Structurally Sound Dynamic Index Futures Hedging*

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Abstract

Portfolio managers use index futures for a variety of reasons. Regardless of their motivation, they will keep a close eye on the relation between the futures returns and their stock portfolio returns. Whenever this relation is perceived to have changed, the manager will decide whether it is worthwhile to rebalance the futures-portfolio mix accordingly. Exact measures as to when and how much rebalancing should occur, have not yet been established. This paper proposes a heuristic algorithm to dynamically update hedged portfolios. This dynamic hedging algorithm is based on a Reverse Order CUSUM-squared (ROC) testing procedure, proposed by Pesaran and Timmermann (2002), to optimally determine forecast estimation windows. In a comparison with standard alternatives (expanding window, EWLS window and rolling window), we find improvements in hedging performance, both in- and out-of-sample.

Keywords: Reverse Order CUSUM-squared test; dynamic index futures hedging

1 Introduction

Measurement and management of price risk continue to occupy academics and practitioners in derivatives markets alike. Whereas price discovery and price dissemination are certainly important functions, the smooth and efficient transfer of risk ultimately justifies the existence of, and steady growth in, derivatives markets. The accurate measurement of risk is fundamental to correctly price derivatives assets and the development of new derivative markets and assets has provided fertile grounds for academic research. The plethora of ARCH-related volatility measurement and forecasting papers dominate any other topic area, not just in the derivatives literature, but in the finance literature at large. A close second (frequently using ARCH models) is the literature on the quest for an optimal hedge ratio to efficiently manage risk using derivative assets. A search for the key words **hedge ratio** on Wiley's Journal of Futures Markets website (admittedly a somewhat biased selection criteria) revealed 50 published papers over the past 8 years.

Hedging is commonly understood to be undertaken to reduce the risk of holding a portfolio of risky assets. This has not always been the case. The founder of modern derivatives research, Working (1953), considered hedging as speculating upon changes in the spot and futures pricing relationship. Our current understanding and interpretation of hedging derives from Johnson's (1960) and Ederington's (1979) papers where the objective of hedging is to minimise total asset portfolio variance. Their methodology specifies an Optimal Hedge Ratio (OHR), the proportion of short futures contracts held for a long spot position, that maximises the agent's expected utility. In a mean-variance framework, and a one-period setting, the optimal hedge ratio minimises the total variance of the hedged portfolio's return and can simply be obtained from an OLS regression of unhedged portfolio returns on futures returns. The optimal hedge ratio is therefore also known as the minimum variance hedge ratio (MVHR).

Dynamic hedging evolved from a recognition of time-variation in the conditional distribution of financial asset returns. Of course, this time-dependency is nowadays most apparent in the conditional variance – or, volatility clustering – of many financial return series. In the late 1970's and early 1980's – the pre-ARCH era – several futures researchers became aware of the potential benefits of dynamic hedging. Early versions of dynamic hedging (see, e.g., Breeden, 1984; Ho, 1984; and Stulz, 1984) exploited the notion that only recent history contained relevant information for the optimal hedge ratio by intertemporally updating the information set, the so-called rolling window methodology.

Post-ARCH, in the latter half of the 1980's, it became increasingly clear that the need for dynamic hedging was primarily due to time-dependency in the (co-)variance of returns, not so much in the levels of returns. Hedging models that account for time-varying covariance are invariably based on an ARCH (Engle, 1982) or a GARCH specification (Bollerslev, 1986). Prime examples of this literature, Cecchetti et al (1988) and Baillie and Myers (1991), find significantly reduced hedged portfolio variance, at least for short hedging horizons. Kroner and Sultan (1993) show that even after accounting for transaction costs, there is still a significant out-of-sample advantage for GARCH based currency hedging. Sim and Zurbruegg (2001), on the other hand, illustrate that GARCH driven changes in the stock index futures hedge ratio are frequent and large and may therefore incur prohibitive transaction costs. Simpler and less frequently updated dynamic hedging strategies, like the rolling window hedge, may then still be preferable.

Yet another line of research pursued the possibility of spot-futures arbitrage whenever futures and/or spot prices violate the cost-of-carry relationship. The subsequent arbitrage flows would drive futures and/or spot prices back to their cost-of-carry equilibrium. This predictable component in the level of futures/spot returns is captured by error-correction hedging models (with or without GARCH variance processes), based on the notion of cointegration (see e.g., Kroner and Sultan; 1993, Brenner and Kroner, 1995; and Low et al., 2002).

The choice between the static and the various dynamic hedging methodologies has direct implications for the size of the information set to be used in estimating the hedge ratio. The traditional, static hedging model would suggest to use new information whenever it becomes available. This expanding window method adds new observations to the estimation sample when time progresses, improving the efficiency of the hedge ratio estimate (see e.g., Harris and Shen, 2002). The simple rolling window dynamic hedging method uses relatively short samples. Its main drawback is that each observation in this window is assigned equal weight. Observations are omitted as soon they drop out of the window. On any given day, an observation is allocated as much importance as any other observation in the window, but the next day it is deemed to be of no importance and disappears from the sample altogether. Such an arbitrary allocation process would only be appropriate to model a truly unstable relationship (Pesaran and Timmermann, 2002). To avoid this particular problem, discounted least squares (DLS) assigns decreasing weights to 'older,' less timely, observations. In the spirit of JP Morgan's (1996) popular EWMA volatility model, for example, Exponentially Weighted Least Squares (EWLS) assigns exponentially declining weights to historical obervations. Brooks and Chong (2001) show that an EWLS hedging model outperforms GARCH, implied volatility, and static hedging models.

These static and dynamic hedging models assume that the unconditional joint distribution of portfolio and futures returns is stable. For perfect hedge scenarios (i.e., where the futures contract perfectly matches the unhedged portfolio) this assumption seems reasonable. The empirical hedging literature is predominantly based on perfect hedges. In practice, imperfect or cross-hedges are much more common. The cross hedger uses a futures contract whose returns are most correlated with the portfolio returns. Butterworth and Holmes (2001) demonstrate the instability of these cross hedges for portfolios of Investment Trusts hedged with FTSE 100 futures contracts. Similarly, Kavussanos and Nomikos (2000) investigate the BIFFEX freight futures contract – used for cross hedging the risk in transportation costs – and find the risk reduction to vary from a low of 4% to a maximum of 19% (with perfect hedges this figure is commonly close to 90%). Hence, cross hedges exhibit higher and more volatile basis risk, see also Benet (1992). Not only is the cross hedge relationship less 'robust,' it is also prone to structural change. Consider, for example, a European sugar-beet farmer who wishes to hedge uncertain output prices with New York Board of Trade sugar-11 futures. These futures call for the delivery of cane sugar, FOB from any of twenty-nine countries of origin (not including EC countries). Apart from qualitydriven distortions, the occasional change in EC agricultural policy has the potential to significantly (and persistently) change the futures-spot price relationship.

If the spot-futures relationship is subject to these structural breaks, the expanding window, rolling window and EWLS models are all inappropriate. None of them explicitly condition on structural breaks. Of course, to distinguish a "discrete" structural break from a continuously changing hedge ratio, we first need to identify possible structural breaks. This paper will follow the recently proposed Reverse-Ordered CUSUM-squared (ROC) testing methodology, see Pesaran and Timmermann (2002). Their procedure is based on the standard CUSUM-squared test of Brown et.al. (1975), which allows identification of structural breaks in the dataset. The ROC test reverses the order of the observations and analyses the structural stability of the relationship backwards in time.

ROC models are not unique in accounting for structural breaks. Kalman filter and/or Markov regime switching models can also be used to capture structural breaks in the hedge ratio. Sarno and Valente (2000), for example, demonstrate that non-linear regime switching models capture the dynamics of the spot-futures relationship for stock indexes more effectively than an error correction model. Bai and Perron (1998, 2003) propose a least squares estimation procedure to test for multiple breaks, while Andreou and Ghysels (2003) investigate a range of change-point tests to detect multiple fundamental changes in the relationship between currency returns.

The advantage of the ROC test over these alternative methods, is that the hedger only requires information regarding the most recent break. That is, to find the optimal forecast hedge ratio, the hedger should only condition the forecast on observations that occur after the most recent structural break. This eliminates the cumbersome procedure of testing for multiple (successive) structural breaks up to the most recent break in the available history of returns. If the variance declines after a structural break, conditioning the forecast on post-break observations will significantly reduce forecast errors compared to rolling window and expanding window models.

Of course, to choose between these hedging models, we need to judge their performance out-of-sample. Surprisingly few empirical hedging papers actually do this. Lin et.al. (1994) and Tong (1996), for example, compare various hedging models on their within-sample hedging effectiveness. It comes as no surprise that the more flexible parameterisation outperforms less 'dynamic' models within-sample (Butterworth and Holmes, 2000). In practice, hedgers need to find an optimal forecast hedge ratio and judge its performance after-the-fact. An out-of-sample hedging strategy exposes the agent to possible change in the variance-covariance relationship of spot and futures returns during the hedged period. Thus, it is this model risk ignored by the in-sample literature, yet faced by the hedger in practice, which explains the underestimation of portfolio variance. Benet (1990) finds that out-of-sample hedging effectiveness is substantially less than within-sample effectiveness for a range of foreign currency portfolios. Butterworth and Holmes (2000) find that out-of-sample hedging effectiveness is marginally reduced for perfect hedges, but significantly reduced for cross hedges. Sim and Zurbruegg (2001) find that GARCH hedging models' out-of-sample effectiveness diminishes as the hedger's holding period increases.

