Time-Consistency of Optimal Fiscal Policy in an Endogenous Growth Model

Begoña Domínguez Manzano*

Departament d'Economia i d'Història Econòmica,

Universitat Autònoma de Barcelona.

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Abstract

This paper analyses the time-consistency of optimal fiscal policy in a model with private capital and endogenous growth achieved via public capital. When a full-commitment technology is assumed, the optimal policy is obviously sustainable. Nevertheless, in the absence of such a commitment, it is well known that the debt restructuring method cannot make the optimal fiscal policy time-consistent in economies with private capital. Under a zero-tax rate on capital income, we prove that debt restructuring can solve the time-inconsistency problem of fiscal policy. We find that the policy under debt-commitment is quite close to the full-commitment policy both in growth and in welfare terms.

Keywords: Endogenous Growth; Optimal Fiscal Policy; Time Consistency.

JEL classification: E61, E62, H21, O41.

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1 Introduction.

This paper investigates under which conditions an optimal fiscal policy can be made time-consistent in an economy with both private and public capital, and how far in terms of growth and welfare this policy would be with respect to the full-commitment policy. The economy is modelled in an endogenous growth framework where public capital is not only essential for production, but also the engine of growth. Given the relevant role that government plays, it will be crucial to make fiscal policy time-consistent at a small welfare cost.

A benevolent government in office chooses both public spending and taxation plans for the current and future periods so as to maximise the welfare of the representative individual. Most studies on optimal taxation assume that there exists a full-commitment technology that enables the current government to bind the actions of future governments. Nevertheless, actual policy design is better described as a policy plan that is selected sequentially through time by a succession of governments. In this case, the resulting policy will in general not coincide with the announced optimal policy. Therefore, as shown by Kydland and Prescott (1977), the optimal policy will be time-inconsistent.

An optimal policy selected by a government at a given period is said to be time-inconsistent when it is no longer optimal when reconsidered at some later date, even though no relevant information has been revealed. The time-inconsistency problem of optimal fiscal policy arises under very general conditions; it appears in dynamic economies populated by individuals with rational expectations. In particular, in a representative agent model with a benevolent government, the optimal policy will be time-inconsistent when the government has no lump-sum taxes at its disposal. Capital taxation illustrates this problem clearly. In order to encourage investment, a government should promise low future taxes on capital income. In contrast, once this investment has taken place, capital is in inelastic supply and should be taxed heavily. Chamley (1986) showed that optimal taxation involves taxing capital at current confiscatory rates but at future low rates. However, when future governments select their current policy, capital should be taxed, so those future low rates would never occur. This is known as the capital levy problem. In view of these incentives to abandon the policy selected, governments face a credibility problem.[1] Therefore, in the absence of full-commitment, the optimal policy cannot be implemented and this time-inconsistency implies a welfare loss.

In a barter economy without capital, Lucas and Stokey (1983) showed that an optimal fiscal policy could be made time-consistent through debt restructuring when governments commit to honouring public debt, and are allowed to issue this debt with a sufficiently rich maturity structure.[2] This analysis was extended to an open economy by Persson and Svensson (1986) and by Faig (1991) and to a monetary economy by Persson, Persson and Svensson (1987). In a model with endogenous government consumption and public capital, Faig (1994) made the optimal fiscal policy time-consistent through this method. Nevertheless, in economies with privately owned capital, debt restructuring is found unable to solve the time-inconsistency of fiscal policies because of the capital levy problem.[3] Notice that debt restructuring consists of selecting debt so that the incentives to abandon the previously selected policy are neutralised. For example, taxing the labour income that is received at a given period has a different degree of distortion depending on the planning date. Debt can neutralise this change in the degree of distortion. In contrast, taxation of capital income at a given date is lump-sum from the perspective of the current government but distortionary when it is considered by previous governments. Debt cannot change the non-distortive nature of the initial tax on capital income. Hence, in economies with private capital, debt restructuring cannot solve the time-inconsistency of optimal policy and thus full-commitment policies and sustainable ones differ. This problem has been widely recognised by the literature, and it has led to limit the debt restructuring method to quite simple models which, for in-

[1] In Faig's (1994) words "optimal plans depend on the endowment vectors held by the private sector. Since different endowment vectors call for different tax plans, the government has always an incentive to change tax rates after endowment vectors have changed."

[2] By debt restructuring is meant the redesigning of the level and maturity calendar of the public debt that will be inherited by next period government.

[3] Zhu (1995) solves the time-inconsistency problem in an economy with private capital, but this capital is assumed to have an endogenous rate of utilisation, so it is never in inelastic supply.

stance, have no private capital and display no growth. However, no solution and no appropriate measure of how important this problem is have been provided yet.

In this paper we investigate under which conditions an optimal fiscal policy can be made time-consistent in an economy with private capital, abstracting from reputational issues, and what effects on growth and welfare this policy would have. In order to do so, an economy with private capital and endogenous growth achieved via public capital is modelled. First, the policy under full-commitment is studied. Next, we show that a restricted optimal fiscal policy, subject to a zero-tax rate on capital income, can be made time-consistent through debt restructuring *à-la-Lucas and Stokey.* On the one hand, this restriction allows us to find a policy plan that is sustainable. But on the other hand, when it is compared with the full-commitment policy, it implies a welfare loss. Studies on tax reform may give rise to the idea that this welfare loss could be large. In a model with exogenous government spending and no growth, Chari, Christiano and Kehoe (1994) showed that about 80% of the welfare gains in a Ramsey system comes from high tax rates on old capital income. Thus, since the policy under debt-commitment does not make use of capital taxation, it may imply a dramatic welfare loss. This argument raises the need for measuring the welfare differential. Then, in order to compare this policy with the one under full-commitment, we use a numerical solution method for non-linear rational expectations models, in particular the eigenvalue-eigenvector decomposition method suggested by Novales et al. (1999), which is based in turn on Sims (1998). Both models are solved and we find that the policy under debt-commitment is quite close to the full-commitment policy both in growth and in welfare terms.

The remainder of the paper is organised as follows. Section 2 presents the model. In section 3, the policy plans under full-commitment and under debt restructuring are described, solved, and compared. Section 4 concludes with a summary of the main findings. Finally, the appendices include proofs and explain the numerical solution method.

2 The model.

The model we present is a version of the endogenous growth model with public spending developed by Barro (1990). Our version departs from the original model in the way of approaching the policy selection. We shall also consider a second best framework, i.e. no lump-sum taxation is available, but in our model government spending will take the form of a public investment that can be financed through time-variant tax rates on labour and capital income and through the issue of debt.

Consider an economy populated by identical infinitely-lived individuals endowed with a given initial capital k_0, initial debt claims maturing at $t \geq 0$, and one unit of time per period that can be either devoted to leisure $1 - l_t$, or to output production l_t. The representative individual derives utility from consumption c_t, and leisure so that its objective is to choose both goods and investment on assets for every period in order to maximise the discounted sum of utilities,

$$\sum_{t=0}^{\infty} \beta^t U\left(c_t, 1 - l_t\right), \tag{1}$$

with $\beta \in (0, 1)$, and $U(\cdot, \cdot)$ takes the following functional form:

$$U(c_t, 1 - l_t) = \begin{cases} \theta \ln c_t + (1 - \theta) \ln\left(1 - l_t\right), & \text{if } \sigma = 1, \\ \frac{\left[c_t^{\theta}(1-l_t)^{(1-\theta)}\right]^{1-\sigma} - 1}{1-\sigma}, & \text{otherwise,} \end{cases} \tag{2}$$

where $\sigma > 0$ is the inverse of the elasticity of intertemporal substitution and $\theta \in (0, 1)$ reflects preferences between leisure and consumption. Taking sequences of prices and policy instruments as given, the consumer maximises welfare (1) restricted by the budget constraint,

$$\begin{aligned} & p_t \left[c_t + k_{t+1} + \sum_{s=t+1}^{\infty} \frac{p_s}{p_t} \left({}_{t+1}b_s^c - {}_t b_s^c \right) + \sum_{s=t+1}^{\infty} \frac{p_s}{p_t} q_s \left({}_{t+1}b_s^w - {}_t b_s^w \right) \right] \\ \leq \; & p_t \left[{}_t b_t^c + (1 - \tau_t^l) w_t \left[l_t + {}_t b_t^w \right] + R_t k_t \right], \end{aligned} \tag{3}$$

and by the no-Ponzi-game condition on assets,

$$\lim_{t \to \infty} \sum_{s=t}^{\infty} p_s \, {}_t b_s^c = 0, \; \lim_{t \to \infty} \sum_{s=t}^{\infty} p_s q_s \, {}_t b_s^w = 0, \; \lim_{t \to \infty} p_t k_{t+1} = 0, \tag{4}$$

where p_t is the price of a final good in period t, w_t is the real wage received for the fraction of time that the individual devotes to work at t, τ_t^l is the labour income tax rate at t, R_t is the gross return on capital k_t, after tax τ_t^k, and depreciation rates δ_k, and r_t is the net return on capital at t, that is, $R_t = \left\{1 + \left(1 - \tau_t^k\right) r_t - \delta_k\right\}$. The debt structure is as follows: $\{ \, _tb_s^c, \, _tb_s^w \}_{s=0}^{\infty}$ is a sequence of claims existing at t to be paid at $s \geq t$ on debt indexed to consumption and to after-tax wage.[4] Finally, q_t is the price of a bond indexed to after-tax wage in terms of final goods in period t.

