Welfare Effects of Monetary Policy Rules in a Model with Nominal Rigidities and Credit Market Frictions *

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Abstract

This paper evaluates monetary policy rules in a business cycle model with staggered prices and wage setting a la Calvo and asymmetric information in the credit market. Rules are compared in a utility based welfare metric, the effects of the model's nonlinear dynamics are captured by a quadratic approximation to the policy function. The firms net worth crucially affects the terms of obtaining outside finance. Financial frictions dampen the economy's response to shocks and make them more persistent. For the baseline calibration, the welfare costs of price stickiness are found to be less than 0.04 per cent of steady state consumption. However, wage stickiness can induce welfare costs of up to 0.85 per cent of steady state consumption. An interest rate rule that places high weight on stabilizing wage inflation can eliminate most of these costs. These findings are by and large independent of the existence of other real distortions in the model, namely credit frictions.

Keywords: Monetary policy rules, nominal rigidities, welfare evaluation, quadratic approximation, agency costs, credit frictions.

JEL Classification: E32, E52, E58

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1 Introduction

The literature on optimal monetary policy is huge. The seminal works by Rotemberg and Woodford (1999) have built a foundation for monetary policy analysis based on completely specified models with optimizing, forward looking firms and households and infrequent price adjustment. This framework allows to derive model consistent objectives for the central bank from the preferences of the agents and the constraints imposed by the model. Woodford (2003) shows that for a particular class of models, the central bank's problem of welfare maximization can be reduced to a constrained minimization of variances of inflation and a properly defined output gap term. The constraints are given by the linearized equilibrium conditions of the model economy. Such a framework essentially allows to operate within the framework of linear-quadratic control, optimal monetary policy can be easily characterized.

It is well known that the aforementioned methodology is limited to a restricted class of models. Special assumptions are needed to ensure that the central bank's objective is free of first order terms, as these terms may introduce a bias when evaluated using a first order approximation to the policy function.[1] The bias may arise, since these terms are independent of policy when evaluated by use of linear policy function, but in fact depend on higher order terms in the approximate policy function. Among these special assumptions are an output subsidy that ensures that the steady state level of output is efficient (despite the presence of monopolistic competition in the goods market), and the absence of capital accumulation. Clearly, these assumptions rule out the analysis of models that feature several non-monetary distortions and incorporate aspects that improve a models fit with the data, such as capital adjustment costs, or variable capacity utilization.

However, recent advances in numerical methods allow to analyze welfare effects of monetary policy rules for a larger class of models by departing from the linear quadratic framework through second order Taylor approximations to the policy functions. This paper uses the numerical procedures and MATLAB code provided by Schmitt-Grohé and Uribe (2004) to evaluate the welfare effects of different monetary policy rules in realistic dynamic general equilibrium model. The main characteristics of this model are staggered price and wages setting, capital accumulation as well as informational frictions in the credit market.

The objective of this paper is to quantify the welfare effects of different versions of Taylor rules as well nominal income targeting rules. The paper is organized as follows. Section 1 derives the models equilibrium conditions. Subsequently, the models main effect of credit frictions on the model's dynamics is illustrated via impulse-response analysis. In section 3, the solution method and the welfare measure are described. Section 4 presents the welfare effects of monetary policy rules for various degrees of nominal rigidities and credit frictions. Finally, section 5 concludes.

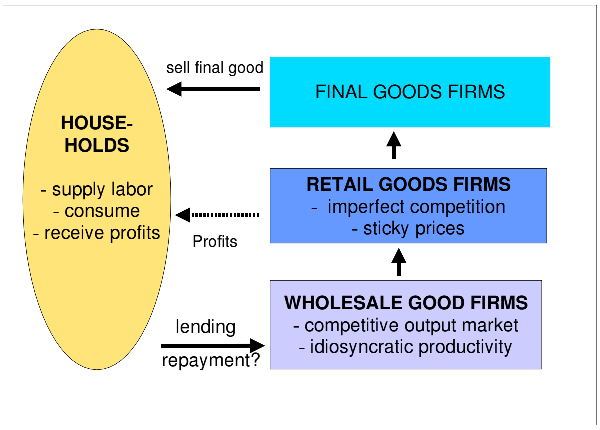
2 The model

The model is an extension of Carlstrom and Fuerst (2000) to include nominal wage and price stickiness. Carlstrom and Fuerst (2000) incorporate an asymmetric information problem between borrowers and lenders into a standard business cycle model

[1] Sometimes it is possible to eliminate these first order terms by approximating a small number of equilibrium conditions to second order. Whenever this is possible, there is no need to compute a second order approximation to the policy functions.

with money. Before going into detail, figure 1 sketches the economy's structure and serves as a road map for the future exposition. Households work, consume and lend

Figure 1: Graph of the model



funds to firms. The borrower-lender relationship is characterized by asymmetric information about idiosyncratic firm productivity. The production structure has three levels as shown in the graph. At the lowest level, a continuum of firms produce a homogeneous wholesale good. The asymmetric information problem takes place between entrepreneurs and households at this level. The wholesale good is used as input of production by a continuum of imperfectly competitive firms that set price in a staggered fashion at the second level. Finally, a competitive bundler aggregates these intermediate varieties into the final output goods, which can be used for consumption and investment. This structure is designed to keep the asymmetric information problem separate from the price stickiness, following Bernanke, Gertler, and Gilchrist (2000). The set up of the model will be explained in more details in the next subsections.

2.1 Household

A cashless limit of a money in the utility function framework a la Woodford (2003) is considered. Since money plays no role in the money in the utility function framework other than to back out the money supply that supports a given interest rate, this can be seen as an innocuous simplification. Households are infinitely lived, supply labor N and receive real wage w, consume final goods C, rent capital goods K to firms at the real rental rate r. Furthermore, they hold government bonds B and receive profits Π from the monopolistic retailers. The utility function is assumed to be separable in consumption and labor. The problems of household h is to:

$$\max_{\{B_{t+1+i}(h),K_{t+1+i}(h),C_{t+i}(h),N_{t+i}(h)\}_{j=0}^{\infty}} E_t \sum_{i=0}^{\infty} \beta^{t+i}\left[U(C_{t+i}(h)) + V(N_{t+i}(h))\right] \quad \text{s.t.}$$

$$C_t(h) = \frac{R_t B_t(h) - B_{t+1}(h)}{P_t} + w_t N_t(h) + \Pi_t(h) + (1+r_t)K_t(h) - K_{t+1}(h).$$

The first order conditions with respect to bonds and capital are

$$U_C(C_t(h)) = E_t\beta\left\{\frac{P_t}{P_{t+1}}U_C(C_{t+1}(h))\right\}R^n_{t+1}, \tag{2.1}$$

$$U_C(C_t(h)) = E_t\beta\{(1+r_{t+1})U_C(C_{t+1}(h))\}. \tag{2.2}$$

Although households also lend to firms, lending does not appear in this budget constraint. It is assumed to be intra-period. This assumption is made in order to facilitate comparison with the model of Carlstrom and Fuerst (2000).

The framework for motivating wage rigidities is standard. A continuum of households supply differentiated labor $N_t(h)$, which is aggregated according to the Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 [N_t(h)]^{\frac{\kappa-1}{\kappa}}\ dh\right]^{\frac{\kappa}{\kappa-1}}. \tag{2.3}$$

Here W_t is the Dixit-Stiglitz wage index. The demand function for differentiated labor is:

$$N_t(h) = \left[\frac{W_t(h)}{W_t}\right]^{-\kappa}L_t \tag{2.4}$$

Nominal rigidities are introduced according to the Calvo (1983) specification. With constant probability θ_w a randomly chosen household is allowed to re-optimize its nominal wage in a given period. We add backward looking components into wage setting following Gali and Gertler (1999). There are two types of wage setters, that differ only in their behavior when receiving a signal to re-optimize the wage. A fraction of households of measure φ_w is backward looking, the remaining households are forward looking. A forward looking households that receives a signal to re-optimize its price, maximizes expected utility through choice of the nominal wage subject to the demand curve and the budget constraint. The FOC for this problem is:

$$E_t\sum_{j=0}^{\infty}(\theta_w\beta)^j N_{t+j}(h)U_C(C_{t+j})\left[\frac{W_t^*(h)}{P_{t+j}} + \frac{\kappa}{\kappa-1}\frac{V_N(N_{t+j}(h))}{U_C(C_{t+j})}\right] = 0 \tag{2.5}$$

As is well known, this condition implies that nominal wages are front loaded. Since prices are fixed with certain probabilities in futures periods, expectation of future price levels as well as marginal rates of substitution influence current wage setting.[2]

For the numerical implementation, the following functional forms are chosen:

$$U(C_t) = \frac{C_t^{1-\tau}}{1-\tau}$$

$$V(N_t) = -\nu\frac{N_t^{1+\lambda}}{1+\lambda}$$

Using the demand function $N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\kappa}L_t$ and $V_N(N_t(h))_t = -\nu N_t(h)^\lambda$ as well as $U_C(C_t(h)) = C_t^{-\tau}$ this condition can be expressed in terms of the optimal

