Maximal Invariant Likelihood Based Testing of Semi-Linear Models

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May 2004

Abstract

In this paper, we use a maximal invariant likelihood (MIL) to construct two likelihood ratio (LR) tests. The first involves testing for the inclusion of a non-linear regressor and the second involves testing of a linear regressor against the alternative of a non-linear regressor. We report the results of a Monte Carlo experiment that compares the size and power properties of the traditional LR tests with those of our proposed MIL based LR tests. Our simulation results show that in both cases the MIL based tests have more accurate asymptotic critical values and better behaved (i.e., better centred) power curves than their classical counterparts.

1. Introduction

Bhowmik and King (2002) investigated the quality of estimates of different parameters for selected non-linear models based on the two-step maximum MIL method and the traditional full maximum likelihood (FML) method. Their empirical results show that the estimators based on MIL are better than those based on the FML method. This result may also have implications for testing regression coefficients based on maximal invariant likelihood ratio (MILR) tests using the MIL functions. These tests might be expected to have better properties than those based on FML. Certainly, the simulation results reported by Moulton and Randolph (1989), Ara and King (1993, 1995), Ara (1995), Grose (1997, 1998), Rahman and King (1998) and Laskar and King (1998) provide evidence of improved test properties in the case of the marginal likelihood over the traditional likelihood.

The likelihood ratio (LR) test is a popular hypothesis testing procedure originally from the work of Neyman and Pearson (1933). Unfortunately, it is not always well behaved in small sample sizes, at least in some cases. The classical LR test has often been found to have inaccurate critical values for many econometric testing problems (see for example Dent (1973), King (1987), Breusch and Schmidt (1988), McManus *et al.* (1994), Ara (1995) and Dobler (2002)). Ara (1995) advocated the use of marginal likelihood based tests in order to improve small sample accuracy. For small sample sizes, econometricians have found that a marginal likelihood based test is more reliable than its classical counterpart (LR), in terms of sizes and powers (see Corduas (1986), Moulton and Randolph (1989), Mukerjee (1992a, 1992b), Ara (1995) and Laskar and King (2001)).

In this paper we construct MILR tests using MIL functions to test the parameters in the non-linear component of non-linear models and we compare the size and power properties of the new tests with those of the traditional full likelihood ratio (LR) test. The MILR tests might be expected to be superior to the traditional LR test with respect to size and power given the evidence in the literature outlined above. Bhowmik and King (2001) derived two MIL functions for two different non-linear models and constructed an LBI test for one-sided alternatives. In a further study Bhowmik and King (2002) denoted these functions as MIL1 and MIL2 where, MIL1 stands for the linear model with a general non-linear component and MIL2 for a linear model with a regressor which is a non-linear function of unknown parameter(s). In this paper, we denote MILR tests derived from the MIL1 and MIL2 functions as MILR1 and MILR2 respectively. For the general non-linear regression model, we are testing for the inclusion of a function that is possibly non-linear using the MILR1 test. However, for the more specific model, we are testing for linearity of the included component against the alternative of non-linearity using the MILR2 test.

The organisation of the paper is as follows. In Section 2, we derive the MILR (MILR1 and MILR2) and LR tests for two testing problem outlined above. In Section 3, we derive these tests for the three specific non-linear models studied by Bhowmik and King (2002). Monte Carlo experiments to investigate the size and power properties of these tests in the context of these three non-linear models are reported in Section 4. Some concluding remarks are made in the final section.

2. Construction of the tests

2.1. Test for the inclusion of a non-linear regressor (LR and MILR1)

In this case, we consider the following non-linear regression model

$$y = X_1 \beta_1 + g(X_2, \beta_2) + u, \; u \sim N(0, \sigma^2 I_n) \quad (1.1)$$

where y is an $n \times 1$ vector, X_1 is an $n \times q$ nonstochastic matrix of n observations on q variables, X_2 is an $n \times p$ nonstochastic matrix of n observations on p variables and $g(X_2, \beta_2)$ is a non-linear function of the $r \times 1$ parameter vector β_2 and X_2 such that $g(X_2, \beta_2)\big|_{\beta_2 = 0} = 0$. Note r and p are different for flexibility and that if $g(X_2, \beta_2)\big|_{\beta_2 = 0} \neq 0$, it can be made zero by replacing $g(X_2, \beta_2)$ with $g_*(X_2, \beta_2) = g(X_2, \beta_2) - g(X_2, \beta_2)\big|_{\beta_2 = 0}$ and y with $y_* = y - g(X_2, \beta_2)\big|_{\beta_2 = 0}$. We wish to test the hypothesis $H_{10}\!:\beta_2 = 0$ against $H_{1a}\!:\beta_2 \neq 0$. In this case, we are testing for the inclusion of a function that is possibly non-linear.

For this model and testing problem, we can derive the traditional LR test. The traditional full likelihood and log likelihood functions for this model are respectively

$$l_1 = \left(2\pi\sigma^2\right)^{-\frac{n}{2}} \exp\left[-\frac{1}{2\sigma^2}\left(y - X_1\beta_1 - g(X_2, \beta_2)\right)'\left(y - X_1\beta_1 - g(X_2, \beta_2)\right)\right] \quad (1.2)$$

and

$$L_1(\beta_1, \beta_2, \sigma^2) = -\frac{n}{2}\ln\left(2\pi\sigma^2\right) - \frac{1}{2\sigma^2}\left(y - X_1\beta_1 - g(X_2, \beta_2)\right)'$$
$$\left(y - X_1\beta_1 - g(X_2, \beta_2)\right). \quad (1.3)$$

The LR test statistic to test $H_{10}\!:\!\beta_2 = 0$ against $H_{1a}\!:\!\beta_2 \neq 0$ is given by

$$LR = 2\left[L_1(\hat{\beta}_1, \hat{\beta}_2, \hat{\sigma}^2) - L_0(\hat{\beta}_{10}, \hat{\sigma}_0{}^2)\right] \tag{1.4}$$

where $\hat{\beta}_1$ $\hat{\beta}_2$ and $\hat{\sigma}^2$ are maximum likelihood estimates of β_1 β_2 and σ^2 under H_{1a}, $\hat{\beta}_{10}$ and $\hat{\sigma}_0{}^2$ are the maximum likelihood estimates of β_1 and σ^2 under H_{10}, $L_1(\hat{\beta}_1, \hat{\beta}_2, \hat{\sigma}^2)$ is defined by equation (1.3) and

$$L_0(\hat{\beta}_{10}, \hat{\sigma}_0{}^2) = -\frac{n}{2}\ln\left(2\pi\hat{\sigma}_0{}^2\right) - \frac{1}{2\hat{\sigma}_0{}^2}\left(y - X_1\hat{\beta}_{10}\right)'\left(y - X_1\hat{\beta}_{10})\right) \tag{1.5}$$

where

$$\hat{\sigma}_0{}^2 = \frac{\left(y - X_1\hat{\beta}_{10}\right)'\left(y - X_1\hat{\beta}_{10})\right)}{n}. \tag{1.6}$$

Bhowmik and King (2001) derived the MIL1 function which will be treated here as a likelihood function for the parameter vector β_2 in order to construct the MILR1 test. The MIL1 function is

$$l_1(\beta_2) = \frac{1}{2}\Gamma(\frac{m}{2})\pi^{-m/2}\exp\{c(w,\beta_2)\}\ \{{}_1F_1[\frac{m}{2},\frac{1}{2},\frac{a^2(w,\beta_2)}{2}] + \sqrt{2}\,a(w,\beta_2)\ \eta\ {}_1F_1[\frac{1+m}{2},\frac{3}{2},\frac{a^2(w,\beta_2)}{2}]\} \tag{1.7}$$

and log of (1.7) is

$$L_1(\beta_2) = \ln\left(\frac{1}{2}\Gamma(\frac{m}{2})\pi^{-m/2}\right) + c(w,\beta_2) + \ln\{{}_1F_1[\frac{m}{2},\frac{1}{2},\frac{a^2(w,\beta_2)}{2}] + \sqrt{2}\,a(w,\beta_2)\ \eta\ {}_1F_1[\frac{1+m}{2},\frac{3}{2},\frac{a^2(w,\beta_2)}{2}]\} \tag{1.8}$$

where

$$a(w,\beta_2) = w'Pg^*(X_2,\beta_2)\,, \tag{1.9}$$

$$c(w,\beta_2) = b(w,\beta_2) - \frac{a^2(w,\beta_2)}{2} = -\frac{1}{2}g^{*\prime}(X_2,\beta_2)M_1 g^*(X_2,\beta_2), \tag{1.10}$$

$$w = z/(z'z)^{1/2}\,, \tag{1.11}$$

$$\eta = \frac{\Gamma(\frac{1+m}{2})}{\Gamma(\frac{m}{2})}\,, \tag{1.12}$$

1F1[.,.,.] is the confluent hypergeometric function, which has the form

$$\text{1F1}[c,d,z] = 1 + \frac{cz}{d} + \frac{c(c+1)}{d(d+1)}\frac{z^2}{2!} + \ldots = \sum_{k=0}^{\infty}\frac{(c)_k}{(d)_k}\frac{z^k}{k!} \tag{1.13}$$

and $m = n - q$.

The MILR1 test statistic to test $H_{10}\!:\beta_2 = 0$ against $H_{1a}\!:\beta_2 \neq 0$ in model (1.1) is

$$MILR1 = 2\left[L_1(\hat{\beta}_2{}^1) - L_0\right] \tag{1.14}$$

where $\hat{\beta}_2{}^1$ is the maximum maximal invariant likelihood (MMIL1) estimate of parameter vector β_2 under H_a, $L_1(\hat{\beta}_2{}^1)$ is defined by equation (1.8) and

$$L_0 = \ln(\frac{1}{2}\Gamma(\frac{m}{2})\pi^{-m/2})\,. \tag{1.15}$$

Under appropriate regularity conditions (see Amemiya, 1985, Godfrey, 1988, or Ara, 1995) and H_0, these LR and MILR1 test statistics asymptotically follow a chi-square distribution with r degrees of freedom.

2.2. Tests of a linear regressor against a non-linear regressor (LR and MILR2)

In this section we consider the following slightly more specific non-linear model,

$$y = X_1\beta_1 + \beta_2 g(X_2, \beta_3) + u\,,\ u \sim N(0, \sigma^2 I_n)\,, \tag{1.16}$$

where X_1 is an $n \times q$ nonstochastic matrix, X_2 is an $n \times p$ nonstochastic matrix and $g(X_2, \beta_3)$ is a non-linear function of β_3 and X_2. In this case, $g(X_2, \beta_3) = \text{constant}$ or more realistically a function of X_2 when $\beta_3 = 0$. Our aim is to test the hypothesis $H_{20}{:}\beta_3 = 0$ against $H_{2a}{:}\beta_3 \neq 0$, where the parameter vector β_3 is of order $r \times 1$. This is a case of testing for linearity against the alternative of non-linearity, in the sense that under H_{20}, we have a linear relationship that can be estimated by OLS, but this is not the case under H_{2a}.

We can derive the traditional LR test for this testing problem. The traditional full likelihood and log likelihood functions for this model are respectively

$$l_1(\beta_1, \beta_2, \beta_3, \sigma^2) = \left(2\pi\sigma^2\right)^{-\frac{n}{2}} \exp[-\frac{1}{2\sigma^2}\left(y - X_1\beta_1 - \beta_2 g(X_2, \beta_3)\right)'$$
$$\left(y - X_1\beta_1 - \beta_2 g(X_2, \beta_3)\right)] \tag{1.17}$$

and

$$L_1(\beta_1, \beta_2, \beta_3, \sigma^2) = -\frac{n}{2}\ln\left(2\pi\sigma^2\right) - \frac{1}{2\sigma^2}[\left(y - X_1\beta_1 - \beta_2 g(X_2, \beta_3)\right)'$$
$$\left(y - X_1\beta_1 - \beta_2 g(X_2, \beta_3)\right)]\,. \tag{1.18}$$

The LR test statistic for the above model to test $H_{20}{:}\beta_3 = 0$ against $H_{2a}{:}\beta_3 \neq 0$ is given by

$$LR = 2\left[L_1(\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\sigma}^2) - L_0(\hat{\beta}_{10}, \hat{\beta}_{20}, \hat{\sigma}_0{}^2)\right] \tag{1.19}$$

where $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$ and $\hat{\sigma}^2$ are the maximum likelihood estimates of β_1, β_2, β_3 and σ^2 under H_{2a}, $\hat{\beta}_{10}$, $\hat{\beta}_{20}$ and $\hat{\sigma}_0{}^2$ are the maximum likelihood estimates of β_1, β_2 and σ^2 under H_{20}, $L_1(\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\sigma}^2)$ is defined by equation (1.18) and

$$L_0(\hat{\beta}_{10}, \hat{\beta}_{20}, \hat{\sigma}_0{}^2) = -\frac{n}{2}\ln\left(2\pi\hat{\sigma}_0{}^2\right) - \frac{1}{2\hat{\sigma}_0{}^2}\left(y - X_1\hat{\beta}_{10} - \hat{\beta}_{20}g(X_2, 0)\right)'$$
$$\left(y - X_1\hat{\beta}_{10} - \hat{\beta}_{20}g(X_2, 0)\right). \tag{1.20}$$

Bhowmik and King (2001) also derived the MIL2 function for model (1.16) which will be treated here as a likelihood function for the parameter β_3 in order to construct the MILR2 test. The MIL2 and log of this function are respectively

$$l_2(\beta_3, \sigma^2) = \left(2\pi\sigma^2\right)^{-\frac{n-q-1}{2}} \exp\left[-\frac{1}{2\sigma^2}\left(y'P'M_g Py\right)\right], \tag{1.21}$$

and

$$L_2(\beta_3, \sigma^2) = -\frac{n-q-1}{2}\ln\left(2\pi\sigma^2\right) - \frac{1}{2\sigma^2}\left(y'P'M_g Py\right), \tag{1.22}$$

where $M_g = I - g^*(X_2, \beta_3)\{g^*(X_2, \beta_3)'g^*(X_2, \beta_3)\}^{-1}g^*(X_2, \beta_3)'$, P is an $m \times n$ matrix such that $PP' = I_m$, $P'P = M_1$, $M_1 = I_n - X_1(X_1{}'X_1)^{-1}X_1{}'$ and $m = n - q$.

