Alternative Beta Risk Estimators in Emerging Markets: The Latin American Case

Robert D Brooks*, Robert W Faff, Tim Fry***, Diana Maldonado Rey***

Abstract

In this paper, using a Latin American database, we investigate the empirical performance of an alternative beta risk estimator, which is designed to be superior to its conventional counterparts in situations of extreme thin trading. The estimator used is based on the sample selectivity model, which includes a two-step method: a selectivity equation and a regression component applied to the non-censored data. The study compares the resultant selectivity-corrected beta to the standard OLS beta and the Dimson Beta. We demonstrate the empirical behaviour of the selectivity corrected beta estimator using a sample of stocks in seven countries which are part of the emerging markets of Latin America. The results indicate that the selectivity-corrected beta does correct the downward bias of the OLS estimates and is likely to better estimate stock risk.

*Research Development Unit, RMIT Business

**Department of Accounting and Finance, Monash University

***School of Economics and Finance, RMIT Business

Acknowledgment: The research on this project was funded by ARC Discovery Grant DP0345680.

1. **Introduction**

The increasing globalisation of the world's financial markets has led to a greater emphasis on the pursuit of the benefits of international diversification. In turn, this has led to consideration of a broader range of capital markets as possible investment opportunities. One such group of capital markets are those in Latin America. The Latin American markets as a group have a degree of homogeneity given their trade and economic linkages. This feature of the Latin American markets has been studied in a cointegration framework by Choudhry (1997). Choudhry (1997) found cointegrating relationships between the key six Latin American markets (Argentina, Brazil, Chile, Colombia, Mexico, Venezuela) using data from 1989 to 1993. Chen, Firth and Rui (2002) extend Choudhry's (1997) analysis by examining whether the finding of cointegration is robust to a broader sample (1995 to 2000) that includes data covering both the Asian and Russian financial crises. For the same six markets they find that the cointegration result is robust to inclusion of the crises. Christofi and Pericli (1999) extend Choudhry's (1997) work and conduct their analysis in a Vector Autoregressive (VAR) framework examining spillovers in both mean and volatility of returns covering the period from 1992 to 1997. For five Latin American markets (Argentina, Brazil, Chile, Colombia, Mexico) they found relationships in both the mean and volatility, and particularly so for volatility. Moreover, Christofi and Pericli (1999) found the volatility relationship between the Latin American markets to be stronger than the relationship for other regions of the world.

A key issue for an investor in the Latin American markets will be the determination of an appropriate risk measure for individual stocks. An obvious starting point is to use the Capital Asset Pricing Model (CAPM) and then systematic risk (beta) is the measure of risk. In the context of the valuation of closely held companies in Latin America, Pereiro (2001) argues that the CAPM is problematic owing to market illiquidity given that most of the markets are small and have concentrated investment patterns. One alternative is to use different measures, such as that emanating from the downside risk model suggested by Estrada (2002). Estrada's (2002) results clearly illustrate that the CAPM beta understates the risk relative to the downside risk measure.

An interesting issue is whether the problem of the understatement of risk by the CAPM beta is a result of data censoring associated with thin trading and/or illiquidity. In both the Australian market (see Brooks, Faff, Fry and Gunn (2004)) and the Canadian market (see Brooks, Faff, Fry and Bissoondoyal-Bheenick (2004)), thin trading has been found to introduce a censoring problem that leads to OLS estimates of beta risk being downward biased. Brooks, Faff, Fry and Gunn (2004) argue that this can be overcome by using a sample selectivity model to estimate betas. If this issue is present in developed and relatively liquid markets such as Australia and Canada then the problem is likely to be accentuated for the emerging and illiquid markets of Latin America. Accordingly, an investigation of the impact of censoring on individual stock betas in Latin American markets is the key objective of this paper.

The plan of this paper is as follows. Section 2 describes the characteristics of the Latin American markets under study. Section 3 outlines the modelling framework to be used in this paper. Section 4 presents the empirical results. Section 5 contains concluding remarks.

2. Latin American Markets

We chose Latin American countries as the focus of our analysis because they represent fast developing economies that are linked by cultural heritage and by some common business conditions. In addition, relatively little is known about these markets. The specific countries included in the sample are Argentina, Brazil, Chile, Mexico, Colombia, Peru and Venezuela; they represent countries in the Americas which have established but emerging stock markets. Moreover, the economies of these countries are considered to be developing rather than developed. These Latin American countries have close ties, are characterised by considerable intra-regional trade, similar commodity exports and important cross-country investment.

