

Title: Models of Foreign Exchange Intervention: Estimation and Testing
Author: Bryan W. Brown and Douglas J. Hodgson
Address: Rice University and Université du Québec à Montréal

Correspondence: Douglas J. Hodgson
Dept. des sciences économiques, Univ. du Québec à Montréal
C.P. 8888, succ. Centre-Ville
Montréal, Québec, Canada H3C 3P8
hodgson.douglas-james@uqam.ca
Phone 1-514-987-3000 (ext 4310#)
Fax 1-514-987-8494

Abstract: We propose a general non-linear simultaneous equations framework for the econometric analysis of models of intervention in foreign exchange markets by central banks in response to deviations of exchange rates from possibly time-varying target levels. We consider efficient estimation of possibly non-linear response functions and tests of functional form, the latter making use of the recent econometric literature on testing in the presence of nuisance parameters unidentified under a null hypothesis. The methodology is applied in an analysis of recent activity of the Bank of Canada with regard to the Canada-U.S. exchange rate.

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INTRODUCTION

There exists a substantial empirical literature seeking to estimate the function characterizing the policy response of a central bank to deviations of an exchange rate from a target level (for surveys, see Almekinders and Eijffinger (1991) and Sarno and Taylor (2001)). A number of measures of policy intervention have been suggested in the literature, often based on the changes in a central bank's holdings of foreign exchange. Various ways have also been envisaged of modeling the target rate, which may be time-varying in response to a variety of economic factors, and may have time series structure as its current level may depend on its own recent values and on recent departures of the exchange rate from target. The basic long-run determinant of the target will be some set of economic fundamentals, and the target itself is best seen as containing a unit root, although it may remain unchanged for substantial periods of time, as in an explicit target zone regime. In such a regime, the target will change periodically in response to fundamentals, which may manifest themselves empirically through an exchange rate that has been tending towards a boundary of the explicit target zone for some stretch of time. Postulating the presence of a unit root in the exchange rate series itself, the latter should be cointegrated with the target rate, with a cointegrating coefficient of one. The monetary authority responds to deviations of the exchange rate from its target level through variations in the policy variable that have the intended effect of countering the deviations, although there is not unanimity in the literature regarding the appropriate policy response function. A number of reaction functions are imaginable. Due to a lack of economic theory specifying the functional form, the response function is often specified on an ad hoc basis in empirical work. Although linear specifications are often employed in practice, various sources of nonlinearity are plausible, such as, for example, asymmetry (if a central bank places greater weight on depreciations than on appreciations), convexity of the reaction function (the reaction becomes increasingly strong the greater is the deviation of the exchange rate from its target), and threshold effects (intervention doesn't occur unless the deviation from target is sufficiently large), or combinations of the above.

This framework presents a number of potential econometric complications. First, the issue of simultaneity should be addressed, as one would expect changes in a correctly-chosen policy variable to have a fairly rapid feedback into movements of the exchange rate itself. Hence, it would be desirable to specify a non-linear simultaneous equations model in which a second equation characterizing this feedback effect is included. Second, the ad hoc nature of the specification of the functional form suggests the desirability of a test of functional form. Due to the non-linearity of the model, however, such a test would be likely to have non-standard properties, as it would belong to the category of tests for which there exist nuisance parameters that are unidentified under the null hypothesis. In recent years, econometric methods have been developed to handle such situations (for example, Andrews and Ploberger (1994) and Hansen (1996)). A third issue that may be important in many applications is the possible problem of instrument relevance and weak instruments. An additional issue arises from the possible non-normality that is typically present when working with exchange rate data.

The next section outlines the basic non-linear simultaneous equations econometric framework in which these issues will be addressed. Section 3 will discuss issues concerned with the estimation of this model, and describes an algorithm for computing a semiparametric efficient estimator. We will then consider in Section 4 the issue of testing for the presence and specification of the policy reaction function.

THE GENERAL FRAMEWORK

The category of models we are interested in will be described here in general terms. For each period $t = 1, \dots, n$, we have data on the exchange rate (generally expressed in logs), s_t , and on a policy instrument, i_t . The target exchange rate (also generally expressed in logs) is s_t^* , and policy reacts to deviations from target according to the following basic relationship:

$$i_t = \alpha g(d_t, \beta), \quad 1$$

where $d_t = s_t - s_t^*$, $g(\cdot)$ is a specified nonlinear function with unknown parameter vector β , and the slope parameter α will equal zero if there is no policy reaction or if the functional form of $g(\cdot)$ is incorrectly specified. The null hypothesis $\alpha = 0$ will thus be of particular interest in this model. We note here that in a fully specified econometric model, the parameter vector β will be unidentified under the null, which will create problems in the testing of this null, as has been observed in the literature. We introduce a sequence of σ -fields $\{\mathcal{F}_t\}$, and assume that the pair (s_t, i_t) is measurable with respect to \mathcal{F}_t . In addition, suppose that a vector z_t , measurable with respect to \mathcal{F}_{t-1} , of auxiliary variables is observed, which may contain lags of (s_t, i_t) , in addition to other economic variables that may be relevant to our model.

We will assume the target rate is a function of z_t , $s_t^* = h(z_t)$, say. For the moment, we will say no more about the function $h(\cdot)$ or the time series variation of s_t^* , except to state our general assumption that the latter is integrated of order one, i.e. that it possess a unit root, so that we have

$$s_t^* = s_{t-1}^* + v_t,$$

where v_t is a stationary process. In the case of an explicit target zone regime, the marginal distribution of v_t can be thought of as having a probability mass at zero, in which case changes in the target zone could be thought of as random events that occur only occasionally. It is reasonable to assume that the same set of economic fundamentals that drive the exchange rate itself also drive the target rate, so that these two variables will be assumed to be cointegrated with a cointegrating coefficient of unity, so that the deviation from target, d_t , is stationary. We also assume that the policy variable i_t is stationary. More precisely, we will assume that the bivariate sequence $\{y_t\} = \{d_t, i_t\}$ is stationary and ergodic.

