Vertical Contracting When Competition for Orders Precedes Procurement*

by

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This paper reverses the standard order between input supply negotiations and downstream competition and assumes that competition for orders takes place prior to procurement of inputs in a vertical chain. In an environment where procurement negotiations involve no private information and no restrictions on the form of pricing, it is found that oligopolistically competitive outcomes will result despite the presence of an upstream monopolist. It is demonstrated that vertical integration is a means by which the monopolist can leverage its market power downstream to the detriment of consumers. However, it does so, not by foreclosing on independent downstream firms, but by softening the competitive behaviour of its own integrated units. Thus, the paper provides a simple rationale for anti-competitive vertical integration in an environment that respects the usual Chicago school assumptions. Journal of Economic Literature Classification Number: L42

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1. **Introduction**

Virtually all models of vertical supply contracting assume that an upstream firm commits to the terms of input supply contracts prior to downstream firms making their own production and marketing decisions. This means that downstream firms compete with one another with full knowledge of at least their own supply conditions. While this timing it rarely given any justification, it would appear to apply most naturally in environments where input supply terms can be changed or renegotiated less frequently than customer orders (for instance, where customers are relatively impatient or goods are relatively perishable so that downstream firms need to have secure supply terms in place before competing for them).

There are many instances, however, where this standard timing does not apply and where, in reality, competition for customer orders precedes commitment to procurement terms. Here are four clear examples:

- Building and architectural contracts where input requirements are generally unknown prior to receiving a customer order.
- The provision of large scale services to government or firms (in defense or information technology) involves competition for contracts that precede procurement decisions regarding capital equipment supplies and human capital expertise in on-going legal and consulting services (Kamien, Li and Samet, 1989).
- If customer switching costs are substantial, competition for those customers takes place at a time well in advance of when consumption of the requisite services will be required.
- Electricity and gas retailing where customers (both residential and industrial) are sold forward contracts before supply agreements to obtain the necessary stock or supplies are set (Stahl, 1988).

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1 This occurs for models where upstream firms simply post input prices (e.g., Waterson, 1980) or where they negotiate those input prices with downstream firms (e.g., Rey and Tirole, 2003).
Indeed, it is quite conceivable that procurement contracts may be subject to hold-up and ex post renegotiation once customer contracts are in place. In this situation, common to analyses of vertical relationships in the incomplete contracts literature (Hart, 1995), modeling procurement negotiations as occurring following the marketing of services to customers would be necessary to fully capture the lack of commitment power inherent in some supply agreements.

For these reasons, this paper reverses the stages in the standard approach to vertical contracting and explores the impact on this on two key questions in that literature: (1) if there is a monopoly in upstream supply, in a perfect information environment, can that monopoly power be leveraged downstream? and (2) does vertical integration have an anti-competitive or strategic effect in this setting? The first question is one which the Chicago school has consistently answered affirmatively leading to a conclusion that there is no further anti-competitive role for the actions in the second question (Bork, 1978). It is only where there is incomplete information where there is both a negative answer to the first question and, consequently, a cause for concern over vertical integration (see Rey and Tirole, 2003, for a recent survey).

To this end, competition for orders is assumed to take place prior to negotiations over procurement with a single upstream supplier. It is demonstrated that this modeling change has profound implications for the level of competition in downstream markets. Specifically, even in an environment of complete (public) information and flexibility regarding the nature of ex post procurement, oligopolistically competitive outcomes result in the industry. In contrast, for the same contractual and informational space, the standard approach would yield a monopoly outcome with the upstream supply able to commit to input supply terms that maximise industry profit (i.e., the Chicago school conclusion). In the standard approach, it is only by restricting the information or contract space that more competitive outcomes emerge.²

² Hart and Tirole (1990), O’Brien and Shaffer (1992), McAfee and Schwartz (1994) and Segal (1999) analyse the outcomes that emerge when supply negotiations between one downstream firm and the upstream monopolist are not observed by other downstream firms. In this setting, depending upon the contract space, different types of oligopolistic outcomes can emerge (see Rey and Tirole, 2003). Indeed, McAfee and Schwartz (p.220) conjectured that if sales were determined prior to input negotiations, a simple oligopolistically competitive outcome would not emerge. It is demonstrated here that this is not the case and a Cournot outcome is an equilibrium outcome.
The intuition behind the result here is simple. Downstream firms anticipate supply negotiations when they compete ex ante. If those ex post negotiations are efficient (i.e., they maximise bilateral surplus), so long as the order price exceeds the marginal cost of the input requirements for that order, procurement will take place. Moreover, as an efficient surplus is shared by the downstream firm, that firm internalises the efficient upstream supply choice when competing for orders with other downstream firms. The result is an oligopolistically competitive outcome with upstream supply taking place in a productively efficient manner.

The existence of competitive outcomes generates scope for vertical integration to raise industry profits. It is demonstrated that vertical integration can achieve an increase in industry profits by softening downstream competition. However, it does this, not by foreclosing on non-integrated downstream firms, but instead by providing incentives for integrated units to weaken their own competitive choices.\(^3\) That is, vertical integration results in a contraction in internal supply and an expansion in external supply. So while integration under standard vertical contracting can generate a situation where “rival’s costs rise” to the detriment of competition, here rivals’ competitive advantages are stimulated. Nonetheless, by virtue of its monopoly position, the upstream firm appropriates part of the increase in industry rents that occurs. This is a novel mechanism for anti-competitive vertical integration and, all the more significant in that it does not arise from problems associated with private information or restrictions on the form of input prices.

The remainder of the paper proceeds as follows. Next, several papers, where competition for orders precedes procurement in vertical contracting, are identified and their relationship to the present paper explored. In the next section, vertical contracting when the upstream monopolist is not integrated downstream is examined and the main result that, even in an environment of complete information, oligopolistically competitive outcomes result despite the presence of a monopoly bottleneck. Section 3 then considers the effect, profitability and welfare effects from vertical integration in this context.

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\(^3\) This stands in contrast to the post-Chicago approach to vertical mergers that focuses on the way a vertical integrated firm can commit to ‘raise rival’s costs.’ (see Riordan and Salop, 1995). In general, this is done by committing to higher input prices to independent downstream firms than is implicit within the integrated firm (see Ordover, Saloner and Salop, 1990; Chen, 2001 and Rey and Tirole, 2003).
Section 4 extends the baseline model to the case of Bertrand price competition downstream and to consider the roles of contingent agreements and exclusive dealing. A final section concludes.