[2]Since households set different nominal wages, they will generally supply different amounts of labor and receive different wage income. Typically, the existence of a complete contingent claims market is assumed in order to be able to ensure equality of consumption across households. Such a market is implicitly assumed in this paper as well, although not modeled explicitly.

nominal wage W_t^* (we focus on a symmetric equilibrium and henceforth drop the index h) and aggregate variables only[3]

$$E_t \sum_{j=0}^{\infty} (\theta_w \beta)^j \left(\frac{W_t^*}{W_{t+j}}\right)^{-\kappa} L_{t+j} C_{t+j}^{-\tau} \left[\frac{W_t^*}{P_{t+j}} - \frac{\kappa\nu}{\kappa-1} \frac{\left(\frac{W_t^*}{W_{t+j}}\right)^{-\kappa\lambda} L_{t+j}^{\lambda}}{C_{t+j}^{-\tau}}\right] = 0 \tag{2.6}$$

In order to apply the MATLAB routines provided by Schmitt-Grohé and Uribe (2004) it is necessary to cast the first order condition for wage setting in a companion form, i.e. involving only variables dated $t+1$ and t. We express the Calvo wage setting condition in terms of the aggregate real wage w_t, the optimal real wage for optimizing firm w_t^* and CPI inflation π_t instead of nominal wage and the price level in the following way. Define two auxiliary variables A_t and B_t by the expressions

$$A_t = E_t \sum_{j=0}^{\infty} (\theta_w \beta)^j \left(\frac{W_t^*}{W_{t+j}}\right)^{-\kappa(1+\lambda)} L_{t+j}^{1+\lambda} \tag{2.7}$$

$$B_t = E_t \sum_{j=0}^{\infty} (\theta_w \beta)^j \left(\frac{W_t^*}{W_{t+j}}\right)^{-\kappa} L_{t+j} C_{t+j}^{-\tau} \frac{W_t^*}{P_{t+j}} \tag{2.8}$$

The first-order condition then reads $B_t = \frac{\kappa\nu}{\kappa-1} A_t$. The auxiliary variables have the following recursive representation:

$$A_t = \left(\frac{w_t^*}{w_t}\right)^{-\kappa(1+\lambda)} L_t^{1+\lambda} + \theta_w \beta E_t \left(\frac{w_t^*}{w_{t+1}^* \pi_{t+1}}\right)^{-\kappa(1+\lambda)} A_{t+1} \tag{2.9}$$

$$B_t = \left(\frac{w_t^*}{w_t}\right)^{-\kappa} w_t^* L_t C_t^{-\tau} + \theta_w \beta E_t \left(\frac{w_t^*}{w_{t+1}^* \pi_{t+1}}\right)^{-\kappa+1} B_{t+1} \tag{2.10}$$

To see that this is correct, repeatedly substitute A_{t+j} for $j = 1, 2, ..$

$$\begin{aligned} A_t = &\left(\frac{w_t^*}{w_t}\right)^{-\kappa(1+\lambda)} L_t^{1+\lambda} \\ &+ \theta_w \beta E_t \left(\frac{w_t^*}{w_{t+1}^* \pi_{t+1}}\right)^{-\kappa(1+\lambda)} \left\{ \left(\frac{w_{t+1}^*}{w_{t+1}}\right)^{-\kappa(1+\lambda)} L_{t+1}^{1+\lambda} \right. \\ &\left. + \ \theta_w \beta E_{t+1} \left(\frac{w_{t+1}^*}{w_{t+2}^* \pi_{t+2}}\right)^{-\kappa(1+\lambda)} \left[\left(\frac{w_{t+2}^*}{w_{t+2}}\right)^{-\kappa(1+\lambda)} L_{t+2}^{1+\lambda} + ... \right] \right\} \end{aligned}$$

Similarly, repeatedly substitute B_{t+j} for $j = 1, 2, ...$

$$\begin{aligned} B_t = &\left(\frac{w_t^*}{w_t}\right)^{-\kappa} w_t^* L_t C_t^{-\tau} \\ &+ \theta_w \beta E_t \left(\frac{w_t^*}{w_{t+1}^* \pi_{t+1}}\right)^{-\kappa+1} \left\{ \left(\frac{w_{t+1}^*}{w_{t+1}}\right)^{-\kappa} w_{t+1}^* L_{t+1} C_{t+1}^{-\tau} \right. \\ &\left. + \ \theta_w \beta E_{t+1} \left(\frac{w_{t+1}^*}{w_{t+2}^* \pi_{t+2}}\right)^{-\kappa+1} \left[\left(\frac{w_{t+2}^*}{w_{t+2}}\right)^{-\kappa} w_{t+2}^* L_{t+2} C_{t+2}^{-\tau} + ... \right] \right\} \end{aligned}$$

[3]As is standard in the literature, the assumption of a complete contingent claims market ensures that all household consume the same amount, despite their different wages income.

A backward looking households that receives a signal in period t to change its wage sets W^b_{t-1} as a geometric average of the wages posted in the last period adjusted for last period's wage inflation.[4]

$$W^b_{t-1} = \frac{W_{t-1}}{W_{t-2}} (W^*_{t-1})^{1-\varphi_w} (W^b_{t-1})^{\varphi_w} \tag{2.11}$$

Defining lower case variable as their upper case counterparts, divided by the price level and dividing by P_{t-1}, we have that

$$w^b_{t-1} = \pi^w_{t-1} (w^*_{t-1})^{1-\varphi_w} (w^b_{t-1})^{\varphi_w}$$

The aggregate wage index evolves according to the usual formula

$$\begin{aligned} W_t^{1-\kappa} &\equiv \int_0^1 W_t(h)^{1-\kappa} dh \\ &= (1-\theta_w)(1-\varphi_w)\left(W^*_t\right)^{1-\kappa} + (1-\theta_w)\varphi_w \left(W^b_t\right)^{1-\kappa} + \theta_w W^{1-\kappa}_{t-1} \end{aligned}$$

Dividing through by P_t, one obtains the evolution of the aggregate wage index in real terms

$$w_t^{1-\kappa} = (1-\theta_w)(1-\varphi)\left(w^*_t\right)^{1-\kappa} (1-\theta_w)\varphi \left(\frac{w^b_{t-1}}{\pi_t}\right)^{1-\kappa} + \theta_w \left(\frac{w_{t-1}}{\pi_t}\right)^{1-\kappa}.$$

2.2 Production structure

The economy has three different layers of production. At the lowest level, a continuum of firms produce a homogeneous wholesale good. This good is used as input of production by a continuum of imperfectly competitive firms at the second level. These firms buy the wholesale good and costlessly differentiate it into an intermediate good of variety j. A competitive bundler aggregates these intermediate varieties into the final output goods, which can be used for consumption and investment. At the wholesale goods level, there is an asymmetric information problem between borrowers and lenders. At the intermediate goods level, there is imperfect competition and asynchronized price setting. Following Bernanke, Gertler, and Gilchrist (2000), this layered production structure is designed to separate the agency problem from price stickiness, which facilitates aggregation in this model.

2.2.1 The asymmetric information problem

Since the firms' problems are static, time subscripts are suppressed for the ease of exposition in this subsection. There is a continuum of firms with unit mass indexed by i. Each firm is owned by an infinitely lived entrepreneur, who has a probability γ of dying in each period. The production function $F(K_i, L_i)$ for the wholesale good Y^W displays constant returns to scale in labor L_i, and capital K_i. The idiosyncratic productivity ω_i has distribution function $\Phi(\omega_i)$ and density function $\phi(\omega_i)$ with nonnegative support and mean of unity.

$$Y^W_i = \omega_i \cdot A \cdot F(K_i, L_i) \tag{2.12}$$

[4]Throughout this paper, the timing notation is as follows. All variables are subscripted with time when they are known, i.e. predetermined variables are denoted with $t-1$.

Assumption 1. *Lender and borrower contract after the realization of aggregate uncertainty A and before the realization of idiosyncratic uncertainty ω. Once realized, ω is private information of the entrepreneur. The lender's cost of verifying ω is a fraction μ of the firm's output.*

A denotes the level of technology, which is common across all entrepreneur, aggregate total factor productivity. It is assumed that this variable is known to both lenders and entrepreneurs at the time of the loan contract, which eliminates aggregate uncertainty for the contracting problem. For simplicity of the exposition, this term is normalized to unity in the subsequent paragraphs. It follows an AR(1) process in the simulations. In order to generate a role for external finance, it is assumed that the firm has to lay out its production costs before receiving the payment for the output good. Let $X_i \equiv wL_i + rK_i$ denote the cost for the input bill, determined by the real wage w and the real rental rate of capital r as well as factor demands. The firm then borrows $X_i - N_i$ from a financial intermediary. Here, N_i is the net worth of the firm i. For convenience, it is assumed that borrowing is intra-period, so the relevant opportunity cost for the household per unit of lending is one.

