Progressive Taxation and the Real Business Cycle

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Abstract

The paper aims to study the impact of the introduction of a progressive labor income tax scheme over the real business cycle in contrast to a model economy with proportional labor income tax.

While in most recent quantitative business cycle studies the proportional tax is introduced due to the easy tractability of this tax struture, the presence of a progressive tax on labor income, which is common to all economies the business cycle studies try to mimic, could introduce some important and distictive features into the cycle's statistical properties under analysis.

The summary statistics of the dynamic pattern of the main aggregate variables show a clear effect of the introduction of progressive labor income taxes. This model economy generates a higher volatility for the aggregate series compared to the ones obtained by the basic model with proportional taxes, displaying some characteristics present in the U.S. time series data.

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1 Introduction

Hansen and Prescott (1995) present extensions of the linear quadratic (LQ) approach for computing a recursive competitive equilibrium (RCE) for model economies in which the competitive equilibrium need not be Pareto optimal to study their real bussiness cycle characteristics.

Bugarin (1998) analyses an algorithm to apply a piece-wise LQ approximation in an attempt to analytically compute an equilibrium for a model economy distorted by progressive taxation. It is shown that given the particular structure of the model, namely a linear deterministic law of motion for capital accumulation and the exogenously given productivity shock together with the progressive tax structure on labor income imposed by the government, lead to a tractable problem for a computationally efficient application of the piece-wise LQ approach.

This paper aims to study the impact of the introduction of a progressive labor income tax scheme over the real business cycle in contrast to a model economy with proportional labor income tax.

While in most recent quantitative business cycle studies the proportional tax is introduced due to the easy tractability of this tax struture, the presence of a progressive tax on labor income, which is common to all economies the business cycle studies try to mimic, could introduce some important and distictive features into the cycle's statistical properties under analysis. In other words, the introduction of the progressive tax struture could cause some business cycle characteristics that cannot be analyzed if the model economy used in the study does not allow for such a scheme.

From the methodological point of view, the numerical computation of a recursive equilibrium leads to a choice, basically, between different approaches for aproximating the value function of the dynamic programing problem. On one hand, the standard value function iteration method, highly non-linear and computationally inefficient, is an approximation based on the dicretization os the state variables to overcome with the problem known as the "curse of dimensionality", through the construction of a grid for the state variable capital and the approximation of the exogenous stochastic shock by a finite order Markov chain. On the oder hand, using the LQ approach one can trade-off the above "curse of dimensionality" by a numerical aproximation of the value function by using the three first terms of the second order Taylor series expansion of the return function around its steady state variable values and, the optimal (linear) decision rules associated with the approximated linear quadratic value function are explicitly derived, according to the definition of recursive competitive equilibrium.

The paper is organized as follow. Setion 2 introduces a model economy distorted with progressive taxation on labor income, the equivalent dynamic programing problem, the definition of RCE, the implemented computational procedure as well as the parameter values used to calibrate the model. Section 3 presents the main results, the steady state results and the optimal (linear) decision rules associated with the defined RCE. Section 4 analyses the cyclical properties under the introduction of progressive labor income taxes into the model economy, contrasting them with the results obtained using a model distorted by proportional taxes. Finally, in Section 5 some concluding remarks about the impact over the business cycle due to the introduction of progressive labor income taxes will be stated.

2 Introducing a Progressive Labor Income Tax

2.1 The Model Economy

Consider the commonly used model in the business cycle literature, which consists of a large number of identical agents each endowed with capital k_0 in period 0 and one unit of time per period that can be spent working or enjoying leisure. The representative agent's (RA) problem is to choose optimal sequences for consumption $\{c_t\}$ and labor $\{h_t\}$ (the complement of leisure l_t), such that the expected discounted stream of utility is maximized given a budget constraint. Assume the following about agent's preferences:

Assumption 1 *The period utility function $u(.,.) : R_+ \to R$ is bounded, continuously differentiable, strictly concave, strictly increasing in both arguments and satisfies the Inada conditions, i.e. $(u_1/u_2) \to \infty$ as $(c_t/(1-h_t)) \to 0$ and $(u_1/u_2) \to 0$ as $(c_t/(1-h_t)) \to \infty$. Hence both consumption and leisure are normal goods.*

The firm hires labor h_t^f and capital services k_t^f at every t to produce output y_t^f, according to a constant return to scale technology. Thus, without loss of generality assume there is only one firm, operating at zero profit, using the technology given by:

$$y_t^f = z_t f(k_t^f, h_t^f) \tag{1}$$

where z_t is an exogenous shock to the technology realized at time t.

