INTERTEMPORAL EQUIVALENCE SCALES
Measuring the Life-Cycle Costs of Children

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ABSTRACT

This paper provides a preliminary investigation into the lifetime cost of children upon a household's lifetime wealth. By comparing the lifetime cost function of a household with children compared to the lifetime cost function of a household without children, an intertemporal equivalence scale can be constructed. By allowing the rate of time preference to vary according to demographics, more specifically with the number of children, the demographic effect on intertemporal allocations can be examined. Solving the model as a function of wealth allows the estimation of the rate of time preference and lifetime equivalence scale in a single cross section of data without the need for panel data on expenditures. The model is estimated for Australian data and finds that households with children have significantly higher rates of time preference than those without.

Keywords: Equivalence Scales, Intertemporal Cost Function, Demand Systems

JEL Classification: D1, D9, J1

I. INTRODUCTION

The use of equivalence scales has become common practice in order to make welfare or resource comparisons between households that differ in size and composition. Equivalence scales can be used to assess policy implications or compensation for households with children relative to those without. Using equivalence scales from static demand systems for welfare analysis ignores households' lifetime welfare and the allocation of their expenditure over their lifetime. For example when determining the appropriate level of government benefits for households with children relative to those without, the static analysis ignores that the household with children will eventually become a household without children.

Equivalence scales typically give the 'cost' of children relative to an adult or adult couple in terms of the additional expenditure required to keep the household at the level of welfare it would enjoy without children. Muellbauer (1974) was the first to advocate the estimation of equivalence scales in a utility theoretic framework, through the estimation static demand systems. This procedure has become a popular method of estimating equivalences amongst economists.

While the static analysis of household expenditure can provide evidence of the way household spending patterns respond to different demographics, it can not identify preferences over demographics, without making assumptions about those preferences, see Pollak and Wales (1979), Blackorby and Donaldson (1991) and Blundell and Lewbell (1991). Banks, Blundell and Preston (1994) show that in an intertemporal framework preferences over demographics independent of demands can be identified. This brings us much closer to establishing the true lifetime 'cost' of children on lifetime expenditure.

Pashardes (1991) was the first to explicitly examine the cost of children over the life-cycle and notes that households may reduce current consumption when children are not present saving for when children enter the household. Static comparisons of expenditure between demographically different households will be affected by the how willing and able parents are able to save and borrow for their child raising years. Pashardes terms an equivalence scale estimated in a static framework as an *equivalent expenditure scale* and an *equivalent income scale* as an equivalence scale developed in an intertemporal framework.

Banks, Blundell and Preston (1994) followed with a study on the intertemporal costs of children using pseudo-panel data constructed from the UK's FES from 1969 to 1988. Through simulations from the estimated parameters the authors constructed scales lifetime scales as the difference in total lifetime sum utility of a household with children and without, but found them too high. By adding an arbitrary linear contribution to lifetime based on the number of children Banks, Blundell and Preston were able to estimate the cost of child born when the household head is 26 years old and leaving 18 years later as a proportion of an adult couple over the life-cycle as being about approximately 16%. An additional child born when the head is 28 years old increases the cost to 40% or 20% for each child. A third child born at 30 raises the total cost of having three children to 75% or 25% per child.

By allowing the rate of time preference to vary according to demographics, more specifically with the number of children, the demographic effect on intertemporal allocations can be easily be examined. Coupled with assumptions about the household's expectations of their future demographic profile allows the intertemporal model to be solved for consumption as a function of wealth. This allows the estimation of demographically varying rate of time preference, from consumption and wealth data without the need for panel data on expenditures. The parameters estimates can then be used to construct an intertemporal or lifetime equivalence scale.

The plan of this paper is as follows. The theoretical framework is presented and the estimating equations are derived in Section II. It begins with; i) a quick review of atemporarl equivalence scales, followed by ii) a discussion of intertemporal scales, before iii) where the intertemporal model is specified and solved for the consumption function and the intertemporal equivalence scale. The data and estimation are briefly described in Section III. The results are presented and analysed in Section IV. The paper ends on the concluding note of Section V.

II. THEORETICAL FRAMEWORK

i) Atemporal Equivalence Scales

Traditionally equivalence scales have been specified as the ratio of consumption expenditure of a household with demographic variables, $\mathbf{z}$, to a reference household R with demographic variables, $\mathbf{z}^R$, to achieve the utility of the reference household, u^R. The household cost function, $c(\ )$ of obtaining a certain level of utility, u, given prices $\mathbf{p}$ and demographics $\mathbf{z}$, can be recovered from the estimation of demand systems and used to construct the equivalence scale,

$$m(u^R, \mathbf{p}, \mathbf{z}) = \frac{c(u^R, \mathbf{p}, \mathbf{z})}{c(u^R, \mathbf{p}, \mathbf{z}^R)}. \qquad (1)$$

If demographic variables directly affect utility, $u = f\left(\mathrm{g}(\mathbf{q}, \mathbf{z}), \mathbf{z}\right)$ rather than through its interaction with demands, $\mathbf{q}$, then demand data can only identify preferences the $\mathrm{g}(\mathbf{q}, \mathbf{z})$, which are conditional on the household's demographic vector[1]. Demand data can not provide information about $f\left(\mathrm{g}(\mathbf{q}, \mathbf{z}), \mathbf{z}\right)$ which is required for the construction of unconditional equivalence scales that give the true cost of demographic. This was first noted by Pollak and Wales (1979), and further investigated by Pollak and Wales (1979), Blackorby and Donaldson (1991) and Blundell and Lewbel (1991).