Latin America has a long history of capitalism and reliance on foreign debt to finance its development, and foreign investors are increasingly familiar with the region and the financial risks associated with its composition of countries. The Latin American emerging markets included in the study are those which are the largest and longest established in South and Central America. Among these countries the two biggest markets are Mexico and Brazil. According to the Iberoamerican Federation of Stock Exchanges (FIABV), at the end of 2002 the Brazil Stock Exchange had more than 412 listed companies, with a market capitalization exceeding US$124 billion and an equity trading volume of around 42 billion. Similarly Mexico, with 169 listed companies, had a market capitalization exceeding US$286 billion and a trading volume of around US$28 billion.

In terms of their 2002 performance, a wide variety of outcomes transpired across these Latin American equity markets. On the one hand, some markets had very favourable outcomes: most notable is the case of Argentina (91.2 per cent), as well as

Colombia (50.22 per cent) and Peru (14.87 per cent). On the other hand, the equity markets of Brazil, Chile and Mexico were poor performers in 2002: Brazil (-17.01 per cent), Chile (7 per cent) and Mexican (-3.85 per cent).

Apart from having the largest stock markets in the region, these countries also are major international borrowers with high levels of outstanding foreign debt making them financial risk premium countries. In addition, these are countries in which the stock market has suffered from the effects of economic adjustment. For some time, Argentina and Brazil have experienced a high two digit inflation rate, Argentina having the highest inflation among these countries. Furthermore, all of these countries have suffered from a depreciation of their currencies over recent years, with Argentina having the highest change, decreasing by three times its value against the US dollar. Venezuela and Brazil have also suffered an important depreciation of their currencies, whereas the fluctuations of the currencies in Mexico and Chile were modest.

Overall, the Latin Economies have expanded their GDP by 1.5 per cent in 2003, in contrast to the 0.4 per cent reduction in GDP in 2002. The relative importance of the stock markets compared to the national economies varies somewhat across countries. The Brazil and Mexico equity market capitalization is about 30 per cent of GDP, while in Chile the market capitalization is 77 percent of GDP.

3. Modelling Framework

Recognition of the inherent underdevelopment of Latin American markets leads us to expect a relatively high incidence of thin trading, which causes problems because of the presence of the zero returns in the estimation in the betas when using time series approaches. In this type of setting, an alternative approach – separating the modelling of zero return observations from the nonzero return observations in the sample – is needed. Such an alternative is now described.

The presence of zero returns in the observed data for an asset means that there will be a "spike" or ("pile up") in the distribution of returns at zero. Such data is referred as censored data. It is argued that least squares regressions under these circumstances produce inconsistent estimates of beta. In the context of thinly traded stocks, the model proposed to deal with censored data is presented in the paper by Blundell and Meghir (1987), *the sample selectivity model*. The model comprises of two components: a selectivity component and a regression model. The first component deals with the "spike", or discreteness, in the observed data and the second component applies to the continuous data on returns (the non zero return data).

In the selectivity component we assume that underlying the observed data is a latent variable, labelled z_{it}^*. If this variable exceeds some threshold value then the second regression component will apply to the observed data on an individual asset's returns, r_{it}. We assume that z_{it}^* is determined via an underlying regression model with explanatory variables $\mathbf{w}_{it}$. The issue then becomes what variable(s) to use. In the current setting we assume that $\mathbf{w}_{it}$ comprises of a constant and trading volume. The choice of trading volume as the explanatory variable in the selectivity component appeals to the literature that has investigated the stock price – volume relation [see for example, Karpoff (1987); Gallant, Rossi and Tauchen (1992); and Hiemstra and Jones (1994)]. If the latent variable, z_{it}^*, is sufficiently large then we observe a non-zero return. In other words, we need a sufficiently large trading volume on a given day to trigger a price change and, hence, yield a non-zero return.

