Modified Tests for a Change in Persistence*

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Abstract

In this paper we propose a set of new persistence change tests which are based on modified versions of the ratio-based statistics of Kim (2000), Kim *et al.* (2002) and Busetti and Taylor (2004). These statistics are used to test the null hypothesis that a time series displays constant trend stationarity ($I(0)$) behaviour against the alternative of a change in persistence from trend stationarity to difference stationarity ($I(1)$), or *vice versa*. Here, we demonstrate that the existing tests are unable to adequately discern between a change in persistence and a constant $I(1)$ process. Our proposed modification yields tests which, by design, have the same critical values regardless of whether the process is $I(0)$ or $I(1)$ throughout. Hence, our null hypothesis is that of constant persistence (either constant $I(0)$ or constant $I(1)$). Tests directed against both $I(1)$ to $I(0)$ and $I(0)$ to $I(1)$ persistence change series are considered, together with tests where the direction of change under the alternative is unspecified. Our modified tests retain the same rates of consistency against persistence change processes as their un-modified counterparts. Numerical evidence suggests that our procedure works extremely well in practice, with the modified ratio-based tests approximately correctly sized under both constant $I(0)$ and constant $I(1)$ environments, and displaying only small losses in power, relative to the un-modified tests, against persistence change processes.

Keywords: Persistence change; ratio-based tests; variable addition tests; Brownian motion.

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1 Introduction

Being able to correctly characterize a time series into its separate difference stationary, $I(1)$, and trend stationary, $I(0)$, components, should they exist, has important implications for effective model building and forecasting in applied economics and finance. Recently, a number of testing procedures have been suggested that aim to distinguish such behavior. These include the ratio-based persistence change tests of, *inter alia*, Kim (2000), Kim *et al.* (2002) and Busetti and Taylor (2004), and the sub-sample augmented Dickey-Fuller-type tests of Banerjee *et al.* (1992) and Leybourne *et al.* (2003). The first three of these assume a null hypothesis of $I(0)$ throughout, while the last two assume a null of $I(1)$ throughout. For each the alternative is a change from $I(0)$ to $I(1)$, or *vice versa*. Busetti and Taylor (2004) also propose locally best invariant (LBI) tests of the constant $I(0)$ null against a change in persistence and explore the behaviour of both full-sample and sub-sample Kwiatkowski *et al.* (1992) (KPSS) stationarity tests against persistence change processes.

An important issue which concerns all of these test procedures is the behaviour of the statistics in the *absence* of a change in persistence, yet when the null hypothesis is not true. That is, for a test with a null of $I(0)$ ($I(1)$) throughout, what happens when a series is actually $I(1)$ ($I(0)$) throughout? In the case of the ratio-based tests with an $I(0)$ null, the statistics upon which these are based can be shown to be $O_p(1)$ when applied to a series which is $I(1)$ throughout and whilst the tests are therefore not consistent, as we will later demonstrate they are severely biased, leading to frequent spurious rejections of the $I(0)$ null in favour of a change in persistence. The sub-sample ADF tests with the $I(1)$ null, are consistent when applied to a series which is $I(0)$ throughout, so that the bias grows asymptotically; that is, the probability of spuriously rejecting the $I(1)$ null in favour of a change in persistence approaches one in the limit. Finally, the LBI-based tests and the full and sub-sample KPSS tests are all consistent against persistence change series, but also against series which are $I(1)$ throughout. Thus, extant tests are unable to adequately discriminate between a change in persistence and constant persistence of the form not covered by their respective null hypothesis.

In view of this important shortcoming, in this paper we propose a set of new tests which are based on modified versions of the ratio-based statistics of Kim (2000), Kim *et al.* (2002) and Busetti and Taylor (2004). Our modification uses the variable addition approach of Vogelsang (1998a,1998b) and yields tests which, by design, have the same critical values regardless of whether the process is $I(0)$ or $I(1)$ throughout (although it should be stressed that the test statistics upon which the tests are based do *not* possess identical limiting distributions). It is important to notice that this technique can only be used with the ratio-based tests because the other tests of the $I(0)$ ($I(1)$) null are divergent under constant $I(1)$ ($I(0)$) processes. Hence, our null hypothesis is that of constant persistence (either a constant $I(0)$ process or a constant $I(1)$ process), and the alternative is that of a change in persistence. As such, a rejection by our test can, subject to the usual Type I error, be unambiguously interpreted as indicating a change in persistence has occurred. Tests are designed which are consistent against $I(0)$ to $I(1)$ changes, $I(1)$ to $I(0)$ changes, or where the direction of change under the alternative is unknown. A nice property of our modified tests is that they retain the same rates of consistency against persistence change processes as their unmodified counterparts. The point at which a change in persistence occurs under the alternative is not assumed to be known.

The paper is organized as follows. Section 2 outlines the model of persistence change which we focus on. In Section 3 we provide a brief review of the variance ratio persistence change tests of Kim (2000), Kim *et al.* (2002) and Busetti and Taylor (2004). In Section 4 we introduce and motivate our modified test statistics and derive their large sample properties. In Section 5, using Monte Carlo simulation, we provide finite sample critical values and compare the properties of the modified tests with the original tests of Kim (2000), Kim *et al.* (2002) and Busetti and Taylor (2004), highlighting the inherent bias problems these latter tests exhibit against constant $I(1)$ processes and also the improvements offered by our proposed modifications. Section 6 applies the tests discussed in the paper to a variety of U.S. macroeconomic and financial data-sets. Section 7 concludes.

2 The Persistence Change Model

As a model for a possible change in persistence, we follow Busetti and Taylor (2004) and adopt the following data generating process (DGP):

$$\begin{aligned} y_t &= d_t + \varepsilon_t, \\ \varepsilon_t &= v_t + w_t, \quad t = 1, ..., T, \end{aligned} \tag{2.1}$$

with either

$$w_t = w_{t-1} + \eta_t 1(t > \lfloor \tau^* T \rfloor) \tag{2.2}$$

or

$$w_t = w_{t-1} + \eta_t 1(t \leq \lfloor \tau^* T \rfloor) \tag{2.3}$$

with $\tau^* \in [0, 1]$, and where $1(\cdot)$ denotes the indicator function and $\lfloor \cdot \rfloor$ denotes the integer part of its argument.

In (2.1), the deterministic kernel, $d_t = \mathbf{x}_t' \beta$, where $\mathbf{x}_t$ is a $(k+1) \times 1$, $k < T - 1$, fixed sequence whose first element is fixed at unity throughout (so that (2.1) always contains an intercept term), with associated parameter vector β. The vector $\mathbf{x}_t$ is assumed to satisfy the mild regularity conditions of Phillips and Xiao (1998): precisely, it is assumed that there exists a scaling matrix δ_T and a bounded piecewise continuous function $\mathbf{x}(\cdot)$ on $[0, 1]$ such that $\delta_T \mathbf{x}_{\lfloor \cdot T \rfloor} \to \mathbf{x}(\cdot)$ uniformly on $[0, 1]$, and $\int_0^1 \mathbf{x}(s) \mathbf{x}(s)' ds$ is positive definite. A leading example satisfying these conditions is given by the k-th order polynomial trend, $\mathbf{x}_t = (1, t, ..., t^k)'$, within which the constant ($d_t = \beta_0$) and constant plus linear time trend ($d_t = \beta_0 + \beta_1 t$) are special cases. For the polynomial trend case $\delta_T = \text{diag}(1, T^{-1}, ..., T^{-k})$ and, hence, $\mathbf{x}(r) = (1, r, ..., r^k)'$. The disturbances $\{v_t\}$ and $\{\eta_t\}$ are mutually independent mean zero processes satisfying the familiar α-mixing conditions of Phillips and Perron (1998,p.336), with strictly positive and bounded long-run variances $\omega_v^2 \equiv \lim_{T\to\infty} E\left(\sum_{t=1}^T v_t\right)^2$, and $\omega_\eta^2 \equiv \lim_{T\to\infty} E\left(\sum_{t=1}^T \eta_t\right)^2$, respectively.

Within this model, we consider four possibilities. The first of these is that y_t is $I(0)$ throughout the sample period. This is represented by (2.1) and (2.2) with $\tau^* = 1$ (or (2.3) with $\tau^* = 0$). We denote this H_0. Secondly, y_t may be $I(1)$ throughout; represented by (2.1) and (2.2) with $\tau^* = 0$ (or (2.3) with $\tau^* = 1$), denoted H_1. Thirdly, there may be a change from $I(0)$ to $I(1)$ at time $t = \lfloor \tau^* T \rfloor$; as given by (2.1) and (2.2) with the changepoint fraction $0 < \tau^* < 1$, denoted H_{01}. Finally, a change from $I(1)$ to $I(0)$ at time $t = \lfloor \tau^* T \rfloor$ is represented by (2.1) and (2.3) with $0 < \tau^* < 1$, denoted H_{10}.

3 Kim's Ratio-Based Tests

Kim (2000), and subsequent modifications proposed independently by Kim *et al.* (2002) and Busetti and Taylor (2004), develop tests for the constant $I(0)$ DGP (H_0) against the $I(0)$-$I(1)$ change DGP (H_{01}) which are based on the ratio statistic

$$K(\tau) = \frac{(T - \lfloor \tau T \rfloor)^{-2} \sum_{t=\lfloor \tau T \rfloor+1}^T (\sum_{i=\lfloor \tau T \rfloor+1}^t \breve{v}_{i,\tau})^2}{\lfloor \tau T \rfloor^{-2} \sum_{t=1}^{\lfloor \tau T \rfloor} (\sum_{i=1}^t \hat{v}_{i,\tau})^2}, \tag{3.1}$$

where, in order to obtain exact invariance to β (the vector of parameters characterising d_t), $\hat{v}_{t,\tau}$ in the denominator of (3.1) is the residual from the OLS regression of y_t on $\mathbf{x}_t$, for observations up to $\lfloor \tau T \rfloor$. In the constant case ($\mathbf{x}_t = 1$), for example, $\hat{v}_{t,\tau} = y_t - \bar{y}(\tau)$ with $\bar{y}(\tau) = \lfloor \tau T \rfloor^{-1} \sum_{t=1}^{\lfloor \tau T \rfloor} y_t$; that is, the data are de-meaned over $t = 1, ..., \lfloor \tau T \rfloor$. Similarly, $\breve{v}_t$ in the numerator of (3.1) is the OLS residual from the regression of y_t on $\mathbf{x}_t$ for $t = \lfloor \tau T \rfloor + 1, ..., T$.