The MILR2 test statistic for model (1.16) to test $H_{20}\colon \beta_3 = 0$ against $H_{2a}\colon \beta_3 \neq 0$ is

$$MILR2 = 2\left[L_2(\hat{\beta}_3{}^1, \hat{\sigma}_1{}^2) - L_0(\hat{\sigma}_{10}{}^2)\right] \tag{1.23}$$

where $\hat{\beta}_3^{\,1}$ and $\hat{\sigma}_1^{\,2}$ are the MMIL2 estimates of β_3 and σ^2 under H_{2a}, $\hat{\sigma}_{10}^{\,2}$ is the MMIL2 estimate of σ^2 under H_{20}, $L_2(\hat{\beta}_3^{\,1}, \hat{\sigma}_1^{\,2})$ is defined by equation (1.22),

$$L_0(\hat{\sigma}_{10}^{\,2}) = -\frac{n-q-1}{2}\ln\left(2\pi\hat{\sigma}_{10}^{\,2}\right) - \frac{n-q-1}{2}, \tag{1.24}$$

$$\hat{\sigma}_{10}^{\,2} = \frac{y'P'M_{g^*}Py}{n-q-1}, \tag{1.25}$$

$$M_{g^*} = I_m - g^*(X_2,0)\{g^*(X_2,0)'g^*(X_2,0)\}^{-1}g^*(X_2,0)', \tag{1.26}$$

and $g^*(X_2, \beta_3) = Pg(X_2, \beta_3)$.

Under appropriate regularity conditions (see Amemiya, 1985, Godfrey, 1988 or Ara, 1995) and H_0, these LR and MILR2 test statistics asymptotically follow a chi-square distribution with r degrees of freedom.

3. Construction of the tests for three different specific models

We will be considering the following three semi-linear models, namely

$$K_t = \gamma V_t + \frac{\beta}{R_t - \alpha} + u_t,\ u_t \sim IN(0, \sigma^2), \tag{2.1}$$

$$Y_t = f(X_t, \theta) = \theta_1 X_{1t} + \theta_2 X_{2t} + \theta_4 \exp(\theta_3 X_{3t}) + u_t,\ u_t \sim IN(0, \sigma^2), \tag{2.2}$$

$$C_t = \alpha U_t + \beta W_t^{\gamma} + u_t,\ u_t \sim IN(0, \sigma^2),\ t = 1,2,\ldots,n. \tag{2.3}$$

Model (2.1) is a non-linear money demand function used by Konstas and Khouja (1969), where

K_t = quantity of money demanded,

V_t = national income,

R_t = rate of interest,

γ, β and α are three unknown parameters such that $0 < \alpha < \infty$, $\beta > 0$ and $\gamma > 0$. Here γ is the nuisance parameter and β and α are non-linear parameters. Model (2.2) was given by Gallant (1975), where X_{1t}, X_{2t} and X_{3t} are three input variables, $Y_t = f(X_t, \theta)$ is the output variable, and θ_1, θ_2, θ_3 and θ_4 are unknown parameters. Model (2.3) is a modified model of the general consumption function from Greene (1997), where

W_t = aggregate income,

C_t = consumption,

U_t = regressor of independent random variables from $N(0,1)$,

α, β and γ are three unknown parameters such that $\alpha > 0$, $0 < \beta < 1$ and $\gamma > 0$. We are interested in estimating all parameters and we are doing it in two steps.

Note each model has a linear and a non-linear component. Because of an identification problem and to avoid complex mathematical computation, we consider model (2.3) which is different than the general consumption function. In Greene's model, U_t is a vector of ones but in our case U_t is an $n \times 1$ vector of independent random variables from $N(0,1)$. There is no intercept term in model (2.3) but in the original consumption function (Greene 1997), α is the intercept term. For the original consumption function, if $\gamma = 0$ or $\gamma = \infty$, then α and β are not identifiable, and if $\beta = 0$, then γ is not identifiable. For these values of the parameters, the original consumption function cannot be estimated properly due to this identification problem. Hence, we used model (2.3).

In this section, we derive the MILR (MILR1 and MILR2) and LR tests in the context of testing parameters of the non-linear component for these models with the aim of investigating the comparative performance of these tests. For three different models, firstly, we are testing for the inclusion of a function that is possibly non-linear and constructing the LR and MILR1 tests. Secondly, we are testing for linearity against the alternative of non-linearity and constructing the LR and MILR2 tests.

Bhowmik and King (2002) derived the various likelihood functions for these three different non-linear models. For the money demand function of Konstas and Khouja (1969), the traditional full likelihood, MIL1 and the MIL2 functions are respectively

$$l_1 = (2\pi\sigma^2)^{-n/2} \exp\{-\frac{1}{2\sigma^2}(K-\gamma V-\beta Z(\alpha))'(K-\gamma V-\beta Z(\alpha))\}, \qquad (2.4)$$

$$l_{11} = f(w) = \frac{1}{2}\Gamma(\frac{m}{2})\pi^{-m/2} \exp\{-\frac{\beta^2}{2\sigma^2}(Z(\alpha)' M_1 Z(\alpha))\}$$
$$\{1F1[\frac{m}{2},\frac{1}{2},\frac{a^2(w,\beta,\alpha)}{2}] + \sqrt{2}a(w,\beta,\alpha)\eta 1F1[\frac{1+m}{2},\frac{3}{2},\frac{a^2(w,\beta,\alpha)}{2}]\} \qquad (2.5)$$

and

$$l_{21} = f(d) = \left(2\pi\sigma^2\right)^{-\frac{n-q-1}{2}} \exp\left[-\frac{1}{2\sigma^2}\left(K'P'M_g PK\right)\right], \qquad (2.6)$$

where $Z(\alpha)_t = \frac{1}{R_t-\alpha}$, K is an $n\times 1$ vector, V is an $n\times 1$ vector, $a(w,\beta,\alpha) = w'Pg^*(Z(\alpha),\beta)$, $g^*(Z(\alpha),\beta) = \frac{\beta}{\sigma}Z(\alpha)$, $w = \frac{PK}{(K'M_1K)^{1/2}}$, P is an $m\times n$ matrix such that $PP' = I_m$, $P'P = M_1$, $M_1 = I_n - V(V'V)^{-1}V'$, $m = n-1$,

$M_g = I_n - PZ(\alpha)\{Z(\alpha)' M_1 Z(\alpha)\}^{-1} Z(\alpha)' P'$, $M_1 = I_n - V(V'V)^{-1}V'$ and $Z(\alpha)_t = \frac{1}{R_t - \alpha}$.

The log of these functions are respectively

$$L_1 = -\frac{n}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}(K - \gamma V - \beta Z(\alpha))'(K - \gamma V - \beta Z(\alpha)), \quad (2.7)$$

$$L_{11} = \ln(\frac{1}{2}\Gamma(\frac{m}{2})\pi^{-m/2}) - \frac{\beta^2}{2\sigma^2}(Z(\alpha)' M_1 Z(\alpha)) + \ln[\{1F1[\frac{m}{2}, \frac{1}{2}, \frac{a^2(w,\beta,\alpha)}{2}] + \sqrt{2}a(w,\beta,\alpha)\eta 1F1[\frac{1+m}{2}, \frac{3}{2}, \frac{a^2(w,\beta,\alpha)}{2}]\}], \quad (2.8)$$

and

$$L_{21} = -\frac{n-q-1}{2}\ln(2\pi) - \frac{n-q-1}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\left(K'P'M_g PK\right). \quad (2.9)$$

For Gallant's (1975) model, log of the full likelihood, MIL1 and the MIL2 functions are respectively

$$L_1 = -\frac{n}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}(Y - \theta_1 X_1 - \theta_2 X_2 - \theta_4 Z(\theta_3))' (Y - \theta_1 X_1 - \theta_2 X_2 - \theta_4 Z(\theta_3)), \quad (2.10)$$

$$L_{12} = \ln(\frac{1}{2}\Gamma(\frac{m}{2})\pi^{-m/2}) - \frac{\theta_4^{\ 2}}{2\sigma^2}(Z(\theta_3)' M_1 Z(\theta_3)\} + \ln[\{1F1[\frac{m}{2}, \frac{1}{2}, \frac{a^2(w,\theta_3,\theta_4)}{2}] + \sqrt{2}a(w,\theta_3,\theta_4)\eta 1F1[\frac{1+m}{2}, \frac{3}{2}, \frac{a^2(w,\theta_3,\theta_4)}{2}]\}] \quad (2.11)$$

and

$$L_{22} = -\frac{n-q-1}{2}\ln(2\pi) - \frac{n-q-1}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\left(Y'P'M_g PY\right), \quad (2.12)$$

where

$$a(w,\theta_3,\theta_4) = w'Pg^*(z(\theta_3),\theta_4)\,,$$

$$g^*(z(\theta_3),\theta_4) = \frac{\theta_4}{\sigma} z(\theta_3)\,,$$

$$w = \frac{P\,Y}{(Y'M_1Y)^{1/2}}\,,$$

$$M_1 = I_n - X_*(X_*{}'X_*)^{-1}X_*{}'\,,$$

$$M_g = I_n - PZ(\theta_3)\{Z(\theta_3)'M_1Z(\theta_3)\}^{-1}Z(\theta_3)'P'\,,$$

X_* is an $n\times 2$ nonstochastic matrix which is comprised of the two regressors X_1, and X_2 of model (2.2), $Z(\theta_3)_t = \exp(\theta_3 X_{3t})$, Y is a vector of order $n\times 1$ and $m = n-2$.

Similarly for the modified model of the general consumption function from Greene (1997), log of the full likelihood MIL1 and the MIL2 functions are respectively

$$L_1 = -\frac{n}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}(C-\alpha\,U-\beta\,Z(\gamma))'(C-\alpha\,U-\beta\,Z(\gamma)) \tag{2.13}$$

$$L_{13} = \ln(\frac{1}{2}\Gamma(\frac{m}{2})\pi^{-m/2}) - \frac{\beta^2}{2\sigma^2}(Z(\gamma)'M_1Z(\gamma)) + \ln[\{1F1[\frac{m}{2},\frac{1}{2},\frac{a^2(w,\beta,\gamma)}{2}]+$$

$$\sqrt{2}a(w,\beta,\gamma)\eta 1F1[\frac{1+m}{2},\frac{3}{2},\frac{a^2(w,\beta,\gamma)}{2}]\}] \tag{2.14}$$

and

$$L_{23} = -\frac{n-q-1}{2}\ln(2\pi) - \frac{n-q-1}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\left(C'P'M_gPC\right), \tag{2.15}$$

where

$$a(w,\beta,\gamma) = w'Pg^*(Z(\gamma),\beta)\,,$$

$$g^*(Z(\gamma),\beta) = \frac{\beta}{\sigma} Z(\gamma),$$

$$w = \frac{PC}{(C'M_1C)^{1/2}},$$

$$M_g = I_n - PW(W'M_1W)^{-1}W'P',$$

$$M_1 = I_n - U(U'U)^{-1}U'.$$

C and U are a vectors of order $n \times 1$, $Z(\gamma)_t = W_t^{\gamma}$, and $m = n-1$.

Using (1.4), (1.14), (1.19), (1.23) and the above log likelihood functions, we can construct LR, MILR1 and MILR2 tests.

3.1. Tests for the inclusion of a non-linear regressor

Using (1.4) and (1.14), the LR and the MILR1 tests for three different models can be constructed. The LR statistic for the first model defined by equation (2.1) to test $H_{10}^{\;1}\!:\!(\beta,\alpha)' = 0$ against $H_{1a}^{\;1}\!:\!(\beta,\alpha)' \neq 0$ is given by

$$LR = 2\left[L_1(\hat{\gamma},\hat{\beta},\hat{\alpha},\hat{\sigma}_{11}^{\;2}) - L_0(\hat{\gamma}_0,\hat{\sigma}_{20}^{\;2})\right] \tag{2.16}$$

where $\hat{\gamma}$, $\hat{\beta}$, $\hat{\alpha}$ and $\hat{\sigma}_{11}^{\;2}$ are the maximum likelihood estimates of γ, β, α and σ^2 under $H_{1a}^{\;1}$, $\hat{\gamma}_0$ and $\hat{\sigma}_{20}^{\;2}$ are the maximum likelihood estimates of γ and σ^2 under $H_{10}^{\;1}$, $L_1(\hat{\gamma},\hat{\beta},\hat{\alpha},\hat{\sigma}_{11}^{\;2})$ is defined by equation (2.7),

$$L_0(\hat{\gamma}_0,\hat{\sigma}_{20}^{\;2}) = -\frac{n}{2}\ln(2\pi\hat{\sigma}_{20}^{\;2}) - \frac{n}{2}, \tag{2.17}$$

$$\hat{\sigma}_{20}{}^2 = \frac{(K - \hat{\gamma}_0 V)'(K - \hat{\gamma}_0 V)}{n}, \tag{2.18}$$

and K and V are vectors of order $n \times 1$.

For model 2 defined by equation (2.2), the LR statistic to test $H_{10}{}^2 : (\theta_4, \theta_3)' = 0$ against $H_{1a}{}^2 : (\theta_4, \theta_3)' \neq 0$ is given by

$$LR = 2\left[L_1(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\sigma}_{12}{}^2) - L_0(\hat{\theta}_{10}, \hat{\theta}_{20}, \hat{\sigma}_{30}{}^2)\right] \tag{2.19}$$

where $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$, $\hat{\theta}_4$ and $\hat{\sigma}_{12}{}^2$ are the maximum likelihood estimates of θ_1, θ_2, θ_3, θ_4 and σ^2 under $H_{2a}{}^2$, $\hat{\theta}_{10}$, $\hat{\theta}_{20}$ and $\hat{\sigma}_{30}{}^2$ are the maximum likelihood estimates of θ_1, θ_2 and σ^2 under $H_{20}{}^2$, $L_1(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_4, \hat{\sigma}_{12}{}^2)$ is defined by equation (2.10),

$$L_0(\hat{\theta}_{10}, \hat{\theta}_{20}, \hat{\sigma}_{30}{}^2) = -\frac{n}{2}\ln(2\pi\hat{\sigma}_{30}{}^2) - \frac{n}{2}, \tag{2.20}$$

$$\hat{\sigma}_{30}{}^2 = \frac{(Y - \hat{\theta}_{10} X_1 - \hat{\theta}_{20} X_2)'(Y - \hat{\theta}_{10} X_1 - \hat{\theta}_{20} X_2)}{n}, \tag{2.21}$$

in which X_1, X_2 and Y are vectors of order $n \times 1$.