Assumption 2. *There is enough anonymity in the credit market, such that agents can only sign one period contracts.*

The above assumption rules out reputation considerations and other aspects of long term credit relationships. This is done for the sake of model tractability. Dynamic credit relationships require the use of much more involved numerical techniques; they also challenge the conclusion from one period models, see Smith and Wang (2000).

Since the realization of the productivity shock is private information of the firm and verification of the true productivity is costly to the lender, the firm has an incentive to misreport its productivity.

The finding of Gale and Hellwig (1985) is that the optimal incentive compatible contract with costly state verification is risky debt. The authors characterized the contract that maximizes expected entrepreneurs payoff subject to the risk neutral lender breaking even in expectation.[5] The contract requires a fixed repayment when the firm is solvent and allows the creditor to recoup as much of the debt as possible in case of bankruptcy. The flat payoff schedule in case of solvency follows directly from incentive compatibility. Allowing the lender to seize all obtainable assets in case of default minimizes the number of states in which monitoring must occur for the lender to break even.

The contract is characterized by a non default value $\bar{\omega}_i$, loan size $X_i - N_i$ and an implicit interest rate $\tilde{r}$. A firm with productivity $\omega_i \geq \bar{\omega}_i$ is able to repay its obligation $(1 + \tilde{r}_i)(X_i - N_i)$ and no monitoring takes place. The non default value is therefore defined by $(1+\tilde{r}_i)(X_i - N_i) = \bar{\omega}_i F(K_i, L_i)$. For lower productivity, the firm defaults, the lender monitors and seizes all of the projects output. The pair $(\bar{\omega}_i, X_i)$, is sufficient for the description of the optimal contract, the interest rate follows immediately as $1 + \tilde{r}_i = \bar{\omega}_i F(K_i, L_i)/(X_i - N_i)$. The optimal financial contract maximizes the expected payoff for the borrower, subject to the participation constraint for the lender. Let P^W denote the nominal price of the firms output good and P the consumer price index. The

[5]The household is not risk-neutral, however without aggregate risk, he can perfectly diversify all the risk. With aggregate risk, it remains to be shown that the firm absorbs all aggregate risk.

problem of the firm can be written as follows.

$$\max_{K_i, L_i, \bar{\omega}_i} P^W F(K_i, L_i) \int_{\bar{\omega}_i}^{\infty} (\omega_i - \bar{\omega}_i)\phi(\omega_i) d\omega_i \qquad \text{s.t.} \tag{2.13}$$

$$F(K_i, L_i) \left\{ (1-\mu) \int_0^{\bar{\omega}_i} \omega_i \phi(\omega_i) d\omega_i + [1 - \Phi(\bar{\omega}_i)]\bar{\omega}_i \right\} \geq \frac{P}{P^W}(rK_i + wL_i - N_i) \tag{2.14}$$

The firm expects as residual income the conditional mean $E[(\omega_i - \bar{\omega}_i)|\omega_i \geq \bar{\omega}_i]F(K_i, L_i)$ when it is solvent and nothing otherwise. The lender receives the negotiated repayment $\bar{\omega}_i F(K_i, L_i)$ in states of solvency and expects a conditional mean $E[\omega_i|\omega_i \leq \bar{\omega}_i]F(K_i, L_i)$ corrected for monitoring costs in states of bankruptcy. The first order conditions for the firm's problem can be conveniently expressed as:

$$r\frac{P}{S_i P^W} = F_K(K_i, L_i) \tag{2.15}$$

$$w\frac{P}{S_i P^W} = F_L(K_i, L_i) \tag{2.16}$$

Here the term S_i is given by:

$$S_i \equiv 1 - \mu \left\{ \bar{\omega}_i \phi(\bar{\omega}_i) \left[\frac{\int_{\bar{\omega}_i}^{\infty} \omega_i \phi(\omega_i) d\omega_i}{1 - \Phi(\bar{\omega}_i)} - \bar{\omega}_i \right] + \int_0^{\bar{\omega}_i} \omega_i \phi(\omega_i) d\omega_i \right\} \tag{2.17}$$

Finally, the break even constraint holds with equality. Inspecting (2.15) and (2.16) reveals the crucial role of monitoring costs for the inefficiency of the economy. When there are no monitoring costs ($\mu = 0$), the usual efficiency condition holds in this model: marginal product equals factor cost. S_i is unity in this case. When monitoring costs are strictly positive however, the economy is distorted. Real marginal product is above the real wage, S_i is below unity. Note, that S_i must be strictly below one with positive monitoring costs, otherwise revenues would be exhausted by factor payments alone and monitoring costs could not be covered. However, the inefficiency goes further than that. Entrepreneurs are making positive profits, revenues are strictly higher than factor payments plus monitoring costs. The firm would like to expand production, but is not able to obtain further external finance.

2.2.2 A note on aggregation

Remark 1. *In order to find the model solution it is not necessary to keep track of the distribution of net worth of the entrepreneurs, the means are a sufficient statistic.*

Entrepreneurs are identical ex-ante with respect to their expected productivity, but heterogeneous ex-post once idiosyncratic productivity has materialized. Therefore, even when starting with identical initial endowment of wealth, over time the entrepreneurs will have different levels of wealth. Thanks to linear monitoring technology and constant returns to scale in production, this heterogeneity does not require to keep track of the distribution of wealth as a state of the system.

First note from (2.15) and (2.16) that the ratio of marginal products of capital and labor are equal for all firms. This implies for linear homogeneous production functions such as the Cobb-Douglas, that the capital-labor-ratios are equal across firms. But the capital-labor-ratio uniquely determines the marginal product of a factor with linear homogeneity, which implies that marginal products are equal across firms and therefore S_i is independent of i. One can conclude from (2.17) that $\bar{\omega}$ must be independent of i. The lender break even constraint (2.14) can be rearranged to show that all firms must choose the same ratio of external finance to project size.

$$1 - \frac{N_i}{X_i} = \frac{(1-\mu)\int_0^{\bar{\omega}} \omega\phi(\omega)d\omega + [1-\Phi(\bar{\omega})]\bar{\omega}}{1-\mu\left\{\bar{\omega}\phi(\bar{\omega})\left[\frac{\int_{\bar{\omega}}^{\infty}\omega\phi(\omega)d\omega}{1-\Phi(\bar{\omega})} - \bar{\omega}\right] + \int_0^{\bar{\omega}} \omega\phi(\omega)d\omega\right\}} \tag{2.18}$$

The essential terms of the debt contract, leverage and cutoff value are thus equal across firms. It follows that the FOC for the optimal contract also hold for aggregate variables, defined as the integral over all i variables. Subscripts i are therefore dropped in what follows. Call the right hand side of (2.18) $\Psi(\bar{\omega})$, the projects leverage is given by:

$$\frac{X_i}{N_i} = [1-\Psi(\bar{\omega})]^{-1} \tag{2.19}$$

2.2.3 The retailer

A continuum of retailers on the unit line, indexed by z buy the entrepreneurs' wholesale good at price P_t^W and costlessly differentiate it into a good of variety z.

Standard Calvo pricing is assumed in goods market just as it was in the labor market. We add backward looking components by following Gali and Gertler (1999). There are two types of firms, that differ only in their pricing behavior when receiving a signal to re-optimize. A fraction of measure φ is backward looking, the remaining firms are forward looking. A forward looking firm that receives a signal to re-optimize its price, faces the following problem

$$\max_{P_t^*(z)} E_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,t+i} \left\{ \left[\frac{P_t(z)}{P_{t+i}}\right]^{1-\epsilon} Y_{t+i} - \frac{P_{t+i}^W}{P_{t+i}} \left[\frac{P_t(z)}{P_{t+i}}\right]^{-\epsilon} Y_{t+i} \right\} \tag{2.20}$$

Here, $\Lambda_{t,t+i}$ is the household's pricing kernel. The first order condition is

$$E_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,t+i} Y_{t+i} \left\{ \left[\frac{P_t(z)}{P_{t+i}}\right]^{-\epsilon} - \frac{\epsilon}{\epsilon-1}\frac{P_{t+i}^W}{P_{t+i}} \left[\frac{P_t(z)}{P_{t+i}}\right]^{-\epsilon-1} \right\} \tag{2.21}$$

Defining $p_t^w \equiv \frac{P_t^W}{P_t}$, $p_t^* \equiv \frac{P_t^*}{P_t}$ and $\pi_{t,t+s} \equiv \pi_{t+1} \dots \pi_{t+s}$ (with $\pi_{t,t} \equiv 1$), the Calvo price setting condition can be expressed in terms of stationary variables

$$E_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,t+i} Y_{t+i} \left\{ \left[\frac{p_t^*}{\pi_{t,t+j}}\right]^{-\epsilon} - \frac{\epsilon}{\epsilon-1} p_{t+i}^w \left[\frac{p_t^*}{\pi_{t,t+i}}\right]^{-\epsilon-1} \right\}. \tag{2.22}$$