The first equation in our econometric model is derived from (1) by writing

$$i_t = \alpha_0 + \alpha_1 g(d_t, \beta) + q_1(z_{1t}, \delta_1) + u_{1t}, \quad 2$$

where $q_1(z_{1t}, \delta_1)$ is a known function, z_{1t} is the sub-vector of z_t containing those elements that are not excluded from $q_1(z_{1t}, \delta_1)$ on a priori grounds, δ_1 is an unknown parameter vector with p_1 elements, to be estimated, and $\{u_{1t}\}$ is an iid sequence of disturbances with density $f_1(u_1)$. We assume that u_{1t} is independent of z_t , but not necessarily of d_t . The possible endogeneity of the regressor d_t arises from the fact that simultaneity can be present in our system if the instrument i_t feeds back into the equation determining the exchange rate s_t (we would expect such feedback to exist if i_t is an effective instrument). The inclusion of the term $q_1(z_{1t}, \delta_1)$ reflects the presence of factors other than the current exchange rate deviation that may influence the behavior of i_t . We would expect, for example, that lags of i_t would enter z_{1t} if this variable exhibits any degree of persistence. Those elements of z_t that are excluded from $q_1(z_{1t}, \delta_1)$ furnish possible instruments in the instrumental variables estimation of β .

To fix ideas, consider a model in which i_t is the policy instrument of the central bank of a small open economy, and s_t is the domestic-currency price of a unit of the currency of a larger foreign economy, so that a positive value of d_t indicates that the domestic currency is undervalued relative to the central bank's target rate s_t^* . For example, if i_t were the change in reserve holdings of the foreign currency by the domestic central bank (a positive value of

which would then be expected to have a depressing effect on the value of the domestic currency), and if $g(d_t, \beta)$ were an increasing function of d_t , then one would expect α_1 to be less than zero. Although there is little theoretical basis to prefer one specification of the functional form of $g(d_t, \beta)$ to another, the following strikes us as intuitively reasonable and will be used in the empirical analysis reported below:

$$g(d_t, \beta) = d_t^\gamma I(d_t > 0) - \eta |d_t|^\gamma I(d_t < 0), \quad 3$$

where $I(\cdot)$ denotes the indicator function. In this example, the parameter vector of $g(d_t, \beta)$ is $\beta = (\eta, \gamma)$. The members of β have economically interesting interpretations. The shape of the response function will be governed by γ , with a value of unity indicating linear policy response and progressively larger values representing a convex policy response, i.e. one that is less responsive to small deviations from target and more responsive to larger deviations. We would not generally expect γ to be less than one. η is expected to be positive, with a value of unity reflecting policy which responds equally strongly to a relatively devalued and relatively overvalued currency. There may be reason to expect the central bank to be more sensitive to devaluations of the currency, in which case we would have $0 < \eta < 1$.

The simultaneous equations system is completed with the following equation characterizing the feedback of the instrument into the exchange rate:

$$d_t = \alpha_2 + \alpha_3 i_t + q_2(z_{2t}, \delta_2) + u_{2t}, \quad 4$$

where $q_2(z_{2t}, \delta_2)$ is a known function, z_{2t} contains the elements of z_t that are not excluded from $q_2(z_{2t}, \delta_2)$, δ_2 is an unknown parameter vector with p_2 elements, to be estimated, and $\{u_{2t}\}$ is an iid sequence of disturbances with density $f_2(u_2)$. Note that z_{1t} and z_{2t} are not prohibited by definition from having common elements. We assume here that the feedback of the instrument into the exchange rate is linear, an assumption that can easily be relaxed. The term $q_2(z_{2t}, \delta_2)$ will often contain lags of d_t . We assume that u_{2t} is independent of z_t , but not necessarily of i_t . The bivariate sequence $u_t = (u_{1t}, u_{2t})^T$ is iid from the density $f(u)$. The superscript T denotes transposition of a vector or matrix.

As mentioned, the terms $q_1(z_{1t}, \delta_1)$ and $q_2(z_{2t}, \delta_2)$ are included to capture time series dynamics that may be present in the series $\{i_t\}$ and $\{d_t\}$, respectively. One possible approach to the specification of these terms would be as autoregressions in the dependent variable of the respective equations, so that we would have

$$q_1(z_{1t}, \delta_1) = \sum_{j=1}^{p_1} \delta_{1j} i_{t-j} = \delta_1^T z_{1t} \quad 5$$

and

$$q_2(z_{2t}, \delta_2) = \sum_{j=1}^{p_2} \delta_{2j} d_{t-j} = \delta_2^T z_{2t}, \quad 6$$

where $z_{1t} = (i_{t-1}, \dots, i_{t-p_1})$ and $z_{2t} = (d_{t-1}, \dots, d_{t-p_2})$. We would then have the lags of the excluded variables available as instruments for the consistent estimation of (2) and (4). For example, lagged values of i_t could be used as instruments for the estimation of α_3 in (4), and nonlinear functions of lagged values of d_t could be used as instruments for the IV estimation of α_1 and β in (2). We shall assume throughout the paper that valid instruments are available, so that equations (2) and (4) are identified. We note here that a question that may need to be addressed in practice, particularly with respect to estimation of (2), is the quality of the instruments employed. Can we find functions of lagged d_t that are correlated with $g(d_t, \beta)$ (in

the case of estimation of α_1) and $\frac{\partial g(d_t, \beta)}{\partial \beta}$ (in the case of estimation of β)?

ESTIMATION OF THE MODEL

Although in practice one should test for the presence and validity of one's specification of the policy function $g(d_t, \beta)$ in (2) (i.e. test the null hypothesis that $\alpha_1 = 0$) before proceeding to estimate the parameter β , we will discuss the issue of estimation before that of testing (which is considered in the next section), as an understanding of the former is necessary to an understanding of the latter. We will begin by analyzing estimators of the model in the case where β is known to the investigator, then proceed to consider the case of unknown β . Of course, β will rarely, if ever, be known, but consideration of the case of known β will play an essential role in our derivation of tests of the null hypothesis that $\alpha_1 = 0$. For purposes of exposition, we assume that the functions $q_1(z_{1t}, \delta_1)$ and $q_2(z_{2t}, \delta_2)$ are linear, as in (5) and (6), although relaxing this assumption will complicate our exposition without adding any essential changes or difficulties.