Similarly for model 3 which is defined by equation (2.3), the LR statistic to test $H_{10}{}^3 : (\beta, \gamma)' = 0$ against $H_{1a}{}^3 : (\beta, \gamma)' \neq 0$ is given by

$$LR = 2\left[L_1(\hat{\alpha}_{11}, \hat{\beta}_{11}, \hat{\gamma}_{11}, \hat{\sigma}_{13}{}^2) - L_0(\hat{\alpha}_0, \hat{\sigma}_{40}{}^2)\right] \tag{2.22}$$

where $\hat{\alpha}_{11}$, $\hat{\beta}_{11}$, $\hat{\gamma}_{11}$ and $\hat{\sigma}_{13}{}^2$ are the maximum likelihood estimates of α, β, γ and σ^2 under $H_{1a}{}^3$, $\hat{\alpha}_0$ and $\hat{\sigma}_{40}{}^2$ are the maximum likelihood estimates of α and σ^2 under $H_{10}{}^3$, $L_1(\hat{\alpha}, \hat{\beta}, \hat{\gamma})$ is defined by equation (2.13),

$$L_0(\hat{\alpha}_0, \hat{\sigma}_{40}{}^2) = -\frac{n}{2}\ln(2\pi\hat{\sigma}_{40}{}^2) - \frac{n}{2}, \qquad (2.23)$$

$$\hat{\sigma}_{40}{}^2 = \frac{(C - \hat{\alpha}\,U)'(C - \hat{\alpha}U)}{n}, \qquad (2.24)$$

and C and U are as defined earlier.

Using (1.14) we can derive the MILR1 tests for these models. For the first model which is defined by equation (2.1), to test $H_{10}{}^1 : (\beta, \alpha)' = 0$ against $H_{1a}{}^1 : (\beta, \alpha)' \neq 0$, the MILR1 test statistic is

$$MILR1 = 2\left[L_1(\hat{\beta}, \hat{\alpha}) - L_0\right] \qquad (2.25)$$

where $\hat{\beta}^1$ and $\hat{\alpha}^1$ are the MMIL1 estimates of β and α under $H_{1a}{}^1$, $L_1(\hat{\beta}^1, \hat{\alpha}^1)$ and L_0 are defined by equations (2.8) and (1.15), respectively.

To test $H_{10}{}^2 : (\theta_4, \theta_3)' = 0$ against $H_{1a}{}^2 : (\theta_4, \theta_3)' \neq 0$ in model 2 defined by equation (2.2), the MILR1 test statistic is

$$MILR1 = 2\left[L_1(\hat{\theta}_3{}^2, \hat{\theta}_4{}^2) - L_0\right] \qquad (2.26)$$

where $\hat{\theta}_3{}^2$ and $\hat{\theta}_4{}^2$ are the MMIL1 estimates of θ_3 and θ_4 under $H_{1a}{}^2$, $L_1(\hat{\theta}_3{}^2, \hat{\theta}_4{}^2)$ and L_0 are defined by equations (2.11) and (1.15), respectively.

Similarly for model 3 which is defined by equation (2.3), the LR statistic to test $H_{10}{}^3 : (\beta, \gamma)' = 0$ against $H_{1a}{}^3 : (\beta, \gamma)' \neq 0$ is

$$MILR1 = 2\left[L_1(\hat{\beta}^*, \hat{\gamma}^*) - L_0\right] \qquad (2.27)$$

where $\hat{\beta}^*$ and $\hat{\gamma}^*$ are the MMIL1 estimates of β and γ under $H_{1a}{}^3$, $L_1(\hat{\beta}^*, \hat{\gamma}^*)$ and L_0 are defined by equations (2.14) and (1.15), respectively.

3.2. Tests of a linear regressor against a non-linear regressor

Using (1.19) and (1.23), the LR and the MILR2 tests for our three different models can be constructed. Following (1.19), the LR statistic for the first model to test $H_{20}{}^1 : \alpha = 0$ against $H_{2a}{}^1 : \alpha \neq 0$ is

$$LR = 2\left[L_1(\hat{\gamma}, \hat{\beta}, \hat{\alpha}, \hat{\sigma}_{14}{}^2) - L_0(\hat{\gamma}_0, \hat{\beta}_0, \hat{\sigma}_{50}^2)\right] \tag{2.28}$$

where $\hat{\gamma}$, $\hat{\beta}$, $\hat{\alpha}$ and $\hat{\sigma}_{14}{}^2$ are the maximum likelihood estimates of γ, β, α and σ^2 under $H_{2a}{}^1$, $\hat{\gamma}_0$, $\hat{\beta}_0$ and $\hat{\sigma}_{50}{}^2$ are the maximum likelihood estimates of γ, β and σ^2 under $H_{20}{}^1$, $L_1(\hat{\gamma}, \hat{\beta}, \hat{\alpha}, \hat{\sigma}_{14}{}^2)$ is defined by equation (2.7),

$$L_0(\hat{\gamma}_0, \hat{\beta}_0, \hat{\sigma}_{50}{}^2) = -\frac{n}{2}\ln(2\pi\hat{\sigma}_{50}{}^2) - \frac{n}{2}, \tag{2.29}$$

$$\hat{\sigma}_{50}{}^2 = \frac{(K - \hat{\gamma}_0 V - \hat{\beta}_0 Z)'(K - \hat{\gamma}_0 V - \hat{\beta}_0 Z)}{n} \tag{2.30}$$

and $Z_t = \dfrac{1}{R_t}$.

To test $H_{20}{}^2 : \theta_3 = 0$ against $H_{2a}{}^2 : \theta_3 \neq 0$ in model 2 which is defined by equation (2.2), the LR statistic is

$$LR = 2\left[L_1(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\sigma}_{15}{}^2) - L_0(\hat{\theta}_{10}, \hat{\theta}_{20}, \hat{\theta}_{40}, \hat{\sigma}_{60}{}^2)\right] \tag{2.31}$$

where $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$, $\hat{\theta}_4$ and $\hat{\sigma}_{15}{}^2$ are the maximum likelihood estimates of θ_1, θ_2, θ_3, θ_4 and σ^2 under $H_{2a}{}^2$, $\hat{\theta}_{10}$, $\hat{\theta}_{20}$ and $\hat{\sigma}_{50}{}^2$ are the maximum likelihood estimates of θ_1, θ_2 and σ^2 under $H_{20}{}^2$, $L_1(\hat{\theta}_1,\hat{\theta}_2,\hat{\theta}_3,\hat{\theta}_4,\hat{\sigma}_{15}{}^2)$ is defined by equation (2.10),

$$L_0(\hat{\theta}_{10},\hat{\theta}_{20},\hat{\theta}_{40},\hat{\sigma}_{60}{}^2) = -\frac{n}{2}\ln(2\pi\hat{\sigma}_{60}{}^2) - \frac{n}{2}, \tag{2.32}$$

$$\hat{\sigma}_{60}{}^2 = \frac{(Y-\hat{\theta}_{10}X_1-\hat{\theta}_{20}X_2-\hat{\theta}_{40}Z^*)'(Y-\hat{\theta}_{10}X_1-\hat{\theta}_{20}X_2-\hat{\theta}_{40}Z^*)}{n} \tag{2.33}$$

and Z^* is an $n \times 1$ vector of ones.

Similarly, to test $H_{20}{}^3\!:\gamma = 0$ against $H_{2a}{}^3\!:\gamma \neq 0$ in model 3, the LR statistic is

$$LR = 2\left[L_1(\hat{\alpha}^a,\hat{\beta}^a,\hat{\gamma}^a,\hat{\sigma}_{16}{}^2) - L_0(\hat{\alpha}_0,\hat{\beta}_0,\hat{\sigma}_{70}{}^2)\right] \tag{2.34}$$

where $\hat{\alpha}^a$, $\hat{\beta}^a$, $\hat{\gamma}^a$ and $\hat{\sigma}_{16}{}^2$ are the maximum likelihood estimates of α, β, γ and σ^2 under $H_{2a}{}^3$, $\hat{\alpha}_0$, $\hat{\beta}_0$ and $\hat{\sigma}_{70}{}^2$ are the maximum likelihood estimates of α, β and σ^2 under $H_{20}{}^3$, $L_1(\hat{\alpha}^a,\hat{\beta}^a,\hat{\gamma}^a,\hat{\sigma}_{16}{}^2)$ is defined by equation (2.13),

$$L_0(\hat{\alpha}_0,\hat{\beta}_0,\hat{\sigma}_{70}{}^2) = -\frac{n}{2}\ln(2\pi\hat{\sigma}_{70}{}^2) - \frac{n}{2}, \tag{2.35}$$

$$\hat{\sigma}_{70}{}^2 = \frac{(C-\hat{\alpha}_0U-\hat{\beta}_0\,Z^*)'(C-\hat{\alpha}_0U-\hat{\beta}_0\,Z^*)}{n} \tag{2.36}$$

and Z^* is an $n \times 1$ vector of ones.

Using logs of MIL2 functions, we can derive the MILR2 tests for these models. For the first model, the MILR2 test statistic to test $H_{20}{}^1\!:\alpha = 0$ against $H_{2a}{}^1\!:\alpha \neq 0$ is

$$MILR2 = 2\left[L_2(\hat{\alpha}_1,\hat{\sigma}_{18}{}^2) - L_0(\hat{\sigma}_{80}{}^2)\right] \tag{2.37}$$

where $\hat{\alpha}_1$ and $\hat{\sigma}_{18}{}^2$ are the MMIL2 estimates of α and σ^2 under $H_{2a}{}^1$, $\hat{\sigma}_{80}{}^2$ is the MMIL2 estimate of σ^2 under $H_{20}{}^1$, $L_1(\hat{\alpha}_1, \hat{\sigma}_{18}{}^2)$ is defined by equation (2.9),

$$L_0(\hat{\sigma}_{80}{}^2) = -\frac{n-q-1}{2}\ln(2\pi) - \frac{n-q-1}{2}\ln(\hat{\sigma}_{80}{}^2) - \frac{n-q-1}{2}, \tag{2.38}$$

$$\hat{\sigma}_{80}{}^2 = \frac{K'P_1{}' M_{g^*} P_1 K}{n-q-1}, \tag{2.39}$$

$$M_{g^*} = I_m - P_1 Z\{Z'M_1 Z\}^{-1} Z'P_1{}', \tag{2.40}$$

$$P_1{}' P_1 = M_1 = I_n - K(K'K)^{-1} K'$$

and $$Z_t = \frac{1}{R_t}.$$

The MILR2 test statistic to test $H_{20}{}^2 : \theta_3 = 0$ against $H_{2a}{}^2 : \theta_3 \neq 0$ in model 2 defined by equation (2.2) is

$$MILR2 = 2\left[L_1(\hat{\theta}_3{}^1, \hat{\sigma}_{19}{}^2) - L_0(\hat{\sigma}_{90}{}^2)\right] \tag{2.41}$$

where $\hat{\theta}_3{}^1$ and $\hat{\sigma}_{19}{}^2$ are the MMIL2 estimates of θ_3 and σ^2 under $H_{2a}{}^2$, $\hat{\sigma}_{90}{}^2$ is the MMIL2 estimate of σ^2 under $H_{20}{}^2$, $L_1(\hat{\theta}_3{}^1, \hat{\sigma}_{19}{}^2)$ is defined by equation (2.12),

$$L_0(\hat{\sigma}_{90}{}^2) = -\frac{n-q-1}{2}\ln(2\pi) - \frac{n-q-1}{2}\ln(\hat{\sigma}_{90}{}^2) - \frac{n-q-1}{2}, \tag{2.42}$$

$$\hat{\sigma}_{90}{}^2 = \frac{Y'P_2{}' M_{g^*} P_2 Y}{n-q-1}, \tag{2.43}$$

$$M_{g^*} = I_m - P_2 Z^* \{Z^{*\prime} M_1 Z^*\}^{-1} Z^{*\prime} P_2{}' , \tag{2.44}$$

$$P_2{}' P_2 = M_1 = I_n - Y(Y'Y)^{-1} Y'$$

and Z^* is an $n \times 1$ vector of ones.

Similarly for model 3 that is defined by equation (2.3), the MILR2 statistic to test $H_{20}{}^3 : \gamma = 0$ against $H_{2a}{}^3 : \gamma \neq 0$ is

$$MILR2 = 2\left[L_1(\hat{\gamma}^1, \hat{\sigma}_{21}{}^2) - L_0(\hat{\sigma}_{100}{}^2)\right] \tag{2.45}$$

where $\hat{\gamma}^1$ and $\hat{\sigma}_{21}{}^2$ are the MMIL2 estimates of γ and σ^2 under $H_{2a}{}^3$, $\hat{\sigma}_{100}{}^2$ is the MMIL2 estimate of σ^2 under $H_{20}{}^3$, $L_1(\hat{\gamma}^1, \hat{\sigma}_{21}{}^2)$ is defined by equation (2.15),

$$L_0(\hat{\sigma}_{100}{}^2) = -\frac{n-q-1}{2}\ln(2\pi) - \frac{n-q-1}{2}\ln(\hat{\sigma}_{100}{}^2) - \frac{n-q-1}{2} , \tag{2.46}$$

$$\hat{\sigma}_{100}{}^2 = \frac{C' P_3{}' M_{g^*} P_3 C}{n-q-1} , \tag{2.47}$$

$$M_{g^*} = I_m - P_3 Z^* (Z^{*\prime} M_1 Z^*)^{-1} Z^* P_3{}' , \tag{2.48}$$

$$P_3{}' P_3 = M_1 = I_n - C(C'C)^{-1} C'$$

and Z^* is an $n \times 1$ vector of ones.

4. Monte Carlo size and power comparisons

The aim of this section is to investigate the size and power of MIL based tests as compared to the classical LR test using Monte Carlo simulation. In order to compare the size and power of the LR, MILR1 and MILR2 tests for testing for the inclusion of a function that is possibly non-linear and testing for linearity against the alternative of non-linearity, we used the experimental framework as outlined below.

We have three non-linear models (non-linear in parameters) given by (2.1), (2.2) and (2.3). Model (2.1), is a money demand function. For the purpose of our experiment, we used generated data to construct the design matrix with V_t and R_t being independent observations from the [0,1] uniform distribution.

For model (2.2), Gallant (1975), used simulated data for X_{1t}, X_{2t} and X_{3t}. In our study,

X_{1t} was independently generated from $N(0,1)$,

X_{2t} was independently generated from $N(0,1)$,

X_{3t} was independently generated from the [0,1] uniform distribution.