Define the expressions N_t and D_t by

$$N_t \equiv E_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,t+i} p_{t+j}^w \left(\frac{p_t^*}{\pi_{t,t+i}}\right)^{-\epsilon-1} Y_{t+i}, \tag{2.23}$$

$$D_t \equiv E_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,t+i} \left(\frac{p_t^*}{\pi_{t,t+i}}\right)^{-\epsilon} Y_{t+i}. \tag{2.24}$$

These can be recursively expressed as

$$N_t = (p_t^*)^{-1-\epsilon} P_t^w Y_t + \beta\theta \frac{C_{t+1}^{-\tau}}{C_t^{-\tau}} \pi_{t+1}^{\epsilon} \left(\frac{p_t^*}{p_{t+1}^*}\right)^{-1-\epsilon} N_{t+1} \tag{2.25}$$

$$D_t = (p_t^*)^{-\epsilon} Y_t + \beta\theta \frac{C_{t+1}^{-\tau}}{C_t^{-\tau}} \pi_{t+1}^{\epsilon} \left(\frac{p_t^*}{p_{t+1}^*}\right)^{-\epsilon} D_{t+1} \tag{2.26}$$

These two equations together with $\frac{\epsilon-1}{\epsilon} = \frac{N_t}{D_t}$ allow the pricing condition to be written in companion form, as needed by the MATLAB codes.

A backward looking firm that receives a signal in period t to change its price P_{t-1}^b sets a geometric average of the prices posted in the last period adjusted for last period's inflation.

$$P_{t-1}^b = \pi_{t-1} (P_{t-1}^*)^{1-\varphi} (P_{t-1}^b)^{\varphi} \tag{2.27}$$

Dividing by P_{t-1}, we have that

$$p_{t-1}^b = \pi_{t-1} (p_{t-1}^*)^{1-\varphi} (p_{t-1}^b)^{\varphi}$$

The price index evolves according to the standard formula

$$P_t \equiv \left[\int_0^1 P_t(z)^{1-\epsilon} dz\right]^{\frac{1}{1-\epsilon}}$$

$$P_t = [\theta P_{t-1}^{1-\epsilon} + (1-\theta)(1-\varphi)\left(P_t^*\right)^{1-\epsilon} + (1-\theta)\varphi\left(P_{t-1}^b\right)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}$$

Dividing by P_t, we have that

$$1 = \theta\pi_t^{-1+\epsilon} + (1-\theta)(1-\varphi)\left(p_t^*\right)^{1-\epsilon} (1-\theta)\varphi\left(\frac{p_{t-1}^b}{\pi_t}\right)^{1-\epsilon}$$

2.2.4 Final goods production

Final goods are produced using the continuum of differentiated input goods using a standard technology.

$$Y_t = \left[\int_0^1 [y_t(z)]^{\frac{\epsilon-1}{\epsilon}} \, dz\right]^{\frac{\epsilon}{\epsilon-1}} \tag{2.28}$$

The competitive firms choose $Y_t(z)$ as to maximize profits:

$$\Pi_t = P_t Y_t^f - \int_{z=0}^1 Y_t(z) P_t(z) dz \tag{2.29}$$

The demand function for intermediate good z resulting from this problem is:

$$Y_t(z) = \left[\frac{P_t(z)}{P_t}\right]^{-\epsilon} Y_t. \tag{2.30}$$

The final output good may be consumed by households C, by entrepreneurs C^E, or be used for investment I.

$$Y_t^f = C_t + C_t^E + I_t \tag{2.31}$$

2.3 Aggregation

Aggregate variables are defined as the integral over the individual variables indexed by z or h

$$K_t = \int_0^1 K_t(z)dz$$
$$L_t = \int_0^1 L_t(z)dz$$
$$N_t = \int_0^1 N_t(h)dh$$

We have the following relation between aggregate factor inputs K_t, L_t and aggregate output Y_t.

Proposition 1 (following Christiano, Eichenbaum, and Evans (2001)). *Under Calvo-pricing, aggregate output is related to aggregate factor input through a measure of price dispersion in the following way*

$$Y_t \equiv \left[\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}} = \frac{A_t}{\tilde{P}_t} K_t^{\alpha} L_t^{1-\alpha}. \tag{2.32}$$

Here, the price dispersion index $\tilde{P}_t$ *is defined as* $\tilde{P}_t \equiv \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\epsilon} dz$. *Since the fraction of price setters that receive a signal to re-optimize its price is randomly chosen and by the law of large numbers, we have the following recursion for the price dispersion index*

$$\begin{aligned}\tilde{P}_t &= (1-\theta)(1-\varphi)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + (1-\theta)\varphi\left(\frac{P_{t-1}^b}{P_t}\right)^{-\epsilon} + \theta\int_0^1 \left(\frac{P(z)_{t-1}}{P_t}\right)^{-\epsilon} dz \\ &= (1-\theta)(1-\varphi)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + (1-\theta)\varphi\left(\frac{P_{t-1}^b}{P_t}\right)^{-\epsilon} + \theta\left(\frac{P_{t-1}}{P_t}\right)^{-\epsilon} \tilde{P}_{t-1}\end{aligned} \tag{2.33}$$

In terms of stationary variables

$$\tilde{P}_t = (1-\theta)(1-\varphi)(p_t^*)^{-\epsilon} + (1-\theta)\varphi\left(\frac{p_{t-1}^b}{\pi_t}\right)^{-\epsilon} + \theta\pi^{\epsilon}\tilde{P}_{t-1}. \tag{2.34}$$

(Proof of Proposition 1). *Since all intermediate good firms face the same relative price of capital and labor, they choose the same capital to labor ratio. Hence,* $Y_t^* \equiv \int_0^1 Y_t(z)dz = A_t K_t^{\alpha} L_t^{1-\alpha}$ *(neglecting monitoring costs for the moment). But from the demand function for variety z, integrating over z and using the definition of* $\tilde{P}_t$ *we have that*

$$\int_0^1 Y_t(z)dz = \int_0^1 \left[\frac{P_t(z)}{P_t}\right]^{-\epsilon} Y_t dz = \tilde{P}_t Y_t.$$

Solving for Y_t *and substituting* $Y_t^* = A_t K_t^{\alpha} L_t^{1-\alpha}$ *then yields the desired result*

$$Y_t \equiv \left[\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}} = \frac{A_t}{\tilde{P}_t} K_t^{\alpha} L_t^{1-\alpha}.$$

□

2.4 Entrepreneurs

Entrepreneurs have the same time preferences rate as the households. Their objective is to maximize

$$\max_{\{C^E_{t+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} \beta^{t+i} E_0 C^E_{t+i} \tag{2.35}$$

Assumption 3. *With constant probability $1-\gamma$, an entrepreneur dies in a given period. Entrepreneurs can consume all their net worth just before death.*

This assumption allows to limit the size of aggregate net worth in an infinite horizon set up. Since the return to internal funds is higher than that to the external funds, risk neutral entrepreneurs want to postpone consumption, until they can self-finance their entire project. To avoid the irrelevance of financial frictions in the long run, exogenous death probabilities are introduced. When an individual entrepreneur receives a signal about his death he consumes all his net worth. Aggregate entrepreneurial consumption is thus given by:

$$C^E_t = \gamma F(K_t, L_t) \int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega}_t) \tag{2.36}$$

The evolution of the borrowers net worth plays a crucial role for the dynamics of the model. Net worth is determined as follows. Let Z_{t-1} denote the units of physical capital owned by the entrepreneur at the beginning of period t. The entrepreneurs' net worth N_t is then simply defined as the market value of Z_{t-1}, where Z_{t-1} is equal to the entrepreneurs' last period's project share minus their consumption in last period C^E_{t-1}.

$$N_t = Z_{t-1} \left[1 - \delta + S_t \frac{P^W_t F_K(K_t, L_t)}{P_t} \right] \tag{2.37}$$

$$Z_t = F(K_t, L_t) \int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega}_t) d\omega - C^E_t \tag{2.38}$$

3 Model calibration and analysis

Most of the model's parameter are calibrated to match certain time series averages or set to standard values in the literature.[6] The capital share in output α is standard 0.36. The discount factor β is set to 0.99 as to induce a steady state real interest rate of roughly 4 per cent on an annual basis. The depreciation rate is set to 0.02 corresponding to an annual rate of 8 per cent. The arrival rates of the signal to re-optimize prices and wages,θ and θ_w are both set to 2/3, implying an average duration of price and wage contracts of 3 quarters. This choice is roughly consistent with the estimates in Christiano, Eichenbaum, and Evans (2001) for US data, who find $\theta = 0.5$ and $\theta_w = 0.7$. The intertemporal elasticity of substitution is set to unity, i.e. utility is logarithmic in consumption.

