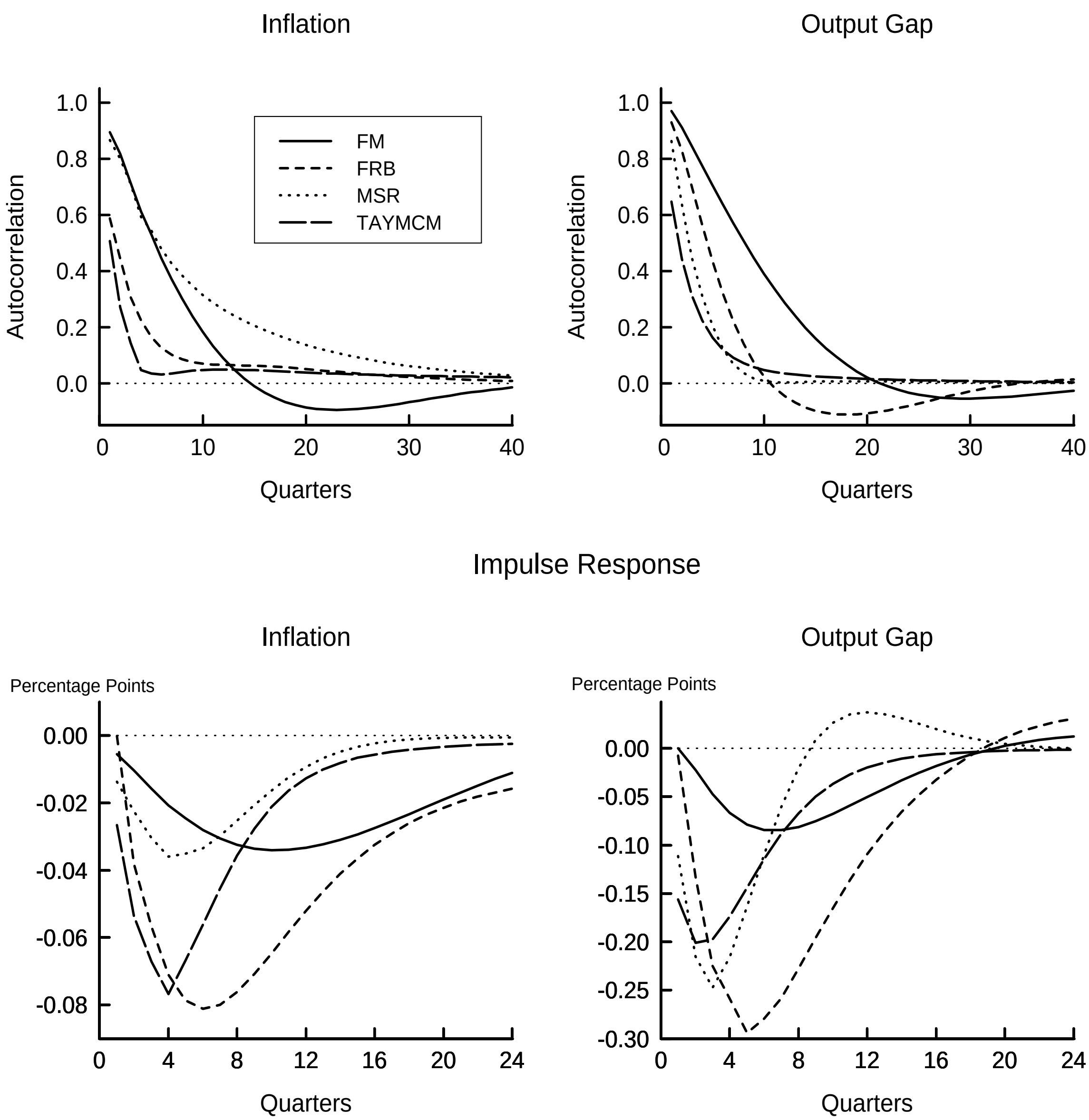
**Table 11: Robustness of FB4 Rules with Model-Inconsistent Forecasts**

| FB4 Rule From | Weight $\lambda$ | Performance in MSR | | | | Best FB4 Rule in MSR | | |
|---|---|---|---|---|---|---|---|---|
| | | $\sigma_{\Delta r}$ | $\sigma_y$ | $\sigma_\pi$ | $V_{FB4}$ | $\sigma_y$ | $\sigma_\pi$ | $V^*_{FB4}$ |
| | 0.00 | 0.13 | 1.15 | 0.36 | **0.358** | 1.51 | 0.30 | 0.304 |
| | 0.25 | 0.35 | 0.75 | 0.49 | 0.566 | 0.71 | 0.45 | 0.524 |
| FM | 0.50 | 0.42 | 0.64 | 0.58 | **0.610** | 0.58 | 0.52 | 0.554 |
| | 0.75 | 0.45 | 0.59 | 0.69 | **0.613** | 0.52 | 0.61 | 0.545 |
| | 1.00 | 0.37 | 0.56 | 1.04 | **0.561** | 0.51 | 0.94 | 0.508 |