Model (2.3) is a consumption function. In our study, U_t is independently generated from $N(0,1)$ and W_t is generated from the [0,1] uniform distribution.

We used two sample sizes, namely $n = 30$ and $n = 60$ for each of the models.

4.1. Experimental design

The Monte Carlo experiment was conducted in four parts. The first part involved calculating sizes of different tests using asymptotic critical values at the five percent level. The size is the proportion of replications for which the test rejects the null hypothesis, using its asymptotic critical value, when the test statistics are calculated from the data generated under the null hypothesis. This process enables us to see whether the use of different MIL (MIL1 and MIL2) based tests in the context of non-linear regression models can have better sizes than those based on the traditional full likelihood function. In the second part, the Monte Carlo method was used to estimate critical values of each of the tests in order to compare sizes and powers of all tests at the same significance level. This was done as follows. For each combination of a range of different parameter values under the null hypothesis, values of the dependent variable were generated and the test statistic calculated. For each set of 2000 replications, the 95th percentile of the test statistic was taken as the simulated critical value (CV) for that particular combination of parameter values. Across all parameter combinations, the final simulated CV is the largest CV because at that point the size of the test will be 0.05 and not greater than 0.05 at other points. In the third part, sizes of different tests for each of the models were calculated using these simulated critical values. In fourth part, powers of all the tests were calculated using the simulated critical values.

The experimental design was conducted for two types of testing problems for three different non-linear models given by equations (2.1), (2.2) and (2.3). For our two general but different non-linear models we want to test the null hypotheses H_{10} and H_{20} against the alternatives H_{1a} and H_{2a} respectively, i.e. for H_{10} we want to test for the inclusion of a function that is possibly non-linear and for H_{20} we test for linearity against the alternative of non-linearity. First, we consider H_{10} against H_{1a} for the three different models. To calculate these two test statistics for model (2.1), data was generated for 36 different sets of values for γ and σ^2 under $H_{10}{}^{1}$, namely, the combinations of

$\gamma = 0.25, 0.35, 0.40, 0.55, 0.70, 0.80, 0.90, 1.00, 1.50, 1.75, 2.00, 2.50$

and $\sigma^2 = 0.05, 0.25, 0.75.$

To overcome problems with local maxima, we used five different sets of starting values for γ, β, α and σ^2 and these were (0.05, 0.02, 0.05, 0.05), (0.5, 0.05, 0.07, 0.3), (1.2, 0.2, 0.1, 0.5), (1.9, 0.5, 0.5, 0.75) and (2.75, 0.9, 0.75, 0.95) for FML and $(\beta, \alpha) = (0.05, 0.1), (0.1, 0.3), (0.3, 0.7), (0.7, 0.8)$, and (0.9, 0.9) for MMIL1.

For model (2.2), data was generated for 36 different sets of values for θ_1, θ_2 and σ^2 under $H_{10}{}^2$. These were

$(\theta_1, \theta_2, \sigma^2) =$ (− 1, 1, 0.05), (− 1, 1, 0.25) (− 1, 1, 0.9), (− 0.85, 0.5, 0.05), (− 0.85, 0.5, 0.25), (− 0.85, 0.5, 0.9), (− 0.75, − 0.25, 0.05), (− 0.75, − 0.25, 0.25), (− 0.75, − 0.25, 0.9), (− 0.5, 0.5, 0.05), (− 0.5, 0.5, 0.25), (− 0.5, 0.5, 0.9), (− 0.5, 0.75, 0.05), (− 0.5, 0.75, 0.25), (− 0.5, 0.75, 0.9), (− 0.25, − 0.5, 0.05), (− 0.25, − 0.5, 0.25), (− 0.25, − 0.5, 0.9), (0.5, 0.25, 0.05), (0.5, 0.25, 0.25), (0.5, 0.25, 0.9), (0.5, 0.5, 0.05), (0.5, 0.5, 0.25), (0.5, 0.5, 0.9), (0.75, 0.25, 0.05), (0.75, 0.25, 0.25), (0.75, 0.25, 0.9), (0.75, 0.75, 0.05), (0.75, 0.75, 0.25), (0.75, 0.75, 0.9), (1, 0.25, 0.05), (1, 0.25, 0.25), (1, 0.25, 0.9), (1, − 1, 0.05), (1, − 1, 0.25), (1, − 1, 0.9).

Again to overcome the problems with local maxima, we used five different sets of starting values for θ_1, θ_2, θ_3, θ_4 and σ^2 and these were (− 1, − 1, − 1.3, − 1, 0.05), (−0.75, − 0.75, − 0.9, − 0.75, 0.25), (− 0.2, − 0.3, − 0.5, − 0.2, 0.5), (0.45, 0.45, 0.5, 0.5, 0.75), and (1, 1, 1.2, 1, 0.95), for FML and (θ_3, θ_4) = (-1, -1), (-0.5, -0.5), (0.25, 0.25), (0.5, 0.5) and (1, 1) for MMIL1.

For model (2.3), data was generated for 36 different sets of values for α and σ^2 under $H_{10}{}^3$, namely, the combinations of

$$\alpha = 0.20, 0.50, 0.75, 0.85, 1, 1.50, 1.75, 2, 2.50, 2.75, 3, 3.50$$

and $\sigma^2 = 0.05, 0.25, 0.75.$

To overcome problems with local maxima, we used five different sets of starting values for α, β, γ and σ^2 and these were (0.05, 0.05, 0.5, 0.05), (0.5, 0.07, 0.75, 0.25), (1.2, 0.1, 1, 0.5), (1.9, 0.5, 1.2, 0.75) and (3, 0.85, 1.5, 0.95) for FML and (β, γ) = (0.05, 0.1), (0.1, 0.5), (0.5, 0.75), (0.75, 1) and (0.85, 1.5) for MMIL1.

Similarly we investigated the properties of our test of H_{20} against H_{2a} for the above models. To calculate the LR and the MILR2 tests in the context of model (2.1), data was generated for 36 different sets of values for γ, β and σ^2 under $H_{20}{}^1$, namely, the combinations of

$$(\gamma, \beta) = (0.3,\ 0.05),\ (0.3,\ 0.1),\ (0.4,\ 0.08),\ (0.5,\ 0.05),\ (0.5,\ 0.1),\ (0.7,\ 0.08),\ (0.7, 0.1), (0.7, 0.12), (1, 0.01), (1.4, 0.06), (1.5, 0.08), (1.75, 0.08)$$

and $\sigma^2 = 0.05, 0.25, 0.75.$

To overcome the problems with local maxima, we used five different sets of starting values for γ, β, α and σ^2 and these were (0.2, 0.05, 0.05, 0.05), (0.5, 0.07, 0.2, 0.25), (0.95, 0.1, 0.5, 0.5), (1.5, 0.5, 0.75, 0.75) and (2.5, 0.85, 1.5, 0.95) for FML and $(\alpha,\sigma^2) = (0.05,\ 0.05),\ (0.1,\ 0.3),\ (0.2,\ 0.5),\ (0.3,\ 0.75)$, and (0.4, 0.95) for MMIL2.

For model (2.2), data was generated for 36 different sets of values for θ_1, θ_2, θ_4 and σ^2 under ${H_{20}}^2$. These were the combinations of

$(\theta_1,\theta_2,\theta_4) = (-1,\ 1,\ 0.75),\ (-0.85,\ 0.50,\ 0.50),\ (-0.75,\ -0.25,\ -0.50),$ $(-0.50, 0.50, -0.75)$, $(-0.50, 0.75, 0.50)$, $(-0.25, -0.50, 0.25)$, (0.50, 0.25, 0.05), (0.50, 0.50, 0.25), (0.75, 0.25, 0.50), (0.75, 0.75, 0.75), $(1, 0.25, -0.50)$, $(1, -1, -0.25)$

and $\sigma^2 = 0.05, 0.25, 0.90.$

Again to overcome the problems with local maxima, we used five different sets of starting values θ_1, θ_2, θ_3, θ_4 and σ^2 and these were $(-1,\ -1,\ -1.3,\ -1,\ 0.05)$, $(0.75,\ -0.75,\ -0.9,\ -0.75,\ 0.25)$, $(-0.2,\ -0.3,\ -0.5,\ -0.2,\ 0.5)$, (0.45, 0.45, 0.5, 0.5, 0.75), and (1, 1, 1.2, 1, 0.95), for FML and (θ_3, σ^2) = $(-1, 0.05)$, $(-0.75, 0.25)$, $(-0.5, 0.5)$, (0.25, 0.75) and (1, 0.95) for MMIL2.

For model (2.3), data was generated for 33 different sets of values for α, β and σ^2 under ${H_{20}}^3$, namely, the combinations of

$(\alpha,\beta) = (0.25,\ 0.25),\ (0.5,\ 0.25),\ (0.5,\ 0.5),\ (1,\ 0.25),\ (1.5,\ 0.25),\ (1.5,\ 0.5),$ (1.5, 0.85), (2.5, 0.25), (2.5,0.5), (2.5, 0.75), (3, 0.25)

and $\sigma^2 = 0.05, 0.25, 0.75.$

To overcome problems with local maxima, we used five different sets of starting values for α, β, γ and σ^2 and these were (0.05, 0.05, 0.5, 0.05), (0.5, 0.07, 0.75, 0.25), (1.2, 0.1, 1, 0.5), (1.9, 0.5, 1.2, 0.75) and (3, 0.85, 1.5, 0.95) for FML and $(\gamma, \sigma^2) = (0.2,\ 0.05),\ (0.5,\ 0.25),\ (0.75,\ 5),\ (0.95,\ 0.75)$, and (1.75, 0.95) for MMIL2.

For each case, 2000 iterations were used to simulate the distributions of the test statistics. We used two sample sizes, $n = 30$ and $n = 60$. In order to maximize the likelihood functions (full likelihood and MIL), the Gauss (see Aptech, 1995 and Gauss, 1998) Co-optimisation routine was used.

4.2. Size results for asymptotic critical values

The estimated sizes of the LR, MILR1 and MILR2 tests for the three different specific models based on their asymptotic critical values are presented in Tables 6.1-6.6. As 2,000 replications were used, estimated sizes in the range of 0.0405-0.0596 are not significantly different from their nominal level of 0.05 at the five percent level. In all tables, a star denotes an estimated size that is significantly different from 0.05 at the five percent level.

The results in Tables 1-6 reflect that most of the sizes of all the tests based on classical likelihood and MIL are significantly higher or lower than the nominal size at the 5% level. These results also show that in general, the estimated size for any of the MIL functions is typically closer to 0.05 than the corresponding estimated size for the classical likelihood function. We observe that there is generally an appreciable improvement in accuracy when an MIL is used in place of the classical approach, especially if it is the MIL1 function.

Table 1 shows the size results for model (2.1) to test ${H_{10}}^{1}$ against ${H_{1a}}^{1}$. An analysis of the results shows that the overall average values of the absolute differences of estimated sizes of the LR and MILR1 tests for $n = 30$ from the nominal size 0.05 are 0.125 and 0.026, respectively. When $n = 60$, these are 0.093 and 0.026, respectively. For model (2.2) when testing ${H_{10}}^{2}$ against ${H_{1a}}^{2}$ the overall average values of the absolute differences of estimated sizes of the LR and MILR1 tests for $n = 30$ are 0.087 and 0.022, respectively. When $n = 60$, these are 0.055 and 0.023, respectively. Similarly, for model (2.3), when testing ${H_{10}}^{3}$ against ${H_{1a}}^{3}$, these are 0.027 and 0.015, for $n = 30$. When $n = 60$, these are 0.023 and 0.015, respectively.

Table 4 shows the size results for model (2.1) to test ${H_{20}}^{1}$ against ${H_{2a}}^{1}$. In this case, the overall average values of the absolute differences of estimated sizes of the LR and MILR2 tests for $n = 30$ from the nominal size 0.05 are 0.015 and 0.016 respectively, and for $n = 60$, they are both 0.016. The results reported in Table 5 show that for testing ${H_{20}}^{2}$ against ${H_{2a}}^{2}$, the overall average values of the absolute differences of estimated sizes of the LR and MILR2 tests for $n = 30$ are both 0.011. When $n = 60$, they are both 0.013. Similarly, for model (2.3), when testing ${H_{20}}^{3}$ against ${H_{2a}}^{3}$ for $n = 30$, these are 0.022 and 0.018, and when $n = 60$, these are 0.021 and 0.019 respectively. These results show that the overall average values of the estimated sizes of the MILR1 test are generally closer to the nominal size 0.05 than the corresponding estimated sizes of the classical LR test, particularly for $n = 30$. On the other hand, the overall average estimated sizes of the MILR2 and LR tests are almost always similar.

The maximum and minimum absolute differences of estimated sizes of LR and MILR tests from the nominal size 0.05 can also be observed from the results in Tables 6.1-6.6. For model (2.1) when testing ${H_{10}}^1$ against ${H_{1a}}^1$, we observe that for $n = 30$, the maximum absolute differences of estimated sizes of the LR and MILR1 tests from the nominal size 0.05 are 0.142 and 0.038, respectively. The corresponding minimum absolute differences are 0.113, and 0.013, respectively. When $n = 60$, 0.104 and 0.036 are the maximum absolute differences respectively, and the corresponding minimum absolute differences are 0.083 and 0.014, respectively. For model (2.2) when testing ${H_{10}}^2$ against ${H_{1a}}^2$, for $n = 30$, the maximum absolute differences are 0.100 and 0.036 and the minimum absolute differences are 0.080 and 0.014, respectively. Similarly, for model (2.3) when testing ${H_{10}}^3$ against ${H_{1a}}^3$, the corresponding maximum absolute differences for $n = 30$ are 0.032 and 0.021 and the minimum absolute differences are 0.019 and 0.009, respectively. For $n = 60$, these are 0.026, 0.019 and 0.014, 0.008, respectively. These results show that the maximum and the minimum absolute differences of the classical LR test are typically larger than the MILR1 test. For the cases between maximum and minimum, we also observe that the absolute differences for the classical LR test are generally larger than for the MILR1 test. Therefore, the use of MIL based tests results in improved size as compared to the use of the LR test.