We assumed that public debt can be issued with a sufficiently rich structure. This debt structure is said to be sufficiently rich in terms of maturity calendar and debt-type variety. More precisely, governments can issue debt that matures at any moment in the future and that can be indexed to consumption and to after-tax wage.[5] By doing so, governments promise debt payments, interest and principal, that are valued a consumption good and a unit of time devoted to production in a future period respectively.[6] We assume that lump-sum taxation is not available. Thus, the policy design takes the form of a second best problem. Governments provide a public spending which is financed through flat-rate taxes on capital and labour income and through the issue of debt. This government spending is productive and takes the form of public investment which accumulates over time and amounts to a stock of public capital g_t that depreciates at a rate δ_g. The government intertemporal budget constraint is

$$\sum_{t=0}^{\infty} p_t z_{0t} \geq 0, \tag{5}$$

where

$$z_{0t} \equiv \left[\tau_t^l w_t l_t + \tau_t^k r_t k_t - \left(g_{t+1} - \left(1 - \delta_g\right) g_t\right) - \, _0b_t^c - q_t \, _0b_t^w\right] \tag{6}$$

[4]If debt were indexed to before-tax wage, another source of time-inconsistency would appear.

[5]This variety of debt can be also found in Faig (1994) and Zhu (1995).

[6]Debt indexed to consumption can be identified with Treasury Inflation-Protected Securities that are issued since 1997 by the U.S. Treasury and which vary with the consumer price index. We may identify debt indexed to after-tax wage with the promise of future social security pensions that are closely linked to the wage rate.

which, in the terminology of Persson, Persson and Svensson (1987), is the government "cash-flow" that at date 0 becomes

$$\sum_{s=1}^{\infty} \frac{p_s}{p_t} \left({}_0b_s^c - {}_1b_s^c \right) + \sum_{s=1}^{\infty} \frac{p_s}{p_t} q_s \left({}_0b_s^w - {}_1b_s^w \right), \tag{7}$$

which can be defined either as the excess of tax revenues over public spending and debt payments or as the real value of the net issue of new debt.

The first order conditions for the individual maximisation problem are the following:

$$\frac{U_{1-l}\left(c_t, 1-l_t\right)}{U_c\left(c_t, 1-l_t\right)} = \left(1-\tau_t^l\right) w_t, \tag{8}$$

$$\frac{U_c\left(c_t, 1-l_t\right)}{U_c\left(c_{t+1}, 1-l_{t+1}\right)} = \beta \left\{1 + \left(1-\tau_{t+1}^k\right) r_{t+1} - \delta_k\right\}, \tag{9}$$

$$\beta^t \frac{U_c\left(c_t, 1-l_t\right)}{U_c\left(c_0, 1-l_0\right)} = \frac{p_t}{p_0}, \tag{10}$$

where $U_c\left(c_t, 1-l_t\right)$ and $U_{1-l}\left(c_t, 1-l_t\right)$ denote the marginal utility with respect to consumption and to leisure at t respectively. Following this notation, second-order derivatives of the utility function will be denoted by $U_{c_tc_t}$, $U_{c_t1-l_t}$ and $U_{1-l_t1-l_t}$.

In this economy there is a final good that can be either consumed or invested. This good is produced through the following technology:

$$y_t = f(k_t, l_t, g_t) = Ak_t^{\alpha}\left(l_t g_t\right)^{1-\alpha}, \tag{11}$$

where $A > 0$ and $\alpha \in (0,1)$. In this produciton function, public capital is introduced as an essential input that enhances both private capital and labour marginal products, and that allows for endogenous growth. Notice also that our technology includes two state variables. This fact could make our model exhibit transitional dynamics. Nevertheless, since negative net capital investments are not ruled out, the private-public capital ratio can adjust instantaneously to its steady state value.[7] Therefore, in the absence of any other disturbances, e. g. restrictions on taxation, our model would behave as an "AK" one from period 1 on. Thus, feasible allocations

[7] Investments could be thought as reversible, that is old units of public capital could be converted into private capital and alternatively. This could be justified by the privatisations of industry.

are described by the resource constraint,

$$c_t + k_{t+1} + g_{t+1} \leq Ak_t^{\alpha} \left(l_t g_t\right)^{1-\alpha} + \left(1-\delta_k\right) k_t + \left(1-\delta_g\right) g_t. \tag{12}$$

A representative firm produces the final good and maximises profits given factor prices. Necessary conditions for this optimisation program are

$$r_t = f_{k_t} \text{ and } w_t = f_{l_t}, \tag{13}$$

where f_{k_t} and f_{l_t} denote marginal products of capital and labour at t respectively. Likewise, a similar notation will be followed by the marginal product of public capital and second-order derivatives of the production function.

A competitive equilibrium for this economy is defined as follows:

Definition 1 *For a given policy* $\left\{g_{t+1}, \tau_t^k, \tau_t^l\right\}_{t=0}^{\infty}$ *and initial values of public* g_0*, and private capital* k_0*, and debt* $\left\{ {}_0b_t^c,\ {}_0b_t^w\right\}_{t=0}^{\infty}$*, an allocation* $\left\{c_t, l_t, k_{t+1}\right\}_{t=0}^{\infty}$ *is a competitive equilibrium allocation if and only if there exists a price sequence* $\left\{p_t, q_t, r_t, w_t\right\}_{t=0}^{\infty}$ *such that the representative individual maximises* (1) *subject to* (3) *and* (4)*, factors are paid their marginal products according to* (13)*, and all markets clear.*

3 The policy selection.

Once the private agents behaviour has been described, we shall turn to the policy selection. First, the policy under the full-commitment technology will be analysed. Next, we compute the optimal policy under some restrictions and discuss whether debt-restructuring can make it time-consistent. This policy plan will be named the policy under debt-commitment. Analitically, both policies can be slightly characterised in the short and long run. Nevertheless, welfare and growth comparisons are intractable. Thus, we recurr to numerical solution methods. Finally, both policies under full-commitment and debt-restructuring are numerically solved and compared.

3.1 The full-commitment policy.

In this section, the policy under full-commitment is analysed. For the time being, we assume that future governments do commit to the optimal tax policy chosen by the current government. This assumption would be equivalent to the existence of a full-commitment technology that makes the optimal policy planned at date 0 sustainable. This policy is such that once the government at 0 selects a fiscal plan for $t \geq 0$, successive governments will be bound to set the policy that is the continuation of the original plan chosen at 0. In this policy scheme, tax rates do not need to be constant over time and may take positive or negative values. But we shall restrict this policy by an upper bound on capital tax rates set equal to unity.[8]

In a second best problem, the government chooses an allocation among the different possible competitive equilibria in order to maximise the welfare of the representative individual. This allocation together with the conditions for a competitive equilibrium provide the optimal tax policy that supports this allocation. A competitive equilibrium allocation is characterised by the upper bound on capital tax rates and two restrictions: the resource constraint (12), and the implementability condition

$$\sum_{t=0}^{\infty}\beta^t \left[\left(c_t - {}_0b_t^c\right) U_c\left(c_t, 1-l_t\right) - \left(l_t + {}_0b_t^w\right) U_{1-l}\left(c_t, 1-l_t\right)\right] \leq W_0 U_c\left(c_0, 1-l_0\right), \tag{14}$$

where W_0 is the individual's initial wealth, that is $W_0 = R_0 k_0$. The implementability condition (14)results from adding the budget constraint of the individual (3) over time and introducing the first order conditions, $(8)-(10)$ and (13), into the resulting expression. Note also that if equations (12) and (14) hold, then the intertemporal government budget constraint is satisfied.

Now, let us define the government optimisation program. This problem can be expressed as choosing the initial tax rate on capital income τ_0^k, and the sequences $\{c_t, l_t, k_{t+1}, g_{t+1}\}_{t=0}^{\infty}$ that maximise the welfare of the representative individual (1) subject to the resource constraint (12), the implementability condition (14), the upper bound on capital tax rates,[9] which is

$$U_{c_t} \geq \beta U_{c_{t+1}} \left\{1 - \delta_k\right\}, \tag{15}$$

[8]This upper bound could be justified by means of limited-liability, there is a limit to the capital income that the individual can be taxed away.

[9]This constraint results from combining $\tau_{k_t} \leq 1$ and the first order condition for capital (9).

and the transversality conditions on debt and on private and public capital,

$$\lim_{t\to\infty}\sum_{s=t}^{\infty}\beta^{s}U_{c_s}\ {}_tb_s^c=0,\ \lim_{t\to\infty}\sum_{s=t}^{\infty}\beta^{s}U_{c_s}\left(1-\tau_s^l\right)f_{l_s}\ {}_tb_s^w=0,$$
$$\lim_{t\to\infty}\beta^{t}U_{c_t}k_{t+1}=0,\ \lim_{t\to\infty}\beta^{t}U_{c_t}g_{t+1}=0, \tag{16}$$

given initial values for debt, private and public capital.

A solution to this problem is characterised by the constraints (12), (14) and (15) together with the following first order conditions for consumption, labour, private and public capital respectively:

$$\mu_{0t} = W_c\left(c_t, 1-l_t,\ {}_0b_t^c,\ {}_0b_t^w, \phi_{0t}, \lambda_0\right), \tag{17}$$
$$f_{l_t}\mu_{0t} = W_{1-l}\left(c_t, 1-l_t,\ {}_0b_t^c,\ {}_0b_t^w, \phi_{0t}, \lambda_0\right), \tag{18}$$
$$\mu_{0t} = \beta\mu_{0t+1}\left\{1+f_{k_{t+1}}-\delta_k\right\}, \tag{19}$$
$$\mu_{0t} = \beta\mu_{0t+1}\left\{1+f_{g_{t+1}}-\delta_g\right\}, \tag{20}$$

with

$$W_{c_t}=(1+\lambda_0)U_{c_t}+\lambda_0\left[U_{c_tc_t}\left(c_t-\ {}_0b_t^c+\Theta_t\right)-U_{c_t1-l_t}\left(l_t+\ {}_0b_t^w\right)\right],$$
$$W_{1-l_t}=(1+\lambda_0)U_{1-l_t}+\lambda_0\left[U_{1-l_tc_t}\left(c_t-\ {}_0b_t^c+\Theta_t\right)-U_{1-l_t1-l_t}\left(l_t+\ {}_0b_t^w\right)\right],$$

and

$$\Theta_t=\begin{cases}-W_0+\frac{\phi_{00}}{\lambda_0}, & \text{for } t=0,\\ \frac{1}{\lambda_0}\left(\phi_{0t}-\{1-\delta_k\}\phi_{0t-1}\right), & \text{for } t>0,\end{cases}$$

where μ_{0t}, λ_0, ϕ_{0t} are the multipliers associated to the constraints (12), (14) and (15) respectively.[10]

The optimal initial tax rate on capital income reaches obviously the upper bound, that is the unity. This is so because initial capital revenues constitute pure economic rents, and therefore it is optimal to tax them as high as possible. As mentioned above, the optimal tax policy for $t\geq 1$ results from combining the optimal allocation and the competitive equilibrium conditions. In a BGP, this policy can be

[10] As pointed out by Lucas and Stokey (1983), second order conditions are not clearly satisfied given that they involve third and second derivatives of the utility function. Then, let us assume that an optimal solution exists and that this solution is interior.

characterised under quite general conditions. However, the short run characterisation of this policy needs more restricted assumptions. A more detailed solution of the full-commitment policy will be later given under more realistic assumptions through a numerical solution methods.

