PRICING LME COMMODITY FUTURES CONTRACTS[1]

By Richard Heaney

Abstract

It is generally argued that there is a link between commodity prices and stock levels and this paper provides a test of two economic models that attempt to explain commodity pricing, the stock-out model with two separate pricing states and the convenience yield model. Global stock levels are collected and interest-adjusted basis is calculated for the LME commodities, copper, lead and zinc spanning the period November 1964 to December 2003. A two-regime Markov model with an added stock variable appears to fit the data reasonably well, providing evidence supporting the existence of two separate commodity pricing regimes and the existence of a convenience yield effect that is inversely related to the level of stocks on hand.

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1. Introduction

The commodities, lead, copper and zinc play an important role in the world economy. Copper is well known for its use in electrical goods and in plumbing and zinc has a range of uses including in paints, in galvanising other metals and in die-casting. Further, both copper and zinc are used in the manufacture of coins and metal currency. Lead is used in batteries, radiation shielding, cable covering, ammunition, plumbing and in the manufacture of glass. While these commodities can be stored for considerable periods of time, their prices can be quite volatile and the price level and price volatility appears to be linked to the level of stocks on hand.

There are two models used to explain the variation in commodity prices over time. The first focuses on the impact of stock outs modelled in Scheinkman and Schechtman (1983) with further analysis in Chambers and Bailey (1996), Deaton and Laroque (1992, 1995, 1996) and Wright and Williams (1989). The second is based on concept of convenience yields (Brennan, 1958, Kaldor, 1939, Stein, 1961, Telser, 1958 and Working, 1949). More recent attempts at combining the two models appear in Ng and Ruge-Murcia (2000) and Routledge, Seppi, and Spatt (2000).

Empirical analysis focuses on the monthly observations of the interest-adjusted basis rather than quoted prices due to the time series behaviour of this variable. LME copper, lead and zinc prices and stocks are collected over the period from November 1964 to December 2003 with matching three-month risk free interest rates. Evidence is provided in this paper to support the existence of two regimes in commodity pricing for the three London Metals Exchange commodities, copper, lead and zinc. A stock variable is also included in the analysis to assess the impact of convenience yields and the parameter estimates support the existence of convenience yield effects. Finally, the level of serial correlation evident in the estimated model suggests that there still some way to go in explaining time series variation in commodity prices. While a brief review of the literature is provided in the next section, the data is described in the Section 3. Section 4 is devoted to analysis of the data and a summary is provided in section 5.

2. Literature Review

Keynes (1950) provides one of the early discussions of the behaviour of commodity prices and the relationship between commodity price, production and stock levels. If we ignore hedging costs consistent with Telser (1958), then Keynes' argument suggests that when stocks are high the difference between futures prices and the underlying asset price (spot price) reflects the cost of storing or carrying the underlying asset but when stocks are low commodity prices tend to reflect the value of immediate consumption and the link to the value of storage is broken. For example, it is quite possible that the spot price could exceed the futures price when stocks are low.[2] Arguments relating to the impact of stock outs on commodity prices are further clarified and extended in Scheinkman and Schechtman (1983) with additional modelling and testing evident in the work of Chambers and Bailey (1996), Deaton and Laroque (1992, 1995, 1996) Routledge, Seppi, and Spatt (2000) and Wright and Williams (1989).

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[2] For spot price to exceed futures price it is implicit that there is sufficient time prior to futures contract maturity for production to adjust to the current shortages.

Convenience yields provide another explanation for the changes in spot prices that occur when stocks are low. The convenience yield is said to arise from the benefit that producers obtain from physically holding stocks, a benefit not available to individuals holding a long futures or forward contract. The benefits are generally couched in terms of the value to the producer of "smoothing production, avoiding stock outs and facilitating the scheduling of production and sales" (Pindyck, 1993, p. 511). It is found that when stocks are low, commodity futures prices do not follow the spot price, or indeed earlier maturing futures contracts, as closely as the simple arbitrage based cost of carry pricing model suggests. It is argued that the convenience value obtained from holding stocks during periods of commodity shortage explains the additional variation in prices that is not explained by observed storage costs (Brennan, 1958, Fama and French, 1988, Kaldor, 1939, Ng and Pirrong, 1994, Pindyck, 1993, 1994, 2002, 2003, Stein, 1961, Telser, 1958 and Working, 1949).

While much of the early literature focuses on describing convenience yields and fitting non-linear functions to the data there was little economic modelling of convenience yields. Options based models were developed by Heinkel, How and Hughes (1990), Litzenberger and Rabinowitz (1995) and Milonas and Thomadakis, (1997a and 1997b) to explain convenience yield in terms of a timing option where the producer (stock holder) holds a put option on the stored commodity that provides them with the right to sell the commodity at the marginal cost of production at some future time. The combination of the put option and the underlying stock holding creates a call option whose value is increasing in commodity price much like the convenience yield effects depicted in the earlier literature. As the holder of a forward or a futures contract does not have this right these models provide a theoretical explanation for the existence of convenience yields in commodity prices.

Futures contract valuation is generally based on the cost of carry model.[3] In its most basic form this model captures the storage value noted by Keynes, with the futures price, F_{tT}, quoted at time t for a contract maturing at time T, expressed in terms of the underlying commodity price, P_t, quoted at time t, and the costs of storage which include, r, the continuously compounding risk free rate of return for the period t to T, the physical costs of storage, s, continuously compounding for the period t to T and the exponential function term, e. We include the convenience yield, cy, for the period from time t to T in the model below though it should be noted that neither Keynes (1950) nor Scheinkman and Schechtman (1983) recognised convenience yields. The cost of carry model, adjusted for convenience yield, takes the form:

$$F_{tT} = P_t e^{r+s-cy} \tag{1}$$

The interest-adjusted basis is obtained by rearranging equation (1) (Fama and French, 1988). Taking natural logs, *ln(.)*, the interest-adjusted basis is written as:

$$IAB_{tT} = \ln\left(P_t / F_{tT}\right) + r \tag{2}$$

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[3] This is a forward contract pricing model. Although the use of a forward pricing model to value futures contracts can lead to errors in pricing arising from marking to market adjustments (Cox, Ingersoll and Ross, 1981) Pindyck (1994) shows that this error is economically small for LME metals. As a result the futures/forward difference is ignored in the following discussion and analysis.

If the traditional cost of carry model adjusted for convenience yield applies to commodity prices then $IAB_{tT} = cy - s$.

Drawing on the cost of carry model and extending it to deal with the impact of stock outs, Scheinkman and Schechtman (1983) show that it is possible to model commodity prices in terms of two pricing regimes, value in consumption and value in storage, much like the process that Keynes described. Scheinkman and Schechtman (1983) provide a formal rational expectations equilibrium model of the firm that holds stocks with applications and extensions of the model appearing in Chambers and Bailey (1996), Deaton and Laroque (1992, 1995, 1996) Routledge, Seppi, and Spatt (2000) and Wright and Williams (1989). The process driving the underlying commodity price is written as:

$$P_t = \max\left(E_t P_T \exp(-r - s), P\{z_t + (1-d)I_{t-1}\}\right) \tag{3}$$

where $P\{x\}$ is the commodity price in the market for immediate consumption given x units of commodity are available in the market. The available commodity is defined as the sum of z, the level of current production, and I_{t-1}, the value of the beginning inventory. If we assume a risk neutral world then the futures price, F_{tT}, is equal to the expected price, $E_t P_T$ and it is possible to rewrite the relationship as:

$$P_t = \max\left(F_{tT} \exp(-r - s), P\{x\}\right) \tag{4}$$

Considerable measurement difficulties may arise when analysing the two pricing models based on stock outs and/or convenience yields when using readily available data. For example, Wright and Williams (1989) take considerable care to highlight the difficulty of measuring the actual level of commodity stocks in a market. Commodity stocks are often spread across the globe and though the LME is based in London the actual stocks are not. For example it is possible in commodity markets for a stock out to occur in one region with a consequent explosion of spot prices for delivery in that region with little or no effect elsewhere. The problem for the researcher lies with the tendency for recorded prices to reflect the average price and for recorded stocks to reflect total stocks regardless of location. For example, a simple average of prices taken across all markets for a commodity may suggest stock out behaviour even though there may be considerable stocks available in all but one region. This limitation should be noted in the following analysis though the LME does have an active warrant market to deal with stock location mismatches and the costs and time required for shipping would not preclude arbitrage where sufficient stocks exist on one area to meet shortages in other locations.