Known β

In this case, equations (2) and (4) form a linear simultaneous equations system, so that, assuming the existence of valid instruments, consistent 2-stage and 3-stage least squares estimation is possible. Define $g_t(\beta) = g(d_t, \beta)$, where β belongs to the parameter space \mathcal{B} , and suppose that there exists an instrument $g_t^*(\beta)$ that is independent of u_t but for which $E[g_t(\beta)g_t^*(\beta)] \neq 0$. For equation (2), define the regressor vector

$$v_{1t}(\beta) = (1, g_t(\beta), z_{1t}^T)^T$$

and the instrument vector

$$v_{1t}^*(\beta) = (1, g_t^*(\beta), z_{1t}^T)^T.$$

For equation (4), the respective regressor and instrument vectors are

$$v_{2t} = (1, i_t, z_{2t}^T)^T$$

and

$$v_{2t}^* = (1, i_{t-1}, z_{2t}^T)^T,$$

where we assume that i_{t-1} is optimal among available valid instruments for i_t . Keeping β fixed, we will now consider estimation of the parameter vectors $\theta_1 = (\alpha_0, \alpha_1, \delta_1^T)^T$ and $\theta_2 = (\alpha_2, \alpha_3, \delta_2^T)^T$. Define the regressor and instrument matrices as follows:

$$V_{1n}(\beta) = \begin{pmatrix} v_{11}(\beta)^T \\ \cdot \\ \cdot \\ v_{1n}(\beta)^T \end{pmatrix},$$

with $V_{1n}^*(\beta)$, V_{2n} , and V_{2n}^* being defined analogously. We also have the dependent variable vector given by $Y_{1n} = (i_1, \dots, i_n)^T$ and $Y_{2n} = (d_1, \dots, d_n)^T$. The two stage least squares (2SLS) estimator of (4) does not depend on β and is given by

$$\hat{\theta}_{2n} = (V_{2n}^T V_{2n})^{-1} V_{2n}^T Y_{2n}$$

whereas the 2SLS estimator of (2) does depend on β , and is computed as follows:

$$\hat{\theta}_{1n}(\beta) = (V_{1n}^{*T}(\beta) V_{1n}(\beta))^{-1} V_{1n}^{*T}(\beta) Y_{1n}.$$

Defining the 2SLS residuals as

$$\hat{U}_{1n}(\beta) = Y_{1n} - V_{1n}(\beta)\hat{\theta}_{1n}(\beta)$$

and

$$\hat{U}_{2n} = Y_{2n} - V_{2n}\hat{\theta}_{2n},$$

with $\hat{U}_n(\beta) = (\hat{U}_{1n}(\beta), \hat{U}_{2n})$, we can estimate the error covariance matrix Σ by

$$\hat{\Sigma}_n(\beta) = n^{-1}\hat{U}_n(\beta)^T\hat{U}_n(\beta).$$

We can then compute the three stage least squares (3SLS) estimator, after introducing some additional notation:

$$Y_n = (Y_{1n}^T, Y_{2n}^T)^T,$$

$$\theta = (\theta_1^T, \theta_2^T)^T,$$

$$V_n(\beta) = \begin{bmatrix} V_{1n}(\beta) & 0 \\ 0 & V_{2n} \end{bmatrix},$$

and

$$V_n^*(\beta) = \begin{bmatrix} V_{1n}^*(\beta) & 0 \\ 0 & V_{2n}^* \end{bmatrix}.$$

We then compute 3SLS as

$$\tilde{\theta}_n(\beta) = \left(V_n^*(\beta)^T \left(\hat{\Sigma}_n(\beta)^{-1} \otimes I_n \right) V_n(\beta) \right)^{-1} V_n^*(\beta)^T \left(\hat{\Sigma}_n(\beta)^{-1} \otimes I_n \right) Y_n,$$

where I_n denotes the identity matrix of order n . If the disturbance vector is normally distributed, i.e. $u_t \sim iidN(0, \Sigma_u)$, then 3SLS will be asymptotically efficient and will be equivalent to the full information maximum likelihood estimator.

We note here that for such volatile economic time series as exchange rates, the assumption of normality may not be a good approximation to the distribution generating the disturbances, so we will take a brief look at the issue of semiparametric efficient estimation of the model when the error density is unknown. Semiparametric efficiency bounds for nonlinear simultaneous equations models in the context of iid data have been derived by Newey (1989) (see also Newey (1990)) under a number of different assumptions on the otherwise unknown distribution of the error vector u_t , including the case of symmetry. Brown and Hodgson (2004) extend Newey's analysis to allow for time series data and they also consider the case of elliptically symmetric errors. The results of these papers are applicable both to the known β and unknown β version of our model. Here, we give expressions for semiparametric efficient estimators for our model, as derived in the above papers.

When β is known, the model is linear and therefore adaptively estimable, so that the expression for an efficient estimator is relatively simple. Suppose that, for a given value of β , we have a \sqrt{n} -consistent preliminary estimator $\theta_n^+(\beta)$ (2SLS and 3SLS are examples of such estimators). Recalling that the unknown joint density function of the disturbance vector u is given by $f(u)$, we can denote the (negative of the) score vector of this density by

$$\psi(u) = \frac{\partial f(u)/\partial u}{f(u)}, \text{ with associated information of } \Omega_f = \int \frac{(\partial f(u)/\partial u)^2}{f(u)} du. \text{ The single-observation}$$

score vector of the log-likelihood function with regard to the parameter vector θ , as evaluated at the point (θ, β) belonging to the parameter space $\Theta \times \mathcal{B}$, can be shown to equal

$$s_t(\theta, \beta) = J_\theta(\theta, \beta) + \rho_{\theta_t}^T(\theta, \beta)\psi(u_t(\theta, \beta)), \quad 7$$

where $J_\theta(\theta, \beta) = (0, (\alpha_1 - \alpha_3)^{-1}, 0_{p_1}^T, 0, (\alpha_3 - \alpha_1)^{-1}, 0_{p_2}^T)^T$, 0_{p_i} is a vector of p_i elements, and

$$\rho_{\theta_t}(\theta, \beta) = \begin{bmatrix} -1 & -g_t(\beta) & -z_{1t}^T & 0 & 0 & 0_{p_2}^T \\ 0 & 0 & 0_{p_1}^T & -1 & -i_t & -z_{2t}^T \end{bmatrix}.$$

Assuming that the functional form of $f(u)$, and therefore of $\psi(u)$, is known, a fully efficient estimator can be computed as follows:

$$\theta_n^*(\beta) = \theta_n^+(\beta) + \left(n^{-1} \sum_{t=1}^n s_t(\theta_n^+(\beta), \beta) s_t^T(\theta_n^+(\beta), \beta) \right) \sum_{t=1}^n s_t(\theta_n^+(\beta), \beta). \quad 8$$

If the functional forms of f and ψ are unknown, then one can proceed semiparametrically by substituting into (7) and (8) a nonparametric kernel estimator $\hat{\psi}_t(u_t(\theta, \beta))$, the computation of which, for the symmetry case, is derived by, for example, Jeganathan (1995), while the computation for the elliptical symmetry case is described by Hodgson, Linton, and Vorkink (2002). We refer the reader to these papers for details on the computation of these semiparametric estimates. The following Proposition can be proved as a special case of Theorem 1 of Brown and Hodgson (2004).