Since the true changepoint, τ^*, is assumed unknown, Kim (2000), Kim *et al.* (2002) and Busetti and Taylor (2004) consider three statistics based on the sequence of statistics $\{K(\tau),\ \tau \in \Lambda\}$, where $\Lambda = [\tau_l, \tau_u]$ is a compact subset of $[0, 1]$. These are:

$$\begin{aligned} K_1 &= T_*^{-1} \sum_{s=\lfloor \tau_l T \rfloor}^{\lfloor \tau_u T \rfloor} K(s/T) \\ K_2 &= \ln \left\{ T_*^{-1} \sum_{s=\lfloor \tau_l T \rfloor}^{\lfloor \tau_u T \rfloor} \exp(\frac{1}{2} K(s/T)) \right\} \\ K_3 &= \max_{s \in \{\lfloor \tau_l T \rfloor, ..., \lfloor \tau_u T \rfloor\}} K(s/T), \end{aligned}$$

where $T_* \equiv \lfloor \tau_u T \rfloor - \lfloor \tau_l T \rfloor + 1$. The first of these uses Hansen's (1991) mean score statistic, the second Andrews and Ploberger's (1994) mean-exponential statistic, and the third the maximum over the sequence of statistics, after Andrews (1993).

When H_0 is true, which is the null hypothesis adopted by Kim (2000), Kim *et al.* (2002) and Busetti and Taylor (2004), and denoting weak convergence by "$\Rightarrow$", these authors demonstrate the result that for $0 < \tau < 1$, and under the condition that both $\int_0^\tau \mathbf{x}(s)\mathbf{x}(s)' ds$ and $\int_\tau^1 \mathbf{x}(s)\mathbf{x}(s)' ds$ are positive definite,

$$K(\cdot) \Rightarrow \frac{N(\cdot)}{D(\cdot)} \equiv L_v(\cdot) \tag{3.2}$$

where $N(\tau) \equiv (1-\tau)^{-2} \int_\tau^1 N_v(r)^2 dr$ and $D(\tau) \equiv \tau^{-2} \int_0^\tau D_v(r)^2 dr$, and where,

$$N_v(r) \equiv W_v(r) - W_v(\tau) - \int_\tau^1 \mathbf{x}(r)' dW_v(r) \left(\int_\tau^1 \mathbf{x}(r)\mathbf{x}(r)' dr \right)^{-1} \int_\tau^r \mathbf{x}(s) ds, \quad r \in (\tau, 1], \tag{3.3}$$

and

$$D_v(r) \equiv W_v(r) - \int_0^\tau \mathbf{x}(r)' dW_v(r) \left(\int_0^\tau \mathbf{x}(r)\mathbf{x}(r)' dr \right)^{-1} \int_0^r \mathbf{x}(s) ds, \quad r \in [0, \tau]. \tag{3.4}$$

with $W_v(\cdot)$ a standard Brownian motion process on $[0,1]$: here defined by $\omega_v^{-1} T^{-1/2} \sum_{t=1}^{\lfloor rT \rfloor} v_t \Rightarrow W_v(r)$, $r \in [0,1,]$. In the de-meaned case ($d_t = \beta_0$), for example, (3.3) and (3.4) reduce to

$$\begin{aligned} N_v(r) &\equiv W_v(r) - W_v(\tau) - (r-\tau)(1-\tau)^{-1}\{W_v(1) - W_v(\tau)\} \\ D_v(r) &\equiv W_v(r) - r\tau^{-1} W_v(\tau). \end{aligned}$$

The limiting distributions of the three statistics K_1, K_2 and K_3 under H_0 then follow from (3.2) and applications of the continuous mapping theorem (CMT); *viz.*,

$$K_1 \Rightarrow \int_{\tau_l}^{\tau_u} L_v(\tau) d\tau \tag{3.5}$$

$$K_2 \Rightarrow \ln \left\{ \int_{\tau_l}^{\tau_u} \exp(\frac{1}{2} L_v(\tau) d\tau) \right\} \tag{3.6}$$

$$K_3 \Rightarrow \sup_{\tau \in [\tau_l, \tau_u]} L_v(\tau). \tag{3.7}$$

Notice that these limiting representations are free of nuisance parameters.

In order to test H_0 against the $I(1)$-$I(0)$ change DGP (H_{10}), Busetti and Taylor (2004) propose further tests based on the sequence of *reciprocals* of $K(\tau)$, $\tau \in \Lambda$;

precisely,

$$
\begin{aligned}
K_1' &= T_*^{-1} \sum_{s=\lfloor \tau_l T \rfloor}^{\lfloor \tau_u T \rfloor} K(s/T)^{-1} \\
K_2' &= \ln \left\{ T_*^{-1} \sum_{s=\lfloor \tau_l T \rfloor}^{\lfloor \tau_u T \rfloor} \exp(\frac{1}{2} K(s/T)^{-1}) \right\} \\
K_3' &= \max_{s \in \{\lfloor \tau_l T \rfloor, \ldots, \lfloor \tau_u T \rfloor\}} K(s/T)^{-1},
\end{aligned}
$$

and, in order to test against an unknown direction of change (that is, either a change from $I(0)$ to $I(1)$ or *vice versa*), they also propose

$$
\begin{aligned}
K_4 &= \max(K_1, K_1'), \\
K_5 &= \max(K_2, K_2'), \\
K_6 &= \max(K_3, K_3')
\end{aligned}
$$

and again the limiting distributions of these six statistics under H_0 follow straightforwardly from (3.2) and applications of the CMT. Again these distributions are seen to be pivotal.

As regards test consistency, under H_{01} it is shown in Busetti and Taylor (2004) that K_1 through K_6 are each of $O_p(T^2)$, while K_1' through K_3' are each of $O_p(1)$. Similarly, under H_{10}, Busetti and Taylor (2004) show that K_1' through K_3' and K_4 through K_6 are each of $O_p(T^2)$, while K_1 through K_3 are of $O_p(1)$. Thus, tests which reject for large values of K_1 through K_3 can be used to detect H_{01}, while tests which reject for large values of K_1' through K_3' can be used to detect H_{10}, and, finally, tests which reject for large values of K_4 through K_6 can be used to detect either H_{01} or H_{10}.

We now consider the behaviour of $K(\tau)$ under the constant $I(1)$ case, H_1. Noting that

$$
K(\tau) = \frac{T^{-2}(T - \lfloor \tau T \rfloor)^{-2} \sum_{t=\lfloor \tau T \rfloor+1}^{T} (\sum_{i=\lfloor \tau T \rfloor+1}^{t} \breve{v}_{i,\tau})^2}{T^{-2} \lfloor \tau T \rfloor^{-2} \sum_{t=1}^{\lfloor \tau T \rfloor} (\sum_{i=1}^{t} \hat{v}_{i,\tau})^2},
$$

it is entirely straightforward to establish that for $0 < \tau < 1$,

$$
K(\cdot) \Rightarrow \frac{N^*(\cdot)}{D^*(\cdot)} \equiv L_\eta^*(\cdot) \tag{3.8}
$$

where $N^*(\tau) = (1-\tau)^{-2} \int_\tau^1 N_\eta^*(r)^2 dr$, $D^*(\tau) = \tau^{-2} \int_0^\tau D_\eta^*(r)^2 dr$, and where, in the de-meaned case,

$$
\begin{aligned}
N_\eta^*(r) &\equiv W_\eta^*(r) - W_\eta^*(\tau) - (r-\tau)(1-\tau)^{-1}\{W_\eta^*(1) - W_\eta^*(\tau)\}, \\
D_\eta^*(r) &\equiv W_\eta^*(r) - r\tau^{-1} W_\eta^*(\tau), \\
W_\eta^*(r) &\equiv \int_0^r W_\eta(s) ds
\end{aligned}
$$

and $W_\eta(\cdot)$ is a standard Brownian motion process on $[0,1]$, independent of $W_v(\cdot)$, here defined by $\omega_\eta^{-1} T^{-1/2} \sum_{t=1}^{\lfloor rT \rfloor} \eta_t \Rightarrow W_\eta(r)$, $r \in [0,1]$. In the general $\mathbf{x}_t$ case, $N_\eta^*(r)$ and $D_\eta^*(r)$ are given by the right members of (3.3) and (3.4), respectively, on replacing $W_v(r)$ by $W_\eta^*(r)$ throughout. Notice, crucially, that the sequence $\{L_\eta^*(\tau),\ 0 < \tau < 1\}$ is free of nuisance parameters.

The limiting distributions of the K_1 through K_6 and K_1' through K_3' statistics under H_1 then follow by replacing $L_v(\tau)$ with $L_\eta^*(\tau)$ of (3.8) in the representations pertaining to H_0 for these statistics. Consequently, all of the above statistics are of $O_p(1)$ under H_1 with pivotal limiting distributions. Whilst the tests based on these statistics are therefore not consistent (since they do not diverge to positive infinity with T), this does not mean that they will not display higher rejection frequencies than are observed under H_0 for a given nominal significance level. This implies that, in trying to detect a change in persistence, we could just accept the potential extra rejections that would result against a constant $I(1)$ process (i.e., a size problem). Alternatively, we could use conservative critical values (precisely, those obtained under the constant $I(1)$ process), as has been suggested in a different testing context by Vogelsang (1998b). With an obvious notation we will denote the resulting (conservative) tests by N_1 through N_6 and N_1' through N_3'. As we demonstrate numerically in Section 5, neither of these approaches is satisfactory in practice. The former approach yields tests which are grossly over-sized, even asymptotically, while the latter results in tests with very poor finite sample power properties.