Assume the following about the technology:

Assumption 2 *The technology shock is described by $z_t = e^{\omega_t}$, where ω_t is a random term which evolves according to the first order autoregressive process $\omega_t = \rho\omega_{t-1} + \epsilon_t$, $0 < \rho < 1$ and $\epsilon_t \overset{i.i.d.}{\sim} N(0, \sigma_\epsilon^2)$, $\sigma_\epsilon^2 < \infty$.*

The term ω_t captures the productivity shock which is the source of uncertainty for this economy.

Assumption 3 *The constant return to scale production function* $f(.,.) : R_+^2 \to R_+^1$ *is twice continuously differentiable, strictly increasing and strictly concave with* $zf(0,h^f) = zf(k^f,0) = 0$ *and* $\lim_{k^f \to 0} zf_1(k^f,h^f) = \infty$ *for* $h^f > 0$ *and for all* $z \in R^1$, $z \neq 0$.

The per period wage rate w_t and the rate of return for capital services r_t are derived from the first order conditions of the firm's profit maximization. They are given by:

$$w_t = z_t f_2(k_t^f, h_t^f) \tag{2}$$

$$r_t = z_t f_1(k_t^f, h_t^f) \tag{3}$$

Use the market clearing conditions $k_t^f = K_t N$ and $h_t^f = H_t N$, and denote by N the total number of households. The following equilibrium conditions result from (1.3) and (1.4):

$$w_t = z_t f_2(K_t, H_t) \tag{4}$$

$$r_t = z_t f_1(K_t, H_t) \tag{5}$$

Assume that the aggregate per capita capital stock evolves according to the law of motion given by:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{6}$$

Assumption 4 *Progressive taxes with non decreasing marginal rates* $\tau_1 < \tau_2$ *are levied on labor income. Hence, two tax brackets are created with* H^* *denoting the kink point of time spent working which occurs at the intersection of these tax brackets.*

The government in this economy collects taxes from labor and capital income at rates τ_1, τ_2 and τ_k respectively. This revenue is returned to the households as a lump sum transfer.

Assumption 4 leads to two after tax wage rates, $w_i = (1-\tau_i)w, i = 1, 2$ where τ_i is the labor income tax rate, such that $w_1 > w_2$. Hence, the per period budget set for the RA will still be convex but with a kink at the intersection of the two budget lines corresponding to the marginal nondecreasing tax rates τ_1 and τ_2.

With the introduction of this progressive tax regime, the income tax is determined by two different brackets defined by H^*, with corresponding marginal tax rates τ_1 and τ_2. The agent's labor income subject to taxation is $w_t h_t$ for any t. Consequently the labor income tax levied on period t is given by:

$$\Phi_t = \begin{cases} \tau_1 h_t w_t & \text{if } 0 \leq h < H^* \\ \tau_1 H^* w^* + \tau_2 (h_t w_t - H^* w^*) & \text{if } 1 > h \geq H^* \end{cases}$$

where $H^* w^*$ is the labor income at the kink point. Observe that for $1 > h \geq H^*$, the corresponding expression for the amount of taxes paid assures no discontinuity of net income at the kink point.

With the above assumptions, the representative agent (RA)'s problem can be expressed as:

$$\max_{c_t, h_t} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(u(c_t, 1-h_t) \right) \right\} \tag{7}$$

such that,

$$c_t + i_t \leq w_t h_t - \Phi_t + (1-\tau_k) r_t k_t + \tau_k \delta k_t + \gamma_t, \quad \text{for } t \in [0, \infty)$$

given the law of motion for capital formation $K_{t+1} = (1-\delta)K_t + I_t$, $k_{t+1} = (1-\delta)k_t + i_t$, initial capital stock k_0 and initial productivity shock z_0.