PROPOSITION: *The asymptotic distribution of the estimator $\theta_n^*(\beta)$ is invariant to the substitution for the true score function $\psi(u_t(\theta, \beta))$ in (7) and (8) of the nonparametric scores $\hat{\psi}_t(u_t(\theta, \beta))$ as described in, for example, Jeganathan (1995) and Hodgson, Linton, and Vorkink (2002).*

Unknown β

We will now discuss the estimation of the model when the parameter vector β is unknown and so must also be estimated. To this end, we redefine the parameter vector $\theta_1 = (\alpha_0, \alpha_1, \beta^T, \delta_1^T)^T$. As before, the full parameter vector is $\theta = (\theta_1^T, \theta_2^T)^T$, and the 2SLS estimator $\hat{\theta}_{2n}$ can be computed as in Section 3.1. Estimation of θ_1 is now a more complicated matter, as we must use a nonlinear instrumental variables procedure to estimate (2). To this end, we require an instrument vector $v_{1t}^*(\theta_1)$ that is independent of the disturbance u_{1t} while still being correlated with the derivative vector

$$v_{1t}(\theta_1) = \left(1, g_t(\beta), \alpha_1 \frac{\partial g_t(\beta)}{\partial \beta^T}, z_{1t}^T \right)^T.$$

In practice, there may be available an excess of valid instruments, in which case it would be possible in principle to select a vector of optimal instruments and compute a non-linear analogue of 2SLS. Our nonlinear IV estimator $\hat{\theta}_{1n}$ would then be the solution to the following nonlinear system:

$$n^{-1} \sum_{t=1}^n u_{1t}(\hat{\theta}_{1n}) v_{1t}^*(\hat{\theta}_{1n}) = 0. \quad 9$$

Numerical methods are required to solve (9) in practice. Our nonlinear IV estimator of the full

parameter vector θ is therefore $\hat{\theta}_n = \left(\hat{\theta}_{1n}^T, \hat{\theta}_{2n}^T \right)^T$.

As in the model where β is known, it is of interest to consider the issue of efficient estimation. How one goes about this problem will depend on the distributional assumptions one wants to make about the error density $f(u)$. If it is assumed to be Gaussian, then standard Gaussian ML procedures are available. Newey (1990) analyzes a general nonlinear simultaneous equations model and derives a formula for a semiparametric efficient estimator if f is assumed to be symmetric about zero, but otherwise unrestricted. Newey (1990) only considers the case of iid data, but his results should extend to the time series case. Brown and Hodgson (2004) analyze essentially the same nonlinear simultaneous equations model as Newey (1990), but in a time series context and with the further restriction on f of elliptical symmetry. Brown and Hodgson (2004) derive the semiparametric efficiency bound under these conditions and suggest a method of computing an efficient estimator. We will now give a brief description of this approach.

We begin by introducing some notation and definitions. Assuming that the density $f(u)$ is elliptical, we can write it as follows:

$$\begin{aligned} f(u) &= |\Sigma|^{-1/2} p(u^T \Sigma^{-1} u) \\ &= |\Sigma|^{-1/2} p(\varepsilon^T \varepsilon), \end{aligned}$$

where $\varepsilon = \Sigma^{-1/2} u$ and Σ is a scalar multiple of the covariance matrix Σ_u , normalized so that $\det(\Sigma) = 1$. Define $\sigma = \text{vech}(\Sigma)$, and redefine the full parameter vector as $\theta = (\theta_1^T, \theta_2^T, \sigma^T)^T$. We can compute estimates $\hat{\theta}_n$ in the usual way using the residuals from the nonlinear IV estimators $\hat{\theta}_{1n}$ and $\hat{\theta}_{2n}$, and can then define the IV estimator of θ as being $\hat{\theta}_n = \left(\hat{\theta}_{1n}^T, \hat{\theta}_{2n}^T, \hat{\sigma}_n^T \right)^T$. Recalling that $y_t = (i_t, d_t)^T$, we can define the following functions:

$$\rho(y_t, z_t, \theta) = \Sigma^{-1/2} \begin{bmatrix} i_t - \alpha_0 - \alpha_1 g(d_t, \beta) - q_1(z_{1t}, \delta_1) \\ d_t - \alpha_2 - \alpha_3 i_t - q_2(z_{2t}, \delta_2) \end{bmatrix} = \varepsilon_t(\theta),$$

$$J(y_t, z_t, \theta) = \ln \left| \det \left(\frac{\partial \rho(y_t, z_t, \theta)}{\partial y_t} \right) \right|,$$

and

$$\phi(\varepsilon^T \varepsilon) = \frac{2p'(\varepsilon^T \varepsilon)}{p(\varepsilon^T \varepsilon)}.$$

As shown by Brown and Hodgson (2004), the *efficient score function* for this model is given by

$$\begin{aligned} s_t(\theta) &= J_\theta(y_t, z_t, \theta) - E[J_\theta(y_t, z_t, \theta) | \varepsilon^T \varepsilon] \\ &\quad + \{ \rho_\theta^T(y_t, z_t, \theta) \varepsilon_t(\theta) - E[\rho_\theta^T(y_t, z_t, \theta) \varepsilon_t(\theta) | \varepsilon^T \varepsilon] \} \phi(\varepsilon(\theta)^T \varepsilon(\theta)), \end{aligned} \tag{10}$$

where the θ subscripts denote partial derivatives. The semiparametric efficiency bound is given by $\mathcal{B} = E[s_t(\theta) s_t(\theta)^T]^{-1}$, and, if the two conditional expectations in (10) were known, along with the functional form of $\phi(\varepsilon^T \varepsilon)$, then one could compute the following iterative estimator, which would achieve the bound asymptotically,

$$\theta_n^* = \hat{\theta}_n + \left(n^{-1} \sum_{t=1}^n s_t(\hat{\theta}_n) s_t^T(\hat{\theta}_n) \right) \sum_{t=1}^n s_t(\hat{\theta}_n). \tag{11}$$

Of course, the conditional expectations and $\phi(\varepsilon^T \varepsilon)$ will generally not be known. However, it may be possible to compute nonparametric estimates $\hat{E}[J_\theta(y_t, z_t, \theta)|\varepsilon^T \varepsilon]$, $\hat{E}[\rho_\theta^T(y_t, z_t, \theta)\varepsilon_t(\theta)|\varepsilon^T \varepsilon]$, and $\hat{\phi}(\varepsilon^T \varepsilon)$ that can be substituted into (10) and (11) without affecting the first-order asymptotic distribution of the iterative estimator θ_n^* . Details of the computation of these nonparametric estimates are given by Brown and Hodgson (2004). The following Proposition can be proved as a special case of Theorem 1 of Brown and Hodgson (2004).