*Related Literature*

There are a handful of antecedents to this paper in the literature. Stahl (1988) examines a model of price setting downstream firms who procure inputs from a competitive set of upstream suppliers. In one variant of his model, it is assumed that competition downstream is for forward contracts and bidding for inputs takes place later. He demonstrates that this leads to (Walrasian) competitive outcomes across the entire vertical chain. As will be demonstrated below (Section 4), his model is distinct in that a single input price is set (there is no price discrimination in the wholesale market) and downstream firms otherwise have all the power in that market. In contrast, the model here presumes that the single upstream firm bargains one-on-one with each downstream firm, giving it some bargaining power and also permitting discriminatory input pricing outcomes. Consequently, in this setting the Walrasian outcomes derived by Stahl do not hold. Moreover, Stahl does not analyse how vertical restrictions impact on downstream market outcomes.

Kamien, Li and Samet (1989) also consider an environment where bidding for customer (in their case a single customer contract) precedes procurement. Their model focuses on a situation where firms are vertically integrated but that, for efficiency reasons, if one firm wins a contract it may want to subcontract part of the contract with the losing firm. They demonstrate that the distribution of bargaining power impacts on the competitiveness of bid competition ex ante. Specifically, if the losing firm has lots of bargaining power ex post, this weakens ex ante price competition. Similarly, below it is demonstrated that strong upstream bargaining power (under vertical separation) weakens ex ante price competition. The strength of the contribution here relates to its comparison with the standard vertical contracting literature and its focus on how changes in vertical
structure impact on downstream competition. Neither of the earlier papers examine these issues.\(^4\)

2. Vertical Contracting

This section provides the baseline result of the paper that when competition for orders precedes procurement negotiations, oligopolistically competitive outcomes (namely, Cournot competition) result. Moreover, this outcome is unique.

**Notation**

Let \( i \in \{1,\ldots,N\} \) denote an individual downstream firm. Firm \( i \) has inverse demand curve, \( P_i(x_i, x_{-i}) \), which is a function of its own output, \( x_i \), and a vector of the output of others, \( x_{-i} \). It is assumed that \( P_i(\cdot) \) is non-increasing in each of its arguments, for all \( i \), and that \( \frac{\partial P_i}{\partial x_j} \leq \frac{\partial^2 P_i}{\partial x_i \partial x_j} x_j \) for all \( i \) and \( j \neq i \).\(^5\) Downstream firms face no other costs, save for the costs of procuring inputs, \( p_i x_i \). In order to sell \( x_i \) units of output, firm \( i \) requires \( y_i = x_i \) units of the input.

Initially, it is supposed there is a single provider of inputs to all downstream firms (\( U \)). \( U \) has costs described by a non-decreasing function, \( C(x_1,\ldots,x_N) \). Denote \( C_i(x_1,\ldots,x_N) \) as the marginal cost of supplying an additional unit to \( i \). Sometimes it is convenient to refer to \( C(x_i, x_{-i}) \) and \( C_i(x_i, x_{-i}) \) in order to focus attention in variations in \( x_i \) holding \( x_{-i} \) as given. It is assumed that, \( P_i(0,0) > C_i(0,\ldots,0) \).

**Timeline**

STAGE 1: Each downstream firm competes for orders in the downstream market. An order comprises an average price over a fixed quantity of orders

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\(^4\) Rey and Tirole (2003) look at a “make to order” specification whereby input pricing terms are agreed upon ex ante but actual input supply quantities are determined following downstream competition. The key feature of the approach here is to assume that no supply commitments (in either price or quantity) are made prior to downstream competition for orders.

\(^5\) These assumptions are made to guarantee the existence and uniqueness of an interior Cournot equilibrium (Vives, 1999).
\((P_i, x_i)\). Competition involves downstream firms choosing \(x_i\) taking the order quantity of other firms \((x_{-i})\) as given.

**STAGE 2:** Each firm who wishes to procure a positive quantity negotiates with \(U\) over the price paid per unit of input required \((p_i, y_i)\). These negotiations are bilateral.

**STAGE 3:** Downstream firms who reach an agreement with \(U\) produce. Those who do not reach an agreement with \(U\) for all of their orders incur default costs.

**STAGE 4:** Payments are made and payoffs are realised.

The Cournot case – where downstream firms choose the order quantities they would like to achieve and the market price adjusts accordingly (as assumed for Stage 1) – is the focus of the paper. However, in a later section, it will be demonstrated that all of the main results of the paper carry over to the Bertrand case.

In Stage 2, bargaining takes place bilaterally. Fix an order for the sequence of negotiations, say beginning with 1 and then through to \(N\). The precise order turns out not to matter. Bilateral negotiations take the following form: with probability \(\lambda\), \(U\) makes a take-it-or-leave-it offer of \((p_i, y_i)\) to downstream firm \(i\) but with probability \(1-\lambda\), \(i\) gets to make a take-it-or-leave-it offer to \(U\). If the offer is rejected, there is no procurement between \(U\) and \(i\) otherwise the accepted offer becomes the agreed procurement terms. The outcome is observed by all other downstream firms who play the same bilateral bargaining game as their turn comes up. It will be demonstrated that this simple bargaining game implements the bilateral Nash bargaining solution with asymmetric bargaining powers.\(^6\)

It is worth emphasising at this point that it is assumed here that \(\lambda \in \{0,1\}\). This is in contrast to the common assumption in the vertical contracting literature that \(U\) can make take-it-or-leave-it offers to downstream firms (McAfee and Schwartz, 1994; and Segal, 1999). When orders precede procurement, this assumption cannot be meaningfully applied as downstream firms would receive no profits regardless of what they did to compete for orders. While this will not change the oligopolistically competitive outcome

\(^6\) For this reason, any multilateral negotiation game that gives \(U\) and \(i\) the payoffs they would receive under asymmetric Nash bargaining will generate the same equilibrium outcomes in what follows.
below from being an equilibrium, any equilibrium outcome is possible.\textsuperscript{7} As such, it is assumed throughout that $U$'s bargaining power ($\lambda$) is bounded away from 1 although on occasion I look at the outcomes of the limiting case as $\lambda$ approaches 1.

A critical set of assumptions in vertical contracting is what each downstream firm can observe about the outcomes of negotiations with other firms. One possibility is that there is complete observability where each firm knows the order and price level of its rivals. In this situation, each would be able to infer the outcomes of negotiations. Indeed, this is a natural assumption given that the outcomes in Stage 1 will comprise an equilibrium.