The approaches to this dilemma have been; i) To assume that the demographics only enter utility through its interaction with demands and that

[1] This is regardless of whether demographic variables, **z**, are an object of choice. If households do have control over demographic variables, conditional equivalence scales allow for excessive substitution, biasing the estimation of equivalence scales downwards.

conditional preferences and unconditional preferences are the same, ii) To focus on the movements in equivalence scales over time from price movements which can be identified, as shown Blundell and Lewbel (1991). iii) To use other data to provide information on $f\left(g(\mathbf{q},\mathbf{z}), \mathbf{z}\right)$. iv) To assume that the equivalence scale is independent of base level utility.

If we assume that the equivalence scale is 'independent of base' level utility (IB), such that the household cost function can be written $x_h \equiv c(u_h, \boldsymbol{p}, \boldsymbol{z}_h) \equiv m_{IB}(\boldsymbol{p}, \boldsymbol{z}_h) c_R(u_h, \boldsymbol{p})$ then unconditional equivalence scales can be recovered from demand data, see Lewbell (1989), Blackorby and Donaldson (1989), Blundell and Lewbel (1991)[2]. While homothetic preferences are a sufficient condition for IB they are not a necessary condition, which is useful since homothetic preferences have been empirically rejected.

ii) Intertemporal Equivalence Scales

In order to assess the "cost" of children on wealth and intertemporal allocations it is necessary to establish an intertemporal model that incorporates demographics. Assuming additive separability of within period utility, $u(\mathbf{q}_t, \mathbf{z}_t)$, across time, allows lifetime utility to be written:

$$U(u, \mathbf{p}, \mathbf{z}) = F\left[\int_0^T f\left(u\left(q_t, \mathbf{z}_t\right), \mathbf{z}\right) dt,\ \mathbf{z}\right]$$

Banks, Blundell and Preston (1994) point out that while information on intertemporal allocations can provide information on the preferences contained in $f\left(u\left(q_t, \mathbf{z}_t\right), \mathbf{z}\right)$.

[2] Independence of base utility (IB) is referred to as Equivalence Scale Exactness (ESE) by Blackorby and Donaldson's (1988).

It can not identify the preferences over demographic variables that enter the lifetime utility function additively as

$$U\left(w_0,\mathbf{p},\mathbf{z}\right)=\int_0^T f\left(u\left(c_t,\mathbf{p}_t,\mathbf{z}_t\right),\mathbf{z}_t,t\right)dt+D\left(\mathbf{z}\right) \ .$$

In which case the only information on how to restore $\int_0^T f\left(u\left(q_t,\mathbf{z}_t\right),\mathbf{z}\right)dt$ can be obtained not the full lifetime cost of children. For this reason, this paper specifies the lifetime utility function as,

$$U\left(w_0,\mathbf{p},\mathbf{z}\right)=D\left(\mathbf{z}\right)\int_0^T f\left(u\left(c_t,\mathbf{p}_t,\mathbf{z}_t\right),\mathbf{z}\right)dt$$

where $u(c_t,\mathbf{p}_t,\mathbf{z}_t)$ is the within period utility function at period *t*,

$\mathbf{p}$ is a *N* by *T* matrix of current and future prices for the *N* goods through time *t*, so that $\mathbf{p_t}$ is a *n* by 1 vector of prices at period *t*,

$\mathbf{z}$ is a *Z* by *T* matrix of current and future demographic variables through time *t*, so that $\mathbf{z_t}$ is a *Z* by 1 vector of the *Z* demographic variables at period *t*,

$D\left(\mathbf{z}\right)$ is a function of the lifetime demographic profile.

The additive separable lifetime utility function allows the problem to be separated into to two stages, Banks, Blundell and Preston (1994). The first stage is the intertemporal allocation of expenditure over the life cycle and the second the allocation of the given level of expenditure to the goods, which is identical to the static demand model.

Solving the intertemporal problem provides optimal $c_t=c\left(w_0,\mathbf{z}\right)$ and allows the recovery of lifetime utility $U\left(w_0,\mathbf{p},\mathbf{z}\right)$. Which can be solved for $w_0\left(U,\mathbf{p},\mathbf{z}\right)$ the initial lifetime wealth (which is equal to the sum of the stream of optimal consumption) as a function of lifetime utility *U*, for a steam of prices ***p*** and demographic history ***z***.

$$w_0\left(U,\mathbf{p},\mathbf{z}\right)=\underset{w_0}{Min}\left\{\begin{array}{l} w_0=\int_0^T e^{rt}c_t dt \quad \text{subject to} \\ U\left(w_0,\mathbf{p},\mathbf{z}\right)=D(\mathbf{z})\int_0^T f\left(u\left(c_t,\mathbf{p}_t,\mathbf{z}_t\right),\mathbf{z}\right)dt\geq U \end{array}\right\}$$

If $\boldsymbol{z}$ is demographic matrix of variables through time for a particular household and $\boldsymbol{z}^R$ is the matrix of demographic variables through time for the reference household, then the intertemporal equivalence scale can be considered as the ratio of the present value sum of expenditures across the lifetime.

$$M\left(U^R,\mathbf{p},\mathbf{z}\right)=\frac{w_0\left(U^R,\mathbf{p},\mathbf{z}\right)}{w_0\left(U^R,\mathbf{p},\mathbf{z}^R\right)}$$

iii) The Intertemporal Problem

In this section I establish a utility maximising problem in continuous time, which allows for direct effects of the lifetime demographic profile on lifetime utility.