The transitional dynamics of this model can be attributed to a number of factors, namely, the individual's initial wealth, and the restriction on capital taxation. In this model, the initial wealth affects indirectly the whole problem and directly decisions that involve variables at 0. Besides, economic decisions are taken differently depending on whether the restriction on capital taxation is binding. Both sources of transition are clearly reflected by the set of first order conditions $(17)-(20)$. Given the high non-linearity of these conditions, no close-form solution of the variables can be displayed. A number of simplifying assumptions are necessary in order to characterise the transitional dynamics of the model. In the following proposition, the evolution of the consumption growth rate γ_{c_t}, labour, and the tax rate on labour income are described:

Proposition 1 *Let us assume that* $\sigma = 1$, $1 > \delta_g > \delta_k > 0$ *and* $\frac{{}_0b_t^c}{c_t} = {}_0b_t^w = 0$ *for all* $t > 1$, *then we can establish that*

(*i*) *If* $\left(\frac{k_t}{g_t}\right) \leq \left(\frac{k}{g}\right)_{ss}$, *then* $\gamma_{c_t} \leq \gamma_{c_{ss}}$, $l_t \geq l_{ss}$ *and* $\tau_t^l \lesseqgtr \tau_{ss}^l$.

(*ii*) *If* $\left(\frac{k_t}{g_t}\right) \geq \left(\frac{k}{g}\right)_{ss}$, *then* $\gamma_{c_t} \leq \gamma_{c_{ss}}$, $l_t \leq l_{ss}$ *and* $\tau_t^l \leq \tau_{ss}^l$.

Proof. *See the appendix.* ■

Proposition 1 characterises the transitional dynamics of the allocation and policy that solve the full-commitment model. Observe that the upper bound on capital taxation makes the model exhibit a transition with a lower but increasing growth rate. In a BGP, the optimal tax policy can be characterised by the following statement:

Proposition 2 *If* $\left\{\tau_t^k, \tau_t^l\right\}_{t=0}^{\infty}$ *is the optimal tax policy under full-commitment, then* $\tau_{ss}^k = 0$ *in a BGP, and if in addition* $\frac{{}_0b_t^c}{c_t} = {}_0b_t^w = 0$ *in a BGP and* $\sigma = 1$ *are assumed, then* $\tau_{ss}^l = \frac{\lambda_0}{\lambda_0 + 1 - l_{ss}}$ *in a BGP.*

Proof. *See the appendix.* ■

Proposition 2 supports of Chamley's (1986) result. Under full-commitment, current capital should be taxed heavily, whereas in the long run it should not be taxed. Independently of the debt structure, the full-commitment technology would make this optimal policy sustainable with just one-period consumption-indexed debt. But, in the absence of full-commitment, the government should reconsider its actions in period 1. Future governments would choose an optimal fiscal policy different from the one chosen at 0 for those future dates and, hence, the latter policy is time-inconsistent.

The incentives to select an allocation different from the one chosen for that date by previous governments come from different forces: the possibility of defaulting, devaluing debt and changing policy plans. We assume that governments commit to honouring debt, but the two remaining forces are still active. The incentives to "devalue" debt come from the fact that an unexpected change in fiscal policy can lower the present value of outstanding debt. Incentives to change policy plans derive from the following behaviour. A government at 0 chooses a policy for date t. This expected policy will affect private agents' decisions in the time interval from 0 to t. However, when the government at t plans a policy for date t, those private decisions are already bygone and will not be taken into account by the current government. This behaviour influences both labour and capital income taxation. In some contexts, debt restructuring allows us to neutralise all forces that drive time-inconsistency. However, the time-inconsistency of capital taxation cannot be solved through this method. This behaviour comes from the capital levy problem. Taxing the labour income that is received at a given period has a different degree of distortion depending on the planning date. Debt can neutralise this change in the degree of distortion. In contrast, capital income taxation at t is distorting when it is planned at $s < t$, but is lump-sum when planned at t. This change in nature that capital taxation suffers cannot be modified by the debt structure.

3.2 The policy under debt commitment.

In the previous section, we have seen that the optimal fiscal policy cannot be made time-consistent through debt-commitment because of the capital levy problem. From now on, we assume that future governments can reconsider both taxation and spending plans, but commit to honouring debt and are free to redesign the public debt that will be inherited by next period government. In this context, we investigate under which conditions an optimal policy can be made time-consistent. A possible and natural solution would be to restrict our analysis to a zero-tax rate on capital and study whether it can be made sustainable. Now, we compute the policy as if a full-commitment technology were available. And, later, we show that in the absence of such a commitment, this policy can be made time-consistent through debt-restructuring.

The optimisation program solved by the government consists in choosing the sequences $\{c_t, l_t, k_{t+1}, g_{t+1}\}_{t=0}^{\infty}$ that maximise the welfare of the representative individual (1) subject to the resource constraint (12), the implementability condition (14), the zero-tax rate constraint on capital,[11]

$$U_{c_t} = \beta U_{c_{t+1}} \left\{ 1 + f_{k_{t+1}} - \delta_k \right\} \tag{21}$$

and the transversality conditions (16), given initial values for debt, private and public capital. First order conditions for this problem are

$$\begin{aligned}
\mu_{0t} &= W_c \left(c_t, 1 - l_t, k_t, g_t, {}_0b_t^c, {}_0b_t^w, \xi_{0t}, \lambda_0 \right), && (22)\\
f_{l_t}\mu_{0t} &= W_{1-l} \left(c_t, 1 - l_t, k_t, g_t, {}_0b_t^c, {}_0b_t^w, \xi_{0t}, \lambda_0 \right), && (23)\\
\mu_{0t} &= \beta\mu_{0t+1} \left\{ 1 + f_{k_{t+1}} - \delta_k \right\} - \beta\xi_{0t} f_{k_{t1}k_{t+1}} U_{c_{t+1}}, && (24)\\
\mu_{0t} &= \beta\mu_{0t+1} \left\{ 1 + f_{g_{t+1}} - \delta_g \right\} - \beta\xi_{0t} f_{k_{t+1}g_{t+1}} U_{c_{t+1}}, && (25)
\end{aligned}$$

where

$$\begin{aligned}
W_{c_t} &= (1 + \lambda_0) U_{c_t} + \lambda_0 \left[U_{c_t c_t} \left(c_t - {}_0b_t^c + \Theta_{c_t} \right) - U_{c_t 1-l_t} \left(l_t + {}_0b_t^w \right) \right],\\
W_{1-l_t} &= (1 + \lambda_0) U_{1-l_t} + \lambda_0 \left[U_{1-l_t c_t} \left(c_t - {}_0b_t^c + \Theta_{l_t} \right) - U_{1-l_t 1-l_t} \left(l_t + {}_0b_t^w \right) \right],
\end{aligned}$$

[11] Introduce $\tau_{k_t} = 0$ and equation (13) into the first order condition for capital (9).

with

$$\Theta_{c_t} = \begin{cases} -W_0 + \frac{\xi_{00}}{\lambda_0}, \text{ for } t = 0, \\ \frac{1}{\lambda_0}\left(\xi_{0t} - \{1 + f_{k_t} - \delta_k\}\,\xi_{0t-1}\right), \text{ for } t > 0, \end{cases}$$

and

$$\Theta_{l_t} = \begin{cases} -W_0 + \frac{\xi_{00}}{\lambda_0} - f_{kl_0}\frac{U_{c_t}}{U_{c_t 1-l_t}} k_0, \text{ for } t = 0, \\ \frac{1}{\lambda_0}\left(\xi_{0t} - \{1 + f_{k_t} - \delta_k\}\,\xi_{0t-1} - f_{k_t l_t}\frac{U_{c_t}}{U_{c_t 1-l_t}}\xi_{0t-1}\right), \text{ for } t > 0, \end{cases}$$

where μ_{0t}, λ_0, ξ_{0t} are the multipliers associated to the resource (12), the implementability conditions (14) and the zero-tax rate constraint on capital income (21) respectively.[12]

These first order conditions together with (12), (14) and (21) describe the optimal allocation. The optimal tax policy that supports this allocation is obtained through the competitive equilibrium conditions. Now we turn to characterise the short run characteristics of the policy and allocation just computed.

Proposition 3 *Let us assume that* $\sigma = 1$, $1 > \delta_g > \delta_k > 0$, $\frac{{}_0b_t^c}{c_t} = {}_0b_t^w = 0$ *for all* $t > 1$, *then enough close to a BGP we can establish that*

(i) If $\left(\frac{k_t}{g_t}\right) \leq \left(\frac{k}{g}\right)_{ss}$, *then* $\gamma_{c_t} \geq \gamma_{c_{ss}}$, $l_t \geq l_{ss}$ *and* $\tau_t^l \lesseqgtr \tau_{ss}^l$.