Observation 1 *For problem (7) above, a LQ dynamic programing approach can be implemented consisting of solving two LQ problems, for $\tau_w = \tau_1$ and for $\tau_w = \tau_2$ respectively, based on the fact that the domain of the linear quadratic value function, in terms of the state variable capital K, is divided into two fixed intervals $K1 = (0, K^*]$ and $K2 = [K^*, \overline{K})$, where $K^* = 3.6414$, which are independent of the shock, for any $t \in [1, \infty)$.*[2]

2.2 The Associated Dynamic Programing Problem

Given this observation for this problem, the above RA's problem with progressive taxes on labor income can be rewritten as the following dynamic programing problem:

[2] For a numerical verification of this observation refer to Bugarin (1998).

$$v(z,S,s) = \max\{v_1(z,S,s), v_2(z,S,s)\} \tag{8}$$

subject to:
$z' = \rho z + \epsilon'$
$K_j' = (1-\delta)K_j + I_j$, for $j = 1,2$
$k_j' = (1-\delta)k_j + i_j$, for $j = 1,2$

$$I = \begin{cases} I_1 = I_1(z,S) & \text{if } v(z,S,s) = v_1(z,S,s) \\ I_2 = I_2(z,S) & \text{if } v(z,S,s) = v_2(z,S,s) \end{cases}$$

$$H = \begin{cases} H_1 = H_1(z,S) & \text{if } v(z,S,s) = v_1(z,S,s) \\ H_2 = H_2(z,S) & \text{if } v(z,S,s) = v_2(z,S,s) \end{cases}$$

where v_1 is solution to:

$$v_1(z,S,s) = \max\{y'Q_1y + \beta E[v_1(z',S',s')/z]\} \tag{9}$$

subject to $(10-1)$ to $(10-5)$, $d_1 = D_1(H_1(z,S), I_1(z,S))$ for $\tau_w = \tau_1$ and,

v_2 is solution to:

$$v_2(z,S,s) = \max\{y'Q_2y + \beta E[v_2(z',S',s')/z]\} \tag{10}$$

subject to $(10-1)$ to $(10-5)$ and, $d_2 = D_2(H_2(z,S), I_2(z,S))$

The matrices Q_1 and Q_2 are obtained from the quadratic (numerical) approximation of the return function around the steady state variable values corresponding to $\tau_w = \tau_1$ and $\tau_w = \tau_2$ respectively.

2.3 Defining the RCE

A RCE equilibrium for the model economy with progressive taxes levied on labor income described by the RA's problem (1.17), technology (1.2) and government budget constraint (1.8) is:

(i) A set of decision rules for households:

$$i = \begin{cases} i_1 = i_1(z,S,s) & \text{if } v(z,S,s) = v_1(z,S,s) \\ i_2 = i_2(z,S,s) & \text{if } v(z,S,s) = v_2(z,S,s) \end{cases}$$

$$h = \begin{cases} h_1 = h_1(z,S,s) & \text{if } v(z,S,s) = v_1(z,S,s) \\ h_2 = h_2(z,S,s) & \text{if } v(z,S,s) = v_2(z,S,s) \end{cases}$$

(ii) A set of aggregate decision rules:

$$I = \begin{cases} I_1 = I_1(z,S) & \text{if } v(z,S,s) = v_1(z,S,s) \\ I_2 = I_2(z,S) & \text{if } v(z,S,s) = v_2(z,S,s) \end{cases}$$

$$H = \begin{cases} H_1 = H_1(z,S) & \text{if } v(z,S,s) = v_1(z,S,s) \\ H_2 = H_2(z,S) & \text{if } v(z,S,s) = v_2(z,S,s) \end{cases}$$

(iii) A value function $v(z, S, s)$ such that:

1. Given I and H, the value function v satisfies (1.18) above and,
2. The associated decision rules $i_j = i_j(z, S, s)$ and $h_j = h_j(z, S, s)$ are such that:
   $I_j(z, S) = i_j(z, S, S)$
   $H_j(z, S) = h_j(z, S, S)$, for $j = 1, 2$

2.4 Computational Procedure

The computational procedure is based on a two piece-wise LQ approximation of the value function, each piece corresponding to one of the two tax brackets, adapting the methodology suggested by Hansen and Prescott (1995) with the following modifications:

Step 1 Compute the steady state solution for problem (7) separately for τ_j, $j = 1, 2$.

Step 2 Compute the value function v_j with associated policy rules I_j and H_j for problems (9) and (10), with $\tau_w = \tau_j$, $j = 1, 2$.

Step 3 Substitute the obtained optimal rules I_j and H_j into the corresponding linear quadratic value function, choosing the maximizing feasible decision rules for problem (8).

Step 4 Compute the optimal capital, labor, consumption and output paths derived from step 3, given initial conditions on the capital stock k_0 and productivity shock z_0.