PROPOSITION: *The asymptotic distribution of the estimator θ_n^* is invariant to the substitution for the true score and conditional expectation functions in (10) and (11) of the nonparametric versions of these functions.*

TESTING FOR THE PRESENCE AND SPECIFICATION OF THE POLICY REACTION FUNCTION

As mentioned above, we are interested, for various reasons, in the question of testing the null hypothesis of $\alpha_1 = 0$ in (2). This is an unusual and interesting problem because the parameter β is unidentified under the null hypothesis but not under the alternative, creating a nonstandard testing problem which several authors have considered. Andrews and Ploberger (1994) obtain a class of optimal tests, but don't say much about implementation or computation of critical values (the tests have nonstandard distributions). The issue of critical values is addressed by Hansen (1996), who provides an illustration through an empirical example.

Andrews-Ploberger (1994)

The tests of Andrews and Ploberger (1994) involve computing LM, LR, or Wald statistics of the null for various values of β , and then computing a weighted average (over the set of possible values of β) of these statistics. For each choice of β , we can compute a Wald, LM, or LR statistic of the null hypothesis that $\alpha_1 = 0$. The Wald test, for example, would be

$$W_n(\beta) = \frac{n\bar{\alpha}_{1n}^2(\beta)}{\widehat{se}(\bar{\alpha}_{1n}(\beta))}, \quad 12$$

where $\bar{\alpha}_{1n}(\beta)$ is one of the estimators of $\alpha_1(\beta)$ described in Section 3a. The denominator in (12) is a consistent estimator of the asymptotic standard deviation of the relevant estimator. The exponential Wald test as suggested by Andrews and Ploberger then takes the form

$$Exp - W_n = (1 + c)^{-1/2} \int_B \exp\left(\frac{c}{2(1 + c)} W_n(\beta)\right) dJ(\beta), \quad 13$$

where c and $J(\beta)$ are user-defined constant and weight function, respectively, whose choice is discussed by Andrews and Ploberger (1994). The limit of the statistic in (13) as $c \rightarrow 0$ is the "average-Wald" ("ave-W") statistic

$$\int_B W_n(\beta) dJ(\beta), \quad 14$$

while its limit as $c \rightarrow \infty$ is

$$\log \int_B \exp\left(\frac{1}{2} W_n(\beta)\right) dJ(\beta), \quad 15$$

and will be referred to below as "log-exp-W". The resulting statistic will have a nonstandard

limiting distribution, computation of the p -values of which is considered by Hansen (1996). Note that in practice it may be necessary to compute $W_n(\beta)$ for a discrete set of points β belonging to the parameter space. We will now describe a method of conducting inference with the statistic given in (13) which follows the lines of Hansen (1996).

To illustrate the idea behind the procedure suggested by Hansen (1996), we consider the application of the procedure to the 2SLS estimator of the parameter vector θ_1 , which is $\hat{\theta}_{1n}(\beta)$ as defined above. The null hypothesis that $\alpha_1 = 0$ can be expressed as the null that $R^T\theta_1 = 0$, where R is a vector of dimension $2 + p_1$ whose elements are all zeros, excepting the second, which is a one. The ‘‘regression score’’ defining $\hat{\theta}_{1n}(\beta)$ is

$$s_t(\beta) = v_{1t}^*(\beta)u_{1t}$$

and its estimated version is

$$\hat{s}_t(\beta) = v_{1t}^*(\beta)\hat{u}_{1t},$$

where \hat{u}_{1t} is the residual from the 2SLS estimator $\hat{\theta}_{1n}(\beta)$. Now define the following matrices:

$$M_{n,ss}(\beta) = n^{-1} \sum_{t=1}^n \hat{s}_t(\beta)\hat{s}_t(\beta)^T,$$

and

$$M_{n,v*v}(\beta_1, \beta_2) = n^{-1} \sum_{t=1}^n v_{1t}^*(\beta_1)v_{1t}(\beta_2)^T,$$

where β_1 and β_2 are possibly different points in the parameter space \mathcal{B} . The asymptotic covariance matrix of $\hat{\theta}_{1n}(\beta)$ is then consistently estimated by

$$\hat{\Phi}_{\theta_1}(\beta) = M_{n,v*v}^{-1}(\beta, \beta)M_{n,ss}(\beta)M_{n,v*v}(\beta, \beta)^{-1T}.$$

The Wald statistic $W_n(\beta)$ defined in (12) can then be rewritten as follows:

$$W_n(\beta) = n\hat{\theta}_{1n}(\beta)^TR\left(R^T\hat{\Phi}_{\theta_1}(\beta)R\right)^{-1}R^T\hat{\theta}_{1n}(\beta).$$

Now, suppose that we have used a random number generator to supply a sequence of iid standard normal random variables $\{\pi_t\}_{t=1}^n$. Define the statistics

$$\hat{S}_n(\beta) = n^{-1/2} \sum_{t=1}^n \hat{s}_t(\beta)\pi_t \tag{16}$$

and

$$\hat{W}_n(\beta) = \hat{S}_n(\beta)^TM_{n,v*v}^{-1}(\beta, \beta)R\left(R^T\hat{\Phi}_{\theta_1}(\beta)R\right)^{-1}R^TM_{n,v*v}(\beta, \beta)^{-1T}\hat{S}_n(\beta). \tag{17}$$

To compute the p -value of our $Exp - W_n$ statistic given in (13), we generate K different sequences of iid random normals, $\{\pi^k\}_{t=1}^n$, $k = 1, \dots, K$, and for each k , we use the definitions (16) and (17) to compute $\hat{S}_n^k(\beta)$ and $\hat{W}_n^k(\beta)$, the latter of which can be substituted into (13) to give us the statistic

$$Exp - W_n^k = (1 + c)^{-1/2} \int_{\mathcal{B}} \exp\left(\frac{c}{2(1 + c)}\hat{W}_n^k(\beta)\right)dJ(\beta).$$

The asymptotic p -value of the $Exp - W_n$ statistic computed from the data will then be estimated to an arbitrarily high degree of accuracy by the proportion of the simulated $Exp - W_n^k$

statistics that exceed it.

