Scale-Invariant Endogenous Growth*

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Abstract

In this paper we develop a theory of scale-invariant endogenous growth. By this we mean a theory capable of generating a balanced growth path where both the growth rate and the level of GDP per capita are independent of the size of population, where population growth is neither necessary nor conductive for economic growth, and where economic incentives and policy matter for growth. Such a theory arises naturally when endogenous skill formation is added to a basic R&D driven growth model featuring diminishing returns to existing knowledge in creating new ideas.

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1 Introduction

The endogenous growth theory initiated by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) (below referred to as R/GH/AH) predicts a positive impact from the size of population on long-run total factor productivity (TFP) growth and therefore on income per capita growth. This prediction is nearly an inevitable consequence of the central assumption that the production of ideas is linear in the existing stock of knowledge implying that an increase in population and thereby in the number of scientists directly translates into an increase in TFP growth. However, as forcefully argued by Jones (1995a,b), this scale effect is at variance with available evidence and therefore constitutes a serious problem for the theory of endogenous growth.[1]

Consequently, Jones (1995b), Kortum (1997), and Segerström (1998) (henceforth J/K/S) have proposed a theory of so-called semi-endogenous growth.[2] This theory builds on an assumption of diminishing returns to knowledge in creating new ideas. Thus, to achieve a constant growth rate of knowledge one needs to allocate an increasing number of researchers to the R&D sector. This avoids the scale effect on long-run growth but also implies that population growth is necessary to keep growth in momentum.[3] Moreover, economic growth is independent of economic incentives and policy.[4] These features of the J/K/S theory are far from the spirit of the original endogenous growth theory.

Several recent papers (e.g. Young, 1998; Dinopoulos and Thompson, 1998; Peretto, 1998; Aghion and Howitt, 1998, ch. 12; Howitt, 1999) have therefore explored a new framework which eliminates the scale effect by combining vertical (quality improvements) and horizontal (new product lines) innovations. In this framework increases in population expand proportionally the number of sectors in the economy thereby keeping the size of each sector, and more importantly, the number of researchers per sector constant. Thus, the scale effect is absent because both the rate of expansion in quality and variety is independent of population size.

[1] A survey of the empirical evidence on scale effects is to be found in Dinopoulos and Thompsen (1999).

[2] A model exhibits semi-endogenous growth if the growth rate in per capita income is determined by a (some) exogenous − non-technological − growth rate(s).

[3] In fact, this insight can already be found in Groth (1992).

[4] It should be noted though that policy may influence growth by affecting the elasticities of output (Eicher and Turnovsky, 1999).

A salient feature of the approach is that the removal of the scale effect is accomplished without eliminating the ability of economic policy to affect long-run growth. In addition, population growth is not necessary for growth in per capita income. However, as emphasized by Jones (1999), this new approach shares two important predictions with the J/K/S framework. First, population growth stimulates growth in per capita income. Second, the long-run *level* of income per capita is increasing in the size of the population. Thus, the scale of the economy still matters, albeit in a more subtle fashion. Whether these two predictions can be supported by data is at this stage unclear.[5] Therefore, it seems interesting to investigate whether a theory of growth, void of scale effects on growth rates, necessarily leads to the conclusion that scale matters for the long-run level of income per capita and that population growth is conductive for economic growth. Or to put it differently, is it possible to construct a model that generates a balanced growth path where (*i*) both the growth rate and the level of GDP per capita are independent of the size of population, (*ii*) population growth is neither necessary nor conductive for economic growth, and (*iii*) economic incentives and economic policy matter for growth. Our analysis demonstrates that such a theory of 'scale-invariant endogenous growth' arises naturally if the level of human capital per person is endogenized in the J/K/S framework.[6]

One might conjecture that adding accumulation of human capital would aggravate the scale problems since a larger population allows for more teachers thereby speeding up human capital formation. While this is true, human capital formation also introduces an important congestion effect: more students for a given level of expenditures on education reduces the human capital acquired by the average student. Thus, if both R&D and human capital accumulation are necessary for growth then from a modelling perspective it is important to remember that human capital,

[5] Kremer (1993) does give some support for a beneficial effect of population growth on long-run growth in income per capita in the pre-industrial era. Yet, many empirical studies following Barro (1991) have documented a negative effect using post-world war II data. It is, however, unclear whether this negative effect is temporary, as predicted by standard neoclassical growth theory, or permanent as in our model. Hall and Jones (1999) studies the relationship between the long-run level of income and the size of the population. Their analysis reveal no significant effect of population size on the level of income per worker.

[6] Groth (1997) also explores the consequences of the interaction between endogenous R&D and human capital accumulation, complementary to each other. Groth shows – in a model without population growth – that the scale effect on the growth rate is dampened considerably.

contrary to ideas created by R&D, is a rival input which is linked to the human body.

Our model suggests that it is not the quantity of citizens but solely the knowledge of the average citizen that matter for the long-run level of per capita income. The knowledge of the average individual rises over time but, as a consequence of the congestion effect, at a slower pace than the aggregate stock of scientific knowledge. Thus, the average individual tends to become relatively more ignorant over time. In a world featuring increased specialization this implication seems eminently reasonable; contrary to, say, the last century, most people today certainly cannot make, and probably do not know the functioning of, the simple equipment they use daily, for example calculators, washing machines, cars, and so on.

Like in the R/GH/AH frameworks our model features constant returns to reproducible factors of production. Therefore, our model shares the property (*iii*) with these models; for instance, the share of resources used on R&D appears in the equation governing long-run growth. However, in our model the share of resources used on human capital accumulation also affects the growth rate. Moreover, the analysis shows that an increase in the share of resources used on R&D at the expense of resources for human capital formation may be detrimental to growth.[7] Hence, focusing entirely on either R&D or human capital in isolation may be misleading.