The absolute maximum differences of estimated sizes of LR tests for models (2.1), (2.2) and (2.3) to test H_{20} against H_{2a} at the nominal size of 0.05, are respectively 0.039, 0.010, 0.014, based on classical likelihood and 0.016, 0.009, 0.011 based on MIL2. These occur when $n = 30$. The corresponding minimum absolute differences are 0.023, 0.018, 0.030 and 0.021, 0.017, 0.023, respectively. When $n = 60$, the absolute maximum differences of estimated sizes of the LR test and MILR2 tests for these three models are 0.002, 0.008, 0.015, and 0.007, 0.005, 0.008, respectively. The corresponding minimum absolute differences are 0.021, 0.016, 0.028, and 0.020, 0.016, 0.024, respectively. These results show that the maximum and the minimum absolute differences for the LR test are generally larger than those for the MILR2 test.

The overall average values of estimated sizes of the LR and MILR1 tests for model (2.1) when $n = 30$ are 0.175 and 0.076 respectively, and for $n = 60$, these are 0.143 and 0.076, respectively. For model (2.2) and $n = 30$, the average values of estimated sizes of the LR and MILR1 tests are 0.137 and 0.072, respectively, and for $n = 60$, these are 0.105 and 0.073 respectively. Similarly for model (2.3), the overall average estimated sizes of the LR and MILR1 tests are 0.077 and 0.065 for $n = 30$ and 0.073 and 0.065, respectively, for $n = 60$. For these models, the average values of estimated sizes for $n = 30$ are respectively 0.035, 0.039, 0.028, for the LR and 0.034, 0.039, 0.032 for the MILR2. When $n = 60$, these are 0.034, 0.037, 0.029, and 0.034, 0.037, 0.031, respectively.

These results show that the average, maximum and minimum absolute differences for all the tests based on classical likelihood are, in general, large compared to those based on MIL functions. These results also confirm the poorer performance of the LR test compared to the MILR (MILR1 and MILR2) tests, with respect to size based on asymptotic critical values. Among the MILR tests, the MILR1 test is best, with the smallest average, maximum and minimum absolute differences.

In most of the cases, the LR tests have sizes significantly higher or lower than those of the MILR tests, and there is a clear sign of improvement as n increases from 30 to

60. With respect to the MIL based tests, the estimated sizes of the MILR1 and MILR2 tests are closer to their nominal size as compared to the LR test, especially for $n = 30$. We observe that for both sample sizes ($n = 30$ and $n = 60$), the MILR1 test typically produced better size results, compared to the traditional LR test. However, for $n = 60$, in most of the cases, the traditional LR test produced better size results compared to the MILR2 test.

On the whole, sizes of the LR tests based on MIL functions are better than those based on the classical likelihood. Therefore, it seems very clear that the use of MIL improves the accuracy of the LR test at least for $n = 30$ and this likelihood does reasonably well in testing problems.

4.3. Size results for simulated critical values

The estimated sizes of the LR, MILR1 and MILR2 tests for the three different specific models, based on their simulated critical values, are presented in Tables 7-12.

The results in Tables 7-12 reflect that the estimated size results of the MILR tests are, in general, closer to 0.05 than the corresponding estimated size for the classical LR test. Table 7 shows the size results for model (2.1) when testing H_{10}^{1} against H_{1a}^{1}. A thorough analysis shows that the ranges of the estimated sizes of the LR and MILR1 tests for $n = 30$ are 0.039-0.050 and 0.042-0.050, respectively. The overall average values of the estimated sizes are 0.045 and 0.047, respectively. When $n = 60$, the ranges are both 0.040-0.050, and the overall average values are both 0.046. For model (2.2), when testing H_{10}^{2} against H_{1a}^{2}, 0.035-0.050 and 0.040-0.050 are the ranges of the estimated sizes of the LR and MILR1 tests for $n = 30$. While, the overall average values of the estimated sizes are 0.044 and 0.047, respectively. When $n = 60$, the ranges are 0.045-0.050 and 0.046-0.050 respectively, and the averages are both 0.048. Similarly, the ranges of the estimated sizes of these tests for $n = 30$, when testing ${H_{10}}^3$ against ${H_{1a}}^3$, are 0.039-0.050, and 0.042-0.050 respectively. The overall averages are 0.044 and 0.045 respectively. For $n = 60$, the ranges are 0.040-0.050, and 0.041-0.050 respectively and the averages are 0.044 and 0.045, respectively. These results are as might be expected.

Table 10 shows the size results for model (2.1) when testing ${H_{20}}^1$ against ${H_{2a}}^1$. In this case, the ranges of the estimated sizes of the LR and MILR2 tests for $n = 30$ are 0.021-0.050 and 0.032-0.050, respectively. The overall average values of the estimated sizes of these tests for $n = 30$ are 0.033 and 0.037, respectively. When $n = 60$, 0.034-0.050 and 0.039-0.050 are the ranges of the estimated sizes for these tests. Also, the overall averages are both 0.045. For testing ${H_{20}}^2$ against ${H_{2a}}^2$, results reported in Table 11 show that the ranges of the estimated sizes of the LR and MILR2 tests for $n = 30$ are 0.026-0.050 and 0.033-0.050, respectively. The average values of the estimated sizes are 0.033 and 0.041, respectively. While, 0.040-0.050 and 0.038-0.050 are the ranges of the estimated sizes of these tests for $n = 60$, and the overall averages are 0.043 and 0.041, respectively. Similarly, for testing ${H_{20}}^3$ against ${H_{2a}}^3$, when $n = 30$, the ranges of the estimated sizes of these tests are 0.027-0.050 and 0.032-0.050, respectively, and the overall averages are 0.035 and 0.039, respectively. When $n = 60$, the ranges are 0.042-0.050 and 0.039-0.050, respectively, and the overall averages are both 0.044.

The above results show that the average values of the estimated sizes of all the tests based on MIL (MIL1 and MIL2) functions are larger compared to those based on the classical likelihood function, except for a few exceptions. We also observe that the ranges of the estimated sizes of the tests based on MIL functions are typically smaller compared to those of their classical counterpart. Therefore, the overall results show that there is a slight improvement in size results when an MIL based test is used in place of the classical approach, although the differences are typically not significant.

4.4. Power results for simulated critical values

Estimated powers of the LR, MILR1 and MILR2 tests for the three different specific models using simulated critical values at the five percent level are presented in Tables 13-18. Generally, the powers of all tests increase as the sample size increases, *ceteris paribus*.

From Tables 13-18, we observe that there is typically an improvement in the power results of the MILR tests when compared to those of the traditional LR test. Over half of the unrestricted parameter space, the MILR tests have higher power than the LR test. A feature of the results is the wide fluctuation in powers for the LR test. Some of the estimated powers of this test are less than 0.05, indicating that it is a biased test.

Table 13 shows the power results for model (2.1) when testing $H_{10}{}^{1}$ against $H_{1a}{}^{1}$. An analysis shows that the overall average values of the estimated powers of the LR and MILR1 tests for $n = 30$ are 0.262 and 0.302, respectively. When $n = 60$, these are 0.562 and 0.560, respectively. The ranges of the estimated powers of the LR and MILR1 tests for $n = 30$ are 0.119-0.645 and 0.152-0.657, respectively. When $n = 60$, 0.188-0.998 and 0.202-0.975 are the respective ranges of the estimated powers of these tests. For model (2.2) and testing ${H_{10}}^2$ against ${H_{1a}}^2$, the overall average values of the estimated powers of the LR and MILR1 tests for $n = 30$ are 0.415 and 0.431, respectively. When $n = 60$, these are 0.565 and 0.568, respectively. For $n = 30$, the ranges of the estimated powers of these tests are 0.059-0.962 and 0.097-0.958, respectively. When $n = 60$, these are 0.093-0.985 and 0.133-0.964, respectively. Similarly, for model (2.3) and testing ${H_{10}}^3$ against ${H_{1a}}^3$, the overall average values of the estimated powers are 0.347 and 0.367, for $n = 30$ and when $n = 60$ these are 0.490 and 0.491, respectively. The ranges of the estimated powers are 0.036-0.985 and 0.062-0.977, for $n = 30$ and when $n = 60$ these are 0.047-0.977 and 0.082-0.954, respectively. In this case, four of the estimated power results of the LR test are less than 0.05.

The overall average estimated powers of the LR and MILR2 tests for model (2.1), when $n = 30$, are 0.375 and 0.386 respectively. For $n = 60$, these are 0.694 and 0.692, respectively. When $n = 30$, the ranges of the estimated powers for the LR and MILR2 tests are 0.060-0.998 and 0.105-0.898, respectively. For $n = 60$, these are 0.324-0.998 and 0.339-0.980, respectively. For model (2.2) and $n = 30$, overall average estimated powers of the LR and MILR2 tests are 0.177 and 0.180, respectively, and for $n = 60$, these are 0.267 and 0.266, respectively. In this case, the ranges of the estimated powers of these tests for $n = 30$ are 0.036-0.765 and 0.042-0.724, respectively. When $n = 60$, these are 0.049-0.908 and 0.054-0.894, respectively. Similarly for model (2.3), overall average values of estimated powers of the LR and MILR2 tests are 0.177 and 0.180 for $n = 30$, and 0.356 and 0.370, respectively, when $n = 60$. The ranges of the estimated powers of these tests are 0.064-0.968 and 0.076-0.944, for $n = 30$ and when $n = 60$ these are 0.076-0.985 and 0.080-0.961, respectively. In this case, for model (2.2), five of the estimated powers of the LR test and three of the MILR2 test are less than 0.05.

These results indicate that the average power of all the tests based on classical likelihood are smaller compared to those based on MIL functions, especially for $n = 30$. We also observe that the ranges of the estimated powers of the tests based on MIL functions are typically smaller compared to those of their classical counterpart. We notice that in most of the cases, the lower limit of the ranges of the estimated powers of the LR test is smaller than for the MILR tests. Therefore, it is clear from the results that the use of MIL functions results in power curves that are better centred around H_0. In some cases, there is no improvement in power for the MILR2 test over the classical LR test, particularly when $n = 60$. On the whole, the performance of LR tests based on MIL functions is generally better than those based on the classical likelihood function. For $n = 60$, in most of the cases, average power results of the LR tests based on MIL functions are not better than the classical likelihood based test.

Hence, our results indicate that the use of MIL based tests typically results in increased power under the alternative hypothesis with a few exceptions, at least for $n = 30$. We observe that the power of a MIL based test increases at a fairly even rate as one moves away from the null, no matter in which direction. This is not the case for the classical LR test which is clearly biased in small samples.

5. Concluding remarks

In this paper, we derived the LR tests based on MIL functions in the context of two different general non-linear models. A Monte Carlo experiment was used to investigate the properties of these tests in the context of three specific non-linear models. The size results reflect that sizes of the MILR tests are closer to the nominal size, compared to the LR test, particularly for $n = 30$. In terms of power, we conclude that an MIL based test is more reliable than its classical counterpart. In general, all the tests based on MIL functions typically have more accurate sizes and better-centred power curves, compared to those based on the classical likelihood function, particularly for $n = 30$. Moreover, Evans and King (1985), Corduas (1986), Ara and King (1993), Ara (1995), Rahman and King (1994,1998), Laskar (1998) and Laskar and King (1998, 2001) have found that the marginal likelihood based tests have more accurate asymptotic critical values than their classical counterparts. From Ara's (1995) finding, we also know that the marginal likelihood and the maximal invariant function are equivalent. Our simulation study of likelihood-based tests confirm that the MIL based LR tests (MILR1 and MILR2) produce better small sample properties with respect to size and power in non-linear regression models. Therefore, we conclude that in the non-linear regression model, the MILR tests have better small sample properties than the classical LR test.

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Table 1 Estimated sizes for testing $H_{10}^{1}:(\beta,\alpha)'=0$ against $H_{1a}^{1}:(\beta,\alpha)'\neq 0$ with $n=30$ and $n=60$ for model (3.1) based on the LR and MILR1 tests using asymptotic critical values at the 5% significance level

Parameter	$n=30$						$n=60$					
value	$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.75$		$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.75$	
γ	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1
0.25	0.169*	0.067*	0.176*	0.070*	0.160*	0.078*	0.143*	0.065*	0.147*	0.069*	0.133*	0.077*
0.35	0.170*	0.063*	0.178*	0.076*	0.170*	0.080*	0.138*	0.071*	0.151*	0.075*	0.139*	0.080*
0.40	0.174*	0.067*	0.181*	0.074*	0.170*	0.083*	0.141*	0.065*	0.146*	0.077*	0.136*	0.082*
0.50	0.180*	0.066*	0.176*	0.073*	0.173*	0.086*	0.142*	0.064*	0.150*	0.072*	0.137*	0.084*
0.70	0.172*	0.068*	0.176*	0.075*	0.172*	0.087*	0.142*	0.067*	0.147*	0.082*	0.143*	0.086*
0.80	0.167*	0.073*	0.177*	0.078*	0.176*	0.086*	0.133*	0.071*	0.141*	0.079*	0.143*	0.083*
0.90	0.167*	0.063*	0.174*	0.071*	0.175*	0.079*	0.138*	0.072*	0.150*	0.081*	0.143*	0.085*
1.00	0.163*	0.071*	0.178*	0.080*	0.173*	0.088*	0.134*	0.068*	0.147*	0.079*	0.141*	0.084*
1.50	0.175*	0.075*	0.182*	0.077*	0.179*	0.085*	0.136*	0.072*	0.150*	0.080*	0.151*	0.084*
1.75	0.175*	0.073*	0.188*	0.079*	0.177*	0.083*	0.137*	0.069*	0.152*	0.077*	0.149*	0.081*
2.00	0.164*	0.074*	0.186*	0.079*	0.184*	0.085*	0.140*	0.070*	0.152*	0.076*	0.154*	0.083*
2.50	0.165*	0.067*	0.192*	0.081*	0.186*	0.088*	0.136*	0.066*	0.147*	0.080*	0.149*	0.086*
Average	**0.170***	**0.069***	**0.180***	**0.076***	**0.175***	**0.084***	**0.138***	**0.068***	**0.148***	**0.077***	**0.143***	**0.083***
Overall average for $n=30$, LR = 0.175*, MILR1 = 0.0768*							**Overall average for $n=60$, LR = 0.l43*, MILR1 = 0.076***					

Table 2 Estimated sizes for testing $H_{10}{}^2:(\theta_4,\theta_3)'=0$ against $H_{1a}{}^2:(\theta_4,\theta_3)'\neq 0$ with $n=30$ and $n=60$ for model (3.2) based on the LR and MILR1 tests using asymptotic critical values at the 5% significance level