To summarize our aims, we evaluate a 'simple' dynamic hedging scheme that conditions on continuous changes, as well as on discrete changes in the relationship between unhedged portfolio and futures returns. A conditional window selection methodology will be implemented that recursively updates the hedge ratio through reverse ordering the information set, and searching for possible structural breaks. A simulation experiment highlights the possible benefits from expanding the information set in the absence of structural breaks, and restricting the information set when a structural break is encountered. We apply this methodology to a perfect hedge scenario as well as two cross hedge scenarios for stock indices traded on the Hong Kong Stock Exchange. We find that the ROC hedging model marginally outperforms the alternative hedging models for the cross hedge. For the perfect hedge, however, a static hedging strategy still dominates all the dynamic alternatives.

The next section discusses the structural break identification methodology and its implementation for stock index futures hedging purposes. Section 3 first summarizes the data from the Hong Kong Exchanges, and then presents and discusses the empirical results of our dynamic hedging scheme and finally draws comparisons with common alternatives. We conclude this paper with a range of extensions and a discussion of the shortcomings of our approach.

2 Structural break methodology

We start with the standard hedged portfolio model (Johnson, 1960; Stein, 1961; and Ederington, 1979), where the unhedged portfolio returns (R_P) and the futures returns (R_F) are related as follows

$$R_{P,t} = \alpha_{P,t} + \beta_{P,t} R_{F,t} + \varepsilon_{P,t} \sigma_{P,t} \qquad t = 1,...,T \tag{1}$$

where ε is a standard normally distributed innovation. The conditional beta – better known as the optimal hedge ratio (OHR)[1] – is given by

$$\beta_{P,t} = \frac{E_{t-1}\left(R_{P,t}, R_{F,t}\right)}{E_{t-1}\left(R_{F,t}^2\right)} = \frac{\sigma_{PF,t}}{\sigma_{F,t}^2}$$

where $\sigma_{PF,t}$ is the conditional covariance between futures and unhedged portfolio returns, and $\sigma_{F,t}^2$ is the conditional variance of the futures returns. Conditionality in the variance of both unhedged portfolio and futures returns suggests to first standardize the returns by dividing (1) through by the standard deviation of unhedged portfolio returns, $\sigma_{P,t}$

$$\frac{R_{P,t}}{\sigma_{P,t}} = \left(\frac{\alpha_{P,t}}{\sigma_{P,t}}\right) + \left(\frac{\sigma_{PF,t}}{\sigma_{P,t}\sigma_{F,t}}\right)\left(\frac{R_{F,t}}{\sigma_{F,t}}\right) + \frac{\varepsilon_{P,t}\sigma_{P,t}}{\sigma_{P,t}} \tag{2}$$

which gives us the basic regression model

$$Y_t = Z_t'\theta_t + \varepsilon_t \tag{3}$$

where at time t, Y_t is the observation on the standardized unhedged portfolio return, Z_t is the column vector of observations on the regressors (a constant and the standardized futures return), and $\theta_t = (A_t, B_t)$. This standardization satisfies to some extent the restrictions on the robustness of the CUSUM of squares test described below, see e.g., Andreou and Ghysels (2003). Note that the (OHR) beta parameter has now been 'transformed' into a conditional correlation coefficient between futures and unhedged portfolio returns. Also note that (3) is possibly misspecified if, as the literature suggests, spot and futures prices are cointegrated. In that scenario, Z_t should include an error correction term (and possibly lagged standardized unhedged portfolio and lagged standardized futures returns), see e.g., Myers and Thompson, (1989). We consider this possibility in the empirical application, but keep the notation as general as possible at this stage.

The notation suggests that the pricing relationship (3) is possibly time-varying – beyond the already captured time-variation in volatility. Time-variation can either occur as a level shift (a time-varying constant), as a slope shift (tracking error), or as both. We can specify several hypotheses to test the assumption of time-variation. Our benchmark scenario is a time-invariant relationship. If we reject the benchmark, we distinguish a further two possibilities. The relationship is either continuously time-dependent, or it is occasionally (discretely) time-dependent. The previous section has alluded to abundant empirical evidence of time variation. The models used to capture this time-variability are typically based on the

[1] Or, alternatively, the minimum variance hedge ratio (MVHR).

assumption of continuous time-dependency (and not surprisingly – given the predominance of perfect hedge applications – focus on the intertemporal variability in the slope parameter).

The Pesaran and Timmermann (2002) ROC procedure is similar to Brown, Durbin and Evans' (1975) Cusum of Squares test, except that the order of the observations is reversed in time (from the most recent observation T, to past observation τ):

$$\widetilde{Y}'_{T,\tau} = \left(y_T, y_{T-1}, \ldots, y_{\tau+1}, y_\tau\right) \qquad \widetilde{Z}'_{T,\tau} = \left(z_T, z_{T-1}, \ldots, z_{\tau+1}, z_\tau\right)$$

which allows us to estimate recursively (in reverse order) by OLS

$$\hat{\theta}_\tau = \left(\widetilde{Z}'_{T,\tau}\widetilde{Z}_{T,\tau}\right)^{-1}\widetilde{Z}'_{T,\tau}\widetilde{Y}_{T,\tau} \qquad \tau = T-2,\ldots,1 \tag{4}$$

Now define the one-step "ahead" standardized residuals

$$\hat{\varepsilon}_\tau = \frac{\left(y_\tau - \hat{\theta}'_{\tau+1}z_\tau\right)}{\sqrt{1 + z'_\tau\left(\widetilde{Z}'_{T,\tau+1}\widetilde{Z}_{T,\tau+1}\right)^{-1}z_\tau}} \qquad \tau = T-3,\ldots,1 \tag{5}$$

compute the cusum-square quantities

$$s_{\tau,T} = \frac{\sum_{j=T-\tau}^{T-3}\hat{\varepsilon}_j^2}{\sum_{j=1}^{T-3}\hat{\varepsilon}_j^2} \qquad \tau = T-3,\ldots,1 \tag{6}$$

and the cusum-square statistics

$$\zeta_{\tau,T} = \left| s_{\tau.T} - \left(\frac{T-(\tau+2)}{T-2}\right)\right| \qquad \tau = T-3,\ldots,1 \tag{7}$$

To compare this test statistic with an appropriate critical value, we resort to Edgerton and Wells (1994). They show that the standard results in Durbin (1969) are not sufficient for large sample sizes (>100) and provide the following asymptotic approximation to the critical values.

$$c_{T,\alpha} = \frac{a_{1(\alpha)}}{T^{\frac{1}{2}}} + \frac{a_{2(\alpha)}}{T} + \frac{a_{3(\alpha)}}{T^{\frac{3}{2}}} \tag{8}$$

where $a_{1(\alpha)}$, $a_{2(\alpha)}$, $a_{3(\alpha)}$ are given for different significance levels, α (see Edgerton and Wells, 1994, p.360).

Where $\zeta_{\tau,T}$ exceeds the relevant critical value $c_{T,\alpha}$, we label $\tau=\tau^*$ as a structural break and condition the forecast hedge ratio on the sample $t=\tau^*+1,\ldots,T$. Given this conditionally updated hedge ratio forecast, we can then estimate the out-of-sample performance of the hedge over the next x time periods (i.e., days), by measuring the hedging error (HE) as

$$HE_{T+i} = R_{P,T+i} - \hat{B}_T\left(\tau^*\right)R_{F,T+i} \qquad i = 1,\ldots,x \tag{9}$$

where the hedger's objective is to minimize the variance of *HE*. The hedger will judge the performance of this breakpoint-optimized hedging scheme against the following three standard alternatives. The expanding sample estimator

$$\hat{\theta}_T\left(t=1,\ldots,T\right)=\left(\widetilde{Z}'_{T,1}\widetilde{Z}_{T,1}\right)^{-1}\widetilde{Z}'_{T,1}\widetilde{Y}_{T,1} \tag{10}$$

which assumes intertemporally constant hedge ratios and the availability of additional information over time improves the efficiency of the long-term optimal hedge ratio. The rolling sample estimator

$$\hat{\theta}_T\left(t=T-\lambda,\ldots,T\right)=\left(\widetilde{Z}'_{T,T-\lambda}\widetilde{Z}_{T,T-\lambda}\right)^{-1}\widetilde{Z}'_{T,T-\lambda}\widetilde{Y}_{T,T-\lambda} \tag{11}$$

which assumes a continuously (smoothly) changing time-varying hedge ratio and the intertemporal variability is captured by estimating the hedge ratio over a (relatively) short sample of λ days. A well known shortcoming of the rolling estimator is its equal weighting of old and recent past observations. Therefore, we also consider the exponentially weighted least squares estimator where the weights w decline exponentially the further they are back in time, by choosing an appropriate parameter ω

$$\begin{aligned}&\hat{\theta}_T\left(t=1,\ldots,T;w\right)=\left(\left(\widetilde{Z}_{T,1}w'\right)'\widetilde{Z}_{T,1}w'\right)^{-1}\left(\widetilde{Z}_{T,1}w'\right)'\widetilde{Y}_{T,1}\\&w_t=\omega^t\left(1-\omega\right)\end{aligned} \tag{12}$$

which is a straightforward exercise.