**Notes:** $V_{FB4}$: objective function value for FB4 rule optimized in the "wrong" model, using forecasts from the same "wrong" model.
$V^*_{FB4}$: objective function value for the best-possible forecast-based rule in the "true" model.
**Bold**: value of objective is 10-20% greater than that of the reference rule.
**<u>Bold and Underlined</u>**: value of objective is more than 20% greater than that of the reference rule.

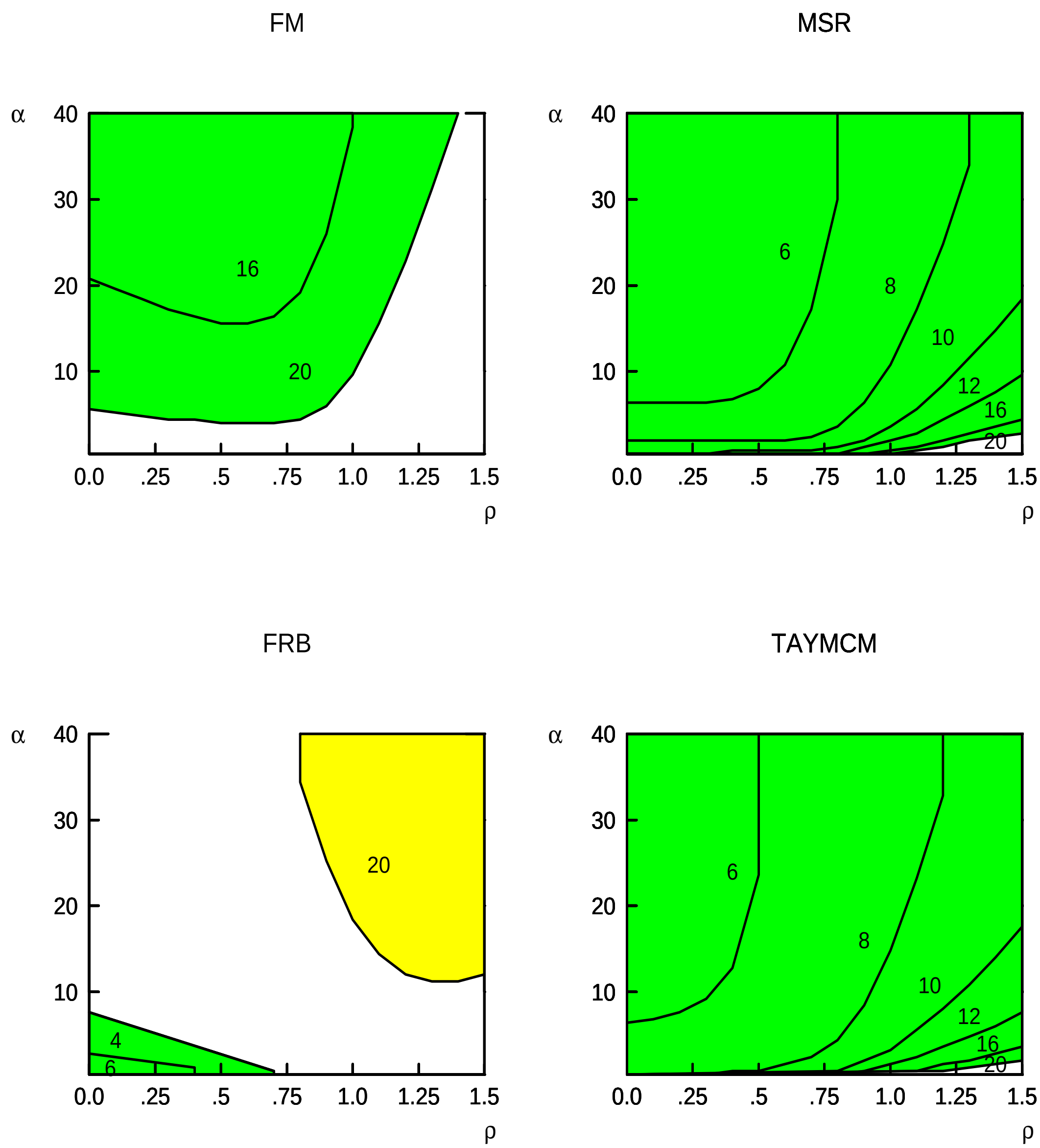
**Figure 1: Output and Inflation Dynamics in the Four Models**