3. Data

Monthly observations of commodity spot price and 3 month futures prices are collected from November 1964 to December 2003. The copper and lead prices are denominated in Great Britain pounds (GBP) and the zinc price is denominated in USA dollars (USD). The spot price and three-month futures contract price are based on the official LME prices determined after the midday trading session each day.[4] Prices are obtained from the Metal Bulletin over the period, November 1964 to December 1988 and from the LME web site for the period from January 1989 to December 2003. All prices supplied on the LME web site are in USDs and so, for consistency, the copper and lead prices are converted to GBP using the foreign exchange rates supplied with the LME web site based data. Although there is some variation among the copper and zinc contracts in terms of the spot asset definition (Sephton and Cochrane, 1991) the lead contract is essentially unchanged over the study period. The copper and zinc prices used in this study reflect an average price taken across the various categories of the metal for which prices are reported on the LME. This is not necessary for the lead contract where only one price series is quoted at any time during the study period. Each of the three commodities traded continuously over the study period.

Descriptive statistics are reported in Table 1. The average 3-month interest-adjusted basis (effective 12-month interest-adjusted basis) estimate is 0.021 (8.4% pa) for copper, 0.015 (6.0% pa) for lead and 0.011 (4.4% pa) for zinc. Stocks vary considerably over the period with a minimum of around 4700 tonne for copper, 2510 tonne for lead and 300 tonne for zinc and maximums of over 972,000 tonne for copper, 372,000 tonne for lead, and 1,234,000 tonne for zinc.

[Insert Table 1 about here]

Time series statistics are also provided in Table 1 for levels, change in levels and squared levels with first order and 10th order correlation coefficients, AR (1) and

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[4] LME prices are quoted as a representative range. The price used in analysis is the mid-point of the range.

AR (10), and chi-square test probabilities for serial correlation at lags 10 and 20. There is evidence of considerable serial correlation in levels, change in levels and in squared levels suggesting that there is considerable serial correlation in the prices and the changes in prices along with time changing variance.

Unit root test statistics are also reported and these include the Phillips-Perron (1988), the Augmented Dickey Fuller (1979, 1981) and the Kwiatowski, Phillips, Schmidt and Shin (1992) test. Results, given 10 lags, are reported in Table 1 though lag length has little impact on the Phillips-Perron and the Augmented Dickey Fuller results. While a unit root null underlies the Phillips-Perron and Augmented Dickey Fuller tests, a stationary null applies to the Kwiatowski, Phillips, Schmidt and Shin test.

Spot and futures prices, stocks levels and interest rate series appear to be non-stationary. For example, the prices, stock levels and interest rates all exhibit first order autocorrelation coefficients that are very close to one. Further, the null of a non-stationary process in the Phillips Perron and Augmented Dickey Fuller tests is rejected on only a couple of occasions and rejection of the null for the Kwiatowski, Phillips, Schmidt and Shin tests occurs in all cases for these variables.

The results for the interest-adjusted basis suggest that this is a stationary variable. The first order autocorrelation coefficients for the interest-adjusted basis are somewhat lower and the null of unit root is rejected for both the Phillips-Perron and the Augmented Dickey Fuller tests in all cases. The Kwiatowski, Phillips, Schmidt and Shin test is only rejected for lead. Thus, while there is some contradiction for lead, it would seem reasonable to assume that the interest-adjusted basis is stationary for the purposes of this study.

Figure 1 shows the relationship between the spot price and futures prices for each of the commodities, copper, lead and zinc. There is a near linear relationship between the futures price and the underlying asset price though some increased spread is observed as price levels increases. Figure 2 compares the level of stocks with the interest-adjusted basis and this suggests that when stocks are high the interest-adjusted basis is close to zero and comparatively stable. When stocks are low the convenience yield is much more volatile and its magnitude tends to increase.

[Insert Figures 1 and 2 about here]

Interest rates are obtained for USD (used for zinc) and for the UK Pound (used for copper and lead). The UK interest rate series consists of the minimum lending bank rate from November 1964 to December 1975, obtained from the Bank of England web site (www.bankofengland.co.uk), and the Euro Currency (London) Sterling 3 month middle rate obtained from Datastream from January 1976 to December 2003. The US interest rate series consists of the three month treasury bill secondary market rates obtained from the USA Federal Reserve Board (www.federalreserve.gov) for the period from November 1964 to December 1975 and the Euro Currency (London) USD 3 month middle rate obtained from Datastream is used for the remainder of the period through to December 2003. The rates are graphed in Figure 3.

[Insert Figure 3 about here]

4. Analysis

4.1 Interest-adjusted basis regime Switches

We ignore the impact of convenience yields in this section and, instead, focus on the stock out model which is based on the argument that commodity price regimes reflect the existence or otherwise of stocks. Rather than rely on available stock data we use the Hamilton regime-switching model to identify whether there are two identifiable states in the interest-adjusted basis data, consistent with stock out effects. This avoids the problem highlighted by Wright and Williams (1989) who argue that stock levels may not accurately reflect the existence of stock outs for the purposes of testing the stock out model.

If we assume that the commodity price reflects either the cost of carry or its value in immediate consumption then it should be possible to model the price distribution as a Markov process with two states of the world, the storage value state and the consumption value state, with each state having a separate distribution. Given the definitions in Chambers and Bailey (1996), Deaton and Laroque (1992, 1995, 1996) and Scheinkman and Schechtman (1983) we restate the pricing function in terms of the interest-adjusted basis (dividing through by the futures price, taking natural logs and adding the risk free rate to both sides). This model suggests a two state process for the interest-adjusted basis.

$$IAB_{tT} = \max\left(-s, \ln\left(\frac{P\{x\}}{F_{tT}}\right) + r\right) = \max\left(-s, IAB^{*}_{tT}\right) \qquad (5)$$

The term, IAB^{*}_{tT}, is based on the immediate consumption value of commodity. This equation suggests that in any period, prices are drawn from one of two possible distributions, either the storage-based distribution, state $S_t=1$, or the value-based distribution, state $S_t=2$. The means for the two distributions are $m(S_t=1) = - s$ which is the negative of the physical storage cost rate and applies in the state where commodity price reflects the value in storage and $m(S_t=2) = IAB^*$ which applies in the state where price reflects the value of immediate consumption. Given the definition of the price process we expect that $m(S_t=1) < m(S_t=2)$, Further, Fama and French (1988) and Ng and Pirrong (1994) observe that the variance in the immediate consumption state is greater than the variance in the storage state and so the variance is defined as $s(S_t)$, with $s(S_t=1)$ for state 1 and $s(S_t=2)$ for state 2 with $s(S_t=1) < s(S_t=2)$. A two state Markov model is fit to the data using the Hamilton (1989, 1990, 1994) model with adjustment for serial correlation and allowing for different mean and variance in each state.

$$IAB_{tT} = m(S_t) + \sum_{i=1}^{n} f_i \left(IAB_{t-iT} - m(S_{t-i})\right) + s\,(S_t)e_t \qquad (7)$$

where $e_t \sim i.i.d.N\left(0, s^{2}\right)$ and two underlying states of the world exist with separate distributions. To model the impact of the lagged values on current values Hamilton defines a variable, s_t, as the outcome from a 2^n-state Markov chain with s_t

independent of e_t for all t and t. Although there are two underlying states, $S_t=1$ and $S_t=2$, with n-lags in the model it is possible to attain the current state in 2^n possible ways (Hamilton, 1994). For example if there were two underlying states and one lag in the model, it is possible to enter the current state from state 1 or from state 2 and so given that the current state is either 1 or 2 there are 4 possible combinations of the current state and the past state. To identify the current state and the relationship with past states we write each state as a vector of ones or twos with the first entry referring to the current state, the second entry referring to the previous period state, and so on.

$$s_t = \begin{cases} 1\ if\ (1,1,\ldots,1,1) \\ 2\ if\ (1,1,\ldots,1,2) \\ \qquad \vdots \\ n\ if\ (2,2,\ldots 2,2) \end{cases} \tag{8}$$

Thus, the vector (1,1,…,1,1) identifies the event where prices are drawn from the state 1 distribution for the current state and all previous states that have an impact on the current realisation of the interest-adjusted basis. Similarly the vector (1,1,…,1,2) is the event where the price was drawn from the state 2 distribution n lags ago and from the state 1 distribution since then (See the Appendix for further details on the density functions underlying this process).