Fortunately, given that the limiting distributions of the test statistics are pivotal under both H_0 and H_1, a third approach is feasible. This involves modifying the statistics *via* a variable addition (pseudo-) test statistic using a technique that was originally devised by Vogelsang (1998a,1998b) in a different testing context. We explore this approach in the next section.

4 Modified Ratio-Based Tests

As demonstrated in the previous section, all of the K_1 through K_6 and K_1' through K_3' statistics possess pivotal limiting distributions under both H_0 or H_1. Consequently, we can employ the approach of Vogelsang (1998a,1998b) to produce tests based on modified versions of these statistics which, for a given test and significance level, have the same critical value in the limit as the corresponding unmodified test under H_0, but where the same limiting critical value is also appropriate under H_1. The modification is such that it has no asymptotic effect under H_0, so that the limiting distribution of the modified statistic is exactly the same as that of the corresponding unmodified statistic. Under H_1, the modification does affect the limiting distribution of the statistic, but is explicitly chosen such that the limiting critical value is precisely the same as under H_0. That is, the modification is performed such that the cdfs of the statistic under H_0 and H_1 coincide asymptotically at a chosen point, that point being the asymptotic critical value associated with a given significance level.

The approach is the same for all the tests, and so we will only analyze the modification of the K_1 statistic in any detail. Let t^k be the highest order polynomial of t in $\mathbf{x}_t$. Then, following the approach of Vogelsang (1998a,1998b), consider a modified variant of K_1 given by

$$M_1 = \exp(-bJ_T)K_1, \tag{4.1}$$

where b is a finite constant and J_T is T^{-1} times the Wald statistic for testing the joint hypothesis $\gamma_{k+1} = \cdots = \gamma_m = 0$ in the regression model

$$y_t = \mathbf{x}_t'\beta + \sum_{i=k+1}^{m} \gamma_i t^i + \text{error}, \quad t = 1, ..., T. \tag{4.2}$$

Under H_0, it is easily demonstrated, via standard results, that $J_T \Rightarrow 0$, and, hence, that $\exp(-bJ_T) \Rightarrow 1$. Consequently, M_1 of (4.1) has the same limiting distribution under H_0 as K_1. That is,

$$M_1 \Rightarrow \int_{\tau_l}^{\tau_u} L_v(\tau)d\tau \equiv G_{v,\Lambda}$$

Under H_1, it follows using the same proof technique as in Vogelsang (1998a) that

$$J_T \Rightarrow \frac{\int_0^1 N_\eta^+(r)^2 dr}{\int_0^1 D_\eta^+(r)^2 dr} - 1 \equiv F_{\eta,k}$$

where $N_\eta^+(r)$ and $D_\eta^+(r)$ are, respectively, the residuals from the projection of $W_\eta(r)$ onto the spaces spanned by $\{\mathbf{x}(r)\}$ and $\{\mathbf{x}(r), r^{k+1}, ..., r^m\}$. So, since

$$K_1 \Rightarrow \int_{\tau_l}^{\tau_u} L_\eta^*(\tau)d\tau \equiv G_{\eta,\Lambda}^*$$

we immediately obtain that

$$M_1 \Rightarrow \exp(-bF_{\eta,k})G_{\eta,\Lambda}^*.$$

Now suppose that we choose the value $b = b_{\lambda,a}$ such that

$$\Pr(G_{v,\Lambda} > \lambda_a) = \Pr\{\exp(-b_{\lambda,a}F_{\eta,k})G_{\eta,\Lambda}^* > \lambda_a\} = a$$

This choice of b ensures that, for a given significance level, $100a\%$, the corresponding asymptotic upper-tail critical value (λ_a) of M_1 under either H_0 or H_1 is identical to the upper-tail $100a\%$ critical value of K_1 under H_0, recalling that M_1 and K_1 have the same limiting distribution (by design) under H_0.

Under the $I(0)$ to $I(1)$ change model, H_{01}, it is easily demonstrated that

$$J_T \Rightarrow \frac{\int_{\tau^*}^1 N_\eta^{\#}(r)^2 dr}{\int_{\tau^*}^1 D_\eta^{\#}(r)^2 dr} - 1$$

where τ^* is the true breakpoint, and $N_\eta^{\#}(r)$ and $D_\eta^{\#}(r)$ are, respectively, the residuals from the projection of $\{W_\eta(r) - W_\eta(\tau^*)\}1(r > \tau^*)$ onto the spaces spanned by $\{\mathbf{x}(r)\}$ and $\{\mathbf{x}(r), r^{k+1}, ..., r^m\}$. Similarly, under the $I(1)$ to $I(0)$ change model, H_{10},

$$J_T \Rightarrow \frac{\int_0^{\tau^*} N_\eta^{\#\#}(r)^2 dr}{\int_0^{\tau^*} D_\eta^{\#\#}(r)^2 dr} - 1$$

where $N_\eta^{\#\#}(r)$ and $D_\eta^{\#\#}(r)$ are, respectively, the residuals from the projection of $\{W_\eta(r)\}1(r \leq \tau^*)$ onto the spaces spanned by $\{\mathbf{x}(r)\}$ and $\{\mathbf{x}(r), r^{k+1}, ..., r^m\}$. Hence, in both cases, J_T is of $O_p(1)$ and, consequently, $\exp(-bJ_T)$ is also of $O_p(1)$ in both cases. Thus, we obtain that a test which rejects for large values of M_1 retains the same rate of consistency as the original test based on K_1. That is, like K_1, M_1 is of $O_p(T^2)$ under H_{01} and is of $O_p(1)$ under H_{10}.

Each of the tests based on K_2 through K_6 and K_1' through K_3' can be modified in exactly the same fashion as was outlined above for K_1. That is, in each case we multiply the original statistic by $\exp(-bJ_T)$, where b not only depends on the significance level together with m and k, but will now also be test-specific. With an obvious notation, we denote the set of modified statistics by M_1 through M_6 and M_1' through M_3', and again our modified tests reject for large values of these statistics. Values of b appropriate for the M_1 through M_6 and M_1' through M_3' tests for both the constant and linear trend case are provided in Section 5.1.

By construction, each of the modified tests has asymptotic critical values which under either H_0 or H_1 are identical to those of the corresponding unmodified test under H_0. Consequently, no new tabulations of critical values are required: those for the original (unmodified) test may be used which, for completeness, are reported in Table 1 of section 5.1. Each of the modified tests will also display the same rate of consistency under H_{01} or H_{10} as the corresponding unmodified test, as should be clear from the previous paragraph.

5 Numerical Results

5.1 Critical Values

Table 1 reports both finite sample and asymptotic upper tail null critical values for the K_1 through K_6 and K_1' through K_3' persistence change tests of Section 3. Precisely, the finite sample critical values of Table 1 were obtained by Monte Carlo simulation using pseudo-data generated according to the pure noise DGP:

$$y_t = \varepsilon_t \sim NIID(0, 1), \quad t = 1, ..., T.$$

Results are reported for de-meaned ($\mathbf{x}_t = 1$) and de-meaned and de-trended ($\mathbf{x}_t = (1,\ t)'$) data in Panels A and B, respectively. In each case finite sample critical values are given for $T = 100, 150, 200, 300$ and 500 while the rows labelled '∞' give asymptotic critical values for the tests, obtained by direct simulation of the appropriate limiting functionals of Section 3 using discrete approximations for $T = 1000$. For each test we used $\Lambda = [0.2, 0.8]$, as is typical in this literature: this choice is applied throughout this and the next section of the paper. The simulation were performed using $50,000$ Monte Carlo replications and the RNDN function of Gauss 3.2.

Table 2 reports the corresponding quantities for the (conservative) N_1 through N_6 and N_1' through N_3' tests discussed at the end of Section 3. These were obtained by Monte Carlo simulation using data generated according to the random walk DGP:

$$y_t = y_{t-1} + \varepsilon_t \sim NIID(0,1),\quad t = 1, ..., T,$$

with $y_0 = 0$.[1]

Tables 1 – 3 about here

In Table 3 we report limiting b values for each of the modified tests M_1 through M_6 and M_1' through M_3' of Section 4. Recall, that, in each case, this choice of b allows us to develop a modified test with an asymptotic critical value (at the given significance level) which under either H_0 or H_1 is identical to that of the corresponding unmodified test under H_0. These values are presented for each of the tests for the 10%, 5% and 1% significance levels and for both the de-meaned and de-meaned and de-trended cases.

In constructing the modified tests we have followed Vogelsang (1998a,1998b) and specified $m = 9$ in the regression equation (4.2). Consequently, J_T of (4.1) is T^{-1} times the Wald statistic for testing either the joint hypothesis $\gamma_1 = \cdots = \gamma_9 = 0$ in the de-meaned case, or the joint hypothesis $\gamma_2 = \cdots = \gamma_9 = 0$ in the de-meaned and de-trended case, in (4.2). The values of b reported in Table 3 were obtained by simulation: for a given test statistic and significance level, the asymptotic $100a\%$ critical value for the appropriate unmodified test under H_0 is first obtained as described above. Asymptotic $100a\%$ critical values are similarly obtained for the corresponding modified test under H_1 for a range of b values; b is then chosen so as to equate the $100a\%$ critical values under H_0 and H_1.

5.2 Properties of the tests under H_1 and H_0

In this section we use Monte Carlo simulation methods to investigate the behaviour of the tests of Section 3 and the modified tests proposed in Section 4 when applied to data generated by the following *constant parameter ARIMA* process

$$y_t = \phi y_{t-1} + \varepsilon_t - \theta\varepsilon_{t-1},\quad t = -100, ..., T,$$

[1]This implies no loss of generality since all of the tests considered are exact invariant to y_0 in the context of this model.

with $\varepsilon_t \sim NIID(0,1)$, and the design parameters $\phi \in \{0.0, 0.50, 0.80, 0.90, 0.95, 1.0\}$ and $\theta \in \{0.0, \pm 0.5\}$.[2] Notice that the degree of persistence in this process is constant throughout the sample and is controlled by ϕ: where $\phi < 1$ ($\phi = 1$) the process is $I(0)$ ($I(1)$) throughout. In order to economise on space, the results reported in this section and in Section 5.3 pertain only to the de-meaned case. However, qualitatively similar results were obtained for the de-meaned and de-trended case and are available on request. Notice that, due to invariance, we have set $d_t = 0$ with no loss of generality.