Unconditional Autocorrelations



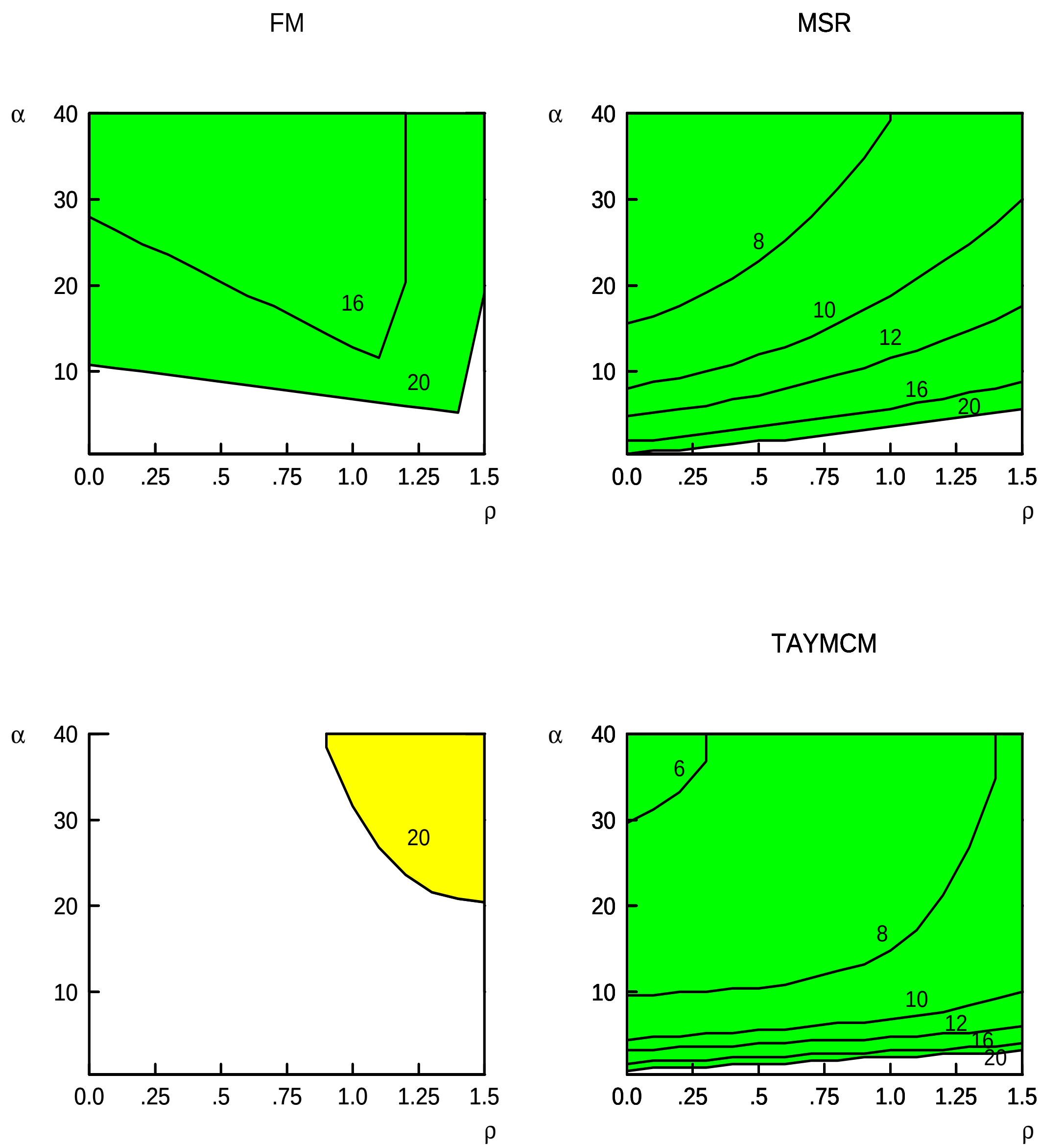

**Notes:** The top two panels show unconditional autocorrelations of output and inflation in each of the four models. The bottom two panels show the impulse responses of output and inflation to a 1 Percentage Point Federal Funds rate shock. Both, the autocorrelations and impulse responses are computed based on the same interest rate reactions, namely, the estimated rule in equation (1).