The remainder of the paper is organized as follows. The next section uses a very simple framework to discuss which of the properties (*i*) to (*iii*) are compatible with the R/GH/AH frameworks and the J/K/S frameworks, respectively. The third section shows how the simple framework can be augmented with endogenous human capital formation which leads to a balanced growth path with the properties (*i*), (*ii*), and (*iii*). The fourth section demonstrates that the 'toy' model in the third section can be microfounded in a Romer (1990) type model. The last section contains concluding remarks.

2 Growth Models with Scale Effects

We start by developing a very simple framework which enables us to illustrate some of the main features of both the endogenous and semi-endogenous growth models.

[7] A similar effect can be found in the model of secondary innovations discussed in Aghion and Howitt (1998, ch. 6).

Time, t, is continuous. Final output, Y_t, is produced using human capital augmented labor input, H_t, ideas, A_t, and some fixed factor of production, say, land, denoted by Z. Below we abstract from this latter factor of production in the analysis by normalizing it to one.[8] Whereas A_t is assumed non-rival, H_t (and Z) is conceived to be rival, since it is inevitably linked to the human body. The production function allows for constant returns to rival inputs but increasing returns to rival and non-rival inputs taken together. Formally, we assume that

$$Y_t = H_t^{\alpha} A_t^{\beta} Z^{1-\alpha}, \quad Z \equiv 1,\ 0 < \alpha < 1,\ 0 < \beta \leq 1, \tag{1}$$

where $H_t \equiv h_t L_t$. L_t is "raw" labor (for example equal to the total number of work hours) whereas h_t is the quality of labor. Next, we assume that ideas are produced using units of final output, that is

$$\dot{A}_t = \sigma_A Y_t, \tag{2}$$

where A_0 is given and where σ_A is the share of total output allocated to R&D. The remaining part of output, $1 - \sigma_A$, is consumed. Hence, we are considering the so-called 'lab-equipment' version of the R/GH/AH models.[9] For simplicity, we assume that σ_A is exogenous and constant. R/GH/AH assume that the production of knowledge is linear in existing knowledge, $\beta = 1$, and that human capital is constant, $H_t = \bar{H} = \bar{h}\bar{L}$. Therefore, the productivity of workers grows at the same pace as knowledge. Using equations (1) and (2) it is easy to show that the balanced growth rate in per capita income, $y \equiv Y/\bar{L}$, equals

$$\frac{\dot{Y}_t}{Y_t} \equiv g_y = \frac{\dot{A}_t}{A_t} \equiv g_A = \sigma_A \left(\bar{h}\bar{L}\right)^{\alpha}. \tag{3}$$

Simple inspection of this equation proves the following proposition:

Proposition 1 ***The R/GH/AH balanced growth path:*** (i) *The growth rate of income per capita is increasing in the size of the population.* (ii) *Population growth is not necessary for a positive growth rate. Positive population growth renders the*

[8] It should be noted that all models discussed below, including ours, exhibits a scale effect from Z. Below we demonstrate that a growth equilibrium can be completely scale invariant with respect to population – the empirical problematic prediction highlighted by Jones (1995a,b). But we do not mean to argue that a growth equilibrium could be (nor should be) invariant with respect to any possible kind of scale effect.

[9] See Rivera-Batiz and Romer (1991).

growth rate explosive. (*iii*) *Economic incentives and policy can affect the growth rate* (*through* σ_A).

As remarked in the introduction the scale effect (*i*) is inconsistent with the available empirical evidence. This raises the question of whether growth can be explained through the accumulation of knowledge without having a positive dependence from population size to growth.

J/K/S have shown that this is in fact possible. To do so, one need simply replace the linearity of existing knowledge in producing new knowledge with diminishing returns, $\beta < 1$, and furthermore assume that the total stock of human capital, H_t, is rising. In the present lab-equipment variant of the J/K/S models – which is closely related to the model developed by Jones and Williamson (1999) – it is also necessary to assume increasing returns to scale to the two growing factors of production, $\beta > 1-\alpha$, in order to ensure positive income per capita growth. In the J/K/S models the average human capital endowment per worker is assumed constant, $h_t = \bar{h}$. Still, the human capital stock rises through time, albeit at a constant rate

$$\frac{\dot{H}_t}{H_t} = \frac{\dot{L}_t}{L_t} \equiv n \geq 0, \tag{4}$$

where the parameter n denotes the exogenous growth rate of population and where $H_0 \equiv \bar{h}L_0$ is given. Using equations (1), (2), and (4) it is easy to show that the growth rate of per capita income along the steady growth path, g_y, is given by

$$g_y = \beta g_A + (\alpha - 1)\, n = \frac{\beta - (1-\alpha)}{1-\beta} n. \tag{5}$$

At this point it is possible to establish the following properties of the J/K/S model.

Proposition 2 ***The J/K/S balanced growth path:*** (*i*) *The growth rate of per capita income is invariant to the size of population but the level of income per capita is increasing in the population size.* (*ii*) *Population growth is necessary and conductive for growth in per capita income.* (*iii*) *The balanced growth rate is invariant to changes in economic incentives and policy* (*e.g. the growth rate is independent of* σ_A).