Maximise $U\left(w_0, \mathbf{p}, \mathbf{z}\right) = D(\mathbf{z}) \int_0^T f\left(u\left(c_t, \mathbf{p}_t, \mathbf{z}_t\right), \mathbf{z}, t\right) dt$

subject to $\dot{w}_t = rw_t - c_t - y_t$

$w_T = 0$

where $u(c_t, \mathbf{p}_t, \mathbf{z}_t)$ is the within period utility function at period *t*.

$\mathbf{p}$ is a N by T matrix of current and future prices for the *N* goods through time *t*, so that $\mathbf{p_t}$ is a *n* by 1 vector of prices at period *t*.

$\mathbf{z}$ is a Z by T matrix of current and future, *Z* demographic variables through time *t*, so that $\mathbf{z_t}$ is a *Z* by 1 vector of demographic variables at period *t*.

$\dot{w}_t$ is the change in financial wealth over time

w_t is financial wealth in period t ,

c_t is consumption in period t ,

y_t is labour income in period t ,

r is the continuous interest rate for saving and borrowing ,

$D(\mathbf{z})$ is a function of the lifetime demographic profile.

Prices are assumed to stay constant at the current level $\mathrm{E}[\mathbf{p_t}] = \mathbf{p_0} = \mathbf{p}$ so that there are no expectations about future price rises. This is appropriate if households believe relative prices stay the same any future rises in the general level of prices will be matched by rises in income and the nominal interest rate. While no expectations of future interest rates are modelled, the assumption that rises in the general level of prices are fully reflected as rises in the nominal interest rate. Expectations about prices and inflation could easily be incorporated into the model

Households are assumed to have static expectations about income (or that it rises with inflation. Income growth can easily be incorporated into the model, merely altering the formula for the preset value of the steam of income. Data on the growth household's income is generally not available with cross-sectional data and requires panel data. For this reason it was not included in the model.

Time at t = 0, can be considered the current point in time in which we observe a household. It is assumed that households without children at time 0, do not plan on having any children. Essentially all children are surprises and there is no forward planning until children arrive. For simplicity households are assumed to have no control over their demographic profile. Thus variables in the demographic vector **z**, are not choice variables.

Specifying the within period utility function as,

$$u\left(c_t,\mathbf{p},\mathbf{z}\right)=\frac{\ln c_t-a\left(\mathbf{z}_0,\mathbf{p}\right)}{b\left(\mathbf{p}\right)} \text{ and } f\left(u\left(c_t,\mathbf{p}_t,\mathbf{z}_t\right),\mathbf{z},t\right)=u\left(c_t,\mathbf{p},\mathbf{z}\right)d\left(\mathbf{z}_t,t\right)$$

gives lifetime utility as,

$$U\left(w_0,\mathbf{p},\mathbf{z}\right)=D\left(\mathbf{z}\right)\int_0^T\left(\frac{\ln c_t-a\left(\mathbf{z}_0,\mathbf{p}\right)}{b\left(\mathbf{p}\right)}\right)d\left(\mathbf{z}_t,t\right)dt$$

where $D\left(\mathbf{z}\right)=\int_0^T d\left(\mathbf{z}_s,s\right)ds$.

Note that while the function $a\left(\mathbf{z}_0,\mathbf{p}\right)$ allows for demographic variables to effect within period demands amongst goods, it is specified as a function of the demographic profile in period 0, that is the current demographic profile. This later simplifies the expression for lifetime utility and allows $d(\mathbf{z}_t, t)$ and $D(\mathbf{z})$ to capture all intertemporal demographic effects on lifetime utility.

The optimal control problem can be solved for optimal consumption, optimal lifetime utility in any period *t* (see the Appendix for more details), to give

$$c_t = w_o \frac{d(\mathbf{z}_t, t)}{D(\mathbf{z})} e^{rt}$$

$$\begin{aligned} U(w_0, \mathbf{p}, \mathbf{z}) = & \frac{1}{b(\mathbf{p})}\{\ln w_0 - a(\mathbf{p}, \mathbf{z}) - \ln D(\mathbf{z})\} \\ & + \frac{r}{D(\mathbf{z})b(\mathbf{p})}\int_0^T \{d(\mathbf{z}_t, t)t\}\,dt \\ & + \frac{1}{D(\mathbf{z})b(\mathbf{p})}\int_0^T \{d(\mathbf{z}_t, t)\ln d(\mathbf{z}_t, t)\}\,dt \end{aligned}$$