Tables 4 – 5 about here

Tables 4 and 5 report the empirical rejection frequencies of the tests of Sections 3 and 4 for $T = 150$ and $T = 300$, respectively. The unmodified tests (K_1 through K_6, K_1' through K_3', N_1 through N_6 and N_1' through N_3') were run at the nominal 5% level using the relevant finite sample critical values from Tables 1 and 2. For the modified tests (M_1 through M_6 and M_1 through M_3') the finite sample 5% critical values for the corresponding unmodified tests from Table 1 were used, and the modifications were performed using the value of b from Table 3 pertaining to each test at the 5% level. In order to control for initial effects, the first 100 observations were discarded. In reading the results in Tables 4 and 5 it is important to recall that $\phi = 1$ corresponds to H_1 (under which the critical values from Table 2 were generated), while values of ϕ less than one correspond to H_0 ($\phi = 0$ being the value of ϕ under which the critical values for the tests of Section 3 were generated).

Consider first the behaviour of the ratio-based K_1 through K_6 and K_1' through K_3' tests of Section 3 which test the constant $I(0)$ null, H_0. These tests display a very pronounced tendency to spuriously reject H_0 in favour of a change in persistence as ϕ increases from zero. For values of ϕ less than unity (cases where H_0 holds), a comparison of the results in Table 4 with those in Table 5 demonstrates that this is a finite-sample effect with the probability of spurious rejection declining (albeit rather slowly) as T increases. For example, for $\phi = 0.95$ and $\theta = 0$, the K_4, K_5 and K_6 tests reject against persistence change around 45%, 54% and 53% of the time respectively for $T = 150$, but these rejection frequencies decline somewhat to around 32%, 40% and 38% for $T = 300$. However, where $\phi = 1$, so that the constant $I(1)$ model H_1 holds, there is no such amelioration as T increases, as predicted by the limiting distribution theory provided in Section 3. The problem is indeed quite severe in this case as, for example, the probability of spuriously rejecting the null in favour of an $I(0) - I(1)$ ($I(1) - I(0)$) persistence change is around 55% using the K_3 (K_3') test, while K_6 (which simultaneously tests against either an $I(0) - I(1)$ or $I(1) - I(0)$ change) rejects around 74% of the time.[3] These rejection frequencies are roughly the same for $T = 150$ and $T = 300$. Other things equal, these distortions are slightly lower for the "mean" tests, K_1, K_1' and K_4, than for the remaining tests. Consequently, although the ratio-based

[2]When $\phi = \theta = 0.5$, a common factor results, thus we did not consider this parameter combination. Other values of the design parameters were also considered but the results for these cases neither add to nor contradict the qualitative conclusions drawn from the cases reported.

[3]In the linear trend case these rejections frequencies worsen to 74% and 90%, respectively.

tests are not consistent against constant $I(1)$ processes, it is nonetheless clear that they reject in favour of a change in persistence far too often under H_1 for them to constitute anything approaching satisfactory tests for changes in persistence.

The conservative tests N_1 through N_6 and N_1' through N_3', by design, avoid the over-rejections seen with the tests discussed in the previous paragraph under H_1. However, under H_0 as a consequence they display rejection frequencies well below the nominal level. This in itself is not a problem, but as we shall see in the next section this under-sizing phenomenon translates into very low finite sample power in these tests where a persistence change occurs.

Turning to the results in Tables 4 and 5, the modified tests, M_1 through M_6 and M_1' through M_3', proposed in Section 4 perform remarkably well indeed. For both the pure random walk case ($\phi = 1$, $\theta = 0$) and the IID case ($\phi = \theta = 0$) the modified tests display empirical rejection frequencies close to the nominal level. Some distortions above the nominal level are seen in the modified tests as ϕ increases from zero. However, these distortions are always considerably smaller than for the corresponding unmodified tests. Under H_1, a negative moving average ($\theta > 0$) can also cause modest over-sizing in the modified tests, although this is merely a finite-sample effect. Interestingly, the conservative tests (N_1 through N_6 and N_1' through N_3') appear relatively robust to moving average errors under H_1.

5.3 Properties of the tests under H_{01} and H_{10}

In this section we report the empirical rejection frequencies of the tests when the data are generated according to the $I(0) - I(1)$ switch DGP

$$\begin{aligned} y_t &= v_t + w_t, \quad t = -100, ..., T \\ v_t &= \phi v_{t-1} + \varepsilon_t - \theta \varepsilon_{t-1}, \quad \varepsilon_t \sim NIID(0,1) \\ w_t &= w_{t-1} + 1(t > \lfloor \tau^* T \rfloor)\eta_t, \quad \eta_t \sim NIID(0, \omega_\eta^2). \end{aligned}$$

We consider the following values for breakpoint parameter: $\tau^* \in \{0.3, 0.5, 0.7\}$, while the signal-to-noise ratio, ω_η^2, was set at 0.05. With regard to the noise process, v_t, we allowed for both stationary AR behaviour and invertible MA behaviour through $\phi \in \{0.0, 0.5\}$ and $\theta \in \{0.0, \pm 0.5\}$.[4] Qualitatively similar conclusions were drawn for other values of τ^* and ω_η^2, and for other stationary and invertible noise processes. Again we have set $d_t = 0$ due to the invariance properties of the tests considered. The first 100 observations were again discarded in order to control for initial effects.

Tables 6 – 9 about here

Tables 6 and 7 report results for $T = 150$ and $T = 300$, respectively, for tests run at the nominal 5% level, again using the relevant finite sample critical values from Tables

[4]The parameter combination $\phi = \theta = 0.5$ is again not considered due to the common factor involved.

1 and 2. The modified tests were constructed in exactly the same manner as described in Section 5.2. In order to control for any size distortions in the tests (cf. Tables 4 and 5), Tables 8 and 9 report the corresponding size-adjusted results.

A striking feature of the results reported in Tables 6 and 7 is that the modified tests M_1 through M_6 appear to display only modest power losses relative to their unmodified counterparts K_1 through K_6, respectively, with the magnitude of these losses tending to decrease as τ^* increases, other things equal. The differences between the powers of the modified and unmodified tests are also smaller, other things equal, for $T = 300$ than for $T = 150$, as might be expected given the large sample properties of the tests; cf. Section 4. The (inconsistent) M_1' through M_3' tests tend to reject the null of no persistence change rather less frequently than their unmodified counterparts. This result is actually quite helpful in that we may more precisely be able to detect the direction of persistence change from considering the outcomes of the modified tests. However, this property also causes a slight but artefactual increase in the difference between the K_4 through K_6 and M_4 through M_6 tests, relative to the differences between the K_1 through K_3 and M_1 through M_3 tests.

The conservative tests, N_1 through N_6 display very poor finite sample power properties. Indeed, in many cases they reject less frequently than the nominal level. These tests are clearly and massively dominated on power by the other tests; therefore, we will not consider them further.

Although not reported, we also considered the corresponding $I(1)$-$I(0)$ switch DGP. These experiments yielded qualitatively similar results to those observed in Tables 5-6 for the K_4 through K_6 tests on switching τ^* for $(1 - \tau^*)$, noting that this model can also be viewed as a process with a switch from $I(0)$ to $I(1)$ at $(1 - \tau^*)$ when the data are taken in reverse order. Moreover, in each case, the relative behaviour of the K_j and K_j', $j = 1, ..., 3$, tests, and the M_j and M_j', $j = 1, ..., 3$, tests was reversed, as one might expect. Full details of these experiments are available on request.

Overall the conclusions to be drawn from the results in this section seem clear. The modified tests proposed in Section 4 of this paper do an excellent job. They virtually eliminate the severe spurious over-rejections seen with the ratio-based persistence change tests of the constant $I(0)$ null hypothesis against data generated according to a constant $I(1)$ DGP. Yet, the cost of this robustness in terms of power loss relative to the unmodified tests against persistence change processes is only modest. In any application of these tests we can therefore be confident (subject to the usual type-I error) that rejections by the modified persistence tests indicate that a change in persistence has actually occurred. The same clearly cannot be said for the unmodified tests. Moreover, where a change in persistence occurs, the chance of the modified tests rejecting is much the same as for the unmodified tests. Indeed, in many ways the application of the approach of Vogelsang (1998a,1998b) to the problem considered in this paper gives even more satisfying results than his original application to the testing of trend function hypotheses.

6 Empirical Applications

We apply the persistence change tests discussed in this paper to a variety of U.S. macroeconomic and financial time-series data sets. Specifically the data sets considered are: U.S. real seasonally adjusted quarterly GDP growth (measured as the annual change in real GDP), observed for 1948Q1 to 2002Q4; U.S. monthly CPI inflation (measured as the annual change in the CPI), observed for 1960M1 to 2002M12; monthly U.S. nominal seasonally adjusted M2 growth (measured as the annual change in M2), observed for 1960M1 to 2002M12; monthly U.S. 20-year Treasury constant maturity rate, observed for 1960M1 to 2002M12, and the U.S. Dollar to German Deutschmark monthly real exchange rate, observed for 1973M1 to 1998M12. All data were obtained from `www.economagic.com` and, with the exception of the 20-year interest rate series, were measured in logarithms.