The model is estimated for the three commodities, copper, lead and zinc and the parameter estimates are reported in Table 2. The probability of being in state one is graphed in Figure 4. Due to serial correlation in the residuals it is necessary to include lagged values of the interest-adjusted basis with the final lag choice resulting from a search beginning with a maximum of four lags and dropping statistically insignificant lags as long as there is no residual serial correlation. This results in the inclusion of two lags for copper and three lags for both lead and zinc. As indicated in

Table 2, there is no evidence of serial correlation in the residuals with the inclusion of these lag choices.

[Insert Table 2 about here]

A two state Markov process appears to fit the data reasonably well (Table 2), consistent with the arguments of Keynes (1950), Chambers and Bailey (1996), Deaton and Laroque (1992, 1995, 1996) Routledge, Seppi, and Spatt (2000) and Wright and Williams (1989). The results of parameter restriction tests are reported in Panel B of Table 2. For all three commodities both the test for equality of means and the test for equality of the standard deviation across the two states are rejected. Given the observed GARCH effects in the raw data (Table 1) it is also important to note that the two state model seems to model the time changing nature of the variance for both copper and lead, though there is still some residual GARCH effects for zinc (Table 2, Panel B).

Thus two states are identified in the data. The first state exhibits a statistically significantly lower mean and standard deviation when compared with the second state. This seems consistent with the existence of a value in storage state and an immediate consumption state as identified in the literature. While a relatively low standard deviation is expected in the value in storage state, equation (5) suggests that the mean value in this state should be equal to the negative of the storage cost rate. Though the estimated storage cost is insignificantly different from zero and accounts for a very small percentage of the price (0.088% for copper, 0.063% for lead and –0.460% for zinc) only zinc exhibits the expected negative sign. Both the mean and the standard deviation are statistically significant in the second state, the immediate consumption state, and both are considerably larger than the mean and standard deviation values reported for the first state, the value in storage state.

[Insert Figure 4 about here]

Both states are quite stable though the value in storage state appears to be the more stable of the two states. This is true for each of the commodities, with the probability of remaining in state 1, the value in storage state, being 0.965 for copper, 0.922 for lead and 0.955 for zinc. The probability of remaining in state 2, the value in consumption state, is somewhat less with 0.950 for copper, 0.832 for lead and 0.902 for zinc. Thus the model suggests that commodity prices spend fairly long periods of time in one or other of the two states with shifts from one state to the other occurring quite rapidly.

4.2 Interest-adjusted basis regimes and the convenience yield

As indicated in the previous section there is support for two price regimes that are consistent with the zero stock constraint models appearing in the literature. While we cannot explicitly identify stock outs, the two price distributions reflect periods of low stocks and periods of high stocks (Figure 4). In the simplest stock out pricing models the level of stocks has no role to play in the pricing of commodities other than through the zero stock constraint. The convenience yield model provides a much more active role for stocks, with convenience yields being a decreasing non-linear function of the level of stocks.

Much of the convenience yield discussion is based on simple graphical analysis with little evidence of time series analysis except for Pindyck (1994). The unit root tests discussed in the data section suggest that the stock variable is integrated of order one and so to regress the interest-adjusted basis on stock levels could lead to problems with statistical tests. As the literature provides little guidance on the most appropriate form of the relationship between convenience yield and stocks the relationship is modelled as a linear function of the change in the natural log of stocks, $Dln(Stk_t)$.

$$cy_t = a + \sum_{k=0}^{K} b_k D \ln\left(Stk_{t-k}\right) \tag{9}$$

When the convenience yield effect is included in the cost of carry model, the interest-adjusted basis takes the form:

$$IAB_{tT} = \ln\left(P_t / F_{tT}\right) + r = -s + a + \sum_{k=0}^{K} b_k D \ln\left(Stk_{t-k}\right) = f + \sum_{k=0}^{K} b_k D \ln\left(Stk_{t-k}\right) \tag{10}$$

The parameter, f, is the sum of the constant term in convenience yield model (equation (9)), a, less the storage rate, s. It is now possible to rewrite equation (5) to give:

$$IAB_{tT} = \max\left(f + \sum_{k=0}^{K} b_k D \ln\left(Stk_{t-k}\right), \ln\left(\frac{P\{x\}}{F_{tT}}\right) + r \right) = \max\left(f + \sum_{k=0}^{K} b_k D \ln\left(Stk_{t-k}\right), IAB_{tT}^{*} \right) \tag{11}$$

Including the stock variable in equation (11) allows a test of some of the assumptions made in the literature with respect to the impact of convenience yields. If there is a statistically significant relationship between stocks and interest-adjusted basis within both regimes then this supports the traditional convenience yield model. Where stocks have no descriptive power at all over commodity prices then this favours the simple model underlying Chambers and Bailey (1996), Deaton and Laroque (1992, 1995, 1996), Routledge, Seppi, and Spatt (2000) and Wright and Williams (1989). If stock effects are observed in the value in storage state but not in the value in consumption state (equation 11) then this supports the Ng and Ruge-Murcia (2000) model.

The impact of stocks is tested using an extended form of the model described in equation (11) and the results are reported in Table 3. Assuming that stocks are exogenous[5], we extend the Hamilton (1994) model to obtain:

$$IAB_{tT} = \mu(S_t) + \sum_{k=0}^{K} \beta_k(S_t)\Delta\ln(Stk_{t-k}) + \sum_{i=1}^{n} \phi_i(IAB_{t-iT} - \mu(S_{t-i})) + \sigma(S_t)\varepsilon_t \tag{12}$$

The parameter, $\beta_k(S_t)$, measures the sensitivity of the interest-adjusted basis to the change in the level of stocks in state S_t in the current period ($k=0$) and prior periods ($k=1, 2, \ldots, K$). To identify the appropriate number of lags to be included for stocks, a search begins with a maximum of the current change in stocks plus 5 lags with

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[5] Heaney (1998), using quarterly data for the commodity lead, finds that both the level of stocks and the futures price are exogenous while the spot price adjusts to shocks to the system.

statistically insignificant lags being dropped. Commodity prices seem to be sensitive to the change in stocks regardless of whether the state reflects pricing under storage or under immediate consumption, contrary to the arguments of Wright and Williams (1989). The sensitivity to the level of stocks accords with the concept put forward in Brennan (1958), Kaldor (1939), Stein (1961), Telser (1958) and Working (1949) though the stocks enter the model in the form of current change in stocks and various lagged stock change terms.

As is evident from Table 3 the stock parameters are generally negative, with some exceptions for copper, consistent with the argument that increases in stock will lead to a decrease in the interest-adjusted basis. It is important to note the variation in the sensitivity of the interest-adjusted basis to the change in stock across the two states. The stock parameters are generally smaller in the value in storage state (state one) than in the immediate consumption value state with some exceptions for copper. Thus a small change in stocks will have a larger impact on prices in the consumption state than in the value in storage state. This result is consistent with the non-linear model that has appeared in the literature. There is little discussion of the impact of lagged stock levels on commodity prices in the literature though it would appear that lagged stock values are important in the models estimated in Table 3.

To assess the impact of the addition of stocks to the model, Spearman rank correlations are calculated using the probability of being in regime one variable drawn from the two models described equation (7) and equation (12). The correlations are 0.893 for copper, 0.922 for lead and 0.976 for zinc. These are statistically significant and positive and suggest greatest correlation in the probability estimates for zinc with lead and copper showing somewhat less correlation. While these differences are not particularly evident in visual comparison of the graphs the fact that the correlation coefficient is not one suggests that the probability estimates may be sensitive to the inclusion of stocks. Further analysis of this question is left to future research.

5. Conclusions

While some of the theoretical commodity price literature points to the possibility of two underlying states determining commodity pricing there is also a considerable literature supporting the existence of convenience yields. To some extent these two models of commodity price were treated as alternatives though more recent modelling has recognised both the two state nature of commodity pricing and existence of convenience yields. This richer approach to modelling commodity prices appears to improve the explanatory power of the theoretical models.