2.5 Parameter Values

The following set of preference, tax policy and technology parameters have been used for calibration purposes.[3]

Preference Parameters:
$\beta = 0.95$
$\alpha = 0.64$

Technology parameters:
$\theta = 0.40$
$\delta = 0.02$
$\rho = 0.95$
$\sigma_\epsilon = 0.007$

Policy parameters:
$\tau_1 = 0.2$, for $0 < h \leq H^*$
$\tau_2 = 0.4$, for $H^* < h < 1$
$H^* = 0.35$ [4]
$\tau_k = 0.4$

3 Results

3.1 Steady State Values

Based on the adopted parameterization, the derived steady state values for capital, labor and consumption are reported in Table 1.

Table 1: Steady State Variable Values

Variables	Tax Rates		Change
	$\tau_1 = 0.2$	$\tau_2 = 0.4$	
$\bar{K}$	4.27107	3.63976	14.78%
$\bar{L}$	0.47965	0.40875	14.78%
$\bar{C}$	1.06477	0.90738	14.78%
$\bar{Y}$	1.15960	0.98820	14.78%

[3] These parameters are the estimates used by Cooley and Prescott (1995).

[4] This break point corresponds to the intersection of the two budget sets using steady state variable values

As expected, all steady state values decrease as the marginal tax rate on labor income increases. A 100% increase in the marginal tax rate on labor income from 20% to 40% leads to decreases in capital, labor, consumption and output at the steady state of 14.78%.

3.2 Optimal (Linear) Decision Rules

The associated linear optimal policy functions for labor are shown in Table 2 and for investment in Table 3. These functions induce the corresponding steady state values for the endogenous state variable capital, and the decision variables labor and investment up to five decimal points.

Table 2: Optimal Labor Decision Rule: $H_{jt} = a_{0j} + a_{1j} z_t + a_{2j} K_t$

Tax Rates	a_0	a_1	a_2
$\tau_1 = 0.2$	0.5068	-0.0194	-0.0018
$\tau_2 = 0.4$	0.4351	-0.0188	-0.0020

Table 3: Optimal Investment Decision Rule: $I_{jt} = b_{0j} + b_{1j} z_t + b_{2j} K_t$

Tax Rates	b_0	b_1	b_2
$\tau_1 = 0.2$	0.0848	4.44e-004	4.16e-005
$\tau_2 = 0.4$	0.0727	3.77e-005	4.15e-006

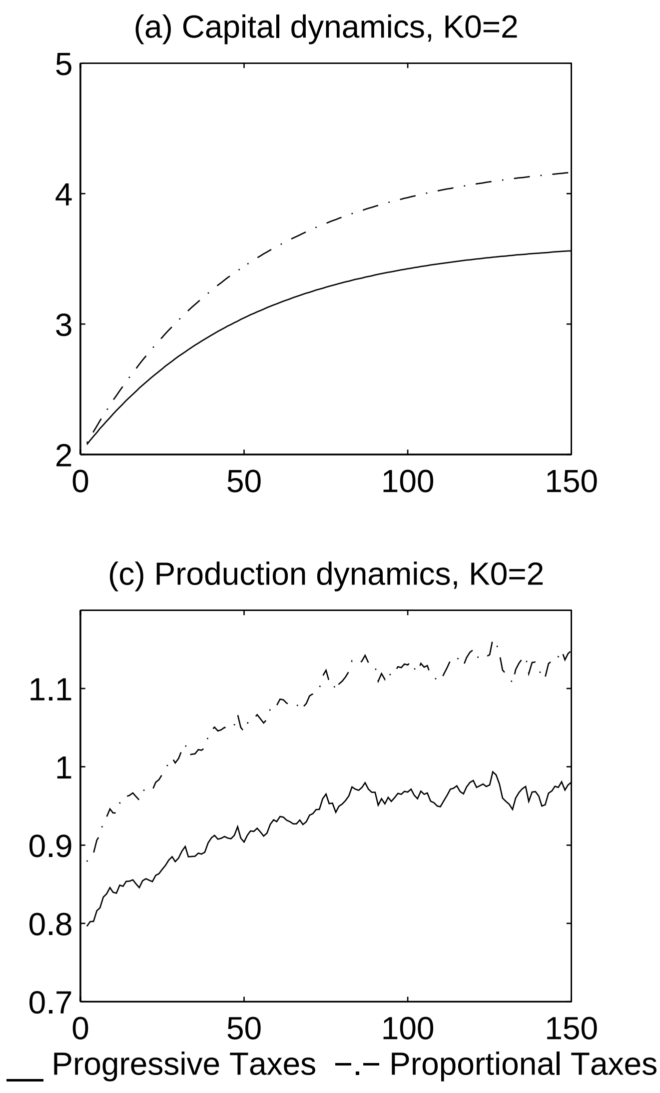
4 Cyclical Properties

4.1 Transition Paths

Figure 1 below shows the dynamic patterns of the aggregate per capita capital, consumption and production series, in (a), (b) and (c) respectively, for $t = 1, ..., 150$, obtained from a progressive and a proprotional labor income tax schemes.