Before we implement our break point identification methodology, we revisit the cost-of-carry index futures pricing relationship.

$$F^*_{t,T^\times}=P_t e^{(r-q)\left(T^\times-t\right)} \tag{13}$$

where F^* is the theoretical futures price, P is the unhedged index portfolio "price", r is an appropriate risk free rate of return, q is the dividend yield of the stocks in the index, and $T^\times$ is the maturity date of the futures contract. Due to convergence between the spot and futures prices towards maturity, the basis will go to zero, and the hedge ratio should go to one. This causes the time dimension effect, see e.g. Castelino (1992), where the OHR is low for hedges lifted far from futures contract maturity and increases monotonically towards maturity. The cost-of-carry relation implies a cointegration relationship between the futures and spot price, appropriately adjusted by the time-proportional basis (or cost-of-carry), see Brenner and Kroner (1995). The time invariant hedging model fails to account for the 'time dimension.' We isolate this predictable dynamic variation from the random variation in the hedge ratio. We can either use the cost-of-carry relation in (13) as a cointegrating relation and specify an error correction variable in Z_t accordingly. Alternatively, we can use theoretical futures returns, $R_{F^*,t} = \ln\left(F^*_{t,T} / F^*_{t-1,T}\right)$, as the dependent variable Y_t. We choose the latter.

The implementation of our methodology is now summarized as follows. First, choose a "training" sample, e.g., the first year of daily observations on futures and portfolio returns. Second, pre-whiten the unhedged portfolio and futures returns. There is some evidence that daily stock index returns display significant and persistent autocorrelation in both levels and in volatility. Hence, we pre-whiten the raw returns with an ARMA(1,1)-GARCH(1,1) filter. The standardized returns might still display excess kurtosis. To further normalize the series, we therefore also apply an extremal filter. Once we are satisfied with the standardized returns, we then proceed in the third stage by estimating the most recent breakpoint with the ROC procedure. If we detect evidence of a structural break in the training sample, we then estimate the dynamic hedge ratio in the fourth stage over the optimized estimation window. If we do not find evidence of a structural break, we simply use the complete training sample as our estimation window for the initial hedge ratio. In the fifth stage we move x days forward, the updating/rebalancing frequency, and compute the out-of-sample hedging error from T+1 to T+x, according to (9). Then, we return to stage 2 and reiterate the procedure until we arrive at the most recent observation. Finally, in stage six, we compute the variance of the accumulated series of hedging errors and express this as the hedged risk reduction, or hedging performance HP

$$HP = \left(1 - \frac{\sigma^2_{R_{HP}}}{\sigma^2_{R_P}}\right) \tag{14}$$

The implementation is identical for the competing models (expanding, rolling, EWLS), except that stage 3 is now ignored. Finally, we compare the HPs of the competing models.

To illustrate the theoretical performance of our methodology, we conduct the following experiment. We generate 2,100 (correlated) futures and unhedged portfolio returns

$$\begin{aligned} R_{F,t} &= \varepsilon_{F,t}\sigma_{F,t} \\ R_{P,t} &= \beta_{P,t}R_{F,t} + \kappa\sqrt{1-\beta^2_{P,t}}\,\varepsilon_{P,t}\sigma_{P,t} \end{aligned} \tag{15}$$

by drawing independent innovations ε_F , ε_P from standard normal distributions. We select the following three hedging regimes

$$\beta_{P,t} = \begin{matrix} 0.8 & & t = 1,...,500 \\ 0.7 & \text{for} & t = 501,...,1500 \\ 0.9 & & t = 1501,...,2100 \end{matrix}$$

and choose a training sample of size 250, a rebalancing frequency x of 5 days, a rolling window size λ of 30 days, and an EWLS weight parameter ω of 0.99. We assess the impact of noise-to-signal variations by scaling the variance of the noise in the portfolio returns by a factor κ (taking values of 0.05, 0.1 and 0.5). To illustrate the impact of the standardization in stage 2, we also experiment with GARCH-type conditional variance in futures and portfolio returns

$$\begin{aligned} \sigma_{F,t}^2 &= \alpha_{0,F} + \alpha_{1,F}\varepsilon_{F,t-1}^2 + \beta_{1,F}\sigma_{F,t-1}^2 \\ \sigma_{P,t}^2 &= \alpha_{0,P} + \alpha_{1,P}\varepsilon_{P,t-1}^2 + \beta_{1,P}\sigma_{P,t-1}^2 \end{aligned} \tag{16}$$

We choose

$$\alpha_{0,F} = \alpha_{0,P} = 0.1, \quad \alpha_{1,F} = \alpha_{1,P} = 0.3, \quad \beta_{1,F} = \beta_{1,P} = 0.6$$

The results are summarized in Table I. For comparative purposes, we also included a static (buy and hold, B&H) hedging strategy where the hedge ratio is determined once on the basis of the training sample and kept in place for the full sample period (t = 251,..2100). Low et al. (2002) find that a static hedging strategy is less sensitive to estimation and model error and therefore outperforms their dynamic hedging strategies. Performance is measured by the percentage reduction in risk (as measured by the variance of returns) of the hedged portfolio relative to the unhedged portfolio.

INSERT TABLE I

The impact of noise is immediately obvious when comparing the three panels for different κ values. When κ increases, the correlation between portfolio and futures returns declines, effectively weakening the cross-hedge. The hedging error increases, hedging performance declines, and the buy and hold strategy looks increasingly promising.

INSERT FIGURE 1

In addition to overall performance, it is also worthwhile to illustrate the structural break detection ability of the hedging strategies. Figure 1, top panel, shows the ROC-optimal estimation window size hedge ratios for three different scenarios (note that breaks were 'inserted' at observations 500 and 1500). Figure 1, bottom panel, shows the matching ROC-optimal hedge ratios for the same three scenarios.

4 Empirical Results

We apply our structural break methodology to a sample of Hong Kong index portfolios over a period from the 3rd of January 1994 to the 29th of July 2003. We consider the 33-stock

Hang Seng Index (HSI) and its companion derivatives contract, the Hang Seng Index Futures (HSIF), as well as the 20-stock Hang Seng Commerce and Industry Index (HSCI), and the 4-stock Hang Seng Finance Index (HSF). A total of 2,247 daily closing prices were obtained from DataStream for each of the Hang Seng indices. The futures contracts are for the nearest maturity with rollover to the next nearest maturity on the last business day prior to the maturity month. Rollover returns were excluded from the sample. After eliminating a few data errors, we are left with 2,087 daily returns.

The HSI comprises the 33 largest stocks by market capitalisation, and is value weighted by the stocks' market capitalisation. The HSCI is a market-capitalisation weighted index of those constituents of the HSI active in the Commerce and Industry sector. Similarly, the HSF is a market-capitalisation weighted index of those constituents of the HSI active in the Finance sector. The constituents of the HSI, and hence also of the HSCI and HSF, are selected by the Stock Exchange of Hong Kong. Their inclusion is subject to review (and hence exclusion) at a quarterly frequency. On the final day of our sample (29 July 2003), the HSCI comprised 20 individual stocks, while the HSF comprised only 4 stocks. In/Exclusions to each index over the course of our sample are reported in Appendix 1.

The HSI is the underlying index for the Hang Seng Index Futures contract, that is also traded within the Hong Kong Exchanges network on the Exchange's Automated Trading System. A HSIF contract matures on the second last business day of each calendar month, with settlement occurring on the final business day of that month. The closing prices used in this paper are those recorded for the nearby futures contract. The HSIF contract is an actively traded security with deep liquidity in the nearby contract. The contract multiplier for the HSIF is HK$50 per index point. There are no matching futures contracts for the HSCI and HSF indices. The HSIF is the obvious candidate for cross-hedging both sub-indices.

Transaction costs for the HSIF have recently undergone considerable change, with the commission fee becoming freely negotiable, as of 30 May 2003[2]. However, as this change only came into effect for the final two months of the sample, the relevant transaction costs are those levied prior to 30 May 2003. These trading costs are an exchange fee of HK$10 per contract per side, an SFC levy of HK$1 per contract per side, an Investor Compensation Levy of HK$0.50 per contract per side, and a minimum commission of HK$60. This gives a total transaction cost for trading one HSIF contract equal to HK$71.50.

[2] We only consider futures transaction costs, as this paper focuses on the activities of hedgers.

INSERT FIGURE 2 AND TABLE II

The index levels of the HSI, HSCI, HSF and HSIF over the sample period are displayed in Figure 2 and descriptive statistics for the full sample are given in Table II. The HSCI and HSI indices track each other reasonably closely, the gap between the two distinctly widening towards the end of our sample period. The clear outlier is of course the HSF index with the most notable divergence occurring from 2000 onwards. These patterns suggest that the HSIF would be a reasonable instrument to cross-hedge the HSCI, but much less so for the HSF. Table II highlights the atypical behaviour of the HSF return series. The HSF series has the lowest standard deviation – although not significantly less than the HSI – and the highest (only positive) mean. It is also the most kurtotic series. All four series share significant excess kurtosis which, not surprisingly leads to a rejection of normality according to the Jarque Bera test. Note in particular the enormous values of the maximum/minimum returns. Whereas the mean and standard deviation are annualized figures, the extremes are daily figures. This equates to 10 standard deviation events! The ARCH test (null of no ARCH) is significantly rejected for all four series, as is the Breusch-Godfrey serial correlation test (null of no serial correlation).