There is little research evident addressing the issue of whether commodity prices actually move between two pricing states, the value state and the consumption state. Further, there is limited time series research concerning the existence of convenience yields under different market conditions. Statistical analysis reported in this paper support the existence of two pricing regimes for the commodities, copper, lead and zinc and the existence of convenience yields that are a decreasing, non-linear function of stocks. Stocks are found to have explanatory power in both regimes and this suggests a more complex process in the consumption state than the simple white noise process often assumed in the literature.

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APPENDIX

Following Hamilton (1994), there are 2^n density functions of the form:

$$f\left(IAB_{tT} \mid IAB_{t-1T},...,IAB_{t-nT}, s_1 : a\right) =$$

$$\frac{1}{\sqrt{2p}\,s_1}\exp\left\{\frac{-\left[\left(IAB_{tT}-m_1\right)-f_1\left(IAB_{t-1T}-m_1\right)-...-f_n\left(IAB_{t-nT}-m_1\right)\right]^2}{2s_1^2}\right\}$$

$$f\left(IAB_{tT} \mid IAB_{t-1T},...,IAB_{t-nT}, s_2 : a\right) =$$

$$\frac{1}{\sqrt{2p}\,s_1}\exp\left\{\frac{-\left[\left(IAB_{tT}-m_1\right)-f_1\left(IAB_{t-1T}-m_1\right)-...-f_n\left(IAB_{t-nT}-m_2\right)\right]^2}{2s_1^2}\right\}$$

$$\cdots$$

$$f\left(IAB_{tT} \mid IAB_{t-1T},...,IAB_{t-n/2T}, s_{n/2} : a\right) =$$

$$\frac{1}{\sqrt{2p}\,s_2}\exp\left\{\frac{-\left[\left(IAB_{tT}-m_1\right)-f_1\left(IAB_{t-1T}-m_2\right)-...-f_n\left(IAB_{t-nT}-m_2\right)\right]^2}{2s_1^2}\right\}$$

$$\cdots$$

$$\cdots$$

$$f\left(IAB_{tT} \mid IAB_{t-1T},..., IAB_{t-(n/2+1)T}, s_{n/2+1} : a\right) =$$

$$\frac{1}{\sqrt{2p}\,s_2}\exp\left\{\frac{-\left[\left(IAB_{tT}-m_2\right)-f_1\left(IAB_{t-1T}-m_1\right)-...-f_n\left(IAB_{t-nT}-m_1\right)\right]^2}{2s_2^2}\right\}$$

$$\cdots$$

$$f\left(IAB_{tT} \mid IAB_{t-1T},...,IAB_{t-nT}, s_n : a\right) =$$

$$\frac{1}{\sqrt{2p}\,s_2}\exp\left\{\frac{-\left[\left(IAB_{tT}-m_2\right)-f_1\left(IAB_{t-1T}-m_2\right)-...-f_n\left(IAB_{t-nT}-m_2\right)\right]^2}{2s_2^2}\right\}$$

With $a = \left(m_1, m_2, f_1, f_2,..., f_n, s_1^2, s_2^2,\right)$ and s_t evolves according to a Markov chain independent of past values of IAB_{tT} (Hamilton, 1994).

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TABLE 1

DATA DESCRIPTION

The price is the mid-point of the reported representative price range reported in the Metals Bulletin for the period 1964 to December 1989. (N = 471). Copper and lead prices are in GBP and the zinc prices are in USD. For the remainder of the period prices are obtained from the LME web site with copper and lead prices converted to GBP using the LME FX rates to maintain consistency. COP3FWD is the 3 month copper forward price (GBP), COPSPOT is the spot price of copper (GBP), CYC is an estimate of the copper interest-adjusted basis, LEAD3FWD is the 3 month lead forward price (GBP), LEADSPOT is the spot price of lead (GBP), CYL is an estimate of the lead interest-adjusted basis, ZINC3FWD is the 3 month zinc forward price (USD), ZINCSPOT is the spot price of zinc (USD), CYZ is an estimate of the zinc interest-adjusted basis, UK interest rate consists of the minimum lending bank rate from November 1964 to December 1975 obtained from the Bank of England web site (www.bankofengland.co.uk) and the Euro Currency (London) Sterling 3 month middle rate obtained from Datastream, US interest rate consists of the three month treasury bill secondary market rates obtained from the USA Federal Reserve Board (www.federalreserve.gov) for the period November 1964 to December 1975 and the Euro Currency (London) USD 3 month middle rate obtained from Datastream is used for the remainder of the period, COPPER is the level of copper stocks in tonnes at all LME warehouses, LEAD is the level of lead stocks in tonnes at all LME warehouses, ZINC is the level of zinc stocks in tonnes at all LME warehouses. Levels refers to the spot price, forward price, interest-adjusted basis estimate or stocks (tonnes). Diff refers to the first differenced series. AR(n) is the nth order autoregression parameter. PrQ(n) is the probability associated with the Ljung-Box Q-Statistic for given n lags. * significant at the 5% level of significance. The cut off value the Phillips-Perron test and the Augmented Dickey-Fuller test is –3.41 and for the KPSS test it is 0.463. While the Phillips-Perron and Augmented Dickey Fuller tests have a null of unit root process the KPSS test has a null of stationary process. The KPSS test reported is the tau test value though there is little variation between the tau test and the mu test statistics.

Descriptive Statistics

	Mean	*Maximum*	*Minimum*	*Std. Dev.*	*Skewness*	*Kurtosis*
COPSPOT	975.620	1946.223	341.000	387.942	0.429	2.579
COP3FWD	975.615	1939.677	314.000	378.692	0.298	2.448
IABC	0.021	0.186	-0.019	0.032	1.717	7.494
Copper stocks	261738.9	971500.0	4700.0	236980.1	1.0	3.2
LEADSPOT	287.405	640.500	79.125	119.672	0.018	2.689
LEAD3FWD	289.266	611.500	79.500	117.931	-0.152	2.514
IABL	0.015	0.208	-0.031	0.032	1.711	8.575
Lead stocks	83743.5	371775.0	2510.0	80734.6	1.5	5.0
ZINCSPOT	638.724	2050.000	95.875	443.192	0.621	2.576
ZINC3FWD	640.168	1943.000	93.688	435.381	0.510	2.271
IABZ	0.011	0.180	-0.057	0.031	1.719	7.797
Zinc stocks	196908.1	1234150.0	300.0	280875.5	1.8	5.8
UK interest rate	9.030	20.875	3.406	3.485	0.577	2.545
US interest rate	6.736	19.938	1.063	3.34	1.321	5.357

Serial correlation analysis on levels and change in levels

	Levels AR(1)	*Levels AR(10)*	*Levels PrQ(10)*	*Levels PrQ(20)*	*Diff AR(1)*	*Diff AR(10)*	*Diff PrQ(10)*	*Diff PrQ(20)*
COPSPOT	0.977	0.815	0.00	0.00	0.009	0.059	0.25	0.03
COP3FWD	0.982	0.832	0.00	0.00	0.037	0.0401	0.34	0.02
IABC	0.848	0.426	0.00	0.00	-0.224	-0.035	0.00	0.00
Copper stocks	0.993	0.768	0.00	0.00	0.527	-0.064	0.00	0.00
LEADSPOT	0.981	0.790	0.00	0.00	0.032	0.039	0.90	0.00
LEAD3FWD	0.984	0.822	0.00	0.00	0.068	0.072	0.51	0.00
IABL	0.768	0.296	0.00	0.00	-0.225	-0.017	0.00	0.00
Lead stocks	0.995	0.811	0.00	0.00	0.438	0.041	0.00	0.00
ZINCSPOT	0.989	0.853	0.00	0.00	0.057	0.001	0.06	0.01
ZINC3FWD	0.992	0.873	0.00	0.00	0.159	-0.061	0.00	0.00
IAB	0.720	0.271	0.00	0.00	-0.366	0.016	0.00	0.00
Zinc stocks	0.998	0.901	0.00	0.00	0.666	0.247	0.00	0.00
UK interest rate	0.975	0.753	0.00	0.00	-0.044	-0.151	0.00	0.00
US interest rate	0.979	0.830	0.00	0.00	0.132	0.088	0.00	0.00