[6] I am currently working on an estimation of the models parameters using Euro area data and the Maximum Likelihood approach following Ireland (2004). Preliminary result suggest a larger degree of wage stickiness than price stickiness.

The parameters governing monopolistic competition ϵ and κ are set to 7 as to induce in markup of 16.7 per cent, roughly consistent with the empirical estimates by Basu and Fernald (1993) who report average markups in U.S. manufacturing close to 15 per cent. The autoregressive parameter in the law of motion for technology is set to the standard 0.95 and $\sigma = 0.007$. The calibration of the key parameter characterizing the credit frictions chosen to emphasize the effects of such frictions. Firms' idiosyncratic productivity is assumed to be uniformly distributed on the interval [0,2], which allows to compute the relevant intervals analytically. The cost of state verification μ, which should be interpreted as a broad cost of bankruptcy is assumed to be 0.25 and taken from Harrison, Sussmann, and Zeira (1999). The parameter γ, governing entrepreneurs' consumption, is set as to induce a steady state default probability of 25 per cent.

3.1 Amplification and persistence?

Does the introduction of credit frictions lead to an amplified or dampened response of the economy to technology and monetary policy shocks? Does it create endogenous persistence into the model, i.e. autocorrelation functions of the models' variables which are not inherited from the exogenous shocks? The response of aggregate output to a technology shock is depicted in figure 2.

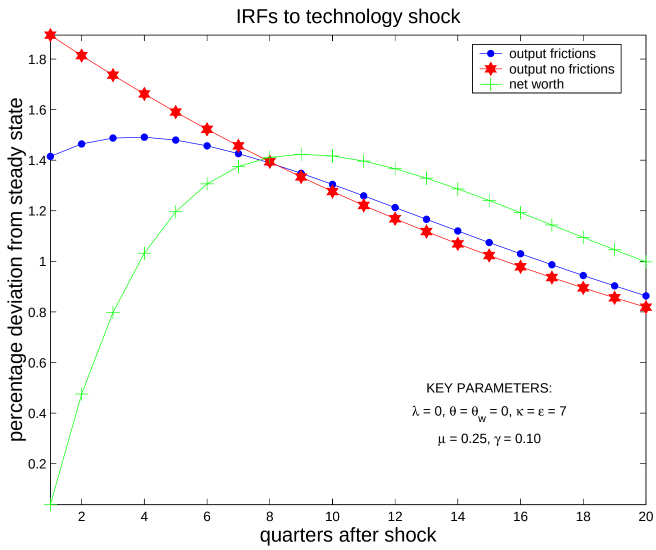


Figure 2: Response of output and net worth to technology shock

Credit frictions dampen the response of output to a technology shock and increase persistence. This effect can be explained by the sluggish behavior of net worth. The firm would like to increase employment of labor and capital, but the loan size is related to net worth of the firm. However, net worth consists largely of profits accumulated in the past and cannot jump much on impact. It increases slowly and reaches its peak after about 2 years. Output persistence in the credit friction model, measured by the half life of the response on impact, is increased through credit frictions. However, the size of this effect depends crucially on the wage elasticity of labor supply. To understand this, log-linearize the aggregate production function. On impact, the output response to a percentage increase in technology equals unity stemming directly from technology plus labor share α times the response in labor. If labor is supplied elastically, then small financial constraints in the firms ability to pay higher wages lead to relatively large fluctuations in labor employed. If labor is supplied very inelastically, then larger variations in net worth are required to bring about a certain change in labor employed. The above graph shows the extreme case $\lambda = 0$, i.e. constant disutility of labor.

4 Welfare measure and solution method

Since the model is built from first principles, one can use the utility of the agents[7] to construct a welfare measure rather than resorting to an ad hoc loss function as in non-structural models.

Several authors have pointed to the pitfalls of using a linear approximation to the policy function when evaluating a second order approximation to the utility function. Kim and Kim (2003) have shown that such a procedure can result in a spurious reversal of welfare comparison in international economics: Autarky appears preferable to full risk sharing.

Essentially the problem is that some second order terms are included in the welfare criterion while other are neglected. A second order approximation to the policy function is used in this paper in order to avoid the bias in welfare comparisons of policy rules. The following subsections briefly summarize the solution methods and the employed welfare measure.

4.1 Model representation and form of the solution

To fix notation, consider the generic representation for rational expectations models introduced by Schmitt-Grohé and Uribe (2004)

$$E_t\, f(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{x}_t) = 0. \tag{4.1}$$

f is a known function describing the equilibrium conditions of the model economy, $\mathbf{y}_t$ is a vector of co-state variables and $\mathbf{x}_t$ a vector of state variables partitioned as $\mathbf{x}_t = [\mathbf{x}_{1,t}; \mathbf{x}_{2,t}]$. $\mathbf{x}_{1,t}$ is a vector of endogenous state variables and $\mathbf{x}_{2,t}$ a vector of state variables following an exogenous stochastic process

$$\mathbf{x}_{2,t+1} = \mathbf{L}\mathbf{x}_{2,t} + \tilde{\mathbf{N}}\sigma\boldsymbol{\epsilon}_t. \tag{4.2}$$

$\mathbf{L}$ and $\tilde{\mathbf{N}}$ are known coefficient matrices, $\boldsymbol{\epsilon}_t$ is a vector of innovations with bounded support, independently and identically distributed with mean zero and covariance matrix I. σ is a parameter scaling the standard deviation of the innovations. The solution to the model described by (4.1) is of the form

$$\mathbf{y}_t = g(\mathbf{x}_t, \sigma), \tag{4.3}$$

$$\mathbf{x}_{t+1} = h(\mathbf{x}_t, \sigma) + \mathbf{N}\sigma\boldsymbol{\epsilon}_{t+1}, \qquad \text{with: } \mathbf{N} = \begin{bmatrix} 0 \\ \tilde{\mathbf{N}} \end{bmatrix}. \tag{4.4}$$

[7] The subsequent optimal policy analysis neglects the utility of entrepreneurs and maximizes only the utility of the households. This is done in order to facilitate comparison with a version of the model in which are no credit frictions.

Schmitt-Grohé and Uribe (2004) derive the second-order Taylor approximation to the policy functions $g(\cdot)$ and $h(\cdot)$ and provide MATLAB codes for the numerical implementation. The approximate model dynamics obtained from their second-order approximation can be compactly expressed as

$$\mathbf{y}_t = \mathbf{G}\mathbf{x}_t + \frac{1}{2}\mathbf{G}^*(\mathbf{x}_t \otimes \mathbf{x}_t) + \frac{1}{2}\mathbf{g}\sigma^2, \tag{4.5}$$

$$\mathbf{x}_{t+1} = \mathbf{H}\mathbf{x}_t + \frac{1}{2}\mathbf{H}^*(\mathbf{x}_t \otimes \mathbf{x}_t) + \frac{1}{2}\mathbf{h}\sigma^2 + \sigma\mathbf{N}\boldsymbol{\epsilon}_{t+1}. \tag{4.6}$$

Here, the vectors $\mathbf{y}_t$ and $\mathbf{x}_t$ denote deviation or log-deviation from the steady state. $\mathbf{G}$ and $\mathbf{H}$ are coefficient matrices representing the linear part of the Taylor approximation. The matrices $\mathbf{G}^*$ and $\mathbf{H}^*$ form the second-order part jointly with the vectors $\mathbf{g}$ and $\mathbf{h}$.

4.2 Unconditional welfare

A natural welfare measure for rankings of monetary policy that can be easily constructed from the second-order approximation is the unconditional expectation of period utility of a randomly drawn household

$$W = E\left\{U(C_t) + \int_0^1 V(N_t(h))dh\right\}. \tag{4.7}$$

where the unconditional expectation E averages across all possible histories of aggregate shocks. The next paragraphs derive the formula for numerically computing welfare given a second order approximation to the policy function.