The alternative is that downstream firms cannot observe negotiated outcomes. This is the usual assumption in the vertical contracting literature; although there it is to capture the notion that $U$ cannot commit not to engage in secret discounting with an individual firm. Here, given that competition for orders precedes procurement negotiations, the secret discounting motive is not present. For this reason, attention is confined to the complete information case throughout this paper – providing the closest comparison with Chicago school assumptions. Nonetheless, it will be readily apparent that all of the results carry over to the incomplete information case.

There is one important restriction on the contracting space that is implicit in the specification of the bargaining game above. An offer from $U$ or a downstream firm is simply a price and quantity pair that does not change regardless of the outcomes of other negotiations. Thus, agreements cannot be made contingent upon the outcome of later negotiations. Nonetheless, the realised agreements in negotiations can still influence the outcomes of later negotiations as those agreements are observed.\textsuperscript{8} This simplifies the

\textsuperscript{7} It should be noted here that in the standard literature with private information, many equilibrium outcomes are possible. The case that is the focus of most attention in that literature is the one that yields a Cournot outcome (see Rey and Tirole, 2003). However, that case requires an assumption of passive beliefs (something that may not be reasonable in many circumstances). Indeed, it is also possible that the integrated monopoly outcome could arise under alternative belief assumptions (McAfee and Schwartz, 1994). The advantage in this paper is that with $\lambda \in (0,1)$, the equilibrium outcome is unique and is the appealing Cournot case.

\textsuperscript{8} In this way, the type of rent shifting analysed by Marx and Shaffer (2002) is still possible. What is not possible is an agreement that allowed the downstream firm to earn a large penalty if a later procurement agreement was reached. It is demonstrated below that such agreements may allow rent shifting (so that the order of negotiations mattered for individual payoffs) but does not change the overall welfare and competitive outcomes.
analysis but as is demonstrated later in the Section 4, the main qualitative conclusions of
the paper continue to hold.

Finally, in Stage 3, two important assumptions are made:

(A1) There is an exogenous default cost, $d > 0$, per unit of orders unfulfilled.

(A2) The total level of default payments cannot exceed the profits of the
downstream firm.

The first assumption simplifies the analysis considerably and, as will be demonstrated, it
does not play a critical role. The second assumption is an assumption that, to some extent,
a downstream firm is judgment proof. That is, any damages award cannot exceed the
ability of the firm to pay that award. In this case, ability to pay is measured by
downstream firm profits. (A2) simplifies the analysis considerably. If it were not present
this would have a minor impact on the strength of downstream competition but otherwise
the results in this paper would be unchanged.

**Bargaining Outcome**

Take a set of orders $\{P_i, x_i\}_{i=1}^N$. Then:

**Proposition 1.** If (i) $P_i + d \geq C_i(x_i, x_{-i})$ for all $i$, and (ii) $\sum_i P_i x_i \geq C(x_i, x_{-i})$, then $y_i = x_i$
and $E[p, x_i] = \lambda P_i x_i + (1 - \lambda)(C(x_i, x_{-i}) - C(0, x_{-i}))$, for all $i$.

All proofs are in the appendix. It will turn out that, in equilibrium, condition (i) is always
met. With this, the proposition states that the solution to any bilateral negotiation is the
same as the Nash bargaining outcome.\(^9\) This requires condition (ii) that, if all orders are
filled, industry profits are positive. Notice that it is conceivable that this might not be the
case even if condition (i) is satisfied if $C(.)$ is not weakly concave (e.g., $U$ has sizeable
fixed costs).

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\(^9\) Note that there is no issue here of the form of upstream pricing. Essentially negotiations are over what
proportion of the order to fulfill and the payment for that. The payment can take a per unit form (as has
been done here) or a non-linear schedule. Essentially, however, it is the lump sum that matters and the form
of the price is simply made for notational convenience.
Ex Ante Competition for Orders

In Stage 1, each $i$ chooses $x_i$ to maximise: $P_i(x_i, x_{-i})x_i - E[p_i(x_i, x_{-i})x_i]$ where the expected procurement costs are as in Proposition 1. Let $\{\hat{x}_i\}_{i=1}^N$ be the set of equilibrium orders. This results in the following:

**Proposition 2.** In any subgame perfect equilibrium, if $\sum_i P_i(\hat{x}_i, \hat{x}_{-i})\hat{x}_i \geq C(\hat{x}_1, ..., \hat{x}_N)$, then $\{\hat{x}_i\}_{i=1}^N$ satisfies:

$$\hat{x}_i \in \arg \max_{x_i} P_i(x_i, \hat{x}_{-i})x_i - C(x_i, \hat{x}_{-i}) \text{ for all } i.$$ 

Thus, the outcome is a Cournot oligopoly outcome where upstream supply is efficiently provided. This mirrors results in the vertical contracting literature where procurement precedes orders and there is incomplete information.\(^{10}\) Here, however, observationally, upstream prices may be linear and not involve lump sum payments. Despite this, for a given order, procurement is efficient and does not involve the negative consequences of double marginalisation that would normally arise when upstream prices are linear and downstream competition is imperfect.

What is also notable here, however, is that this outcome occurs in a complete information environment and is independent of the degree or nature of product differentiation and the allocation of bargaining power. As McAfee and Schwartz (1994) demonstrate, when $\lambda = 1$ (i.e., $U$ has all of the bargaining power) and downstream products are perfect substitutes for one another, the unique equilibrium outcome is the same as would be achieved by an integrated monopoly. That outcome is achieved because in ex ante sequential negotiations for inputs, $U$ can commit to sign an agreement with a single firm (the last one in the sequence) and appropriates all of the industry returns. Here that outcome is only possible if $N = 1$.

In the standard vertical contracting case (where procurement precedes orders), as products become differentiated and/or relative bargaining power changes, it is not possible to provide a characterisation of the outcome; it is likely to depend on both of these parameters. Where orders precede procurement, however, the Cournot outcome occurs regardless of the allocation of bargaining power or the nature of downstream

products and upstream technology. While the characterisation is both sharp and robust, it
does, however, rely on the identification of the nature of downstream competition.\textsuperscript{11,12}

4. Vertical Integration

The key feature of the results derived thus far is the fact that, in an environment
with complete information, oligopolistically competitive outcomes emerge despite the
presence of an upstream monopolist. Competition emerges because there is no
mechanism by which the monopolist can commit to input pricing outcomes that impact
on downstream competition. A lack of commitment also drives the emergence of
competitive outcomes in the standard vertical contracting literature. There, however, an
upstream monopolist cannot commit not to engage in ‘secret discounting’ in an
environment where there is imperfect information. In that environment, a move to
complete information would give the monopolist the ability to embed multilateral
contingencies into individual supply contracts so as to ensure a monopoly outcome
industry-wide. Vertical integration, however, provides an alternative means by which the
monopolist can achieve commitments to softer downstream competition in the absence of
feasible contractual mechanisms. That possibility is examined here.