(ii) If $\left(\frac{k_t}{g_t}\right) \geq \left(\frac{k}{g}\right)_{ss}$, *then* $\gamma_t \leq \gamma_{ss}$, $l_t \leq l_{ss}$ *and* $\tau_t^l \leq \tau_{ss}^l$.

Proof. See the appendix. ■

This proposition characterises the transition of the allocation and policy that solves the debt-commitment model. In a BGP, we can state the following:

Proposition 4 *If* $\left\{\tau_t^l\right\}_{t=0}^{\infty}$ *is the optimal tax policy under a zero-tax rate on capital income, then the zero-tax rate constraint on capital income does not restrict the allocation and policy selection in a BGP, that is,* $\xi_{ss} = 0$ *and, moreover, if* $\frac{{}_0b_t^c}{c_t} = {}_0b_t^w = 0$ *in a BGP and* $\sigma = 1$ *are assumed, then* $\tau_{ss}^l = \frac{\lambda_0}{\lambda_0 + 1 - l_{ss}}$ *in a BGP.*

[12] As in the previous section, we assume that an optimal solution that is interior exists.

Proof. See the appendix. ∎

Let us turn now to the time-inconsistency problem. First, we have assumed that governments commit to honouring debt. Moreover, by restricting our analysis to a zero-tax rate on capital income, the incentives to tax heavily old capital disappeared. Nevertheless, the incentives to lower the present value of debt and change public capital and tax rates on labour income persist. Hence, in the absence of full-commitment, next period government would reconsider the spending and taxation plans, and the policy plan at 0 will be time-inconsistent. This time-inconsistency implies that the allocation and policy described in this section would not take place and that the final result will involve a welfare reduction. In order to prevent this welfare loss, we should consider whether this policy can be made time-consistent, that is, if the policy chosen at time 0 can be sustainable at period 1 and later on.

Proposition 5 *If the sequences $\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}$ and $\{g_{t+1}, \tau_t^l\}_{t=0}^{\infty}$ are respectively the optimal allocation and optimal policy under a zero-tax rate on capital income, then it is always possible to choose $\{{}_1b_t^c, {}_1b_t^w\}_{t=1}^{\infty}$ at market prices* (10) *such that the continuation $\{c_t, l_t, k_{t+1}\}_{t=1}^{\infty}$ and $\{g_{t+1}, \tau_t^l\}_{t=1}^{\infty}$ of the same allocation and policy are a solution for the government problem when it is reconsidered at date* 1*. This could be done through the following debt-structure:*

$$ {}_1b_t^c - {}_0b_t^c = \left[\frac{\lambda_0}{\lambda_1} - 1\right]\left[{}_0b_t^c - \left[1 - \frac{1}{\sigma}\right]c_t\right] + \Gamma_t^c, \tag{26} $$

$$ {}_1b_t^w - {}_0b_t^w = \left[\frac{\lambda_0}{\lambda_1} - 1\right]\left[{}_0b_t^w + \frac{1}{\sigma} + \left[1 - \frac{1}{\sigma}\right]l_t\right] + \Gamma_t^w, \tag{27} $$

where

$$ \Gamma_t^c = \begin{cases} \left[\frac{\xi_{00} - \lambda_1 k_1}{\lambda_0 - \lambda_1}\right]\left[(1 + f_{k_t} - \delta_k) + \theta\left(\frac{(\sigma-1)}{\sigma}\right)(1 - l_t) f_{k_t l_t}\right], & \text{for } t = 1, \\ 0, & \text{for } t > 1, \end{cases} \tag{28} $$

and

$$ \Gamma_t^w = \begin{cases} -\left[\frac{\xi_{00} - \lambda_1 k_1}{\lambda_1}\right]\left[\left(\frac{\theta}{1-\theta}\right)\left(\frac{1+(\sigma-1)\theta}{\sigma}\right)\frac{(1-l_t)^2}{c_t} f_{k_t l_t}\right], & \text{for } t = 1. \\ 0, & \text{for } t > 1. \end{cases} \tag{29} $$

By induction, the same is true for all later periods.

Proof. See the appendix. ■

The present value of the new debt issues at date 0 is determined by the government budget constraint (7) and there are infinite pairs of sequences $\{\ {}_1b_t^c,\ {}_1b_t^w\}_{t=1}^{\infty}$ that satisfy the restriction, but just one that enables the government to make the optimal policy time-consistent. Proposition 5 allows us to guarantee that under that debt structure the allocation and policy chosen for $t \geq 1$ by the government at 0, and described by propositions 3 and 4, will be sustainable. Hence, in economies with privately owned capital, the optimal fiscal policy under a zero-tax rate constraint on capital income can be made time-consistent through debt restructuring.

How is the debt structure that enables the government to make the optimal policy time-consistent? In our model, this debt structure is difficult to characterise because the sign of $(\xi_{00} - \lambda_1 k_1)$ is unknown. Nevertheless, it can be seen that debt indexed to consumption and to after-tax-wage maturing at date $t > 1$ should follow a constant pattern with respect to consumption and to labour respectively. On the other hand, it can be seen that debt maturing at date 1 follows a different pattern. This difference may come from the aim of neutralising the effect that the initial wealth, capital income, at 1 has in the policy selection when it is planned at 1, but it is not captured in the policy plan selected at 0. In all previous studies, the debt structure that ensured time-consistency had always the same pattern independently of the maturing date.This different behaviour in the debt maturing at date 1 is a distintive factor in this paper and shows that in economies with private capital the debt structure that ensures time consistency should take into account the effect of the initial wealth.

In this section, a restricted policy has been made time-consistent. However, it is not known yet how desirable this policy is. In a model with exogenous government spending and no growth, Chari, Christiano and Kehoe (1994) studied the effects from a tax reform. These authors compare a Ramsey system with and without a zero-tax rate restriction on capital income. The Ramsey system without this restriction is characterised by high initial capital tax rates that are followed by a zero-tax rate from then on. However, the models with and without capital taxation differ greatly in welfare terms. Thus, they conclude that most of welfare gains would be provided by capital tax revenues. In view of this result, they also argue that since the temptation to renege on the capital tax policy is so large, the time-inconsistency problem of capital taxation is quantitatively severe. In our model, the policy under debt commitment does not make use of capital taxation. This may lead us to think that our restricted tax policy, though time-consistent, could imply dramatic welfare losses. Besides, once we have computed these welfare losses, they will provide us a measure of how important the capital levy problem is.

3.3 Numerical solution for both models.

In order to compare this restricted policy with the one under full-commitment, we use a numerical solution method for non-linear rational expectations models, in particular the eigenvalue-eigenvector decomposition method that is suggested by Novales et al. (1999) which is based in turn on Sims (1998). This method consists of the following. First, the necessary conditions are obtained and transformed so that are functions of either ratios or variables that are constant in a BGP.[13] Their steady state values are found, as table 1 shows. Then, these conditions are linearised around the steady state in order to find the stability conditions. They are obtained by imposing orthogonality between each eigenvector associated with an unstable eigenvalue of the linear system and the variables of this system. These stability conditions are imposed into the original non-linear model to compute a numerical solution.

[Insert Table 1 about here.]

In order to simulate the series, some realistic parameter values are chosen.[14] The discount rate β is 0.99. The coefficient A in the production function equals 0.48. The parameter α is 0.25 as in Barro (1990). Depreciation rates for private δ_k, and public capital δ_g, are 0.025 and 0.03 respectively.[15] Preference parameters σ and θ are respectively 2 and 0.3, the last number is chosen in order to have reasonable

[13] Note that all variables that are not constant grow at the same rate in a BGP.

[14] All simulations are carried out with the program GAUSS-386.

[15] Clearly, a public highway would deprecite faster than a private one.

values for leisure. Initial values for private and public capital are respectively 15 and 45. Initial debt values are zero for all periods. Both models share the same set of parameter values and depart from the same initial state.

Series for both policies, under full-commitment and with debt restructuring, are simulated. We report the main results. As figure (1) shows, the growth rate differential between both policies is small. In the short run, the policy under debt-commitment yields a higher growth rate. Long run rates of growth are very similar and the growth rate under debt restructuring approaches from below the rate attained under full-commitment. In the BGP, growth rates under full-commitment and under debt restructuring are respectively 4.74 and 4.64 per cent.

[Insert Figure 1 about here.]

This similarity between growth rates in the medium and long run leads us to think that the differences in welfare may not be so dramatic. Numerically, the welfare attained under full-commitment and under debt restructuring take values of 54.1458 and 52.8691 respectively; then, this debt restructuring policy involves a welfare reduction of 2.35%. Therefore, the debt-commitment policy is quite close in terms of growth and welfare to the policy under full-commitment. One could argue that these results could hinge on the upper bound on capital tax rates in the full-commitment policy. Nevertheless, under the same parameter values and initial conditions, an unrestricted full-commitment policy yields a 5.24 per cent growth rate and welfare sized by 55.6524.

Our results contrast with Chari, Christiano and Kehoe's (1994) findings. This difference may come from two facts: first, our model allows for endogenous growth, so future allocations play a great role for welfare; second, our taxes are levied to finance a productive public investment rather than an exogenous stream of government spending. In the present model, public spending may be more important for welfare than the way of financing it. Moreover, under both policies this stream of public investment behaves similarly; in the short run, the public capital rate of growth is higher under the full-commitment policy than with debt restructuring, this inequality reverses in the medium term, and both become quite similar in the long run as figure 2 shows.

[Insert Figure 2 about here.]

The way of financing this spending differs in both models. Under full-commitment, tax rates on capital income are high in the first periods and zero thereafter. On the other hand, tax rates on labour income are negative in the initial period and positive from then on. In comparison with the policy under debt restructuring, the tax rate on labour income is smaller both in the short and in the long run under full-commitment.