The same general procedure can be used to determine p -values when the 3SLS or adaptive estimators are used, with suitable redefinition of the relevant statistics. In the case of 3SLS, we are estimating the full parameter vector θ , of dimension $4 + p_1 + p_2$, and our null hypothesis can be expressed as $R^T\theta = 0$, where now R is a vector of dimension $2 + p_1$ whose elements are all zeros, excepting the second, which is a one. We can now write the regression score defining $\hat{\theta}_n(\beta)$ as

$$s_t(\beta) = v_t^*(\beta)^T \Sigma^{-1} u_t,$$

where

$$v_t^*(\beta) = \begin{bmatrix} v_{1t}^*(\beta)^T & 0 \\ 0 & v_{2t}^{*T} \end{bmatrix}.$$

The estimated form of the regression score is

$$s_t(\beta) = v_t^*(\beta)^T \hat{\Sigma}(\beta)^{-1} \hat{u}_t(\beta).$$

Turning to the adaptive estimator, the form of the regression score is given by $s_t(\theta, \beta)$ in (7), and its estimated form can be obtained by substituting into (7) preliminary estimators $\hat{\theta}_n$ and $\hat{\psi}_t(u_t(\hat{\theta}_n, \beta))$.

INTERVENTION BY BANK OF CANADA

Canada is a classic example of a small open economy, the lion's share of whose foreign trade is with its mammoth neighbour, the United States. The exchange rate between the Canadian and U.S dollars is thus of great interest and importance to Canada, and it is plausible that the rate is closely monitored, and possibly influenced, by the Bank of Canada. A number of attempts have been made to econometrically measure the nature and extent of the Bank of Canada's intervention in the foreign exchange market (for example, Longworth (1980), Weymark (1995), and Rogers and Siklos (2003)). According to the following quotation, taken from the Bank's website and dated July 2001, it has in recent years refrained from such intervention:

The Bank of Canada influences the exchange rate only indirectly. This can happen when the Bank changes its Target for the Overnight Rate, which affects short-term interest rates. As of 1998, the Bank no longer intervenes in foreign exchange markets to ensure an orderly market, but rather reserves such actions for times of major international crisis or a clear loss of confidence in the currency or Canadian-dollar-denominated securities.

The test outlined in the preceding section, applied to Canadian data from the post-1998 period, would thus constitute a test of the null hypothesis that the Bank's public utterance of a no-intervention policy is an accurate reflection of its true behaviour. We proceed with such an analysis in this section. Before presenting our results, we discuss various details relating to the application of the methodology.

Data and Measurement of Variables

The first step in any study of foreign exchange market intervention is to define precisely what will be meant by "intervention". How is it measured in practice, using available data series? Secondly, in estimating the response of the intervention variable to deviations of the exchange rate from its target, we must somehow measure or estimate a (generally time-varying) target exchange rate. Various approaches have been taken to the definition of

both of these variables, as can be seen by a quick perusal of the literature survey of Almekinders and Eijffinger (1991).

Many authors use changes in foreign reserve holdings, possibly modified to account for changes in reserves due to factors other than intervention, as a measure of intervention. Dornbusch (1980, p.173), for example, in a study of the markets for several currencies (including the Canadian) vis-à-vis the U.S. dollar, uses as his measure “an adjusted series that subtracts from changes in reserves an amount equal to the U.S. Treasury bill rate times the lagged stock of reserves. This series is measured as a fraction of lagged reserves.” The intervention measure used by Longworth (1980, p.285) is “the change in foreign exchange reserves less revaluation items (and strategic drawing reserve allocation) plus the change in net undelivered contracts in U.S. dollars”. Weymark (1995, p.281), in the context of a fully-specified five-equation model of the macroeconomy, proposes “an index of exchange market intervention that measures the intervention activity of the policy authority in terms of the proportion of exchange market pressure relieved by exchange market intervention,” constructed as follows:

$$\frac{\Delta r_t}{(1/\eta)\Delta s_t + \Delta r_t},$$

where s_t is the log-exchange rate, $\Delta r_t = [h_t R_t - h_{t-1} R_{t-1}]/M_{t-1}$, h_t is the money multiplier, R_t the stock of foreign exchange reserves, M_{t-1} the inherited money stock in period t , and $\eta = -\partial e_t / \partial r_t$ is a parameter that depends on the specification of the model.

We use as our measure of intervention the first difference in the Bank of Canada’s official international reserves of U.S. dollars. We use weekly observations on reserves running from July 7, 1997 to January 8, 2003, for 169 observations on reserves and 168 on first differences. The series was obtained from the Statistics Canada CANSIM database, series v15943317.

Various approaches have also been taken to the specification of the target exchange rate s_t^* . In the absence of an explicitly stated target rate, the specification here is largely left to the discretion of the researcher. The target could simply be the previous period’s exchange rate, so that intervention is modelled as being a reaction to any change in the exchange rate (Longworth (1980), for example, takes this approach, while also considering the possibility that parity of the U.S. and Canadian dollars is an objective of the Bank of Canada, a much less outlandish question at the time than it may seem today). In a similar fashion, a moving average of recent exchange rate levels could also be used. Other approaches, such as considering deviations from purchasing power parity, or deviations of the exchange rate from its expected value according to uncovered interest parity, have also been considered (see Almekinders and Eijffinger (1991)).

We use weekly observations of the Canada-U.S. exchange rate, obtained from the Bank of Canada web site, for the same dates as for the reserve series mentioned above. As a measure of the target rate, we use an equally-weighted moving average of recent (log) exchange rate levels, considering a number of lags ranging from 4 to 12 weeks.