Proof. In order to prove the second part of (*i*), note that per capita income – using equation (1) – can be written as

$$\frac{Y_t}{L_t} \equiv y_t = A_t^{\beta} L_t^{\alpha-1} \bar{h}^{\alpha}. \tag{6}$$

On the steady growth path, the growth rate of knowledge, g_A, is constant. Taking equations (2) and (5) into account it must hold that

$$g_A = \frac{\alpha}{1-\beta}n = \frac{\sigma_A Y_t}{A_t} = \sigma_A \frac{L^\alpha \bar{h}^\alpha}{A_t^{1-\beta}}.$$

Isolating A_t and substituting the result into equation (6) yields

$$y_t = \left(\frac{1-\beta}{\alpha} \frac{\sigma_A}{n} \right)^{\frac{\beta}{1-\beta}} \bar{h}^{\frac{\alpha}{1-\beta}} L_0^{\frac{\beta-(1-\alpha)}{1-\beta}} e^{\frac{\beta-(1-\alpha)}{1-\beta}nt}, \tag{7}$$

which reveals that the size of the population, for given $\bar{h}$, matters for the long-run level of per capita income. □

Thus, while the J/K/S framework is able to eliminate the scale effect on long-run growth, it still features a scale effect on the level of income. Moreover, the J/K/S approach removes the identifying characteristic of endogenous growth theory, namely that policy matters for long-run growth.[10]

Observe that in the limiting case of constant returns to the two growing factors of production, $\beta = 1 - \alpha$, the scale effect on the level of income is in fact eliminated from the J/K/S framework (see equation (7)). However, in this case the long-run growth rate of per capita income is zero. But this result rests on the assumption that the quality of researchers, that is h_t, is assumed to be constant. On the intuitive level, all that is needed to overcome the diminishing technological opportunities is that the skills of researchers rise over time. And this need not entail a growing population as assumed in the J/K/S models. Therefore we investigate the implications of allowing h_t to be endogenous in the next section.

3 A Simple Scale-Invariant Endogenous Growth Model

In this section we reconsider the J/K/S model above, albeit with two modifications. First, we allow the average level of human capital, h_t, to be endogenous in the model. Thus, the aggregate stock of human capital rises, not only because of an

[10] There are, however, two corollaries to this result. First, policy affects the level of income, which is apparent from equation (7). Thus policy can have *transitional* growth effects. Secondly, policy can have permanent growth effects insofar as it can affect fertility (see Jones, 1997).

increasing number of individuals in the economy, but also because of a rising quality of each individual. Second, we assume, like in the R/GH/AH models, constant returns to reproducible factors of production, in our case A_t and H_t. This requires that β equals $1-\alpha$ in equation (1). Note, that the model therefore allows for diminishing returns to knowledge as in the J/K/S models. However, increases in the quality and quantity of researchers tend to mitigate this effect. If the number of researchers grows, measured in efficiency units, a constant flow of research results can be obtained. If the quality adjusted amount of researchers does not grow, growth will eventually come to a halt. The model thus captures that a high level of A_t is less useful, when it comes to producing the next idea, if the level of human capital is low. Following Mankiw, Romer and Weil (1992) we assume that the equation governing the evolution of the human capital stock is

$$\dot{H}_t = \sigma_H Y_t, \tag{8}$$

where σ_H is the share of output used to produce human capital and where $H_0 \equiv h_0 L_0$ is given. Thus, the share $\sigma_C \equiv 1-(\sigma_A+\sigma_H)$ of output is consumed. Observe that equation (8) contains a congestion effect which, as will become clear later, ensures that the accumulation of human capital does not introduce any new scale effects neither on the growth rate of per capita income nor on the level of per capita income. This congestion effect is evident if we use the definition of the human capital stock, $H_t = h_t L_t$, to eliminate $\dot{H}_t$ in the above equation. This yields

$$\dot{h}_t = \frac{\sigma_H Y_t}{L_t} - n h_t, \tag{9}$$

where h_0 is given. The numerator, $\sigma_H Y_t$, represents the input in the human capital sector which consists of land, ideas, and human capital. The denominator in the first term can be interpreted as the number of students (here equal to the entire population). As is apparent from the equation, the smaller the ratio of expenditures on education to the number of students, the less quality expands. If a given growth rate of individual human capital is to be attained, expenditures have to be growing relatively to the inflow of students; otherwise the sheer number of students will tend to 'crowd out' quality growth. The second term on the RHS of (9) reflects the costs of bringing the skill level of the newcomers up to the average level of the existing population. This implies that population growth, ceteris paribus, tends to reduce the 'quality' of the average individual in the population.

To solve the model we define

$$\chi_t \equiv H_t/A_t.$$

The dynamic evolution of χ_t can subsequently be derived from equations (1), (2), and (8):

$$g_\chi \equiv g_H - g_A = \sigma_H \chi_t^{\alpha-1} - \sigma_A \chi_t^{\alpha}. \tag{10}$$

Along a steady growth path, where g_y is constant, human capital and knowledge have to grow at the same rate, that is $g_\chi = 0$. Therefore, the steady state human capital to knowledge ratio is given by

$$\chi = \frac{\sigma_H}{\sigma_A}. \tag{11}$$

It is easy to see from equation (10) that this steady state is indeed stable. Using (11) it is possible to derive the growth rates of human capital, g_H, and knowledge, g_A, along the balanced growth path using equations (2) and (8):

$$g_H = g_A = \sigma_H^{\alpha} \sigma_A^{1-\alpha} \equiv \tilde{g}. \tag{12}$$

Using equation (1) it can be confirmed that the growth rate of per capita income equals

$$g_y = \tilde{g} - n. \tag{13}$$

The characteristics of the balanced growth path are summarized in the following proposition

Proposition 3 ***The growth path of the simple scale-invariant model:*** (i) *The long-run level and growth rate of per capita income is invariant to the size of the population.* (ii) *Population growth is neither necessary nor conductive for long-run growth.* (iii) *Economic incentives and policy can affect the long-run growth rate (through σ_A and σ_H).*

Proof. To see that there is no scale effect on the level of income, we compute the balanced growth path of per capita income. Using equations (1) and (12), we get

$$\frac{Y_t}{L_t} = \frac{A_t^{\alpha} H_t^{1-\alpha}}{L_t} = \left(\frac{A_t}{H_t}\right)^{\alpha} h_t = \left(\frac{\sigma_A}{\sigma_H}\right)^{\alpha} h_0 e^{(\tilde{g}-n)t},$$

which is independent of L_0 for given h_0. $\square$

Hence, the present model demonstrates how diminishing returns to knowledge and endogenous human capital accumulation allow for the simultaneous removal of the scale effect on the growth rate and on the level of income while allowing for positive per capita growth.