Once a non-zero return is observed – equivalently $z_{it}^{*} > 0$ - then the regression component will apply to the data. That is, for all non-zero returns the traditional market model (a regression model) applies. In essence then the binary choice component is concerned with sample selection and the regression component is concerned with modelling the (non-zero) returns data. Formally we have the following:

Selectivity Component

$$z_{it}^{*} = \mathbf{w}_{it}^{'}\gamma_{i} + u_{it} \qquad \text{. . . (1)}$$

where:

$$z_{it} = \begin{cases} 1 \text{ if } z_{it}^{*} > 0, \\ 0 \text{ otherwise.} \end{cases}$$

Equivalently,

$$z_{it} = \begin{cases} 1 \text{ if non-zero return,} \\ 0 \text{ if zero return.} \end{cases}$$

This yields a discrete choice model for the zero versus the non-zero return variable, z_{it}. If we assume normality for the underlying distribution then we have a probit model with $P(z_{it} = 1) = \Phi(\mathbf{w}_{it}^{'}\gamma_{i})$ and $P(z_{it} = 0) = 1 - \Phi(\mathbf{w}_{it}^{'}\gamma_{i})$.

The regression component of the sample selectivity model applies when $z_{it} = 1$. That is, when we have a non-zero return. For simplicity we will assume that this regression component can be specified as the traditional market model, but it is possible to apply the non-synchronous trading arguments to justify another specification (*e.g.* Dimson). In our case we have:

$r_{it} = á_i + â_i r_{mt} + v_{it}$

when $z_{it} = 1$.

To complete the specification we make an assumption concerning the stochastic parts of the sample selectivity model. In particular, we assume that the vector of stochastic variables, (u_{it}, v_{it}), follows a bivariate normal distribution $[0,0,1,\sigma_v,\rho]$. Thus the selectivity and regression components may be correlated $(\rho \neq 0)$.

In this model we have:

$$E[r_{it} \mid z_{it} = 1] = á_i + â_i r_{mt} + ñó_v ë(\mathbf{w}'_{it} ã_i) = á_i + â_i r_{mt} + \theta_i ë(\mathbf{w}'_{it} ã_i) \qquad (2)$$

where the “Inverse Mill’s Ratio” (IMR) is given by:

$$ë(\mathbf{w}'_{it} ã) = \frac{ö(\mathbf{w}'_{it} ã_i)}{\Phi(\mathbf{w}'_{it} ã_i)}.$$

The source of the bias and inconsistency is now apparent – it is caused by the omission of the IMR from the regression model of (4) (see Greene (1997, p. 977)).

The sample selectivity model may be estimated either by maximum likelihood techniques or by a two-step procedure due to Heckman (see Heckman (1979)). The two-step procedure is easy to implement in practice and yields an estimator that is unbiased, consistent but not fully efficient. Thus in this paper, we will use the two-step estimation procedure. The procedure is as follows:

(1) Estimate the probit selection equation by maximum likelihood to obtain $\hat{\mathbf{a}}_i$ and, hence, estimate the Inverse Mill’s Ratio.

(2) Estimate the regression model $r_{it} = á_i + â_i r_{mt} + èë(\mathbf{w}'_{it} ã_i) + e_{it}$, replacing the Inverse Mill’s ratio with the estimated version from step (1). This

second step regression has heteroscedastic errors and, thus, should be estimated by generalised least squares. However, an ordinary least squares estimation will still yield consistent and unbiased estimators.

For comparative purposes however, the beta is estimated using the market model as well as by the Dimson (1979) model. The Dimson approach treats the thin trading problem as being caused by asynchronous movements in individual stock returns as compared to the market return. This is then overcome via the inclusion of lead and lag terms. The Dimson model has been used with two leads and two lags of the market return:

$$r_{it} = \alpha_i + \beta_{i-2} r_{mt-2} + \beta_{i-1} r_{mt-1} + \beta_{i0} r_{mt} + \beta_{i1} r_{mt+1} + \beta_{i2} r_{mt+2} + e_{it} \qquad (3)$$

using least squares to estimate the Dimson beta from the relationship: $\beta_{iDIM} = \sum_{k=-2}^{2} \beta_{ik}$.

Hence, the focus of this study is to compare: (1) the standard OLS beta (β_{OLS}); (2) the standard Dimson beta with two leads and two lags (β_{DIM}); (3) the OLS beta with a selectivity correction $\left(\beta_{OLS}^{SEL}\right)$; and (4) the Dimson beta with a selectivity correction $\left(\beta_{DIM}^{SEL}\right)$. We examine the Dimson beta with selectivity correction to assess the impacts of correcting for two elements of thin trading, both censoring and synchronicity.