Suppose that $U$ purchases one or more downstream firms. In this situation, the
integrated units will be able to base their orders directly on the $U$’s marginal cost.
However, as in Chen (2001), integrated units will also have regard to the potential profits
$U$ might earn from non-integrated firms.

Nonetheless, it is first useful to note that for a given set of orders, the bargaining
outcomes between $U$ and any individual downstream will be as in Propositions 1. This is
because, in each bilateral negotiation, joint surplus depends only upon the value of the

\textsuperscript{11} Other models of vertical contracting rely on this too. Generally, downstream competition is assumed to
be Bertrand with the outcomes driven by the types of input pricing contracts that are feasible.
\textsuperscript{12} It is worth noting that the above equilibrium also remains an equilibrium when there is incomplete
information regarding procurement contract outcomes. As is commonly assumed in the standard vertical
contracting literature, suppose that retailers hold passive beliefs regarding the outcomes of other
negotiations. In this case, if a retailer receives an offer other than that consistent with the Cournot
equilibrium outcome, it does not revise its beliefs about the outcomes of other negotiations. For this reason,
a deviation by the manufacturer cannot impact on other negotiations and hence, will give it no advantage.
order and the impact on upstream costs. It does not depend upon the terms achieved by other independent firms or U’s own downstream divisions.

Let \( \hat{x}_i(I) \) be the equilibrium order and price of an independent \( i \) when \( I \) firms are integrated and let \( \hat{z}_j(I) \) be those for an integrated unit. It is easy to see that \( \hat{z}_i(N) = x_i^* \); that is, if all downstream firms are integrated, a monopoly outcome is possible; so complete integration will be (weakly) preferred by all firms relative to non-integration or partial integration. What is more interesting are the comparative static results regarding the effect of partial integration.

**Proposition 3.** Suppose that each \( i \) is symmetric and \( I < N \). In any subgame perfect equilibrium, if \( \partial E[p, \hat{x}_i]/\partial z_j < 0 \) (for all \( i \) and \( j \)) then, (i) \( \hat{x}_j(I) < \hat{x}_j(0) \), (ii) \( \hat{x}_j(0) < \hat{x}_j(I) \), (iii) \( \frac{1}{N} < \frac{\hat{z}(I)}{E(z(I))} \); and (iv) \( N\hat{x}_j(0) > \hat{z}_j(I) + (N-I)\hat{x}_j(I) \). A necessary condition for (i) to (iv) not to hold is that \( \partial E[p, \hat{x}_i]/\partial z_j > 0 \).

The proposition demonstrates that if the revenues \( U \) expects to receive from independent downstream firms falls as integrated output expands then industry output and hence, consumer surplus will be lower as a result of integration. Moreover, this occurs because \( U \) has an incentive to contract the output of its integrated units relative to what they would sell if they were not integrated. This, in turn, leads to an expansion in independent sales and an increase in their market share. Thus, the anti-competitive effect of integration here is not foreclosure (a reduction in independent output) but precisely the opposite. Indeed, independent firms benefit from integration.

The intuition behind Proposition 3 can be illustrated by focusing on the Cournot case with symmetric downstream firms selling a homogenous product. In this situation, independents still solve:

\[
\hat{x}(I) \in \arg \max_x P(I\hat{x} + (N-I-1)\hat{x} + x)x - C(I\hat{x} + (N-I-1)\hat{x} + x)
\]

while an integrated firm solves:

\[
\hat{x}(I) \in \arg \max_x P(I\hat{x} + (N-I)\hat{x})x + (N-I)E[\hat{p}(Iz)\hat{x} - C(Iz + (N-I)\hat{x})] \]

where \( \hat{p}(Iz)\hat{x} = \lambda P(Iz + (N-I)\hat{x})\hat{x} + (1-\lambda)\left(C(Iz + (N-I)\hat{x}) - C(Iz + (N-I-1)\hat{x})\right) \).

Integration means that when considering their sales level, integrated units take into account their impact on (a) the revenues received externally from independent units; and
(b) the revenues achieved internally by other integrated units. The first effect may be positive or negative and, as it plays a critical role in Proposition 3, it is examined in detail next. On the other hand, the second effect is the same as would occur if downstream firms were to merge and, ceteris paribus, implies that integrated firms will contract output relative to when they were not integrated. It is for this reason that a sufficient condition for an overall reduction in integrated firm and overall output is that integrated sales reduce $U$’s external revenue and that an output expansion necessities as positive external revenue impact.

When $I = 1$, the external revenue impact drives the outcomes entirely (as there is no internal revenue impact). In this situation, what is the impact of integrated sales on external revenue? Note that:

\[
\frac{\partial E[\hat{\mu}(z)\hat{x}]}{\partial z} = \lambda \frac{p_x^\lambda}{\text{Increased Competition}} + (1 - \lambda)\left(C'(z + (N-1)\hat{x}) - C'(z + (N-2)\hat{x})\right) \tag{1}
\]

There are two effects here. First, if an increase in sales by integrated units raises the incremental cost of supplying a given independent unit, this improves $U$’s relative bargaining position. If supply to independents is less attractive at the margin this raises the price extracted from them. Second, an increase in integrated sales reduces the downstream price. This reduction is shared by the independent firm and $U$, reducing $U$’s external revenues.

These two effects – a bargaining effect and a competition effect – (potentially) have opposing impacts on the incentives of an integrated firm to increase its sales downstream. Indeed, if the competition effect outweighs the bargaining effect, that incentive is reduced; integrated units will generate fewer orders, in equilibrium, than their independent downstream rivals. In the end, overall downstream quantity is reduced following integration, lowering consumer surplus. On the other hand, a relatively strong bargaining effect may turn integration into what Fudenberg and Tirole (1985) termed a ‘top dog’ strategy. Ceteris paribus, this effect causes the integrated firm to be more aggressive downstream, raising internal supply at the expense of external supply. While this harms independent downstream firm, it benefits consumers as total quantity increases.

When will one effect dominate? Re-arranging (1), and assuming that $\hat{x} > 0$:
Thus, the competition effect dominates the bargaining effect when (i) incremental upstream costs are increasingly convex and (ii) $U$’s bargaining power is high. Note that when upstream costs exhibit increasing returns or have a constant marginal cost, external revenues necessarily fall as the integrated firm generates more downstream orders. As such, increased integrated sales make it more desirable to reach independent deals at the margin (improving an independent’s bargaining position). In contrast, when upstream costs are convex, it is increasingly costly for $U$ to serve another independent unit when it has a high level of internal supply. This commits it to a higher price from those independent sales.