[Insert Figures 3 and 4 about here.]

In order to spread the excess of burden, debt is issued in an equivalent way in both models as figure 5 shows. In the short run, the government incurs in cash flow surpluses, whereas in the medium and in the long run this surplus vanishes. The main difference is that under full-commitment the first period involves a cash flow deficit that becomes surplus after few periods.

[Insert Figures 5 about here.]

Finally, it would be interesting to see how the debt-structure that ensures time-consistency behaves. As we have just mentioned, under debt-commitment, the government incurs in a surplus at date 0. How is this surplus structure over time?

[Insert Figures 6 and 7 about here.]

4 Conclusions.

We have investigated the time-inconsistency problem of optimal fiscal policies in a model with private capital and endogenous growth achieved via public capital. A policy restricted to a zero-tax on capital income has been made time-consistent through debt restructuring. The Modigliani-Miller theorem breaks down and debt serves as an incentive device to continue with the selected allocation and optimal policy. We conclude that the tax policy under debt restructuring is quite close both in growth and in welfare terms to the optimal policy under full-commitment. In this sense, we can also argue that the time-inconsistency of capital taxation is not quantitatively so severe.

In economies with privately-owned capital, it is well known that the debt restructuring method cannot make the optimal policy plan time-consistent. We have reconsidered this issue and we have established that in economies with private capital, a time-consistent policy plan requires both debt-commitment and a rule on capital taxation. In this paper, we have chosen a zero-tax rate rule on capital income. One could ask that once this tax-rate constraint is allowed to be chosen by the government, whether this rule would be set different from zero.

A criticism that often emerges in this literature is that such a rich debt structure in debt-variety and maturity calendar has no clear counterpart in actual economies. One can argue that nowadays financial products indexed to very different economic variables are present in financial markets. Besides, as pointed out by Faig (1994), future social security pensions form an example that resembles debt indexed to after-tax wage. Nevertheless, it would be interesting to study what the government could better do without such a rich debt structure. Then, it may be the case that the optimal policy cannot be made time-consistent. Then, policy selection should be restricted to sustainable policies. In the choice of the best sustainable policy, it would be interesting to see whether debt restructuring of the remaining instruments still matters. The economy performance could provide a measure for the importance of a rich public financial structure.

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Appendix A.

Proof of Proposition 1.

Obviously, when the capital tax rate restriction is binding the model exhibits $\tau_t^k = 1$. First, by simple inspection of equation (15), we have that $\gamma_{c_r} < \gamma_{c_{ss}}$. Second, if the RHS of (19) and (20) are equated, labour can be expressed as

$$l_t = \left[\frac{\delta_g - \delta_k}{A}\right]^{\frac{1}{1-\alpha}} * X_t^{-\frac{1}{1-\alpha}}$$

with

$$X_t = \left[(1-\alpha)\left(\frac{k_t}{g_t}\right)^{\alpha} - \alpha\left(\frac{k_t}{g_t}\right)^{-(1-\alpha)}\right]. \tag{30}$$

It can be easily seen that $\frac{\partial l_t}{\partial X_t} < 0$ and $\frac{\partial X_t}{\partial\left(\frac{k_t}{g_t}\right)} > 0$, hence by the chain rule $\frac{\partial l_t}{\partial\left(\frac{k_t}{g_t}\right)} < 0$. Finally, combining equations (17) and (18) with (8), the labour income tax rate takes the following value:

$$\tau_t^l = \frac{\frac{\lambda_0}{1-l_t} - \left[\frac{1}{\beta}\frac{\phi_{ot-1}}{c_{t-1}} - \frac{\phi_{ot}}{c_t}\right]}{1 + \frac{\lambda_0}{1-l_t}} \tag{31a}$$

where $\left[\frac{1}{\beta}\frac{\phi_{ot-1}}{c_{t-1}} - \frac{\phi_{ot}}{c_t}\right] \geq 0$ given that $\frac{\phi_{ot}}{c_t}$ approaches zero from above. Then it can be seen that when l_t approaches from below its steady state value so does τ_t^l.

■

Proof of Proposition 2.

First, let us demonstrate that the capital tax rate is zero in the long run. This will be proven in a more general framework without taking into account the assumptions stated in the proposition. Considering (2) and (17), we can write the first order condition for capital for the government maximisation problem (19) as

$$W_{c_t} = \beta W_{c_{t+1}} \left\{ 1 + f_{kt+1} - \delta_k \right\}, \tag{32}$$

where

$$W_{c_t} = U_{c_t} H_{c_t}, \tag{33}$$

with

$$H_{c_t} = (1+\lambda_0) + \lambda_0 \left[(\theta (1-\sigma) - 1) \left(1 - \frac{{}_0b_t^c}{c_t} + \Theta_t' \right) - (1-\theta)(1-\sigma) \frac{(l_t + {}_0b_t^w)}{(1-l_t)} \right], \tag{34}$$

and

$$\Theta_t' = \left(\frac{1}{\lambda_0} \left(\frac{\phi_{0t}}{c_t} - \{1-\delta_k\} \frac{\phi_{0t-1}}{c_{t-1}} \frac{c_{t-1}}{c_t} \right) \right).$$

Notice that all terms in (34) are either variables or ratios that are constant in a BGP. Hence, $H_{c_t} = H_c$, and therefore, $\frac{W_{c_t}}{W_{c_{t+1}}} = \frac{U_{c_t} H_c}{U_{c_{t+1}} H_c} = \frac{U_{c_t}}{U_{c_{t+1}}}$ in the long run. Taking this into account, it is easy to see from comparing (9) and (32) that the tax rate on capital income is zero in a BGP. Note also that as the capital tax rate becomes zero, the upper bound on capital tax rates will not be binding and the multiplier ϕ_{0t} will be zero.

Second, let us compute the labour income tax rate in a BGP. Given that $\frac{\phi_{ot}}{c_t}$ is zero in the long run, then (31a) becomes

$$\tau_{ss}^l = \frac{\lambda_0}{\lambda_0 + 1 - l_{ss}}. \tag{35a}$$

■

Proof of Proposition 3.

When the RHS of (24) and (25) are equated, we solve for labour which takes the value

$$l_t = \left[\frac{\delta_g - \delta_k}{A} \right]^{\frac{1}{1-\alpha}} * Q_t^{-\frac{1}{1-\alpha}}$$

where

$$Q_t = X_t - \frac{\frac{\xi_{0t}}{c_t}}{\left[1+\frac{1}{\beta}\frac{\xi_{0t-1}}{c_{t-1}} - \frac{\xi_{0t}}{c_t}\right]}\left[\alpha(1-\alpha)\frac{c_t}{k_t}\left(\left(\frac{k_t}{g_t}\right)^{\alpha} + \left(\frac{k_t}{g_t}\right)^{-(1-\alpha)}\right)\right]$$

From equation (30), we know that $\frac{\partial X_t}{\partial\left(\frac{k_t}{g_t}\right)} > 0$, and since $\frac{\xi_{0t}}{c_t}$ approaches zero, as it will be proven, we can guarantee that enough close to a BGP $\frac{\partial Q_t}{\partial\left(\frac{k_t}{g_t}\right)} > 0$ and hence $\frac{\partial l_t}{\partial\left(\frac{k_t}{g_t}\right)} < 0$. Taking this into account, it is easy to see from equation (21) that $\frac{\partial \gamma_{t+1}}{\partial\left(\frac{k_t}{g_t}\right)} < 0$. Finally, combining equations (22) and (23) with (8), the labour income tax rate takes the value

$$\tau_t^l = \frac{\frac{\lambda_0}{1-l_t} - \left[\frac{1}{\beta}\frac{\xi_{ot-1}}{c_{t-1}} - \frac{\xi_{ot}}{c_t}\right]}{1+\frac{\lambda_0}{1-l_t}} \tag{36}$$

where $\left[\frac{1}{\beta}\frac{\xi_{ot-1}}{c_{t-1}} - \frac{\xi_{ot}}{c_t}\right] \geq 0$ given that $\frac{\xi_{ot}}{c_t}$ approaches zero from above. Then it is obvious that when l_t approaches from below its steady state value so does τ_t^l.

■

Proof of Proposition 4.

First, we prove that ξ_{0t} is zero in a BGP. Under a zero-tax rate on capital income, given (2) and (17), we can write the first order condition for capital from the government maximisation problem (24) as

$$W_{c_t} = \beta W_{c_{t+1}}\left\{1 + f_{kt+1} - \delta_k\right\} - \beta\xi_{0t} f_{kk_{t+1}} U_{c_{t+1}}, \tag{37}$$

where $W_{c_t} = U_{c_t} H_{c_t}$, with

$$H_{c_t} = (1+\lambda_0) + \lambda_0\left[(\theta(1-\sigma)-1)\left(1 - \frac{{}_0b_t^c}{c_t} + \Theta'_{c_t}\right) - (1-\theta)(1-\sigma)\frac{(l_t + {}_0b_t^w)}{(1-l_t)}\right], \tag{38}$$

and

$$\Theta'_{c_t} = \left(\frac{1}{\lambda_0}\left(\frac{\xi_{0t}}{c_t} - \left\{1 + f_{k_t} - \delta_k\right\}\frac{\xi_{0t-1}}{c_{t-1}}\frac{c_{t-1}}{c_t}\right)\right).$$

It can be checked that $H_{c_t} = H_c$ in a BGP. Then (37) can be now written as

$$U_{c_t} = \beta U_{c_{t+1}}\left\{R_{k_{t+1}} + \frac{\xi_{0t}}{c_t}\frac{c_t}{c_{t+1}}\frac{c_{t+1}}{k_{t+1}}\alpha(1-\alpha)\frac{A}{H_c}\left(\frac{k_{t+1}}{g_{t+1}}\right)^{\alpha-1} l_{t+1}^{1-\alpha}\right\}. \tag{39}$$

Note that $\frac{c_{t+1}}{k_{t+1}}, \frac{c_t}{c_{t+1}}, \frac{k_{t+1}}{g_{t+1}}$, and l_{t+1} take values different from zero. Then, combining equations (21) and (39), the ratio $\frac{\xi_{0t}}{c_t}$ must be zero in a BGP. Moreover, $\frac{\xi_{0t}}{c_t} = 0$ implies that the zero-tax rate constraint (21) is satisfied through the government first order condition on capital (39) , then ξ_{0t} is zero in a BGP. Note also that since $\frac{\xi_{0t+1}}{c_{t+1}} = 0$, we have that $\Theta'_{l_{t+1}} = \Theta'_{c_{t+1}} = 0$ in the long run.