Observe from equation (13) that population growth actually reduces growth in income per capita. This effect is permanent - contrary to, say, standard neoclassical growth theory where increases in n only has a temporary negative impact on growth. However, in the microfounded version of the model, developed in the next section, increases in n may have either none or a negative effect on long-run growth depending on household preferences.

In addition our model allows policy to play a role in shaping long-run growth. This occurs in a similar fashion as in the R/GH/AH models where σ_A also enters the growth equation. However, contrary to these models increases in σ_A need not spur growth in our framework. Consider the following corollary

Corollary 1 *For any given σ_C, growth in per capita income is maximized when*

$$\frac{\sigma_A}{\sigma_H} = \frac{1-\alpha}{\alpha}.$$

Proof. The problem is to $\max_{\sigma_A} \tilde{g} = \sigma_H^{\alpha} \sigma_A^{1-\alpha}$ subject to the identity $\sigma_H = 1 - \sigma_C - \sigma_A$, which immediately yields the result. $\square$

Hence, growth will be reduced if the share of resources used on R&D is increased at the expense of resources for human capital formation beyond the point where $\frac{\sigma_A}{\sigma_H}$ equals $\frac{1-\alpha}{\alpha}$. The intuition is that R&D and human capital accumulation complement each other; if too little emphasis is placed on human capital production, the average quality of researchers suffers, and so does growth. Thus, the corollary suggests that looking at the share of resources on R&D or human capital in isolation may be misleading. Another unique implication of the model is the following

Corollary 2 *For $n > 0$, individuals tend to become relatively more ignorant over time, since $g_A - g_h > 0$.*

Thus the 'knowledge frontier' grows faster than the average knowledge of any given individual. In a world featuring increasing specialization this implication is eminently reasonable. This will become more evident when we, in the next section, reconfirm the conclusions of the above simple analysis in a model of growth through specialization.

4 The Decentralized Model

In the model developed below, technical progress manifests itself as increasing specialization, that is through an increasing variety of intermediate inputs. The basic structure follows Rivera-Batiz and Romer (1991), but we depart from the basic framework by allowing the stock of human capital to be endogenously determined as in the toy model above. Therefore, growth persists in our model for two reasons − R&D and human capital accumulation.

At the more detailed level, the model comprises three sectors; a final goods sector, an intermediate good sector, and finally, an R&D sector. While the final goods sector and the R&D sector are competitive, we assume that the intermediate goods sector is monopolistic. Final goods are produced using two rival inputs; intermediate goods and human capital. Over time production becomes more involved, in the sense that the number of intermediate goods rises. The output from the final goods sector is used for consumption and investment. Furthermore, we assume that investments can be made in patents, that is funding for R&D and intermediate goods production, and human capital. This implies that final goods are used for three kinds of production purposes; R&D, intermediate good production and production of human capital. While firms decide on how many resources to employ in R&D and in production of intermediate goods, it is the household that decides on investing in human capital.

We start by examining the final goods sector in Section 4.1. In Section 4.2 we solve the monopolists' problem in the intermediate goods sector and characterize the incentives to innovate. Then, in Section 4.3, we solve the consumers' problem. Lastly, we derive the balanced growth path and state our main results in Section 4.4.

4.1 The Final Goods Sector

We assume that final goods, Y_t, are produced using human capital augmented labor input, $H_t \equiv h_t L_t$, a fixed factor Z, and specialized inputs x_{jt}. The latter is indexed by j. We denote by A_t the total number of varieties used in production at time t. Specifically, the production function of the representative firm is given by

$$Y_t = \left(\frac{H_t}{A_t}\right)^{\alpha} \int_{j=0}^{A_t} x_{jt}^{\gamma} dj \cdot Z^{1-\alpha-\gamma}, \tag{14}$$

where α and γ are positive parameters fulfilling $\alpha + \gamma < 1$. The term $(H_t/A_t)^{\alpha}$ is thought to capture that production tends to become more human capital intensive through time as production complexity increases (see e.g. Howitt, 1999). Technically, it allows for an aggregate production function which exhibits constant returns to rival inputs (H_t, x_{jt}, and Z) and to human capital and ideas (H_t and A_t). The former property ensures that the aggregate production function is consistent with the well-known replication argument whereas the latter represent a sufficient condition for endogenous growth. Like in the toy model we normalize Z to one.

The representative firm maximizes profits. The price of final goods act as numeraire. Therefore, the firm employ intermediate goods and human capital to the point where marginal productivity equals the price, denoted by p_{jt} and r_t^H, respectively. Thus,

$$\frac{\partial Y_t}{\partial x_{jt}} = \gamma \left(\frac{H_t}{A_t}\right)^{\alpha} x_{jt}^{\gamma-1} = p_{jt}, \tag{15}$$

$$\frac{\partial Y_t}{\partial H_t} = \alpha H_t^{\alpha-1} A_t^{-\alpha} \int_{j=0}^{A_t} x_{jt}^{\gamma} dj = r_t^H. \tag{16}$$