INSERT FIGURE 3

To illustrate the volatility clusters, we estimate a GARCH(1,1) specification for the full sample of HSI returns and plot the conditional standard deviations in Figure 3. The Asian crisis and its aftermath (in particular the intervention by the Hong Kong Authority) is very evident. There is also some evidence of a (short-lived) volatility increase post-Y2K. The most recent volatility surge (in our sample period) is related to the September 11, 2001 events. The volatility patterns are rather similar for the other (HSCI, HSF and HSIF) series. There is strong evidence of volatility clustering which suggests that the dynamic hedging methodologies might outperform the static hedging methodology.

We start our hedging exercise on the 21st of February 1995 (hedging decision time T), allowing for a 'training' sample of 250 daily returns (t=1,…T). The hedge will be updated on a weekly basis, so the initial hedging decision at T will remain in place until T+5. The time line looks as follows:

1 $\longleftarrow \tau \longrightarrow$ T T+5 …… N

The window size [τ,T] will be determined for the different optimization methods. Parameter τ is fixed at observation 1 for the expanding window, while for the rolling methodology, we choose a fixed window size of 30 business days, $\tau = T$-29 . We also choose a weighting parameter for the EWLS scheme that "matches" the initial GARCH(1,1) pattern estimated over the training sample. Hence, a parameter $\omega = 0.99$ in (12) generates a similar half-life of shocks as the estimated GARCH parameters for the portfolio return series, i.e., a short effective estimation window with a large weight on the most recent observations.

We standardise the portfolio and futures return series by estimating an ARMA(1,1)-GARCH(1,1) specification over the full history available at optimization time $T+\ i$ (i = 0,5,10,….,N), i.e., over a window [1,T+i]. A sample of the recursive standardisation parameters is given in Figure 4.

INSERT FIGURE 4

For the ARMA(1,1) parameters, the first order autoregressive parameter – top panel – is significantly positive for the portfolio returns (converging to about 0.04), but not for the futures returns. This feature is rather typical for stock index portfolio returns. The first order moving average parameter was never significantly different from zero. The GARCH parameters were all significant. The middle panel illustrates the recursive empirical estimates for the α_1-parameter in (16) for the portfolio and futures returns, respectively. Its value varies between 0.06 and 0.1 with a few significant jumps in March 1996, October 1997, and September 2001. These events are also reflected in the GARCH-implied recursive unconditional standard deviation estimates in the bottom panel. The three events cause obvious jumps in the level of volatility, but perhaps more imporant is the observation of persistently higher volatility after October 1997. A final observation on Figure 4 is the higher level of volatility in futures returns (the gap widening after 1999), and the smoother evolution of the immediate impact α_1-parameter for futures returns.

Having standardised the returns, we then test for structural breaks $\hat{\tau}$ to optimize the estimation window [$\hat{\tau}$,T+i] following the ROC procedure.

INSERT FIGURES 5 AND 6

Figures 5 and 6 illustrate our findings for the HSI(coc-adjusted returns) and HSF(coc-adjusted returns) vis-à-vis the HSIF returns. For the perfect hedge (HSI), we find limited evidence of structural breaks. The only clear candidate ($\hat{\tau}$ = 22 March 2000) is detected on the optimization date T = 8 May 2001. The estimation window therefore increases reasonably smoothly to almost 1,000 observations until that break date when it drops back to about 250 observations. For the imperfect hedge (HSF), there seems to be more distinct evidence of structural breaks identified as 12 January 1995, 6 August 1996, 7 March 2000, and 5 November 2001. Note that none of these structural break dates coincide with (or are near) the index composition change dates listed in the Appendix. The optimized window sizes (T- $\hat{\tau}$) are given in Figure 6.

Based on these optimized window sizes, we then compute the ROC-optimized hedge ratios and compare those with alternative methodologies. The dynamic hedge ratios are illustrated in Figure 7 for the imperfect hedge, HSF-HSIF (coc-adjusted returns). The static buy&hold hedge ratio is 0.85. The expanding window hedge ratio is fairly stable, dropping from 0.85 to 0.79. The ROC hedge ratio is much more variable with a particularly noteworthy drop in January 2000 to 0.46, and remains persistently below the expanding hedge ratio after that date. The rolling hedge ratio is, hardly surprising, very volatile with enormous short-lived swings. The EWLS hedge ratio behaves like a moving average of the rolling hedge ratio.

INSERT FIGURE 7

Our final step is to incorporate transaction costs in the dynamic hedging optimization. The basic idea is straightforward: the investor will tradeoff the (non-linear) benefits from rebalancing the hedge against the (linear) costs involved. This suggests the existence of a threshold deviation between the existing hedge ratio and the optimized hedge ratio. The implementation of this tradeoff depends on the hedger's attitude to risk. We follow Masters (2003), who defines a rebalancing trigger point TH as

$$TH_T = \frac{2\gamma C}{\sigma_P^2 + \hat{B}_T^2 \sigma_F^2 - 2\hat{B}_T \sigma_{PF}} \tag{17}$$

where γ is the risk tolerance parameter (expressed as a percentage), C is the transaction cost (expressed as a percentage of the futures contract value), and the denominator is the variance of the hedged portfolio return. Hence, the benefit from hedging is a positive function of the hedger's risk aversion (the inverse of γ) and of the reduction in portfolio variance. Lafuente and Novales (2003) use a similar mean-variance utility setup, but express the benefits from hedging in terms of utility changes.

Whenever the optimal hedge ratio adjustment, $\Delta\hat{B}_T = \left|B_{T-i} - \hat{B}_T\right| / \hat{B}_T$, exceeds the threshold *TH* at rebalancing date *T*, the futures position is adjusted to $\hat{B}_T$. Otherwise, the existing hedge position is maintained at B_{T-i} until the next rebalancing date *T*+*i*. Note that we abstract from the opportunity for the hedger to realign the hedge whenever a rollover occurs into the next nearest futures maturity contract. The transaction cost adjusted dynamic hedge ratios are given in the bottom panel of Figure 7. Given our choice of parameters γ (=0.05) and C (=0.02), we find that little rebalancing adjustment remains in the hedge ratio. Of course, these parameter values were chosen in a rather ad hoc fashion.

Another noteworthy feature of the optimization exercise is illustrated in Figure 8, which compares the precision of the hedge ratio estimates for the different optimization procedures.

INSERT FIGURE 8

As we would expect, the precision of the expanding window hedge ratio dominates the alternatives. The ROC window hedge ratio is less precise at times when the window is 'reset' due to an identified structural break, but then improves in efficiency when the window is subsequently expanded. The rolling windows are particularly inefficient with standard errors more than three times as large, and while the EWLS hedge ratios are generally more efficient than the ROC hedge ratios, they are less efficient than the expanding window hedge ratios.

Which leaves us with the question, is dynamic hedging worthwhile? Or, to quote the conclusion in Brooks, Burke and Persand (2003, p.733): "*Thus, whilst the benefit from engaging in hedging is clear, it does not matter which package you use to calculate the OHRs and you are just as well not to bother with MGARCH models at all but to stick to OLS!*"

INSERT FIGURE 9

Figure 9 tracks the cumulative out-of-sample performance of the ROC method and compares this against the Buy&Hold performance. Performance is measure by the hedged portfolio standard deviation of returns. For the perfect hedge (HSI-coc), the static Buy&Hold strategy clearly outperforms the ROC dynamic alternative, with dynamic performance deteriorating after January 1998. For the cross hedge (HSF-coc), the dynamic ROC hedge outperforms the static Buy&Hold alternative, although the benefits only occur after January 2000. This coincides with the dating of the only significant rebalance in the hedge after adjusting for transaction costs. To further identify these benefits, we also computed the daily performance difference (ROC – B&H), which is given in the bottom panel. All the dynamic hedging gains seem to be clustered around January-February 2000, while the static hedge was clearly outperforming the dynamic hedge in the aftermath of October 1997.

INSERT TABLE III

Table III formalizes the hedging performance results. Performance is expressed by the standard deviation of the hedged portfolio returns and by the hedging performance measure, defined in (14). Entries in bold print indicate the 'best' performer. For the perfect HSI-HSIF hedge, the static Buy&Hold hedging strategy outperforms the dynamic alternatives. For the imperfect HSCI-HSIF hedge, the dynamic EWLS hedging strategy very marginally outperforms the dynamic and static alternatives. For the imperfect HSF-HSIF hedge, the dynamic ROC model hedge outperforms the dynamic and static alternatives. The latter outperformance is still marginal, but larger than for the other imperfect hedge. The results are consistent whether we use raw returns or cost-of-carry adjusted returns. The comparative hedging performance changes somewhat when hedge ratios are only adjusted when the change exceeds the threshold in (17). For the (near)perfect hedges, any distinction disappears, and static buy&hold hedging is preferred. For the cross hedge (HSF coc), however, the ROC model performs better than before, while the dynamic alternatives fare worse.

5 Conclusion

A clearly outperforming dynamic hedging strategy remains elusive as ever. Despite overwhelming evidence of time-varying behaviour in the variance-covariance matrix of portfolio and futures returns, it is still diffult to capture this in a 'profitable' manner for out-of-sample hedging. This paper proposes a comprehensive, yet simple, dynamic hedging model. Through careful selection of the estimation sample size, the ROC model strikes a balance between the efficiency gains from expanding windows and the precision gains from rolling or EWLS windows. Selection occurs on the basis of running recursive regressions on reverse ordered observations and computing CUSUM-squared tests.