Table 10 about here

Table 1 reports the application of the ratio-based tests K_1 through K_6 and K_1' through K_3', and the modified versions thereof: M_1 through M_6 and M_1' through M_3', to the afore-mentioned data sets. Modified tests run at both the nominal 5% and 10% levels are reported (recall that different modified tests obtain for different choices of the significance level) using the relevant values of b from Table 3. For all of the data sets we set $\mathbf{x}_t = 1$, since none of the series appears to display a trend. The superscripts * and ** are used to indicate a significant outcomes at the 10% and 5% level, respectively, using the critical values in Table 1 for the value of T closest to the sample size.[5]

The ratio-based tests for a change in persistence of Section 3 and their modified counterparts of Section 4 are broadly in agreement in the case of the first three series. For real GDP growth, there are no rejections of the constant $I(0)$ null, H_0, anywhere; this is in line with expectation, as a rejection would imply that at least part of the real GDP series in levels displayed $I(2)$ behaviour. Unreported simulations confirm that the change in persistence tests considered in this paper do not reject the null hypothesis when the true DGP entails a change from $I(0)$ to $I(-1)$ or *vice-versa*, as would occur if the levels of real GDP exhibited a change in persistence from $I(1)$ to $I(0)$ or $I(0)$ to $I(1)$. Thus real GDP growth appears to be integrated of order at most zero throughout its history. For CPI inflation, H_0 is rejected and the overwhelming balance of evidence points to a change in persistence from $I(1)$ to $I(0)$; Busetti and Taylor (2004) draw similar conclusions from a quartely time-series on U.S. CPI inflation. Finally, for M2 growth there is strong evidence in favour of a change in persistence from $I(0)$ to $I(1)$, indicating a change from $I(1)$ to $I(2)$ in the levels of M2.[6]

On the other hand, the outcomes of the unmodified *vis-à-vis* modified tests differ in the case of the final two series. In the case of the 20-year interest rate series, the

[5]Exactly the same conclusions are drawn throughout using the asymptotic critical values from Table 1.

[6]King *et al.* (1991) also found evidence of $I(1)$ ($I(2)$) behaviour in US (per capita) M2 growth (levels).

unmodified K_1 through K_6 and K_1' through K_3' all comfortably reject H_0. However, none of the modified tests reject H_0, even at the 10% level. Given the numerical evidence from Section 5, it seems not unreasonable to conjecture on the basis of these results that the 20-year interest rate series is a constant $I(1)$ process and that the unmodified tests are delivering spurious rejections, as they are prone to do under H_1. Broadly the same conclusions as for the 20-year interest rate series are also drawn for the dollar-DM exchange rate series, highlighting the importance of the modified tests and their size robustness in the presence of constant $I(1)$ behaviour.

7 Conclusions

In this paper we have shown that the ratio-based persistence change tests of Kim (2000), Kim *et al.* (2002) and Busetti and Taylor (2004) have the highly undesirable property that they display a very strong tendency to spuriously reject the constant $I(0)$ null in favour of a change in persistence when the data are generated as a constant $I(1)$ process, and that this effect does not vanish asymptotically. However, we have proposed simple modifications of the ratio-based tests which, by design, circumvent this shortcoming. The proposed modifications were based on the variable addition technique originally proposed in Vogelsang (1998a,1998b). The modified tests were designed such that they have the same limiting distribution as the corresponding unmodified original test, thereby obviating the need for any new tables of critical values. We have also demonstrated that the modified tests attain the same rate of consistency against persistence change series as the corresponding unmodified tests. Numerical evidence presented has suggested that our modified tests perform excellently in practice, virtually eliminating the severe spurious over-rejections against constant $I(1)$ processes seen with the unmodified tests, yet displaying only modest power losses relative to the unmodified tests against true persistence change processes. Applications of the original and modified tests to data on a variety of U.S. macroeconomic and financial data sets were considered. These demonstrated the practical relevance of the issues discussed in this paper and the usefulness of the modifications which we have made.

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Table 1. Critical values for tests of stationarity against a change in persistence.

Panel A. Mean case										
		K_1	K_2	K_3	K_1'	K_2'	K_3'	K_4	K_5	K_6
$T = 100$	10%	3.56	3.48	12.91	3.56	3.48	12.88	4.66	5.23	17.00
	5%	4.67	5.31	17.24	4.64	5.25	17.00	5.91	7.38	21.72
	1%	7.75	11.02	29.38	7.67	10.49	28.37	9.26	13.34	34.31
$T = 150$	10%	3.55	3.48	13.16	3.55	3.46	13.15	4.67	5.23	17.40
	5%	4.66	5.29	17.48	4.68	5.27	17.51	5.92	7.40	22.16
	1%	7.72	10.43	28.73	7.73	10.89	29.55	9.22	13.30	34.67
$T = 200$	10%	3.51	3.36	13.14	3.54	3.47	13.37	4.62	5.11	17.31
	5%	4.58	5.06	17.18	4.68	5.27	17.65	5.85	7.24	22.06
	1%	7.56	10.21	28.58	7.82	10.69	29.64	9.21	13.20	34.82
$T = 300$	10%	3.52	3.41	13.37	3.59	3.44	13.41	4.63	5.11	17.56
	5%	4.62	5.14	17.64	4.63	5.17	17.73	5.80	7.21	22.27
	1%	7.59	10.37	29.06	7.65	10.44	29.40	9.24	13.21	34.96
$T = 500$	10%	3.51	3.35	13.42	3.53	3.46	13.70	4.60	5.07	17.69
	5%	4.60	5.12	17.80	4.60	5.11	17.85	5.79	7.10	22.36
	1%	7.52	10.37	29.75	7.42	10.29	29.36	8.94	12.93	35.06
$T = \infty$	10%	3.51	3.41	13.81	3.51	3.41	13.81	4.63	5.16	18.15
	5%	4.61	5.21	18.34	4.61	5.21	18.34	5.88	7.28	23.15
	1%	7.69	10.56	30.34	7.69	10.56	30.34	9.24	13.14	35.71
Panel B. Trend case										
		K_1	K_2	K_3	K_1'	K_2'	K_3'	K_4	K_5	K_6
$T = 100$	10%	2.38	1.55	6.71	2.38	1.53	6.66	2.91	2.01	8.28
	5%	2.91	2.02	8.39	2.90	2.02	8.28	3.48	2.61	10.04
	1%	4.28	3.50	12.52	4.24	3.49	12.66	4.87	4.29	14.50
$T = 150$	10%	2.38	1.52	6.70	2.37	1.53	6.73	2.92	1.98	8.31
	5%	2.92	2.00	8.30	2.92	1.99	8.40	3.47	2.54	10.01
	1%	4.23	3.43	12.54	4.23	3.42	12.46	4.88	4.29	14.73
$T = 200$	10%	2.35	1.50	6.75	2.37	1.51	6.72	2.88	1.96	8.34
	5%	2.87	1.96	8.37	2.90	1.98	8.40	3.43	2.50	10.07
	1%	4.18	3.33	12.46	4.22	3.41	12.59	4.80	4.10	14.53
$T = 300$	10%	2.35	1.50	6.83	2.37	1.51	6.86	2.88	1.96	8.42
	5%	2.89	1.95	8.40	2.88	1.98	8.55	3.40	2.50	10.23
	1%	4.13	3.31	12.54	4.17	3.47	12.79	4.77	4.09	14.64
$T = 500$	10%	2.35	1.50	6.89	2.37	1.50	6.91	2.87	1.95	8.50
	5%	2.87	1.96	8.58	2.88	1.96	8.51	3.42	2.47	10.26
	1%	4.20	3.32	12.69	4.21	3.36	12.72	4.79	4.08	14.59
$T = \infty$	10%	2.36	1.50	6.98	2.36	1.50	6.98	2.86	1.95	8.57
	5%	2.86	1.96	8.62	2.86	1.96	8.62	3.42	2.49	10.33
	1%	4.20	3.30	12.77	4.20	3.30	12.77	4.79	4.14	14.77

Table 2. Critical values for tests of a unit root against a change in persistence.

Panel A. Mean case										
		N_1	N_2	N_3	N_1'	N_2'	N_3'	N_4	N_5	N_6
$T = 100$	10%	36.65	121.56	251.31	36.62	121.93	251.98	64.15	232.12	472.47
	5%	66.67	245.77	499.27	65.17	242.44	493.11	109.29	423.94	856.09
	1%	186.11	793.08	1594.39	184.48	776.83	1561.88	263.48	1169.46	2347.14
$T = 150$	10%	37.00	127.78	264.50	37.45	129.20	267.20	65.31	243.30	495.59
	5%	66.76	256.31	521.63	67.09	254.84	518.65	109.59	432.89	874.81
	1%	195.08	796.85	1602.71	193.88	805.36	1619.73	272.54	1201.71	2412.45
$T = 200$	10%	36.47	126.74	262.57	37.43	127.71	264.54	65.06	240.52	490.63
	5%	66.34	250.52	510.61	66.73	253.47	516.54	107.66	431.15	871.89
	1%	189.84	824.12	1657.84	189.00	811.19	1631.96	276.16	1237.93	2485.45
$T = 300$	10%	35.82	126.26	262.04	36.49	124.20	258.29	64.52	239.30	488.98
	5%	65.23	249.31	508.59	66.13	247.72	505.26	107.08	428.27	866.86
	1%	185.59	826.48	1663.36	184.80	830.88	1672.16	266.55	1275.33	2561.06
$T = 500$	10%	36.46	127.45	264.75	36.50	126.39	263.08	64.95	242.01	494.82
	5%	65.74	252.49	515.79	67.11	250.22	511.39	106.12	441.36	893.80
	1%	188.89	842.23	1695.88	182.46	840.40	1692.22	262.18	1222.74	2456.89
$T = \infty$	10%	36.08	123.39	257.91	36.08	123.39	257.91	63.59	238.41	488.26
	5%	65.00	248.84	509.10	65.00	248.84	509.10	106.24	437.13	885.54
	1%	186.39	851.06	1714.91	186.39	851.06	1714.91	270.71	1226.25	2465.29

Panel B. Trend case										
		N_1	N_2	N_3	N_1'	N_2'	N_3'	N_4	N_5	N_6
$T = 100$	10%	16.70	53.51	115.09	16.74	53.59	115.12	25.05	85.97	179.97
	5%	25.69	90.12	188.47	25.59	87.39	182.83	36.63	135.06	278.34
	1%	57.22	227.87	463.90	54.19	215.80	439.51	72.67	299.07	606.37
$T = 150$	10%	17.00	55.91	120.36	16.96	55.28	119.15	25.65	91.52	191.79
	5%	26.12	93.95	196.83	26.13	93.45	195.91	37.86	143.99	296.72
	1%	57.88	233.52	475.89	58.04	238.22	485.19	77.15	330.33	669.68
$T = 200$	10%	16.96	56.31	121.43	16.97	55.42	119.76	25.77	92.70	194.45
	5%	26.24	95.02	199.28	26.24	94.43	198.20	36.97	144.77	299.10
	1%	55.67	234.87	479.34	57.40	236.82	483.23	74.10	324.19	657.98
$T = 300$	10%	16.90	56.24	121.75	17.07	57.14	123.62	25.72	93.39	196.42
	5%	26.17	95.64	200.67	26.05	96.06	201.63	37.42	148.32	306.72
	1%	57.30	244.76	499.90	56.71	244.34	499.08	75.42	339.38	687.98
$T = 500$	10%	16.68	56.23	122.45	17.25	57.95	125.46	25.69	94.17	198.59
	5%	25.98	94.87	200.30	26.44	97.98	205.69	37.32	148.49	307.90
	1%	56.93	247.71	506.36	57.85	255.27	520.59	75.01	352.73	716.06
$T = \infty$	10%	16.55	54.86	119.58	16.55	54.86	119.58	24.18	85.24	181.10
	5%	25.31	93.29	197.07	25.31	93.29	197.07	35.34	132.14	275.46
	1%	56.91	243.63	499.02	56.91	243.63	499.02	69.78	300.98	614.65

Table 3. Asymptotic b-values for modified tests of stationarity or a unit root against a change in persistence.