Solving for $\ln \tilde{w}_0$ to give lifetime wealth as a function of lifetime utility *U*, gives

$$\begin{aligned} \ln w_0(U, \mathbf{p}, \mathbf{z}) = & \, a(\mathbf{p}, \mathbf{z}_0) + b(\mathbf{p})U(w_0, \mathbf{p}, \mathbf{z}) + \ln D(\mathbf{z}) \\ & - \frac{r}{D(\mathbf{z})b(\mathbf{p})}\int_0^T \{d(\mathbf{z}_t, t)t\}\,dt - \frac{1}{D(\mathbf{z})}\int_0^T \{d(\mathbf{z}_t, t)\ln d(\mathbf{z}_t, t)\}\,dt \end{aligned}$$

then the intertemporal equivalence scale is

$$\begin{aligned} \ln M(\mathbf{z}, \mathbf{z}^R) = & \ln w_0(U^R, \mathbf{p}, \mathbf{z}) - \ln w_0(U^R, \mathbf{p}, \mathbf{z}^R) \\ = & \left\{\ln D(\mathbf{z}) - \ln D(\mathbf{z}^R)\right\} - \left\{\frac{r}{D(\mathbf{z})}\int_0^T \{d(\mathbf{z}_t, t)t\}\,dt - \frac{r}{D(\mathbf{z}^R)}\int_0^T \{d(\mathbf{z}_t^R, t)t\}\,dt\right\} \\ & - \left\{\frac{1}{D(\mathbf{z})}\int_0^T \{d(\mathbf{z}_t, t)\ln d(\mathbf{z}_t, t)\}\,dt - \frac{1}{D(\mathbf{z}^R)}\int_0^T \{d(\mathbf{z}_t^R, t)\ln d(\mathbf{z}_t^R, t)\}\,dt\right\} \end{aligned}$$

If $d(\mathbf{z}_t^R, t) = 1$ then $D(\mathbf{z}^R) = T$ the intertemporal equivalence wealth scale reduces to

$$\ln M(\mathbf{z}) = \{\ln D(\mathbf{z}) - T\} - \left\{\frac{r}{D(\mathbf{z})}\int_0^T \{d(\mathbf{z}_t, t)t\}\,dt - \tfrac{1}{2}rT\right\} - \left\{\frac{1}{D(\mathbf{z})}\int_0^T \{d(\mathbf{z}_t, t)\ln d(\mathbf{z}_t, t)\}\,dt\right\}$$

Note that if $f\left(c_t,\mathbf{p}_t,\mathbf{z}_t\right)=u\left(c_t,\mathbf{p},\mathbf{z}\right)d\left(\mathbf{z}_t,t\right)$ without $D(\mathbf{z})$ then the intertemporal equivalence wealth scale is dependent upon lifetime utility $M\left(U^R,\mathbf{p},\mathbf{z},\mathbf{z}^R\right)$. See appendix for details.

One possible specification for $d\left(\mathbf{z}_t,t\right)$ is to allow demographics to adjust the discount rate such that

$$d\left(\mathbf{z}_t,t\right)=\mathrm{Exp}\left[\delta\left(\mathbf{z}\right)t\right]=\mathrm{Exp}\left[\left(\delta_0+\boldsymbol{\delta}'\mathbf{z}_t\right)t\right] .$$

Which gives the current consumption function that can be estimated from consumption, wealth and income data to recover an estimate of $\delta(\mathbf{z})$ that can be used to construct a lifetime equivalence scale.

$$c_0=\delta\left(\mathbf{z}\right)\left(w_0+\frac{y_0}{r}\right) \qquad \text{as an approximation to } c_0=\frac{d\left(\mathbf{z_0}\right)}{D\left(\mathbf{z}\right)}\tilde{w}_0$$

III. DATA, ESTIMATION AND METHODOLOGY

The Household Expenditure Survey (HES) confidentialised unit record files (CURFs) from the Australian Bureau of Statistics (ABS), for 1975-76, 1984,1988-89, and 1993-94 were pooled to form a pooled data set of about 25,649 observations.

The estimation involves regressing optimal expenditure against financial wealth and human capital in the current period across for all *h* households

$$c_h = \delta(\mathbf{z}_h)\left(w_h + \frac{y_h}{r}\right) + \varepsilon_h$$

where $$\delta(\mathbf{z}) = \delta_0 + \delta_k \sum_{k=1}^{K} nc_k$$

and δ_0 and δ_k are parameters to be estimated and nc_k is specified as the number of children in each age bracket. This allows the examination of the affect of children on intertemporal expenditure and thus the construction of an intertemporal equivalence scale *M*.

The HES datasets do not contain data on wealth but do contain property income, financial income (income from financial institutions) and capital income (income from investments in capital such as dividends, trusts, debentures). By dividing the income from an asset by the rate of return, an estimate of the level of assets can be obtained. The rate of return on property was assumed to be 5% for all surveys. The rate of return for the latter two of these variables was taken by a weighted sum of the rates or return of the investments that comprised them, with the weights being taken from a supplement to the 1993-94 HES on the proportion of investment types in the two measures.

Table 1 Rates of Return by Year

Year	Nominal Rate of Return on Financial Assets	Nominal Rate of Return on Capital Assets
1975/76	6.71%	9.47%
1984	7.97%	8.87%
1988/89	9.77%	10.04%
1993/94	3.43%	4.48%

The constant interest rate used to obtain human wealth was also chosen to be 5% and this is the figure used the calculation of the equivalence scales.

Estimating $c_h = \delta(\mathbf{z}_h)\left(w_h + \frac{y_h}{r}\right) + \varepsilon_h$ were $\delta(\mathbf{z}) = \delta_0 + \delta_1 z_1$ by non-linear OLS provides the following results.

IV. RESULTS

Table 2 Parameter Estimates

	δ_0	δ_1
Estimate	0.042573	0.001753
SE	0.0001885	0.0001245
t-ratio	225.84	14.09
R^2	0.4352	
$\overline{R}^2$	0.4352	

The model preforms reasonably well for cross section estimation over many households in many different situations that have not been modelled with 44% of the variation in spending explained by the model. More importantly the estimate of the rate of time preference seems reasonable at 4.3% and is significant. The effect of a child on the rate of time preference is significant and raises it by approximately 0.2% for each child. Thus a household with a child spends $\frac{\delta(\mathbf{z})}{\delta(\mathbf{z}^R)} = \frac{0.04432}{0.04257} = 1.04$ than a household without children.