The second-order approximation to an arbitrary objective function $u(\mathbf{y}_t)$ of co-states $\mathbf{y}_t$ is

$$u(\mathbf{y}_t) \approx u(\overline{\mathbf{y}}) + \nabla u(\overline{\mathbf{y}})\mathbf{y}_t + \frac{1}{2}vec\left(\nabla^2 u(\overline{\mathbf{y}})\right)'(\mathbf{y}_t \otimes \mathbf{y}_t), \tag{4.8}$$

such that upon taking expectations

$$E(u(\mathbf{y}_t)) \approx u(\overline{\mathbf{y}}) + \nabla u(\overline{\mathbf{y}})\boldsymbol{\mu}_\mathbf{y} + \frac{1}{2}vec\left(\nabla^2 u(\overline{\mathbf{y}})\right)' vec(\boldsymbol{\Sigma}_\mathbf{y} + \boldsymbol{\mu}_\mathbf{y}\boldsymbol{\mu}_\mathbf{y}'). \tag{4.9}$$

Here, $\boldsymbol{\mu}_\mathbf{y}, \boldsymbol{\Sigma}_\mathbf{y}$ denote unconditional mean and covariance matrix of $\mathbf{y}$, respectively. To construct first and second moments of the co-state variables assume covariance stationarity and take expectation of (4.5) and (4.6)

$$\boldsymbol{\mu}_\mathbf{y} = \mathbf{G}\boldsymbol{\mu}_\mathbf{x} + \frac{1}{2}\mathbf{G}^* vec(\boldsymbol{\Sigma}_\mathbf{x} + \boldsymbol{\mu}_\mathbf{x}\boldsymbol{\mu}_\mathbf{x}') + \frac{1}{2}\mathbf{g}\sigma^2, \tag{4.10}$$

$$\boldsymbol{\mu}_\mathbf{x} = \mathbf{H}\boldsymbol{\mu}_\mathbf{x} + \frac{1}{2}\mathbf{H}^* vec(\boldsymbol{\Sigma}_\mathbf{x} + \boldsymbol{\mu}_\mathbf{x}\boldsymbol{\mu}_\mathbf{x}') + \frac{1}{2}\mathbf{h}\sigma^2. \tag{4.11}$$

Note that while under the linear approximation unconditional means do not differ from the steady state values, the second-order approximation is able to capture the effect of variances on means. Since variances can be computed accurately up to second-order from the linear part of the policy function, it is sufficient to approximate $vec(\boldsymbol{\Sigma}_\mathbf{x} + \boldsymbol{\mu}_\mathbf{x}\boldsymbol{\mu}_\mathbf{x}') \approx vec(\boldsymbol{\Sigma}_\mathbf{x})$ and $vec(\boldsymbol{\Sigma}_\mathbf{y} + \boldsymbol{\mu}_\mathbf{y}\boldsymbol{\mu}_\mathbf{y}') \approx vec(\boldsymbol{\Sigma}_\mathbf{y})$. It is then possible to construct these using the simple formulas

$$vec(\boldsymbol{\Sigma}_\mathbf{y}) = (\mathbf{G} \otimes \mathbf{G})vec(\boldsymbol{\Sigma}_\mathbf{x}), \tag{4.12}$$

$$vec(\boldsymbol{\Sigma}_\mathbf{x}) = \sigma^2(\mathbf{I} - \mathbf{H} \otimes \mathbf{H})^{-1}(\mathbf{N} \otimes \mathbf{N})vec(\mathbf{I}). \tag{4.13}$$

Given these approximations for the variances, the means can be computed from (4.10) and (4.11). The described welfare measure has the following compact representation, which can easily be verified by applying the rules of the partitioned inverse.

$$E(u(y_t)) \approx u(\overline{y}) + \left[\nabla u(\overline{y}), \frac{1}{2} vec\left(\nabla^2 u(\overline{y})\right)'\right] \times$$
$$\left(\begin{bmatrix} \mathbf{G} & \frac{1}{2}\mathbf{G}^* \\ 0 & \mathbf{G} \otimes \mathbf{G} \end{bmatrix} \left[\mathbf{I} - \begin{bmatrix} \mathbf{H} & \frac{1}{2}\mathbf{H}^* \\ 0 & \mathbf{H} \otimes \mathbf{H} \end{bmatrix} \right]^{-1} \begin{bmatrix} \frac{1}{2}\mathbf{h} \\ \mathbf{N} \otimes \mathbf{N} vec(\mathbf{I}) \end{bmatrix} \sigma^2 + \begin{bmatrix} \frac{1}{2}\mathbf{g} \\ 0 \end{bmatrix} \sigma^2 \right) \quad (4.14)$$

The task of computing optimal monetary policy then amounts to numerically optimizing this welfare measure through choice of the coefficients in the policy rule.

Remark 2. *If the second order approximation to the welfare function can be rewritten as to involve quadratic terms only, then linear and quadratic approximations to the policy functions will yield the same level of welfare. I.e. up to second order, there is no bias in welfare calculations based on linear policy rules.*

The effect of the higher order terms of the model's dynamics on the first order terms in the welfare criterion is the origin of the bias. Woodford (2003) carefully re-arranges the second order expansion to the utility function as to eliminate the first order terms. Whenever such a procedure is possible, using a first order approximation to the model's dynamics does not introduce a bias in welfare comparison.

When the model involves endogenous state variables or is not efficient in the steady state, it is often not possible to eliminate the first order terms in the welfare criterion and a higher order solution to the models dynamics is needed.

4.3 Rewriting the objective function

The average disutility from working $-\frac{\nu}{1+\lambda}\int_0^1 N_t(h)^{1+\lambda} dh$ depends on an appropriately constructed measure of wage dispersion in the following way.

Define a measure of wage dispersion as $\tilde{W}_t$ as $\tilde{W}_t \equiv \int_0^1 \left(\frac{W_t(h)}{W_t}\right)^{-(1+\lambda)\kappa} dh$. Starting from the demand function for variety h, integrating over h and using the definition of $\tilde{W}_t$ yields

$$\int_0^1 N_t(h)^{(1+\lambda)} dh = \int_0^1 \left[\frac{W_t(h)}{W_t}\right]^{-(1+\lambda)\kappa} L_t^{1+\lambda} dh = \tilde{W}_t L_t^{1+\lambda}. \quad (4.15)$$

Since the fraction of wage setters that receive a signal to re-optimize its wage is randomly chosen and by the law of large numbers, we have the following recursion for the wage index

$$\begin{aligned}
\tilde{W}_t &= (1-\theta_w)\left(\frac{W_t^*}{W_t}\right)^{-(1+\lambda)\kappa} + \theta_w \int_0^1 \left(\frac{W(h)_{t-1}}{W_t}\right)^{-(1+\lambda)\kappa} dh \\
&= (1-\theta_w)(1-\varphi_w)\left(\frac{W_t^*}{W_t}\right)^{-(1+\lambda)\kappa} + (1-\theta_w)\varphi_w \left(\frac{W_{t-1}^b}{W_t}\right)^{-(1+\lambda)\kappa} \\
&\quad + \theta_w \left(\frac{W_{t-1}}{W_t}\right)^{-(1+\lambda)\kappa} \tilde{W}_{t-1}
\end{aligned}$$

We obtain a formulation in terms of stationary variables

$$\tilde{W}_t = (1-\theta_w)(1-\varphi_w)\left(\frac{w_t^*}{w_t}\right)^{-(1+\lambda)\kappa} + (1-\theta_w)\varphi_w\left(\frac{w_{t-1}^b}{\pi_t w_t}\right)^{-(1+\lambda)\kappa}$$
$$+\theta_w\left(\frac{w_{t-1}}{w_t\pi_t}\right)^{-(1+\lambda)\kappa}\tilde{W}_{t-1} \tag{4.16}$$

The objective of the central bank is then to maximize the unconditional expectation of O_t

$$O_t \equiv \frac{C_t^{1-\tau}}{1-\tau} - \frac{\nu}{1+\lambda}\tilde{W}_t L_t^{1+\lambda}. \tag{4.17}$$

4.4 Accuracy check: Inspection of Euler residuals

Judd (1998) advocates the inspection of Euler equations residuals to assess the accuracy of the obtained solution. One plots the residual in the Euler equation as a function of the state variables of the system. Let $x_{t+1} = h^s(x_t)$ denote transition function for the state variables obtained under solution method s and $y_t = g^s(x_t)$ the policy function for time t decision variables. The residual arising from the Euler equation for capital holdings is:

$$R^s(x_t) = 1 - \frac{\left\{\beta E_t\left[C\left(h^s(x_t)\right)^{-\sigma}\left\{\alpha P^W\left(h^s(x_t)\right)Y\left(h^s(x_t)\right)/K\left(h^s(x_t)\right)+1-\delta\right\}\right]\right\}^{-\frac{1}{\tau}}}{C(x_t)} \tag{4.18}$$

The residual gives the error from following the approximated policy rule as a fraction of current period consumption. Under certain conditions the approximation error of the policy function is of the same order of magnitude as the Euler equation residual as pointed out by Santos (2000). Figure (A) in the appendix plots the absolute value of the consumption Euler equation residual obtained from the second-order accurate solution method over a range of deviations of the state variables from −30% to +30% from their steady state levels. The residual is expressed in base 10 logarithms for the ease of visualization. [8] The maximum Euler residual is on the order of magnitude of 10^{-4}. This may be interpreted as 10 cents error for every 100 $ spent. By comparision, the Euler equation residual obtained from the first order approximation is on average an order of magnitude larger.

5 Welfare analysis of policy rules

The model outlined so fare features several real and monetary distortions. Price inflation is welfare reducing since the dispersion of relative nominal prices across retailers leads to a dispersion of relative quantities demanded by the bundler. Under the efficient allocation relative quantities are equal to unity, essentially since all the retailer face the same production function and the same demand function. The welfare costs of a given amount of price dispersion is crucially related to the price elasticity of demand ϵ. The stronger the degree of substitutability among varieties, the larger is the resulting quantity dispersion resulting from price dispersion. Up to a log-linear approximation, price dispersion and quantity dispersion are proportional with the factor of proportionality

[8] The expectation is computed using Gaussian quadrature with 10 nodes.

being equal to ϵ squared. Exactly the same argument applies to wage dispersion. Since workers supply whatever amount of labor demanded by the bundler at the nominal wage, the welfare costs are additionally influenced by the λ, determining the marginal utility of labor. The higher λ, the higher the welfare costs of wage inflation.