Serial correlation analysis on squared levels – A test for ARCH

	Sq'd Levels AR(1)	*Sq'd Levels AR(10)*	*Sq'd Levels PrQ(10)*	*Sq'd Levels PrQ(20)*
COPSPOT	0.968	0.768	0.00	0.00
COP3FWD	0.974	0.783	0.00	0.00
IABC	0.619	0.169	0.00	0.00
Copper Stocks	0.991	0.652	0.00	0.00
LEADSPOT	0.965	0.635	0.00	0.00
LEAD3FWD	0.971	0.68	0.00	0.00
IABL	0.426	0.112	0.00	0.00
Lead Stocks	0.994	0.728	0.00	0.00
ZINCSPOT	0.974	0.722	0.00	0.00
ZINC3FWD	0.983	0.756	0.00	0.00
IABZ	0.375	0.097	0.00	0.00
Zinc Stocks	0.996	0.757	0.00	0.00
UK interest rate	0.960	0.676	0.00	0.00
US interest rate	0.957	0.774	0.00	0.00

Unit root tests

	Levels PP(10)	*Levels ADF(10)*	*Levels KPSS(10)*	*Diff PP(10)*	*Diff ADF(10)*	*Diff KPSS(10)*
COPSPOT	-3.46*	-2.97	2.97*	-21.56*	-7.00*	0.03
COP3FWD	-3.35	-3.03	3.39*	-20.85*	-6.81*	0.03
IABC	-6.31*	-3.63*	0.40	-31.24*	-8.52*	0.03
Copper stocks	-2.96	-3.35	1.67*	-12.70*	-5.74*	0.03
LEADSPOT	-3.09	-3.46*	2.17*	-20.95*	-6.28*	0.03
LEAD3FWD	-2.90	-3.22	2.31*	-20.19*	-6.12*	0.04
IABL	-8.75*	-4.37*	0.69*	-32.95*	-8.09*	0.02
Lead stocks	-2.78	-3.61*	2.14*	-14.90*	-4.95*	0.04
ZINCSPOT	-3.19	-3.46*	3.22*	-20.62*	-6.72*	0.03
ZINC3FWD	-3.06	-3.19	3.35*	-18.74*	-6.65*	0.03
IAB	-9.77*	-4.25*	0.44	-39.13*	-8.92*	0.02
Zinc stocks	-2.02	-3.29	2.22*	-10.88*	-4.34*	0.10
UK interest rate	-2.70	-2.30	0.93*	-22.81*	-6.81*	0.11
US interest rate	-2.12	-2.06	0.78*	-18.93*	-6.05*	0.17

TABLE 2
HAMILTON TWO STATE REGIME SWITCHING MODEL FOR COPPER, LEAD AND ZINC

The parameter estimates are obtained from the Hamilton two state switching regime model using maximum likelihood estimation over the interest-adjusted basis expressed as a percentage per month. P11 is the probability of being in state 1. P22 is the probability of being in state 2. The intercept term is the average interest-adjusted basis, $m(S_t)$, and it takes on a value of $m(S=1)$ in state 1 and a value of $m(S=2)$ in state 2. The terms, $f_1, f_2, f_3,$ are the lag coefficients. Lag choice is based on a general 4-lag model with exclusion of statistically insignificant lags subject to the requirement that there be no residual serial correlation. The residual term is the product of the state dependent standard deviation scale parameter, $s(S_t)$, which takes on values of $s(S=1)$ in state one or $s(S=2)$ in state 2, and a mean zero, unit variance residual term e_t. The equation takes the form:

$$IAB_{tT} = m(S_t) + \sum_{i=1}^{n} f_i \left(IAB_{t-iT} - m(S_{t-i})\right) + s(S_t)e_t$$

PrQ(20) is the probability associated with the Ljung-Box Q-Statistic for 20 lags and PrQ(1) is the probability associated with the parameter restriction . * significant at the 5% level of significance.

Panel A: Parameter estimates

	Copper Parameter	*Copper t-statistic*	*Lead Parameter*	*Lead t-statistic*	*Zinc Parameter*	*Zinc t-statistic*
μ (S=1)	0.088	0.56	0.063	0.17	-0.460	-1.77
μ (S=2)	2.228*	6.05	2.222*	4.19	1.310*	3.56
ϕ_1	0.592*	14.01	0.569*	12.03	0.705*	19.57
ϕ_2	0.227*	5.77	0.071	1.41	0.123*	3.57
ϕ_3	-		0.200*	6.17	0.057*	2.13
P11	0.965*	76.65	0.922*	43.35	0.955*	75.65
P22	0.950*	60.23	0.832*	17.78	0.902*	32.52
σ(S=1)	0.448*	16.17	0.821*	13.40	0.538*	20.55
σ(S=2)	2.424*	18.55	3.110*	13.46	3.489*	16.56

Panel B: Tests of restrictions and residual tests

	Copper	Lead	Zinc
Tests of parameter restrictions			
$m(S=1) = m(S=2)$, PrQ(1)	0.00*	0.00*	0.00*
$s(S=1) = s(S=2)$, PrQ(1)	0.00*	0.00*	0.00*
Test for serial correlation			
Std. residual, PrQ(20)	0.52	0.26	0.45
Std. residual sqrd., PrQ(20)	0.86	0.93	0.00*

TABLE 3
HAMILTON TWO STATE REGIME SWITCHING MODEL FOR COPPER, LEAD AND ZINC INCLUDING THE IMPACT OF STOCKS

The parameter estimates are obtained from the Hamilton two state switching regime model using maximum likelihood estimation over the interest-adjusted basis expressed as a percentage per month. P11 is the probability of being in state 1. P22 is the probability of being in state 2. The intercept term is the average interest-adjusted basis, $m(S_t)$, and it takes on a value of $m(S=1)$ in state 1 and a value of $m(S=2)$ in state 2. Similarly, the stock parameter, $b_k(S_t)$, is estimated for both states with a value of $b_k(S=1)$ in state 1 and a value of $b_k(S=2)$ in state 2 with current (k=0) and lag terms, k=1, 2, … K. The lag terms are added to the model in pairs, one pair for each lag, until additional pair of lag terms is no longer statistically significant. The terms, $f_1, f_2, f_3,$ are the lag coefficients. Lag choice is based on a general 4-lag model with exclusion of statistically insignificant lags subject to the requirement that there be no residual serial correlation. The residual term is the product of the state dependent standard deviation scale parameter, $s(S_t)$, which takes on values of $s(S=1)$ in state one or $s(S=2)$ in state 2, and a mean zero, unit variance residual term e_t. The equation takes the form:

$$IAB_{tT} = m(S_t) + \sum_{k=0}^{K} b_k(S_t) D\ln(Stk_{t-k}) + \sum_{i=1}^{n} f_i\left(IAB_{t-iT} - m(S_{t-i})\right) + s(S_t)e_t$$

PrQ(20) is the probability associated with the Ljung-Box Q-Statistic for 20 lags and PrQ(1) is the probability associated with the parameter restriction. * significant at the 5% level of significance.