In terms of bargaining power ($\lambda$), note that when $\lambda$ is close to 1, an integrated firm places most weight on the downstream value from independent sales (that is the downstream price received). As such, it internalises this competitive externality when choosing sales quantity from its own division; reducing its own sales in order to soften downstream competition. Interestingly, in equilibrium, this will mean that independent firms will generate higher orders and profits relative to the situation where $U$ was not integrated.

The relative salience of the competition and bargaining effects can be easily seen in a specific example. Suppose that $N = 2$, $I = 1$, $P = 1 - z - x$ and $C(z + x) = \alpha(z + x) + \beta(z + x)^2$ with $\alpha < 1$. With this, the respective equilibrium quantities are:

$$\hat{x} = \frac{1-\alpha}{3-4\beta(1-\lambda)+\beta(4\lambda-6)-\lambda} \quad \text{and} \quad \hat{z} = \frac{(1-\alpha)(1+\beta)(1-\lambda)}{3-4\beta(1-\lambda)+\beta(4\lambda-6)-\lambda}.$$

Notice that when $\beta = 0$ (costs are linear), then $\hat{x} = \frac{1-\alpha}{3-\lambda} > \hat{z} = \frac{(1-\lambda)(1-\alpha)}{3-\lambda}$ and as $\lambda$ goes to 1, partial integration achieves the integrated monopoly outcome. This is achieved by $U$ completely foreclosing on its internal division ($\hat{z} = 0$). Note also that $\hat{x} + \hat{z} = \frac{(2-\lambda)(1-\alpha)}{3-\lambda} \geq \frac{2(1-\alpha)}{3}$, the non-integrated outcome (with equality at $\lambda = 0$).

Considering now the case where $\beta > 0$, note that (2) becomes:

$$\lambda > (\leq) \frac{2\beta}{1+2\beta}$$

(3)
The right hand side of (3) is increasing in $\beta$ (a measure of the convexity of upstream costs). (3) is a necessary and sufficient condition for $\hat{x} > \langle \hat{z}, \hat{\epsilon} \rangle$ and $\hat{x} + \hat{\epsilon} > \langle \hat{z}, \frac{2(1-\alpha)}{3+4\beta} \rangle$, the non-integrated outcome. Note, however, that at $\lambda = 1$, integration always results in a reduction in industry output.

In summary, when competition for orders precedes negotiations over procurement, there is a novel mechanism for anti-competitive vertical integration. Depending upon the nature of upstream costs, so long as $U$'s bargaining power is sufficiently high, it will contract its internal supply (perhaps completely) in order to mitigate negative effects from intense competition downstream. In contrast, in traditional models of vertical contracting, integration results in an expanded market share for integrated firms at the expense of independent ones (potentially foreclosing on them). This is because the competitive externality in those models is taken into account in bargaining with independent firms while integrated firms are supplied only on the basis of their own downstream revenues. In effect, reversing the timing of procurement and competition for orders switches the supply agreements (external and internal) competitive externalities are internalised.

*Equilibrium Integration*

While the preceding analysis has examined the consequences of vertical integration, there is a question over the level of vertical integration $U$ might choose. To be sure, as complete integration achieves the industry monopoly outcome, there appear to be gains to trade from all downstream firms being acquired by $U$. So, at first blush, one might be tempted to conclude that a similar logic might apply to integration of a single firm. However, as already noted, if upstream costs are convex and $U$'s bargaining power is sufficiently low, industry profits fall as a result of partial integration. For such integration to be profitable $U$'s improved bargaining position would have to outweigh any loss as a result of higher competition downstream, even before taking into account any acquisition costs for downstream firms.

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13 See Hart and Tirole (1990) and also de Fontenay and Gans (2002) for the case where downstream retailers have some bargaining power.
To see these issues more clearly it is worthwhile returning to the simple 2 firm example used above. Note first that when $\beta = 0$ (so upstream costs are linear), the profits of an integrated $U$ and an independent downstream firm are:

$$\pi_U(1) = \frac{(1-\alpha)^2}{(3-\lambda)^2}$$  and  $$\pi_i(1) = \frac{(1-\alpha)^2(1-\lambda)}{(3-\lambda)^2},$$

while, under non-integration, they were:

$$\pi_U(0) = \frac{2}{3}(1-\alpha)^2\lambda$$  and  $$\pi_i(0) = \frac{1}{3}(1-\alpha)^2(1-\lambda).$$

Notice that: $\pi_U(1) > \pi_U(0)$ and $\pi_i(1) \geq \pi_i(0)$ with equality at $\lambda = 0$. Thus, integration in this case is always welfare reducing. However, $\pi_U(1) > \pi_U(0) + \pi_i(0)$ only for $\lambda > 0.697$. This means that even if $U$ could purchase a downstream firm at its non-integrated profit level, it may not be worthwhile as integration only raises bilateral surplus when $U$’s bargaining power is sufficiently high.

Turning to the convex cost case, it is straightforward to demonstrate that $\pi_i(1) \geq \pi_i(0)$ if and only if $\lambda > \frac{2\beta}{1+2\beta}$. This is not surprising as otherwise the independent firms’ output would be reduced by integration. Figure 1 depicts the ranges of $(\lambda, \beta)$ over which integration is bilaterally profitable and socially desirable (where the $\beta = 0$ vertices corresponds to the constant marginal cost case). Thus, for intermediate levels of $U$’s bargaining power, integration is not bilaterally profitable. However, as $\beta$ gets larger, the range where integration is not bilaterally profitable lies between $\left(\frac{2\beta}{1+2\beta}, 1\right]$. Thus, when costs are convex, integration will not be bilaterally profitable unless $\lambda$ is sufficiently low (where there is a strong bargaining effect from integration; something not present in the constant marginal cost case). Moreover, in this situation, integration will result in lower industry output and tend to involve foreclosure on the independent downstream firm. Nonetheless, as costs become increasing convex, vertical integration will only occur if it is socially desirable.