In this section, the welfare effects of different interest rate rules are compared. Initially, attention is restricted to a class of rules targeting wage and price inflation, since these are most clearly related to welfare.

$$R_t^N = \bar{R}\pi_t^{\chi_0} (\pi_{w,t})^{\chi_1} \tag{5.1}$$

Five different versions of these Taylor-type rules are considered: *weak CPI inflation targeting* ($\chi_0 = 1.5, \chi_1 = 0$), *weak wage inflation targeting* ($\chi_0 = 0, \chi_1 = 1.5$), *weak CPI and wage inflation targeting* ($\chi_0 = 1.5, \chi_1 = 1.5$), *complete goods inflation stabilization* ($\chi_0 = 10000, \chi_1 = 0$), and *complete wage inflation stabilization* ($\chi_0 = 0, \chi_1 = 10000$). Whereas this choice of policy rules considered is somewhat arbitrary, it is chosen for tractability.[9]

Welfare effects of policy rules are compared as percentage points of equivalent variation of steady state consumption. What percentage of consumption in a deterministic version of the model must an agent be given in order to achieve the same level of welfare as in a particular rule under consideration? If the utility in a deterministic version of the model is V^A and consumption and labor supply under the rule B are C_t^B and L_t^B, then the required percentage variation in consumption ψ is defined by the relation:

$$V^B = V^A((1+\psi)C_t^A, L_t^A) \tag{5.2}$$

For the log utility specification in this paper, we have $V^B = \log(1+\psi) + V^A$ It follows that $\psi = \exp(V^B - V^A) - 1$. Whereas a number of authors such as Schmitt-Grohé and Uribe (2003) and Kim and Kim (2003) argue against the use of an unconditional welfare measure, this paper finds virtually no differences in the level of welfare computed from unconditional expectation of period utility or from the expectation of discounted expected lifetime utility conditional on initial values for mean and variance of the state vector. [10] The welfare analysis of policy rules begins by considering a baseline model with staggered price and wage setting, but no asymmetric information problem. In the subsequent subsection, credit frictions are added. In the current version of the paper, we condider only forward looking price and wage setting. [11]

5.1 A baseline model without financial frictions

Inspection of table 1[12] shows that weak CPI inflation targeting entails the largest and significant welfare costs. [13] Weak targeting of both wage and price inflation can improve welfare by roughly three quarters of a percentage point of period consumption

[9]A search for the optimal rule through choice of χ_0 and χ_1 is difficult, since the welfare surface is very flat in many regions.

[10]The initial mean and covariance matrix are set to zero, implying that the initial is the deterministic steady state of the model.

[11]An extension to the general case of forward and backward looking price and wage setting is work in progress.

[12]std refers to the unconditional standard deviation of a variable.

[13]Welfare in the baseline model absent any nominal rigidities ($\theta = \theta_w = 0$) is 0.0273. This implies that the welfare costs of nominal rigidities are roughly 0.85 percentage points of consumption. Note that welfare costs expressed not expressed in decimal notation, i.e. 1 is only 1 per cent.

relative to weak CPI inflation targeting. Completely stabilizing CPI inflation entails considerable welfare costs of one third of a percentage point of period consumption, it improves welfare relative to targeting CPI inflation only weakly. From the results of the next table we will see that this is mainly due to the reduction in the variance of wage inflation. The next table shows that welfare costs of price inflation are by and large negligible. Wage inflation is partially driven by price inflation, so that targeting price inflation stronger improves welfare by its indirect effect on the variance of wage inflation. Finally, the winner among the considered policy rules is complete wage inflation stabilization. It can be seen that complete wage inflation stabilization results in considerable variance in price inflation, but this does not seem to matter much for welfare. Table 1 gives compensating variations (cv) for various scenarios relating to the degree of backward-lookingness in price and wage setting denoted by I through IV. Scenario I refers to the baseline case with purely forward-looking price and wage setters ($\varphi = \varphi_w = 0$). II refers to the case of half of all price setters being backward-looking and wage setters fully forward-looking ($\varphi = 0.5, \varphi_w = 0$). III is the balanced case with 50 per cent of both wage and price setters being backward-looking ($\varphi = \varphi_w = 0.5$). Finally, IV refers to price setters being fully forward-looking and half of wage setters backward-looking ($\varphi = 0, \varphi_w = 0.5$). The standard deviations of wage and price inflation refer to the baseline scenario I.

Table 1: Welfare effects of monetary policy rules in baseline model

χ_0	χ_1	std(π)	std(π_w)	compensating variation
1.5	0	0.17	0.40	-0.82
1.5	1.5	0.15	0.12	-0.06
0	1.5	0.41	0.14	-0.13
10000	0	0.00	0.25	-0.35
0	10000	0.24	0.00	0.00

To isolate the effect of CPI inflation, table 2 reports welfare effects from a variant of the baseline model, which features completely flexible wages ($\theta_w = 0$). In this model, completely stabilizing the variance of price inflation is the welfare maximizing policy rule, in fact it results in the same level of welfare as in the model without any nominal rigidities. However, differences in welfare across any policy rule considered are very small. Targeting only CPI inflation weakly, leads to a reduction in welfare of only 0.004 percentage points of consumption. This finding is in line with a recent paper by Schmitt-Grohé and Uribe (2003) also reporting negligible welfare differences among Taylor rules in sticky price framework.

Since the welfare costs of CPI inflation are negligible, shutting of price stickiness ($\theta = 0$) in the baseline model should lead to similar welfare costs as in the model with both types of nominal rigidities. This conjecture is confirmed in table 3. The reason that welfare of the sticky wage model and the sticky price model don't always add up to the welfare from the model with sticky wages and prices is that wage setting is different when prices are also sticky but not perfectly stabilized.

5.2 The model with credit frictions

In this subsection, welfare is computed for the full model involving credit frictions as well as staggered wage and price setting ($\theta = \theta_w = 2/3$).[14] [15] Results in table 4 show that the basic conclusions from the analysis of the model without credit frictions still hold. That is, complete stabilization of the wage inflation is still the winner among the different policy rules considered. Targeting CPI inflation weakly still entails considerable welfare losses. However, it appears that the welfare costs of the different policy rules relative to full stabilization of wage inflation are somewhat smaller than in the case without credit frictions. The explanation for this is not yet obvious. However, it is possible that to some extent the effect of credit frictions and wage inflation have mutually offsetting effects.

[14] Welfare in the credit friction model absent any nominal rigidities ($\theta = \theta_w = 0$) is 0.026.

[15] Note that the steady state of the model with credit frictions is different from the one without credit frictions.

Table 2: Welfare effects of monetary policy rules in sticky price model

χ_0	χ_1	std(π)	std(π_w)	compensating variation
1.5	0	0.11	0.72	0.0236
1.5	1.5	0.18	0.14	0.0152
0	1.5	0.41	0.18	-0.0284
10000	0	0.00	0.66	0.0273
0	10000	0.29	0.00	0.0000

Table 3: Welfare effects of monetary policy rules in sticky wage model

χ_0	χ_1	std(π)	std(π_w)	compensating variation
1.5	0	0.16	0.37	-0.71
1.5	1.5	0.17	0.14	-0.10
0	1.5	0.58	0.14	-0.06
10000	0	0.00	0.25	-0.34
0	10000	0.66	0.00	0.03

Table 4: Welfare effects of monetary policy rules credit friction model with sticky wages and sticky price model

χ_0	χ_1	std(π)	std(π_w)	compensating variation
1.5	0	0.32	0.37	-0.69
1.5	1.5	0.29	0.11	-0.04
0	1.5	0.82	0.13	-0.24
10000	0	0.00	0.24	-0.27
0	10000	0.54	0.00	0.001

6 Conclusion

This paper has evaluated the welfare effects of 5 different Taylor type rules in a New Keynesian model with staggered price and wage setting. A model consistent objective for the central bank is derived from the utility of a randomly drawn household and evaluated subject to a second-order approximation of the policy function. Consistent with recent findings of Schmitt-Grohé and Uribe (2003) this paper finds negligible welfare differences among different Taylor rules when only prices are set in a staggered fashion. The maximum difference in welfare among the Taylor rules is roughly 0.05 per cent of period consumption.