3. Data and Empirical Results

3.1 Data and Descriptive Statistics

The daily stock price for companies in Latin American countries is obtained from the Datastream database for a period of three years from 1 January 2000 to 31

December 2002. Data from seven Latin countries was included in the sample; Brazil, Argentina, Chile, Mexico, Colombia, Peru and Venezuela. The total number of time series observations in the sample varied across countries; 724 for Argentina, 742 for Brazil, 744 for Chile, 750 for Mexico, 726 for Colombia, 741 for Peru, and 703 for Venezuela. This reflects different holidays and market closures in the different countries.

In total for all seven countries, there are 131 companies in the sample. The degree of censoring varies greatly across the 131 companies. The lowest censoring is 0.027 per cent (two zero return observations out of 750 total observations) for Alfa (Mexico) and the highest censoring is 62.71 percent (454 zero return observations out of 724 total observations) for Comercio del Plata (Argentina). The mean level of censoring is 13.35 per cent and the median is 7.87 per cent. There are 130 companies with less than 50 per cent censoring and 77 companies with less than 10 per cent censoring. Compared to the Australian data (see Brooks, Faff, Fry and Bissondoyal-Bheenick 2003) and Canadian data (see Brooks, Faff, Fry and Gunn 2003), the sample for the Latin American market is composed of smaller and more frequently traded companies, that results in less censoring. This is primarily due to intentionally filtering out all companies with very high censoring where prices did not change for months on end. This will rule out finding extreme beta estimates

The full sample of Latin American companies range in size from US$28,000 to US$20 million, measured by market value. The mean company size is US$1.6 million and the median company size is US$878,000. In general, company size is negatively correlated with censoring (ρ=-0.3216). The sample of Latin American companies has an average trading volume ranging from 1,927 shares to 25 million shares. The mean of the average daily trading volume is 1,324,000 shares and the median is 412,000 shares. In general, daily trading volume is weakly negatively correlated with censoring (ρ=-0.027), but positively correlated to size (ρ=0.3986).

2.2 Comparison of the Betas

The four different betas were calculated for the 131 companies. The betas estimated were the standard OLS beta; the standard Dimson beta with two leads and two lags; the OLS beta with the selectivity correction; and the Dimson beta with the selectivity correction.

The results are shown in Tables 1 through Table 3 in three categories; the degree of censoring, the firm size measured by the market value and the average trading volume. Table 1 shows the average, high and low beta estimates across five censoring categories. Table 2 shows the average, high and low beta estimates across six identified categories according to firm size. Finally, Table 3 shows the average, high and low estimates across seven categories according to trading volume.

An examination of Tables 1 through Table 3 confirms the expected negative relationship between censoring and trading volume and between censoring and size. Table 1 confirms this statement showing that among the sample, there was just one company (Comercio de la Plata) ranked with an average degree of censoring above 0.5, 62.7 per cent, and an average trading volume of 480,000 shares; while the majority of firms show an average degree of censoring less than 10 per cent, with an average size of US$2 million and an average trading volume of 1,331,340 shares. It should however be noted that companies with an average degree of censoring between 0.2 and 0.3 have an average size of US$ 685,000 and an average trading volume of

2,501,970 shares. This can be explained by the fact that Banco Santander (Chile) is the company with the highest average trading volume of the sample and is included in this range of censoring.

A further illustration is given in Table 2 and Table 3, as small firms with a market value less than US$1 million, which are the majority of companies included in the sample, have an average censoring of 18.1 per cent and an average trading volume of 739,000 shares, while the largest firm with a market value over $15 million has an average censoring of 0.4 per cent and an average trading volume of 17 million shares. In addition, the firms with small average daily trading volumes (less than 100,000) have a degree of censoring of 16.6 per cent, while companies with a trading volume between 800,000 and 1 million shares have a degree of censoring of 6.1 percent. However, the volume category with a trading volume higher than 1 million have a degree of censoring of 11 percent, which exceeds the volume category between 800,000 and 1 million. Nevertheless, these results generally confirm the inverse relation between size and trading volume, and the direct relation between size and trading volume.

The standard OLS beta has a general downward bias in the case of thinly traded stocks. Thus, we now compare the Dimson Beta and the selectivity corrected OLS beta to the standard OLS beta. On average, both the Dimson and the selectivity-corrected beta have resulted in adjustments for the general downward bias in the OLS beta. As expected the selectivity-corrected OLS beta exceeds the standard OLS beta in all censoring, trading volume and size categories as revealed in Table 1, Table 2, and Table 3. This is unlike previous results for Australia and Canada and perhaps illustrates the importance of making such corrections in emerging markets. This emphasises the need for greater risk adjustment in emerging markets in the estimation of individual stock risk.