4.2 The Intermediate Goods Sector

The intermediate goods sector operates under monopoly. We assume that once monopoly status is acquired, by purchasing a blueprint from the R&D sector, it lasts indefinitely. Additionally, it is assumed that the production of one unit of intermediary input costs one unit of final output. Thus, the jth monopolists problem of maximizing profits, π_{jt}, can be stated as

$$\max_{x_{jt}} \pi_{jt} = (p_{jt} - 1)\, x_{jt}, \tag{17}$$

subject to the demand schedule, equation (15). On this basis it can readily be shown that the profit maximizing price and output level are given by

$$p_{jt} = p_t \equiv 1/\gamma, \quad x_{jt} = x_t \equiv \gamma^{\frac{2}{1-\gamma}} \left(\frac{H_t}{A_t} \right)^{\frac{\alpha}{1-\gamma}}. \tag{18}$$

Notice that the production of each intermediary input is an increasing function of the supply of human capital. This follows from the complementarity between x_t and H_t in the production of final goods. As output and prices are identical for all j, it follows that profits are the same for all j :

$$\pi_{jt} = \pi_t \equiv (1/\gamma - 1) \gamma^{\frac{2}{1-\gamma}} \left(\frac{H_t}{A_t} \right)^{\frac{\alpha}{1-\gamma}}. \tag{19}$$

Notice that insofar as H_t/A_t is constant, as it will be in equilibrium, x_t, p_t, and π_t will be constant.

The decision to engage in innovative activities is determined by comparing marginal costs to marginal benefits. Assuming that spending one unit of output (deterministically) leads to the discovery of a new variety, one can state the arbitrage condition as

$$1 = V_t = \int_{t=0}^{\infty} \pi_t e^{-\int_{s=0}^{t} r_s^A ds} dt, \tag{20}$$

where V_t is marginal benefit from engaging in production of intermediary inputs and r_t^A is the rate of return on research and development. Differentiating (20) one can derive the condition on flow form:

$$1 = \frac{\pi_t}{r_t^A}. \tag{21}$$

4.3 The Households

The total number of households is constant through time, and normalized to unity. However, the size of the household increases through time at the rate of population growth, n. The representative household maximizes

$$U_0 = \int_{t=0}^{\infty} \frac{c_t^{1-\varepsilon} - 1}{1 - \varepsilon} e^{-\theta t} dt, \tag{22}$$

where c_t is the consumption level of each individual in the household and where θ and ε are the rate of time preference and the coefficient of relative risk aversion, respectively.[11] The optimization problem of the representative household consists of dividing income, Q_t, between consumption, $c_t L_t$, and investments, and furthermore, in allocating investments between human capital, I_t^H, and patents, I_t^A. Thus,

$$Q_t = I_t^H + I_t^A + c_t L_t, \tag{23}$$

where $I_t^H \equiv \dot{H}_t$ and $I_t^A \equiv \dot{A}_t$.

Total household income derives from the return from human capital $r_t^H H_t$ and from investing in the production of ideas, the proceeds of which is returned to the households in the shape of dividends, $r_t^A A_t$. In order to parameterize policy, we allow for subsidies to human capital and R&D investments at the proportional rates τ^H and τ^A (which may be negative corresponding to a tax). The subsidies are financed trough a lump sum tax, T_t, and we assume that the government balances the budget at all times. Hence,

$$Q_t = \left(1 + \tau^H\right) r_t^H H_t + \left(1 + \tau^A\right) r_t^A A_t - T_t. \tag{24}$$

The representative household chooses $\left\{c_t, I_t^A, I_t^H\right\}_{t=0}^{\infty}$ in order to maximize (22) subject to (23), (24), and the non-negativity constraints $c_t \geq 0$, $I_t^A \geq 0$, and $I_t^H \geq 0$. The solution to this problem is provided in Appendix A.

4.4 The Balanced Growth Equilibrium

The above non-negativity constraints on the household may give rise to transitional dynamics, that is the household may choose temporary to invest in only one of the two assets. However, the economy converges to a unique balanced growth path in finite time (see Appendix B). On this path the household invests in both human capital and patents and the returns on these two investments are equalized, that is

$$\left(1 + \tau^H\right) r_t^H = \left(1 + \tau^A\right) r_t^A \equiv r, \tag{25}$$

which corresponds to a standard no-arbitrage condition. Additionally, the path is characterized by the Keynes-Ramsey rule

$$\frac{\dot{c}_t}{c_t} = g_c = \frac{1}{\varepsilon}\left(r - \theta - n\right). \tag{26}$$

[11] As usual we assume that discounted utility is bounded, implying that $\theta > (1 - \varepsilon) g_y$, where g_y is the long-run growth rate in per capita income, derived below.

The research arbitrage equation (21), the equilibrium condition for the asset market (25), along with the expression for the rental rate of human capital (16), and the expression for profits (19), pin down the H_t/A_t ratio along the balanced growth path:

$$\frac{A_t}{H_t} = \Phi \equiv \frac{\gamma(1-\gamma)}{\alpha}\frac{1+\tau^A}{1+\tau^H}. \tag{27}$$

The immediate implication of this equation is that $g_A = g_H$ along the balanced growth path. From the accounting equations (23) and (24), it can readily be confirmed that aggregate output and consumption grow at the same rate. Hence, the Keynes-Ramsey rule pins down growth in total (and per capita) income. Notice, that the ratio of R&D expenditures to GDP will be constant through time, as $g_A = g_Y$; an implication which conforms with the available empirical evidence (see e.g. Howitt, 1999).