Let us now find the labour income tax rate in a BGP. Since $\frac{\xi_{0t}}{c_t}$ is zero in the long run, then (36) becomes

$$\tau^l_{ss} = \frac{\lambda_0}{\lambda_0 + 1 - l_{ss}}. \tag{40a}$$

Note that since the debt-commitment policy and under the full-commitment policy have different λ_0 and l_{ss}, then the steady-state tax rates on labour income, $(40a)$ and $(35a)$ differ under both policies.

■

Proof of Proposition 5.

We consider the policy plans under the governments at 0 and at period 1. If both decisions can be solved for the same allocation, the solution can be generalised for all later periods, and hence the policy plan at period 0 is made time-consistent. The government at period 0 can make its policy plan time-consistent by selecting debt values such that the same allocation $\{c_t, l_t, k_{t+1}\}_{t=1}^{\infty}$ and policy $\{g_{t+1}, \tau_t^l\}_{t=1}^{\infty}$ would solve both optimisation problems at 0 and 1. Therefore, this allocation must solve the different sets of restrictions and first order conditions that describe each plan. Let us now present both sets of conditions. When the government at date 0 plans a policy for $t \geq 0$, the policy and tax-induced allocation will be characterised by the set of constraints,

$$\sum_{t=0}^{\infty} \beta^t \left[U_{c_t} \left(c_t - {}_0b_t^c\right) - U_{1-l_t} \left[l_t + {}_0b_t^w\right]\right] \leq U_{c_0} W_0, \tag{41}$$

$$c_t + k_{t+1} + g_{t+1} \leq Ak_t^{\alpha} \left(l_t g_t\right)^{1-\alpha} + \left(1 - \delta_k\right) k_t + \left(1 - \delta_g\right) g_t, \text{ for all } t \geq 0, \tag{42}$$

$$U_{c_t} = \beta U_{c_{t+1}} \left\{1 + f_{k_{t+1}} - \delta_k\right\}, \text{ for all } t \geq 0, \tag{43}$$

the first order condition from the individual,

$$U_{1-l_t} = \left(1 - \tau_{l_t}\right) U_{c_t} f_{l_t}, \text{ for all } t \geq 0, \tag{44}$$

and the government plan,

$$W_{1-l}\left(c_t, 1-l_t, k_t, g_t, \ {}_0b_t^c, \ {}_0b_t^w, \xi_{0t}, \lambda_0\right) = f_{l_t} W_c\left(c_t, 1-l_t, k_t, g_t, \ {}_0b_t^c, \ {}_0b_t^w, \xi_{0t}, \lambda_0\right), \text{ for all } t \geq 0, \tag{45}$$

$$\mu_{0t} = \beta\mu_{0t+1}\left\{1 + f_{k_{t+1}} - \delta_k\right\} - \beta\xi_{0t} f_{k_{t+1}k_{t+1}} U_{c_{t+1}}, \text{ for all } t \geq 0, \tag{46}$$

$$\mu_{0t} = \beta\mu_{0t+1}\left\{1 + f_{g_{t+1}} - \delta_g\right\} - \beta\xi_{0t} f_{k_{t+1}g_{t+1}} U_{c_{t+1}}, \text{ for all } t \geq 0. \tag{47}$$

The set of equations, (41)-(47), forms the system that government at date 0 solves at announcing its policy plan.

When government at period 1 selects a policy for $t \geq 1$, this policy and the corresponding allocation satisfy the system of constraints,

$$\sum_{t=1}^{\infty} \beta^t \left[U_{c_t}\left(c_t - \ {}_1b_t^c\right) - U_{1-l_t}\left[l_t + \ {}_1b_t^w\right]\right] \leq U_{c_1} W_1, \tag{48}$$

$$c_t + k_{t+1} + g_{t+1} \leq Ak_t^{\alpha}\left(l_t g_t\right)^{1-\alpha} + \left(1-\delta_k\right) k_t + \left(1-\delta_g\right) g_t, \text{ for all } t \geq 1, \tag{49}$$

$$U_{c_t} = \beta U_{c_{t+1}}\left\{1 + f_{k_{t+1}} - \delta_k\right\}, \text{ for all } t \geq 1, \tag{50}$$

and first order condition for the individual,

$$U_{1-l_t} = \left(1 - \tau_{l_t}\right) U_{c_t} f_{l_t}, \text{ for all } t \geq 1, \tag{51}$$

and the government program,

$$W_{1-l}\left(c_t, 1-l_t, k_t, g_t, \ {}_1b_t^c, \ {}_1b_t^w, \xi_{1t}, \lambda_1\right) = f_{l_t} W_c\left(c_t, 1-l_t, k_t, g_t, \ {}_1b_t^c, \ {}_1b_t^w, \xi_{1t}, \lambda_1\right), \text{ for all } t \geq 1, \tag{52}$$

$$\mu_{1t} = \beta\mu_{1t+1}\left\{1 + f_{k_{t+1}} - \delta_k\right\} - \beta\xi_{1t} f_{k_{t+1}k_{t+1}} U_{c_{t+1}}, \text{ for all } t \geq 1, \tag{53}$$

$$\mu_{1t} = \beta\mu_{1t+1}\left\{1 + f_{g_{t+1}} - \delta_g\right\} - \beta\xi_{1t} f_{k_{t+1}g_{t+1}} U_{c_{t+1}}, \text{ for all } t \geq 1. \tag{54}$$

Equations, (48)-(54), would be the system that the government at date 1 solves when computing its policy plan.

Now, let us prove that the allocation $\{c_t, l_t, k_{t+1}\}_{t=1}^{\infty}$ and policy $\left\{g_{t+1}, \tau_t^l\right\}_{t=1}^{\infty}$ that solve the policy plan at 0 for $t \geq 1$ can solve the policy plan at 1. Since the sequences $\left\{c_t, l_t, k_{t+1}, g_{t+1}, \tau_t^l\right\}_{t=1}^{\infty}$ solve the former, it is also a solution for equations $(49)-(51)$. On the one hand, to solve (52) for the same allocation, we need to impose a condition for which one debt instrument at each period is needed. On the other hand, it can be checked that another debt instrument is needed in order to make (53) and (54) time-consistent. If both conditions are equated, ξ_t can be expressed as

$$\xi_t = \mu_{t+1}\left[\frac{R_{t+1} - \left(1 + f_{g_{t+1}} - \delta_g\right)}{U_{c_{t+1}}\left(f_{k_{t+1}k_{t+1}} - f_{k_{t+1}g_{t+1}}\right)}\right]. \tag{55}$$

Equation (55) implies that conditions (53) and (54) can be written as

$$\begin{aligned}
\mu_t &= \beta\mu_{t+1}\left[R_{t+1} - f_{k_{t+1}k_{t+1}}\left[\frac{R_{t+1} - \left(1 + f_{g_{t+1}} - \delta_g\right)}{f_{k_{t+1}k_{t+1}} - f_{k_{t+1}g_{t+1}}}\right]\right], \\
\mu_t &= \beta\mu_{t+1}\left[R_{t+1} - f_{k_{t+1}g_{t+1}}\left[\frac{R_{t+1} - \left(1 + f_{g_{t+1}} - \delta_g\right)}{f_{k_{t+1}k_{t+1}} - f_{k_{t+1}g_{t+1}}}\right]\right].
\end{aligned}$$

Therefore, once μ_{1t} and μ_{0t} take the same value, the same allocation solves the policy plan at 0 and at 1. In order to make μ_{1t} take that value, one extra debt instrument is needed. Note that when $\mu_{1t} = \mu_{0t}$, we have $\xi_{1t} = \xi_{0t}$ for the same allocation. So far, it has been shown that two debt instruments are needed in order to solve all equations, but (48). The path for this debt will be function of λ_1. Once the government at 0 finds these functions, they will be imposed into the budget constraint (3), which leads to a specific debt structure. Since the budget constraint is fulfilled, it can be seen that the implementability condition (48) is satisfied. Thus, under that debt structure, the continuing allocation and policy planned at 0 solves the policy plan at 1.