Now considering a comparison between the standard OLS with the Dimson beta, the results show that the average standard OLS beta exceeds the average Dimson counterpart in some categories. According to the level of censoring in Table 1, the average Dimson beta exceeds the standard OLS beta in all but one category - the category of lowest censoring($c<0.1$), wherein the average Dimson Beta (β= 1.1920) is lower than the average standard OLS beta (β=1.2542). Table 2 reveals that in all categories the average standard OLS beta exceeds the average Dimson beta in all categories except the category with companies having a size less than $1 million (Dimson β=1.14). According to the volume data in Table 3, the average Dimson beta does not exceed the average standard OLS in the three categories with the highest trading volume (exceeding 600,000 shares), ie. categories with a volume >1,000,000 (Dimson β= 1.2055), for volume between 800,000 and 1,000,000 shares (Dimson β=1.2905); and for volume in between 600,000 and 800,000 shares (Dismson β=1.0690). This suggests that the Dimson beta is not making a full correction for the impacts of censoring. As such, this is potentially evidence that the need to correct for censoring is more important than asynchronicity in these markets.

We now consider a comparison of the average selectivity-corrected OLS Beta with the average standard Dimson Beta. With respect to the censoring categories (Table 1), the average selectivity-corrected OLS beta exceeds the Dimson beta in all categories. As expected the higher selectivity corrected OLS betas tend to occur for the higher censoring groups ($c>0.50$). For example, in the sample the category with the highest degree of censoring is between 60 per cent and 70 per cent with an average selectivity corrected beta of 2.03 considerably larger than the average standard Dimson of 1.51 as compared to other censoring categories. Similar results were obtained with Canadian data where the average selectivity OLS exceeds the average Dimson beta in the categories with the highest degree of censoring. However in absolute terms the extent to which selectivity exceeds the average Dimson beta in the Canadian data is much higher, largely because that sample includes more extremely censored data.

As far as firm size (Table 2) and trading volume (Table 3) are concerned, the average selectivity-corrected OLS beta exceeds the average Dimson beta in all six categories according to market size and all seven categories of trading volume. Compared to the Australian and Canada data, a similar pattern is shown, however the Australian and Canadian data had some exceptions where the average Dimson exceeded the average selectivity corrected OLS beta.

Now we consider a comparison of the average selectivity-corrected Dimson beta with the other three betas. The results vary across Tables 1, 2 and 3. With regard to the censoring categories of Table 1, the average selectivity-corrected Dimson beta exceeds the average Dimson beta in all categories, while the average selectivity-corrected Dimson beta exceeds the average standard OLS beta in all but one category and is less than the selectivity-corrected OLS beta in 2 categories. Under the size categories in Table 2, the average selectivity-corrected Dimson beta exceeds all the other betas in just one category, the smaller firm size category with firms under $1 million. The average selectivity-corrected Dimson beta is larger than the average standard OLS in just one size category, is larger than the average Dimson beta in all but one size category and is larger than the selectivity-corrected OLS beta in just one size category. Under the volume categories, the average selectivity-corrected Dimson beta exceeds all other betas in 4 categories. Compared to each of the betas the average selectivity-corrected Dimson Beta exceeds in the same four categories the average standard OLS, the average Dimson Beta and the average selectivity OLS.

Considering Panel B of Tables 1, 2, and 3 which shows the high/low betas, it should be noted that the difference between the low and high beta across the different categories is not as extensive as was the case for Canada or Australia. Across all categories single figure betas were always achieved with a maximum of 4.21. This is in part due to the filtering out of extreme censoring.

3.3 Beta Correlation Analysis

Table 4 shows the correlation matrix between censoring, firm size, trading volume and the different beta estimates. OLS beta estimates are expected to increase as market size and trading volume increase, and as the degree of censoring decreases. Accordingly the results in Table 4 show a positive correlation of OLS beta estimates with firm size (ρ=0.1353) and trading volume (ρ=0.0467) and a negative correlation with the degree of censoring (ρ=-0.3339). This is consistent with the expectation that OLS is more likely to provide more accurate beta estimates for large liquid stocks.