However, wage stickiness may induce considerable differences among Taylor rules. These differences can amount to up to 0.8 per cent of consumption. However these costs can be largely eliminated by employing a Taylor type of rule that strongly targets wage inflation. The reason is simple. Since CPI inflation has welfare costs that are very very small, fully stabilizing wage inflation, which induces relatively large CPI inflation, has small costs. This finding is consistent with papers such as Mankiw and Reis (2002), who also point to a substantial weight for wage inflation in central bank policy. This finding is robust to the inclusion of other real frictions such as credit market imperfections into the model. However, a thorough investigation of optimal monetary policy when credit markets are imperfect has not been conducted in this paper

A number of caveats need to be noted. Most importantly, whether nominal rigidities in wage setting have strong allocational effects may be doubted. Recent research on the welfare costs of wage stickiness when workers can choose their level of effort as in Chang and Bills (2003) questions such large costs as found in this paper. Furthermore, there is a large empirical debate over the relative importance of forward-looking versus backward looking aspects in price and wage setting. Empirical models of Philips curves seem to perform poorly without the inclusion of backward looking terms. A more general welfare analysis that takes into account backward looking components in wage and price setting thus seems desirable in future research.

A Appendix: Euler equation residuals

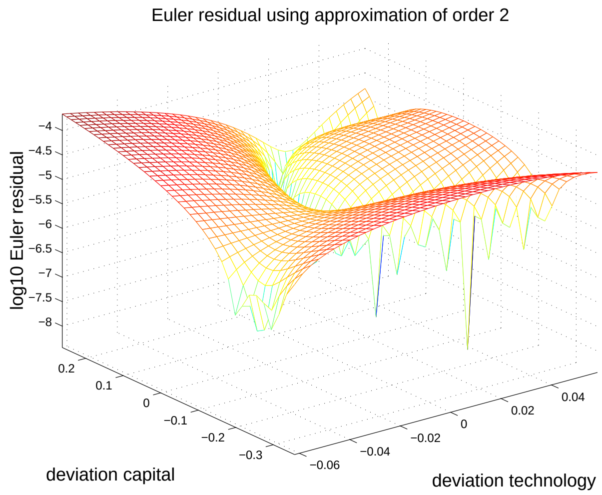


Figure 3: Euler equation residual for the baseline model

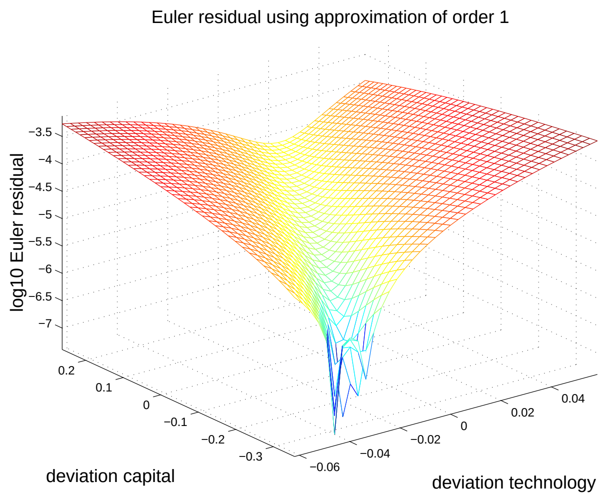


Figure 4: Euler equation residual for the baseline model

B The model with sticky prices, sticky wages and credit frictions

Households:

$$w_t^b = \pi_{w,t}(w_t^*)^{1-\varphi_w}\left(\frac{w_{t-1}^b}{\pi_t}\right)^{\varphi_w}$$
$$\pi_{w,t} = \frac{w_t \pi_t}{w_{t-1}}$$
$$w_t^{1-\kappa} = (1-\theta_w)(1-\varphi)\,(w_t^*)^{1-\kappa}\,(1-\theta_w)\varphi\left(\frac{w_{t-1}^b}{\pi_t}\right)^{1-\kappa} + \theta_w\left(\frac{w_{t-1}}{\pi_t}\right)^{1-\kappa}$$
$$B_t = \frac{\kappa\nu}{\kappa-1}A_t$$
$$A_t = \left(\frac{w_t^*}{w_t}\right)^{-\kappa(1+\lambda)} L_t^{1+\lambda} + \theta_w\beta E_t\left(\frac{w_t^*}{w_{t+1}^*\pi_{t+1}}\right)^{-\kappa(1+\lambda)} A_{t+1}$$
$$B_t = \left(\frac{w_t^*}{w_t}\right)^{-\kappa} w_t^* L_t C_t^{-\tau} + \theta_w\beta E_t\left(\frac{w_t^*}{w_{t+1}^*\pi_{t+1}}\right)^{-\kappa+1} B_{t+1}$$
$$C_t^{-\tau} = \beta E_t\left\{C_{t+1}^{-\tau}\left[1-\delta+p_t^w S_{t+1}A_{t+1}F_K(K_{t+1},L_{t+1})\right]\right\}$$
$$C_t^{-\tau} = E_t\beta\left\{\frac{R_{t+1}^n}{\pi_{t+1}}C_{t+1}^{-\tau}\right\}$$

Optimal contract:

$$S_t = 1-\mu\left\{\bar{\omega}_t\phi(\bar{\omega}_t)\left[\frac{\int_{\bar{\omega}_t}^{\infty}\omega_t\phi(\omega_t)d\omega_t}{1-\Phi(\bar{\omega}_t)} - \bar{\omega_t}\right] + \int_0^{\bar{\omega}_t}\omega_t\phi(\omega_t)d\omega_t\right\}$$
$$rK_t + wL_t - N_t = p_t^w A_t F(K_t,L_t)\left\{(1-\mu)\int_0^{\bar{\omega}_t}\omega_t\phi(\omega_t)d\omega_t + [1-\Phi(\bar{\omega}_t)]\bar{\omega_t}\right\}$$

Entrepreneurs:

$$N_t = Z_t\left[1-\delta+p_t^w S_t A_t F_K(K_t,L_t)\right]$$
$$Z_{t+1} = p_t^w A_t F(K_t,L_t)\int_{\bar{\omega}}^{\infty}(\omega-\bar{\omega}_t) - C_t^E$$
$$C_t^E = \gamma p_t^w A_t F(K_t,L_t)\int_{\bar{\omega}}^{\infty}(\omega-\bar{\omega}_t)$$

Market clearing:

$$Y_t = C_t + C_t^E + K_{t+1} - (1-\delta)K_t$$
$$Y_t = \tilde{P}_t^{-1}Y_t^W$$
$$\tilde{P}_t = (1-\theta)(1-\varphi)\,(p_t^*)^{-\epsilon} + (1-\theta)\varphi\left(\frac{p_{t-1}^b}{\pi_t}\right)^{-\epsilon} + \theta\pi^{\epsilon}\tilde{P}_{t-1}$$

Production:

$$Y_t^W = A_t K_t^{\alpha} L_t^{1-\alpha}\left(1-\mu\int_0^{\bar{\omega}_i}\omega_i\phi(\omega_i)d\omega\right)$$

Price setting

$$1 = \theta\pi_t^{-1+\epsilon} + (1-\theta)(1-\varphi)\left(p_t^*\right)^{1-\epsilon}(1-\theta)\varphi\left(\frac{p_{t-1}^b}{\pi_t}\right)^{1-\epsilon}$$

$$p_t^b = \pi_t(p_t^*)^{1-\varphi}\left(\frac{p_{t-1}^b}{\pi_t}\right)^{\varphi}$$

$$\frac{\epsilon-1}{\epsilon} = \frac{N_t}{D_t}$$

$$N_t = (p_t^*)^{-1-\epsilon}\,p_t^w Y_t + \beta\theta\frac{C_{t+1}^{-\tau}}{C_t^{-\tau}}\pi_{t+1}^{\epsilon}\left(\frac{p_t^*}{p_{t+1}^*}\right)^{-1-\epsilon}N_{t+1}$$

$$D_t = (p_t^*)^{-\epsilon}\,Y_t + \beta\theta\frac{C_{t+1}^{-\tau}}{C_t^{-\tau}}\pi_{t+1}^{\epsilon}\left(\frac{p_t^*}{p_{t+1}^*}\right)^{-\epsilon}D_{t+1}$$

Welfare computation:

$$O_t = \frac{C_t^{1-\tau}}{1-\tau} - \frac{\nu}{1+\lambda}\tilde{W}_t L_t^{1+\lambda}$$

$$\tilde{W}_t = (1-\theta_w)(1-\varphi_w)\left(\frac{w_t^*}{w_t}\right)^{-(1+\lambda)\kappa} + (1-\theta_w)\varphi_w\left(\frac{w_{t-1}^b}{\pi_t w_t}\right)^{-(1+\lambda)\kappa} + \theta_w\left(\frac{w_{t-1}}{w_t\pi_t}\right)^{-(1+\lambda)\kappa}\tilde{W}_{t-1}$$

Monetary policy rule:

$$R_t^N = \bar{R}\pi_t^{\chi_0}\left(\pi_{w,t}\right)^{\chi_1}$$

Exogenous stochastic processes:

$$\log(A_{t+1}) = (1-\rho_a)\log(\bar{A}) + \rho_a\log(A_t) + u_{t+1}$$

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