Panel A: Parameter estimates

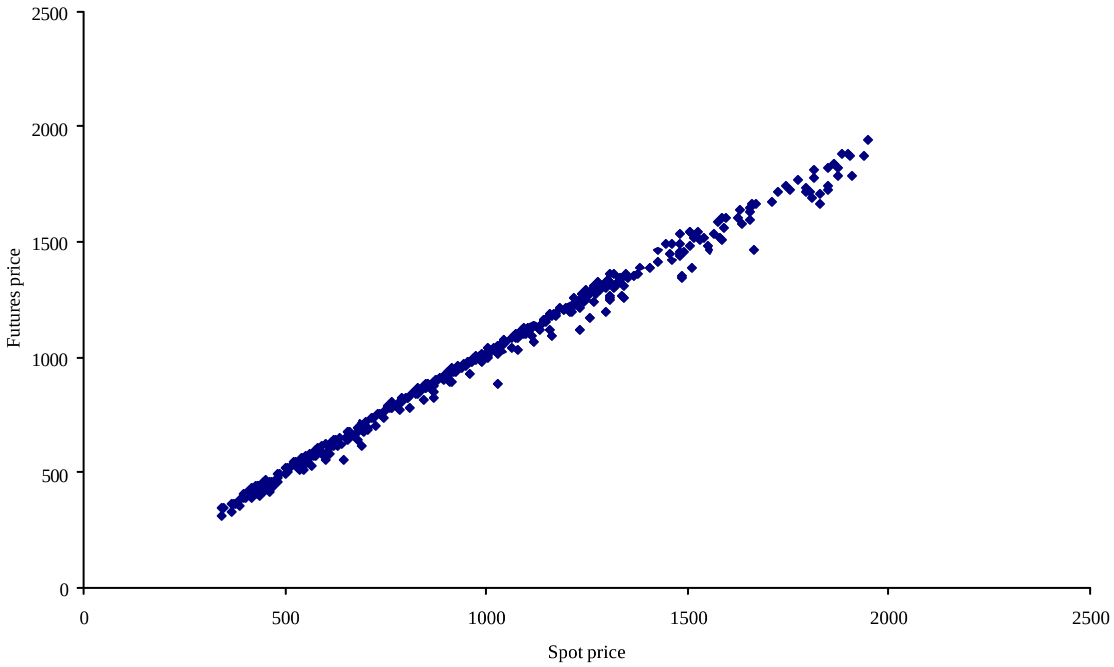
	Copper Parameter	*Copper t-statistic*	*Lead Parameter*	*Lead t-statistic*	*Zinc Parameter*	*Zinc t-statistic*
μ (S=1)	0.727*	2.04	0.457	1.18	-0.155	-0.48
μ (S=2)	1.879*	4.60	2.657*	5.20	1.376*	3.24
ϕ_1	0.590*	12.95	0.513*	12.00	0.649*	14.58
ϕ_2	0.329*	7.30	0.143*	3.25	0.151*	4.02
ϕ_3	-		0.218*	6.53	0.101*	2.99
	-					
P11	0.926*	52.17	0.926*	48.63	0.954*	69.67
P22	0.889*	27.95	0.804*	15.93	0.904*	33.87
σ(S=1)	0.363*	13.65	0.814*	17.83	0.504*	21.90
σ(S=2)	2.178*	17.49	2.641*	15.30	3.315*	17.07
β_0(S=1)	-2.617*	-9.68	-1.655*	-3.68	-1.027*	-4.60
β_0(S=2)	-4.408*	-5.67	-8.480*	-6.78	-1.341*	-2.61
β_1(S=1)	0.066	0.25	-1.991*	-4.31	-0.658*	-3.16
β_1(S=2)	-3.084*	-3.90	-3.741*	-2.98	-0.981*	-1.66
β_2(S=1)	0.518*	2.20	-	-	-0.023	-0.10
β_2(S=2)	-0.793	-1.05	-	-	-1.278*	-2.30
β_3(S=1)	-0.118	-0.64	-	-	-	-
β_3(S=2)	0.695	0.87	-	-	-	-

Panel B: Tests of restrictions and residual tests

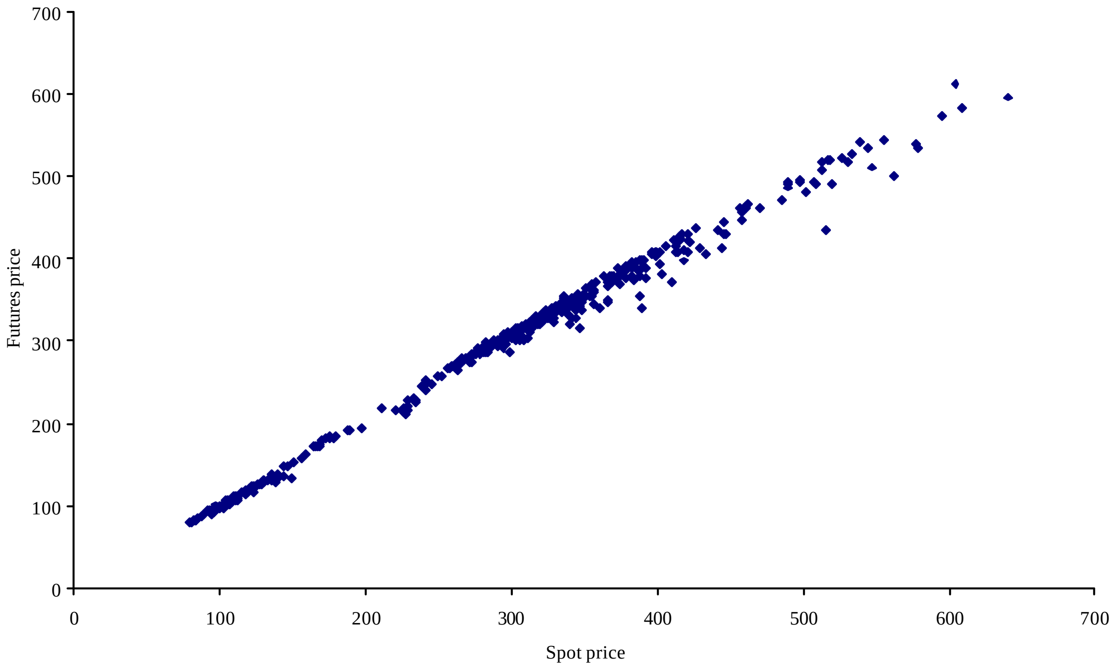
	Copper	Lead	Zinc
Tests of parameter restrictions			
μ (S=1) = μ (S=2), PrQ(1)	0.00*	0.00*	0.00*
σ(S=1) = σ(S=2), PrQ(1)	0.00*	0.00*	0.00*
Test for serial correlation			
Std. residual, PrQ(20)	0.61	0.21	0.06
Std. residual sqrd., PrQ(20)	0.94	0.58	0.00*

FIGURE 1
THE RELATIONSHIP BETWEEN SPOT PRICE AND FUTURES PRICE (WEEKLY OBSERVATIONS, 1964 TO 2003)

Panel A: Copper spot vs. forward prices (GBP)



Panel B: Lead spot vs. forward prices (GBP)



Panel C: Zinc spot vs. forward prices (USD)