To solve for the growth rate in per capita income, we derive the equilibrium interest rate, r, from equations (16) and (18), and insert the expression into equation (26). This yields

$$g_y = \frac{1}{\varepsilon}\left(\left(1+\tau^H\right)\alpha\Phi^{\frac{1-\alpha-\gamma}{1-\gamma}}\gamma^{\frac{2\gamma}{1-\gamma}} - \theta - n\right), \tag{28}$$

which we assume is positive. The entire balanced growth path of per capita income can now be derived from equation (14) and the equilibrium expressions for x_t and H_t/A_t, which gives

$$y_t = y_0 e^{g_y t} = \left(\frac{A_0}{H_0}\right)^{1-\alpha} x_0^{\gamma} h_0 e^{g_y t} = \gamma^{\frac{2\gamma}{1-\gamma}}\Phi^{\frac{1-\alpha-\gamma}{1-\gamma}} h_0 e^{g_y t}. \tag{29}$$

The properties of the balanced growth path of the model are summarized in the following proposition

Proposition 4 (i) *The long-run level and growth rate of per capita income are invariant to the size of the population.* (ii) *Population growth is neither necessary nor conductive for long-run growth.* (iii) *Policy affects the long-run growth rate of income per capita* (*through* τ^A *and* τ^H).

The proposition shows that it is possible to construct a microfounded theory of scale-invariant endogenous growth.

Observe from equation (28) that increases in population growth, n, decreases per capita income growth like in the toy model of Section 3. However, this result is not robust as it depends crucially on the specification of preferences. Suppose that the household, instead of using the current welfare criteria, had used the Benthamite welfare criteria.[12] Under such circumstances θ would be replaced by $\rho - n$, where ρ reflects the "pure" rate of time preference. It is easy to see from equation (28) that growth of per capita income in this case would be independent of n. The intuition is simply that, under total utility maximization (the Bentham criteria), the household becomes more patient, because the size of the family in the future is taken into account. This tends to increase the propensity to save, and as a result, exactly cancels the detrimental effect of population growth on the rate of income expansion which is present in equation (28). This is, however, the only difference in results between the general model and the toy model discussed above.

The decentralized model also features the result that there exists a non-linear relationship between growth and the share of output used for R&D, cf. Corollary 1. To see this consider the special case of the model where lump sum taxes, T_t, are zero implying that the subsidies/taxes, τ^H and τ^A, fulfill

$$\frac{r_t^H H_t}{Y_t}\tau^H + \frac{r_t^A A_t}{Y_t}\tau^A = \alpha\tau^H + (1-\gamma)\gamma\tau^A = 0, \tag{30}$$

where the first equality follows from equations (14), (16), (19), (21), and (27). Maximizing growth, equation (28), with respect to τ^H and τ^A subject to (30) yields

$$\hat{\tau}^H = \frac{\alpha}{1-\gamma} - (1-\gamma) \in (-1, 1),$$

$$\hat{\tau}^A = \frac{\alpha}{\gamma}\left(1 - \frac{\alpha}{(1-\gamma)^2}\right) \in (-1, \infty),$$

where we have used the parameter restriction $\alpha + \gamma < 1$. Now, if the share of output used for R&D, $\dot{A}_t/Y_t$, is increased by raising τ^A (and lowering τ^H correspondingly) then this only increases growth if $\tau^A < \hat{\tau}^A$. Increasing τ^A beyond $\hat{\tau}^A$ is detrimental to growth, and the intuition is exactly as in the toy model: if the quality of researchers does not increase sufficiently over time income growth will suffer.

[12] That is, if we had used $L_t\left(c_t^{1-\varepsilon} - 1\right)/(1-\varepsilon)$ as instantaneous utility function in equation (22).

The result of "increasing relative ignorance", cf. Corollary 2, can also be confirmed within the present model, since the following holds:

$$g_A = g_H = g_h + n.$$

Thus, $g_A > g_h$. This result is a product of the balanced growth property, that knowledge is increased as the *effective* amount of labor input rises. Thus, in a relative sense, each person in the economy becomes increasingly ignorant as time passes by.

5 Concluding Remarks

In this paper, we offer a novel explanation for the absence of the scale effect on growth which, like recent contributions, preserves a role for policy in shaping economic growth and allows for perpetual growth when the population size is fixed. In contrast, however, our explanation does not imply that the population size matters for the long-run level of income per capita and that population growth is conductive for growth in per capita income. Hence, the present paper demonstrates that the elimination of all scale effects from population size in no way makes endogenous growth impossible.

These results arise naturally when allowing for endogenous accumulation of both ideas and human capital. Human capital production is, contrary to the production of ideas, associated with an important congestion effect: an increase in the number of students will, ceteris paribus, tend to crowd out the quality of the average student. This simple congestion effect is crucial for eliminating the scale effects.

Several empirical questions remain. First, does population size matter for long-run income? It seems to be an important topic for future empirical work to asses whether a scale effect on income levels is supported by the evidence. Second, is population growth conductive for long-run growth in income per capita? According to recent theoretical contributions the tentative answer is affirmative – on the contrary, our theory suggests that the answer is in the negative.

A The Households' Problem

The households' problem is to

$$\max_{\left\{c_t, I_t^A, I_t^H\right\}_{t=0}^{\infty}} U_0 = \int_{t=0}^{\infty} \frac{c_t^{1-\varepsilon} - 1}{1-\varepsilon} e^{-\theta t} dt,$$

subject to the following set of constraints:

$$\begin{aligned}
\dot{H}_t &= I_t^H \geq 0, \quad H_0 \equiv h_0 L_0 \text{ given}, \\
\dot{A}_t &= I_t^A \geq 0, \quad A_0 \text{ given}, \\
c_t &\geq 0, \\
I_t^H + I_t^A &= \left(1+\tau^H\right) r_t^H H_t + \left(1+\tau^A\right) r_t^A A_t - T_t - c_t L_t, \\
A_t &> 0 \quad \text{for all } t, \\
H_t &> 0 \quad \text{for all } t.
\end{aligned}$$