**Figure 2: Stability Regions in Coefficient Space ($\beta = 0$)**



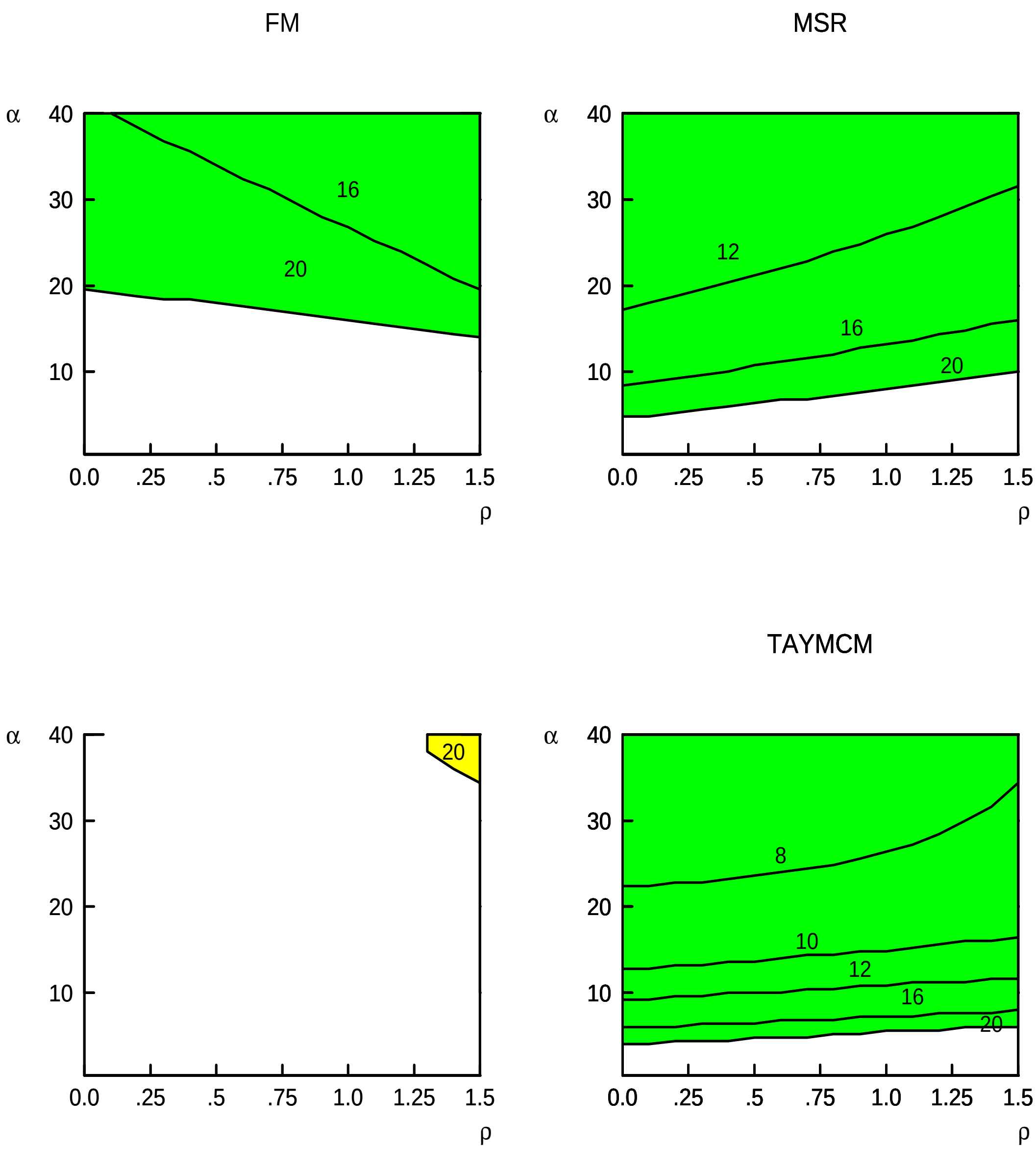

**Notes:** For FM, MSR, and TAYMCM, the dark shaded regions indicate the combinations of $\rho$ and $\alpha$ where multiple equiilbria exist when the forecast horizon for inflation is 20 quarters. The rules respond to the four-quarter change in the price level, $\pi^{(4)}$. The other contours indicate the lower bounds of the regions of mutltiple equilbria for shorter forecast horizons (indicated by the number on the chart). Results for FRB are preliminary and incomplete (the light shaded region indicates that no equilibrium exists for a forecast horizon of 20 quarters.)