Furthermore, Table 4 also shows that the Dimson beta has a weak positive correlation with size (ρ=0.0582) and a weak negative correlation with the degree of censoring (ρ=-0.0873). The selectivity-corrected OLS betas have a very low positive correlation with trading volume, size and the degree of censoring. The selectivity-corrected Dimson beta has a very low and negative correlation with size and volume and a positive correlation with degree of censoring. Finally, as expected the betas are positively correlated with each other.

Table 5 shows beta correlations for the subcomponents in the sample i.e. the low censoring sub-sample (censoring<0.10); the large firm sub-sample (market value>$4 million); and the high trading volume sub-sample (volume>1,000,000 shares). Panel A of Table 5 shows that the highest correlation among the four categories of beta estimates is $\rho=0.9231$ between the Dimson beta and selectivity-corrected Dimson beta. In Panel B, we observe that, in the case of low censoring, the correlation between the four categories of beta is substantially higher compared to the full sample of firms' category. The correlation between the standard and the selectivity corrected betas are very strong with $\rho=0.9958$ for the least squares estimates and $\rho=0.9976$ for the Dimson estimates. In addition Panel C of Table 5 indicates that betas across the large firm subsample are highly correlated. The highest correlation is $\rho=0.9968$ between the Dimson beta and its selectivity-corrected counterpart. Finally, the last panel of Table 5 shows that in the high trading category, the betas are highly correlated. The strongest correlation is $\rho=0.9791$ between the Dimson beta and the selectivity-corrected Dimson beta.

The empirical results can be summarized as follows. First on average, both the Dimson and the selectivity-corrected betas exceed the standard OLS beta. Second, the results confirm the negative relation between censoring and trading volume, but shows a positive relation between trading volume and size; and censoring and size. These results confirm our expectations derived from theory.

4. Conclusions

In this paper we have presented an alternative method of computing the beta risk estimator, which is designed to deal with thin trading situations, and apply it to a Latin American dataset. This new type of estimator adjusts for the presence of zero return observations, as the presence of these observations will cause a spike of zero returns which is the case of extreme thin trading situations. The approach used - the sample selectivity model - includes two components, a selectivity equation that deals with the spike and a regression model that is applied to the non-censored data. The resultant selectivity-corrected beta is designed to have the desirable statistical properties.

Given the institutional features of Latin American markets we, informed by Estrada's (2002) view expect that the standard OLS beta is very likely to underestimate the risk of Latin American countries. Our analysis corrects for this feature via the use of a sample selectivity model. This increases the estimated beta risk of individual securities and appears to make this correction more effectively than the standard Dimson correction. This is consistent with the expectations of having used an estimator with desirable statistical properties.

The model is applied in this case to a sample of daily data for 131 companies in Latin America for a period of three years (1 January 2000 to 31 December 2002). The empirical analysis can be described as follows. First we found that on average the Dimson and the selectivity-corrected betas exceed the standard OLS betas. Second, the results confirm a negative relationship between censoring and trading volume, and between censoring and size. It also confirms a positive relation between size and trading volume.

Also, for this particular exercise, a positive and single figure beta was achieved across all categories for both selectivity-corrected OLS beta and for selectivity-corrected Dimson beta. The results suggest that these trading adjusted betas correct for the general downward bias in OLS betas.

This paper explored the use of alternative beta risk estimators in the presence of thin trading, which is typical in markets of the Latin American region. The aim of the paper was to show the application of a technique with theoretical merit, and the results suggest that the selectivity betas are more appropriate in thin trading situations providing superior statistical properties.

References

Blundell, R. and Meghir, C., (1987), "Bivariate Alternatives to the Univariate Tobit Model", *Journal of Econometrics* 33, 179-200.

Brooks, R., Faff, R., Fry, T. and Bissoondoyal-Bheenick, E. (2004), Alternative Beta Risk Estimators in Cases of Extreme Thin Trading: Canadian Evidence, unpublished manuscript, RMIT.

Brooks, R., Faff, R., Fry, T. and Gunn, L., (2004), Censoring and its Impact on Beta Risk Estimation, *Advances in Investment Analysis and Portfolio Management,* forthcoming.

Chen, G., Firth, M. and Rui, O. (2002), Stock Market Linkages: Evidence from Latin America, *Journal of Banking and Finance* 26, 1113-1141.

Choudhry, T. (1997), Stochastic Trends in Stock Prices: Evidence from Latin American Markets, *Journal of Macroeconomics* 19, 285-304.