Estimation and Testing - Details

We will report results for the application of the statistics given in (14) and (15) for the test of the null hypothesis that $\alpha_1 = 0$ in equation (2), with $q_1(z_{1t}, \delta_1) = 0$ and the function g specified as in (3). For given values of β , (2) is estimated by OLS and IV, and (2) and (4) jointly by 3SLS, with $g(d_{t-1}, \beta)$ being used as the instrument for $g(d_t, \beta)$. The target rate s_t^* is defined as the equally-weighted moving averages of the first 12 lags of s_t (some experimentation with different settings of the moving average order produced very similar

results). For the tests, 2500 possible values of the bivariate vector β are considered, with γ values taken from the range 1-4, divided into a grid of 50 points, with the range of considered η values being .75-1, with a grid of 50 points. The weights $J(\beta)$ decline linearly in γ from 1 to 4, and decline linearly in η from 1 to .75, so that maximal weight is placed on $(\gamma, \eta) = (1, 1)$, and weight zero placed on $(\gamma, \eta) = (4, .75)$. The number of simulation draws used in the computation of the p -values is $K = 1000$.

We first present some details on the data series to be employed in the analysis, in Table 1 and Figure 1 (where the reserves variable has been scaled down to approximately the same scale as the exchange rate variable). As Table 1 indicates, the two series are slightly negatively correlated, with a correlation coefficient of -.087. The degree of autocorrelation in the reserve changes is slight, but is quite strong in the exchange rate deviations. The Jarque-Bera (1980) statistic is small for the latter variable, with some evidence of skewness, whereas the reserve change is very highly leptokurtic. This characteristic is evident from a glance at Figure 1, where a handful of large outliers, both positive and negative, stand out.

Table 2 contains the results of the application of the log-exp-W and ave-W statistics, computed as described above. The results for both tests, and for all three estimators considered, strongly suggest that there is no policy response to deviations of the exchange rate from recent levels. The p -values of the statistics range from .46 to .64, lending strong support to the null hypothesis that $\alpha_1 = 0$. It is of some interest to note that although the test statistics and p -values for IV and 3SLS are quite close to one another, they both differ somewhat from the corresponding OLS numbers. Although not constituting a formal test of the exogeneity of the regressors, these figures suggest the possibility that the regressors are not exogenous in the policy equation.

The point estimates and standard errors of the estimation of the parameters of equations (2) and (4) are reported in Table 3. Equation (2) was estimated by NLLS and NLIV, whereas (4), being linear, was estimated by OLS and IV (the instruments used are as described above). The starting value in the nonlinear estimation algorithm for the parameter vector $(\alpha_0, \alpha_1, \gamma, \eta)$ in (2) was (0, -0.1, 1, 1). Some alternative starting values were also considered as a check on the robustness of the results. The estimate of the intercept α_0 is positive and not sensitive to the estimator (a finding that was robust to variation in the starting value). The estimates of α_1 tend to be fairly small and negative, but with quite large standard errors. There was some sensitivity of this estimate to starting values, not surprising considering the imprecision of the estimate. Perhaps the most interesting result here is a finding of γ in the 2-3 range, and fairly precisely estimated. Although there was some sensitivity of the standard error estimate to changes in starting value, the point estimate obtained is fairly robust. Note that the η estimates are far too imprecise to be of any interest. This result suggests that reliable information about the value of η cannot be extracted from this data set, and suggests redoing the analysis above in assuming a symmetric policy response, fixing $\eta = 1$, and focusing attention on the degree of nonlinearity as represented by γ . (to come)

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Table 1 – Summary Statistics

	Change in reserves (i_t)			Exchange rate deviation (d_t)		
Mean	5.50×10^{-5}			2.22×10^{-3}		
Var	1.07×10^{-5}			1.86×10^{-4}		
Corr	-.087					
$\rho(1)$	-.078			.817		
$\rho(2)$	-.119			.634		
$\rho(3)$.061			.455		
$\rho(4)$.013			.309		
$\rho(5)$	-.050			.212		
J-B (sk, kurt,sk+kurt)	.99	782.09	783.09	6.90	0.47	7.37

Note: $\rho(j)$ indicates the autocorrelation at lag j , and J-B refers to the Jarque-Bera (1980) skewness, kurtosis, and skewness-kurtosis statistics

Table 2 – Tests of null of no intervention ($\alpha_1 = 0$ in equation (2))

Estimator	Log-exp-W	p-value	Ave-W	p-value
OLS	.29	.46	.54	.47
IV	.10	.63	.20	.63
3SLS	.11	.64	.22	.64

Table 3 – Parameter estimates

Policy equation (2)				
Estimator	α_0	α_1	γ	η
NLLS	2.54×10^{-5} (3.66×10^{-4})	-.0365 (5.46)	2.43 (.110)	62.9 (9250)
NLIV	2.96×10^{-5} (4.85×10^{-4})	-.144 (19.8)	2.59 (.572)	11.4 (1230)
Feedback equation (4)				
Estimator	α_2	α_3	δ_1	δ_2
OLS	4.45×10^{-4} (6.36×10^{-4})	-.196 (.191)	.911 (.0801)	-.115 (.0799)
IV	4.52×10^{-4} (6.46×10^{-4})	-.347 (2.20)	.905 (.114)	-.109 (.110)

Note: Standard errors are in parentheses

Fig. 1 – Change in Foreign Reserve Holdings (Solid) and Deviation of Log Exchange Rate from Target (Broken)