We compare the performance of this ROC hedging model against common dynamic hedging alternatives including the rolling window and EWLS models. We also compare ROC performance against the static buy-and-hold strategy. To enhance the practical value of the exercise, we compare on out-of-sample hedging effectiveness, and also operationalise transaction cost restrictions to excessive rebalancing. With a simulation experiment, we highlight the possible benefits obtained by the ROC model. These benefits do, however, disappear when the noise to signal ratio (that is, the ‘perfection’) of the hedge relationship dimishes. That is unfortunately somewhat of a “catch-22” since we also know that the dynamic models are only really useful for cross hedge scenarios. Our empirical results for the Hang Seng stock indices verify this result. For a (near)perfect hedge scenario (HSI and HSCI), there is very little evidence of any dynamic strategy significantly outperforming the simple buy-and-hold strategy. For a genuine cross hedge scenario (HSF), there is limited evidence of the ROC model outperforming the dynamic alternatives as well as the static alternative. The gain, however, is small.

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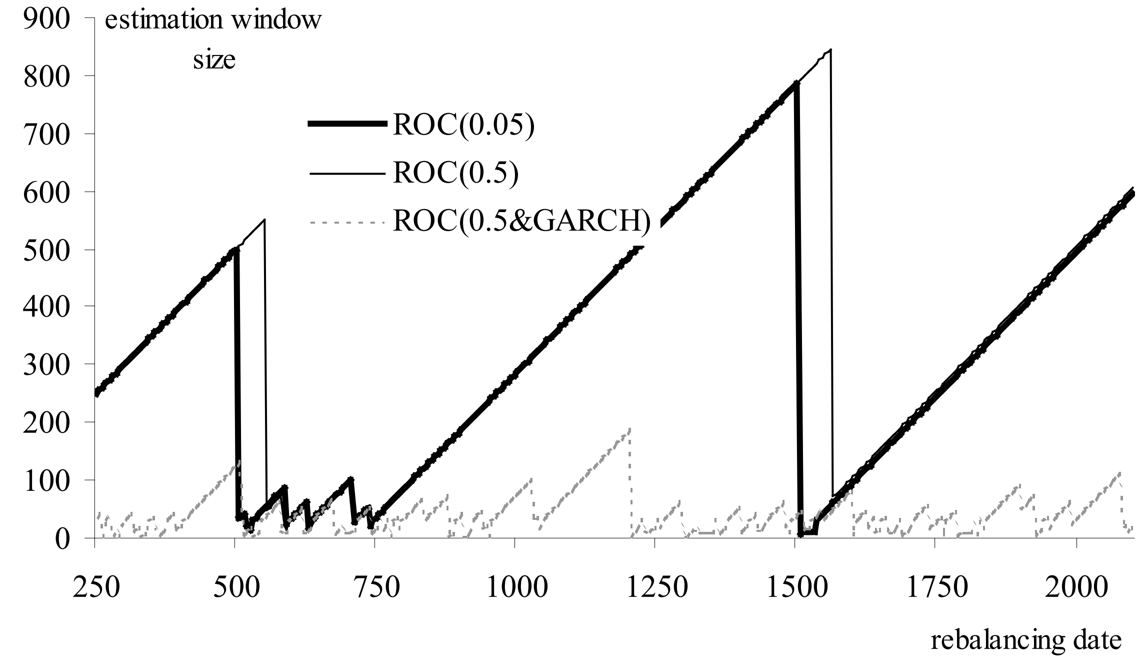
Figure 1: ROC Optimized Estimation Windows and Hedge Ratios



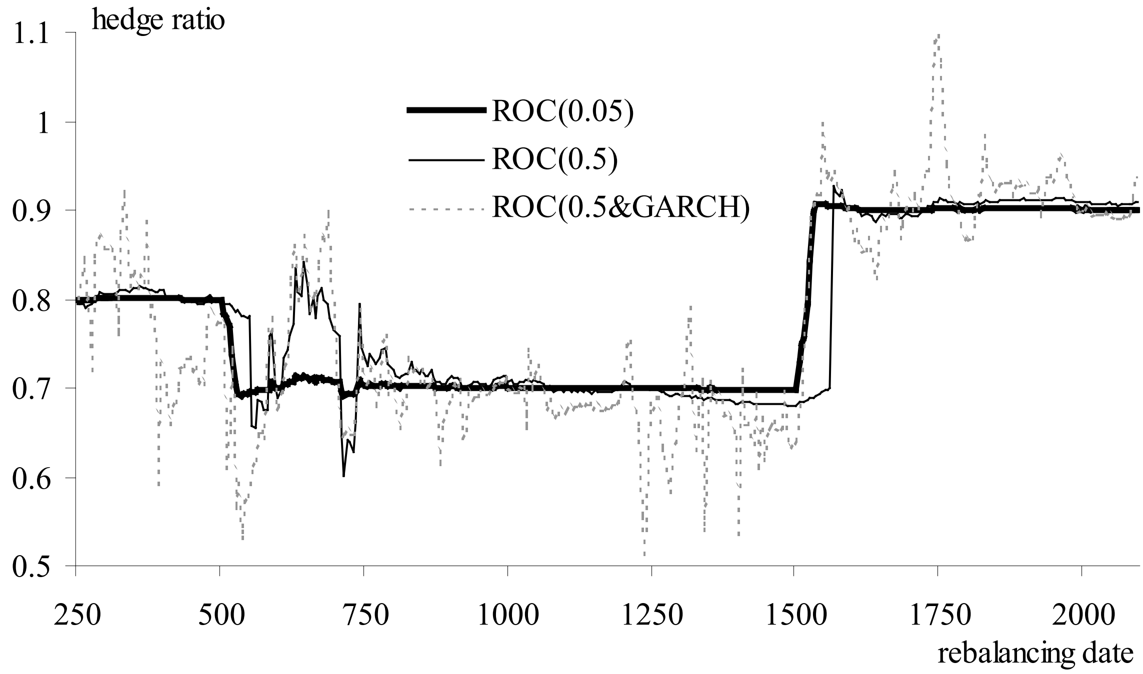
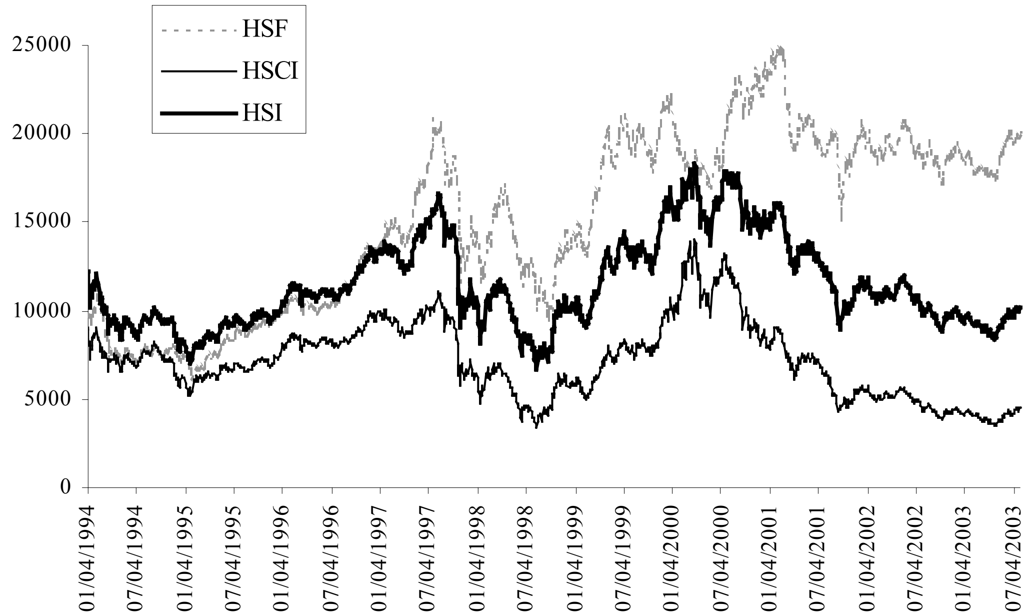
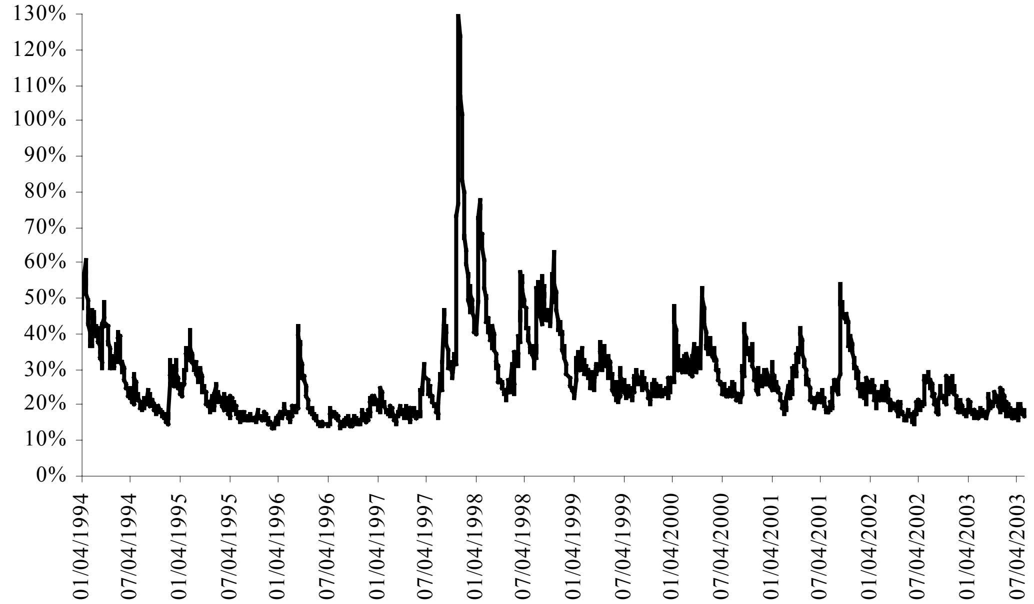


Figure 2: Closing Prices Data plot for Hang Seng Indices, 1994-2003



HSF = Hang Seng Finance Index, HSCI = Hang Seng Commerce & Industry Index, HSI = Hang Seng Index.