Christofi, A. and Pericli, A. (1999), Correlation in Price Changes and Volatility of Major Latin American Stock Markets, *Journal of Multinational Financial Management* 9, 79-93.

Estrada, J. (2002), Systematic Risk in Emerging Markets: The D-CAPM, *Emerging Markets Review* 3, 365-379.

Greene, W. H., (1997) *Econometric Analysis,* 3rd edition, Prentice Hall International, London.

Heckman, J., (1979) "Sample Selection Bias as a Specification Error", *Econometrica* 47, 153-161.

Pereiro, L. (2001), The Valuation of Closely-Held Companies in Latin America, *Emerging Markets Review* 2, 330-370.

Table 1: Average and High/Low Estimates across Censoring Categories

This table presents the average, low and high for each of the four different beta estimates when partitioned into one of six categories according to the degree of censoring in the data. The censoring measure (c) is defined as the proportion of the total sample period for which zero return observations are recorded for each stock.

Panel A: Average size, Trading Volume and Betas across Censoring Categories

Category	Number of firms	Size MV ($000s)	Trading Volume (000s)	Beta OLS	Beta dim	Beta Sel OLS	$\left(\beta_{DIM}^{SEL}\right)$
0.4<c < 0.5	5	461.14	1018.87	0.8895	1.1237	1.4480	1.7359
0.3 < c < 0.4	13	494.21	916.79	0.7986	1.0218	1.1151	1.3793
0.2 < c < 0.3	14	685.48	2501.97	1.05	1.1113	1.2546	1.2529
0.1 < c < 0.2	21	1102.98	878.09	1.00	1.020	1.0715	1.0657
c<0.1	77	2325.05	1331.34	1.25	1.1920	1.2646	1.1948

Panel B: Low/High Betas across Censoring Categories

Category	Beta OLS		Beta Dimson		Beta Sel OLS		Beta Sel Dim	
	Low	High	Low	High	Low	High	Low	High
0.4 < C < 0.5	0.4391	2.0949	0.4407	2.5214	0.6492	3.6124	0.6518	4.2170
0.3 < c < 0.4	0.1693	1.3749	0.2047	1.9084	0.2532	1.9628	0.2733	2.8468
0.2 < c < 0.3	0.4686	1.9472	0.4486	1.8853	0.5070	2.2206	0.5119	2.2347
0.1 < c < 0.2	0.2665	1.5013	0.1961	1.8584	0.2999	1.7030	0.1544	2.1958
c<0.1	0.4073	1.8761	0.4883	1.8328	0.4198	1.9737	0.4823	1.8258

Table 2: Average and High/Low Beta Estimates across Size Categories

This table presents the average, low and high for each of the four beta estimates when portioned into one of six categories according to firm size. The firm size measure (M) is the average market value of equity (US$000) across the sample period for each stock.

Panel A: Average Degree of Censoring. Trading Volume and Betas across Size Categories

Category	Number of firms	Deg of censoring	Trading Volume (000s)	Beta OLS	Beta dim	Beta Sel OLS	Beta Sel Dim
M < 1000	72	0.1807	739.09	1.0793	1.1432	1.2224	1.2780
1000 < M < 3000	41	0.0906	1647.22	1.1707	1.0998	1.2214	1.1383
3000 < M < 4000	6	0.0479	974.98	1.2557	1.2001	1.2735	1.2105
4000 < M < 10000	10	0.0449	2885.32	1.2665	1.2200	1.3048	1.2270
10000 < M < 15000	1	0.0485	848.35	1.1536	1.0943	1.1555	1.0944
M> 15000	1	0.0040	17596.06	1.3280	1.2809	1.3280	1.2793

Panel B: Low / High Betas across Size Categories

Category	Beta OLS		Beta Dimson		Beta SEL OLS		Beta Sel Dimson	
	Low	High	Low	High	Low	High	Low	High
M < 1000	0.1693	2.0949	0.1961	2.5214	0.2532	3.6124	0.1643	4.2170
1000 < M < 3000	0.4317	1.8761	0.2117	1.8328	0.4434	1.9737	0.1544	1.9329
3000 < M < 4000	0.7262	1.8054	0.5779	1.6260	0.7657	1.8497	0.5825	1.6260
4000 < M < 10000	0.6583	1.6997	0.5440	1.7168	0.6584	1.6998	0.5448	1.7190
10000 < M < 15000	1.1536	1.1536	1.0943	1.0943	1.1555	1.1555	1.0944	1.0944
M> 15000	1.3280	1.3280	1.2809	1.2809	1.3280	1.3280	1.2793	1.2793

Table 3: Average and High/Low Beta Estimates across Trading Volume Categories

This table presents the average, low and high for each of the four different beta estimates when partitioned into one of seven categories according to trading volume. The trading volume measure (V) is the average daily volume (000s) of traded shares across the total sample period for each stock.