		M_1	M_2	M_3	M_1'	M_2'	M_3'	M_4	M_5	M_6
Mean case	10%	0.239	0.408	0.308	0.237	0.409	0.311	0.290	0.470	0.376
	5%	0.293	0.480	0.383	0.290	0.481	0.382	0.336	0.538	0.446
	1%	0.389	0.635	0.548	0.392	0.632	0.542	0.431	0.696	0.609
Trend case	10%	0.511	1.062	0.805	0.497	1.014	0.771	0.579	1.189	0.904
	5%	0.595	1.248	0.953	0.577	1.187	0.899	0.658	1.367	1.046
	1%	0.773	1.699	1.325	0.714	1.538	1.186	0.812	1.738	1.371

Table 4. Empirical sizes of nominal 5%-level tests against a change in persistence: mean case, $T = 150$.

ϕ	θ	K_1 N_1 M_1	K_2 N_2 M_2	K_3 N_3 M_3	K_1' N_1' M_1'	K_2' N_2' M_2'	K_3' N_3' M_3'	K_4 N_4 M_4	K_5 N_5 M_5	K_6 N_6 M_6
0.00	0.00	4.81	4.92	4.93	4.84	4.87	4.88	4.96	4.79	4.84
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		4.47	4.59	4.59	4.52	4.55	4.52	4.58	4.38	4.29
	−0.50	5.75	6.01	6.03	5.73	6.12	6.01	5.98	6.38	6.29
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		5.17	5.37	5.34	5.24	5.49	5.32	5.21	5.37	5.25
	0.50	1.86	1.44	1.57	1.71	1.49	1.54	1.20	1.01	1.19
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		1.81	1.40	1.54	1.68	1.46	1.52	1.17	0.98	1.13
0.50	0.00	7.34	8.21	8.14	7.59	8.52	8.27	8.46	9.63	9.52
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		6.47	7.06	6.84	6.54	7.18	6.87	6.97	7.93	7.54
	−0.50	7.85	8.79	8.61	8.04	9.14	8.85	9.14	10.55	10.31
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		6.48	7.21	6.94	6.71	7.45	6.98	7.17	8.10	7.76
0.80	0.00	13.53	16.91	16.49	13.61	17.02	16.47	17.88	22.48	22.03
		0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		9.40	12.02	11.33	9.84	12.22	11.44	11.14	14.63	13.84
	−0.50	13.83	17.18	16.81	13.96	17.52	16.81	18.27	23.08	22.48
		0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
		9.03	11.49	10.67	9.40	11.73	10.85	10.51	13.79	12.99
	0.50	11.36	14.13	13.91	11.80	14.24	13.94	14.65	18.08	17.85
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		9.64	12.08	11.60	9.93	12.19	11.65	11.90	14.57	14.07
0.90	0.00	21.49	27.39	26.68	21.30	27.23	26.60	30.23	38.54	37.73
		0.07	0.07	0.07	0.08	0.03	0.03	0.01	0.01	0.01
		11.34	14.85	13.97	11.62	14.94	13.89	13.68	18.13	17.02
	−0.50	21.72	27.58	26.80	21.52	27.45	26.77	30.45	39.02	38.12
		0.08	0.07	0.07	0.07	0.05	0.05	0.01	0.02	0.02
		10.34	13.64	12.75	10.70	13.59	12.73	12.10	16.16	15.25
	0.50	19.82	25.51	25.06	19.54	25.11	24.61	27.63	35.21	34.67
		0.02	0.01	0.01	0.03	0.00	0.00	0.01	0.01	0.01
		14.57	19.38	18.30	14.58	19.11	18.23	18.88	24.97	23.69
0.95	0.00	30.34	37.81	36.99	29.95	37.85	36.92	44.77	53.85	52.96
		0.54	0.45	0.45	0.46	0.50	0.50	0.27	0.27	0.27
		11.02	13.54	13.12	10.89	13.38	13.00	12.31	15.74	15.27
	−0.50	30.48	37.91	37.06	30.01	38.02	37.12	45.03	53.92	53.01
		0.57	0.46	0.46	0.47	0.53	0.53	0.30	0.30	0.30
		9.44	11.60	11.35	9.47	11.65	11.43	10.33	13.10	12.90
	0.50	29.13	36.38	35.93	28.68	36.36	35.77	42.48	51.72	51.08
		0.39	0.22	0.22	0.28	0.22	0.22	0.13	0.11	0.11
		17.73	23.06	22.08	17.71	23.15	21.88	23.31	30.19	28.87
1.00	0.00	46.53	55.67	55.03	46.88	55.81	55.11	68.80	75.02	74.26
		5.08	4.79	4.79	5.22	5.16	5.16	5.00	5.06	5.06
		5.44	5.57	5.84	5.67	5.74	5.92	5.75	5.85	6.19
	−0.50	46.57	55.75	55.21	46.95	55.95	55.26	68.95	75.22	74.45
		5.15	4.96	4.96	5.33	5.24	5.24	5.09	5.18	5.18
		4.38	4.50	4.75	4.71	4.55	4.85	4.45	4.59	4.87
	0.50	45.86	54.90	54.29	46.18	55.09	54.45	67.85	74.36	73.63
		4.54	4.11	4.11	4.63	4.39	4.39	4.22	4.02	4.02
		14.69	15.71	15.95	15.25	16.50	16.65	18.00	18.97	19.45

Table 5. Empirical sizes of nominal 5%-level tests against a change in persistence: mean case, $T = 300$.

ϕ	θ	K_1 N_1 M_1	K_2 N_2 M_2	K_3 N_3 M_3	K_1' N_1' M_1'	K_2' N_2' M_2'	K_3' N_3' M_3'	K_4 N_4 M_4	K_5 N_5 M_5	K_6 N_6 M_6
0.00	0.00	5.03	5.18	5.22	4.93	4.96	4.98	5.16	5.24	5.38
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		4.88	5.03	5.10	4.80	4.83	4.77	4.97	5.08	5.13
	−0.50	5.42	5.74	5.71	5.48	5.46	5.30	5.81	5.97	5.97
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		5.22	5.49	5.33	5.18	5.21	5.01	5.38	5.61	5.46
	0.50	2.84	2.70	2.82	2.73	2.43	2.61	2.50	2.14	2.30
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		2.81	2.67	2.78	2.69	2.42	2.58	2.48	2.13	2.25
0.50	0.00	6.35	6.88	6.63	6.35	6.68	6.40	6.97	7.34	7.11
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		5.91	6.28	6.14	5.85	6.18	5.89	6.25	6.58	6.35
	−0.50	6.56	7.03	6.79	6.48	6.92	6.60	7.32	7.80	7.59
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		5.92	6.34	5.96	5.88	6.27	5.92	6.29	6.74	6.42
0.80	0.00	9.75	11.49	10.84	9.28	11.24	10.58	11.61	14.11	13.45
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		7.89	9.35	8.65	7.76	9.20	8.52	8.82	10.82	10.17
	−0.50	9.74	11.67	10.94	9.35	11.30	10.66	11.73	14.21	13.66
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		7.72	9.14	8.42	7.58	9.04	8.28	8.66	10.61	9.85
	0.50	8.97	10.35	9.97	8.71	10.17	9.74	10.45	12.27	11.88
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		8.10	9.29	8.92	7.94	9.20	8.64	9.09	10.71	10.25
0.90	0.00	14.43	18.62	17.66	14.36	18.10	17.15	19.16	24.80	23.81
		0.01	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00
		9.87	12.86	11.88	9.84	12.79	11.70	11.68	15.78	14.58
	−0.50	14.46	18.68	17.66	14.44	18.27	17.24	19.27	24.84	23.91
		0.01	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00
		9.54	12.52	11.45	9.50	12.35	11.24	11.24	15.25	13.88
	0.50	13.90	17.66	17.07	13.68	17.43	16.74	18.31	23.35	22.50
		0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		11.13	14.43	13.43	11.13	14.15	13.29	13.75	18.01	16.93
0.95	0.00	22.02	28.19	27.05	21.87	28.29	27.00	31.77	39.87	38.45
		0.10	0.07	0.07	0.07	0.07	0.07	0.03	0.01	0.01
		11.34	15.20	13.99	11.64	15.03	13.68	13.95	18.75	17.11
	−0.50	22.07	28.37	27.07	21.93	28.37	27.00	31.94	39.90	38.45
		0.11	0.07	0.07	0.07	0.07	0.07	0.03	0.01	0.01
		10.72	14.45	13.28	11.16	14.22	13.01	13.07	17.52	16.11
	0.50	21.55	27.82	26.64	21.47	27.73	26.55	31.14	39.23	38.02
		0.08	0.05	0.05	0.05	0.04	0.04	0.01	0.01	0.01
		14.30	19.28	17.78	14.71	19.11	17.57	18.49	25.09	23.40
1.00	0.00	46.00	55.27	54.41	46.63	55.91	55.01	68.52	74.91	73.94
		5.14	4.84	4.84	4.65	4.74	4.74	4.90	4.81	4.81
		4.94	5.10	5.26	5.54	5.50	5.62	5.42	5.50	5.65
	−0.50	46.13	55.34	54.48	46.73	55.89	55.07	68.48	75.02	73.96
		5.18	4.88	4.88	4.70	4.70	4.70	4.94	4.87	4.86
		4.42	4.59	4.72	4.91	4.84	5.03	4.75	4.81	4.99
	0.50	45.98	55.14	54.29	46.42	55.65	54.85	68.29	74.53	73.77
		5.01	4.71	4.71	4.46	4.54	4.54	4.66	4.61	4.61
		10.03	10.64	10.67	10.50	11.37	11.38	12.14	12.57	12.81

Table 6. Estimated powers of nominal 5%-level tests against a change in persistence: mean case, $T = 150$, $\sigma_u^2 = 0.05$.