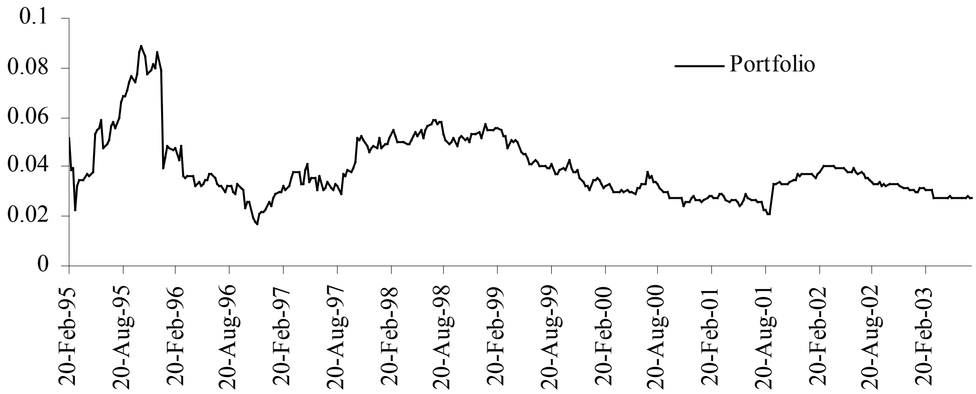
Figure 3: Conditional Standard Deviation for the Hang Seng Index, 1994-2003



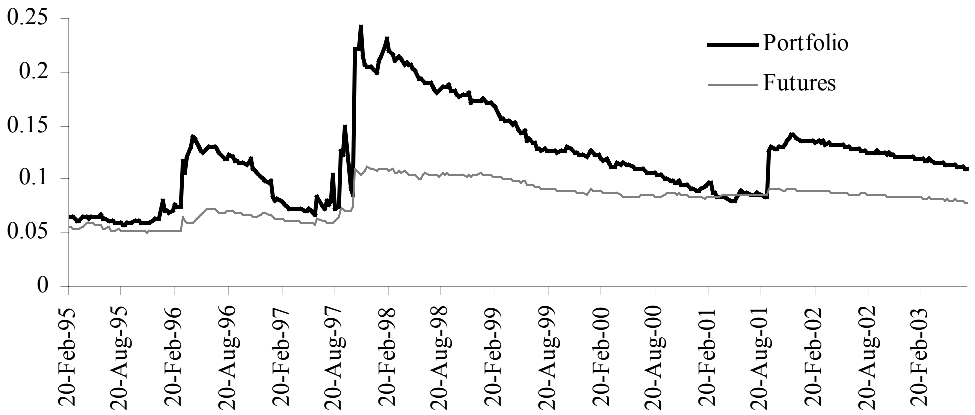
Conditional standard deviations obtained from fitting a GARCH(1,1) specification to HSI returns.

Figure 4: Recursive Parameter Estimates

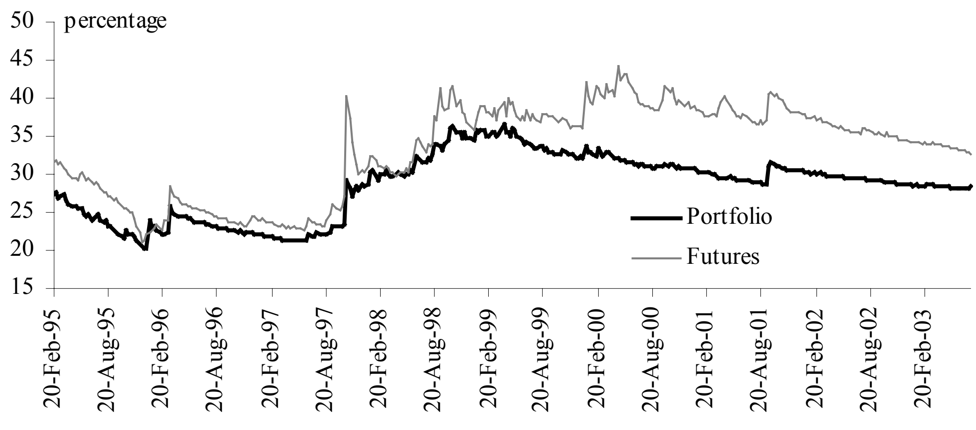
AR(1) parameter



GARCH α_1 parameter



Unconditional Standard Deviation



Portfolio refers to the HSI return series, while Futures refers to the HSIF return series. The recursive unconditional standard deviations are computed as