Panel A: Average Degree of Censoring, Size and Betas across Trading Volume Categories

Category	Number of firms	Deg of censoring	Size	Beta OLS	Beta dim	Beta Sel OLS	Beta Sel Dim
V < 100	23	0.166	1071.692	0.979	1.035	1.078	1.164
100 < V < 250	26	0.160	723.205	1.089	1.098	1.238	1.212
250 < V < 400	16	0.106	1786.702	1.257	1.272	1.356	1.370
400 < V < 600	20	0.175	1619.271	1.032	1.086	1.174	1.214
600 < V <800	9	0.062	1155.618	1.163	1.069	1.170	1.068
800 < V < 1000	6	0.061	1026.864	1.301	1.291	1.310	1.280
V> 1000	31	0.110	3211.585	1.243	1.206	1.314	1.252

Panel B: Low/High Betas across Volume Categories

Category	Beta OLS		Beta Dimson		Beta SEL OLS		Beta Sel Dimson	
	Low	High	Low	High	Low	High	Low	High
V < 100	0.2404	1.4671	0.2457	1.9084	0.4869	1.9628	0.5119	2.8468
100 < V < 250	0.1693	2.0949	0.1961	2.5214	0.2532	3.6124	0.1643	4.2170
250 < V < 400	0.4317	1.7281	0.5779	1.6907	0.4434	1.7382	0.5825	2.2347
400 < V < 600	0.3659	1.8054	0.2117	1.6992	0.5105	2.0863	0.1544	2.0775
600 < V <800	0.6208	1.7259	0.4883	1.6877	0.6209	1.7255	0.4888	1.6902
800 < V < 1000	0.7262	1.7208	0.7296	1.7368	0.7828	1.7209	0.7628	1.7372
V> 1000	0.5669	1.8761	0.4260	1.8584	0.6877	1.9737	0.5439	2.1958

Table 4: Correlation Matrix of Censoring, Firm size, Trading Volume and Betas

This table presents the correlation matrix for the full sample of stocks.

	% of Zeroes	Size	Trading Volume	OLS Beta	Dimson Beta	Sel OLS Beta	Sel Dimson Beta
% of Zeroes	1						
Size	-0.32162	1					
Trading Volume	-0.02701	0.398629	1				
OLS Beta	-0.33386	0.135302	0.046696	1			
Dimson Beta	-0.08731	0.058204	-0.00809	0.88381	1		
Sel OLS Beta	0.03023	0.045306	0.031088	0.900995	0.897858	1	
Sel Dimson Beta	0.208591	-0.02236	-0.02468	0.723177	0.923062	0.890253	1

Table 5: Correlation Matrix of Different Betas: Full Sample, Low Censoring Sub-sample, Large Firm Sub-sample and High Trading Volume Sub-sample

This table presented the correlation matrix amongst the different beta estimates. Results are presented for the full sample, low censoring sample; large firms sample and the high trading volume sample.

	OLS beta	Dimson Beta	Sel OLS Beta	Sel Dimson Beta
Panel A: Full Sample				
OLS beta	1			
Dimson Beta	0.8838	1		
Sel OLS Beta	0.9010	0.8979	1	
Sel Dimson Beta	0.7232	0.9231	0.8903	1
Panel B: Low Censoring Sample (c< 0.1)				
OLS Beta	1			
Dimson beta	0.9336	1		
Sel OLS Beta	0.9958	0.9314	1	
Sel Dimson Beta	0.9285	0.9976	0.9287	1
Panel C: Large Firm Size (MV>$4,000,000)				
OLS Beta	1			
Dimson beta	0.9790	1		
Sel OLS Beta	0.9619	0.9385	1	
Sel Dimson Beta	0.9867	0.9968	0.9544	1
Panel D: High Trading Volume(V>1,000,000)				
OLS Beta	1			
Dimson beta	0.9117	1		
Sel OLS Beta	0.9699	0.9094	1	
Sel Dimson Beta	0.8432	0.9791	0.8776	1