**Figure 1: Profitability and Social Desirability of Vertical Integration**

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14 In fact, the range where integration is profitable is where:

$$\lambda \in \left[0, \frac{2\beta}{1+2\beta}\right] \cup \left(\frac{\beta(5+10\beta+22\beta^2-9\beta^3+3\sqrt{3}\beta(1+2\beta)^2(1+10\beta+10\beta^2+104\beta^3+56\beta^4-3\beta^5)}{2(1+\beta)(1+2\beta)^2}, 1\right].$$
These considerations carry over to the many firm case but involve an additional negative impact on bilateral surplus that comes from the horizontal integration impact of vertical integration. As demonstrated by Salant, Switzer and Reynolds (1983), horizontal mergers between Cournot competitors may be unprofitable when the main beneficiaries of the merger are outside rivals. Here, if vertical integration involves more than one downstream unit, a similar effect occurs. Those units’ pricing are now coordinated and in equilibrium will, ceteris paribus, lead to a reduction in their market share and earnings. For integration to be profitable, the impact on external revenue will have to outweigh any losses associated with horizontal integration.

5. Other Considerations

This section considers various extensions of the baseline model to demonstrate the robustness of the qualitative results above.
Bertrand Price Competition

Consider a situation where price is the strategic variable in Stage 1; that is, downstream firms are Bertrand competitors. In this case, if there is sufficient product differentiation so that an equilibrium exists where each downstream firm has positive orders with price chosen above upstream marginal cost, the outcome is similar to that in Proposition 2. That is:

**Proposition 4.** Let \( \hat{P} \equiv \{\hat{P}_i\}_{i=1}^N \) where:

\[
\hat{P}_i \in \arg \max_{\hat{P}} P X_i(\hat{P}) - C(x_i(\hat{P}), \hat{x}_{-i}(\hat{P})) \text{ for all } i.
\]

If \( x_i(\hat{P}) > 0 \) for all \( i \), and \( \sum_i \hat{P}_i \hat{x}_i(\hat{P}) \geq C(\{\hat{x}_i(\hat{P})\}) \), then in any subgame perfect equilibrium, where retailer \( i \) chooses \( P_i \) in Stage 1, realised orders are \( \{x_i(\hat{P})\}_{i=1}^N \).

The key condition here is that despite price competition, there is an interior solution where all retailers take positive orders in equilibrium.

Of course, whenever final goods are close enough substitutes an interior Bertrand equilibrium may not exist. In the perfect substitutes case, some downstream firms may realise no orders in equilibrium and this will, in turn, impact upon ex post bargaining over procurement. To assist in analysing this case, it is supposed here that when two or more downstream firms choose the same lowest price, then each has an equal chance in providing all of the orders; that is, these are not shared. This assumption means that some indeterminacy that arises with Bertrand competition under convex costs will be avoided.

For the homogenous goods case, let \( X(P) = \sum_i x_i(P) \) denote the aggregate demand function. The following proposition characterises the resulting equilibrium.

**Proposition 5.** Suppose that downstream products are homogenous and upstream costs are strictly convex. Then, in any subgame perfect equilibrium, only one retailer receives orders where:

\[
\hat{P} \in \{P | (P + d) y(P) - dX(P) = C(\tilde{y}(P), 0) \} \text{ and } \tilde{y}(\hat{P}) = \min[X(\hat{P}), C^{-1}(\hat{P} + d)].
\]

The intuition behind this result is very simple. Downstream firms will bid the price down as far as possible in order serve the entire market; indeed, they will bid the price down to a point where they expect to earn zero profits. Notice that if there are convex upstream costs, this will involve a price less than marginal cost and could also involve a situation where some orders are not fulfilled (for \( d \) sufficiently low). Finally, given that a
downstream firm with orders will share any remaining surplus with $U$, in bidding their profits to zero, the downstream firm necessarily dissipates any industry rents (or quasi-rents). Thus, there are always some orders unfulfilled and the damage payments associated with this balance any industry profits from orders that are fulfilled.

There are several important things to note about this equilibrium. First, notice that a pure strategy equilibrium exists even when upstream costs are convex. This is in contrast to the usual analyses of Bertrand competition in that case: where there is typically a mixed strategy equilibrium involving price above marginal cost. A key feature of the model that drives uniqueness of the equilibrium outcome is that $d > 0$. If $d = 0$, then the equilibrium existence problems that occur for Bertrand competition between firms with convex costs re-emerge (Vives, 1999).

Second, Stahl (1988) found that when upstream supply was perfectly competitive (arising from price taking firms with an upward sloping supply curve), that when orders proceeded procurement, there existed a subgame perfect equilibrium where final good price equaled industry marginal cost. Stahl confined attention to the case where $d$ was so high that downstream firms found it optimal never to default on orders. However, more critically, he assumed that procurement entailed the same price for each unit supplied. Thus, a retailer setting a price below upstream marginal cost was sure to make a loss; driving his Walrasian equilibrium outcome.

Here, a Walrasian equilibrium does not exist when upstream costs are convex. This is because, in contrast to Stahl (1988), bargaining does not constrain the per unit input price paid to equal upstream marginal cost.\footnote{Another difference between the model here and that of Stahl (1988) is the need for and sensitivity to tie breaking rules. While Stahl investigates alternate rules, here no such rule is needed as ex post bargaining drives outcomes towards a single retailer.} Even when downstream firms have all of the bargaining power (i.e., $\lambda = 0$), they may find it optimal to price below upstream marginal cost – incurring losses on marginal units or damages for unfulfilled orders. In this case, the equilibrium is socially inefficient involving over-production relative to the Walrasian case.\footnote{The ability to effectively price discriminate in procurement is the key factor driving the non-Walrasian outcome. Even if there were more than one upstream firm, if the procurement price allowed inframarginal payments below upstream marginal cost, the same qualitative outcomes would result.}
Moreover, it could also be the case that consumer surplus is lower than might be achieved in an integrated monopoly. Whether this is so depends upon the rationing rule for consumers as to who receives orders and who does not. In particular, it is possible to imagine an equilibrium with $d$ arbitrarily small whereby all orders are received by a retailer with a very low price but very few consumers are actually served. In this situation, consumer surplus generated in the industry is low so that consumers as a whole would be better off with a downstream monopoly. However, this type of equilibrium relies on the lack of sophistication of consumers who willingly accept the lowest price offer without thinking further about what they might receive. An extension of the model to the case of sophisticated consumers is left for future work.

Finally, it is worth considering the effect of vertical integration in the Bertrand environment. When an interior equilibrium exists, then we cannot say whether there is a change towards internal versus external supply following integration. Nonetheless, prices of all downstream firms rise as the integrated firm softens price competition. In the homogenous goods case, an anti-competitive effect relies on there being only one independent downstream firm. This is because downstream competition cannot be softened in that case if there are two independent firms. In that case, the integrated downstream firm is completely foreclosed in the downstream market.