$$\sigma_{T+i} = \sqrt{\frac{\alpha_{0.}}{1-\alpha_{1.}-\beta_{1.}}}$$

Figure 5: Structural Break Identification

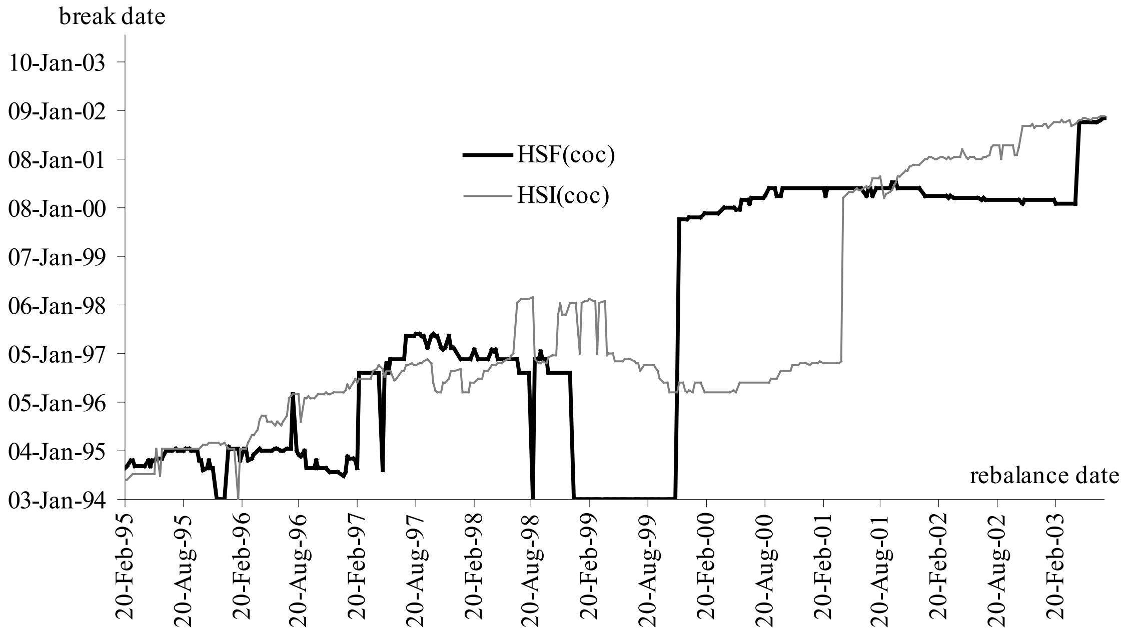


Figure 6: Optimized Window Size

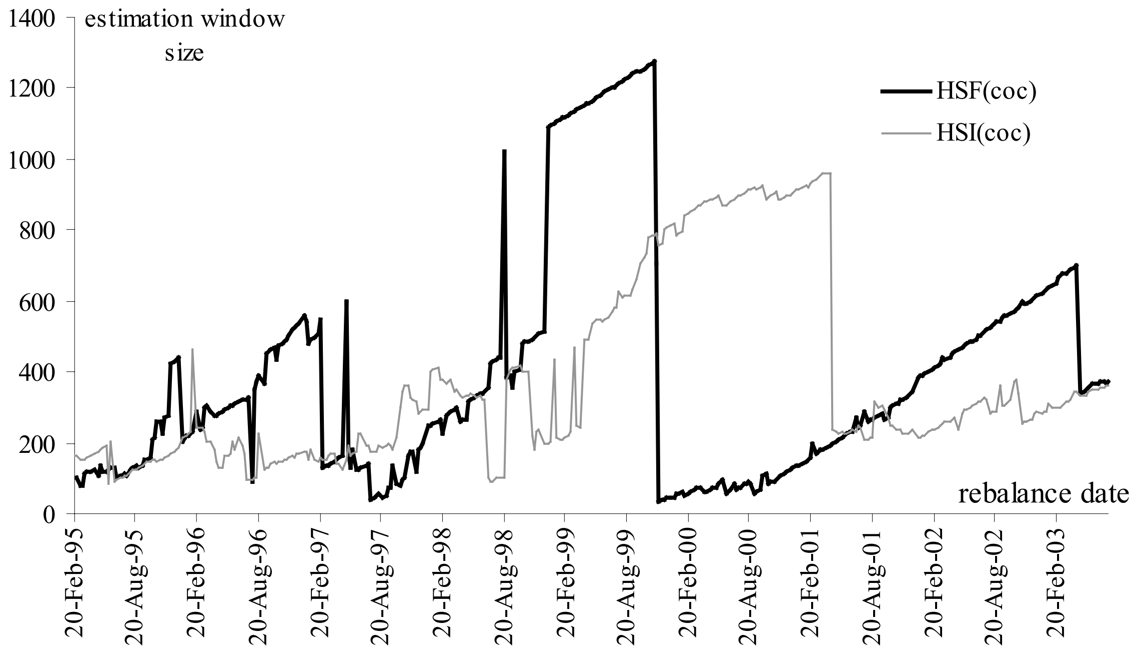


Figure 7: Hedge Ratios

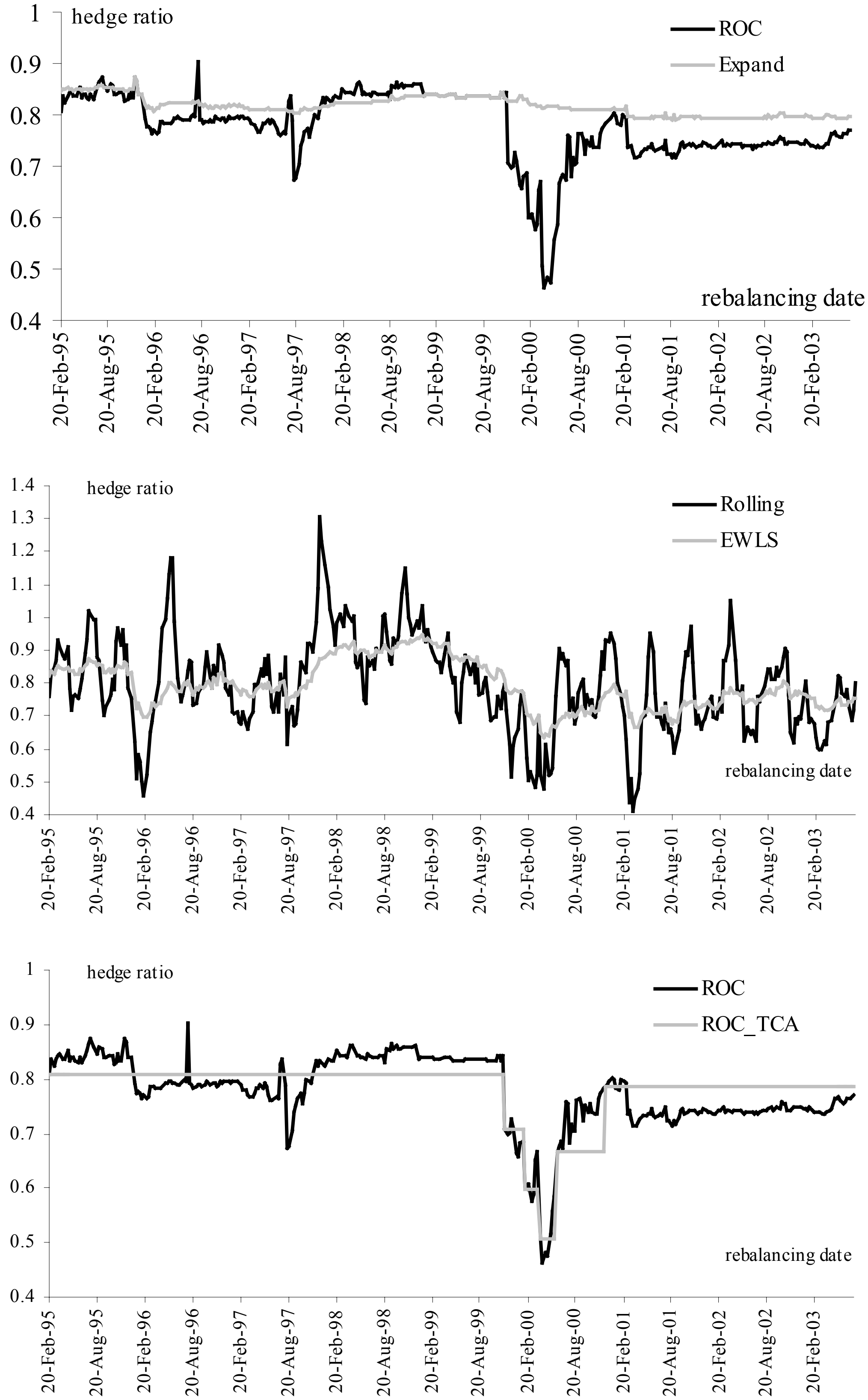
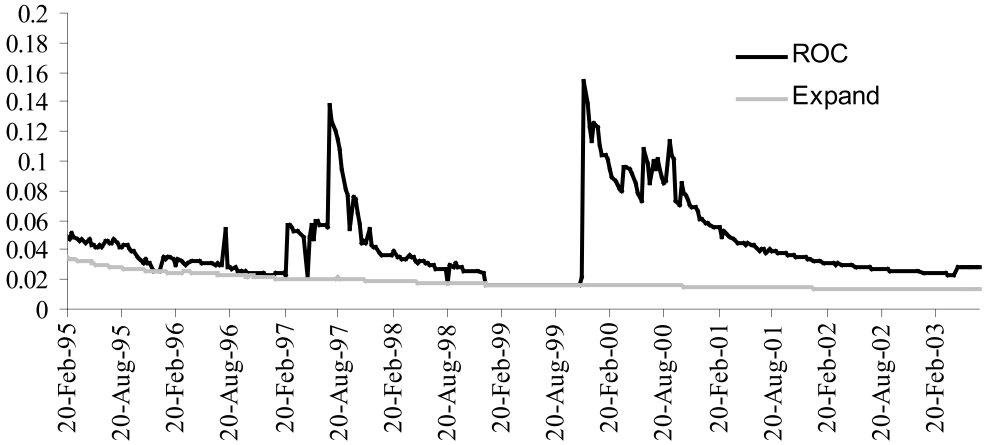


Figure 8: Standard Errors of Hedge Ratios



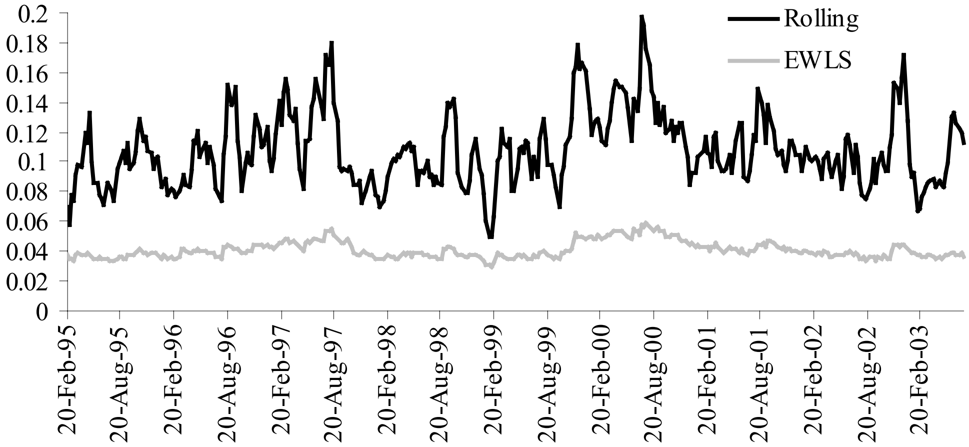
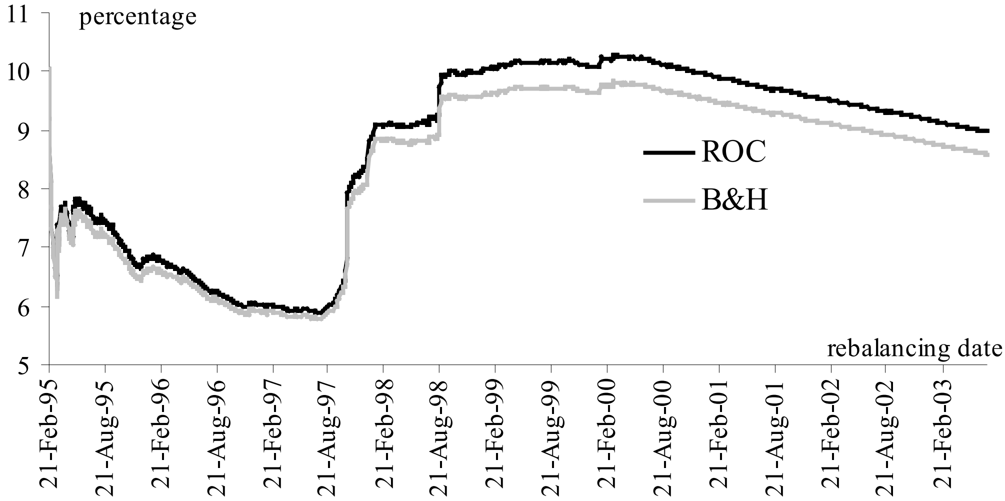
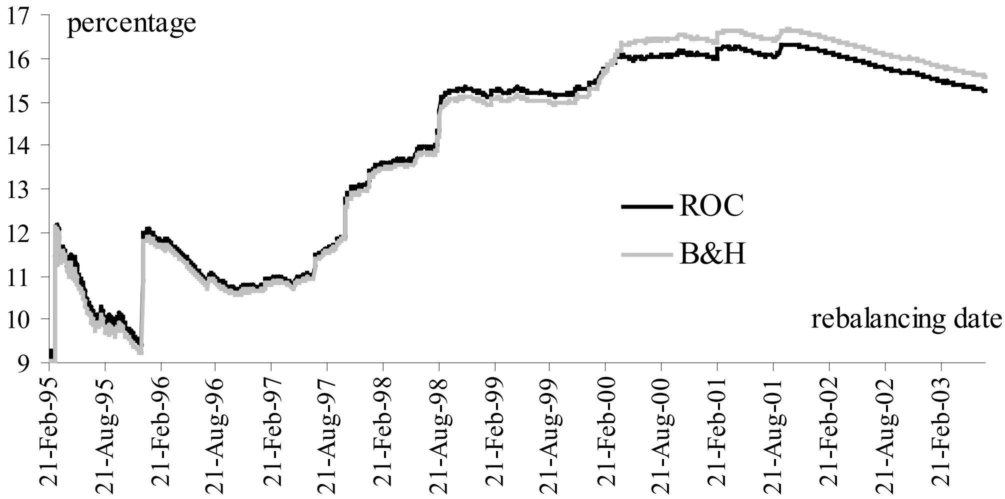