Let us now find the debt structure that provides time consistency. Four types of debt need to be found: (i) the new inherited debt indexed to consumption for the first period; (ii) debt indexed to consumption for second period and on; (iii) the new inherited debt indexed to after-tax-wage for the first period; (iv) debt indexed to after-tax-wage for second period and on. Now, we find the evolution of debt indexed to after-tax-wage from the second period on. Let us consider the first order conditions for consumption and leisure under both plans (45) and (52),

$$W_{1-l}\left(c_t, 1 - l_t, k_t, g_t, {}_0b_t^c, {}_0b_t^w, \xi_{0t}, \lambda_0\right) - f_{l_t}W_c\left(c_t, 1 - l_t, k_t, g_t, {}_0b_t^c, {}_0b_t^w, \xi_{0t}, \lambda_0\right) = 0 \tag{56}$$

$$W_{1-l}\left(c_t, 1 - l_t, k_t, g_t, {}_1b_t^c, {}_1b_t^w, \xi_{1t}, \lambda_1\right) - f_{l_t}W_c\left(c_t, 1 - l_t, k_t, g_t, {}_1b_t^c, {}_1b_t^w, \xi_{1t}, \lambda_1\right) = 0 \tag{57}$$

When (56) and (57) are satisfied, we can equate their LHS, and find

$$\begin{aligned}
&[\lambda_0-\lambda_1]\left[(U_{1-l_t}-f_{l_t}U_{c_t})+(U_{c_t1-l_t}-f_{l_t}U_{c_tc_t})\,c_t-(U_{1-l_t1-l_t}-f_{l_t}U_{c_t1-l_t})\,l_t-\left[\tfrac{\xi_{0t-1}-\xi_{1t-1}}{\lambda_0-\lambda_1}\right]U_{c_t}f_{k_tl_t}\right]\\
&\quad=-\left(U_{c_t1-l_t}-f_lU_{c_tc_t}\right)\left[\lambda_1\;{}_1b_t^c-\lambda_0\;{}_0b_t^c+\left[(\xi_{0t}-\xi_{1t})-\left(1+f_{k_{t-1}}-\delta_k\right)\left(\xi_{0t-1}-\xi_{1t-1}\right)\right]\right]\\
&\qquad\qquad-\left(U_{1-l_t1-l_t}-f_{l_t}U_{c_t1-l_t}\right)\left[\lambda_1\;{}_1b_t^w-\lambda_0\;{}_0b_t^w\right].
\end{aligned}\tag{58}$$

Let us divide equation (58) by $(U_{1-l_t1-l_t}-f_{l_t}U_{c_t1-l_t})\,\lambda_1$, take into account that $\xi_{0t}=\xi_{1t}$ and then add ${}_0b_t^w\left[\frac{\lambda_0}{\lambda_1}-1\right]$, and rearrange terms to obtain

$$\begin{aligned}
&\left[\tfrac{U_{1-l_t1-l_t}-f_{l_t}U_{c_t1-l_t}}{U_{c_t1-l_t}-f_{l_t}U_{c_tc_t}}\right]\left\{\left[\tfrac{\lambda_0}{\lambda_1}-1\right]\left[l_t+\;{}_0b_t^w-\tfrac{U_{1-l_t}-f_{l_t}U_{c_t}}{U_{1-l_t1-l_t}-f_{l_t}U_{c_t1-l_t}}\right]-\left[{}_1b_t^w-\;{}_0b_t^w\right]\right\}\\
&\qquad=\left[{}_1b_t^c-\tfrac{\lambda_0}{\lambda_1}\;{}_0b_t^c+\left[\tfrac{\lambda_0}{\lambda_1}-1\right]c_t\right].
\end{aligned}\tag{59}$$

Now, let us find the equation for $\mu_{0t}=\mu_{1t}$. Substitute μ_t by its value, equation (22), divide by $-U_{1-l_t1-l_t}\lambda_1$, take into account that $\xi_{0t}=\xi_{1t}$ and add ${}_0b_t^w\left[\frac{\lambda_0}{\lambda_1}-1\right]$, we obtain

$$\begin{aligned}
&\left[\tfrac{U_{1-l_t1-l_t}}{U_{c_t1-l_t}}\right]\left\{\left[\tfrac{\lambda_0}{\lambda_1}-1\right]\left[l_t+\;{}_0b_t^w-\left[\tfrac{U_{1-l_t}}{U_{1-l_t1-l_t}}\right]\right]-\left[{}_1b_t^w-\;{}_0b_t^w\right]\right\}\\
&\qquad=\left[{}_1b_t^c-\tfrac{\lambda_0}{\lambda_1}\;{}_0b_t^c+\left[\tfrac{\lambda_0}{\lambda_1}-1\right]c_t\right].
\end{aligned}\tag{60}$$

Notice that the RHS of equations (59) and (60) are equal, we can thus equate both. Rearranging terms we find

$${}_1b_t^w-\;{}_0b_t^w=\left[\frac{\lambda_0}{\lambda_1}-1\right]\left[{}_0b_t^w+l_t-\left[\frac{U_{c_t1-l_t}U_{c_t}-U_{c_tc_t}U_{1-l_t}}{U_{c_tc_t}U_{1-l_t1-l_t}-U^2_{c_t1-l_t}}\right]\right],$$

and taking into account the specific utility function form (??), we have

$${}_1b_t^w-\;{}_0b_t^w=\left[\frac{\lambda_0}{\lambda_1}-1\right]\left[{}_0b_t^w+\frac{1}{\sigma}+\left[1-\frac{1}{\sigma}\right]l_t\right],$$

which is the new inherited debt indexed to after-tax-wage for $t>1$. This procedure needs to be replicated for the three remaining debt types.

Appendix B: Numerical Solution Method.

In this appendix, we outline a more precise explanation of the eigenvalue-eigenvector decomposition method. The equations that will be given concerned the debt-commitment policy, they will serve as an example for the remaining policies. This method consists of the following:

i) Some reasonable parameter values are given.

ii) Stability conditions from the linearised system of first order conditions and restrictions are computed. To this end, the model is solved for a steady state, then the conditions are linearised around this steady state and the unstable eigenvalues of the linear system are computed. The stability conditions are the result of eliminating those unstable paths.

a) The solution of the debt-commitment model, $\{c_t, l_t, g_{t+1}, k_{t+1}, \tau_{l_t}, \mu_{0t}, \xi_{0t}\}_{t\geq 0}$ and λ_0, is characterised by the set of necessary conditions, (8) and (22) – (25), and the restrictions, (12) , (14) and (21). If the number of total periods were T, the system would have $T*4+(T-1)*3+1$ equations and the same number of unknown variables. However, our economy extends over an infinity of periods. For each period, there are four equations involving variables at that date, three equations that link future to current variables and one that is a function of variables in all dates. For the purpose of solving the system, the equations that link current and future variables need to be replaced by the stability conditions that depend on variables at that date. In an endogenous growth model, steady-state levels of the variables change over time. These variables are transformed such that they take constant values in a steady state, for example, $w_t^{kg} = \frac{k_t}{g_t}$, $w_t^{cc} = \frac{c_t}{c_{t-1}}$, $w_t^{ck} = \frac{c_t}{k_t}$, $w_t^{\xi c} = \frac{\xi_{0t}}{c_t}$ and $w_t^{cb^c} = \frac{b_t^c}{c_t}$. However, notice that to find the value of λ_0, that is independent of the moment of time, we need to know the whole series of variables and plugged them into (14). To solve this, a steady-state for a given value of λ_0 can be computed, and later, we search for the value of λ_0 that solves (14). Once we have a new set of constant variables in steady-state and a value for λ_0, all the conditions and restrictions that define a solution, but (14) , are transformed so that are functions of either ratios or variables that are constant in a BGP. Since all new transformed variables are constant in a BGP, we can take away the t index, and find the steady-state values of these new variables.

b) Next, all restrictions and conditions are linearised around the steady state. These equations are viewed as a function $f\left(w_t^{ck}, l_t, w_t^{kg}, w_t^{cc}, w_t^{\phi c}, w_t^{cb^c}\right)$, then one can define $y_t = \left(w_t^{ck} - w_{ss}^{ck}, l_t - l_{ss}, w_t^{kg} - w_{ss}^{kg}, w_t^{cc} - w_{ss}^{cc}, w_t^{\phi c} - w_{ss}^{\phi c}, w_t^{cb^c} - w_{ss}^{cb^c}\right)$.

Next, we do a first order Taylor approximation around the steady state

$$\left.\frac{\partial f}{\partial y_t}\right|_{ss} y_t + \left.\frac{\partial f}{\partial y_{t-1}}\right|_{ss} y_{t-1} = Ay_t + By_{t-1} = 0.$$

c) The unstable eigenvalues of the linear system are found. We find the set of eigenvalues and eigenvectors of the matrix $-(A^{-1}) * B$. We consider that an unstable eigenvector is one that takes an absolute value greater than $\beta^{-\frac{1}{2}}$, this number is chosen so that the objective function in the optimisation problem is bounded.

d) The stability conditions are computed. They are obtained by imposing orthogonality between each eigenvector associated with an unstable eigenvalue and the variables of this system, that is,

$$Cy_t = 0,$$

where C is the matrix of eigenvectors associated to unstable eigenvalues. These stability conditions are needed in order to guarantee that transversality conditions will hold.

iii) The stability conditions are imposed into the original non-linear model and then, a numerical solution is computed. The equations that linked current to future variables are replaced by the stability conditions. A solution can be now computed. Notice that since the stability conditions come from the linearised system of conditions, the solution involves some numerical error.

For a more complete review see Sims (1998) and Novales et al. (1999).

■

Figures and tables.

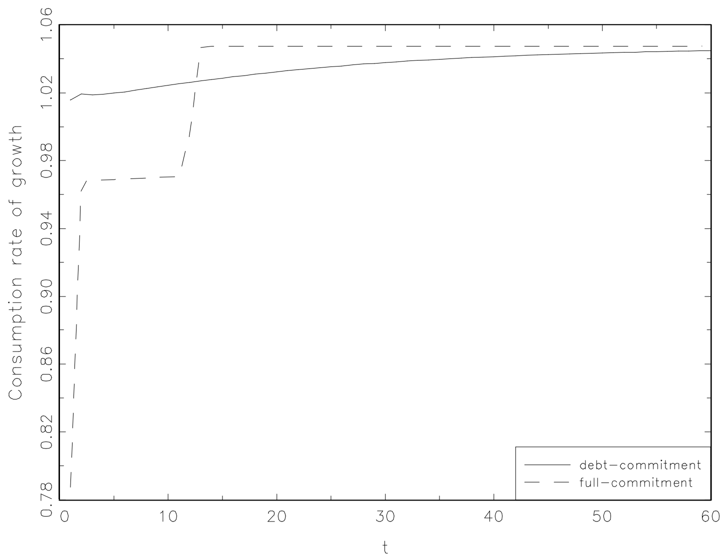

Figure 1: Consumption growth rates.