**Contingent Agreements**

Throughout the paper it has been assumed that procurement agreements cannot be made contingent upon the outcome of later agreements. The impact of that assumption can be readily demonstrated for the $N = 2$ case. In this case, if a contingent agreement were possible, $U$ and the first downstream firm (call this firm 1) in the negotiations could agree that if $U$ were to supply a positive quantity to the second downstream firm, $U$ would have to pay the firm 1 a penalty, $f$. Given the complete information assumptions in this paper, this agreement would be observed by firm 2 and would be seen as reducing its gains from trade with $U$.

Formally, the agreement with firm 2, would involve an expected payment:

$$E[p_2x_2] = \lambda p_2x_2 + (1-\lambda)(f + C(x_1, x_2) - C(x_1, 0))$$  \hspace{1cm} (4)
so long as \( f \leq P_2x_2 - C(x_1, x_2) + C(x_1, 0) \). For a given \( f \) satisfying this constraint, the gains from trade between firm 1 and \( U \) become: \( P_1x_1 - C(x_1, x_2) + C(0, x_2) + f \). In this case, they will agree to \( f = P_2x_2 - C(x_1, x_2) + C(x_1, 0) \) as this maximises their joint profits. Thus,

\[
E[p_i x_i] = \lambda \left( P_1x_1 - C(x_1, x_2) \right) - (1 - \lambda)C(0, x_2) + P_2x_2 + C(x_1, 0)
\]

Given this, \( U \)'s expected payoff is \( \lambda \left( P_1x_1 - C(x_1, x_2) \right) + P_2x_2 - (1 - \lambda)C(0, x_2) \), firm 1’s is \( (1 - \lambda) \left( P_1x_1 + C(0, x_2) - C(x_1, x_2) \right) \) while firm 2’s is 0.\(^{17}\) If we suppose that each downstream firm has a non-zero probability of being the first to negotiate with \( U \), it is easy to see that the equilibrium outcome from ex ante competition is the same in this case as in Proposition 2.

Interestingly, if \( U \) were to vertically integrate with one of the firms, rent shifting would no longer be possible and the equilibrium outcome would be as in Proposition 3. So, as before, integration causes a contraction in industry output as the output of integrated downstream units is reduced. Thus, the effects of vertical integration on competition and welfare remain the same as before. However, as the distribution of industry rents under non-integration has changed – in particular, it has moved in favour of \( U \) – the returns to integration will have been reduced.

**Exclusive Dealing**

In the standard vertical contracting literature, exclusive dealing is seen as a way in which upstream monopolists can restore monopolistic outcomes downstream. Here, an exclusive deal would be a contract signed with a retailer that prevents \( U \) from dealing with other downstream firms. On the face of it, such a contract would allow a single firm to generate orders at the monopoly price, if it excludes other firms from generating orders (as it may if downstream firms believed those orders would not be fulfilled). However, would exclusivity prevent other downstream firms from competing for orders? It won’t if those firms can still procure the necessary inputs to fill those orders.

\(^{17}\) Thus, perfect rent shifting is possible (Marx and Shaffer, 2003).
To consider this, suppose that an exclusivity deal is signed but that some other firms succeed in generating orders. $U$ and its exclusive partner could choose to enforce their agreement and leave those orders unfilled. To do so, however, would involve missing a surplus creating opportunity. As Segal and Whinston (2000) note (when that opportunity is created by new entry), there exist many multilateral bargaining outcomes (between the downstream firm, $U$ and its exclusive partner) that would allow the additional surplus to be split three ways ex post.\textsuperscript{18} Moreover, whether an order was filled or not would be driven by the same efficiency criteria as in Proposition 1; that is, if the order price above upstream marginal cost.

Thus, it is easy to see that a downstream firm who generates an order ex ante will be able to receive a share of the surplus created by that order ex post. Under multilateral bargaining outcomes such as the Shapley value, independent retailers will receive a one third share of this value and hence, will compete to maximise it in Stage 1 competition. Thus, the equilibrium outcome of Proposition 2 will remain. Nonetheless, an exclusive deal will still be advantageous for $U$ and the downstream firm concerned as it allows them jointly to appropriate more rents in ex post bargaining over procurement.\textsuperscript{19} Exclusive dealing, in contrast to vertical integration, does not actually change the nature of competition for orders and hence, has no anti-competitive effect.

5. Conclusions

The contribution of this paper has been to stand the usual approach to vertical contracting on its head and consider competition for orders as taking place prior to procurement negotiations. This change not only captures reality in some vertical markets but also generates sharp and robust predictions regarding the ability of monopolists in a vertical chain to leverage their market power in downstream markets.

\textsuperscript{18} See also Aghion and Tirole (1987) and Spier and Whinston (1995).
\textsuperscript{19} This pure bargaining effect from exclusivity has been noted by Segal and Whinston (2000) and Segal (2003). If there are some stipulated damages then these would be irrelevant in ex post negotiations except in terms of ensuring that $U$ and independent downstream firm did not break off and reach a bilateral agreement and pay the stimulated damages. Thus, exclusivity would work in a similar way to contingent agreements as analysed above. In either case, appropriation of rents is all that is achieved by exclusive dealing.
Significantly, in an otherwise Chicago School environment – with none of the usually considered impediments to procurement negotiations – private information or restricted pricing options – it is demonstrated that the strength of downstream competition constrains the ability of bottleneck monopolists to generate monopoly levels of final good prices. This, in turn, provides a motivation for vertical integration for purely strategic reasons. However, when it comes to anti-competitive effect, that vertical integration is used, not to foreclose on independent firms, but to weaken the competitive impetus coming from integrated downstream units. Thus, the model here generates predictions regarding the effects of vertical integration that are testable and distinct from those arising from the standard vertical contracting literature.

The fact that reversing the timing of competition for orders and procurement negotiations can generate such different results in otherwise identical environments suggests the importance of future research into the determinants of the timing of wholesale and output market contracts. These are undoubtedly related to issues of uncertainty, flexibility and the relative ability to re-negotiate supply contracts based on the realisation of orders in downstream markets. The standard literature assumes that no renegotiation is possible while here it has been assumed that no ex ante contractual commitment can be made. In most industries, the truth no doubt lies somewhere in between and is related to more fundamental conditions in the contracting environment.
Appendix

Proof of Proposition 1

Working backwards, $i$ gets the chance to make an offer, given condition (i), $i$ will make an offer of $y_i = x_i$ and $p_iy_i = C(y_i, x_i) - C(0, x_i)$ which will be accepted by $U$. On the other hand, if it gets a chance, $U$ will make an offer that solves: \[ \max_{p_i, y_i} p_iy_i - C(y_i, x_i) \text{ subject to } (P_i - p_i)y_i \geq d(x_i - y_i). \] Solving the constraint for $p_iy_i$ and substituting this into the objective gives: \[ \max_{y_i} p_iy_i - C(y_i, x_i) - d(x_i - y_i). \] Note that, given condition (i), this implies that the entire order would be filled (i.e., $y_i = x_i$) while evaluating $p_iy_i$ at this point and taking expectations implies $E[p_i x_i]$ as stated in the Proposition.