The intertemporal equivalence scales constructed using a crude approximation gives:

$$M = \frac{\delta(\mathbf{z})}{\delta(\mathbf{z}) - r} \Big/ \frac{\delta(\mathbf{z}^R)}{\delta(\mathbf{z}^R) - r} = \frac{\delta(\mathbf{z})}{\delta(\mathbf{z}^R)} \Big/ \frac{\delta(\mathbf{z}) - r}{\delta(\mathbf{z}^R) - r}$$

the estimate of $\delta(\mathbf{z})$ and for values of the interest rate.

Table 3 Intertemporal Scale Estimates *M*

	$r = 3\%$	$r = 5\%$	$r = 7\%$
Additional lifetime spending for each additional child	0.91	1.36	1.11

The scales are highly dependent upon the interest rate with low interest rates suggesting that households with a child need about 9% less than a household with children, which seems implausible. For higher interest rates the scale seems more realistic. When the interest rate is 5% the same rate as that used to obtain human wealth provides a scales of 1.36 suggesting that a household with a child needs an additional 36% lifetime expenditure or wealth in order to maintain lifetime expenditure.

By splitting children into those under 5 years and those above 5 may provide insight as to whether households spend less when children are very young saving for when children are older and more expensive to maintain.

Table 4 Intertemporal Scale Estimates with Children Age Differences

	δ_0	δ_1	δ_1
Estimate	0.042558	0.003003	0.001488
SE	0.000189	0.000310	0.000138
t-ratio	225.82	9.67	10.76
R^2	0.4356		
$\bar{R}^2$	0.4356		

The results suggest that households are more inclined to spend a greater proportion of their wealth when young children are present than when children are older.

V. CONCLUSION

This paper has proposed a method for estimating an intertemporal or lifetime equivalence scale without the need for panel data, by solving the optimal intertemporal allocations of expenditures as a function of initial lifetime wealth. Demographic variables affect the intertemporal allocations of expenditure by altering the rate of time preference, which is shown to be the marginal propensity to consume out of wealth. This allows the estimation of an intertemporal equivalence scale, as the ratio of lifetime expenditures of a particular household to the reference household's.

The major limitation of the model is it's simple modelling of the intertemporal problem, without allowing for expectations of future prices, demographics (such as family size) or income. The specification of the within period utility as AIDS allows the recovery of evolution of expenditure with ease but has linear Engel curves and no rich versus poor effects of non-linear models. In fact most of the improvements in the intertemporal utility maximising problems such as liquidity constraints, finite lifetimes and uncertainty can be incorporated into the model and should do in order to provide more accurate intertemporal equivalent scales

REFERENCES

Banks, J., Blundell, R. and Preston, I., 1994, 'Measuring the Life-Cycle Consumption Cost of Children', Ch 8 in The Measurement of Household Welfare, Blundell, R., Preston, I and Walker, I. (eds), Cambridge University Press, Melbourne.

Banks, J., Blundell, R. and Preston, I., 1997, 'Life-cycle Expenditure Allocations and the Consumption Costs of Children', *European Economic Review*, vol. 38, no. 2, pp. 1391-1410.

Blackorby, C. and Donaldson, D., 1988, 'Adult Equivalence Scales and the Economic Implementation of Interpersonal Comparisons of Well-being', *University of British Columbia Discussion Paper*, no. 88-27.

Blundell, R. and Lewbel, A., 1991, 'The Information Content of Equivalence Scales', *Journal of Econometrics*, Vol. 50, no. 1-2, pp. 49-68.

Blundell, R., Browning, M. and Meaghir, C., 1994, 'Consumer Demand and the Life-Cycle Allocation of Household Expenditures', *Review of Economic Studies* , vol. 61, no. 4, pp. 57-80.

Browning, M., Deaton, A. and Irish, M., 1985, 'A Profitable Approach to Labour Supply and Commodity Demands over the Life Cycle', *Econometrica*, vol. 53, no. 3, pp. 503-543.

Browning, M., 1992, 'Children and Household Economic Behaviour', *Journal of Economic Literature*, vol. 30, no. 3, pp. 1434-1475.

Fisher, F., 1987, 'Household Equivalence Scales and Intertemporal comparisons', *Review of Economic Studies*, vol. 54, pp. 519-24.

Keen, M., 1990, 'Welfare Analysis and Intertemporal Substitution', *Journal of Public Economics*, vol. 42, pp. 47-66.

Muellbauer, J., 1974, 'Household Composition, Engel Curves and Welfare Comparisons between Households', *European Economic Review*, vol. 5, no. 2, pp. 103-122.

Pashardes, P., 1991, 'Contemporaneous and intertemporal child costs', *Journal of Public Economics*, Vol. 45, pp. 191-213.

Pollak, R.A., and Wales, T.J., 1979, 'Welfare Comparisons and Equivalence Scales', *American Economic Review*, vol. 69, no. 2, pp. 216-221.

APPENDIX

Maximise $U\left(w_0,\mathbf{p},\mathbf{z}\right)=D\left(\mathbf{z}\right)\int_0^T f\left(u\left(c_t,\mathbf{p}_t,\mathbf{z}_t\right),\mathbf{z}_t,t\right)dt$ (23)

subject to $\dot{w}_t = rw_t - c_t - y_t$ (24)

$w_T = 0$ (24)

where $u\left(c_t,\mathbf{p}_t,\mathbf{z}_t\right)$ is the within period utility function at period *t*.