**Figure 3: Stability Regions in Coefficient Space ($\beta = 2.0$)**



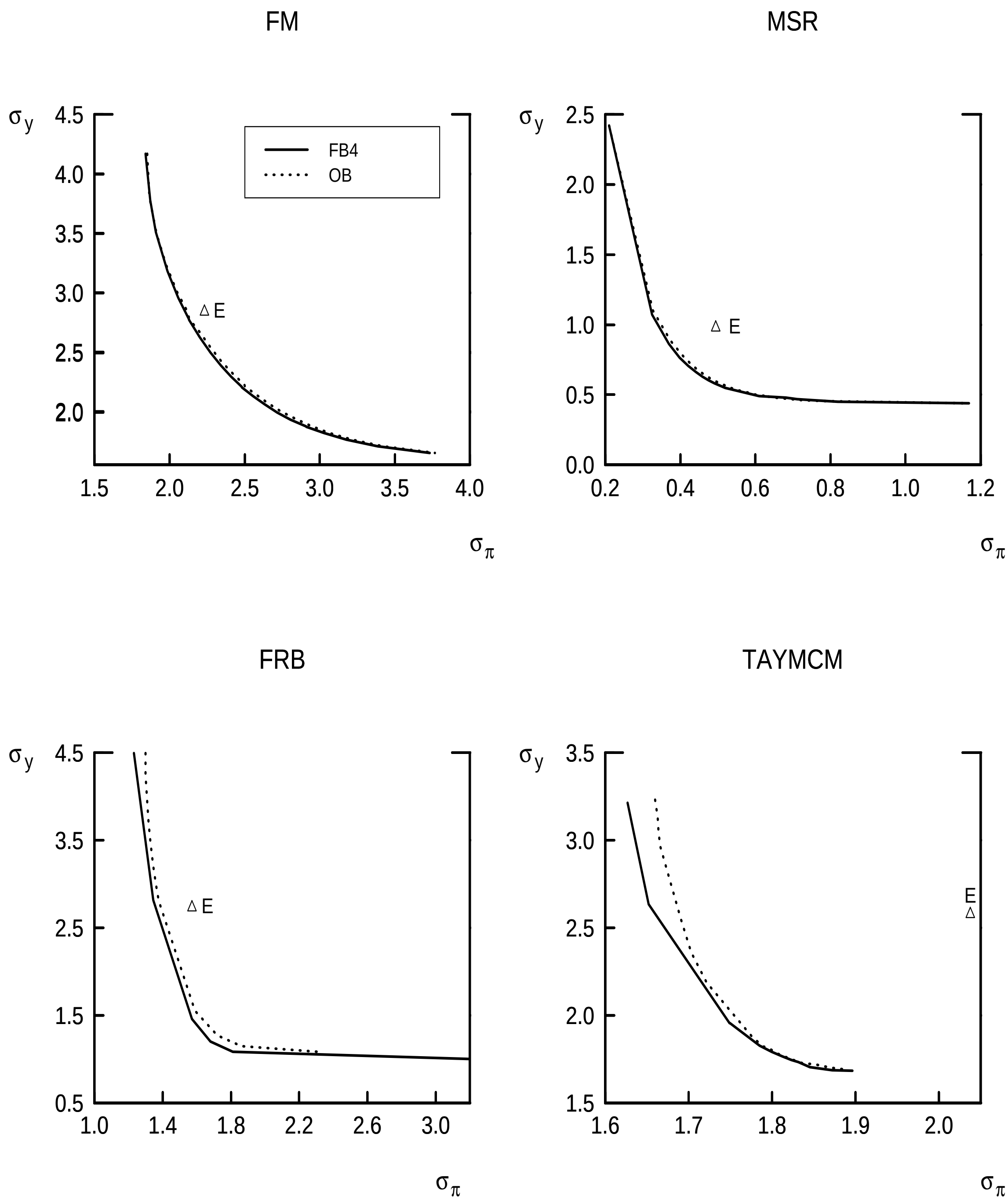

**Notes:** See notes to Figure 2.

**Figure 4: Stability Regions in Coefficient Space ($\beta = 5.0$)**




**Notes:** See notes to Figure 2.

**Figure 5: Policy Frontiers with Outcome-Based and Forecast-Based Rules**



## Figure 6: Optimal Forecast Horizons in the FM Model