The discounted value Hamiltonian, J_t, after insertion of c_t, is given by

$$J_t = \frac{\left(\frac{\left(1+\tau^H\right) r_t^H H_t + \left(1+\tau^A\right) r_t^A A_t - T_t - I_t^H - I_t^A}{L_t}\right)^{1-\varepsilon} - 1}{1-\varepsilon} e^{-\theta t} + \lambda_{Ht} I_t^H + \lambda_{At} I_t^A.$$

Due to the inequalities $I_t^A \geq 0$ and $I_t^H \geq 0$, the first order conditions with respect to I_t^A and I_t^H are

$$\frac{\partial J_t}{\partial I_t^A} \cdot I_t^A = 0, \tag{31}$$

$$\frac{\partial J_t}{\partial I_t^H} \cdot I_t^H = 0, \tag{32}$$

and

$$\frac{\partial J_t}{\partial I_t^A} = \lambda_{At} - c_t^{-\varepsilon} \cdot L_t^{-1} \cdot e^{-\theta t} \leq 0, \tag{33}$$

$$\frac{\partial J_t}{\partial I_t^H} = \lambda_{Ht} - L_t^{-1} c_t^{-\varepsilon} e^{-\theta t} \leq 0. \tag{34}$$

The first order conditions with respect to the state variables are

$$\frac{\partial J_t}{\partial A_t} = L_t^{-1} c_t^{-\varepsilon} e^{-\theta t} \left(1+\tau^A\right) r_t^A = -\dot{\lambda}_{At}, \tag{35}$$

$$\frac{\partial J_t}{\partial H_t} = L_t^{-1} c_t^{-\varepsilon} e^{-\theta t} \left(1+\tau^H\right) r_t^H = -\dot{\lambda}_{Ht}. \tag{36}$$

Finally, the solution has to fulfill the two transversality conditions:

$$\lim_{t \to \infty} \lambda_{At} A_t \leq 0,$$

$$\lim_{t \to \infty} \lambda_{Ht} H_t \leq 0.$$

A.1 Household behavior when $\left(1+\tau^H\right) r_t^H = \left(1+\tau^A\right) r_t^A$

If the return on the two assets are identical, it follows from equations (16), (18), (19), and (21) that

$$\left(1+\tau^H\right) r_t^H = \left(1+\tau^A\right) r_t^A = r \equiv \left(1+\tau^H\right) \alpha \Phi^{\frac{1-\alpha-\gamma}{1-\gamma}} \gamma^{\frac{2\gamma}{1-\gamma}},$$

where Φ is a constant given by equation (27). We now rule out that (a) $I_t^A > 0 \wedge I_t^H = 0$, (b) $I_t^A = 0 \wedge I_t^H > 0$, and (c) $I_t^A = I_t^H = 0$, thus implying that the solution is characterized by $I_t^A > 0 \wedge I_t^H > 0$.

First, note that the transversality conditions and $A_0 > 0$ and $H_0 > 0$ and $I_t^A \geq 0$ and $I_t^H \geq 0$ imply

$$\lim_{T\to\infty} \lambda_{AT} = 0, \tag{37}$$

$$\lim_{T\to\infty} \lambda_{HT} = 0. \tag{38}$$

Next, integrate equations (35) and (36) from time 0 to time T to yield

$$\int_{t=0}^{T} \dot{\lambda}_{At} dt = \lambda_{AT} - \lambda_{A0} = -\int_{t=0}^{T} L_t^{-1} c_t^{-\varepsilon} \left(1+\tau^A\right) r_t^A e^{-\theta t} dt, \tag{39}$$

$$\int_{t=0}^{T} \dot{\lambda}_{Ht} dt = \lambda_{HT} - \lambda_{H0} = -\int_{t=0}^{T} L_t^{-1} c_t^{-\varepsilon} \left(1+\tau^H\right) r_t^H e^{-\theta t} dt. \tag{40}$$

We next subtract equation (40) from (39) and let $T \to \infty$. After using equations (37) and (38), we obtain

$$\lambda_{A0} - \lambda_{H0} = \int_{t=0}^{\infty} L_t^{-1} c_t^{-\varepsilon} \left[\left(1+\tau^A\right) r_t^A - \left(1+\tau^H\right) r_t^H\right] e^{-\theta t} dt. \tag{41}$$

When $\left(1+\tau^A\right) r_t^A$ and $\left(1+\tau^H\right) r_t^H$ are identical, so are λ_{A0} and λ_{H0}. This implies from equations (33) and (34) that $\frac{\partial J_t}{\partial I_t^A} = \frac{\partial J_t}{\partial I_t^H}$, which rules out case (a) and (b). The assumption of positive growth, that is $r > \theta + n$, implies that the household has an incentive to invest thereby ruling out case (c) directly. Hence, $I_t^A > 0$ and $I_t^H > 0$. It is now straightforward to derive the Keynes-Ramsey-rule, equation (26), from the equations (31) to (36).

A.2 Household behavior when $\left(1+\tau^H\right) r_t^H \neq \left(1+\tau^A\right) r_t^A$

Consider the case where $\left(1+\tau^A\right) r_0^A > \left(1+\tau^H\right) r_0^H$ at time zero. We wish to rule out (a) $I_0^H > 0 \wedge I_0^A = 0$, (b) $I_0^H > 0 \wedge I_0^A > 0$, and (c) $I_0^H = 0 \wedge I_0^A = 0$, thus demonstrating that $I_0^A > 0$ and $I_0^H = 0$ holds.