$\mathbf{p}$ is a *N* by *T* matrix of current and future prices for the *N* goods through time *t*, so that $\mathbf{p_t}$ is a *n* by 1 vector of prices at period *t*.

$\mathbf{z}$ is a *Z* by *T* matrix of current and future, *Z* demographic variables through time *t*, so that $\mathbf{z_t}$ is a *Z* by 1 vector of demographic variables at period *t*.

$\dot{w}_t$ is the change in financial wealth over time

w_t is financial wealth in period t ,

c_t is consumption in period t ,

y_t is labour income in period t ,

r is the continuous interest rate for saving and borrowing ,

Prices are assumed to stay constant at the current level $\mathrm{E}\left[\mathbf{p_t}\right]=\mathbf{p_0}=\mathbf{p}$ so that there are no expectations about future price rises.

H1: $\frac{\partial H}{\partial c_t}=0 \quad \Rightarrow \quad \lambda\left(t\right)=\frac{\partial f\left(u\left(c_t,\mathbf{p}_t,\mathbf{z}_t,t\right),\mathbf{z}_t\right)}{\partial c_t}$

H2: $\frac{\partial H}{\partial \lambda}=\dot{w} \quad \Rightarrow \quad \frac{dw\left(t\right)}{dt}=r\ w+y-c$

$$\Rightarrow \quad w\left(t\right)e^{-rt}=w_0+\int_0^t e^{-rs}y_s ds-\int_0^t e^{-rs}c_s ds$$

H3: $\frac{\partial H}{\partial w\left(t\right)}=-\dot{\lambda} \quad \Rightarrow \quad \frac{d\lambda\left(t\right)}{dt}=-r\lambda$

$$\Rightarrow \qquad \lambda(t) = \lambda_0 e^{-rt}$$

Specifying $f\left(v(c_t,\mathbf{p}_t,\mathbf{z}_t),\mathbf{z}_t,t\right) = f\left(c_t,\mathbf{p}_t,\mathbf{z}_t\right) = u\left(c_t,\mathbf{p},\mathbf{z}\right)\dfrac{d\left(\mathbf{z}_t\right)}{D(\mathbf{z})}$ where

$D(\mathbf{z}) = \int_0^T d\left(\mathbf{z}_s\right)ds$ and $u\left(c_t,\mathbf{p},\mathbf{z}\right) = \dfrac{\ln c_t - a(\mathbf{z}_0,\mathbf{p})}{b(\mathbf{p})}$

$$H = \frac{\ln c - a(\mathbf{z}_t,\mathbf{p})}{b(\mathbf{p})}\frac{d\left(\mathbf{z}_t\right)}{D(\mathbf{z})} + \lambda\left(rw + y - c\right)$$

H1: $\dfrac{\partial H}{\partial c} = 0 \qquad \Rightarrow \qquad \lambda(t) = \dfrac{1}{b(\mathbf{p})c_1(t)}\dfrac{d\left(\mathbf{z}_t\right)}{D(\mathbf{z})}$

From H1 when $t = 0$ then $\lambda(t)$ is

$$\lambda_0 = \frac{1}{b(\mathbf{p})c_0}\frac{d\left(\mathbf{z}_0\right)}{D(\mathbf{z})}$$

Combing the above with H3 gives consumption.

$$\frac{1}{b(\mathbf{z})c_t}\frac{d\left(\mathbf{z}_t\right)}{D(\mathbf{z})} = \frac{1}{b(\mathbf{p})c_0}\frac{d\left(\mathbf{z}_0\right)}{D(\mathbf{z})}e^{-rt}$$

$$c_t = c_o\frac{d\left(\mathbf{z}_t\right)}{d\left(\mathbf{z}_0\right)}e^{rt}$$

Inserting the above equation into the equation of motion for wealth H2 gives

$$w(t)e^{-rt} = w_0 + \int_0^t e^{-rs}y(s)ds - \frac{c_0}{d\left(\mathbf{z}_0\right)}\int_0^t d\left(\mathbf{z}_s\right)ds\,.$$

Setting $t=T$ to find c_0.

$$w(T)e^{-rT} = w_0 + \int_0^T e^{-rs}y(s)ds - \frac{c_0}{d\left(\mathbf{z}_0\right)}\int_0^T d\left(\mathbf{z}_s\right)ds$$

Assuming no bequest motive and defining $\tilde{w}_0 = w_0 + \int_0^T e^{-rs} y(s) ds$ then optimal initial consumption is

$$c_0 = \frac{d(\mathbf{z_0})}{D(\mathbf{z})} \tilde{w}_0$$

Inserting optimal consumption in period *t* $c_t = w_0 \frac{d(\mathbf{z}_t)}{D(\mathbf{z})} e^{rt}$ into lifetime utility