		$n=30$						$n=60$					
Parameter value		$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.9$		$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.9$	
θ_1	θ_2	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1
-1	1	0.138*	0.064*	0.139*	0.067*	0.141*	0.071*	0.104*	0.069*	0.103*	0.072*	0.106*	0.081*
-0.85	0.5	0.135*	0.065*	0.135*	0.073*	0.130*	0.078*	0.101*	0.064*	0.101*	0.071*	0.105*	0.077*
-0.75	-0.25	0.135*	0.067*	0.133*	0.071*	0.139*	0.074*	0.102*	0.063*	0.101*	0.072*	0.106*	0.079*
-0.5	0.5	0.133*	0.069*	0.133*	0.074*	0.138*	0.077*	0.106*	0.065*	0.102*	0.073*	0.106*	0.078*
-0.5	0.75	0.137*	0.072*	0.137*	0.079*	0.138*	0.083*	0.104*	0.068*	0.102*	0.077*	0.105*	0.084*
-0.25	-0.5	0.137*	0.067*	0.137*	0.075*	0.139*	0.077*	0.103*	0.066*	0.103*	0.072*	0.106*	0.083*
0.5	0.25	0.134*	0.071*	0.136*	0.073*	0.138*	0.075*	0.105*	0.069*	0.105*	0.069*	0.106*	0.081*
0.5	0.5	0.135*	0.068*	0.135*	0.070*	0.138*	0.074*	0.105*	0.066*	0.107*	0.067*	0.106*	0.074*
0.75	0.25	0.135*	0.067*	0.136*	0.069*	0.135*	0.072*	0.105*	0.067*	0.107*	0.068*	0.106*	0.078*
0.75	0.75	0.135*	0.068*	0.136*	0.070*	0.135*	0.086*	0.105*	0.066*	0.107*	0.068*	0.106*	0.079*
1	0.25	0.136*	0.067*	0.138*	0.071*	0.138*	0.073*	0.105*	0.068*	0.106*	0.071*	0.107*	0.081*
1	-1	0.150*	0.072*	0.148*	0.074*	0.143*	0.080*	0.103*	0.070*	0.106*	0.074*	0.105*	0.080*
Average		**0.137***	**0.068***	**0.137***	**0.072***	**0.138***	**0.077***	**0.104***	**0.067***	**0.104***	**0.071***	**0.106***	**0.080***
Overall average for $n=30$, LR = 0.137* MILR1 = 0.072*								**Overall average for $n=60$, LR = 0.105* MILR1 = 0.073***					

Table 3 Estimated sizes for testing $H_{10}{}^{3}\!:(\beta,\gamma)'=0$ against $H_{1a}{}^{3}\!:(\beta,\gamma)'\neq 0$ with $n=30$ and $n=60$ for model (3.3) based on the LR and MILR1 tests using asymptotic critical values at the 5% significance level

Parameter	$n=30$						$n=60$					
value	$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.75$		$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.75$	
α	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1
0.20	0.070*	0.059	0.070*	0.064*	0.069*	0.065*	0.064*	0.064*	0.065*	0.066*	0.066*	0.065*
0.50	0.075*	0.065*	0.073*	0.067*	0.072*	0.069*	0.070*	0.066*	0.072*	0.067*	0.071*	0.066*
0.75	0.077*	0.063*	0.075*	0.065*	0.074*	0.069*	0.072*	0.062*	0.070*	0.065*	0.069*	0.068*
0.85	0.080*	0.061*	0.075*	0.066*	0.073*	0.070*	0.073*	0.060*	0.071*	0.064*	0.069*	0.067*
1	0.078*	0.065*	0.075*	0.066*	0.074*	0.068*	0.073*	0.063*	0.070*	0.066*	0.070*	0.066*
1.50	0.080*	0.064*	0.078*	0.066*	0.075*	0.068*	0.075*	0.062*	0.072*	0.065*	0.072*	0.067*
1.75	0.077*	0.063*	0.078*	0.066*	0.077*	0.067*	0.076*	0.060*	0.074*	0.064*	0.072*	0.067*
2	0.077*	0.062*	0.079*	0.067*	0.077*	0.069*	0.075*	0.061*	0.074*	0.063*	0.073*	0.068*
2.50	0.077*	0.060*	0.080*	0.065*	0.078*	0.071*	0.075*	0.058	0.075*	0.064*	0.073*	0.069*
2.75	0.075*	0.059	0.080*	0.064*	0.079*	0.068*	0.076*	0.061*	0.076*	0.066*	0.075*	0.067*
3	0.080*	0.059	0.081*	0.065*	0.079*	0.068*	0.076*	0.060*	0.075*	0.063*	0.076*	0.067*
3.50	0.081*	0.062*	0.082*	0.067*	0.081*	0.071*	0.076*	0.061*	0.076*	0.065*	0.075*	0.069*
Average	**0.077***	**0.062***	**0.077***	**0.066***	**0.076***	**0.069***	**0.073***	**0.062***	**0.073***	**0.065***	**0.072***	**0.067***
Overall average for $n=30$, LR = 0.077*, MILR1 = 0.065*							**Overall average for $n=60$, LR = 0.073*, MILR1 = 0.065***					

Table 4 Estimated sizes for testing $H_{20}{}^{1}: \alpha = 0$ against $H_{2a}{}^{1}: \alpha \neq 0$ with $n = 30$ and $n = 60$ for model (3.1) based on the LR and MILR2 tests using asymptotic critical values at the 5% significance level

		$n = 30$						$n = 60$					
Parameter value		$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.75$		$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.75$	
γ	β	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2
0.3	0.05	0.029*	0.030*	0.031*	0.032*	0.027*	0.045	0.033*	0.034*	0.032*	0.033*	0.031*	0.034*
0.3	0.1	0.029*	0.029*	0.029*	0.030*	0.032*	0.033*	0.035*	0.036*	0.034*	0.035*	0.032*	0.031*
0.4	0.08	0.029*	0.030*	0.030*	0.030*	0.032*	0.033*	0.035*	0.035*	0.033*	0.034*	0.032*	0.030*
0.5	0.05	0.029*	0.030*	0.031*	0.034*	0.045	0.039*	0.036*	0.035*	0.035*	0.032*	0.034*	0.031*
0.5	0.1	0.029*	0.029*	0.029*	0.030*	0.031*	0.032*	0.034*	0.035*	0.033*	0.030*	0.031*	0.035*
0.7	0.08	0.029*	0.030*	0.030*	0.032*	0.031*	0.034*	0.034*	0.036*	0.033*	0.033*	0.032*	0.030*
0.7	0.1	0.029*	0.030*	0.029*	0.033*	0.032*	0.033*	0.035*	0.037*	0.034*	0.032*	0.033*	0.035*
0.7	0.03	0.031*	0.032*	0.047	0.037*	0.089*	0.039*	0.048	0.039*	0.043*	0.037*	0.040*	0.037*
1	0.01	0.029*	0.033*	0.061*	0.060*	0.073*	0.066*	0.034*	0.035*	0.035*	0.043	0.040*	0.036*
1.4	0.06	0.029*	0.030*	0.031*	0.033*	0.035*	0.037*	0.034*	0.033*	0.033*	0.034*	0.031*	0.030*
1.5	0.08	0.029*	0.030*	0.030*	0.033*	0.032*	0.034*	0.033*	0.034*	0.032*	0.031*	0.031*	0.030*
1.75	0.08	0.029*	0.031*	0.030*	0.032*	0.031*	0.033*	0.033*	0.035*	0.031*	0.032*	0.029*	0.030*
Average		**0.029**	**0.030**	**0.034**	**0.035**	**0.041**	**0.038**	**0.035***	**0.035***	**0.034***	**0.034***	**0.033***	**0.032***
Overall average for $n = 30$, LR = 0.035*, MILR2 = 0.034*								**Overall average for $n = 60$, LR = 0.034*, MILR2 = 0.034***					

Table 5 Estimated sizes for testing $H_{20}{}^{2}\colon\theta_3 = 0$ against $H_{2a}{}^{2}\colon\theta_3 \neq 0$ with $n = 30$ and $n = 60$ for model (3.2) based on the LR and MILR2 tests using asymptotic critical values at the 5% significance level

			$n = 30$						$n = 60$					
Parameter value			$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.9$		$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.9$	
θ_1	θ_2	θ_4	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2
-1	1	0.75	0.032*	0.033*	0.033*	0.034*	0.035*	0.036*	0.034*	0.034*	0.035*	0.035*	0.038*	0.035*
-0.85	0.50	0.50	0.033*	0.034*	0.034*	0.034*	0.042	0.043	0.035*	0.034*	0.037*	0.034*	0.039*	0.035*
-0.75	-0.25	-0.50	0.033*	0.033*	0.033*	0.033*	0.040*	0.040*	0.037*	0.034*	0.036*	0.034*	0.038*	0.037*
-0.50	0.50	-0.75	0.033*	0.033*	0.034*	0.034*	0.034*	0.035*	0.038*	0.039*	0.035*	0.035*	0.037*	0.037*
-0.50	0.75	0.50	0.033*	0.033*	0.034*	0.033*	0.042	0.042	0.035*	0.034*	0.037*	0.034*	0.040*	0.036*
-0.25	-0.50	0.25	0.034*	0.034*	0.044	0.044	0.051	0.052	0.035*	0.035*	0.041	0.040*	0.035*	0.042
0.50	0.25	0.05	0.054	0.055	0.054	0.055	0.060*	0.059	0.035*	0.040*	0.038*	0.055	0.042	0.042
0.50	0.50	0.25	0.034*	0.033*	0.044	0.043	0.048	0.050	0.035*	0.035*	0.041	0.040*	0.035*	0.041
0.75	0.25	0.50	0.033*	0.033*	0.034*	0.035*	0.041	0.042	0.035*	0.034*	0.037*	0.035*	0.039*	0.036*
0.75	0.75	0.75	0.032*	0.033*	0.033*	0.033*	0.035*	0.036*	0.035*	0.034*	0.035*	0.035*	0.037*	0.037*
1	0.25	-0.50	0.033*	0.034*	0.033*	0.034*	0.039*	0.040*	0.037*	0.037*	0.036*	0.036*	0.039*	0.038*
1	-1	-0.25	0.034*	0.035*	0.041	0.042	0.052	0.051	0.035*	0.034*	0.034*	0.034*	0.041	0.039*
Average			**0.035***	**0.035***	**0.038***	**0.038***	**0.043**	**0.044**	**0.036***	**0.035***	**0.037***	**0.037***	**0.038***	**0.038***
Overall average for $n = 30$, LR = 0.039*, MILR2 = 0.039*									**Overall average for $n = 60$, LR = 0.037*, MILR2 = 0.037***					

Table 6 Estimated sizes for testing $H_{20}{}^{3}: \gamma = 0$ against $H_{2a}{}^{3}: \gamma \neq 0$ with $n = 30$ and $n = 60$ for model (3.3) based on the LR and MILR2 tests using asymptotic critical values at the 5% significance level

		$n = 30$						$n = 60$					
Parameter value		$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.75$		$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.75$	
α	β	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2
0.25	0.25	0.025*	0.027*	0.036*	0.036*	0.030*	0.039*	0.027*	0.027*	0.030*	0.031*	0.035*	0.041
0.5	0.25	0.025*	0.027*	0.036*	0.037*	0.031*	0.039*	0.027*	0.027*	0.030*	0.032*	0.035*	0.042
0.5	0.5	0.026*	0.030*	0.025*	0.029*	0.030*	0.032*	0.029*	0.026*	0.028*	0.027*	0.029*	0.031*
1	0.25	0.025*	0.027*	0.035*	0.036*	0.031*	0.039*	0.027*	0.032*	0.030*	0.032*	0.035*	0.041
1.5	0.25	0.025*	0.027*	0.036*	0.036*	0.031*	0.039*	0.027*	0.027*	0.030*	0.032*	0.035*	0.041
1.5	0.5	0.026*	0.030*	0.025*	0.029*	0.030*	0.032*	0.029*	0.026*	0.028*	0.027*	0.029*	0.031*
1.5	0.85	0.026*	0.031*	0.022*	0.030*	0.020*	0.029*	0.030*	0.026*	0.026*	0.027*	0.022*	0.027*
2.5	0.25	0.025*	0.027*	0.036*	0.036*	0.031*	0.039*	0.027*	0.027*	0.030*	0.032*	0.035*	0.042
2.5	0.5	0.026*	0.030*	0.024*	0.029*	0.030*	0.032*	0.029*	0.026*	0.028*	0.027*	0.029*	0.031*
2.5	0.75	0.026*	0.031*	0.024*	0.029*	0.020*	0.031*	0.020*	0.027*	0.027*	0.026*	0.022*	0.029*
3	0.25	0.025	0.027*	0.035*	0.036*	0.030*	0.039*	0.030*	0.027*	0.030*	0.032*	0.035*	0.042
Average		**0.025***	**0.029***	**0.030***	**0.033***	**0.029***	**0.035***	**0.027***	**0.027***	**0.029***	**0.030***	**0.031***	**0.036***
Overall average for $n = 30$, LR = 0.028*, MILR2 = 0.032*								**Overall average for $n = 60$, LR = 0.029*, MILR2 = 0.031***					

Table 7 Estimated sizes for testing $H_{10}^{1}:(\beta,\alpha)'=0$ against $H_{1a}^{1}:(\beta,\alpha)'\neq 0$ with $n=30$ and $n=60$ for model (3.1) based on the LR and MILR1 tests using simulated critical values at the 5% significance level