Consider, first case (a). When $I_t^H > 0$ equation (34) holds with equality according to equation (32), that is

$$L_t^{-1} c_t^{-\varepsilon} e^{-\theta t} = \lambda_{Ht} > 0.$$

Using this equation and equation (33), it follows that

$$\lambda_{At} - L_t^{-1} c_t^{-\varepsilon} e^{-\theta t} = \lambda_{At} - \lambda_{Ht} \leq 0. \tag{42}$$

If $I_t^H > 0$ and $I_t^A = 0$ then the production technology implies that $\left(1+\tau^H\right) r_t^H$ will be decreasing through time whereas $\left(1+\tau^A\right) r_t^A$ will be increasing through time. Hence, if $\left(1+\tau^A\right) r_0^A > \left(1+\tau^H\right) r_0^H$ implies $I_0^H > 0 \wedge I_0^A = 0$ then $\left(1+\tau^A\right) r_t^A > \left(1+\tau^H\right) r_t^H$ for all $t > 0$. In then follows from equation (41) that $\lambda_{At} - \lambda_{Ht} > 0$, which contradicts equation (42).

Consider case (b). Equation (31) and (32) implies that $\frac{\partial J_t}{\partial I_t^A} = \frac{\partial J_t}{\partial I_t^H} = 0$. Hence, equations (33) and (34) imply that

$$\lambda_{At} = \lambda_{Ht} \text{ for all } t,$$

from which it follows that $\dot{\lambda}_{At} = \dot{\lambda}_{Ht}$ for all t. It then follows from equations (35) and (36) that $\left(1+\tau^A\right) r_t^A = \left(1+\tau^H\right) r_t^H$ $\forall t$ contradicting that $\left(1+\tau^A\right) r_0^A > \left(1+\tau^H\right) r_0^H$.

Finally, consider case (c). If $I_t^A = I_t^H = 0$ then total income is constant and equal to total consumption, that is $Y_t = \bar{Y} = c_t L_t$, implying that

$$\frac{\dot{c}_t}{c_t} = -n.$$

Also, if $I_t^A = I_t^H = 0,$ then A_t/H_t is constant implying that the returns to the two assets are constant, that is $r_t^A = \bar{r}^A$ and $r_t^H = \bar{r}^H$. Using this, equation (39), and the transversality condition, we have

$$\lambda_{A0} = \int_{t=0}^{\infty} L_t^{-1} c_t^{-\varepsilon} \left(1+\tau^A\right) \bar{r}^A e^{-\theta t} dt = L_0^{-1} c_0^{-\varepsilon} \left(1+\tau^A\right) \bar{r}^A \int_{t=0}^{\infty} e^{-(\theta - n(\varepsilon-1))t} dt. \tag{43}$$

For the parameter constellation $\theta - n(\varepsilon - 1) < 0$ this equation implies that $\lambda_{A0} = \infty$. This leads to a contradiction since equation (33) implies that

$$\lambda_{A0} \leq c_0^{-\varepsilon} \cdot L_0^{-1} \tag{44}$$

in case (c). Consider instead the parameter constellation $\theta - n(\varepsilon - 1) > 0$. Then equation (43) yields

$$\lambda_{A0} = \frac{L_0^{-1} c_0^{-\varepsilon} \left(1 + \tau^A\right) \bar{r}^A}{\theta - n(\varepsilon - 1)}. \tag{45}$$

To see that this contradicts (44), insert (45) into (44). This gives

$$\frac{L_0^{-1} c_0^{-\varepsilon} \left(1 + \tau^A\right) \bar{r}^A}{\theta - n(\varepsilon - 1)} \leq c_0^{-\varepsilon} \cdot L_0^{-1},$$

or, equivalently,

$$\left(1 + \tau^A\right) \bar{r}^A - \theta - n \leq -\varepsilon n. \tag{46}$$

Note that $\left(1 + \tau^A\right) \bar{r}^A > \left(1 + \tau^H\right) \bar{r}^H$ implies that $\left(1 + \tau^A\right) \bar{r}^A > r > \left(1 + \tau^H\right) \bar{r}^H$ where r corresponds to the value of A_t/H_t where $\left(1 + \tau^A\right) r_t^A = \left(1 + \tau^H\right) r_t^H \equiv r$. Hence,

$$\left(1 + \tau^A\right) \bar{r}^A - \theta - n > r - \theta - n > 0,$$

where the last inequality follows from the assumption of positive steady state growth. The above inequality contradicts the inequality (46), thereby ruling out case (c).

Thus, if $\left(1 + \tau^A\right) r_0^A > \left(1 + \tau^H\right) r_0^H$ then $I_0^A > 0$ and $I_0^H = 0$. A similar argument as above can be used to show that $\left(1 + \tau^A\right) r_0^A < \left(1 + \tau^H\right) r_0^H$ implies $I_0^A = 0$ and $I_0^H > 0$.

B The Transition to the Balanced Growth Path

If $A_t/H_t = \Phi$, where Φ is given by equation (27), it follows from equations (16), (18), (19), and (21) that $\left(1 + \tau^A\right) r_t^A = \left(1 + \tau^H\right) r_t^H = r$. In this case the economy follows the balanced growth path which exhibits the properties summarized in Proposition 4. We now show that the economy converges to this path if $A_t/H_t \neq \Phi$. Consider first the case where $A_t/H_t < \Phi$. It then follows from equations (16), (18), (19), and (21) that $\left(1 + \tau^A\right) r_t^A > r > \left(1 + \tau^H\right) r_t^H$. In Appendix A we have established that this implies that $I_t^A > 0$ and $I_t^H = 0$ ensuring that A_t/H_t is rising. This process will continue until $A_t/H_t = \Phi$ corresponding to the balanced growth path. A similar argument applies to the case when $A_t/H_t > \Phi$.

The economy reaches the balanced growth path in finite time. To see this, remember that the households only invest in A_t, as long as $A_t/H_t < \Phi$, and that the amount of investment needed to reach the steady state ratio Φ equals $\Phi H_0 - A_0$ which is finite. Thus, the economy will eventually reach the balanced growth path and continue along this path afterwards.

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