$U(w_0, \mathbf{p}, \mathbf{z})$

$$\begin{aligned} U(w_0,\mathbf{p},\mathbf{z}) &= \int_0^T \frac{\ln\left[e^{rt} c_0(w_0)\right] - a(\mathbf{p},\mathbf{z}_0)}{b(\mathbf{p})} \frac{d(\mathbf{z}_t)}{D(\mathbf{z})} dt \\ &= \frac{1}{D(\mathbf{z}) b(\mathbf{p})} \int_0^T \left\{ d(\mathbf{z}_t) \ln d(\mathbf{z}_t) + d(\mathbf{z}_t) rt + d(\mathbf{z}_t) \left\{ \ln w_0 - a(\mathbf{p},\mathbf{z}_0) - \ln D(\mathbf{z}) \right\} \right\} dt \\ &= \frac{1}{D(\mathbf{z}) b(\mathbf{p})} \int_0^T \left\{ d(\mathbf{z}_t) \ln d(\mathbf{z}_t) \right\} dt + r \int_0^T \left\{ d(\mathbf{z}_t) t \right\} dt + \left\{ \ln w_0 - a(\mathbf{p},\mathbf{z}_0) - \ln D(\mathbf{z}) \right\} \int_0^T d(\mathbf{z}_t) dt \\ &= \frac{1}{b(\mathbf{p})} \left\{ \ln w_0 - \ln D(\mathbf{z}) - a(\mathbf{p},\mathbf{z}_0) \right\} + \frac{r}{D(\mathbf{z}) b(\mathbf{p})} \int_0^T \left\{ d(\mathbf{z}_t) t \right\} dt + \frac{1}{D(\mathbf{z}) b(\mathbf{p})} \int_0^T \left\{ d(\mathbf{z}_t) \ln d(\mathbf{z}_t) \right\} dt \end{aligned}$$

Solving for $\ln \tilde{w}_0$ to give lifetime wealth as a function of lifetime utility *U*, gives

$$\begin{aligned} \ln \tilde{w}_0 &= \ln w_0 \left(U^R, \mathbf{p}, \mathbf{z}^R \right) \\ &= a(\mathbf{p},\mathbf{z}_0) + b(\mathbf{p}) U(w_0,\mathbf{p},\mathbf{z}) \\ &\quad + \ln D(\mathbf{z}) - \frac{r}{D(\mathbf{z}) b(\mathbf{p})} \int_0^T \left\{ d(\mathbf{z}_t) t \right\} dt - \frac{1}{D(\mathbf{z})} \int_0^T \left\{ d(\mathbf{z}_t) \ln d(\mathbf{z}_t) \right\} dt \end{aligned}$$

then the intertemporal equivalence scale is

$$\begin{aligned} \ln M &= \ln w_0 \left(U^R, \mathbf{p}, \mathbf{z} \right) - \ln w_0 \left(U^R, \mathbf{p}, \mathbf{z}^R \right) \\ &= \left\{ \ln D(\mathbf{z}) - \ln D(\mathbf{z}^R) \right\} - \left\{ \frac{r}{D(\mathbf{z})} \int_0^T \left\{ d(\mathbf{z}_0) t \right\} dt - \frac{r}{D(\mathbf{z}^R)} \int_0^T \left\{ d(\mathbf{z}_0^R) t \right\} dt - \right\} \\ &\quad - \left\{ \frac{1}{D(\mathbf{z})} \int_0^T \left\{ d(\mathbf{z}_0) \ln d(\mathbf{z}_0) \right\} dt - \frac{1}{D(\mathbf{z}^R)} \int_0^T \left\{ d(\mathbf{z}_0^R) \ln d(\mathbf{z}_0^R) \right\} dt \right\} \end{aligned}$$

The reference household is specified as an adult couple that do not (nor intend to) have children, thus their demographic profile is considered constant (aging is incorporated into the model through the intertemporal framework). If their demographic profile stays constant (or is expected to do so) then we may normalise

$d\left(\mathbf{z}^R\right)$ to one in each period such that $D\left(\mathbf{z}^R\right)=T$. In this case the intertemporal equivalence scale reduces to

$$\ln M\left(\mathbf{z}\right)=\left\{\ln D\left(\mathbf{z}\right)-T\right\}-\left\{\frac{r}{D\left(\mathbf{z}\right)}\int_0^T\left\{d\left(\mathbf{z}_t\right)t\right\}dt-\tfrac{1}{2}rT\right\}-\left\{\frac{1}{D\left(\mathbf{z}\right)}\int_0^T\left\{d\left(\mathbf{z}_t\right)\ln d\left(\mathbf{z}_t\right)\right\}dt\right\}$$

Model I Demographic Discounting

By allowing the rate of time preference to vary according to demographics, more specifically with the number of children, the demographic effect on intertemporal allocations can be easily be examined. The effect on lifetime utility is a little more complicated but it can be obtained from the information on intertemporal allocation along with assumptions about the households' expectations of its demographic profile.

The simplest but naive assumption is to assume that households believe that their current demographic profile will not change. A much more appealing assumption that is still quite simple is to assume that the household believe that each child will leave the house at a certain age, say 18 or 21 years.