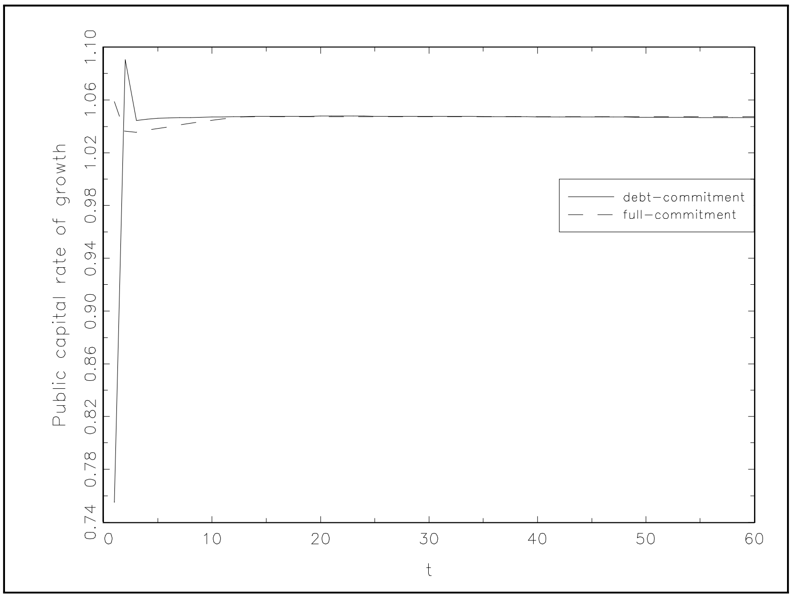

Figure 2: Public capital growth rate.

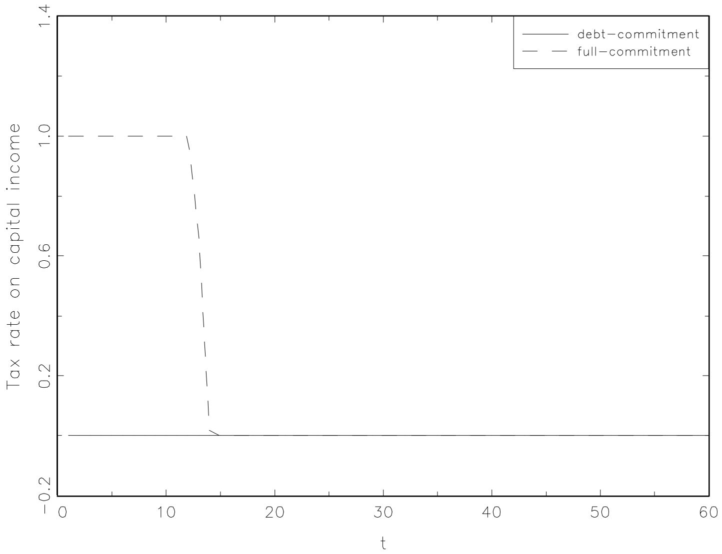

Figure 3: Tax rates on capital income.

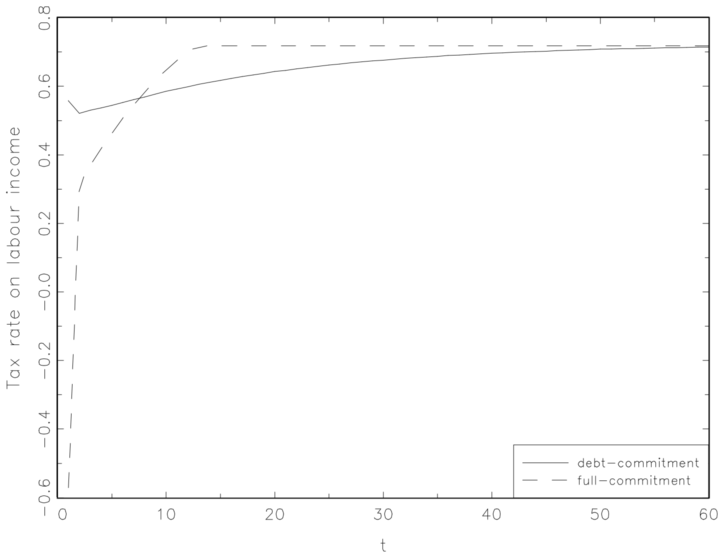

Figure 4: Tax rates on labour income.

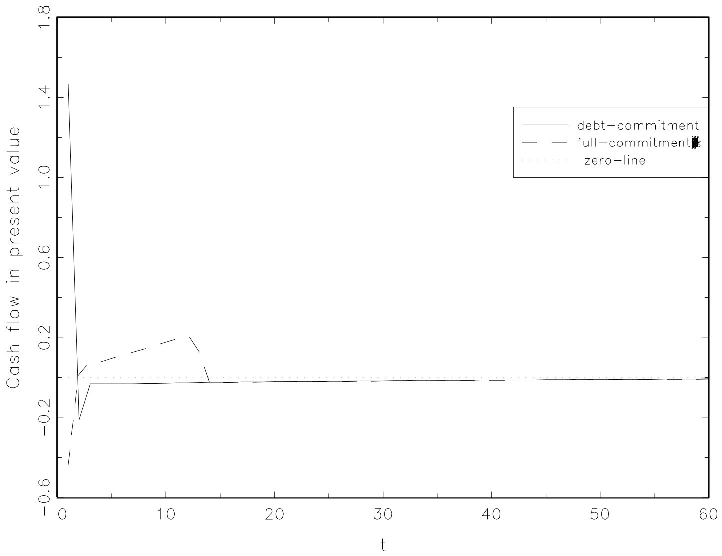

Figure 5: Cash flow in present value

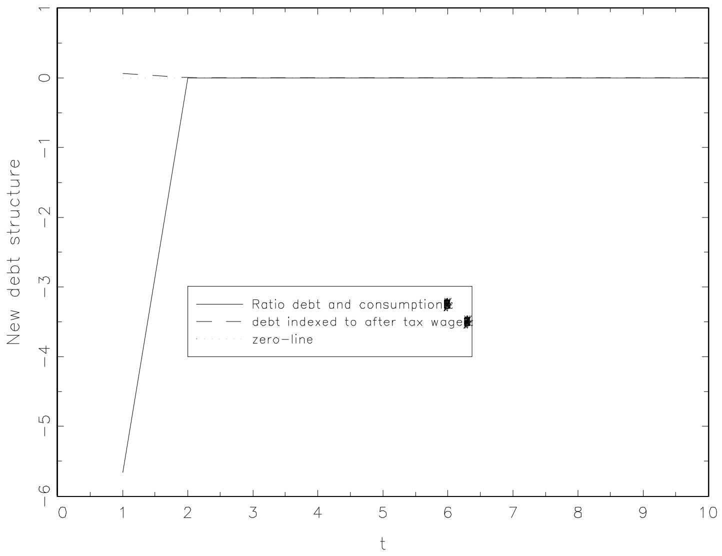

Figure 6: Debt structure that makes the policy time-consistent.

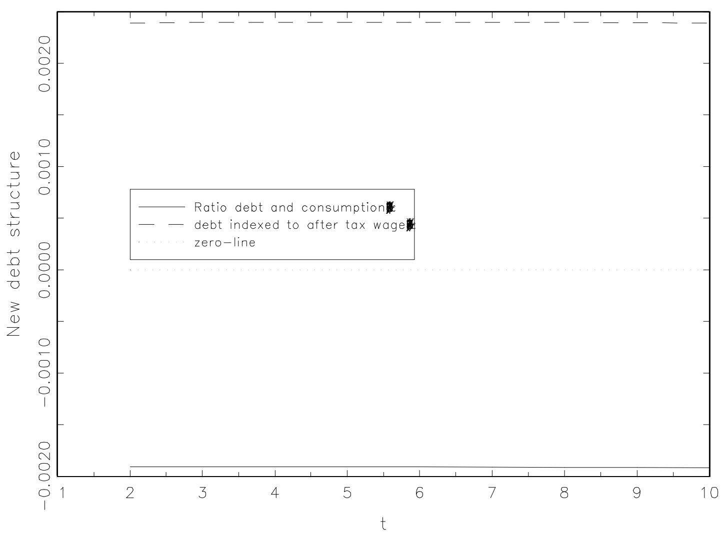

Figure 7: Debt structure.

TABLE 1. Summary of Results from the Numerical Solution Method.

Variables	**Policy 0**	**Policy 1**	**Policy 2**
$\left(\frac{c}{k}\right)_{ss}$	0.09931	0.09338	0.09230
$\mathbf{l}_{ss}$	0.29044	0.26664	0.26235
$\left(\frac{k}{g}\right)_{ss}$	0.34928	0.35037	0.35059
τ^l_{ss}	0.68156	0.71708	0.72242
τ^k_{ss}	0	0	0
M. E. K.	4.52e-9	7.18e-7	0.00691
M. E. G.	2.00e-8	2.19e-6	0.02040
M. E. T.	–	0	0.00043
E. I.	2.1e-13	9.59e-6	8.14e-7
Growth rate	1.05244	1.04739	1.04647
Welfare	55.6524	54.1458	52.8691

Policy 0 is the policy under full-commitment.

Policy 1 is the policy under full-commitment with $\tau_{k_t} \leq 1$.

Policy 2 is the policy under debt-commitment with $\tau_{k_t} = 0$.

M.E.K. and M.E.G. stand for maximum error at satisfying the first order condition for private and public capital respectively in the government program.

M.E.T. stands for maximum error at satisfying the restriction on capital tax rates.

E.I. stands for the error that is made at satisfying the implementability condition.