Given this, we need only check that a deviation from this offer by $U$ would not change the offers made in any subsequent negotiation. Note that the prices of other negotiations do not enter $U$’s problem so any deviation would have to be on the basis of order level. In this case, a deviation could only feasibly involve a reduction in input quantity (i.e., to $x_i < x_i$). For this to be accepted, the price in that order would adjust but more critically, either (i) the revenue received in subsequent orders would fall or (ii) some orders would not be filled in those orders. In either case, $U$ receives a reduction in their overall payoff. Hence, no deviation would be profitable.

Proof of Propositions 2 and 4

Assume that the conditions of Proposition 1 hold. Then, firm $i$ expects to receive: \[ P_i(x_i, x_j) x_i - \lambda P_i(x_i, x_j) x_i - (1 - \lambda) \left(C(x_i, x_j) - C(0, x_j)\right) \] if it generates $x_i$ orders. It chooses $x_i$ to maximise this holding $x_j$ constant. If all downstream firms do the same, this gives $\hat{x}_i$ as defined in the proposition. Note that this implies that: \[ P_i(x_i, \hat{x}_{-i}) - C_i(x_i, \hat{x}_{-i}) = -\frac{\partial C_i(x_i, \hat{x}_{-i})}{\partial x_i} x_i > 0 \] (confirming the condition of Proposition 1).

The proof of Proposition 4 proceeds along the same lines with the interiority of the equilibrium ensuring the relevant condition of Proposition 1.

Proof of Proposition 3

A non-integrated firm, $I$, has the same first order condition as in the non-integrated case: \[ P_i(\hat{x}_i(I), x_{-i}, \hat{z}_{-i}) x_i + P(\hat{x}_i(I), x_{-i}, \hat{z}_{-i}) = C_i(\hat{x}_i(I), x_{-i}, \hat{z}_{-i}) \] (5)
while for a given integrated unit, \( U \) chooses \( z_j \) that satisfies:

\[
P_j(\hat{z}_j(I), \hat{x}_{-j}, \hat{z}_{-j}) \left( \hat{z}_j(I) + \hat{z}_{-j} \right) + P(\hat{z}_j(I), \hat{x}_{-j}, \hat{z}_{-j}) \\
+ (N - I) \frac{\partial E[p_j(\hat{z}_j(I)) \hat{x}_j]}{\partial z_j} = C_j(\hat{z}_j(I), \hat{x}_{-j}, \hat{z}_{-j}) \tag{6}
\]

Evaluating (6) at \( \hat{x}_I = \hat{x}_j(0) \) and \( \hat{z}_j(I) = \hat{z}_j(0) \), it is easy to see that if \( \partial E[p_j(\hat{z}_j(I)) \hat{x}_j] / \partial z_j < 0 \), then the RHS is less than the LHS so that, if \( \hat{x}_I > \hat{x}_j(0) \), then \( \hat{z}_j(I) < \hat{z}_j(0) \) (as quantity choices are strategic substitutes). Evaluating (5) where \( \hat{x}_I = \hat{x}_j(0) \) but \( \hat{z}_j(I) < \hat{z}_j(0) \), implies that the RHS is greater than the LHS, so that \( \hat{x}_I > \hat{x}_j(0) \). This gives (i) and (ii). It is easy to see that (iii) is equivalent to \( I\hat{z}_I < I\hat{z}_j(I) \) which holds given (i) and (ii).

Summing up (5) and (6), and using symmetry, we have:

\[
P_I() (I\hat{z}_I(I) + I(I - 1)\hat{z}_j(I) + (N - I)\hat{z}_j(I)) + NP() \\
+ I(N - I) \frac{\partial E[p_j(\hat{z}_j(I)) \hat{x}_j]}{\partial z_j} = NC_j(\hat{z}_j(I), \hat{x}_{-j}, \hat{z}_{-j}) \tag{7}
\]

If we evaluate (7) at \( \hat{x}_I = \hat{x}_j(0) \) and \( \hat{z}_j(I) = \hat{z}_j(0) \) it is easy to see that if \( \partial E[p_j(\hat{z}_j(I)) \hat{x}_j] / \partial z_j < 0 \) then aggregate equilibrium output under integration must be lower than that under non-integration.

Finally, note that because \( P_I() \hat{z}_j(I) < 0 \), a necessary condition for the inequalities (i) to (iv) not to hold is that \( \partial E[p_j(\hat{z}_j(I)) \hat{x}_j] / \partial z_j > 0 \).

**Proof of Proposition 5**

The proof proceeds in two parts. First, consider the outcomes of stage 2 bargaining when there is a single downstream firm with positive orders (as will occur if two or more firms set the lowest downstream price) and where \( P_i + d < C_i(x_i, 0) \). In this case, \( U \) makes an offer that solves: \( \max_{p_i, y_i} p_i y_i - C(y_i, 0) \) subject to \( (P_i - p_i) y_i - d(x_i - y_i) \geq 0 \). If it gets a chance, the downstream firm makes an offer that solves: \( \max_{p_i, y_i} (P_i y_i - p_i y_i - d(x_i - y_i)) \) subject to \( p_i y_i \geq C(y_i, 0) \). So long as \( (P_i + d) \hat{y}_i - C(\hat{y}_i, 0) \geq dx_i \), both of these yield: \( \hat{y}_i \in \{ y_i | P_i + d = C_i(y_i, 0) \} \) and, in expectation, \( E[p_i \hat{y}_i] = \lambda((P_i + d) \hat{y}_i - dx_i) + (1 - \lambda)C(\hat{y}_i, 0) \).

Consider an equilibrium where all downstream firms set a price equal to \( \hat{P} \) as in the proposition. By assumption, only one firm receives positive orders, that is \( X(\hat{P}) \).
Given that downstream products are homogenous, a firm setting price at $\hat{P}$ cannot raise price because this would not raise profits (above zero) and it cannot lower price because this will result in a loss. For other firms, if they raise their price, they continue to receive no orders (so this is not profitable). At the conjectured equilibrium, if $i$ were to set a price, $P_i < \hat{P}$, it would receive all of the orders but necessarily make a loss.
References