Model II Expectations about Demographics (No Discounting)

If $D\left(\mathbf{z}\right)=\int_0^T d\left(\mathbf{z}_s\right)ds=\int_0^T\left(1+\kappa n_c p_t\right)ds=T+P\kappa n_c$

$$\begin{aligned}\ln w_0&=a\left(\mathbf{p},\mathbf{z}_0\right)+b\left(\mathbf{p}\right)U\left(w_0,\mathbf{p},\mathbf{z}\right)+\ln\left(T+P\kappa n_c\right)-\tfrac{1}{2}r\frac{\left(T^2+\kappa n_c P^2\right)}{T+P\kappa n_c}-\frac{\left(T+P\kappa n_c\right)\ln\left(T+P\kappa n_c\right)}{T+P\kappa n_c}\\&=a\left(\mathbf{p},\mathbf{z}_0\right)+b\left(\mathbf{p}\right)U\left(w_0,\mathbf{p},\mathbf{z}\right)-\tfrac{1}{2}r\frac{T^2+\kappa n_c P^2}{T+P\kappa n_c}\end{aligned}$$

So that the intertemporal equivalence scale is

$$\ln M(\mathbf{z}) = -\tfrac{1}{2} r \frac{T^2 + \kappa n_c P^2}{T + P\kappa n_c} + \tfrac{1}{2} rT$$

$$= \tfrac{1}{2} r \frac{\kappa n_c P(T-P)}{T + P\kappa n_c}$$

Model I Demographic Discounting

By allowing the rate of time preference to vary according to demographics, more specifically with the number of children, the demographic effect on intertemporal allocations can be easily be examined. The effect on lifetime utility is a little more complicated but it can be obtained from the information on intertemporal allocation along with assumptions about the household's expectations of its demographic profile.

The simplest but naive assumption is to assume that households believe that their current demographic profile will not change. A much more appealing assumption that is still quite simple is to assume that the household believe that each child will leave the house at a certain age, say 18 or 21 years.

Initial consumption for the demographic discounting model is given by

$$c_0 = \frac{\delta(\mathbf{z})}{1 - e^{-\delta(\mathbf{z})T}} \{\tilde{w}_0\}$$

$$U(w_0, \mathbf{p}, \mathbf{z}) = \int_0^T e^{-\delta(\mathbf{z})t} \frac{\ln\left[e^{(r-\delta(\mathbf{z}))t} c_0(w_0)\right] - a(\mathbf{p}, \mathbf{z}_0)}{b(\mathbf{p})} dt$$

$$= \frac{1}{b(\mathbf{p})} \int_0^T (r - \delta(\mathbf{z})) t e^{-\delta(\mathbf{z})t} dt \quad + \frac{\ln\left[c_0(w_0)\right] - a(\mathbf{p}, \mathbf{z}_0)}{b(\mathbf{p}, \mathbf{z})} \int_0^T e^{-\delta(\mathbf{z})t} dt$$

If the household believes that **z** will stay constant then

$$U\left(w_0,\mathbf{p},\mathbf{z}\right)=\frac{\left(r-\delta\left(\mathbf{z}\right)\right)}{b\left(\mathbf{p}\right)}\frac{1-\left(1-\delta\left(\mathbf{z}\right)T\right)e^{-\delta(\mathbf{z})T}}{\delta\left(\mathbf{z}\right)^2}+\frac{\ln\left[c_0\left(w_0\right)\right]-a\left(\mathbf{p},\mathbf{z}_0\right)}{b\left(\mathbf{p}\right)}\frac{1-e^{-\delta(\mathbf{z})T}}{\delta\left(\mathbf{z}\right)}$$

Which can be solved for wealth

$$\ln\left[c_0\left(\tilde{w}_0,\mathbf{p},\mathbf{z}\right)\right]=1-\frac{r}{\delta\left(\mathbf{z}\right)}+\frac{\delta\left(\mathbf{z}\right)}{1-e^{-\delta(\mathbf{z})T}}b\left(\mathbf{p}\right)\mathrm{U}+a\left(\mathbf{p},\mathbf{z}_0\right)+\frac{e^{-\delta(\mathbf{z})T}}{1-e^{-\delta(\mathbf{z})T}}\left(r-\delta\left(\mathbf{z}\right)\right)T$$

and then the intertemporal equivalence scale

$$\tilde{w}_0\left(\mathbf{z}\right)=\frac{1-e^{-\delta(\mathbf{z})T}}{\delta\left(\mathbf{z}\right)}\mathrm{Exp}\left[1-\frac{r}{\delta\left(\mathbf{z}\right)}+\frac{\delta\left(\mathbf{z}\right)}{1-e^{-\delta(\mathbf{z})T}}b\left(\mathbf{p}\right)\ U+a\left(\mathbf{p},\mathbf{z}_0\right)+\frac{e^{-\delta(\mathbf{z})T}}{1-e^{-\delta(\mathbf{z})T}}\left(r-\delta\left(\mathbf{z}\right)\right)T\right]$$

To simplify consider what happens as terminal time approaches infinity

$$\tilde{w}_0\left(\mathbf{z}\right)=\frac{1}{\delta\left(\mathbf{z}\right)}\mathrm{Exp}\left[1-\frac{r}{\delta\left(\mathbf{z}\right)}+\delta\left(\mathbf{z}\right)b\left(\mathbf{p}\right)\ U+a\left(\mathbf{p},\mathbf{z}_0\right)\right]$$

$$M=\frac{\dfrac{1}{\delta\left(\mathbf{z}\right)}\mathrm{Exp}\left[1-\dfrac{r}{\delta\left(\mathbf{z}\right)}+\delta\left(\mathbf{z}\right)b\left(\mathbf{p}\right)\ U+a\left(\mathbf{p},\mathbf{z}_0\right)\right]}{\dfrac{1}{\delta\left(\mathbf{z}^R\right)}\mathrm{Exp}\left[1-\dfrac{r}{\delta\left(\mathbf{z}^R\right)}+\delta\left(\mathbf{z}^R\right)b\left(\mathbf{p}\right)\ U+a\left(\mathbf{p},\mathbf{z}_0^R\right)\right]}$$