Figure 9: Out-of-sample Performance

(HSI-coc)



(HSF-coc)



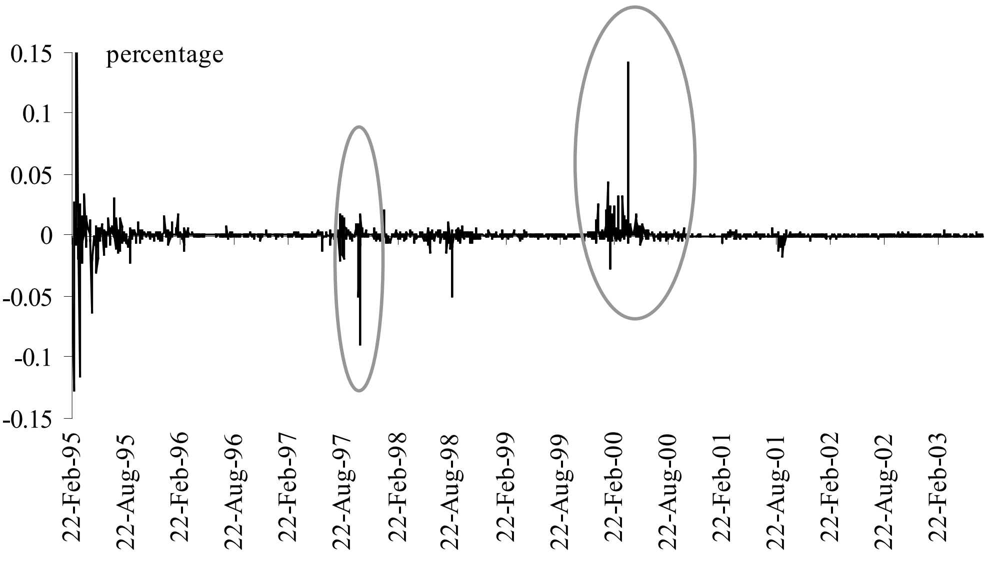


TABLE I

Out-of-Sample Hedging Performance

	Sample Size Methodology				
	ROC	*EXPAND*	*ROLLING*	*EWLS*	*B&H*
			$\kappa = 0.05$		
No GARCH	95.29	87.51	95.26	93.71	87.33
GARCH	95.04	83.54	94.99	92.62	83.45
GARCH_STD	71.94	69.15	71.99	71.61	69.39
			$\kappa = 0.1$		
No GARCH	91.52	85.66	91.48	90.61	85.51
GARCH	91.13	82.47	91.02	89.86	82.44
GARCH_STD	71.92	69.39	72.02	71.67	69.81
			$\kappa = 0.5$		
No GARCH	62.25	61.02	61.99	62.45	61.01
GARCH	60.18	60.13	59.69	61.20	60.23
GARCH_STD	58.40	59.59	58.05	59.80	58.53

Note: Hedging performance is measured by the percentage reduction in risk (the variance of returns) of the hedged portfolio relative to the unhedged portfolio risk. The No GARCH scenario is based on i.i.d. innovations in portfolio and futures returns. The GARCH scenario is based on GARCH innovations in portfolio and futures returns. The GARCH_STD scenario is based on GARCH-standardized innovations in portfolio and futures returns. The value for κ relates to the strength of the correlation between portfolio and futures returns (with increasing κ indicating a decrease in correlation).

TABLE II

Full Sample Descriptive Statistics

	HSI	HSCI	HSF	HSIF
Mean	-5.82%	-11.68%	4.25%	-9.23%
Standard Deviation	29.08%	34.68%	28.61%	33.41%
Skewness	-0.016	0.076*	-0.207*	0.332*
Excess Kurtosis	8.473*	6.032*	12.123*	10.244*
Minimum	-14.73%	-13.40%	-17.69%	-16.09%
Maximum	17.25%	18.42%	18.00%	22.98%
Jarque Bera	6212.16*	3149.74*	12734.67*	9119.83*
ARCH-LM(5)	447.40*	283.25*	614.70*	417.40*
BG-LM(12)	20.38*	33.13*	26.50*	27.83*

Note: Mean and Standard Deviation of returns are annualized figures. Minimum and Maximum returns are daily figures. Significance at the 5% confidence level is indicated with an asterisk. For Skewness and Excess Kurtosis, the null hypothesis is a value of zero. For the Jarque Bera test, the null hypothesis is normality based on the skewness and excess kurtosis measures. For the ARCH LM(5) test, the null hypothesis is no serial correlation up to lag 5 in the squared returns. For the Breusch-Godfrey LM(12) test, the null hypothesis is no serial correlation up to lag 12 in the returns.

TABLE III

Dynamic Hedging Performance

	Dynamic Optimization Method					
	Unhedged	*Buy&Hold*	*Expanding*	*Rolling*	*EWLS*	*ROC*
Hedge Ratios Estimated for Raw Returns						
HIS	28.881	**8.585** **(0.912)**	8.831 (0.907)	9.009 (0.903)	8.987 (0.903)	8.954 (0.904)
HSCI	34.988	14.676 (0.824)	14.740 (0.823)	14.790 (0.821)	**14.587** **(0.826)**	14.708 (0.823)
HSF	28.492	15.581 (0.701)	15.674 (0.697)	15.951 (0.687)	15.569 (0.701)	**15.253** **(0.713)**
Hedge Ratios Estimated for Cost-of-Carry Adjusted Returns						
HSI(coc)	28.726	**8.590** **(0.911)**	8.860 (0.905)	9.039 (0.901)	9.008 (0.902)	8.973 (0.902)
HSCI(coc)	34.835	14.617 (0.824)	14.677 (0.822)	14.775 (0.820)	**14.536** **(0.826)**	14.651 (0.823)
HSF(coc)	28.351	15.581 (0.698)	15.669 (0.695)	15.957 (0.683)	15.562 (0.699)	**15.250** **(0.711)**
Transaction Cost Adjusted Hedge Ratios Estimated for Cost-of-Carry Adjusted Returns						
HSI(coc)	28.726	8.590 (0.911)	8.858 (0.905)	**8.333** **(0.916)**	8.684 (0.909)	8.590 (0.911)
HSCI(coc)	34.835	**14.617** **(0.824)**	14.617 (0.824)	15.145 (0.811)	14.637 (0.823)	14.617 (0.824)
HSF(coc)	28.351	15.581 (0.698)	15.865 (0.687)	16.005 (0.681)	15.672 (0.694)	**15.158** **(0.714)**

Note: The cell entries give the standard deviation of hedged portfolio returns for each of the four dynamic hedge ratio optimization methods, as well as the standard deviation of static buy&hold and unhedged portfolio returns. The numbers between parentheses give the matching hedging performance measures. Hedging performance is measured by the percentage reduction in risk (the variance of returns) of the hedged portfolio relative to the unhedged portfolio risk.

Appendix:
The Hang Seng indices have undergone a number of changes in composition over the period 1994 until 2003. Table A1 lists the inclusion/exclusion dates for each of the three considered indices.

Table A1: Alterations to the Hang Seng indices over the sample period

	HSI		HSCI		HSF	
Date	*No. Of Stocks*	*Change*	*No. Of Stocks*	*Change*	*No. Of Stocks*	*Change*
30th November 1994	33	+4, -4	15	+ 3	3	-
28th February 1995	33	+3, -3	17	+ 2	3	-
30th August 1996	33	+2, -2	17	+1, -1	3	-
31st July 1997	33	+2, -2	17	+2, -2	3	-
27th January 1998	33	+2, -2	17	+2, -2	3	-
6th December 1999	33	+3, -3	16	+2, -3	3	-
2nd August 2000	33	+2, -2	17	+2, -1	3	-
1st June 2001	33	+2, -2	17	+2, -2	3	-
31st July 2001	33	+1, -1	18	+1	3	-
2nd December 2002	33	+2, -2	20	+2	4	+1

Note: The change columns indicate how many stocks were added (+) and/or removed (-) from the index.