			K_1	K_2	K_3	K_1'	K_2'	K_3'	K_4	K_5	K_6
			N_1	N_2	N_3	N_1'	N_2'	N_3'	N_4	N_5	N_6
τ^*	ϕ	θ	M_1	M_2	M_3	M_1'	M_2'	M_3'	M_4	M_5	M_6
0.30	0.00	0.00	76.28	80.89	80.55	27.35	39.75	39.52	77.16	81.39	80.98
			10.06	4.83	4.83	0.64	0.53	0.53	4.48	1.75	1.75
			72.67	77.89	76.96	21.45	32.37	31.59	72.34	77.28	76.45
		−0.50	62.57	67.48	66.55	23.94	34.90	34.55	64.40	69.42	68.67
			3.67	1.36	1.36	0.27	0.20	0.20	1.24	0.32	0.32
			57.17	61.77	60.07	18.37	27.68	26.83	56.53	61.86	60.46
		0.50	88.75	92.02	91.98	29.78	42.83	42.95	88.44	91.36	91.14
			23.89	13.41	13.41	1.06	0.92	0.92	12.98	5.88	5.87
			87.60	91.25	91.02	25.41	37.91	37.40	87.04	90.44	90.16
	0.50	0.00	53.90	58.98	57.86	22.22	32.16	31.90	56.54	62.18	61.36
			2.07	0.75	0.75	0.18	0.14	0.14	0.61	0.17	0.17
			46.38	50.48	48.83	16.07	24.10	23.11	45.66	50.76	49.13
		−0.50	38.73	42.60	41.33	17.38	25.03	24.55	40.84	45.97	45.05
			0.40	0.14	0.14	0.04	0.01	0.01	0.06	0.03	0.03
			31.80	34.45	32.68	12.51	18.86	17.77	30.76	35.09	33.29
0.50	0.00	0.00	81.26	79.47	78.53	7.51	19.06	19.36	76.34	75.76	74.98
			13.24	3.54	3.54	0.01	0.03	0.03	6.20	1.00	1.00
			79.70	77.50	76.21	5.26	14.97	14.61	73.98	72.80	71.61
		−0.50	66.70	65.30	63.81	6.18	15.62	15.84	61.07	61.09	59.74
			4.54	0.83	0.83	0.00	0.01	0.01	1.58	0.14	0.14
			63.67	61.33	59.36	4.08	11.90	11.52	56.67	55.14	53.60
		0.50	93.19	92.31	91.86	8.67	21.63	22.07	90.60	89.67	89.16
			32.32	11.27	11.26	0.01	0.07	0.07	20.20	4.42	4.42
			93.00	91.94	91.42	6.80	18.83	18.74	90.16	89.03	88.47
	0.50	0.00	57.25	55.97	54.41	6.28	14.87	14.84	51.95	52.90	51.80
			2.38	0.43	0.43	0.00	0.01	0.01	0.66	0.07	0.07
			52.38	49.98	47.80	4.12	10.67	10.22	45.31	44.60	42.74
		−0.50	41.16	40.12	38.44	5.63	11.91	11.80	36.00	37.24	36.14
			0.42	0.04	0.04	0.00	0.00	0.00	0.04	0.01	0.01
			35.95	33.65	31.50	3.87	8.60	8.19	28.77	28.65	27.06
0.70	0.00	0.00	70.59	65.71	64.18	0.41	1.09	1.16	64.66	58.89	57.67
			7.19	0.57	0.57	0.00	0.00	0.00	2.73	0.07	0.07
			69.47	64.01	62.23	0.39	0.88	0.91	63.09	56.46	54.97
		−0.50	55.17	49.97	48.12	1.02	1.90	1.93	48.54	42.70	41.42
			1.87	0.04	0.04	0.00	0.00	0.00	0.40	0.00	0.00
			53.11	46.84	44.84	0.90	1.54	1.52	45.53	38.56	36.84
		0.50	87.02	83.44	82.50	0.02	0.44	0.56	83.10	78.57	77.70
			24.12	3.30	3.30	0.00	0.00	0.00	13.12	0.73	0.73
			86.90	83.03	82.09	0.02	0.34	0.39	82.78	78.00	77.12
	0.50	0.00	45.91	41.53	39.72	2.15	3.63	3.64	39.63	35.38	34.39
			0.78	0.03	0.03	0.00	0.00	0.00	0.11	0.00	0.00
			42.56	37.08	34.93	1.79	2.90	2.83	35.25	29.84	28.04
		−0.50	31.05	28.27	26.67	3.74	5.24	5.20	25.93	24.04	22.96
			0.07	0.01	0.01	0.00	0.00	0.00	0.01	0.00	0.00
			27.47	23.57	21.85	2.95	4.11	3.92	21.39	18.59	17.47

Table 7. Estimated powers of nominal 5%-level tests against a change in persistence: mean case, $T = 300$, $\sigma_u^2 = 0.05$.

τ^*	ϕ	θ	K_1 N_1 M_1	K_2 N_2 M_2	K_3 N_3 M_3	K_1' N_1' M_1'	K_2' N_2' M_2'	K_3' N_3' M_3'	K_4 N_4 M_4	K_5 N_5 M_5	K_6 N_6 M_6
0.30	0.00	0.00	93.15	95.60	95.45	30.98	43.53	43.38	92.78	95.13	95.06
			35.72	28.34	28.37	1.86	2.11	2.11	23.97	17.51	17.51
			91.32	94.28	94.17	21.44	31.42	30.64	90.45	92.91	92.76
		−0.50	85.67	89.73	89.33	29.50	41.60	41.33	85.98	89.39	88.96
			20.42	13.42	13.44	1.33	1.33	1.34	11.53	6.56	6.57
			82.16	86.74	85.95	20.75	31.19	30.20	80.99	85.02	84.10
		0.50	98.44	99.25	99.28	32.02	44.73	44.76	98.08	98.97	98.94
			60.80	55.16	55.18	2.33	2.83	2.83	47.61	40.53	40.53
			98.22	99.15	99.17	24.41	35.51	34.66	97.78	98.80	98.78
	0.50	0.00	78.85	83.43	82.68	27.90	39.73	39.45	79.61	83.41	82.73
			12.59	7.50	7.50	0.92	1.03	1.03	6.22	3.16	3.16
			72.97	77.70	76.17	19.15	28.82	27.68	71.55	75.86	74.45
		−0.50	64.49	69.41	68.03	24.04	34.95	34.58	66.10	70.66	69.48
			4.23	1.73	1.72	0.40	0.43	0.43	1.44	0.58	0.58
			58.29	63.10	60.68	17.85	27.16	25.82	57.67	62.51	60.53
0.50	0.00	0.00	95.93	95.52	95.10	10.37	23.86	24.14	94.36	93.81	93.56
			44.77	25.17	25.17	0.10	0.30	0.30	31.97	14.10	14.10
			95.45	95.03	94.45	6.41	17.10	16.79	93.61	92.97	92.54
		−0.50	89.46	88.83	88.02	9.26	21.82	22.10	86.56	85.98	85.27
			26.20	10.73	10.73	0.03	0.14	0.14	15.30	4.45	4.45
			88.44	87.44	86.17	5.65	15.93	15.43	84.78	83.69	82.67
		0.50	99.30	99.16	99.09	11.21	24.99	25.39	98.89	98.67	98.63
			71.34	51.98	51.99	0.15	0.40	0.40	59.48	37.59	37.59
			99.27	99.11	99.02	7.87	19.97	19.68	98.86	98.61	98.54
	0.50	0.00	82.80	82.03	80.56	8.38	20.40	20.57	79.18	78.68	77.66
			15.92	5.41	5.41	0.02	0.10	0.10	8.02	1.86	1.86
			80.64	79.03	77.08	4.92	14.16	13.68	75.68	73.66	72.02
		−0.50	68.55	67.20	65.14	6.66	16.35	16.45	63.28	62.98	61.59
			5.21	1.16	1.16	0.00	0.01	0.01	1.77	0.21	0.21
			65.28	63.09	60.32	4.42	12.34	11.73	58.55	57.19	55.00
0.70	0.00	0.00	90.58	88.50	87.80	0.05	1.03	1.21	87.61	84.84	84.19
			34.13	10.66	10.67	0.00	0.00	0.00	22.19	4.18	4.18
			90.24	88.02	87.13	0.05	0.69	0.71	87.13	84.00	83.14
		−0.50	80.97	77.79	76.33	0.22	1.10	1.21	76.59	72.08	70.76
			16.93	2.86	2.86	0.00	0.00	0.00	8.48	0.76	0.76
			80.20	76.38	74.75	0.19	0.78	0.80	75.28	69.87	68.25
		0.50	97.94	97.13	96.95	0.00	1.00	1.18	96.85	95.83	95.67
			62.22	33.41	33.41	0.00	0.00	0.00	50.02	20.10	20.11
			97.90	97.05	96.91	0.00	0.76	0.83	96.77	95.74	95.55
	0.50	0.00	72.39	68.27	66.16	0.56	1.52	1.60	66.88	62.04	60.44
			8.73	1.15	1.15	0.00	0.00	0.00	3.48	0.24	0.24
			70.76	65.78	63.35	0.50	1.11	1.10	64.72	58.25	56.10
		−0.50	56.57	51.96	49.63	1.08	2.08	2.09	50.35	45.00	43.16
			2.06	0.14	0.14	0.00	0.00	0.00	0.51	0.01	0.01
			54.32	48.76	46.06	0.92	1.67	1.59	47.38	40.61	38.34

Table 8. Estimated size-adjusted powers of nominal 5%-level tests against a change in persistence: mean case, $T = 150$, $\sigma_u^2 = 0.05$.