As can be seen in the figure, all series reache a higher level if the model economy is distorted by a proportional labor income tax compared to the same economy where progressive taxes are applied. In other words, the introduction of a progressive tax on labor income leads to a lower labor choice depressing the capital accumulation process of the economy, hence, leading to a lower level of aggregate production and consumption as well.

Figure 1: Transition Paths with Progressive and Proportional Taxes



Finally, as can be seen in Table 4, regardless of the initial capital condition, the labor sequence shows the smallest standard deviation from its mean value and the capital sequence has the highest volatility among the variables. This matches some of the empirical evidence reported by Cooley and Prescott (1995).

Table 4: Standard Deviations of Labor, Capital Accumulation, Consumption and Output Transition Paths

Initial Capital Stock	Labor	Capital	Consumption	Output
$K_0 = 2$	0.0049	0.4151	0.0875	0.0505
$K_0 = 3$	0.0048	0.1599	0.0333	0.0202
$K_0 = 4$	0.0047	0.0951	0.0263	0.0172
$K_0 = 5$	0.0047	0.3503	0.0676	0.0378

4.2 Real Bussines Cycle Statistical Properties

The summary statistics describing the cyclical behavior of the model economy with proportional and progressive labor income taxes, as well as the statistics corresponding to actual U.S. time series[5], are shown in the Table below, where the initial capital stock is arbitrarily assumed to be $k_0 = K_0 = 3.5$ for both model economies.

Table 5: Standard Deviations and Correlation with Output

Series	U.S.Quarterly series		Model with Proportional Tax		Model with Progressive Tax	
	st. dev.	correlation with output	st. dev.	correlation with output	st. dev.	correlation with output
Output	0.0174	1.00	0.0066	1.00	0.0120	1.00
Consumption	0.0081	0.65	0.0065	0.99	0.0092	0.91
Capital Stock	0.0038	0.28	0.0035	0.91	0.0035	0.27
Labor	0.0141	0.86	0.0007	0.98	0.0048	0.90

The statistics provide a basis for comparison between the model with proportional (labor and capital) income taxes and the model distorted with progressive labor income tax. Note, however, that this study is not a data matching exercise but rather a simulation of an artificial economy under two qualitatively different tax schemes. The optimal decision rules for the particular parameterization above are used, along with the law of motion for capital accumulation and productivity shock process, to generate artificial time series with the same number of observations as in the data sample, for each model economy.

The above Table shows that the aggregate series of the model economy with proportional income taxes display a lower volatility than the series generated by the

[5]The U.S. quarterly time series statistics from 1955:3 to 1984:1 are taken from Cooley and Hansen (1989).

model with progressive labor income tax. The introduction of the latter decreases the correlation coefficient with aggregate output which closely approximated the actual U.S.cyclical behavior. Hence, it is apparent that introducing progressive taxes on labor income affects the cyclical statistical properties of the aggregate variables of the model economy. The variables becomes more volatile, and the capital stock and labor series decreases their correlation with output compared to the series displayed by the basic model with proportional income taxes.

5 Conclusion

This study shows the application of a piece-wise LQ approximation of the value function in a subclass of economies distorted by a progressive tax schedule on labor income in order to study its real business cylce characteristics.

Based on this approach, a set of optimal decision rules are derived, which along with the dynamics for capital accumulation and the productivity shock process, determine the dynamic path for the main aggregate variables labor, the capital stock and consumption for a given set of initial conditions on the capital stock and the productivity shock. All sequences approach the corresponding steady state values for $\tau_w = 0.4$, with their respective standard deviations decreasing as the initial capital stock is closer to its steady state value. Moreover, regardless of the initial capital stock, the capital accumulation path displays the highest volatility among the considered variables.

The summary statistics of the dynamic pattern of the main aggregate variables show a clear effect of the introduction of progressive labor income taxes. This model economy generates a higher volatility for the aggregate series compared to the ones obtained by the basic model with proportional taxes, displaying some characteristics present in the U.S. time series data.

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