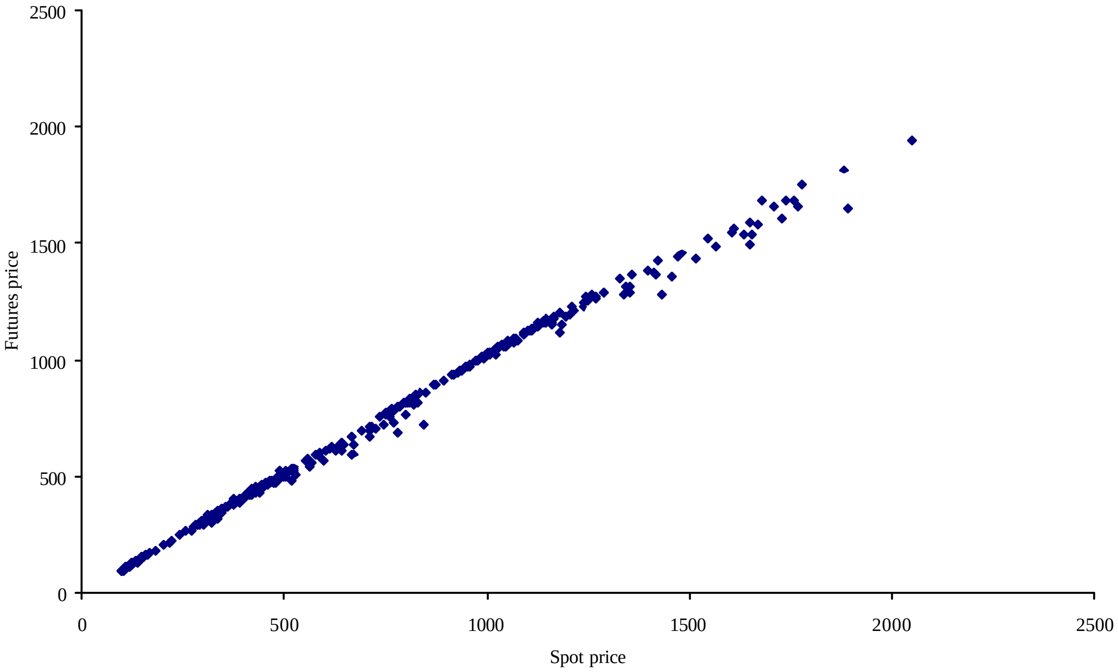
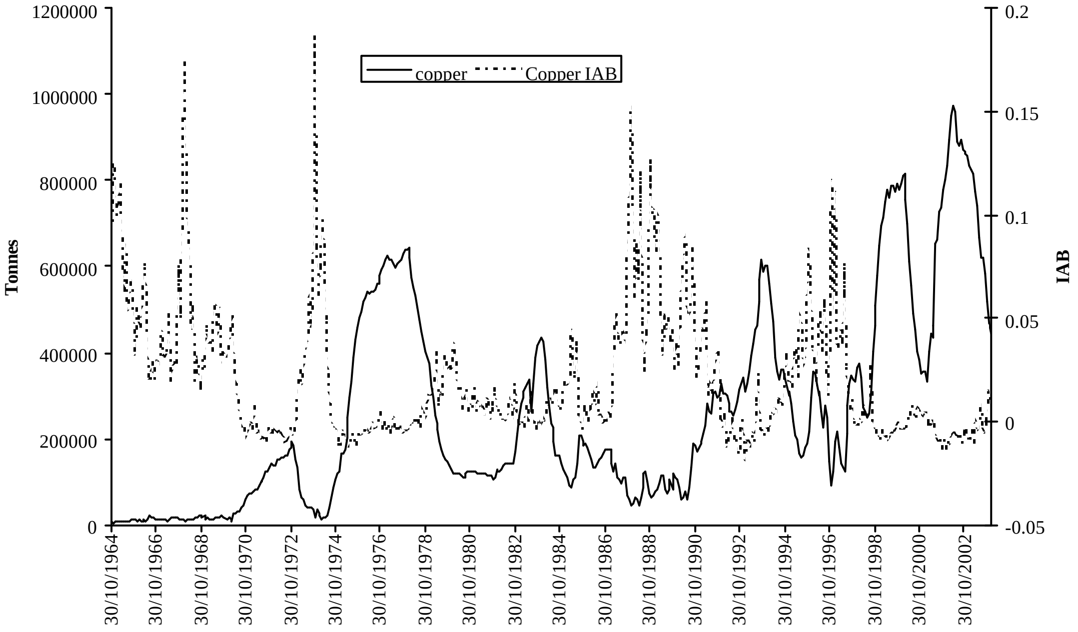
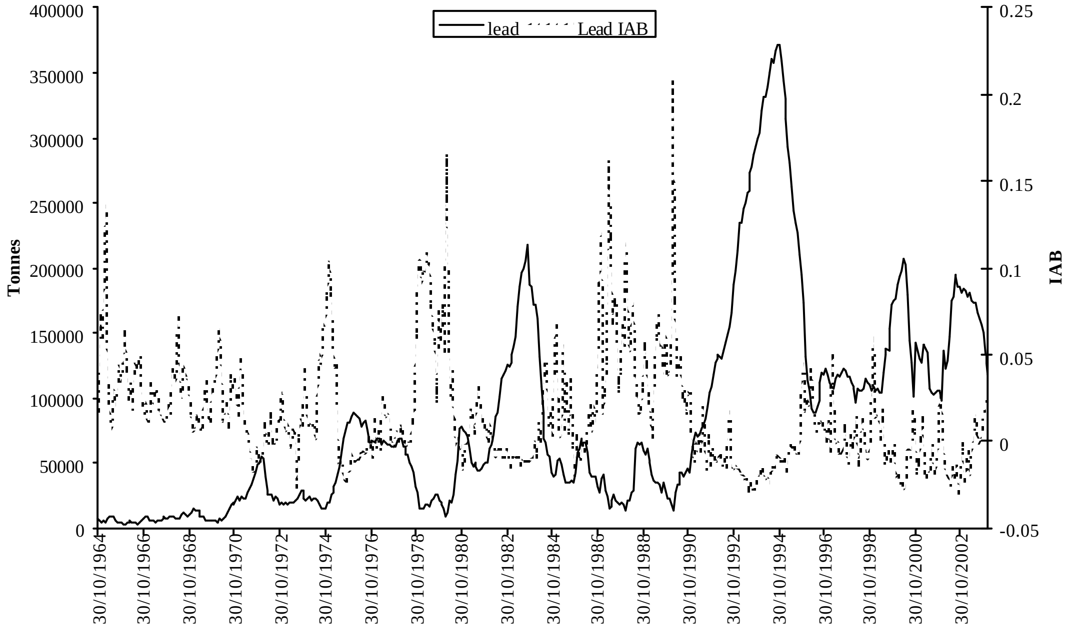


FIGURE 2
INTEREST-ADJUSTED BASIS (IAB) AND PROBABILITY OF STATE = 1

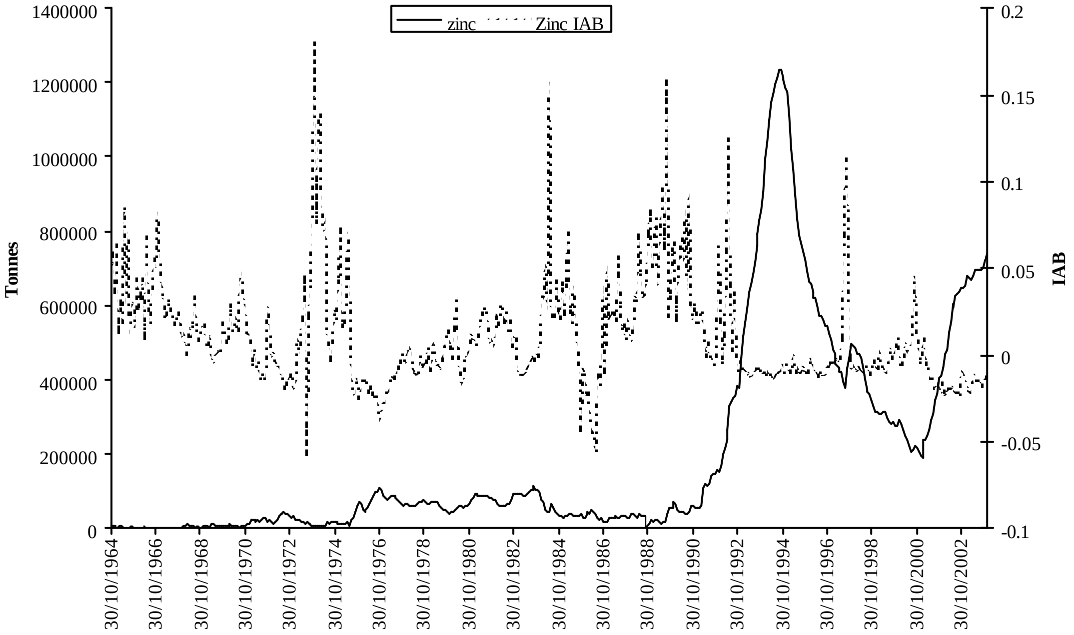
Panel A: Copper



Panel B: Lead



Panel C: Zinc


zinc
Zinc IAB
Tonnes
IAB
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1200000
1000000
800000
600000
400000
200000
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0.2
0.15
0.1
0.05
0
-0.05
-0.1
30/10/1964
30/10/1966
30/10/1968
30/10/1970
30/10/1972
30/10/1974
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30/10/1984
30/10/1986
30/10/1988
30/10/1990
30/10/1992
30/10/1994
30/10/1996
30/10/1998
30/10/2000
30/10/2002

FIGURE 3
INTEREST RATES (WEEKLY OBSERVATIONS, 1964 TO 2003)