τ^*	ϕ	θ	K_1, N_1 / M_1	K_2, N_2 / M_2	K_3, N_3 / M_3	K_1', N_1' / M_1'	K_2', N_2' / M_2'	K_3', N_3' / M_3'	K_4, N_4 / M_4	K_5, N_5 / M_5	K_6, N_6 / M_6
0.30	0.00	0.00	76.58	81.03	80.67	27.60	39.90	39.75	77.31	81.64	81.19
			73.50	78.55	77.63	22.13	33.09	32.39	73.30	78.29	77.46
		−0.50	61.09	65.33	64.53	22.81	33.30	33.09	62.13	67.00	66.20
			56.75	60.52	59.34	18.05	26.82	26.19	55.85	60.98	59.77
		0.50	92.42	95.20	94.99	36.17	49.70	49.70	93.14	95.29	95.06
			91.75	94.90	94.53	31.74	45.34	44.38	92.53	94.88	94.44
	0.50	0.00	48.67	52.00	51.38	18.83	27.31	26.98	49.33	53.50	52.90
			42.91	44.75	43.96	14.09	20.64	20.11	41.24	43.54	42.95
		−0.50	32.50	33.84	33.16	13.35	19.27	19.04	32.27	34.91	34.37
			28.07	28.62	28.06	10.47	15.00	14.82	26.24	27.66	27.17
0.50	0.00	0.00	81.53	79.59	78.65	7.67	19.18	19.52	76.44	76.04	75.31
			80.34	78.34	76.95	5.49	15.42	15.12	74.87	74.00	72.85
		−0.50	65.24	63.08	61.69	5.62	14.49	14.62	59.19	58.36	57.21
			63.30	60.20	58.58	3.88	11.40	11.16	56.07	54.23	52.93
		0.50	95.75	95.53	94.82	12.07	26.66	27.33	94.36	94.21	93.69
			95.69	95.39	94.61	9.93	24.29	24.06	94.18	94.06	93.36
	0.50	0.00	52.07	48.85	47.92	4.64	11.39	11.45	45.53	44.23	43.20
			49.02	44.34	43.39	3.06	8.62	8.42	41.42	37.56	36.85
		−0.50	34.95	31.32	30.31	3.62	8.00	8.05	28.06	26.71	26.25
			32.01	27.79	27.02	2.90	6.37	6.34	24.73	22.07	21.51
0.70	0.00	0.00	70.96	65.88	64.31	0.43	1.11	1.18	64.84	59.29	58.06
			70.38	64.98	63.11	0.43	0.97	1.00	64.03	57.73	56.24
		−0.50	53.72	47.47	45.79	0.86	1.53	1.63	46.45	40.27	38.82
			52.75	45.61	44.14	0.86	1.43	1.44	45.02	37.77	36.24
		0.50	90.94	89.56	88.05	0.05	0.90	1.15	88.33	86.16	84.61
			90.89	89.41	87.88	0.06	0.80	0.92	88.19	86.00	84.35
	0.50	0.00	40.77	33.79	32.91	1.38	2.18	2.29	33.67	27.10	26.25
			39.17	31.45	30.45	1.31	2.01	2.03	31.81	23.78	23.05
		−0.50	25.10	20.10	19.50	2.29	3.06	3.08	19.35	15.23	14.80
			23.85	18.67	18.05	2.21	2.86	2.86	17.91	13.44	13.03

Note: Results for the standard K tests and the conservative N tests are of course identical, since the test statistics are identical and empirical critical values are used to obtain size-adjusted powers.

Table 9. Estimated size-adjusted powers of nominal 5%-level tests against a change in persistence: mean case, $T = 300$, $\sigma_u^2 = 0.05$.

τ^*	ϕ	θ	K_1, N_1 M_1	K_2, N_2 M_2	K_3, N_3 M_3	K_1', N_1' M_1'	K_2', N_2' M_2'	K_3', N_3' M_3'	K_4, N_4 M_4	K_5, N_5 M_5	K_6, N_6 M_6
0.30	0.00	0.00	93.13	95.50	95.31	31.11	43.58	43.43	92.69	94.97	94.80
			91.45	94.27	94.12	21.75	31.77	30.98	90.46	92.80	92.62
		−0.50	85.15	88.97	88.60	28.90	40.81	40.84	85.02	88.34	88.11
			81.84	86.03	85.39	20.52	30.85	30.20	80.27	84.13	83.56
		0.50	98.97	99.57	99.53	35.77	49.20	48.67	98.73	99.32	99.31
			98.78	99.52	99.48	28.13	40.23	38.62	98.59	99.21	99.19
	0.50	0.00	76.64	81.03	80.13	25.87	37.34	37.29	76.56	80.17	79.70
			71.28	75.24	74.19	17.95	26.83	26.46	68.76	72.00	71.30
		−0.50	61.23	65.25	64.31	21.92	32.00	31.75	61.58	65.44	64.92
			55.99	59.59	58.26	16.68	24.90	24.29	54.62	58.00	57.06
0.50	0.00	0.00	95.93	95.40	94.95	10.43	23.89	24.20	94.23	93.64	93.34
			95.50	95.02	94.41	6.56	17.30	16.99	93.62	92.94	92.48
		−0.50	89.06	88.02	87.14	8.89	21.27	21.73	85.82	84.89	84.33
			88.22	86.77	85.62	5.54	15.75	15.43	84.30	82.89	82.08
		0.50	99.56	99.48	99.39	13.35	28.18	28.64	99.27	99.16	99.06
			99.52	99.44	99.35	9.84	23.40	22.61	99.23	99.12	99.00
	0.50	0.00	80.86	79.25	78.09	7.50	18.40	18.77	76.33	74.76	73.98
			79.24	76.56	75.07	4.49	12.96	12.72	73.50	70.08	68.88
		−0.50	65.40	62.73	61.16	5.66	14.35	14.45	59.05	57.56	56.71
			63.13	59.50	57.69	3.82	10.94	10.75	55.64	52.81	51.61
0.70	0.00	0.00	90.55	88.33	87.46	0.06	1.03	1.21	87.47	84.50	83.75
			90.36	88.00	87.00	0.06	0.70	0.75	87.14	83.91	83.03
		−0.50	80.32	76.38	75.12	0.19	1.00	1.13	75.56	70.34	69.17
			79.84	75.49	74.09	0.19	0.73	0.80	74.70	68.81	67.49
		0.50	98.47	98.11	97.80	0.03	1.36	1.72	97.83	97.07	96.89
			98.44	98.08	97.74	0.01	1.05	1.21	97.80	97.02	96.81
	0.50	0.00	70.04	64.75	62.93	0.43	1.14	1.26	63.92	57.20	56.04
			68.95	62.98	60.76	0.42	0.94	0.95	62.57	54.30	52.73
		−0.50	53.26	47.13	45.44	0.80	1.49	1.59	46.46	39.05	38.06
			52.21	45.41	43.38	0.80	1.29	1.34	44.90	36.35	35.06

Note: Results for the standard K tests and the conservative N tests are of course identical, since the test statistics are identical and empirical critical values are used to obtain size-adjusted powers.

Table 10. Application of tests to US macroeconomic time series.

Series	T	K_1 / $M_1(10\%)$ / $M_1(5\%)$	K_2 / $M_2(10\%)$ / $M_2(5\%)$	K_3 / $M_3(10\%)$ / $M_3(5\%)$	K'_1 / $M'_1(10\%)$ / $M'_1(5\%)$	K'_2 / $M'_2(10\%)$ / $M'_2(5\%)$	K'_3 / $M'_3(10\%)$ / $M'_3(5\%)$	K_4 / $M_4(10\%)$ / $M_4(5\%)$	K_5 / $M_5(10\%)$ / $M_5(5\%)$	K_6 / $M_6(10\%)$ / $M_6(5\%)$
Real GDP growth	220	0.58	0.31	1.44	2.49	1.65	8.01	2.49	1.65	8.01
		0.56	0.29	1.39	2.43	1.58	7.73	2.41	1.57	7.68
		0.56	0.29	1.38	2.41	1.57	7.67	2.40	1.56	7.62
CPI inflation	516	2.02	6.64**	23.51**	24.12**	84.18**	177.34**	24.12**	84.18**	177.34**
		1.23	2.84	12.40	14.74*	35.99*	92.94*	13.21*	31.71*	81.20*
		1.10	2.45	10.61	13.21**	30.99**	80.19**	12.00**	27.53**	70.21**
M2 growth	516	7.28**	20.68**	51.21**	0.48	0.26	1.45	7.28**	20.68**	51.21**
		4.94*	10.66*	31.05*	0.33	0.13	0.88	4.55	9.64*	27.81*
		4.52	9.49**	27.49**	0.30	0.12	0.78	4.22	8.63**	24.82**
20-year interest rate	516	5.15**	20.75**	51.93**	4.75**	8.52**	25.55**	5.15*	20.75**	51.93**
		0.81	0.88	4.79	0.76	0.36	2.30	0.55	0.55	2.83
		0.53	0.51	2.68	0.50	0.21	1.33	0.38	0.32	1.65
$:DM exchange rate	312	5.73**	34.39**	79.26**	11.23**	18.12**	44.01**	11.23**	34.39**	79.26**
		1.82	4.85*	18.06*	3.60*	2.54	9.89	2.79	3.60	13.03
		1.40	3.43	12.60	2.79	1.80	7.03	2.24	2.60	9.31

Notes: (i) * and ** denote rejection at the 10% and 5% levels respectively; (ii) The notation $M_1(5\%)$, for example, denotes the modified M_1 test run at the 5% level.