Parameter value	$n=30$						$n=60$					
	$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.75$		$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.75$	
γ	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1
0.25	0.045	0.046	0.045	0.046	0.043	0.046	0.040	0.040	0.045	0.047	0.047	0.045
0.35	0.045	0.047	0.045	0.047	0.046	0.048	0.047	0.046	0.047	0.048	0.048	0.046
0.40	0.044	0.047	0.046	0.047	0.043	0.046	0.047	0.045	0.047	0.047	0.047	0.045
0.50	0.045	0.047	0.046	0.047	0.046	0.047	0.046	0.045	0.047	0.046	0.048	0.046
0.70	0.044	0.046	0.050	0.046	0.048	0.050	0.044	0.044	0.049	0.047	0.049	0.046
0.80	0.047	0.049	0.048	0.048	0.045	0.047	0.045	0.044	0.048	0.047	0.047	0.048
0.90	0.046	0.048	0.046	0.047	0.047	0.048	0.043	0.045	0.045	0.046	0.045	0.044
1.00	0.043	0.045	0.048	0.049	0.048	0.049	0.044	0.045	0.048	0.047	0.048	0.050
1.50	0.045	0.047	0.046	0.047	0.046	0.050	0.042	0.043	0.047	0.045	0.049	0.047
1.75	0.045	0.047	0.045	0.046	0.047	0.048	0.048	0.046	0.047	0.046	0.047	0.046
2.00	0.039*	0.042	0.043	0.047	0.044	0.047	0.047	0.047	0.044	0.045	0.045	0.044
2.50	0.041	0.044	0.042	0.045	0.043	0.047	0.042	0.043	0.045	0.047	0.048	0.046
Average	**0.044**	**0.046**	**0.046**	**0.047**	**0.046**	**0.048**	**0.045**	**0.044**	**0.047**	**0.047**	**0.047**	**0.046**
Overall average for $n=30$, LR = 0.045, MILR1 = 0.047							**Overall average for $n=60$, LR = 0.046, MILR1 = 0.046**					

Table 8 Estimated sizes for testing $H_{10}^{2}:(\theta_4,\theta_3)'=0$ against $H_{1a}^{2}:(\theta_4,\theta_3)'\neq 0$ with $n=30$ and $n=60$ for model (3.2) based on the LR and MILR1 tests using simulated critical values at the 5% significance level

		$n=30$						$n=60$					
Parameter value		$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.9$		$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.9$	
θ_1	θ_2	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1
-1	1	0.048	0.049	0.041	0.045	0.035	0.040	0.046	0.047	0.048	0.047	0.048	0.049
-0.85	0.5	0.046	0.048	0.047	0.048	0.048	0.049	0.047	0.046	0.047	0.046	0.048	0.046
-0.75	-0.25	0.048	0.049	0.049	0.049	0.050	0.048	0.047	0.047	0.049	0.048	0.048	0.048
-0.5	0.5	0.046	0.050	0.044	0.048	0.039	0.044	0.047	0.046	0.049	0.049	0.049	0.049
-0.5	0.75	0.048	0.049	0.043	0.045	0.038	0.043	0.048	0.047	0.048	0.047	0.049	0.048
-0.25	-0.5	0.048	0.049	0.048	0.049	0.047	0.048	0.046	0.047	0.048	0.048	0.048	0.047
0.5	0.25	0.048	0.049	0.038	0.043	0.041	0.045	0.048	0.047	0.046	0.047	0.049	0.048
0.5	0.5	0.047	0.049	0.043	0.046	0.042	0.046	0.045	0.047	0.048	0.048	0.049	0.048
0.75	0.25	0.047	0.048	0.043	0.046	0.041	0.044	0.046	0.047	0.048	0.048	0.049	0.048
0.75	0.75	0.048	0.049	0.047	0.049	0.043	0.047	0.047	0.048	0.048	0.048	0.049	0.050
1	0.25	0.047	0.048	0.043	0.047	0.040	0.043	0.047	0.047	0.048	0.048	0.049	0.047
1	-1	0.046	0.047	0.042	0.045	0.039	0.042	0.047	0.047	0.049	0.048	0.050	0.048
Average		**0.047**	**0.049**	**0.044**	**0.047**	**0.042**	**0.045**	**0.047**	**0.047**	**0.048**	**0.048**	**0.049**	**0.048**
Overall average for $n=30$, LR = 0.044, MILR1 = 0.047								**Overall average for $n=60$, LR = 0.048, MILR1 = 0.048**					

Table 9 Estimated sizes for testing $H_{10}{}^{3}:(\beta,\gamma)' = 0$ against $H_{1a}{}^{3}:(\beta,\gamma)' \neq 0$ with $n = 30$ and $n = 60$ for model (3.3) based on the LR and MILR1 tests using simulated critical values at the 5% significance level

Parameter value	$n = 30$						$n = 60$					
	$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.75$		$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.75$	
α	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1
0.20	0.039*	0.043	0.040	0.044	0.041	0.044	0.040*	0.042	0.041	0.043	0.041	0.042
0.50	0.043	0.045	0.044	0.046	0.045	0.046	0.046	0.045	0.048	0.047	0.049	0.048
0.75	0.043	0.045	0.044	0.045	0.044	0.045	0.043	0.044	0.042	0.047	0.041	0.042
0.85	0.045	0.047	0.045	0.046	0.043	0.045	0.045	0.045	0.045	0.044	0.042	0.043
1	0.040*	0.042	0.044	0.046	0.043	0.046	0.041	0.042	0.044	0.044	0.042	0.044
1.50	0.043	0.045	0.044	0.045	0.045	0.046	0.042	0.042	0.047	0.046	0.046	0.045
1.75	0.045	0.047	0.046	0.048	0.045	0.047	0.048	0.047	0.046	0.046	0.045	0.046
2	0.048	0.050	0.043	0.044	0.043	0.044	0.048	0.050	0.042	0.042	0.042	0.045
2.50	0.042	0.044	0.043	0.044	0.043	0.046	0.041	0.041	0.042	0.042	0.050	0.045
2.75	0.045	0.047	0.046	0.048	0.047	0.047	0.048	0.047	0.047	0.047	0.048	0.046
3	0.039*	0.042	0.043	0.044	0.043	0.044	0.041	0.042	0.042	0.042	0.043	0.047
3.50	0.050	0.049	0.044	0.045	0.044	0.045	0.043	0.042	0.044	0.044	0.044	0.046
Average	**0.044**	**0.046**	**0.044**	**0.045**	**0.044**	**0.045**	**0.044**	**0.044**	**0.044**	**0.044**	**0.044**	**0.045**
Overall average for $n = 30$, LR = 0.044, MILR1 = 0.045							**Overall average for $n = 60$, LR = 0.044, MILR1 = 0.045**					

Table 10 Estimated sizes for testing $H_{20}{}^{1}\!:\alpha = 0$ against $H_{2a}{}^{1}\!:\alpha \neq 0$ with $n = 30$ and $n = 60$ for model (3.1) based on the LR and MILR2 tests using simulated critical values at the 5% significance level

Sizes													
		$n = 30$						$n = 60$					
Parameter value		$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.75$		$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.75$	
γ	β	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2
0.3	0.05	0.030*	0.035*	0.031*	0.032*	0.047	0.050	0.048	0.043	0.047	0.043	0.046	0.040*
0.3	0.1	0.029*	0.034*	0.031*	0.035*	0.032*	0.035*	0.038*	0.044	0.040*	0.046	0.039*	0.040*
0.4	0.08	0.030*	0.035*	0.032*	0.037*	0.032*	0.034*	0.047	0.047	0.049	0.044	0.046	0.043
0.5	0.05	0.029*	0.035*	0.032*	0.034*	0.039*	0.041	0.046	0.045	0.047	0.044	0.047	0.048
0.5	0.1	0.030*	0.034*	0.031*	0.035*	0.033*	0.034*	0.037*	0.047	0.039*	0.040*	0.040*	0.043
0.7	0.08	0.030*	0.035*	0.032*	0.037*	0.031*	0.033*	0.049	0.047	0.048	0.047	0.048	0.045
0.7	0.1	0.030*	0.034*	0.031*	0.035*	0.032*	0.034*	0.048	0.049	0.049	0.048	0.048	0.044
0.7	0.12	0.030*	0.034*	0.030*	0.035*	0.031*	0.035*	0.046	0.050	0.034*	0.045*	0.038*	0.039*
1	0.01	0.021*	0.034*	0.042	0.045	0.050	0.047	0.043	0.045	0.047	0.044	0.050	0.043
1.4	0.06	0.035*	0.039*	0.034*	0.035*	0.035*	0.037*	0.044	0.047	0.046	0.045	0.045	0.045
1.5	0.08	0.030*	0.035*	0.032*	0.037*	0.032*	0.034*	0.045	0.047	0.045	0.044	0.049	0.045
1.75	0.08	0.036*	0.044	0.039*	0.043	0.041	0.044	0.047	0.046	0.045	0.047	0.048	0.046
Average		**0.030***	**0.036***	**0.033***	**0.037***	**0.036***	**0.038***	**0.045**	**0.046**	**0.045**	**0.045**	**0.045**	**0.043**
Overall average for $n = 30$, LR = 0.033*, MILR2 = 0.037*								**Overall average for $n = 60$, LR = 0.045, MILR2 = 0.045**					

Table 11 Estimated sizes for testing $H_{20}^{\ 2}: \theta_3 = 0$ against $H_{2a}^{\ 2}: \theta_3 \neq 0$ with $n = 30$ and $n = 60$ for model (3.2) based on the LR and MILR2 tests using simulated critical values at the 5% significance level

			$n = 30$						$n = 60$					
Parameter value			$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.9$		$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.9$	
θ_1	θ_2	θ_4	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2
-1	1	0.75	0.027*	0.035*	0.026*	0.041	0.030*	0.041	0.043	0.042	0.041	0.039*	0.041	0.039*
-0.85	0.50	0.50	0.028*	0.041	0.028*	0.043	0.036*	0.044	0.042	0.040*	0.041	0.038*	0.043	0.039*
-0.75	-0.25	-0.50	0.029*	0.042	0.031*	0.038*	0.037*	0.044	0.043	0.041	0.044	0.040*	0.044	0.040*
-0.50	0.50	-0.75	0.029*	0.042	0.028*	0.041	0.031*	0.038*	0.042	0.038*	0.042	0.039*	0.043	0.040*
-0.50	0.75	0.50	0.028*	0.033*	0.028*	0.034*	0.036*	0.039*	0.042	0.040*	0.040*	0.039*	0.044	0.042
-0.25	-0.50	0.25	0.026*	0.044	0.038*	0.049	0.045	0.050	0.040*	0.041	0.046	0.043	0.045	0.044
0.50	0.25	0.05	0.048	0.048	0.047	0.049	0.050	0.049	0.042	0.044	0.044	0.046	0.050	0.050
0.50	0.50	0.25	0.034*	0.044	0.038*	0.045	0.045	0.047	0.040*	0.041	0.045	0.043	0.043	0.042
0.75	0.25	0.50	0.028*	0.034*	0.028*	0.035*	0.036*	0.040*	0.042	0.043	0.041	0.042	0.044	0.043
0.75	0.75	0.75	0.027*	0.035*	0.026*	0.042	0.030*	0.044	0.043	0.044	0.041	0.040*	0.041	0.040*
1	0.25	-0.50	0.029*	0.036*	0.031*	0.039*	0.037*	0.043	0.043	0.040*	0.044	0.043	0.043	0.042
1	-1	-0.25	0.029*	0.034*	0.031*	0.039*	0.037*	0.042	0.042	0.039*	0.044	0.042	0.044	0.043
Average			**0.030***	**0.039***	**0.032***	**0.041**	**0.038***	**0.043**	**0.042**	**0.041**	**0.043**	**0.041**	**0.044**	**0.042**
Overall average for $n = 30$, LR = 0.033*, MILR2 = 0.041									**Overall average for $n = 60$, LR = 0.043, MILR2 = 0.041**					

Table 12 Estimated sizes for testing $H_{20}{}^{3}: \gamma = 0$ against $H_{2a}{}^{3}: \gamma \neq 0$ with $n = 30$ and $n = 60$ for model (3.3) based on the LR and MILR2 tests using simulated critical values at the 5% significance level

Parameter value		$n = 30$						$n = 60$					
		$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.75$		$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.75$	
α	β	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2
0.25	0.25	0.028*	0.036*	0.045	0.047	0.036*	0.040	0.042	0.039*	0.044	0.045	0.046	0.046
0.5	0.25	0.027*	0.032*	0.043	0.044	0.047	0.049	0.042	0.040*	0.044	0.043	0.047	0.050
0.5	0.5	0.027*	0.035*	0.029*	0.033*	0.041	0.042	0.044	0.042	0.043	0.044	0.044	0.045
1	0.25	0.027*	0.032*	0.042	0.044	0.047	0.050	0.042	0.043	0.044	0.045	0.047	0.049
1.5	0.25	0.027*	0.032*	0.042	0.044	0.047	0.049	0.042	0.040*	0.044	0.045	0.050	0.048
1.5	0.5	0.027*	0.035*	0.029*	0.033*	0.042	0.043	0.044	0.043	0.042	0.044	0.044	0.045
1.5	0.85	0.027*	0.036*	0.027*	0.035*	0.029*	0.032*	0.044	0.045	0.042	0.043	0.043	0.041
2.5	0.25	0.027*	0.032*	0.042	0.044	0.050	0.049	0.042	0.042	0.044	0.039*	0.048	0.049
2.5	0.5	0.027*	0.035*	0.029*	0.033*	0.041	0.042	0.044	0.039*	0.043	0.042	0.044	0.045
2.5	0.75	0.027*	0.036*	0.027*	0.034*	0.031*	0.033*	0.044	0.043	0.042	0.044	0.043	0.041
3	0.25	0.027*	0.032*	0.042	0.044	0.047	0.049	0.042	0.041	0.044	0.045	0.047	0.049
Average		**0.027***	**0.034***	**0.036***	**0.040***	**0.042**	**0.043**	**0.043**	**0.042**	**0.043**	**0.044**	**0.046**	**0.046**
Overall average for $n = 30$, LR = 0.035, MILR2 = 0.039								**Overall average for $n = 60$, LR = 0.044, MILR2 = 0.044**					

Table 13 Estimated powers for testing $H_{10}^{1}:(\beta,\alpha)'=0$ against $H_{1a}^{1}:(\beta,\alpha)'\neq 0$ with $n=30$ and $n=60$ for model (3.1) based on the LR and MILR1 tests using simulated critical values at the 5% significance level