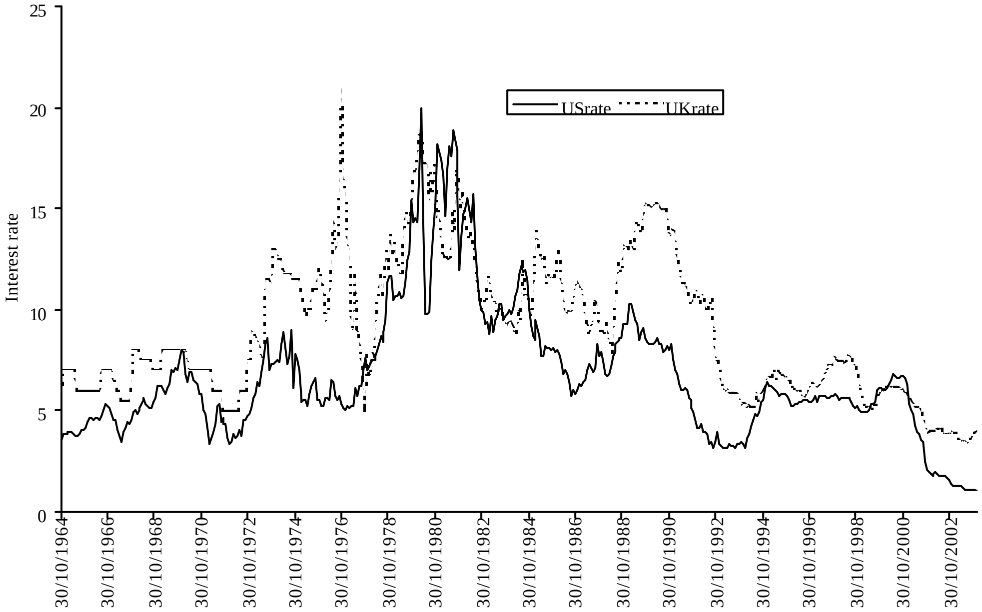
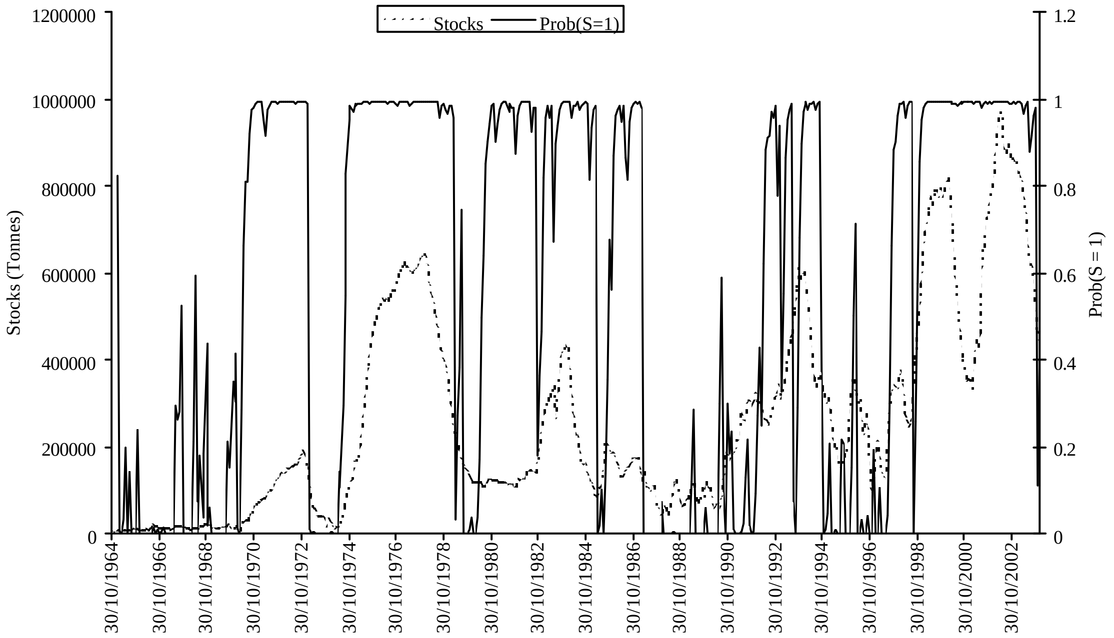
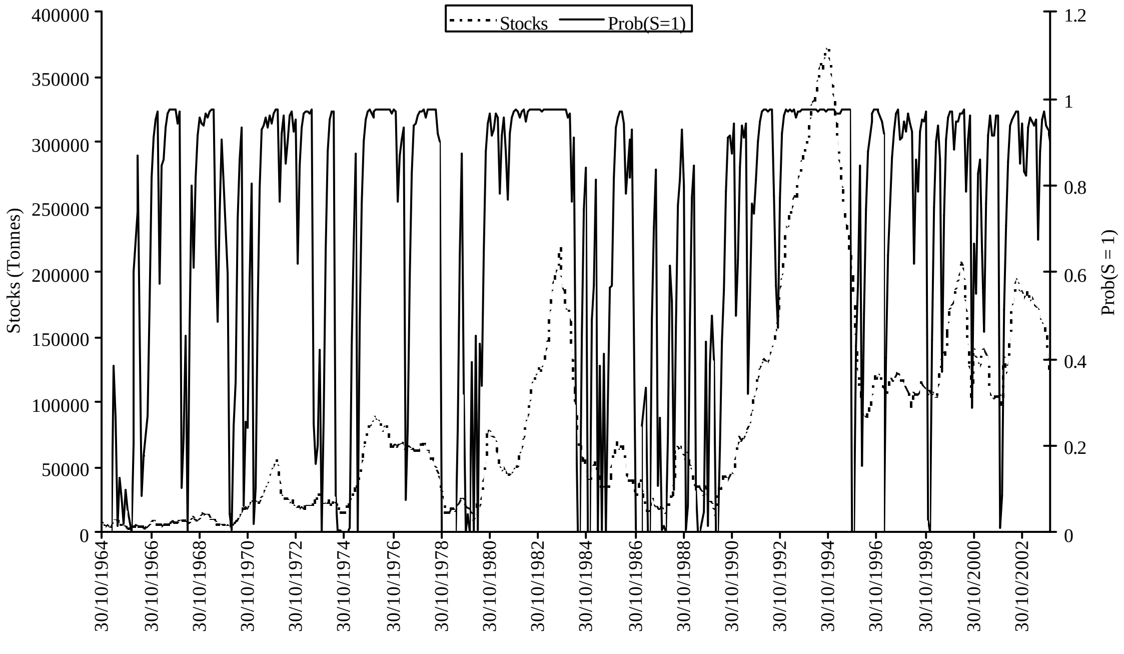


FIGURE 4
STOCK LEVELS AND PROBABILITY OF STATE = 1

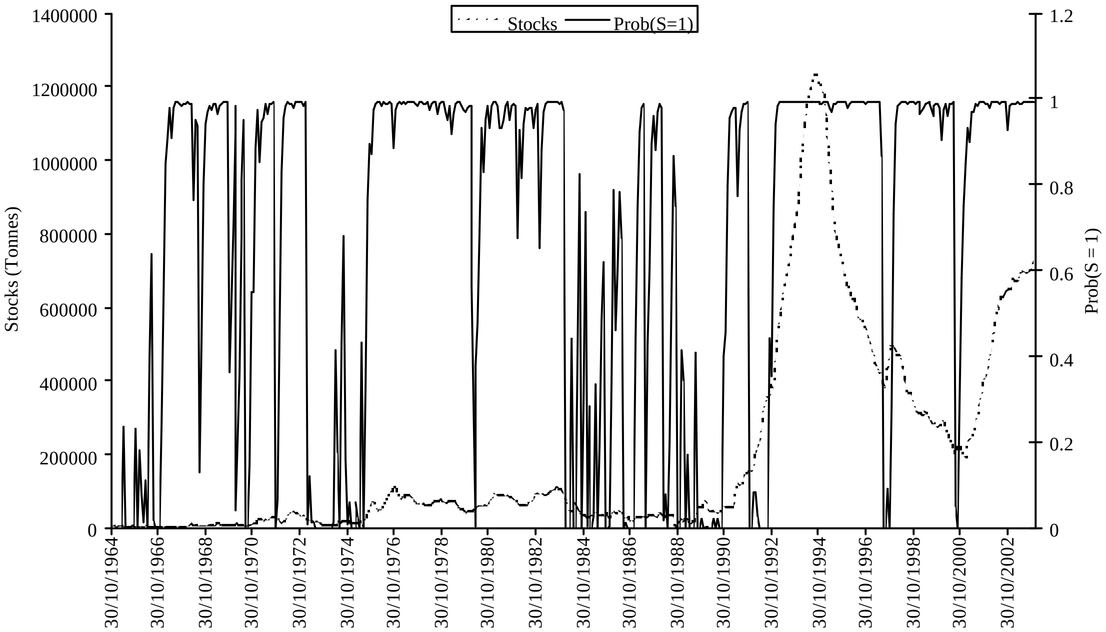
Panel A: Copper



Panel B: Lead



Panel C: Zinc


Stocks
Prob(S=1)
Stocks (Tonnes)
Prob(S = 1)
1400000
1200000
1000000
800000
600000
400000
200000
0
1.2
1
0.8
0.6
0.4
0.2
0
30/10/1964
30/10/1966
30/10/1968
30/10/1970
30/10/1972
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30/10/1998
30/10/2000
30/10/2002