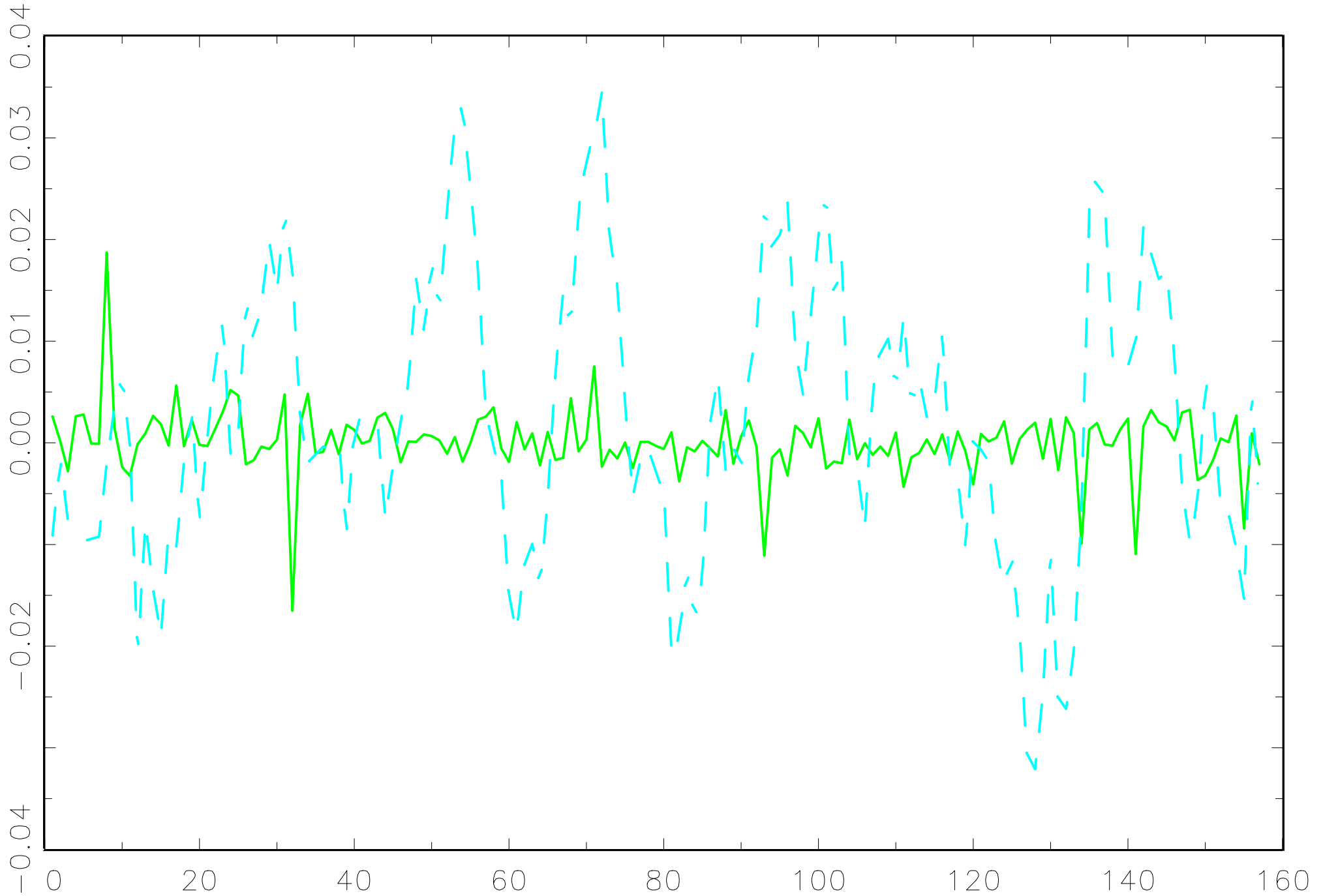


Fig. 2 – IV Residuals, Policy equation

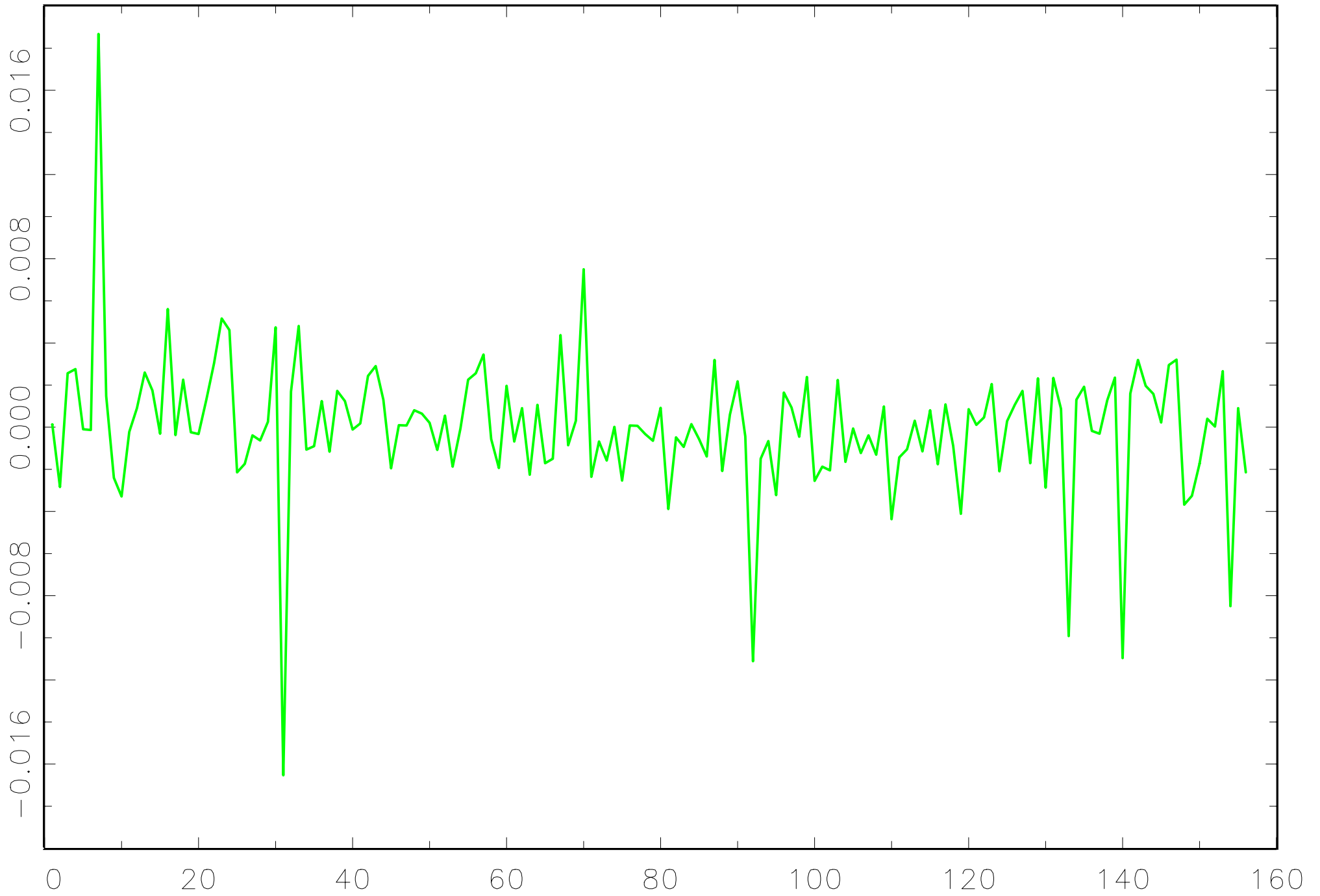


Fig. 3 – IV Residuals, Feedback equation