Parameter value			$n=30$						$n=60$					
			$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.75$		$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.75$	
γ	β	α	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1
0.25	0.1	0.03	0.183	0.257	0.186	0.224	0.198	0.232	0.232	0.255	0.405	0.411	0.509	0.578
0.35	0.08	0.012	0.135	0.235	0.192	0.205	0.183	0.231	0.188	0.202	0.390	0.411	0.512	0.498
0.40	0.05	0.01	0.276	0.401	0.206	0.241	0.219	0.241	0.847	0.856	0.614	0.625	0.483	0.495
0.50	0.03	0.20	0.577	0.603	0.121	0.162	0.232	0.254	0.990	0.895	0.998	0.905	0.997	0.975
0.70	0.03	0.15	0.437	0.442	0.265	0.312	0.198	0.226	0.954	0.865	0.369	0.402	0.257	0.263
0.80	0.20	0.025	0.140	0.201	0.119	0.175	0.125	0.152	0.342	0.365	0.355	0.360	0.401	0.455
0.90	0.01	0.30	0.645	0.657	0.355	0.450	0.254	0.266	0.772	0.765	0.780	0.775	0.805	0.758
1.00	0.05	0.04	0.204	0.235	0.165	0.221	0.162	0.195	0.612	0.635	0.627	0.63	0.730	0.698
1.50	0.15	0.01	0.532	0.556	0.385	0.430	0.257	0.280	0.657	0.637	0.674	0.668	0.700	0.685
1.75	0.08	0.06	0.255	0.308	0.227	0.254	0.205	0.244	0.340	0.355	0.36	0.375	0.524	0.535
2.00	0.04	0.09	0.309	0.330	0.256	0.322	0.205	0.213	0.405	0.452	0.422	0.435	0.53	0.545
2.50	0.03	0.08	0.360	0.421	0.352	0.363	0.320	0.334	0.465	0.470	0.472	0.482	0.502	0.505
Average			**0.338**	**0.387**	**0.236**	**0.280**	**0.213**	**0.239**	**0.567**	**0.563**	**0.539**	**0.540**	**0.579**	**0.583**
Overall average for $n=30$, LR = 0.262, MILR1 = 0.302									**Overall average for $n=60$, LR = 0.562, MILR1 = 0.562**					

Table 14 Estimated powers for testing $H_{10}^{2}:(\theta_4,\theta_3)'=0$ against $H_{10}^{2}:(\theta_4,\theta_3)'\neq 0$ with $n=30$ and $n=60$ for model (3.2) based on the LR and MILR1 tests using simulated critical values at the 5% significance level

				$n=30$						$n=60$					
Parameter value				$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.9$		$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.9$	
θ_1	θ_2	θ_3	θ_4	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1
-1	1	-0.50	0.5	0.955	0.958	0.949	0.912	0.432	0.408	0.978	0.964	0.985	0.884	0.742	0.701
-0.85	0.50	-0.65	0.25	0.670	0.708	0.588	0.613	0.344	0.367	0.890	0.876	0.635	0.668	0.433	0.441
-0.75	-0.25	0.75	0.05	0.295	0.336	0.085	0.192	0.059	0.097	0.617	0.649	0.158	0.260	0.127	0.133
-0.5	0.50	-0.05	0.35	0.940	0.936	0.479	0.468	0.331	0.322	0.965	0.952	0.92	0.902	0.626	0.598
-0.5	0.75	-0.50	0.20	0.900	0.920	0.274	0.291	0.101	0.133	0.956	0.945	0.859	0.862	0.165	0.203
-0.25	-0.5	-0.75	0.15	0.556	0.571	0.153	0.171	0.073	0.113	0.846	0.872	0.241	0.255	0.093	0.136
0.5	0.25	0.05	0.15	0.863	0.881	0.253	0.283	0.093	0.119	0.926	0.930	0.487	0.463	0.163	0.175
0.5	0.50	-0.15	0.25	0.891	0.902	0.516	0.545	0.171	0.202	0.903	0.950	0.857	0.848	0.317	0.297
0.75	0.25	-0.15	0.25	0.962	0.924	0.517	0.475	0.170	0.191	0.975	0.944	0.857	0.861	0.316	0.319
0.75	0.75	0.50	0.05	0.229	0.242	0.175	0.211	0.155	0.172	0.475	0.483	0.277	0.291	0.257	0.261
1	0.25	0.15	0.05	0.162	0.201	0.166	0.192	0.154	0.167	0.321	0.339	0.195	0.191	0.158	0.174
1	-1	0.15	0.15	0.893	0.913	0.274	0.292	0.097	0.103	0.908	0.911	0.542	0.529	0.181	0.192
Average				**0.693**	**0.708**	**0.369**	**0.387**	**0.182**	**0.200**	**0.813**	**0.818**	**0.584**	**0.585**	**0.298**	**0.303**
Overall average for $n=30$, LR = 0.415, MILR1 = 0.431										**Overall average for $n=60$, LR = 0.565, MILR1 = 0.568**					

Table 15 Estimated powers for testing $H_{10}^{3}:(\beta,\gamma)'=0$ against $H_{1a}^{3}:(\beta,\gamma)'\neq 0$ with $n=30$ and $n=60$ for model (3.3) based on the LR and MILR1 tests using simulated critical values at the 5% significance level

			$n=30$						$n=60$					
Parameter value			$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.90$		$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.90$	
α	β	γ	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1	LR	MILR1
0.20	0.12	0.30	0.478	0.512	0.124	0.146	0.057	0.081	0.855	0.863	0.235	0.247	0.104	0.116
0.50	0.25	1.1	0.857	0.832	0.248	0.273	0.109	0.131	0.975	0.952	0.483	0.461	0.177	0.142
0.75	0.10	0.50	0.301	0.419	0.082	0.101	0.047	0.077	0.587	0.651	0.143	0.162	0.080	0.107
0.85	0.015	0.05	0.845	0.867	0.224	0.249	0.095	0.113	0.958	0.944	0.471	0.510	0.171	0.184
1	0.05	0.10	0.127	0.175	0.046	0.062	0.036	0.071	0.254	0.291	0.083	0.113	0.047	0.082
1.50	0.20	0.75	0.774	0.741	0.196	0.244	0.088	0.102	0.978	0.954	0.411	0.402	0.152	0.161
1.75	0.10	0.50	0.303	0.334	0.088	0.109	0.055	0.085	0.588	0.634	0.145	0.156	0.086	0.109
2	0.30	1.15	0.945	0.922	0.346	0.372	0.132	0.174	0.958	0.946	0.618	0.519	0.238	0.211
2.50	0.40	1.50	0.965	0.934	0.485	0.461	0.18	0.217	0.968	0.941	0.781	0.776	0.331	0.328
2.75	0.20	0.05	0.963	0.938	0.39	0.411	0.163	0.205	0.976	0.943	0.736	0.73	0.430	0.417
3	0.20	1.50	0.582	0.612	0.147	0.151	0.072	0.104	0.865	0.859	0.257	0.263	0.113	0.185
3.50	0.50	1.50	0.985	0.977	0.691	0.702	0.278	0.315	0.977	0.954	0.929	0.903	0.495	0.475
Average			**0.677**	**0.689**	**0.256**	**0.273**	**0.109**	**0.140**	**0.828**	**0.828**	**0.441**	**0.437**	**0.202**	**0.210**
Overall average for $n=30$, LR = 0.347, MILR1 = 0.367									**Over all average for $n=60$, LR = 0.490, MILR1 = 0.491**					

Table 16 Estimated powers for testing $H_{20}{}^{1}:\alpha=0$ against $H_{2a}{}^{1}:\alpha\neq 0$ with $n=30$ and $n=60$ for model (3.1) based on the LR and MILR1 tests using simulated critical values at the 5% significance level

			$n=30$						$n=60$					
Parameter value			$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.75$		$\sigma^2=0.05$		$\sigma^2=0.25$		$\sigma^2=0.75$	
γ	β	α	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2
0.3	0.05	0.2	0.942	0.876	0.295	0.316	0.121	0.136	0.977	0.963	0.651	0.712	0.553	0.535
0.3	0.1	0.5	0.641	0.633	0.225	0.251	0.167	0.188	0.982	0.921	0.780	0.791	0.324	0.346
0.4	0.08	0.03	0.535	0.559	0.386	0.363	0.281	0.301	0.771	0.752	0.637	0.691	0.566	0.549
0.5	0.05	0.02	0.300	0.362	0.317	0.338	0.332	0.315	0.428	0.365	0.407	0.356	0.400	0.339
0.5	0.1	0.4	0.102	0.135	0.100	0.121	0.060	0.105	0.655	0.712	0.521	0.535	0.499	0.473
0.7	0.08	0.35	0.251	0.318	0.185	0.209	0.157	0.177	0.723	0.765	0.611	0.633	0.587	0.615
0.7	0.1	0.045	0.639	0.652	0.531	0.556	0.515	0.498	0.979	0.980	0.983	0.986	0.979	0.940
0.7	0.12	0.4	0.156	0.207	0.156	0.199	0.130	0.163	0.554	0.569	0.502	0.541	0.448	0.457
1	0.01	0.1	0.998	0.898	0.937	0.746	0.429	0.401	0.998	0.946	0.952	0.933	0.623	0.657
1.4	0.06	0.04	0.401	0.445	0.39	0.435	0.365	0.390	0.765	0.772	0.780	0.768	0.742	0.733
1.5	0.08	0.015	0.379	0.443	0.436	0.442	0.434	0.427	0.998	0.929	0.874	0.862	0.649	0.707
1.75	0.08	0.07	0.425	0.446	0.400	0.430	0.395	0.420	0.650	0.655	0.701	0.711	0.735	0.720
Average			**0.481**	**0.498**	**0.363**	**0.367**	**0.282**	**0.293**	**0.790**	**0.777**	**0.700**	**0.710**	**0.592**	**0.589**
Overall average for $n=30$, LR = 0.375, MILR2 = 0.386									**Overall average for $n=60$, LR = 0.694, MILR2 = 0.692**					

Table 17 Estimated powers for testing $H_{20}{}^2 : \theta_3 = 0$ against $H_{2a}{}^2 : \theta_3 \neq 0$ with $n = 30$ and $n = 60$ for model (3.2) based on the LR and MILR2 tests using simulated critical values at the 5% significance level

				$n = 30$						$n = 60$					
Parameter value				$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.9$		$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.9$	
θ_1	θ_2	θ_3	θ_4	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2
-1	1	-0.75	0.75	0.765	0.724	0.190	0.201	0.077	0.076	0.908	0.894	0.287	0.264	0.095	0.084
-0.85	0.50	-0.50	0.50	0.228	0.242	0.067	0.07	0.048	0.046	0.371	0.424	0.089	0.112	0.054	0.062
-0.75	-0.25	0.50	-0.50	0.567	0.558	0.130	0.135	0.056	0.058	0.814	0.774	0.230	0.221	0.086	0.074
-0.50	0.50	0.25	-0.75	0.261	0.273	0.069	0.076	0.044	0.046	0.466	0.491	0.108	0.113	0.062	0.065
-0.50	0.75	0.50	0.50	0.574	0.564	0.131	0.127	0.063	0.060	0.823	0.791	0.248	0.228	0.103	0.093
-0.25	-0.50	0.75	0.25	0.440	0.452	0.103	0.101	0.057	0.058	0.697	0.733	0.199	0.179	0.087	0.090
0.50	0.25	-1.75	0.05	0.315	0.331	0.081	0.102	0.054	0.053	0.574	0.592	0.161	0.193	0.074	0.069
0.50	0.50	-0.75	0.25	0.114	0.126	0.058	0.059	0.053	0.054	0.164	0.181	0.067	0.072	0.049	0.054
0.75	0.25	-0.50	0.50	0.228	0.245	0.067	0.071	0.047	0.053	0.371	0.354	0.089	0.103	0.053	0.056
0.75	0.75	0.25	0.75	0.277	0.289	0.073	0.078	0.041	0.049	0.455	0.422	0.128	0.115	0.064	0.069
1	0.25	0.25	-0.50	0.123	0.141	0.045	0.053	0.036	0.042	0.214	0.224	0.070	0.109	0.053	0.057
1	-1	0.90	-0.25	0.67	0.634	0.158	0.152	0.065	0.063	0.906	0.838	0.297	0.301	0.107	0.087
Average				**0.380**	**0.382**	**0.098**	**0.102**	**0.053**	**0.055**	**0.564**	**0.560**	**0.164**	**0.168**	**0.074**	**0.072**
Overall average for $n = 30$, LR = 0.177, MILR2 = 0.180										**Overall average for $n = 60$, LR = 0.267, MILR2 = 0.266**					

Table 18 Estimated powers for testing $H_{20}{}^{3}:\gamma = 0$ against $H_{2a}{}^{3}:\gamma \neq 0$ with $n = 30$ and $n = 60$ for model (3.3) based on the LR and MILR2 tests using simulated critical values at the 5% significance level

			$n = 30$						$n = 60$					
Parameter value			$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.75$		$\sigma^2 = 0.05$		$\sigma^2 = 0.25$		$\sigma^2 = 0.75$	
α	β	γ	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2	LR	MILR2
0.25	0.25	0.25	0.244	0.352	0.102	0.099	0.0695	0.076	0.253	0.291	0.106	0.171	0.076	0.080
0.25	0.25	0.5	0.391	0.413	0.138	0.146	0.081	0.101	0.458	0.477	0.153	0.184	0.094	0.105
0.5	0.25	1.1	0.514	0.522	0.161	0.176	0.089	0.112	0.669	0.695	0.224	0.231	0.127	0.130
0.5	0.5	1.2	0.962	0.934	0.441	0.496	0.193	0.209	0.984	0.961	0.594	0.575	0.272	0.244
1	0.25	0.5	0.391	0.378	0.137	0.146	0.089	0.098	0.458	0.473	0.153	0.159	0.099	0.105
1.5	0.25	0.25	0.224	0.294	0.107	0.133	0.064	0.091	0.249	0.253	0.112	0.121	0.084	0.101
1.5	0.5	1.1	0.962	0.938	0.437	0.447	0.195	0.197	0.982	0.934	0.580	0.604	0.269	0.272
1.5	0.85	0.75	0.896	0.884	0.808	0.872	0.372	0.332	0.980	0.926	0.878	0.846	0.488	0.442
2.5	0.25	0.5	0.391	0.418	0.138	0.146	0.081	0.101	0.458	0.533	0.153	0.170	0.094	0.105
2.5	0.5	1	0.968	0.944	0.427	0.455	0.169	0.177	0.985	0.937	0.562	0.566	0.245	0.248
2.5	0.75	0.75	0.958	0.890	0.703	0.721	0.324	0.339	0.974	0.909	0.802	0.758	0.410	0.395
3	0.25	0.5	0.388	0.434	0.131	0.153	0.064	0.102	0.452	0.485	0.153	0.161	0.100	0.096
Average			**0.607**	**0.617**	**0.311**	**0.333**	**0.149**	**0.161**	**0.659**	**0.656**	**0.373**	**0.379**	**0.197**	**0.194**
Overall average for $n = 30$, LR = 0.177, MILR2 = 0.180									**Overall average for $n = 60$, LR = 0.356, MILR2 = 0.370**					